Analysis of Networks Using S-Parameters

Let's now look at a simple example which will demonstrate how S-parameters can be determined analytically.

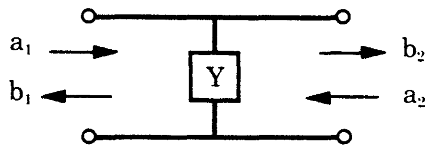
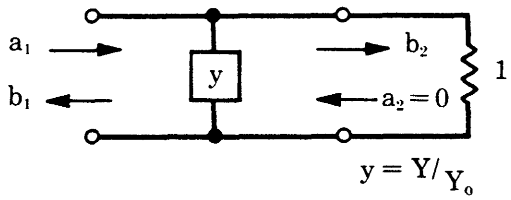


Figure 27

Using a shunt admittance, we see the incident and reflected waves at the two ports (Fig. 27). We first normalize the admittance and terminate the network in the normalized characteristic admittance of the system (Fig. 28a). This sets $a_2 = 0$. S_{11}, the input reflection coefficient of the terminated network, is then: (Fig. 28b).

To calculate S_{21}, let's recall that the total voltage at the input of a shunt element, $a_1 + b_1$, is equal to the total voltage at the output, $a_2 + b_2$ (Fig. 28c). Since the network is symmetrical and reciprocal, $S_{22} = S_{11}$ and $S_{12} = S_{21}$. We have then determined the four S-parameters for a shunt element.

a)



b) $$S_{11} = \frac{1 - y_T}{1 + y_T} = \frac{-y}{y + 2}$$

c) $$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2 = 0} = \left.\frac{a_1 + b_1}{a_1}\right|_{a_2 = 0} = 1 - \frac{y}{y + 2}$$

Figure 28

The Smith Chart

Another basic tool used extensively in amplifier design will now be reviewed. Back in the thirties, Phillip Smith, a Bell Lab engineer, devised a graphical method for solving the oft-repeated equations appearing in microwave theory. Equations like the one for reflection coefficient, $\Gamma = (Z - 1) / (Z + 1)$. Since all the values in this equation are complex numbers, the tedious task of solving this expression could be reduced by using Smith's graphical technique. The Smith Chart was a natural name for this technique.

This chart is essentially a mapping between two planes—the Z or impedance plane and the Γ or reflection coefficient plane. We're all familiar with the impedance plane—a rectangular coordinate plane having a real and an imaginary axis. Any impedance can be plotted in this plane. For this discussion, we'll normalize the impedance plane to the characteristic impedance (Fig. 29a).

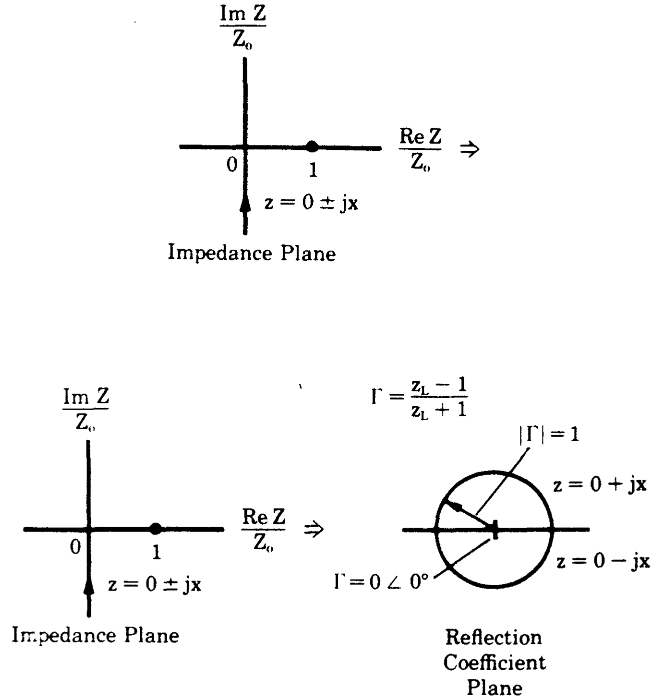


Figure 29

Let's pick out a few values in this normalized plane and see how they map into the Γ plane. Let $z = 1$. In a 50-ohm system, this means $Z = 50$ ohms. For this value, $|\Gamma| = 0$, the center of the Γ plane.

We now let z be purely imaginary; i.e., $z = jx$ where x is allowed to vary from minus infinity to plus infinity. Since $\Gamma = (jx - 1) / (jx + 1)$, $|\Gamma| = 1$ and its phase angle varies from 0 to 360°. This traces out a circle in the Γ plane (Fig. 29b). For positive reactance, jx positive, the impedance maps into the upper half circle. For negative reactance, the impedance maps into the lower half circle. The upper region is inductive and the lower region is capacitive.

Now let's look at some other impedance values. A constant resistance line, going through the point $z = 1$ on the real axis, maps into a circle in the Γ plane. The upper semicircle represents an impedance of $1 + jx$, which is inductive; the lower semicircle, an impedance of $1 - jx$ or capacitive (Fig. 30).

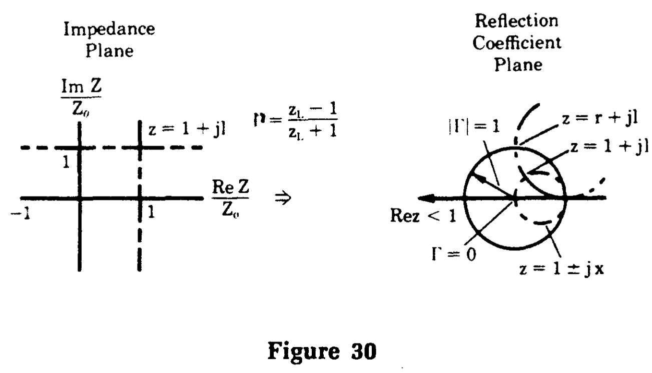


Figure 30

The constant reactance line, r + j1, also maps into the Γ plane as a circle. As we approach the imaginary axis in the impedance plane, Γ approaches the unit radius circle. As we cross the imaginary axis, the constant reactance circle in the Γ plane goes outside the unit radius circle.

If we now go back and look at z real, we see at z = −1, Γ = ∞. When z is real and less than one, we move out toward the unit radius circle in the Γ plane. When the real part of z goes negative, Γ continues along this circle of infinite radius. The entire region outside the unit radius circle represents impedances with negative real parts. We will use this fact later when working with transistors and other active devices which often have negative real impedances.

In the impedance plane, constant resistance and constant reactance lines intersect. They also cross in the Γ plane. There is a one-to-one correspondence between points in the impedance plane and points in the Γ plane.

The Smith Chart can be completed by continuing to draw other constant resistance and reactance circles (Fig. 31).

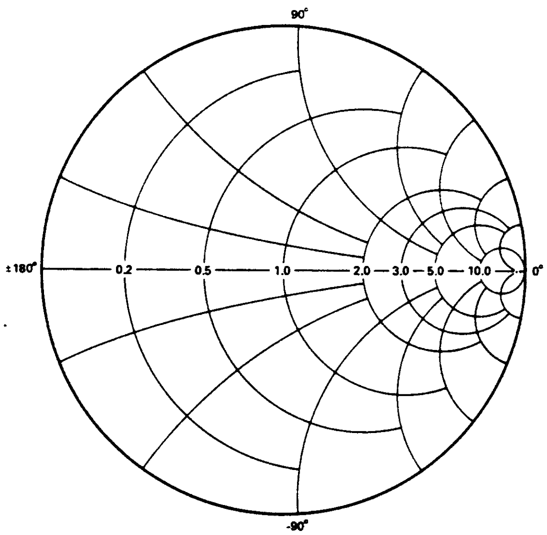
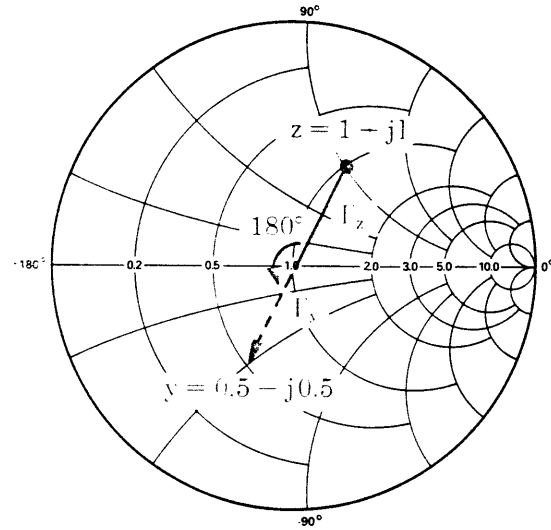


Figure 31



$$\Gamma_z = \frac{z-1}{z+1} \qquad \Gamma_y = \frac{1-y}{1+y} \qquad |y| = \frac{1}{|z|}$$

$$|\Gamma_z| = |\Gamma_y|$$

Figure 32

We can approach this impedance to admittance conversion in another way. Rather than rotate the Γ vector by 180°, we could rotate the Smith Chart by 180° (Fig. 33). We can call the rotated chart an admittance chart and the original an impedance chart. Now we can convert any impedance to admittance, or vice versa, directly.

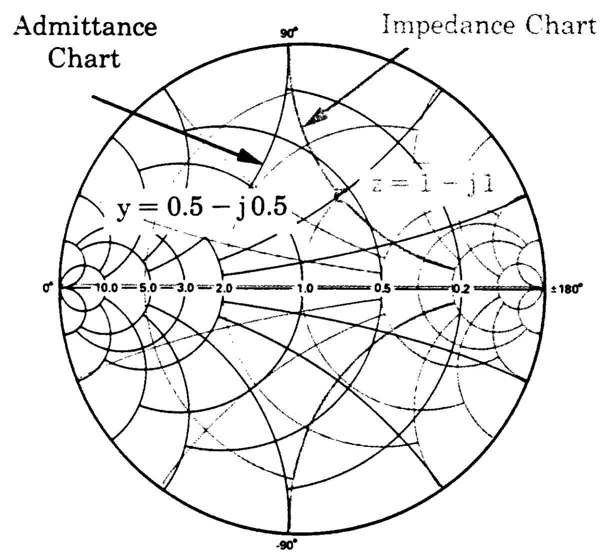


Figure 33

Applications of the Smith Chart

Let's now try a few examples with the Smith Chart to illustrate its usefulness.

1. **Conversion of impedance to admittance:** Converting a normalized impedance of 1 + j1 to an admittance can be accomplished quite easily. Let's first plot the point representing the value of z on the Smith Chart (Fig. 32). From these relationships, we see that while the magnitude of admittance is the reciprocal of the magnitude of impedance, the magnitude of Γ is the same—but its phase angle is changed by 180°. On the Smith Chart, the Γ vector would rotate through 180°. This point could then be read off as an admittance.

2. **Impedances with negative real parts:** Let's now take a look at impedances with negative real parts. Here again is a conventional Smith Chart defined by the boundary of the unit radius circle. If we have an impedance that is inductive with a negative real part, it would map into the Γ plane outside the chart (Fig. 34). One way to bring this point back onto the chart would be to plot the reciprocal of Γ, rather than Γ itself. This would be inconvenient since the phase angle would not be preserved. What was a map of an inductive impedance appears to be capacitive.

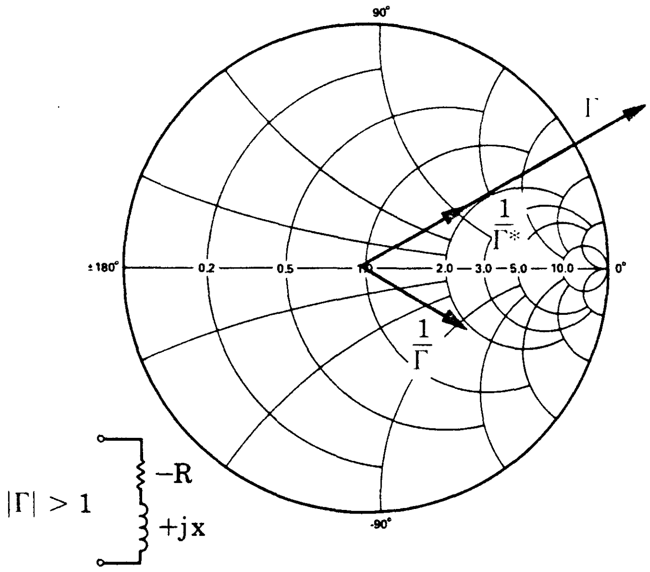


Figure 34

If we plot the reciprocal of the complex conjugate of Γ, however, the phase angle is preserved. This value lies along the same line as the original Γ. Typically in the Hewlett-Packard transistor data sheets, impedances of this type are plotted this way.

There are also compressed Smith Charts available that include the unit radius chart plus a great deal of the negative impedance region. This chart has a radius which corresponds to a reflection coefficient whose magnitude is 3.16 (Fig. 35).

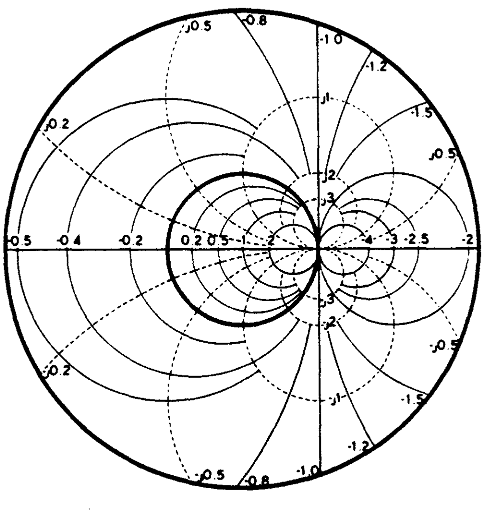


Figure 35

In the rest of this seminar, we will see how easily we can convert measured reflection coefficient data to impedance information by slipping a Smith Chart overlay over the Hewlett-Packard network analyzer polar display.

