We have shown that in a medium in which conduction current dominates (conductors), the EM fields obey a diffusion equation, whereas in a medium in which displacement current dominates (dielectrics), the EM fields obey a wave equation. The implications of the two types of equations, (13.19) and (13.20), will be explored in the next sections.

13.4 SINUSOIDAL PLANE WAVES

Plane waves are waves that vary only in the direction of propagation and are uniform in planes normal to the direction of propagation. In (13.11) we considered such a wave. It propagated in the z direction. The **E** field had only an E_x component which has the same value at every point in a plane parallel to the xy plane.

It appears that as solutions to the general vector wave equation (13.8) are hopelessly complicated, plane waves are introduced primarily to make the mathematics simpler. Fortunately, this is not the case. It is well known in more advanced studies of EM fields that an arbitrary field or wave can always be represented as a spectrum of plane waves.† Therefore, plane waves can be considered as the building blocks in more complicated waves. Even of more importance is that the fields radiated by any transmitting antenna look like plane waves at distances far from the source. This is depicted in Fig. 13.3, where over a finite area ΔA, which is normal to the propagating direction z, the E and H fields are approximately planar. The farther one gets from the antenna, the better the approximation is. The fact that plane waves are simple and obey a scalar wave equation is a welcome mathematical convenience.

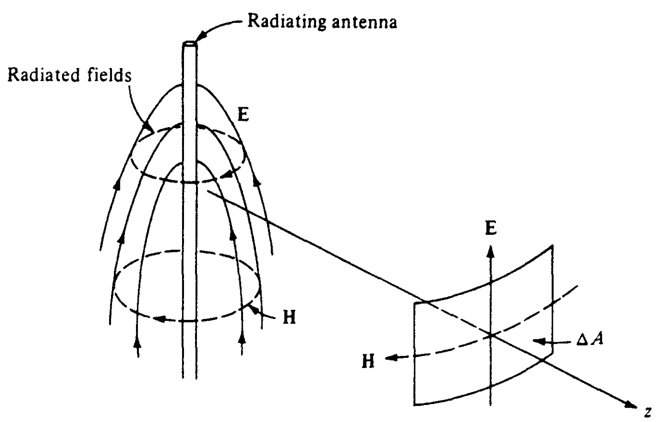

Figure 13.3 A vertical tower antenna radiates a field which spreads out in a radial direction from the antenna. Far from the antenna the field in area ΔA is a plane wave.

† P. C. Clemmow, "The Plane Wave Spectrum Representation of Electromagnetic Fields," Pergamon Press, Oxford, 1966.

Let us now choose the time behavior of the fields as sinusoidal: i.e., the fields oscillate at a single frequency $f = \omega/2\pi$ Hz. Again, the motivation for this is not just to consider simpler fields but is based upon two reasons. One is that many transmitting sources (radio, microwave, and optical) operate at such a narrow band of frequencies that the single-frequency approximation applies. The other is that any periodic wave can be represented as a Fourier series of sinusoids and any nonperiodic wave such as a pulse can be represented as a continuous spectrum of harmonics by the Fourier integral. For example, the pulse shown in Fig. 13.2 can be constructed from an infinite set of sinusoidally varying plane waves. These plane waves interfere constructively at the location of the pulse in such a way as to yield the pulse shape and interfere destructively every place else to give zero. Thus the general time case can be reduced to a problem involving sinusoids, which we will now proceed to develop.

Using the phasor notation,† the sinusoidal time variation of an E field polarized in the x direction can be represented by

$$\mathbf{E}(z, t) = E_x(z, t)\hat{\mathbf{x}} = E_x(z)e^{j\omega t}\hat{\mathbf{x}} \tag{13.21}$$

