EEAP 210 ELECTROMAGNETIC FIELD THEORY

JANUARY 30, 1984

EXAM NO. 1

NAME: ____________________

INSTRUCTIONS & NOTES:

1. A formula sheet is permitted: one side of a single $8\frac{1}{2} \times 11$ sheet that will be collected with the exam.
2. Closed book — no notes except for that allowed in 1.
3. There are seven (7) problems ranked in terms of difficulty, i.e. 7 is much harder than 1.
4. Not all problems are worth the same - see below.

PROBLEM	SCORE	VALUE
1		5
2		5
3		10
4		10
5		20
6		20
7		30
TOTAL		100

ADVISORY GRADE

1. What is $\underline{a}_r \times \underline{a}_\theta$ in spherical coordinates?

   a) $\underline{a}_\phi$
   b) $-\underline{a}_\phi$
   c) zero
   d) $\underline{a}_\theta$
   e) $-\underline{a}_\theta$

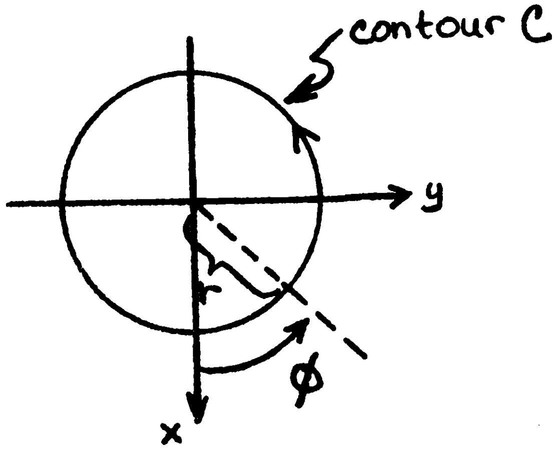
   Circle your answer.

2. What is $\underline{a}_z \times \underline{a}_\phi$ in cyclindrical coordinates?

   a) $\underline{a}_\theta$
   b) $-\underline{a}_\theta$
   c) zero
   d) $-\underline{a}_r$
   e) $\underline{a}_r$

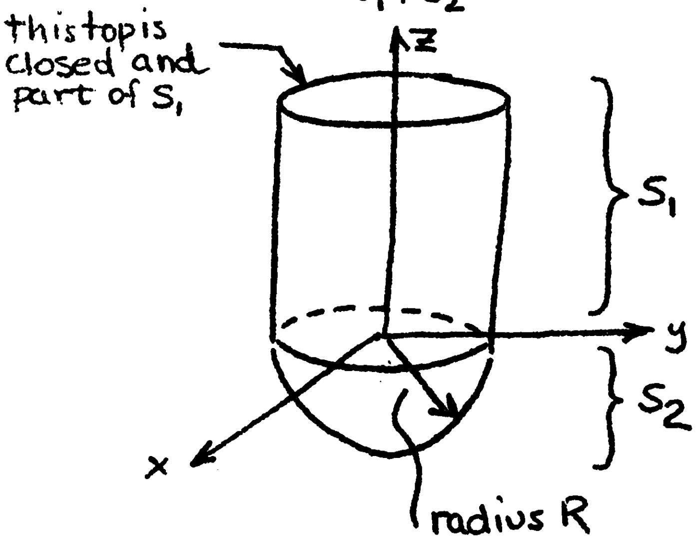
   Circle your answer.

3. For the circular contour shown below what is the unit vector $\underline{dl}$ in cartesian coordinates?



4. What is $\nabla\left(\frac{1}{r}\right)$ in spherical coordinates?

5. Evaluate $\int_{S_1+S_2} (\underline{\nabla} \times \underline{F}) \cdot d\underline{S}$ for the surfaces indicated below.