3. **Frequency response of networks:** One final point needs to be covered in this brief review of the Smith Chart and that is the frequency response for a given network. Let's look at a network having an impedance, z = 0.4 + jx (Fig. 36). As we increase the frequency of the input signal, the impedance plot for the network moves clockwise along a constant resistance circle whose value is 0.4. This generally clockwise movement with increasing frequency is typical of impedance plots on the Smith Chart for passive networks. This is essentially Foster's Reactance Theorem.

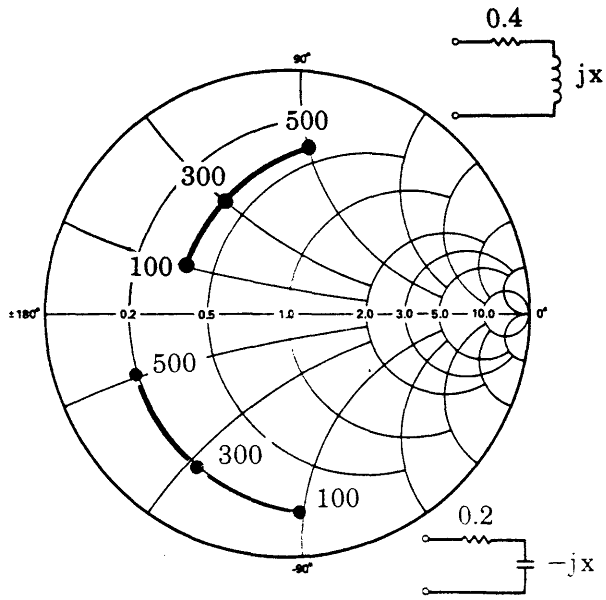


Figure 36

If we now look at another circuit having a real part of 0.2 and an imaginary part that is capacitive, the impedance plot again moves in a clockwise direction with an increase in frequency.

Another circuit that is often encountered is the tank circuit. Here again, the Smith Chart is useful for plotting the frequency response (Fig. 37). For this circuit at zero frequency, the inductor is a short circuit. We start our plot at the point, z = 0. As the frequency increases, the inductive reactance predominates. We move in a clockwise direction. At resonance, the impedance is purely real, having the value of the resistor. If the resistor had a higher value, the cross-over point at resonance would be farther to the right on the Smith Chart. As the frequency continues to increase, the response moves clockwise into the capacitive region of the Smith Chart until we reach infinite frequency, where the impedance is again zero.

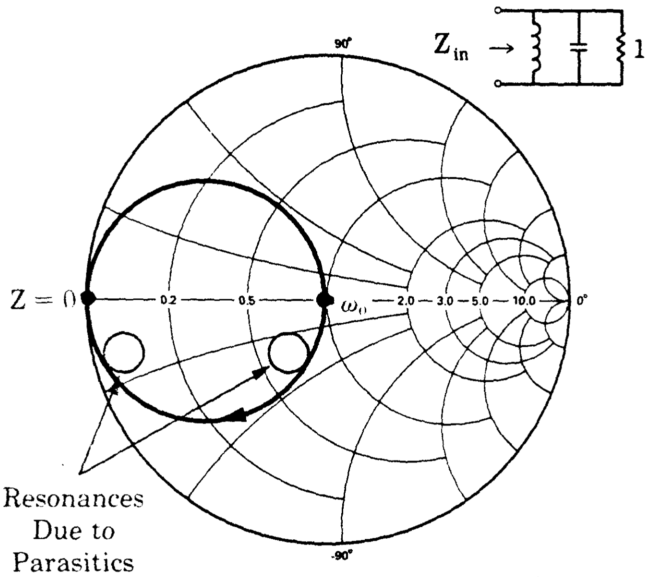
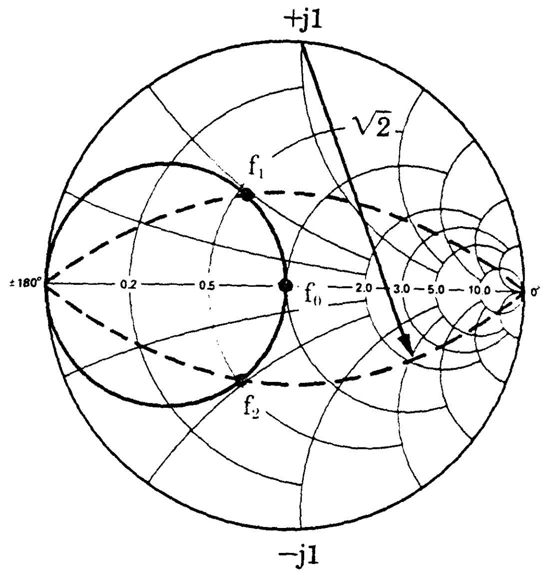


Figure 37



$$Q = \frac{f_0}{f_2 - f_1}$$

Figure 38

In theory, this complete response for a tank circuit would be a circle. In practice, since we do not generally have elements that are pure capacitors or pure inductors over the entire frequency range, we would see other little loops in here that indicate other resonances. These could be due to parasitic inductance in the capacitor or parasitic capacitance in the inductor. The diameter of these circles is somewhat indicative of the Q of the circuit. If we had an ideal tank circuit, the response would be the outer circle on the Smith Chart. This would indicate an infinite Q.

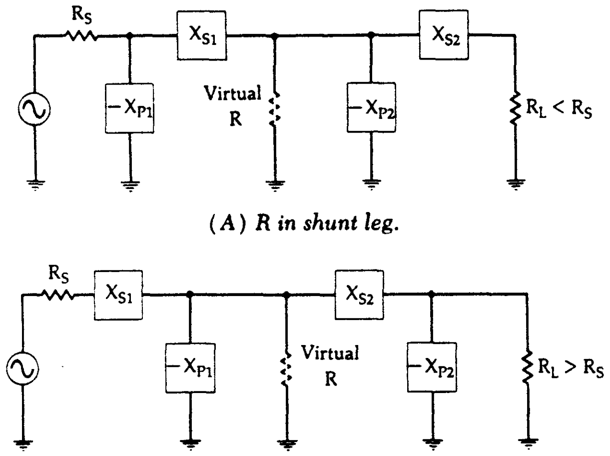
Hewlett-Packard Application Note 117-1 describes other possible techniques for measuring the Q of cavities and YIG spheres using the Smith Chart. One of these techniques uses the fact that with a tank circuit, the real part of the circuit equals the reactive part at the half-power points. Let's draw two arcs connecting these points on the Smith Chart (Fig. 38). The centers for these arcs are at ±j1. The radius of the arcs is $\sqrt{2}$.

We then increase the frequency and record its value where the response lies on the upper arc. Continuing to increase the frequency, we record the resonant frequency and the frequency where the response lies on the lower arc. The formula for the Q of the circuit is simply f_0, the resonant frequency, divided by the difference in frequency between the upper and lower half-power points. $Q = f_0/\Delta f$.

Summary

Let's quickly review what we've seen with the Smith Chart. It is a mapping of the impedance plane and the reflection coefficient or Γ plane. We discovered that impedances with positive real parts map inside the unit radius circle on the Smith Chart. Impedances with negative real parts map outside this unit radius circle. Impedances having positive real parts and inductive reactance map into the upper half of the Smith Chart. Those with capacitive reactance map into the lower half.

In the next part of this S-Parameter Design Seminar, we will continue our discussion of network analysis using S-parameters and flow graph techniques.



(A) *R in shunt leg.*

(B) *R in series leg.*

Fig. 4-25. Two series-connected L networks for lower Q applications.

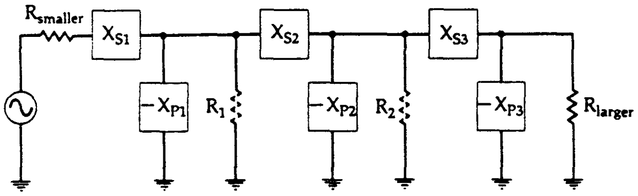


Fig. 4-26. Expanded version of Fig. 4-25 for even wider bandwidths.

resistance needed. Or, to design for an optimally wide bandwidth, solve Equation 4-6 for R. Once R is known, the design is straightforward.

THE SMITH CHART

Probably one of the most useful graphical tools available to the rf circuit designer today is the Smith Chart shown in Fig. 4-27. The chart was originally conceived back in the Thirties by a Bell Laboratories engineer named Phillip Smith, who wanted an easier method of solving the tedious repetitive equations that often appear in rf theory. His solution, appropriately named the Smith Chart, is still widely in use.

At first glance, a Smith Chart appears to be quite complex. Indeed, why would anyone of sound mind even care to look at such a chart? The answer is really quite simple; once the Smith Chart and its uses are understood, the rf circuit designer's job becomes much less tedious and time consuming. Very lengthy complex equations can be solved graphically on the chart in seconds, thus lessening the possibility of errors creeping into the calculations.

Smith Chart Construction

The mathematics behind the construction of a Smith Chart are given here for those that are interested. It is important to note, however, that you do not *need* to know or understand the mathematics surrounding the actual construction of a chart as long as you understand what the chart represents and how it can be used to your advantage. Indeed, there are so many uses for the chart that an entire volume has been written on the subject. In this chapter, we will concentrate mainly on the Smith Chart as an impedance matching tool and other uses will be covered in later chapters. The mathematics follow.

The reflection coefficient of a load impedance when given a source impedance can be found by the formula:

$$\rho = \frac{Z_s - Z_L}{Z_s + Z_L} \qquad \text{(Step 1)}$$

In normalized form, this equation becomes:

$$\rho = \frac{Z_o - 1}{Z_o + 1} \qquad \text{(Step 2)}$$

where Z_o is a complex impedance of the form R +jX.

The polar form of the reflection coefficient can also be represented in rectangular coordinates:

$$\rho = p + jq$$

Substituting into Step 2, we have:

$$p + jq = \frac{R + jX - 1}{R + jX + 1} \qquad \text{(Step 3)}$$

If we solve for the real and imaginary parts of p +jq, we get:

$$p = \frac{R^2 - 1 + X^2}{(R+1)^2 + X^2} \qquad \text{(Step 4)}$$

and,

$$q = \frac{2x}{(R+1)^2 + X^2} \qquad \text{(Step 5)}$$

Solve Step 5 for X:

$$X = \left(\frac{p(R+1)^2 - R^2 + 1}{1 - p}\right)^{½} \qquad \text{(Step 6)}$$

Then, substitute Step 6 into Step 5 to obtain:

$$\left(p - \frac{R}{R+1}\right)^2 + q^2 = \left(\frac{1}{R+1}\right)^2 \qquad \text{(Step 7)}$$

Step 7 is the equation for a family of circles whose centers are at:

$$p = \frac{R}{R+1}$$

$$q = 0$$

and whose radii are equal to:

$$\frac{1}{R+1}$$

These are the constant resistance circles, some of which are shown in Fig. 4-28A.

Similarly, we can eliminate R from Steps 4 and 5 to obtain:

$$(p-1)^2 + \left(q - \frac{1}{X}\right)^2 = \left(\frac{1}{X}\right)^2 \qquad \text{(Step 8)}$$

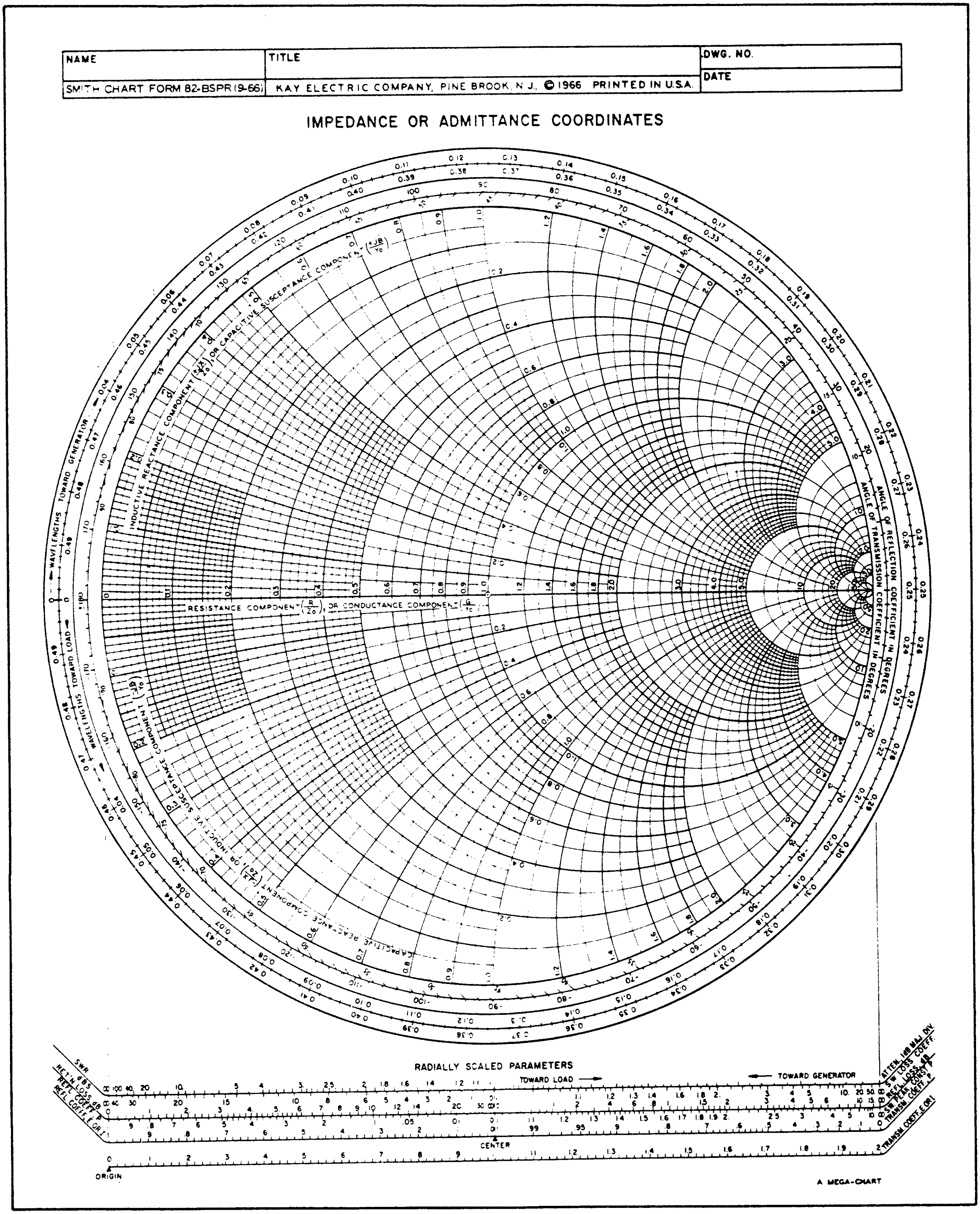
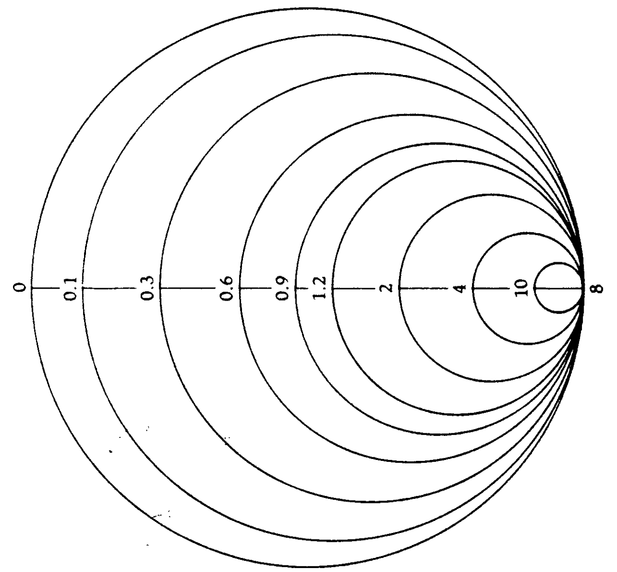
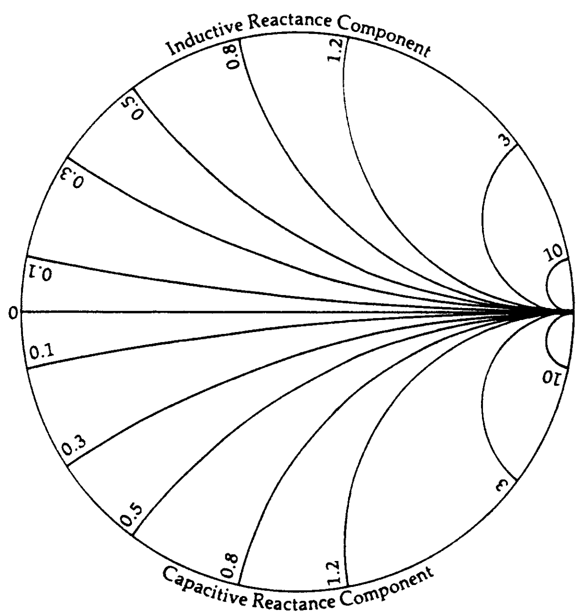


