

FREQUENCY RESPONSE

READING ASSIGNMENT: Horowitz, pgs. 25-35

Abstract:

This lab demonstrates the frequency response of RC and RLC networks. Frequency response is shown to be an alternative to differential equations and time constants for analyzing the time response of RC circuits, specifically a compensated scope probe. A resonant circuit is studied and used to examine the spectral content of a square wave.

Part 1 - Phase and Frequency Response Measurement

Networks which include inductors and capacitors affect both the amplitude and the phase of periodic signals. To measure phase shift, connect your circuit to the scope's channel 1 input (CH1) and connect the pulse output of the signal generator to channel 2 (CH2). Set the display mode knob to DUAL, press the CH2 trigger source button to trigger on the pulse, and make sure that the INV CH2 button is not pressed. Under these conditions, the positive going step of the pulse signal is always synchronized with the negative peak of the generator's waveform output. See Fig. 4.1.

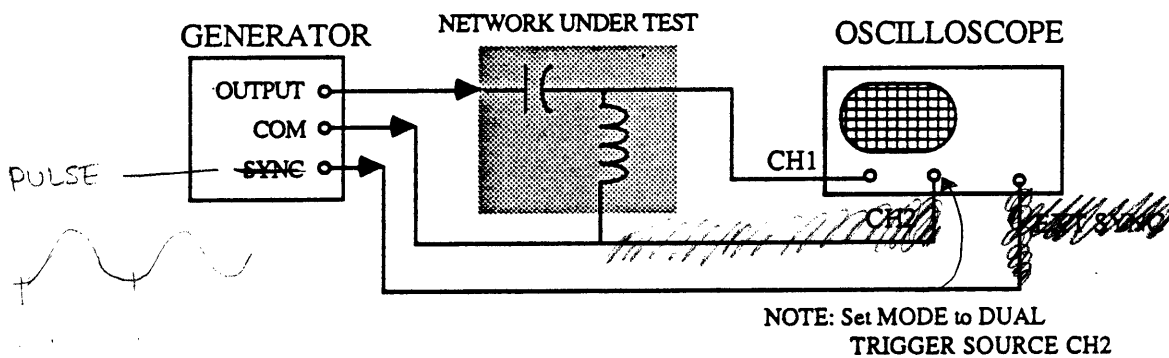


Figure 4.1 - Oscilloscope synchronization to generator output

A convenient way to measure phase shift is to set the scope's VAR SWEEP control at each of your test frequencies so that the distance between rising steps on the pulse signal is eight centimeters. If you do this, each centimeter of displacement between the negative peak at point B and the rising step of the pulse will correspond to a 45 degree phase shift. This phase shift is illustrated in Fig. 4.2.

- (1) Using the scope, set the signal generator to produce a 10 V_{p-p} sine wave at 1 KHz with zero DC offset. Do not change the amplitude setting for the rest of this part. You can assume that the generator will produce a constant 10 V_{p-p} for all of the frequencies used in this part of the lab.
- (2) Build the circuit shown in Fig. 4.3. R_s is the internal output resistance of the generator.