Substituting into the source-free wave equation (13.16), which is applicable to the lossy case ($\sigma \neq 0$) as well as the lossless case ($\sigma = 0$), we obtain

$$\frac{\partial^2 E_x(z)}{\partial z^2} + \omega^2 \mu\varepsilon \left(1 - \frac{j\sigma}{\omega\varepsilon}\right) E_x(z) = 0 \tag{13.22}$$

where the common factor $\hat{\mathbf{x}}e^{j\omega t}$ has been deleted. This is a relatively simple wave equation since it depends only on a single space variable. Equation (13.22) determines the space behavior of a uniform plane wave which varies sinusoidally with time. Using the complex permittivity $\varepsilon^\star$, defined in (11.23) as

$$\varepsilon^\star = \varepsilon\left(1 - \frac{j\sigma}{\omega\varepsilon}\right) \tag{13.23}$$

we can write (13.22) in the form of a lossless wave equation

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} + \omega^2 \mu\varepsilon^\star E_x = 0} \tag{13.24}$$

which is better known as the equation of *simple harmonic motion* and has the solution

$$E_x(z) = E_0^i e^{-j\beta^\star z} + E_0^r e^{j\beta^\star z} \tag{13.25}$$

† Sinusoidal time variation can be represented by the real part (Re) of an exponential; that is, $\cos \omega t = \mathrm{Re}\, e^{j\omega t} = \mathrm{Re}\,(\cos \omega t + j \sin \omega t) = \cos \omega t$. As superposition applies in a linear system (Maxwell's equations and the wave equation are linear in media for which μ, ε, σ are constants), we can drop the operator Re and simply work with $e^{j\omega t}$. After a solution to a problem (using $e^{j\omega t}$) has been worked out, to give it physical meaning, we take the real part which is then referred to as the instantaneous solution.

where $\beta^\star$ is a complex phase-propagation constant† given by $\beta^{\star 2} = \omega^2 \mu \varepsilon^\star$. In general $\beta^\star$ will have real and imaginary parts which are given by $\beta^\star = \omega\sqrt{\mu\varepsilon^\star} = \beta - j\alpha$. E_0^i and E_0^r are the amplitudes of the forward (incident) and backward (reflected) traveling waves, respectively. If we assume there are no reflections ($E_0^r = 0$), we have propagation in one direction only. Putting back the time dependence, the single-frequency uniform-plane wave solution to (13.24) is‡

$$E_x(z, t) = E_0 e^{j(\omega t - \beta^\star z)} = E_0 e^{-\alpha z} e^{j(\omega t - \beta z)} \tag{13.26}$$

where $\omega t - \beta z$ is the phase of the wave.

It is interesting to observe that by introducing the complex permittivity $\varepsilon^\star$, the wave equation (13.24) and its solution (13.26) give the correct behavior of plane waves in dielectric as well as conducting media simply by letting $\varepsilon^\star \to \varepsilon$ and $\varepsilon^\star \to -j\sigma/\omega$, respectively. Thus for a dielectric medium, in which displacement current dominates and conductive current is negligible $[J/(\partial D/\partial t) = \sigma/\omega\varepsilon \ll 1]$, we have

$$\frac{\partial^2 E_x}{\partial z^2} + \beta^2 E_x = 0 \qquad E_x = E_0 e^{j(\omega t - \beta z)} \tag{13.27}$$

where $\beta^2 = \omega^2 \mu\varepsilon$ and the approximation that $\sigma/\omega\varepsilon = 0$ is used. In highly conducting media ($\sigma/\omega\varepsilon \gg 1$), we let $\varepsilon^\star \to -j\sigma/\omega$ and obtain for (13.24) and (13.26)

$$\frac{\partial^2 E_x}{\partial z^2} - j\omega\sigma\mu E_x = 0 \qquad E_x = E_0 e^{-z/\delta} e^{j(\omega t - z/\delta)} \tag{13.28}$$

where $\beta^\star = (1 - j)/\delta$, and where $\delta = (\omega\mu\sigma/2)^{-1/2} = (\pi f \mu\sigma)^{-1/2}$ and is known as the skin depth or depth of penetration of a wave in a conducting medium. Thus if the wave has an amplitude E_0 at some point in the conducting medium, in a distance equal to $z = \delta$, the amplitude of that wave will have decreased by a factor of $1/e$. Since δ can be very small for good conductors even at low frequencies, the wave decreases exponentially very rapidly as it propagates into the medium (see values for δ on page 400 and Table 13.1). Such rapid decrease is more characteristic of diffusion than of propagation. This is as expected, for (13.28) is really a diffusion equation; it is the time-independent form of (13.20) which is a diffusion equation. What is surprising is that for harmonic time variation we can obtain the solution to a diffusion equation from a solution to a wave equation. But note that even