Fig. 4-27. The Smith Chart. (*Courtesy Analog Instruments Co.*)



(A) Constant resistance circles.



(B) Constant reactance circles.

Fig. 4-28. Smith Chart construction.

which represents a family of circles with centers at $p = 1, V = \frac{1}{X}$, and radii of $\frac{1}{X}$. These circles are shown plotted on the p, jq axis in Fig. 4-28B.

As the preceding mathematics indicate, the Smith Chart is basically a combination of a family of circles and a family of arc of circles—the centers and radii of which can be calculated using the equations given (Steps 1 through 8). Fig. 4-28 shows the chart broken down into these two families. The circles of Fig. 4-28A are known as *constant resistance circles*. Each point on a constant resistance circle has the same *resistance* as any other point on the circle. The arcs of circles shown in Fig. 4-28B are known as *constant reactance circles*, as each point on a circle has the same *reactance* as any other point on that circle. These circles are centered off of the chart and, therefore, only a small portion of each is contained within the boundary of the chart. All arcs above the centerline of the chart represent +jX, or inductive reactances, and all arcs below the centerline represent −jX, or capacitive reactances. The centerline must, therefore, represent an axis where X = 0 and is, therefore, called the *real axis*.

Notice in Fig. 4-28A that the "constant resistance = 0" circle defines the outer boundary of the chart. As the resistive component increases, the radius of each circle decreases and the center of each circle moves toward the right on the chart. Then, at infinite resistance, you end up with an infinitely small circle that is located at the extreme right-hand side of the chart. A similar thing happens for the constant reactance circles shown in Fig. 4-28B. As the magnitude of the reactive component increases (−jX or +jX), the radius of each circle decreases, and the center of each circle moves closer and closer to the extreme right side of the chart. Infinite resistance and infinite reactance are thus represented by the same point on the chart.

Since the outer boundary of the chart is defined as the "R = 0" circle, with higher values of R being contained within the chart, it follows then that any point outside of the chart must contain a negative resistance. The concept of negative resistance is useful in the study of oscillators and it is mentioned here only to state that the concept does exist, and if needed, the Smith Chart can be expanded to deal with it.

When the two charts of Fig. 4-28 are incorporated into a single version, the Smith Chart of Fig. 4-29 is born. If we add a few peripheral scales to aid us in other rf design tasks, such as determining *standing wave ratio* (*SWR*), *reflection coefficient*, and *transmission loss* along a transmission line, the basic chart of Fig. 4-27 is completed.

Plotting Impedance Values

Any point on the Smith Chart represents a *series* combination of resistance and reactance of the form Z = R + jX. Thus, to locate the impedance Z = 1 + j1, you would find the R = 1 constant resistance circle and follow it until it crossed the X = 1 constant reactance circle. The junction of these two circles would then represent the needed impedance value. This particular point, shown in Fig. 4-30, is located in the upper half of the chart because X is a positive reactance or an inductor. On the other hand, the point 1 − j1 is located in the *lower* half of the chart because, in this instance, X is a negative quantity and represents a capacitor. Thus, the junction of the R = 1 constant resistance circle and the X = −1 constant reactance circle defines that point.

In general, then, to find any *series* impedance of the form R ± jX on a Smith Chart, you simply find the junction of the R = constant and X = constant circles.

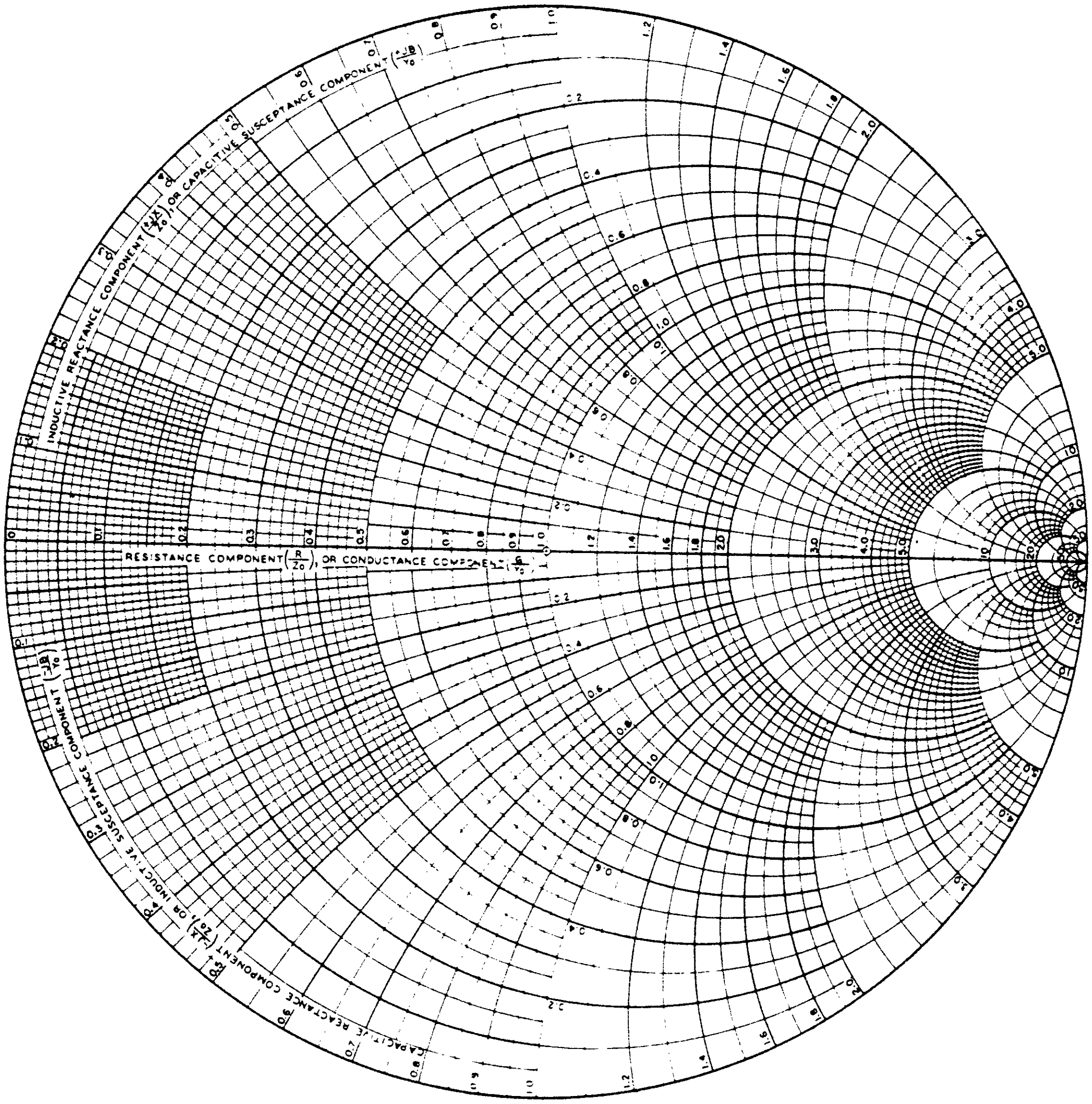


Fig. 4-29. The basic Smith Chart.

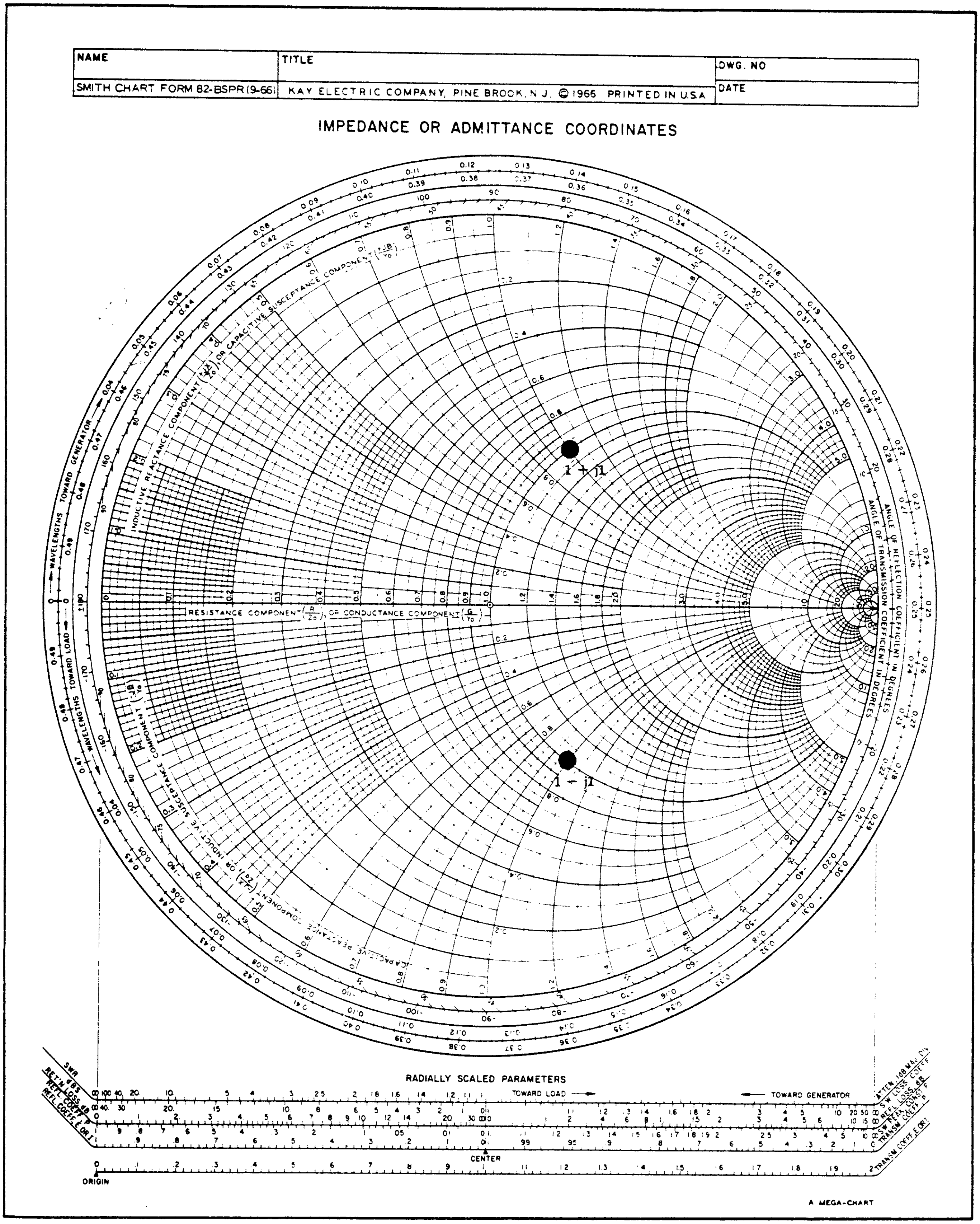


Fig. 4-30. Plotting impedances on the chart.