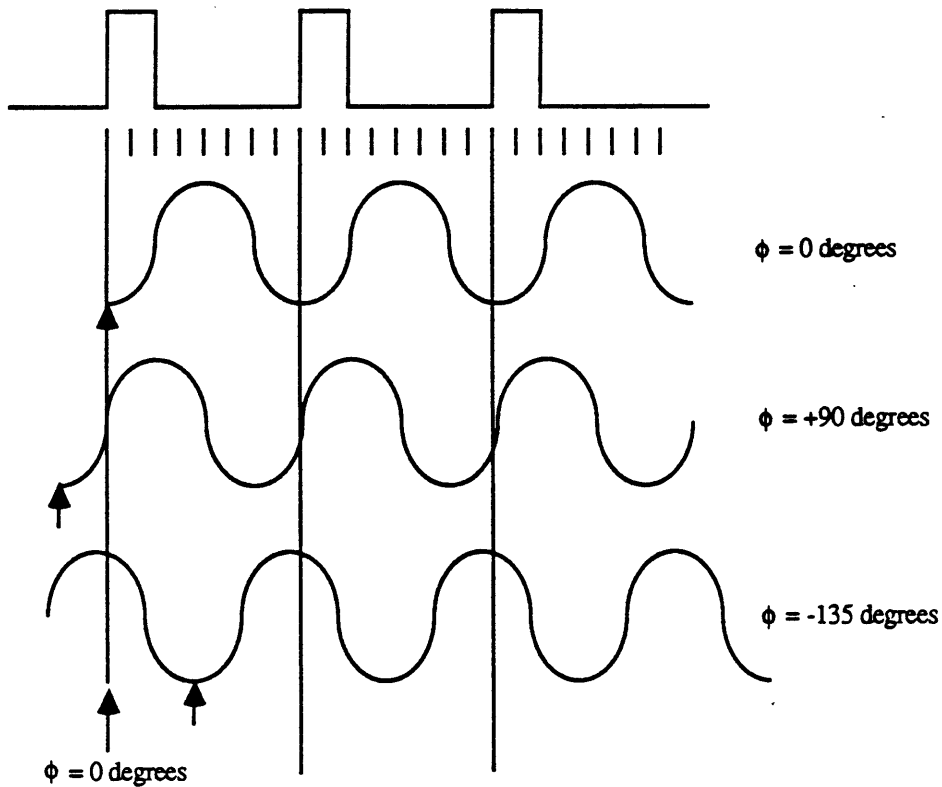


Figure 4.2 - Oscilloscope phase measurements

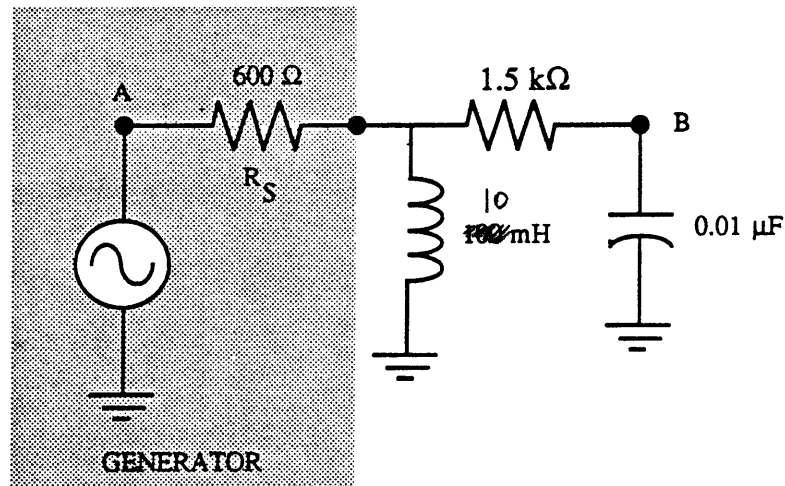


Figure 4.3 - Network for frequency response measurements

- (3) Using the scope, record the amplitude and phase shift for the frequencies from 10 Hz to 100 KHz listed in Table 4.1. Adjust the vertical sensitivity as necessary to make accurate amplitude measurements and try to measure the phase shift to within 9 degrees (one minor division). You should use BRIGHT LINE OFF for your low frequency measurements or else the scope display will not be stable.
- (4) Repeat steps 2 and 3 for the circuit of Fig. 4.4. You may start your measurements at 100 Hz. Record your results in Table 4.2.