† Other books define a complex propagation constant $\gamma = \alpha + j\beta$ by letting $\omega^2\mu\varepsilon^\star = -\gamma^2$. The proper relationship between $\beta^\star$ and γ is $\gamma = j\beta^\star$ or $\beta^\star = \beta - j\alpha$. The term α is known as the *attenuation constant* and β as the *phase constant* or phase-propagation constant. Note that phase and phase constant have meaning only in reference to sinusoidally varying waves (single-frequency waves).

‡ Note that this is a phasor expression. To convert this to a physically meaningful expression, one takes the real part of Eq. (13.26), called the instantaneous value $E_x(z, t)_{\text{inst}} = \text{Re}\ (13.26) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)$.

though the diffusion part in (13.28), which is $e^{-z/\delta}$, heavily dominates the solution, a traveling-wave part is present in the solution. We will elaborate on (13.27) and (13.28) in the following two sections.

The Transverse Nature of Plane Waves

Maxwell's equation (13.2) for free space (or any other homogeneous and isotropic medium for which $\rho = 0$) is

$$\nabla \cdot \mathbf{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \tag{13.29}$$

If we apply this statement to plane waves for which there is no variation of the field with x or y, (13.29) reduces to

$$\frac{\partial E_z}{\partial z} = 0 \tag{13.30}$$

A solution to this equation is that $E_z =$ constant. Therefore, E_z cannot have any variations with x, y, or z. Such a solution cannot be a wave. Since an exactly similar argument holds for **H**, we conclude that $H_z = E_z = 0$ for a wave that travels in the z direction. An EM wave which has only components transverse to the direction of propagation is called a *TEM wave*, an abbreviation for transverse electric and magnetic.

Relation between Electric and Magnetic Fields in a Plane Wave

Starting with (13.17) and using a similar procedure that was followed for E, we can obtain H as

$$H(z, t) = H_0 e^{j(\omega t - \beta^\star z)} \tag{13.31}$$

For a wave traveling in the z direction, H can be H_x or H_y but not H_z, just as in the case of E. For a relationship between E and H, we must go back to Maxwell's equations. Thus, for sinusoidal time variation and for a plane wave which has only an E_x component, (13.1) gives

$$\nabla \times E_x \hat{\mathbf{x}} = -j\omega\mu \mathbf{H} \tag{13.32}$$

which in rectangular coordinates simplifies to

$$\hat{\mathbf{y}} \frac{\partial E_x}{\partial z} = -j\omega\mu \mathbf{H} \qquad \text{or} \qquad \frac{\partial E_x}{\partial z} = -j\omega\mu H_y \tag{13.33}$$

since $\partial/\partial y = \partial/\partial x = 0$ for a z-directed plane wave. This determines that a plane wave which has an E_x component can have only an H_y component of magnetic field. Substituting for E_x from (13.26) and differentiating, we obtain for the above equation

$$-j\beta^\star E_0 e^{j(\omega t - \beta^\star z)} = -j\omega\mu H_y \tag{13.34}$$

or

$$H_y = \frac{\beta^\star}{\omega\mu} E_x = \frac{\beta^\star}{\omega\mu} E_0 e^{j(\omega t - \beta^\star z)} \tag{13.35}$$

Using (13.31), we find that the amplitude H_0 of the magnetic field is related to that of the electric field by $H_0 = E_0(\beta^\star/\omega\mu)$. We can now make the important observation that E and H are at right angles to each other in a plane wave, and furthermore, that the direction of propagation, the direction of the H field, and the direction of the E field are mutually orthogonal to each other. It is common to write the above result in a form often called Ohm's law for a plane wave:

$$\boxed{E_x = \eta^\star H_y} \tag{13.36}$$

where

$$\eta^\star = \frac{\omega\mu}{\beta^\star} \tag{13.36a}$$

and is called the complex characteristic, intrinsic, or wave impedance of the medium. The units of η are volt per ampere or ohm. For vacuum $\eta^\star$ is real and is $\eta_0 = \omega\mu_0/\omega\sqrt{\mu_0\varepsilon_0} = \sqrt{\mu_0/\varepsilon_0} = 377\ \Omega$.