$\underline{F} = r \sin\left(\frac{\theta}{2}\right) \underline{a}_\theta - z \sin\theta \, \underline{a}_z$

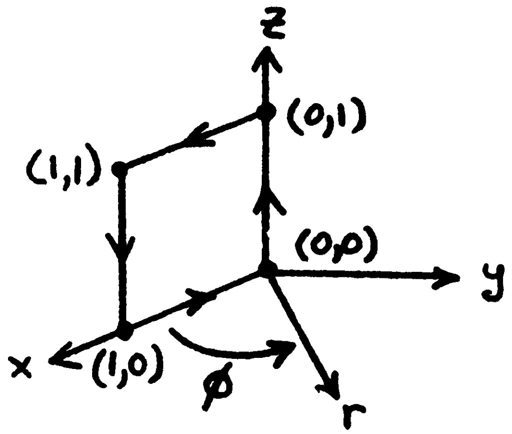
$d\underline{S}$ is always pointed outward

35

6. Is the vector field $\underline{F} = 10 \sin\theta \, \underline{a}_\theta$ (spherical coordinates) a rotational field? Why?

7. (a) Find the curl of the vector field $\underline{F} = r^2 \cos\phi \, \underline{a}_z$ (cyclindrical coordinates).

7. (b) For the field of part (a) What is the line integral of $\underline{F}$ around the contour shown below?



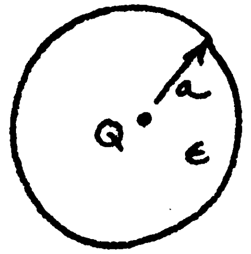
I am looking for a numerical answer.

EEAP 210

ELECTROMAGNETIC FIELD THEORY

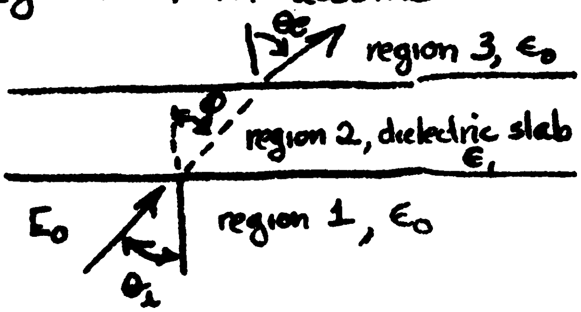
[LAST YEAR'S EXAM ON ELECTROSTATICS
This was a one hour closed-book exam]

1.

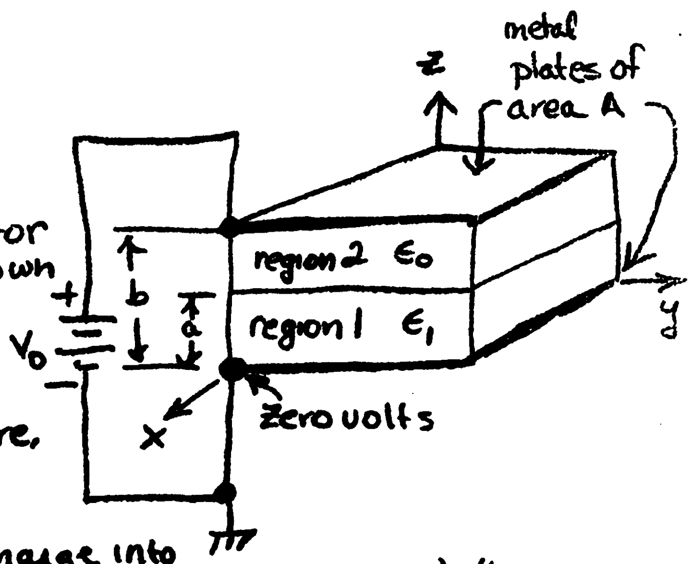


A point charge Q is placed at the center of a dielectric sphere of permittivity ϵ and radius a. Assume the sphere is in free space (ϵ_0). Find $\underline{D}$ and $\underline{E}$ everywhere. (Use Gauss' Law).

2. A dielectric slab is immersed in a uniform electric field in free space. If the entrance angle θ_i for the electric field is known in region 1, find the field components exiting the slab into region 3. Hint: assume an angle θ_e for the exit angle.



3. Consider the parallel plate capacitor with two layers of dielectric as shown at the right.



(a) Calculate the $\underline{E}$ field and the electrostatic potential Φ everywhere. State all assumptions.