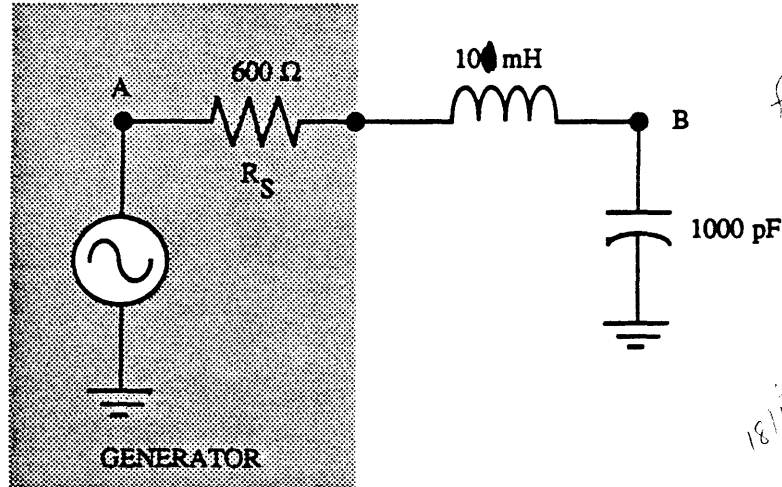


Figure 4.4 - Second frequency response network

- (5) Find the frequency at which the amplitude reaches its peak (probably ~~100~~ KHz) and measure the amplitude and phase shift at that frequency.
- (6) Use the DMM to measure the series resistance of the inductor. Record your result in Table 4.3.

PLEASE CALL A TEACHING ASSISTANT TO CHECK YOUR DATA

Part II - Compensated oscilloscope probe

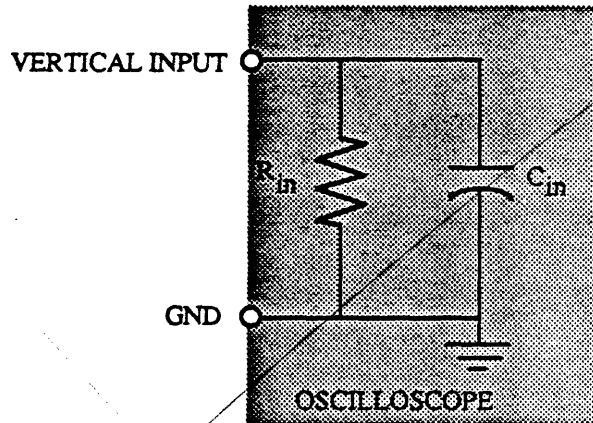


Figure 4.5 - Equivalent input circuit of oscilloscope

As shown in Fig. 4.5 and discussed in Lab #3, the input of the oscilloscope appears as a parallel resistance and capacitance to ground. We compensated for the cable capacitance in Lab #3 using a voltage divider analysis. The same analysis can be done more formally using frequency domain analysis.