If the electric field in the plane wave had only an E_y component, the analogous relationship to (13.36) would be $E_y = -\eta^\star H_x$. We can now generalize as follows: If the direction of propagation is given by the $\hat{\mathbf{z}}$ vector, Ohm's law for plane waves is given by

$$\boxed{\hat{\mathbf{z}} \times \mathbf{E} = \eta^\star \mathbf{H} \qquad \text{or} \qquad \hat{\mathbf{z}} \times \mathbf{H} = -\frac{\mathbf{E}}{\eta^\star}} \tag{13.37}$$

13.5 PLANE WAVES IN INSULATING OR DIELECTRIC MEDIA

This is the case of propagation of plane waves in vacuum, air, or any other dielectric medium which has practically no loss. The displacement current dominates, and the plane wave solution that applies is (13.27), with the other constants being

$$\varepsilon^\star = \varepsilon = \varepsilon_r\varepsilon_0$$

$$\beta^\star = \beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\varepsilon_r} = \beta_0\sqrt{\varepsilon_r}$$

$$\eta^\star = \eta = \frac{E_x}{H_y} = \sqrt{\frac{\mu}{\varepsilon}} = \frac{1}{\sqrt{\varepsilon_r}}\sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{\sqrt{\varepsilon_r}} \tag{13.38}$$

where it was assumed that the approximation $\sigma = 0$ is valid and the permeability μ of the medium (except for ferromagnetic medium) is that of vacuum. For vacuum $\varepsilon_0 = 8.85 \times 10^{-12}\ \text{Fm}^{-1}$, $\mu_0 = 4\pi \times 10^{-7}\ \text{Hm}^{-1}$, $\eta_0 = 377\ \Omega \cong 120\pi\ \Omega$. The electric and magnetic fields are then given by

$$\boxed{\begin{aligned} E_x &= E_0 e^{j(\omega t - \beta z)} \\ H_y &= \sqrt{\frac{\varepsilon}{\mu}} E_0 e^{j(\omega t - \beta z)} \end{aligned}} \tag{13.39}$$