(b) Suppose one can inject free charge into the interface between region 1 and region 2 (at $z=a$). How much charge must be injected to make $\underline{E} \equiv 0$ in region 2? Of what sign must the injected charge be?

(c) What is the capacitance of this parallel-plate capacitor?

65 TOOK EXAM

EEAP 210 ELECTROMAGNETIC FIELD THEORY

FEBRUARY 27, 1984

EXAM No. 2

NAME: SOLUTIONS

INSTRUCTIONS & NOTES:

1. A formula sheet is permitted: one side + $\frac{1}{2}$ of the other side of a single $8\frac{1}{2} \times 11$ sheet.
2. The formula sheet will be collected with the exam.
3. Closed book – no notes except for the formula sheet.
4. There are five (5) problems ranked in terms of difficulty, i.e. 5 is much harder than 1.
5. Not all problems are worth the same – see below

PROBLEM	SCORE	VALUE
1		10
2		15
3		20
4		20
5		35
TOTAL		100

ADVISORY GRADE

1. (a) Mark the following statements true or false. by circling the appropriate letter

i. T F The normal component of the electric flux density is continuous across a non-conducting boundary with an abrupt change in permittivity ϵ

ii. T F The tangential component of the electric field intensity is continuous across <u>all</u> boundaries.

iii. T F The normal component of the current density $\underline{J}$ is continuous across a boundary marking an abrupt change in conductivity σ.

iv. T F The electric potential is continuous across <u>all</u> dielectric interfaces.

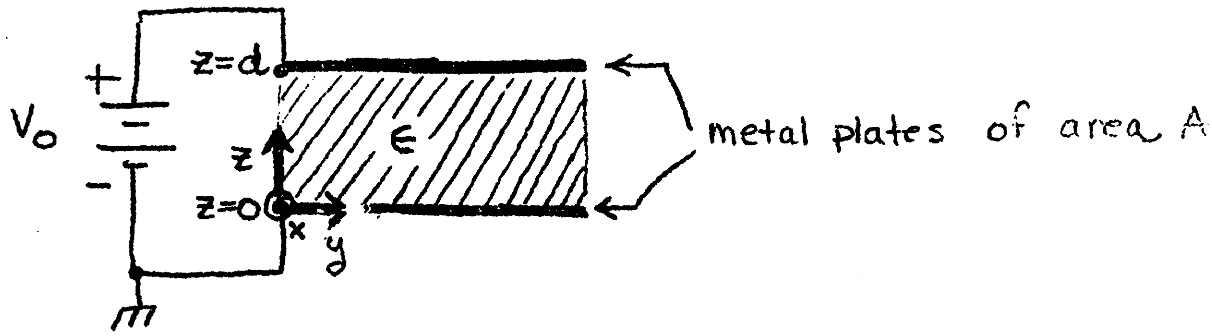
v. T F The electric potential is continuous across <u>all</u> conductor – dielectric interfaces.

(b) Rewrite all statements that you marked false in part (a) to make them true. [just negating the verb, i.e. is continuous to is not continuous, is not acceptable.]

i. The normal component of the electric flux density is is continuous across a non-conducting boundary with an abrupt change in permittivity ϵ and no surface charge density ρ_s.

ii. The tangential component of the electric field intensity is continuous across <u>all</u> dielectric boundaries.

2. For the parallel-plate capacitor shown below circle the appropriate response to the given situation



(a) If the permittivity ϵ increases the capacitance

increases   remains the same   decreases

(b) If V_0 increases the capacitance

increases   remains the same   decreases

(c) If d increases the total charge on each plate

increases   remains the same   decreases

(d) If A increases the charge density on each plate

increases   remains the same   decreases

(e) If the battery is disconnected, (Q constant) then ϵ decreases (reducing C) the potential between the plates

increases   remains the same   decreases

$$C = \frac{Q}{V} \text{ (constant)} \quad E = \frac{V}{d} \text{ so } D = \frac{\epsilon V}{d}$$

$$\rho_s = n \cdot (D_2 - D_1) = -\frac{\epsilon V}{d}$$

(e) $V = \frac{Q}{C}$ (constant) (decreases)

$$Q_{tot} = -\epsilon A \frac{V}{d}$$

$$C = \frac{\epsilon A}{d}$$

3. A raindrop can be modeled as a dielectric (solid) sphere with a uniform surface charge density.

(a) Using Gauss' Law determine the E field everywhere for such a raindrop. (Assume a radius R and a total surface charge +Q).