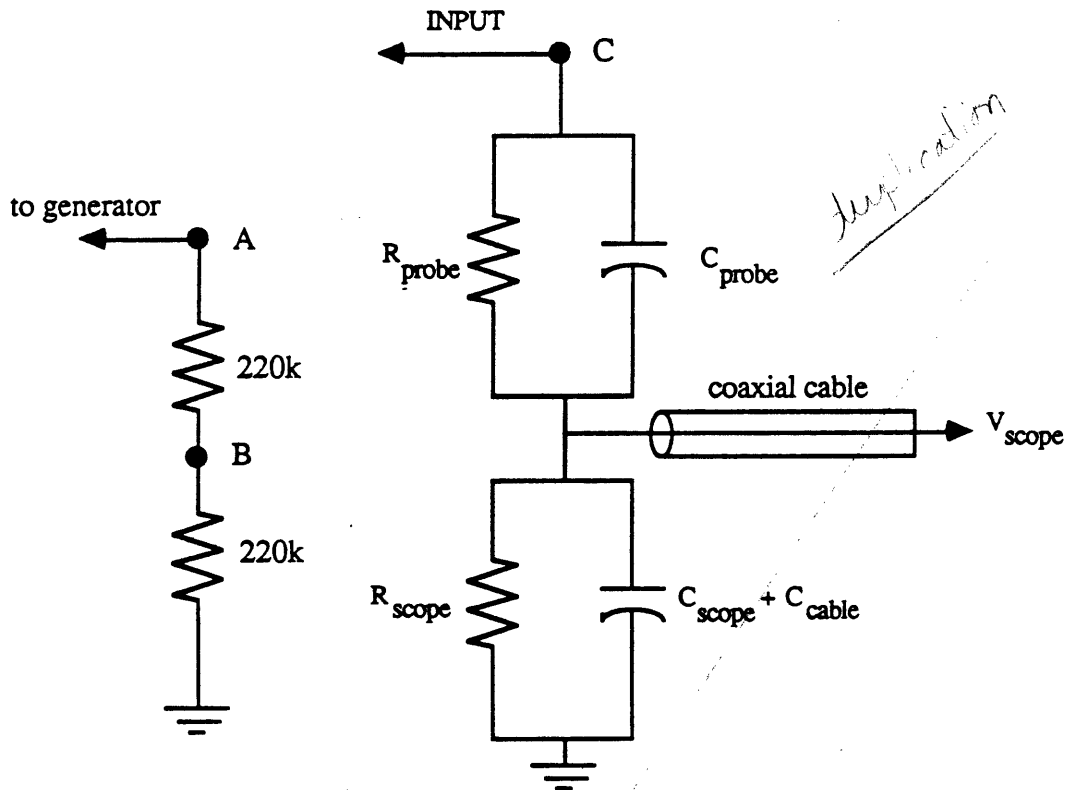


Figure 4.6 - Compensated scope probe circuit

- (6) Set the generator to maximum amplitude with zero DC offset.
- (7) Use the coaxial cable to measure the peak-peak AC voltage at points A and B with the generator set to a sine wave output at the frequencies shown in Table 4.4.

If we define "compensation" as adjusting the probe circuit to produce a waveform equivalent to a resistive voltage divider, we can "compensate" the probe circuit by adjusting the capacitor. When the probe is adjusted to satisfy Eqn.(1), the scope will display amplitude according to Eqn. 4.2.

$$R_{\text{probe}} \times C_{\text{probe}} = R_{\text{scope}} \times (C_{\text{scope}} + C_{\text{coax}}) \quad (4.1)$$

$$\text{displayed } V = \text{actual } V \times \frac{R_{\text{scope}}}{R_{\text{scope}} + R_{\text{probe}}} \quad (4.2)$$

- (1) Construct the circuit shown in Figure 4.3 where $R_{\text{probe}}=10\text{M}$ and C_{probe} is a 1.2-30 pF variable capacitor. This is the same circuit you built in Lab #3.
- (2) Set the generator to a 1 KHz square wave at maximum amplitude and zero DC offset.
- (3) Connect the probe input (point C) to point A. Observe what happens as you turn the variable capacitor a full 360 degrees.
- (4) Adjust the capacitor so that the tops and bottoms of the square wave are perfectly flat and horizontal. (Note that this is how you would adjust any compensated probe. Terminal C-7 on your scope is a square wave source provided for compensating probes.)
- (5) Remove the wire between points A and C and connect points B and C.

- (6) Record the displayed voltages with the generator set to a sine wave for the frequencies shown in Table 4.5.

PLEASE CALL A TEACHING ASSISTANT TO CHECK YOUR DATA BEFORE CONTINUING.

Part 3 - Resonance

Construct the circuit shown in Figure 4.7. Adjust your signal generator for a 15 kHz sine wave output and connect it to the input of your circuit. Measure the peak-peak input and output voltage with an oscilloscope for the frequencies shown below.

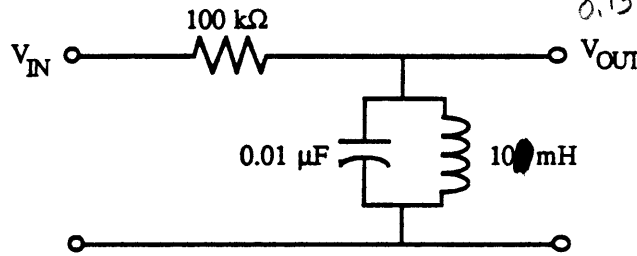


Figure 4.7 - Resonant network

use 10V P-P.
0.13 to 1.6V.
≈ 16 kHz,
 $f = \frac{1}{2\pi\sqrt{LC}}$
 $\frac{1}{6\sqrt{.01 \times 10^{-6} \cdot 10 \times 10^{-3}}}$
 $\frac{1}{6\sqrt{.01 \times 10^{-8}}}$
 $\frac{1}{6(.1) \times 10^{-4}}$
 $\frac{1}{.6 \times 10^{-4}}$
 $\frac{1}{.6} \times 10^4$
 1.5×10^4
 15×10^3

Measure your component values (i.e. L, R and C) using the digital LCR meter in the instrument room.