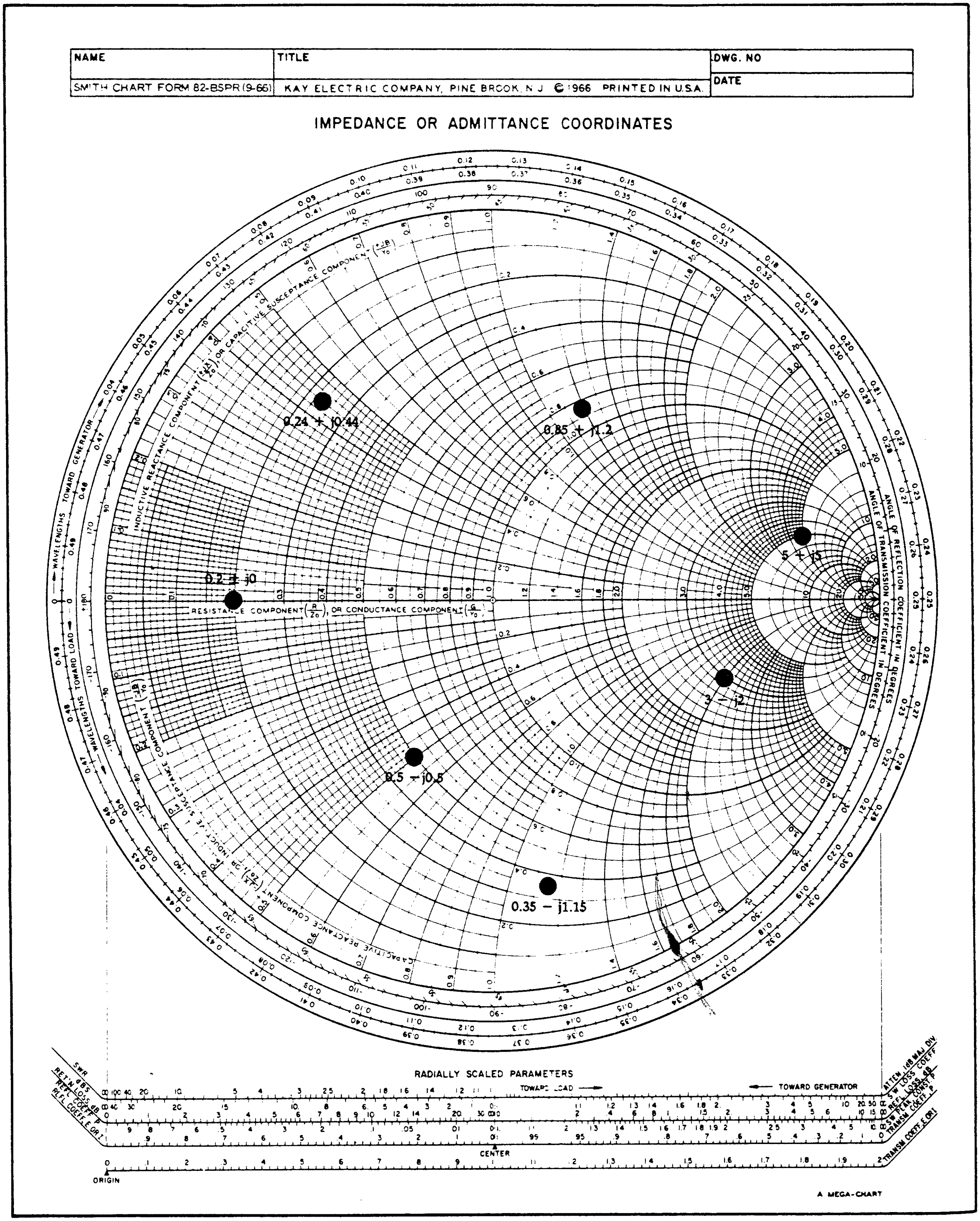


Fig. 4-31. More impedances are plotted on the chart.

In many cases, the actual circles will not be present on the chart and you will have to interpolate between two that are shown. Thus, plotting impedances and, therefore, any manipulation of those impedances must be considered an inexact procedure which is subject to "pilot error." Most of the time, however, the error introduced by subjective judgements on the part of the user, in plotting impedances on the chart, is so small as to be negligible for practical work. Fig. 4-31 shows a few more impedances plotted on the chart.

Notice that all of the impedance values plotted in Fig. 4-31 are very small numbers. Indeed, if you try to plot an impedance of $Z = 100 + j150$ ohms, you will not be able to do it accurately because the $R = 100$ and $X = 150$ ohm circles would be (if they were drawn) on the extreme right edge of the chart—very close to infinity. In order to facilitate the plotting of larger impedances, *normalization* must be used. That is, each impedance to be plotted is divided by a convenient number that will place the new *normalized* impedance near the center of the chart where increased accuracy in plotting is obtained. Thus, for the preceding example, where $Z = 100 + j150$ ohms, it would be convenient to divide Z by 100, which yields the value $Z = 1 + j1.5$. This is very easily found on the chart. Once a chart is normalized in this manner, all impedances plotted on that chart *must be* divided by the *same* number in the normalization process. Otherwise, you will be left with a bunch of impedances with which nothing can be done.

Impedance Manipulation on the Chart

Fig. 4-32 graphically indicates what happens when a series capacitive reactance of $-j1.0$ ohm is added to an impedance of $Z = 0.5 + j0.7$ ohm. Mathematically, the result is

$$\begin{aligned} Z &= 0.5 + j0.7 - j1.0 \\ &= 0.5 - j0.3 \text{ ohm} \end{aligned}$$

which represents a series RC quantity. Graphically, what we have done is move *downward* along the $R =$ 0.5-ohm constant resistance circle for a distance of $X = -j1.0$ ohm. This is the plotted impedance point of $Z = 0.5 - j0.3$ ohm, as shown. In a similar manner, as shown in Fig. 4-33, adding a series inductance to a plotted impedance value simply causes a move *upward* along a constant resistance circle to the new impedance value. This type of construction is very important in the design of impedance-matching networks using the Smith Chart and must be understood. In general then, the addition of a series capacitor to an impedance moves that impedance *downward* (counterclockwise) along a constant resistance circle for a distance that is equal to the reactance of the capacitor. The addition of any series inductor to a plotted impedance moves that impedance *upward* (clockwise) along a constant resistance circle for a distance that is equal to the reactance of the inductor.

Conversion of Impedance to Admittance

The Smith Chart, although described thus far as a family of impedance coordinates, can easily be used to convert any impedance (Z) to an admittance (Y), and vice-versa. In mathematical terms, an admittance is simply the inverse of an impedance, or

$$Y = \frac{1}{Z} \qquad \text{(Eq. 4-9)}$$

where, the admittance (Y) contains both a real and an imaginary part, similar to the impedance (Z). Thus,

$$Y = G \pm jB \qquad \text{(Eq. 4-10)}$$

where,
G = the conductance in mhos,
B = the susceptance in mhos.

The circuit representation is shown in Fig. 4-34. Notice that the *susceptance is positive for a capacitor and negative for an inductor,* whereas, for reactance, the opposite is true.

To find the inverse of a series impedance of the form $Z = R + jX$ mathematically, you would simply use Equation 4-9 and perform the resulting calculation. But, how can you use the Smith Chart to perform the calculation for you without the need for a calculator? The easiest way of describing the use of the chart in performing this function is to first work a problem out mathematically and, then, plot the results on the chart to see how the two functions are related. Take, for example, the series impedance $Z = 1 + j1$. The inverse of Z is:

$$\begin{aligned} Y &= \frac{1}{1 + j1} \\ &= \frac{1}{1.414 \angle 45^\circ} \\ &= 0.7071 \angle -45^\circ \\ &= 0.5 - j0.5 \text{ mho} \end{aligned}$$

If we plot the points $1 + j1$ and $0.5 - j0.5$ on the Smith Chart, we can easily see the graphical relationship between the two. This construction is shown in Fig. 4-35. Notice that the two points are located at exactly the same distance (d) from the center of the chart but in opposite directions (180°) from each other. Indeed, the same relationship holds true for *any* impedance and its inverse. Therefore, without the aid of a calculator, you can find the reciprocal of an impedance or an admittance by simply plotting the point on the chart, measuring the distance (d) from the center of the chart to that point, and, then, plotting the measured result the same distance from the center but in the opposite direction (180°) from the original point. This is a very simple construction technique that can be done in seconds.

Another approach that we could take to achieve the same result involves the manipulation of the actual chart rather than the performing of a construc-

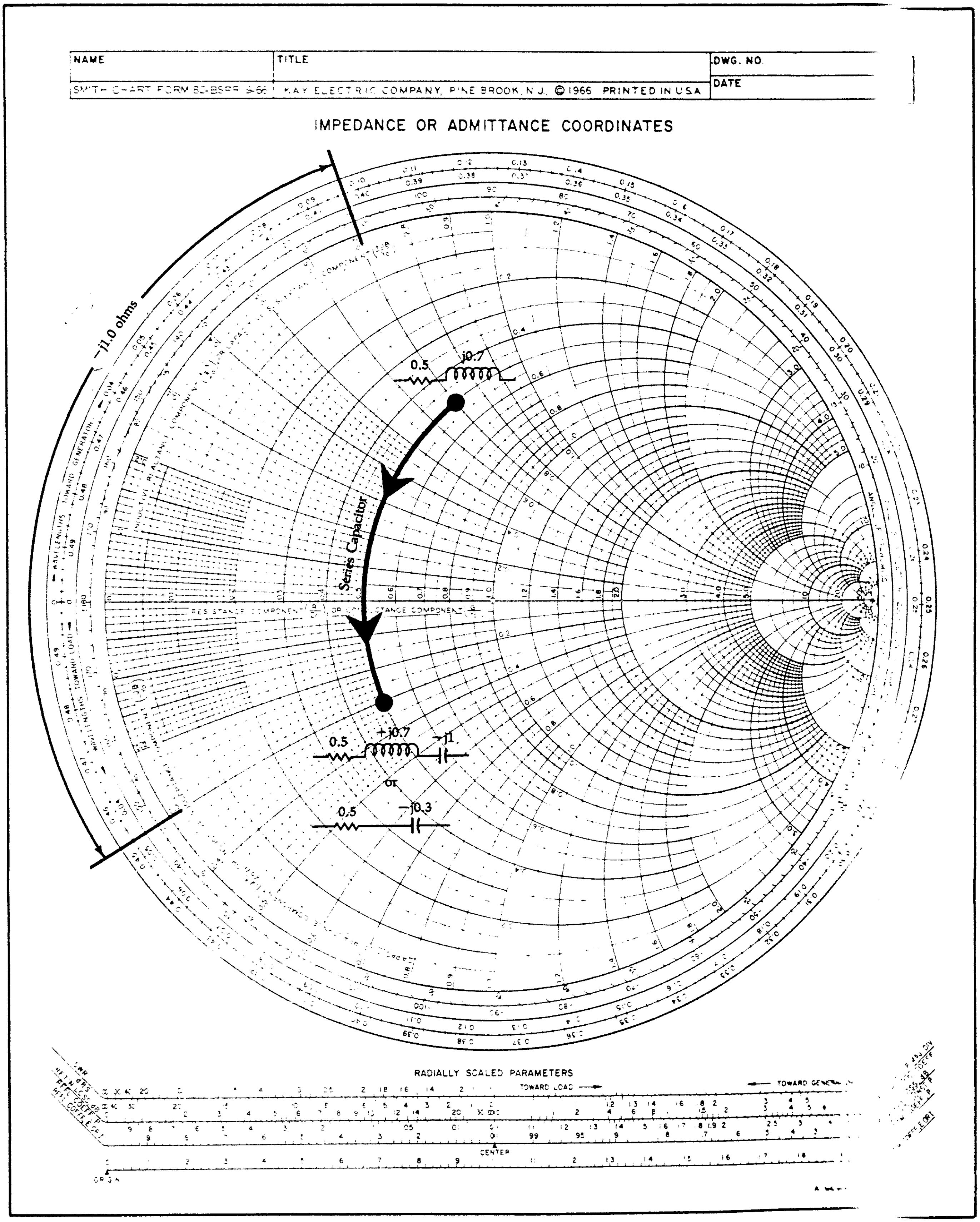


Fig. 4-32. Addition of a series capacitor.

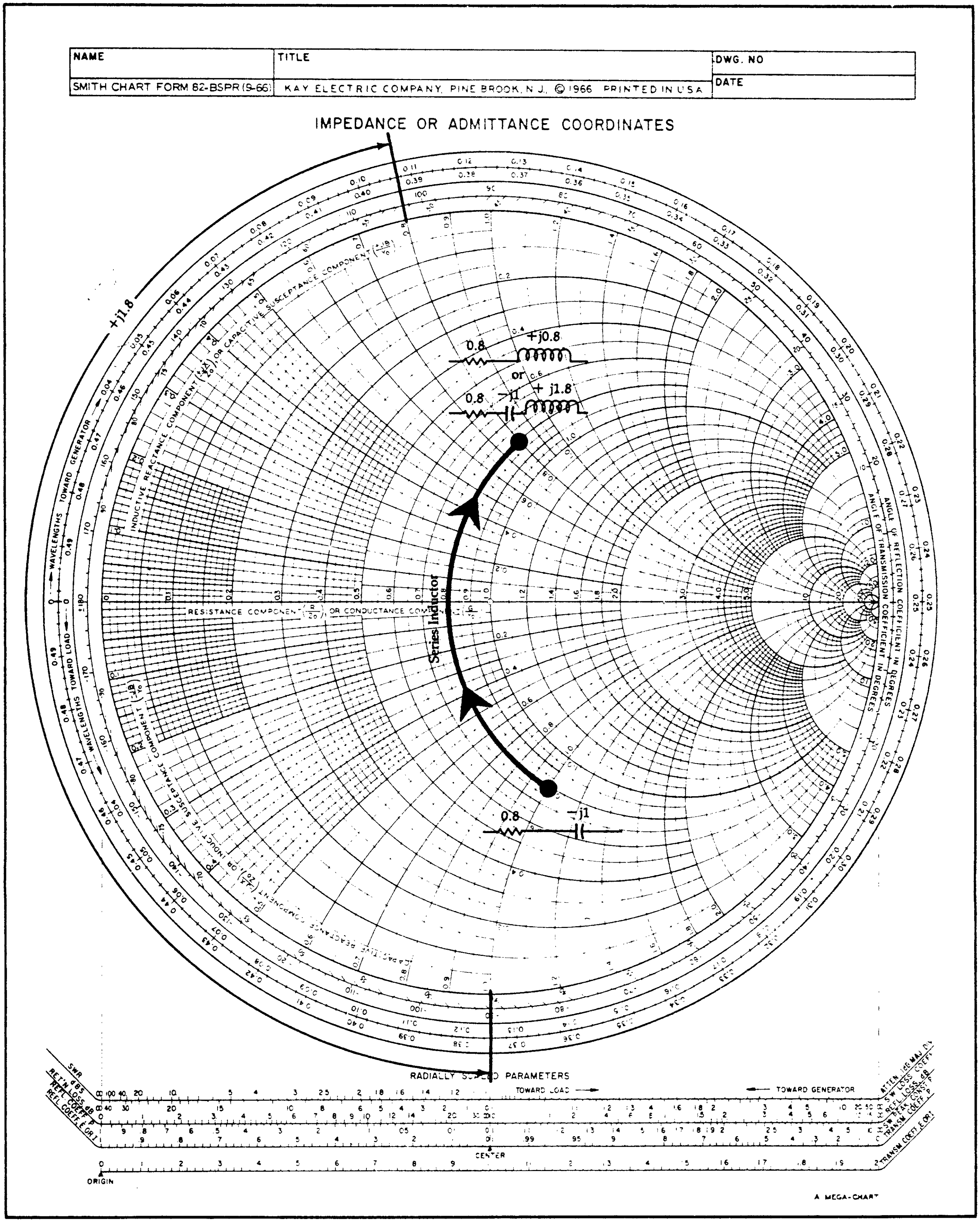


Fig. 4-33. Addition of a series inductor.