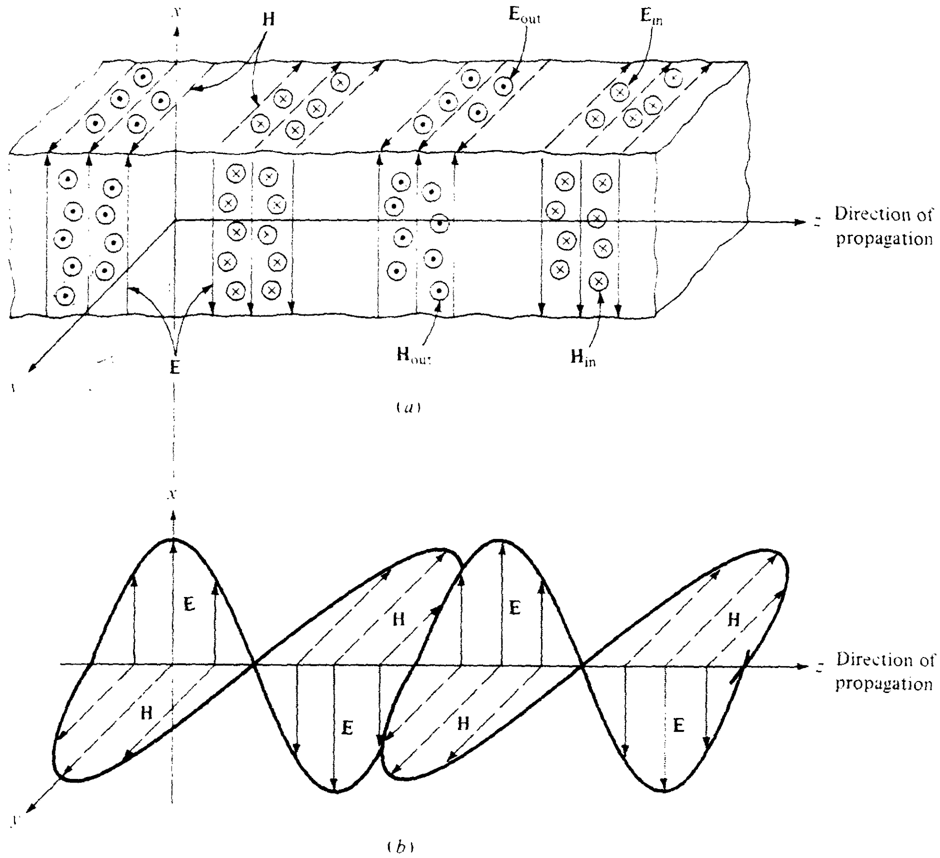

Figure 13.4 (*a*) The **E** and **H** fields in a sinusoidally varying plane wave. A "snapshot" of a three-dimensional section of a plane wave showing the relationship between the **E** and **H** fields. (*b*) An alternative representation of a plane wave showing the sinusoidal nature of the fields and the orthogonality between **E**, **H**, and the direction of propagation.

and their variation along the direction of propagation is shown in Fig. 13.4. The sinusoidal variation which is shown in the figure is obtained by taking the real part of (13.39) in the usual manner when phasor notation is employed; i.e., the instantaneous values are given by Re $E_0 e^{j(\omega t - \beta z)} = E_0 \cos(\omega t - \beta z)$. This figure suggests that once the wave is set in motion, it continues in space unattenuated. The E and H fields are interdependent and should not be thought of as independent sets of waves, but as different aspects of the same phenomenon.

A single-frequency plane wave is thus characterized by its polarization (direction in which the **E** vector points), its amplitude E_0, and its phase $\psi = \omega t - \beta z$, as, for example,

$$\mathbf{E} = \hat{\mathbf{x}} E_0 e^{j\psi} \tag{13.40}$$

All three can be measured, and all three can be used to impose information on the plane wave by modulating polarization, amplitude, or phase. But the wave nature resides strictly in the phase term. Thus at a fixed point along the z axis, an observer could measure a phase change that increases linearly with time, $\psi(t) \propto \omega t$, as the wave moves past. Similarly, if we could freeze time, we would see a phase change $\psi(z) \propto \beta z$ along the axis of propagation. Hence ω is a temporal phase-shift constant (phase shift in radians per unit time), and β is a spatial phase-shift constant (phase shift in radians per unit distance). A *period* is defined as the time T during which a wave undergoes a phase shift of 2π:

$$\omega T = 2\pi \qquad \text{or} \qquad T = \frac{2\pi}{\omega} \tag{13.41}$$

A *wavelength* is defined as the distance λ during which a wave undergoes a phase shift of 2π:

$$\beta\lambda = 2\pi \qquad \text{or} \qquad \lambda = \frac{2\pi}{\beta} \tag{13.42}$$

The wavelength λ plays the same role in the space domain as period T plays in the time domain. The relationship between the phase-propagation constant β and velocity v of the wave is given by the solution to the wave equation (13.25) as

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{v} \tag{13.43}$$

For sinusoidal waves, the velocity v is called the phase velocity. It is the velocity with which a given value of E or H advances along the z axis. Since in a sinusoidal wave a given value of E or H is specified by the value of the phase angle ψ, the velocity of the wave is appropriately referred to as the phase velocity. In other words, an observer moving with velocity v alongside the wave observes a constant phase ψ.

A medium in which the phase velocity remains constant as the frequency of the wave is varied is referred to as a *nondispersive medium*. As $v = 1/\sqrt{\mu\varepsilon}$, a nondispersive medium must have μ and ε which are not functions of frequency; vacuum is an example.