Part 4 - Fourier analysis

All signals can be modeled as sums of sinusoids according to the following Fourier series expansion

$$v(t) = \sum_{i=1}^{\infty} \{a_i \cos(\omega_i t) + b_i \sin(\omega_i t)\} \quad (4.3)$$

where $\omega_i = 2\pi/T$. The signal must repeat every T seconds where T is known as the period of the signal. This is a very important relationship as it allows us to solve problems in the frequency domain. This is especially useful for filters and resonance problems. We will not attempt to prove this relationship or really do anything with it. However, we can establish that it really works by doing a crude experiment. The resonant circuit of Figure 4.7 has a very narrow resonance which means that only a small band of frequencies can pass through it. For example, if the network was resonant at 15 kHz then it would pass only a small band of frequencies near 15 kHz. If we could vary the filter's resonant frequency then we could look at the frequency components of a signal. Unfortunately, we do not have the components to vary the resonant frequency of our network nor to build enough filters with different resonant frequencies to check the spectrum of any real signal. However, we can vary our signal frequency and then scale the results. This sounds more complicated than it really is. Measure the output voltage of your resonant network for a square wave signal at the resonant frequency. Record this voltage and frequency in Table 4.7. Now slowly decrease the generator frequency until you see another output signal from the filter. Find and record at least three more signals. HINT: all signals will be at odd multiples of your resonant frequency, i.e. 1, 3, 5, etc.

*During this lab
 measure output*

Questions:

- ① Make Bode plots of the phase and magnitude response of the circuits of Figures 4.3 and 4.4 using your data from Tables 1 and 2. Using a different color pen or pencil, add straight line approximations to each of the four graphs. Please draw all of your plots on graph paper.
2. (a) What is the transfer function of the circuit shown in Figure 4.4?
(b) Without using your experimental data, calculate the frequency and amplitude of the peak output voltage. (Hint: at what frequency is the slope of the amplitude zero?)
(c) What peak amplitude and frequency are indicated by your Bode plots?
3. Re-draw circuit 4.4 modelling the inductor as an ideal inductor in series with the resistance you measured. Re-calculate the poles and zeros.
4. The frequency dependent terms in the combined probe/scope input circuit transfer function (V_o/V_s) drop out when Eqn. 4.1 is satisfied. Find the transfer function of the circuit in Figure 4.6 and prove that it is frequency independent when Eqn. 4.1 is satisfied.
- ⑤ Calculate the resonant frequency of Figure 4.7 and compare it with your measured value. Explain any discrepancies.
- ⑥ Plot $\log_{10}(V_{OUT}/V_{IN}@f_{res})$ from Table 4.5 on the vertical axis of versus frequency on the horizontal axis. The resulting graph is a realization of the Fourier series equation, Eqn. 4.3, and is called the frequency spectrum of the generator output signal.

BONUS

Write a computer program to sum sine (or cosine) signals at f , $2f$, $3f$ up to $10f$ together according to Eqn. (4.1). Your data ($V_{OUT}/V_{IN}@f_{res}$) from Table 4.1 represents the b_i coefficients in Equation 4.1. Assume that the a_i coefficients are zero. Plot your results for a single sine wave of amplitude 1 at frequency f . Then, add the sine wave of amplitude $V_{OUT}/V_{IN}@f_{res}$ at frequency $3f$. Plot your results. Repeat this process for as many frequency coefficients you measured (up to 10). Discuss your results. What would have happened if you had used your results for the a_i coefficients rather than for the b_i coefficients?