(b) Assuming $\Phi(\infty)=0$, what is the potential at the surface of the sphere?

(c) If it is found that a raindrop has a potential of +100 volts and a total surface charge of 10^{-11} coulombs, what is the raindrop radius? [use $\epsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ farads/meter]

a) $\int \underline{D} \cdot \underline{ds} = \int \rho \, dv = Q$

$\epsilon_0 E \, 4\pi r^2 = Q$

8 $\boxed{E = \begin{cases} \dfrac{Q}{4\pi\epsilon_0 r^2} & \text{for } r > R \\ 0 & r \leq R \end{cases}}$

b) $E = -\nabla\Phi$

$\frac{Q}{4\pi\epsilon_0 r^2} = -\frac{d\Phi}{dr}$

$d\Phi = -\frac{Q}{4\pi\epsilon_0}\frac{dr}{r^2}$

8 $\int_\infty^r d\Phi = -\frac{Q}{4\pi\epsilon_0}\int_\infty^r \frac{dr}{r^2}$

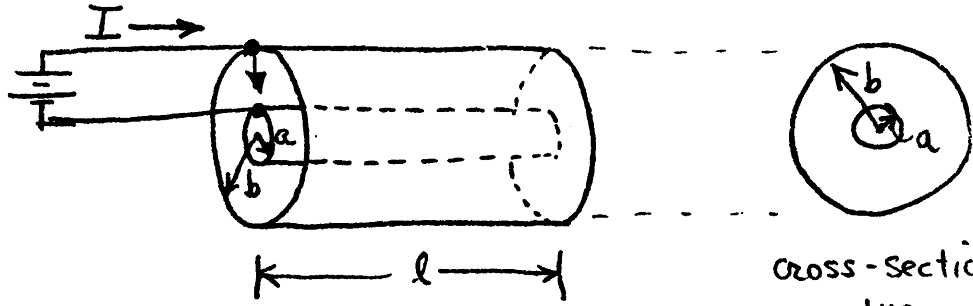
$\Phi(r) - \cancel{\Phi(\infty)}_{0} = +\frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r}\right]_\infty^r = \frac{Q}{4\pi\epsilon_0 r} \Rightarrow \boxed{\Phi(R) = \frac{Q}{4\pi\epsilon_0 R}}$

c) $R = \frac{Q}{4\pi\epsilon_0 \Phi(R)} = \frac{10^{-11}}{\cancel{4\pi}\,\frac{1}{\cancel{36\pi}_{9}} \times 10^{-9} \times 10^2} = 9 \times 10^{-11} \times 10^7$

4 $= 9 \times 10^{-4}$ meters

$\boxed{R = 0.9\,\text{mm}}$

4. What is the resistance of the insulation in a length ℓ of coaxial cable. The total current through the insulation is I amperes.



cross-sectional view

The fields are directed radially.

$$J = \frac{I}{A} = \frac{I}{2\pi r \ell}$$

but $J = \sigma E$ so $E_r = \dfrac{I}{2\pi\sigma r \ell}$

$$\Phi = -\int \underline{E} \cdot \underline{d\ell} = -\int_a^b \frac{I}{2\pi\sigma r\ell}\, dr$$

$$= -\frac{I}{2\pi\sigma\ell}\int_a^b \frac{dr}{r} = -\frac{I}{2\pi\sigma\ell}\left[\ln b - \ln a\right]$$

$$= \frac{I \ln(a/b)}{2\pi\sigma\ell}$$

$$R = \frac{\Phi}{I} = \frac{\ln(a/b)}{2\pi\sigma\ell}$$

5. For the wedge capacitor shown below

(a) Using Laplace's equation find the form of $\Phi(r)$ everywhere. Write your expression(s) for $\Phi(r)$.