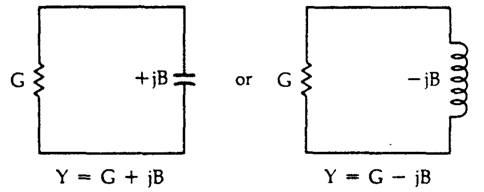


Fig. 4-34. Circuit representation for admittance.

tion on the chart. For instance, rather than locating a point 180° away from our original starting point, why not just rotate the chart itself 180° while fixing the starting point in space? The result is the same, and it can be read directly off of the rotated chart without performing a single construction. This is shown in Fig. 4-36 (Smith Chart Form ZY-01-N)* where the rotated chart is shown in black. Notice that the impedance plotted (solid lines on the red coordinates) is located at $Z = 1 + j1$ ohms, and the reciprocal of that (the admittance) is shown by dotted lines on the black coordinates as $Y = 0.5 - j0.5$. Keep in mind that because we have rotated the chart 180° to obtain the admittance coordinates, the upper half of the admittance chart represents *negative susceptance* ($-jB$) which is *inductive*, while the lower half of the admittance chart represents a *positive susceptance* ($+jB$) which is *capacitive*. Therefore, nothing has been lost in the rotation process.

The chart shown in Fig. 4-36, containing the superimposed impedance and admittance coordinates, is an extremely useful version of the Smith Chart and is the one that we will use throughout the remainder of the book. But first, let's take a closer look at the admittance coordinates alone.

Admittance Manipulation on the Chart

Just as the impedance coordinates of Figs. 4-32 and 4-33 were used to obtain a visual indication of what occurs when a *series* reactance is added to an *impedance*, the admittance coordinates provide a visual indication of what occurs when a *shunt* element is added to an *admittance*. The addition of a shunt capacitor is shown in Fig. 4-37. Here we begin with an admittance of $Y = 0.2 - j0.5$ mho and add a shunt capacitor with a susceptance (reciprocal of reactance) of $+j0.8$ mho. Mathematically, we know that parallel susceptances are simply added together to find the equivalent susceptance. When this is done, the result becomes:

$$\begin{aligned} Y &= 0.2 - j0.5 + j0.8 \\ &= 0.2 + j0.3 \text{ mho} \end{aligned}$$

If this point is plotted on the admittance chart, we quickly recognize that all we have done is to move along a constant conductance circle (G) *downward* (clockwise) a distance of $jB = 0.8$ mho. In other words, the real part of the admittance has not changed, only the imaginary part has. Similarly, as Fig. 4-38 indicates, adding a shunt inductor to an admittance moves the point along a constant conductance circle upward (counterclockwise) a distance ($-jB$) equal to the value of its susceptance.

If we again superimpose the impedance and admittance coordinates and combine Figs. 4-32, 4-33, 4-37, and 4-38 for the general case, we obtain the useful chart shown in Fig. 4-39. This chart graphically illustrates the direction of travel, along the impedance and admittance coordinates, which results when the particular type of component that is indicated is added to an existing impedance or admittance. A simple example should illustrate the point (Example 4-6).

IMPEDANCE MATCHING ON THE SMITH CHART

Because of the ease with which series and shunt components can be added in ladder-type arrangements on the Smith Chart, while easily keeping track of the impedance as seen at the input terminals of the structure, the chart seems to be an excellent candidate for an impedance-matching tool. The idea here is simple. Given a load impedance and given the impedance that the source would like to see, simply plot the load impedance and, then, begin adding series and shunt elements on the chart until the desired impedance is achieved—just as was done in Example 4-6.

Two-Element Matching

Two-element matching networks are mathematically very easy to design using the formulas provided in earlier sections of this chapter. For the purpose of illustration, however, let's begin our study of a Smith Chart impedance-matching procedure with the simple network given in Example 4-7.

To make life much easier for you as a Smith Chart user, the following equations may be used. For a series-C component:

$$C = \frac{1}{\omega XN} \qquad \text{(Eq. 4-11)}$$

For a series-L component:

$$L = \frac{XN}{\omega} \qquad \text{(Eq. 4-12)}$$

For a shunt-C component:

$$C = \frac{B}{\omega N} \qquad \text{(Eq. 4-13)}$$

For a shunt-L component:

$$L = \frac{N}{\omega B} \qquad \text{(Eq. 4-14)}$$

where,

$\omega = 2\pi f$,

X = the reactance as read from the chart,

* Smith Chart Form ZY-01-N is a copyright of Analog Instruments Company, P.O. Box 808, New Providence, NJ 07974. It and other Smith Chart accessories are available from the company.

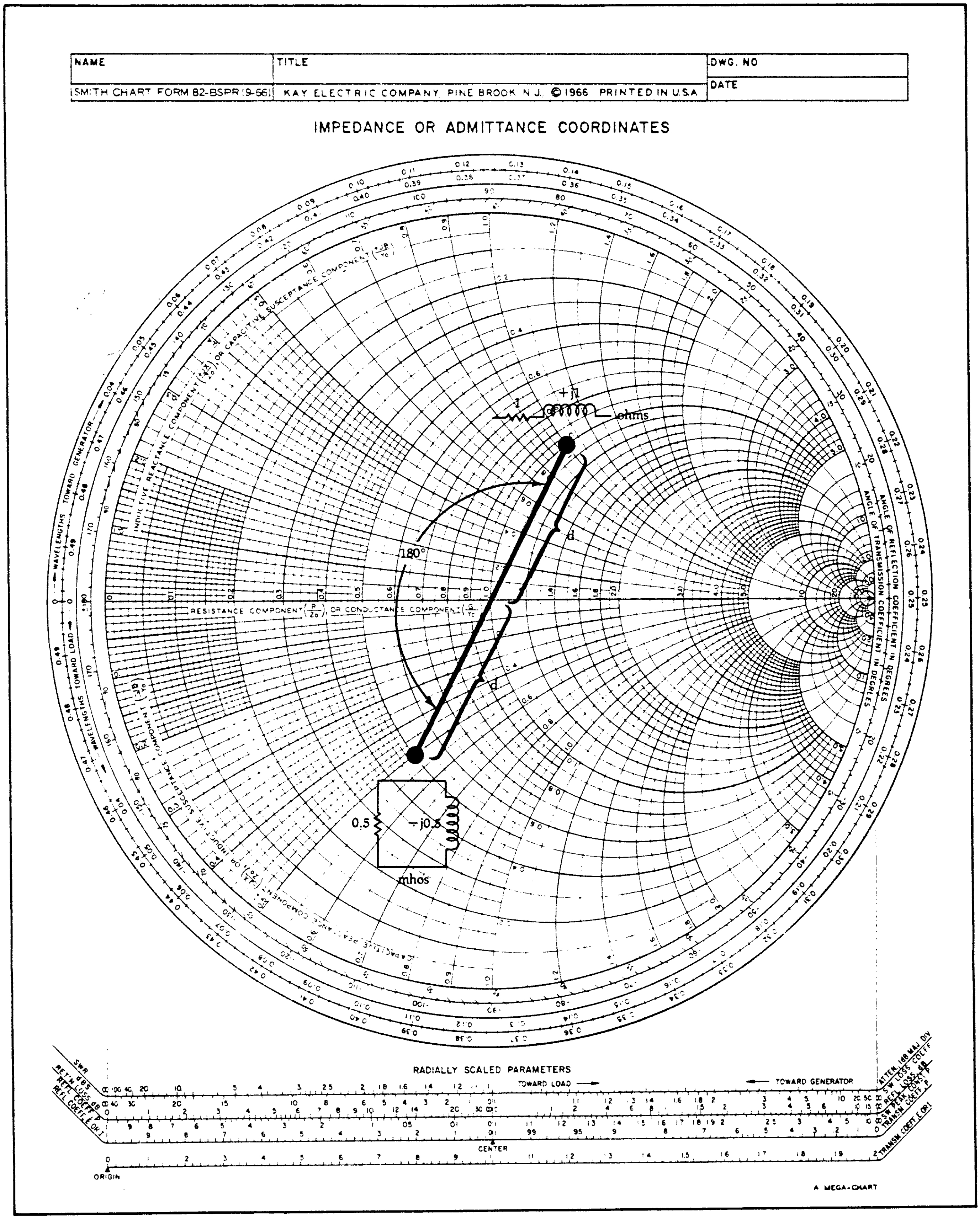


Fig. 4-35. Impedance-admittance conversion on the Smith Chart.

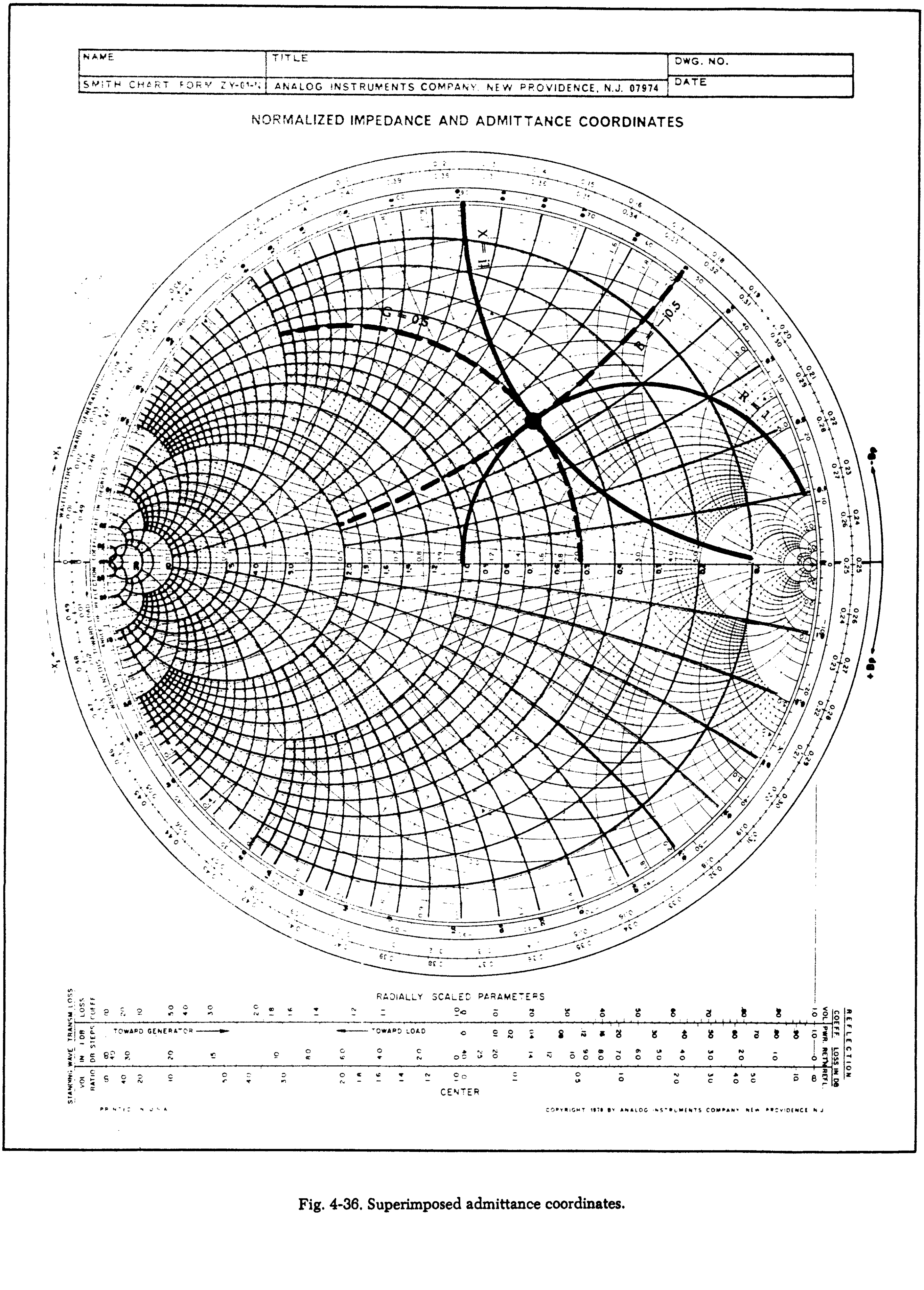


Fig. 4-36. Superimposed admittance coordinates.