Explain the Fourier series!

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LAB 4 EVALUATION

NAME (print) _____ CHECKPOINT #1 ____ DATE _____
GRADE ____/____ CHECKPOINT #2 ____ DATE _____

With respect to the course material, this lab was: (pick one)
___ highly relevant ___ relevant ___ not relevant ___ completely irrelevant

This lab was: (pick one)
___ too long ___ long ___ just right ___ short ___ too short

This lab was: (pick one)
___ too hard ___ hard ___ just right ___ easy ___ too easy

The background material in the lab assignment was: (pick one)
___ too detailed ___ just right ___ sufficient ___ insufficient ___ totally inadequate

The step by step procedures in the lab assignment were: (pick one)
___ too detailed ___ just right ___ sufficient ___ insufficient ___ totally inadequate

Describe any mistakes made in the lab assignment.

Describe anything that just didn't work right.

Describe how this lab could be made better.

QUIZ

NOTE: THE TEACHING ASSISTANT IS TO SELECT BOTH QUESTIONS FROM THE UNDERLINED OPTIONS AT THE SECOND CHECKPOINT

Question #1-3

Assume that $F_x = 0.9$, $V_f = 1.0$, and that the transformer's nominal ratings are correct. What load resistance would be necessary in each of the three rectifier circuits to establish a DC load current of 50 mA ?

_____ half wave rectifier

_____ full wave bridge rectifier

_____ full wave center tap rectifier

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NAMES: _____

Lab 4 Data

frequency	Amplitude, V_{p-p}	Phase, degrees
10 Hz	_____	_____
20 Hz	_____	_____
50 Hz	_____	_____
100 Hz	_____	_____
200 Hz	_____	_____
500 Hz	_____	_____
1 KHz	_____	_____
2 KHz	_____	_____
5 KHz	_____	_____
10 KHz	_____	_____
20 KHz	_____	_____
50 KHz	_____	_____
100 KHz	_____	_____

Table 4.1 Frequency response of Figure 4.3 circuit

frequency	Amplitude, V_{p-p}	Phase, degrees
100 Hz	_____	_____
200 Hz	_____	_____
500 Hz	_____	_____
1 KHz	_____	_____
2 KHz	_____	_____
5 KHz	_____	_____
10 KHz	_____	_____
20 KHz	_____	_____
50 KHz	_____	_____
100 KHz	_____	_____

Table 4.2 Frequency response of RC circuit

DC series resistance of inductor: _____ ohms.

Table 4.3

frequency	$V_{A,P-P}$	$V_{B,P-P}$
1 KHz	_____	_____
2 KHz	_____	_____
5 KHz	_____	_____
10 KHz	_____	_____
20 KHz	_____	_____
50 KHz	_____	_____
100 KHz	_____	_____
200 KHz	_____	_____
500 KHz	_____	_____
1 MHz.	_____	_____

Table 4.4 Frequency response measurements of uncompensated scope cable

frequency	V_{P-P}
1 KHz	_____
2 KHz	_____
5 KHz	_____
10 KHz	_____
20 KHz	_____
50 KHz	_____
100 KHz	_____
200 KHz	_____
500 KHz	_____
1 MHz.	_____

Table 4.5 Frequency response measurements of compensated scope cable

change

	frequency	V _{IN}	V _{OUT}
10	15 kHz	_____	_____
	16 kHz	_____	_____
	17 kHz	_____	_____
	18 kHz	_____	_____
	19 kHz	_____	_____
	20 kHz	_____	_____
	21 kHz	_____	_____
	22 kHz	_____	_____
	23 kHz	_____	_____
	24 kHz	_____	_____
20	25 kHz	_____	_____

Table 4.6 - Frequency response measurements of resonant circuit

frequency	multiple of f _{res}	V _{IN}	V _{OUT}	V _{OUT} /(V _{IN} @f _{res})
_____	_____1_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____
_____	_____	_____	_____	_____

Table 4.7 Frequency Analysis of a Square Wave