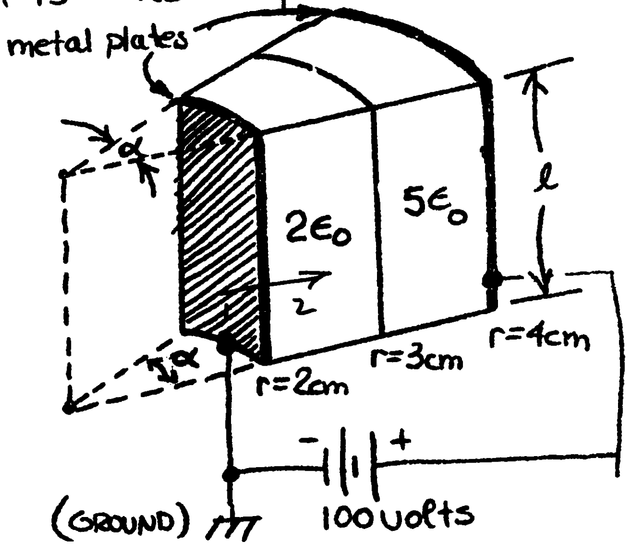
(b) What are the appropriate boundary conditions to determine the coefficients in your expression for $\Phi(r)$?

(c) Determine the values of those coefficients in $\Phi(r)$.

(d) Plot Φ, E as functions of r.

(e) What is the capacitance?

7 pts each



(a)

$$\nabla^2\Phi = 0$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial \Phi}{\partial r}\right) = 0$$

$$r\frac{d\Phi}{dr} = A$$

$$d\Phi = A\,\frac{dr}{r}$$

region 1 $2 \le r < 3$: $\Phi_1 = A \ln r + B$

region 2 $3 \le r < 4$: $\Phi_2 = C \ln r + D$

(b)

$\Phi_1(2) = 0$

$\Phi_2(4) = +100$

$\Phi_1(3) = \Phi_2(3)$

$D_1(3) = D_2(3)$

$E_1 = -\nabla\Phi_1 = -\frac{\partial}{\partial r}\Phi_1$ $\qquad E_2 = -\frac{\partial \Phi_2}{\partial r}$

$E_1 = -\frac{A}{r}$ $\qquad E_2 = -\frac{C}{r}$

$D_1 = -\frac{\epsilon_1 A}{r} = -\frac{2\epsilon_0 A}{r}$ $\qquad D_2 = -\frac{\epsilon_2 C}{r} = \frac{-5\epsilon_0 C}{r}$

(c) From (b)

(i) $A \ln 2 + B = 0$ $\qquad$ (ii) $C \ln 4 + D = 100$

(iii) $A \ln 3 + B = C \ln 3 + D$

(iv) $\frac{2\epsilon_0 A}{3} = \frac{5\epsilon_0 C}{r}$

EEAP 210 - ELECTROMAGNETIC FIELD THEORY

MAKE-UP EXAM #2

NAME: ______________________________

INSTRUCTIONS & NOTES:

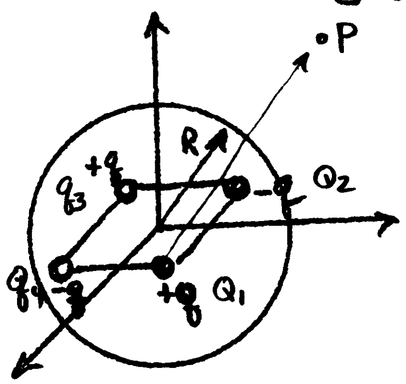
1. A Formula sheet is permitted: one side + $\frac{1}{2}$ of the other side of a single 8.5x11 inch sheet of paper.
2. The formula sheet will be collected with the exam.
3. Closed book
4. Problems are roughly ranked in terms of difficulty.
5. Not all problems are worth the same.

PROBLEM	SCORE	VALUE
1.		10
2.		15
3.		15
4.		30
5.		30
TOTAL		100

1. A sphere of radius R and centered at the origin has four charges inside it. The charges are located on a square of side length "a" lying in the x-y plane as shown below. Two of the charges are +q and two are -q as shown below.