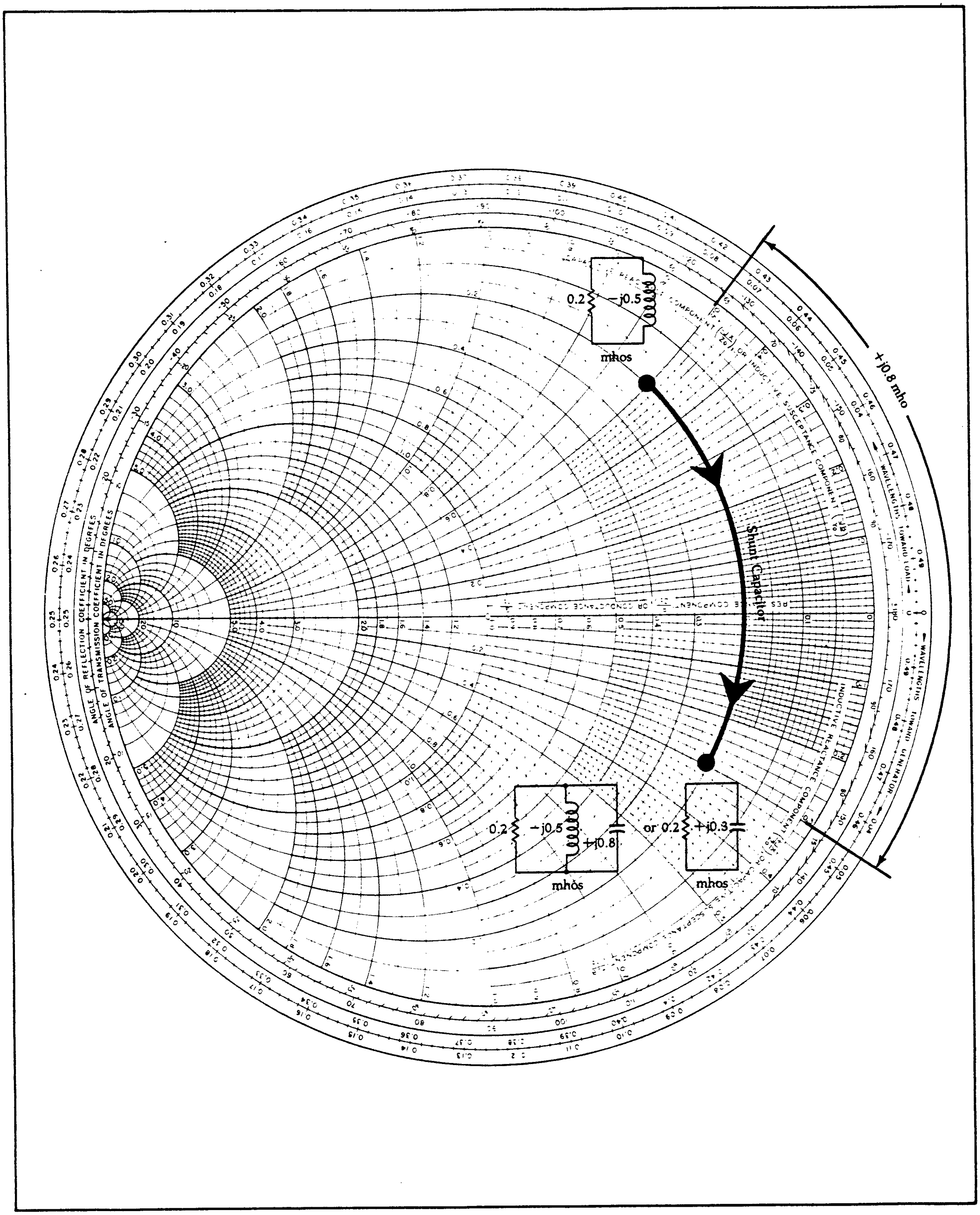


Fig. 4-37. Addition of a shunt capacitor.

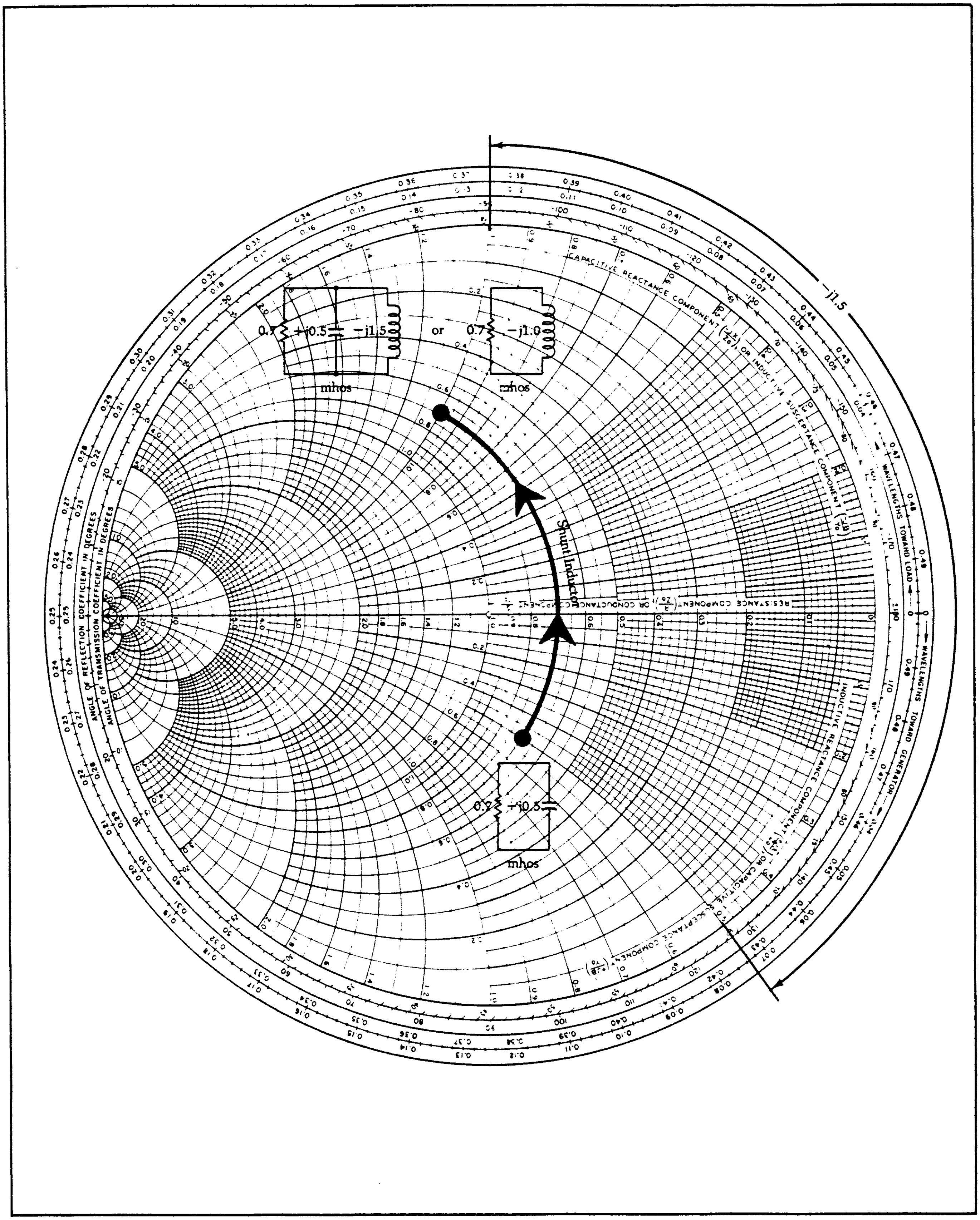


Fig. 4-38. Addition of a shunt inductor.

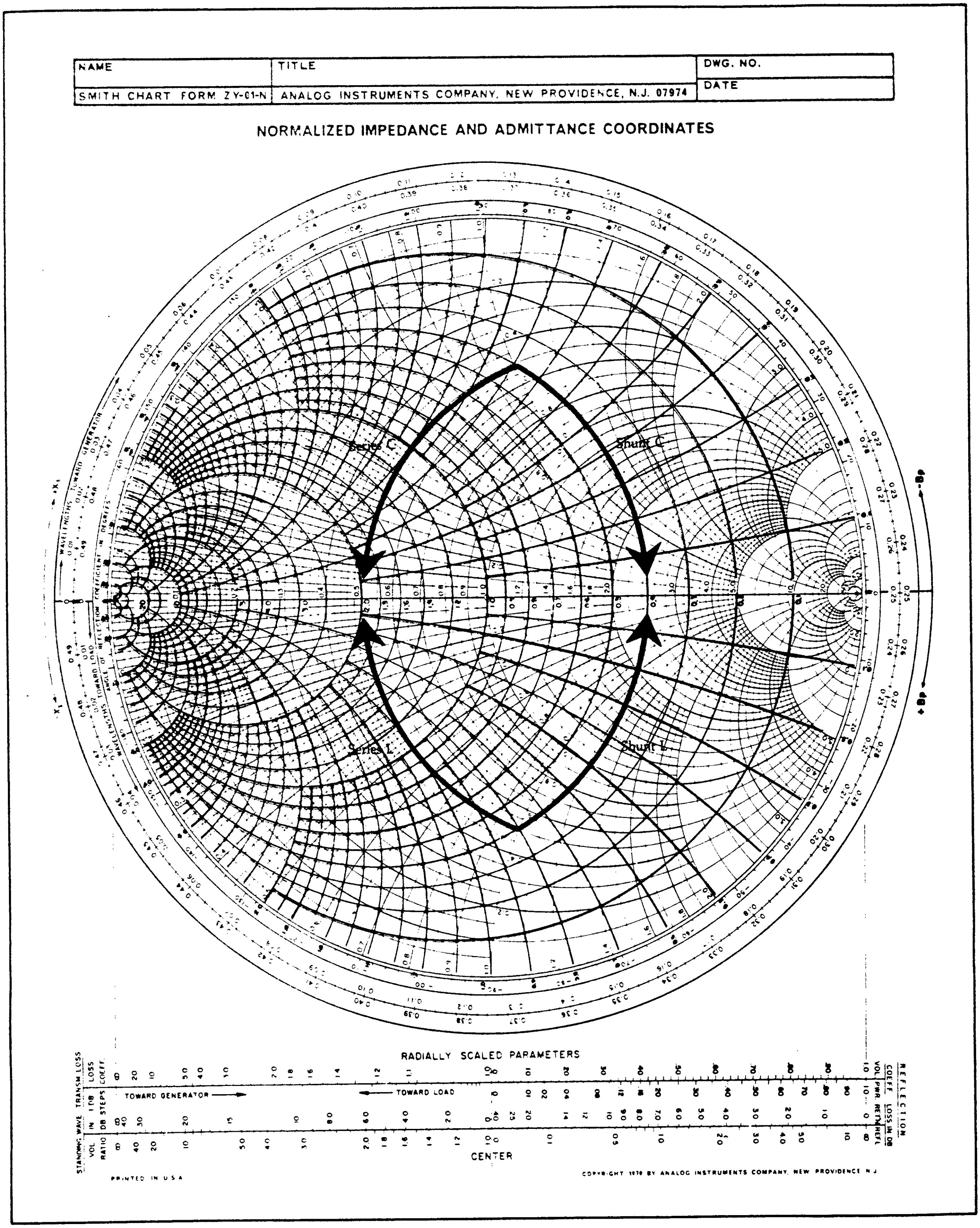


Fig. 4-39. Summary of component addition on a Smith Chart.

B = the susceptance as read from the chart,
N = the number used to normalize the original impedances that are to be matched.

If you use the preceding equations, you will never have to worry about changing susceptances into reactances before unnormalizing the impedances. The equations take care of both operations. The only thing you have to do is read the value of susceptance (for shunt components) or reactance (for series components) directly off of the chart, plug this value into the equation used, and wait for your actual component values to pop out.

Three-Element Matching

In earlier sections of this chapter, you learned that the only real difference between two-element and three-element matching is that with three-element matching, you are able to choose the loaded Q for the network. That was easy enough to do in a mathematical-design approach due to the virtual resistance concept. But how can circuit Q be represented on a Smith Chart?

As you have seen before, in earlier chapters, the Q of a series-impedance circuit is simply equal to the ratio of its reactance to its resistance. Thus, any point on a Smith Chart has a Q associated with it. Alternately, if you were to specify a certain Q, you could find an infinite number of points on the chart that could satisfy that Q requirement. For example, the following impedances located on a Smith Chart have a Q of 5:

$$R + jX = 1 \pm j5$$
$$= 0.5 \pm j2.5$$
$$= 0.2 \pm j1$$
$$= 0.1 \pm j0.5$$
$$= 0.05 \pm j0.25$$

These values are plotted in Fig. 4-45 and form the arcs shown. Thus, any impedance located on these arcs must have a Q of 5. Similar arcs for other values of Q can be drawn with the arc of infinite Q being located along the perimeter of the chart and the Q = 0 arc (actually a straight line) lying along the pure resistance line located at the center of the chart.

The design of high-Q three-element matching networks on a Smith Chart is approached in much the same manner as in the mathematical methods presented earlier in this chapter. Namely, one branch of the network will determine the loaded Q of the circuit, and it is this branch that will set the characteristics of the rest of the circuit.

The procedure for designing a three-element impedance-matching network for a specified Q is summarized as follows:

1. Plot the constant-Q arcs for the specified Q.
2. Plot the load impedance and the complex conjugate of the source impedance.
3. Determine the end of the network that will be used to establish the loaded Q of the design. For T networks, the end with the *smaller* terminating resistance determines the Q. For Pi networks, the end with the *larger* terminating resistor sets the Q.
4. For T networks:

$$R_s > R_L$$

EXAMPLE 4-6

What is the impedance looking into the network shown in Fig. 4-40? Note that the task has been simplified due to the fact that shunt susceptances are shown rather than shunt reactances.

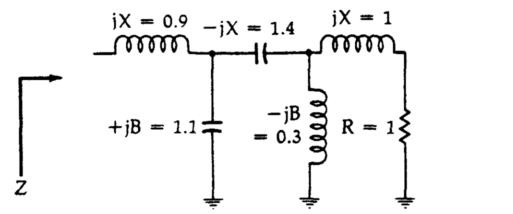


Fig. 4-40. Circuit for Example 4-6.