Wave Propagation in a Dielectric with Small Losses

Wave propagation when displacement current dominates but a small amount of energy is extracted from the wave because the medium is absorbent represents a practical situation. Then

$$\varepsilon^{*} = \varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right) \tag{13.44}$$

where $\sigma/\omega\varepsilon \ll 1$, that is, small but not zero. The complex phase-propagation constant is

$$\beta^* = \omega\sqrt{\mu\varepsilon^*} = \omega\sqrt{\mu\varepsilon}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}}$$

$$\approx \beta\left(1 - j\frac{\sigma}{2\omega\varepsilon}\right)$$

$$= \beta - j\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

$$\boxed{= \beta - j\alpha} \qquad (13.45)$$

where the binomial approximation $(1 \pm \Delta)^{1/2} \approx 1 \pm \frac{\Delta}{2}$ for $\Delta \ll 1$ was used. Hence the electric field is

$$E_x = E_0 e^{j(\omega t - \beta^* z)} = E_0 e^{-\alpha z} e^{j(\omega t - \beta z)} \qquad (13.46)$$

where α is the attenuation coefficient $\alpha = (\sigma/2)\sqrt{\mu/\varepsilon}$ measured in nepers per meter (Np/m). The exponent of e is then in the dimensionless units of neper. The electric field (as well as the magnetic field) now experiences a small exponential attenuation. Small because the decrease in a distance of one wavelength is small; that is,

$$\alpha z\bigg|_{z=\lambda} = \left(\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}\right)\left(\frac{2\pi}{\omega\sqrt{\mu\varepsilon}}\right) = \frac{\pi\sigma}{\omega\varepsilon} \ll 1 \qquad (13.47)$$

Figure 13.5 shows the instantaneous values of the E field with a small attenuation modulating an otherwise sinusoidal spatial variation which has a wavelength $\lambda = 2\pi/\beta$.

The intrinsic or characteristic impedance of the medium which has a finite conductivity σ is

$$\eta^* = \sqrt{\frac{\mu}{\varepsilon^*}} = \sqrt{\frac{\mu}{\varepsilon}}\frac{1}{\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}}} = \eta\left(1 + j\frac{\sigma}{2\omega\varepsilon}\right) \qquad (13.48)$$

Hence, the loss adds a small reactive component to the intrinsic impedance, which for most practical purposes can be ignored; that is, $\eta^* \cong \eta = \sqrt{\mu/\varepsilon}$.

Nomenclature Used in Reference Books

There are two types of loss mechanisms which attenuate a wave. The first (already considered) arises when the dielectric is slightly conducting. The second arises when energy is dissipated in the course of the polarization process even though the conductivity of the dielectric is zero (dipoles experience friction as they

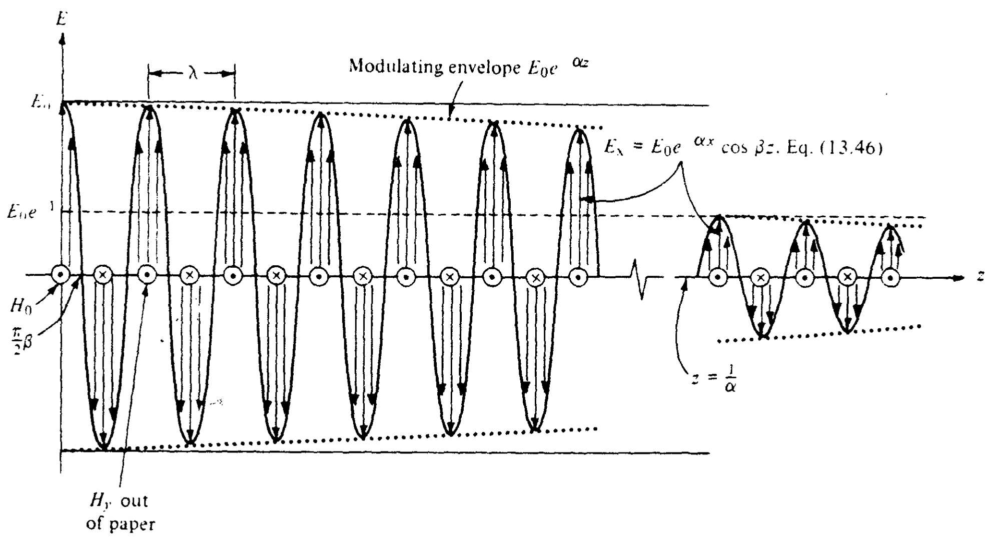

Figure 13.5 A slightly absorbing dielectric medium will impose a small exponential attenuation on the propagating fields. In practical situations the distance $z = 1/\alpha$ at which the field has decayed to E_0e^{-1} is usually very large.