Solution

This problem is very easily handled on a Smith Chart and not a single calculation needs to be performed. The solution is shown in Fig. 4-42. It is accomplished as follows.

First, break the circuit down into individual branches as shown in Fig. 4-41. Plot the impedance of the series RL branch where Z = 1 + j1 ohm. This is point A in Fig. 4-42. Next, following the rules diagrammed in Fig. 4-39, begin adding each component back into the circuit—one at a time. Thus, the following constructions (Fig. 4-42) should be noted:

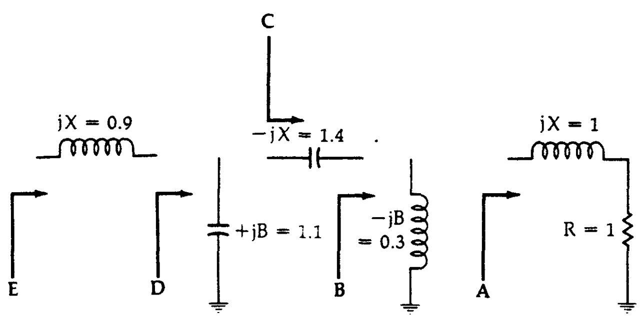


Fig. 4-41. Circuit is broken down into individual branch elements.

Arc AB = shunt L = −jB = 0.3 mho
Arc BC = series C = −jX = 1.4 ohms
Arc CD = shunt C = +jB = 1.1 mhos
Arc DE = series L = +jX = 0.9 ohm

The impedance at point E (Fig. 4-42) can then be read directly off of the chart as Z = 0.2 + j0.5 ohm.

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EXAMPLE 4-6—Cont.

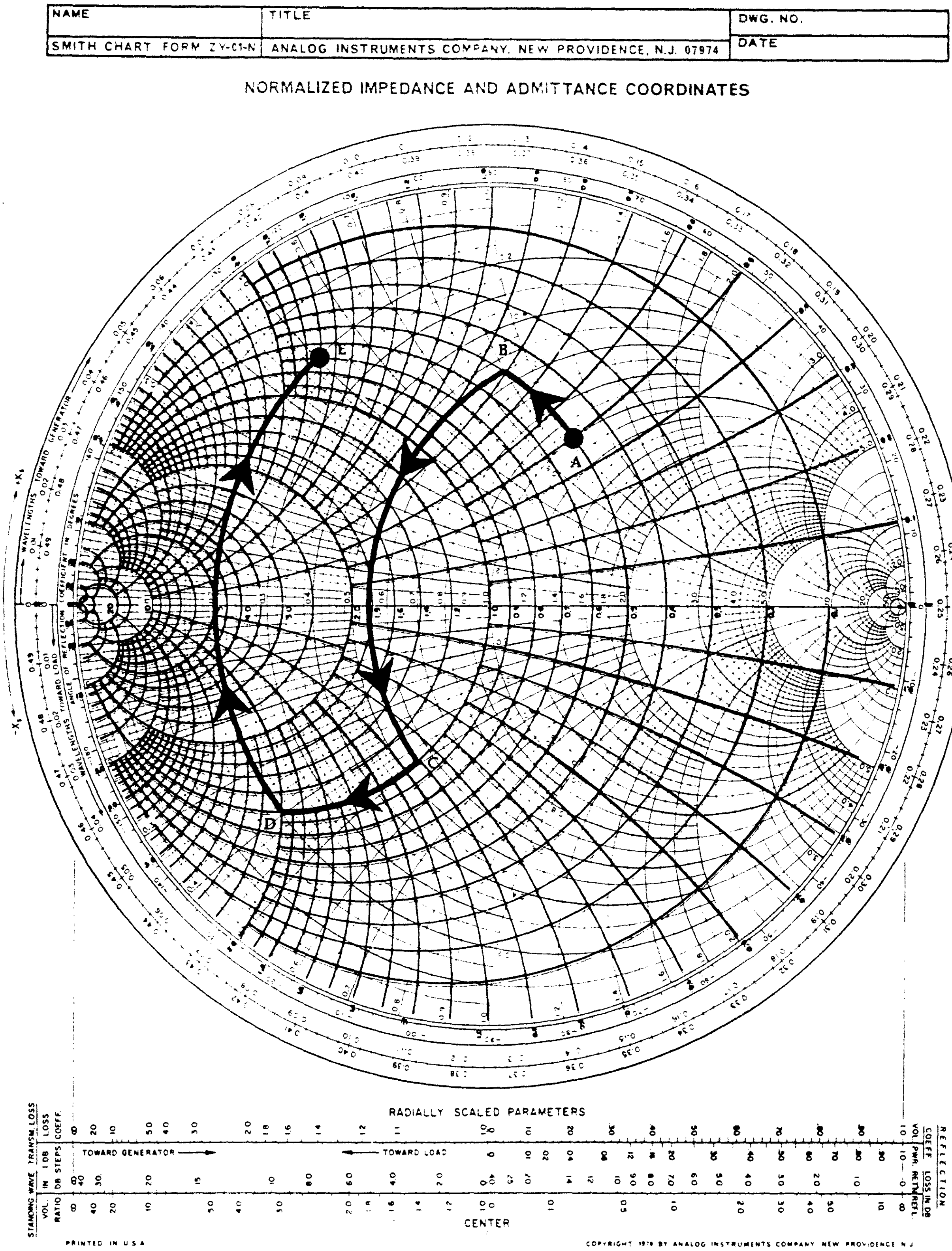

Fig. 4-42. Smith Chart solution for Example 4-6.

Move from the load along a constant-R circle (series element) and intersect the Q curve. The length of this move determines your first element. Then, proceed from this point to Z_s^* ($Z_s^* = Z_s$ conjugate) in two moves—first with a shunt and, then, with a series element.

$$R_s < R_L$$

Find the intersection (I) of the Q curve and the source impedance's R = constant circle, and plot that point. Move *from the load impedance* to point I with two elements—first, a series element and, then, a shunt element. Move from point I to Z_s^* along the R = constant circle with another series element.

5. For Pi networks:

$$R_S > R_L$$

Find the intersection (I) of the Q curve and the source impedance's G = constant circle, and plot that point. Move from the load impedance to point I with two elements—first, a shunt element and, then, a series element. Move from point I to Z_s^* along the G = constant circle with another shunt element.

$$R_S < R_L$$

Move from the load along a constant G circle (shunt element) and intersect the Q curve. The length of this move determines your first element. Then, proceed from this point to Z_s^* in two moves—first, with a series element and, then, with a shunt element.

The above procedures might seem complicated to the neophyte but remember that we are only forcing the constant-resistance or constant-conductance arc, located between the Q-determining termination and the specified-Q curve, to be one of our matching elements. An example may help to clarify matters (Example 4-8).

Multielement Matching

In multielement matching networks where there is no Q constraint, the Smith Chart becomes a veritable

EXAMPLE 4-7

Design a two-element impedance-matching network on a Smith Chart so as to match a 25 − j15-ohm source to a 100 − j25-ohm load at 60 MHz. The matching network must also act as a low-pass filter between the source and the load.

Solution

Since the source is a complex impedance, it wants to "see" a load impedance that is equal to its complex conjugate (as discussed in earlier sections of this chapter). Thus, the task before us is to force the 100 − j25-ohm load to look like an impedance of 25 + j15 ohms.

Obviously, the source and load impedances are both too large to plot on the chart, so normalization is necessary. Let's choose a convenient number (N = 50) and divide all impedances by this number. The results are 0.5 + j0.3 ohm for the impedance the source would like to see and 2 − j0.5 ohms for the actual load impedance. These two values are easily plotted on the Smith Chart, as shown in Fig. 4-44, where, at point A, Z_L is the *normalized* load impedance and, at point C, Z_s^* is the *normalized* complex conjugate of the source impedance.

The requirement that the matching network also be a low-pass filter forces us to use some form of series-L, shunt-C arrangement. The only way we can get from the impedance at point A to the impedance at point C and still fulfill this requirement is along the path shown in Fig. 4-44. Thus, following the rules of Fig. 4-39, the arc AB of Fig. 4-44 is a shunt capacitor with a value of +jB = 0.73 mho. The arc BC is a series inductor with a value of +jX = 1.2 ohms.

The shunt capacitor as read from the Smith Chart is a susceptance and can be changed into an equivalent reactance by simply taking the reciprocal.

$$X_C = \frac{1}{+jB} = \frac{1}{j0.73\ \text{mho}} = -j1.37\ \text{ohms}$$

To complete the network, we must now unnormalize all impedance values by *multiplying* them by the number N = 50—the value originally used in the normalization process. Therefore:

$$X_L = 60\ \text{ohms}$$
$$X_C = 68.5\ \text{ohms}$$

The component values are:

$$L = \frac{X_L}{\omega} = \frac{60}{2\pi(60 \times 10^6)} = 159\ \text{nH}$$

$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi(60 \times 10^6)(68.5)} = 38.7\ \text{pF}$$

The final circuit is shown in Fig. 4-43.

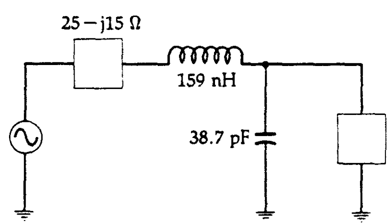

Fig. 4-43. Final circuit for Example 4-7.

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EXAMPLE 4-7—Cont.

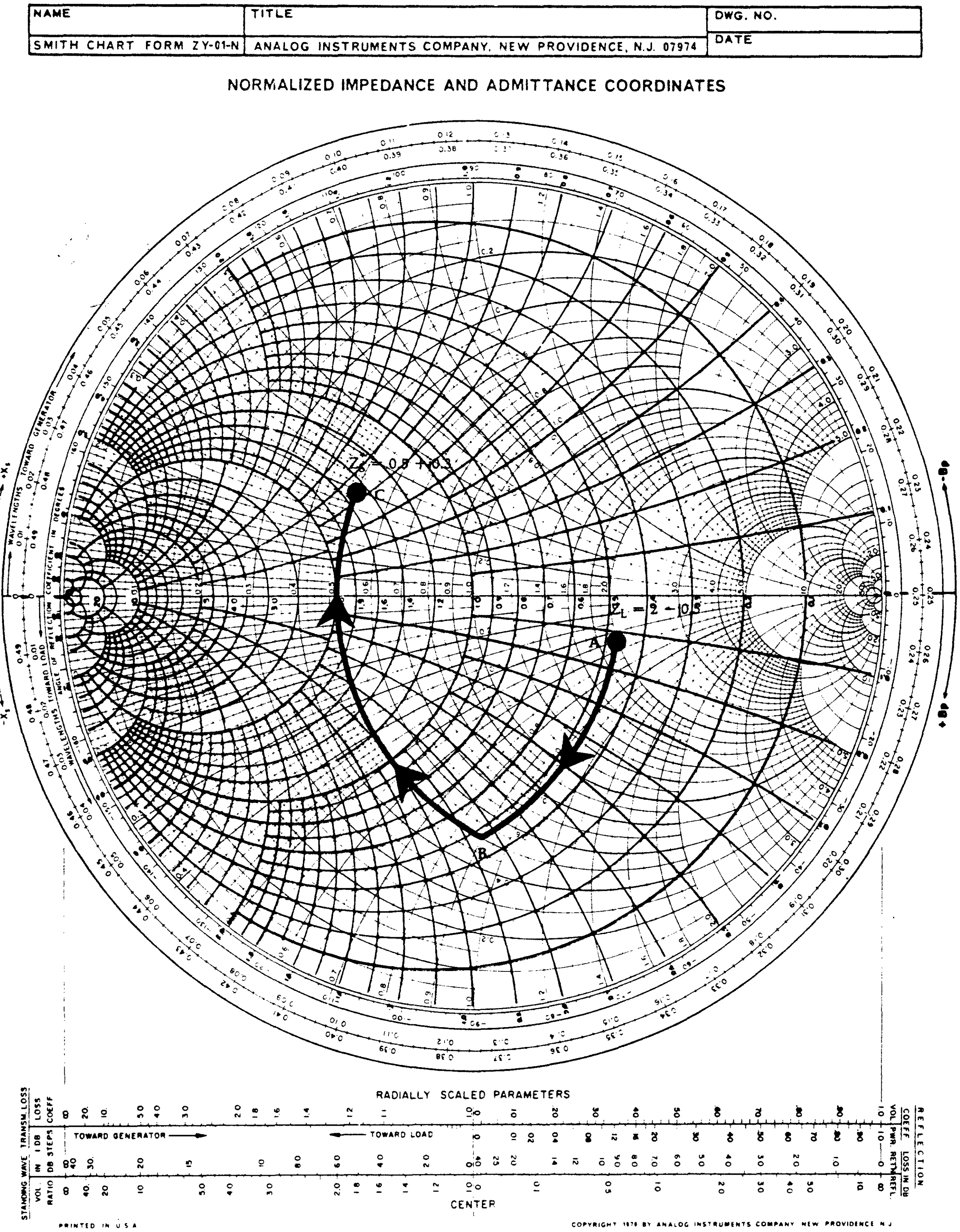

Fig. 4-44. Solution to Example 4-7.