flip back and forth in a sinusoidal field thus extracting energy from the field.† As both loss mechanisms generate heat, each can be represented by a conductivity σ. The complex permittivity Eq. (13.23) can now be generalized to reflect conduction and polarization losses as

$$\boxed{\varepsilon^\star = \varepsilon' - j\varepsilon'' - j\frac{\sigma}{\omega}} \tag{13.49}$$

where $\varepsilon'/\varepsilon_0$ is the dielectric constant of the material and the total effective conductivity is

$$\sigma_{\text{eff}} = \sigma + \omega\varepsilon'' \tag{13.50}$$

The ratio of conduction current to displacement current in the lossy dielectric is called the *loss tangent* or *dissipation factor*:

$$\boxed{\tan\phi = \frac{\sigma_{\text{eff}}}{\omega\varepsilon'} = \frac{\sigma + \omega\varepsilon''}{\omega\varepsilon'}} \tag{13.51}$$

Values of dielectric losses are tabulated in reference books under a variety of names, such as loss tangent, dissipation factor, power factor. The loss tangent

† Because of such friction (polarization damping forces), the polarization vector **P** will lag behind the applied **E** field. The difference in time phase between **P** and **E** is accounted for by a permittivity with an imaginary part; that is $\varepsilon = \varepsilon' - j\varepsilon''$.

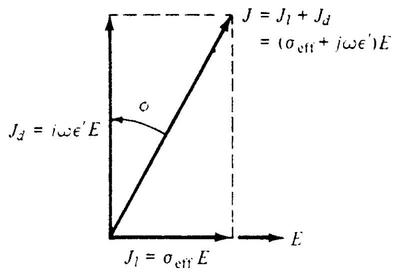

Figure 13.6 The loss angle ϕ. Power factor is $\sin\phi = \cos(\pi/2 - \phi)$, where $\pi/2 - \phi$ is the angle by which **J** leads **E**. The loss tangent is $\tan\phi = J_{loss}/J_{displ.} = \sigma_{eff}/\omega\varepsilon'$.

relates to the power factor which is defined as $\sin\phi$. These relationships are illustrated in Fig. 13.6. Since the losses in most dielectrics are small, we see that loss tangent = dissipation factor $\approx$ power factor $\approx \phi$. The loss tangent (13.51) includes conduction and polarization losses. At microwave frequencies, because of large values of ω, losses due to polarization damping forces dominate ($\omega\varepsilon'' \gg \sigma$) and $\tan\phi \approx \varepsilon''/\varepsilon'$.

Example What is the loss per kilometer for a plane wave propagating in dry earth? The frequency is 1 MHz.

At this frequency, dry soil has a conductivity of $\sigma = 10^{-5}$ S/m and a relative permittivity of $\varepsilon_r = 3$. Hence, $\sigma/\omega\varepsilon \cong 0.06 \ll 1$, which means that displacement current dominates and the effect of the conductivity is to attenuate the propagating wave. The value of the attenuation coefficient, using (13.45), is given as

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \beta\left(\frac{\sigma}{2\omega\varepsilon}\right) = 3.6 \times 10^{-2}(0.03) = 1.1 \times 10^{-3}\ \text{Np/m}$$

where $\beta = \omega/v = 2\pi f/(v_0/\sqrt{\varepsilon_r}) = 2\pi \times 10^6/(3 \times 10^8/\sqrt{3}) = 3.6 \times 10^{-2}$ rad/m. In 1 km of propagation the amplitude will have decreased from one to

$$e^{-(1.1\times 10^{-3})(10^3)} = e^{-1.1} = 0.33$$

or by 20 log (0.33) = 9.5 dB, which for many applications is a tolerable loss.