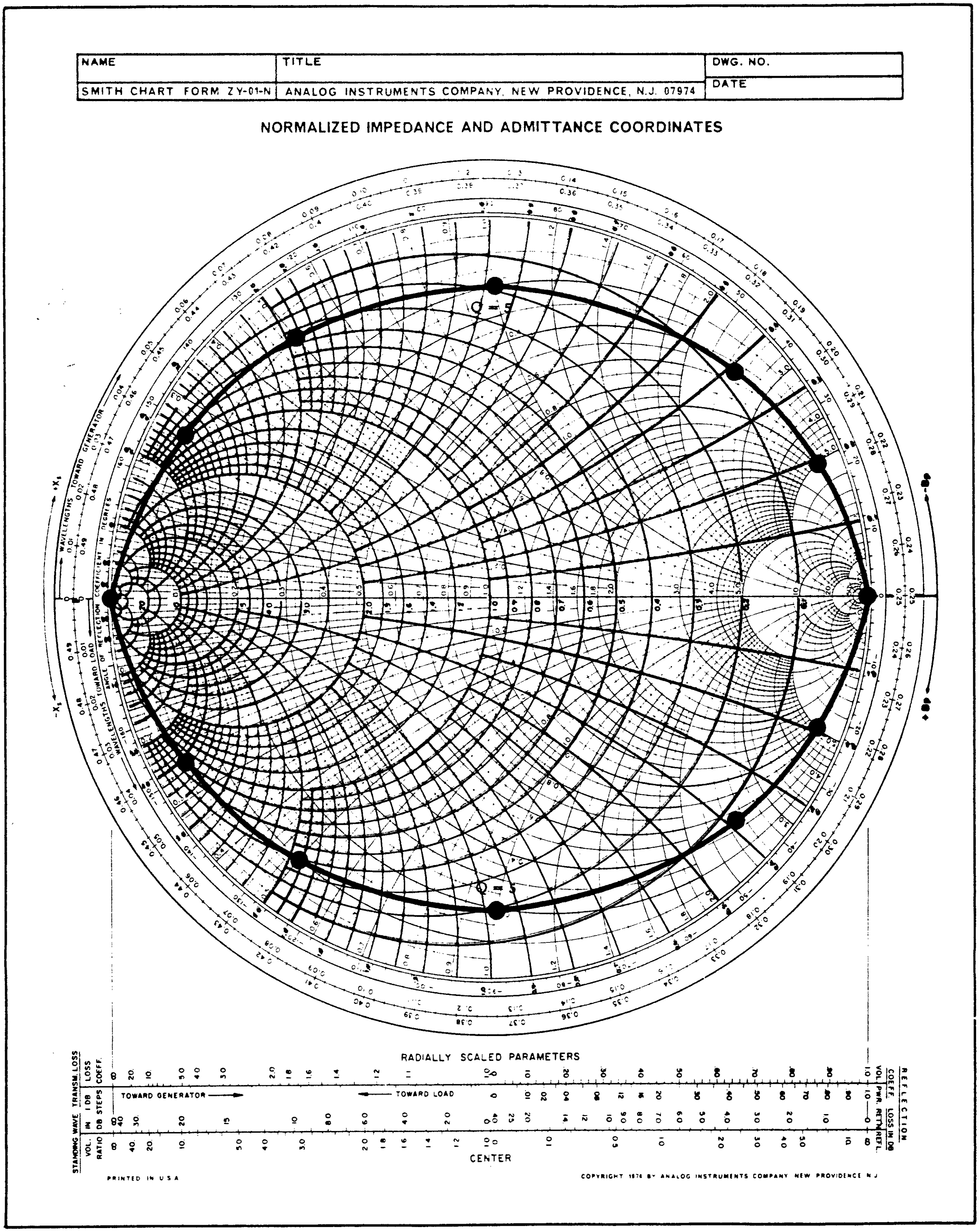

Fig. 4-45. Lines of constant Q.

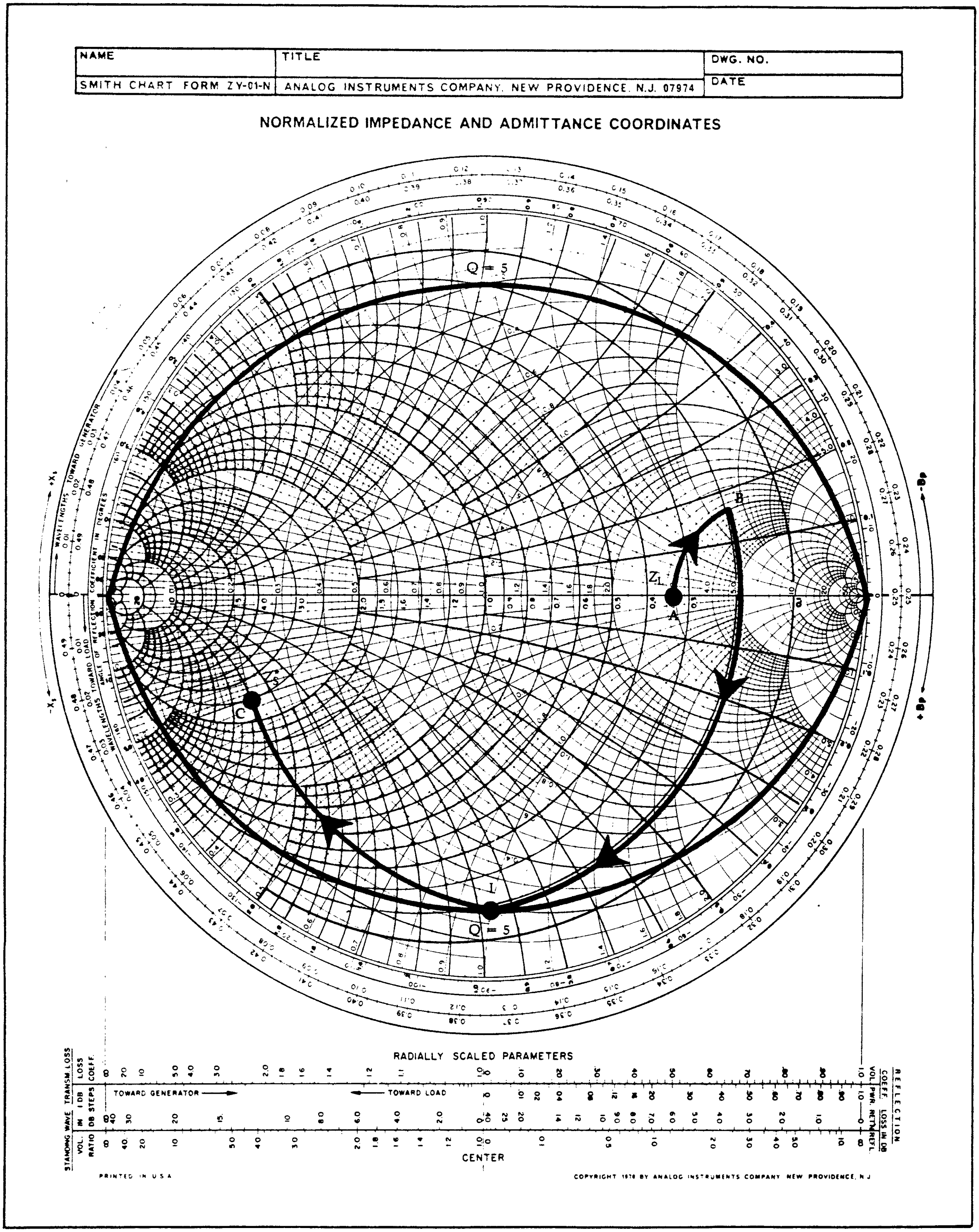

Fig. 4-46. Smith Chart solution for Example 4-8.

EXAMPLE 4-8

Design a T network to match a $Z = 15 + j15$-ohm source to a 225-ohm load at 30 MHz with a loaded Q of 5.

Solution

Following the procedures previously outlined, draw the arcs for $Q = 5$ first and, then, plot the load impedance and the complex conjugate of the source impedance. Obviously, normalization is necessary as the impedances are too large to be located on the chart. Divide by a convenient value (choose $N = 75$) for normalization. Therefore:

$$Z_s^* = 0.2 - j0.2 \text{ ohm}$$

$$Z_L = 3 \text{ ohms}$$

The construction details for the design are shown in Fig. 4-46.

The design statement specifies a T network. Thus, the source termination will determine the network Q because $R_s < R_L$.

Following the procedure for $R_s < R_L$ (Step 4, above), first plot point I, which is the intersection of the $Q = 5$ curve and the R = constant circuit that passes through Z_s^*. Then, move from the load impedance to point I with two elements.

Element 1 = arc AB = series L = j2.5 ohms

Element 2 = arc BI = shunt C = j1.15 mhos

Then, move from point I to Z_s^* along the R = constant circle.

Element 3 = arc IC = series L = j0.8 ohm

Use Equations 4-11 through 4-14 to find the actual element values.

Element 1 = series L:

$$L = \frac{(2.5)75}{2\pi(30 \times 10^6)} = 995 \text{ nH}$$

Element 2 = shunt C:

$$C = \frac{1.15}{2\pi(30 \times 10^6)75} = 81 \text{ pF}$$

Element 3 = series L:

$$L = \frac{(0.8)75}{2\pi(30 \times 10^6)} = 318 \text{ nH}$$

The final network is shown in Fig. 4-47.

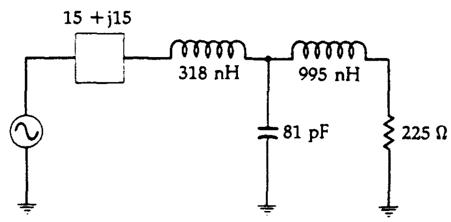

Fig. 4-47. Final circuit for Example 4-8.

treasure trove containing an infinite number of possible solutions. To get from point A to point B on a Smith Chart, there is, of course, an optimum solution. However, the optimum solution is not the only solution. The two-element network gets you from point A to point B with the least number of components and the three-element network can provide a specified Q by following a different route. If you do not care about Q, however, there are 3-, 4-, 5-, 10-, and 20-element (and more) impedance-matching networks that are easily designed on a Smith Chart by simply following the constant-conductance and constant-resistance circles until you eventually arrive at point B, which, in our case, is usually the complex conjugate of the source impedance. Fig. 4-48 illustrates this point. In the lower right-hand corner of the chart is point A. In the upper left-hand corner is point B. Three of the infinite number of possible solutions that can be used to get from point A to point B, by adding series and shunt inductances and capacitances, are shown. Solution 1 starts with a series-L configuration and takes 9 elements to get to point B. Solution 2 starts with a shunt-L procedure and takes 8 elements, while Solution 3 starts with a shunt-C arrangement and takes 5 elements. The element reactances and susceptances can be read directly from the chart, and Equations 4-11 through 4-14 can be used to calculate the actual component values within minutes.

SUMMARY

Impedance matching is not a form of "black magic" but is a step-by-step well-understood process that is used to help transfer maximum power from a source to its load. The impedance-matching networks can be designed either mathematically or graphically with the aid of a Smith Chart. Simpler networks of two and three elements are usually handled best mathematically, while networks of four or more elements are very easily handled using the Smith Chart.

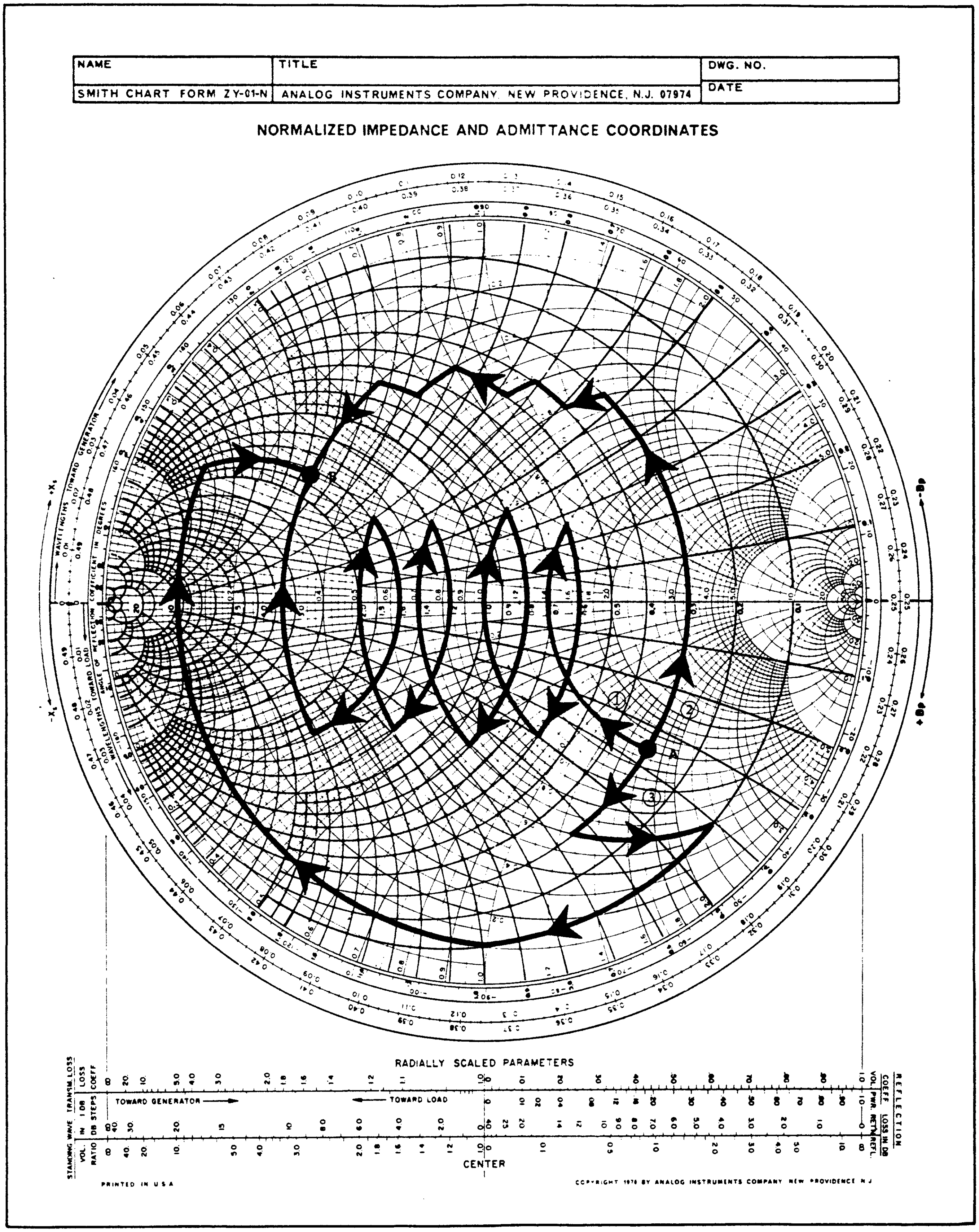

Fig. 4-48. Multielement matching.