Example Calculate the loss per kilometer for a plane wave propagating in distilled water at a frequency of 25 GHz.

The dissipation factor and dielectric constant ε_r at this frequency are given as 0.3 and 34, respectively. Since the dissipation factor is equal to $\sigma_{eff}/\omega\varepsilon = \varepsilon''/\varepsilon'$, we have for the attenuation coefficient, using (13.45)

$$\alpha = \beta\frac{\text{dissipation factor}}{2} = \sqrt{\varepsilon_r}\beta_0\frac{0.3}{2} = (\sqrt{34})(524)(0.15) = 460\ \text{Np/m}$$

where $\beta_0 = \omega/v_0 = 2\pi f/v_0 = 2\pi(2.5 \times 10^{10})/3 \times 10^8 = 524$ rad/m. In 1 km of propagation the amplitude will have decreased from one to

$$e^{-(460)(10^3)} = e^{-4.6\times 10^5} \cong 0$$

or by 4×10^6 dB. Clearly, communication is not possible. Even for a distance of 1 cm, the loss is $20 \log e^{-4.6} = 40$ dB, a very large value. Hence, communication (or radar) which uses such high-frequency microwaves is not possible. Other means of communication, which employ acoustic waves (sonar) or very low-frequency radio waves (see example in next section) must be used. The case for seawater which has higher conductivities than distilled water is even worse. The extreme rapid attenuation of high-frequency waves in water explains why the presence of water in the atmosphere (rain, fog) causes severe attenuation of such waves.

We might point out, that for dissipation factors ($\sigma/\omega\varepsilon$) larger than 0.1, the two-term binomial approximation for the attenuation coefficient α given by (13.45) is not sufficiently accurate. Additional terms in the binomial approximation must be carried, that is, $(1 \pm \Delta)^n = 1 \pm n\Delta + [n(n-1)/2]\,\Delta^2 \pm \cdots$. However, even when $\sigma/\omega\varepsilon = 0.3$ was used in (13.45), as was the case in this example, the error is small.

13.6 PLANE WAVES IN CONDUCTING MEDIA

In conducting media the conduction current dominates the displacement current

$$\frac{J}{\partial D/\partial t} = \frac{\sigma}{\omega\varepsilon} \gg 1 \tag{13.52}$$

to such an extent that we ignore the displacement current completely and substitute for $\varepsilon^* = \varepsilon(1 - j\sigma/\omega\varepsilon)$ simply $\varepsilon^* = -j\sigma/\omega$. The wave equation and its solution† for this case is (13.28), with the other constants being

$$\beta^* = \omega\sqrt{\mu\varepsilon^*} \cong \omega\sqrt{\mu\left(-\frac{j\sigma}{\omega}\right)} = (1-j)\sqrt{\frac{\omega\mu\sigma}{2}} = \frac{1-j}{\delta} = \beta - j\alpha \tag{13.53}$$

where the phase-propagation constant is $\beta = 1/\delta$, the attenuation constant is $\alpha = 1/\delta$,

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{\pi f\mu\sigma}} \tag{13.54}$$

$\sqrt{-j} = e^{-j\pi/4} = (1-j)/\sqrt{2}$, and the intrinsic impedance of the conducting medium is

$$\eta^* = \frac{E_x}{H_y} = \sqrt{\frac{\mu}{\varepsilon^*}} \cong \sqrt{\frac{j\omega\mu}{\sigma}} = (1+j)\sqrt{\frac{\omega\mu}{2\sigma}} \tag{13.55}$$

We still have wave propagation in the conducting medium, since solution (13.28) contains the term $e^{j(\omega t - z/\delta)}$, which is a traveling wave whose phase constant is

† The instantaneous values of the fields are obtained by taking the real part of the phasor expression (13.28):

$$E_x = E_0 e^{-z/\delta} \cos(\omega t - z/\delta)$$
$$H_y = E_0(\sigma/\omega\mu)^{1/2} e^{-z/\delta} \cos(\omega t - z/\delta - \pi/4)$$

and are plotted in Fig. 13.7.