(a) What is the electric flux over the sphere, i.e. $\int \underline{D}\cdot d\underline{S}$?

(b) What is the potential anywhere? Do not simplify your answer.



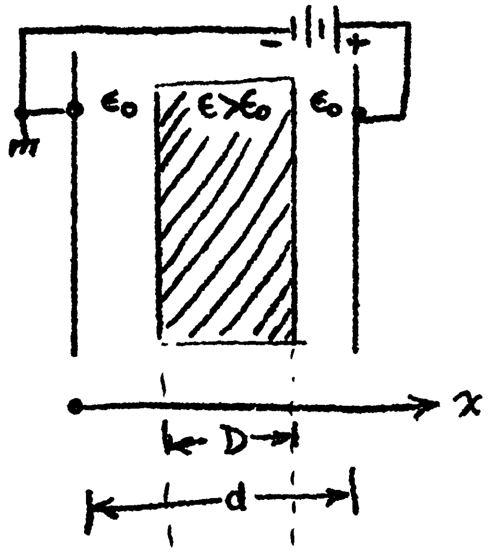
(a) $\int \underline{D}\cdot d\underline{S} = \int \rho \, dv = \Sigma Q.$

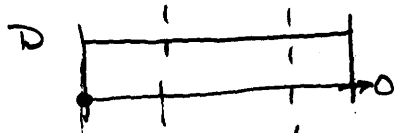
obviously $\Sigma Q = 0$ so the total electric flux = 0.

(b) $\phi = -\int \underline{E}\cdot d\underline{S}$

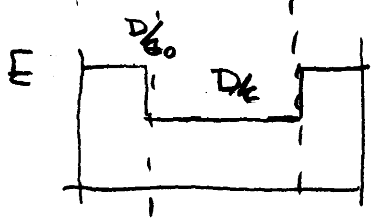
$$\phi = \frac{q_1}{4\pi\epsilon_0|\underline{r_1}|} + \frac{q_2}{4\pi\epsilon_0|\underline{r_2}|} + \frac{q_3}{4\pi\epsilon_0|\underline{r_3}|} + \frac{q_4}{4\pi\epsilon_0|r_3|}$$

2. A voltage V_o is applied between a pair of conducting plates of area A separated by a distance d. A dielectric slab of thickness $D < d$ is between the plates as shown below. Plot $\underline{E}, \underline{D}, \underline{P}, \Phi$ and ρ as functions of x.

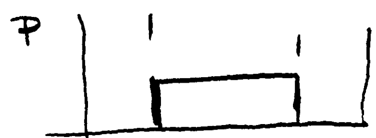




as normal D is continuous

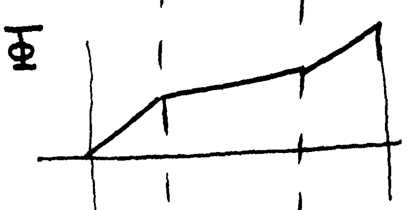


since $\epsilon > \epsilon_0$



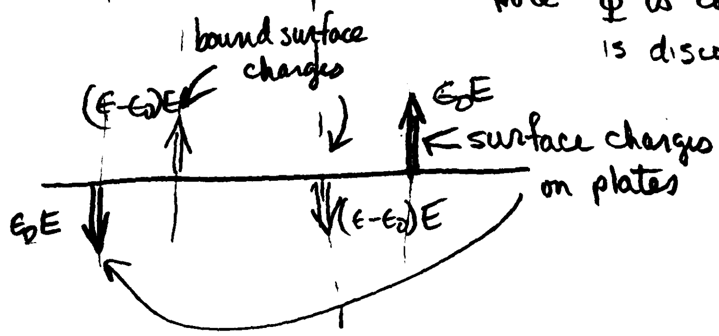
since $D = \epsilon_0 E + P$

$P = \underline{D} - \epsilon_0 \underline{E} = \epsilon \underline{E} - \epsilon_0 \underline{E}$
$= (\epsilon - \epsilon_0)\, \underline{E}$



since $\Phi = -\int E \cdot dS$.

Note Φ is continuous but slope, i.e. E, is discontinuous

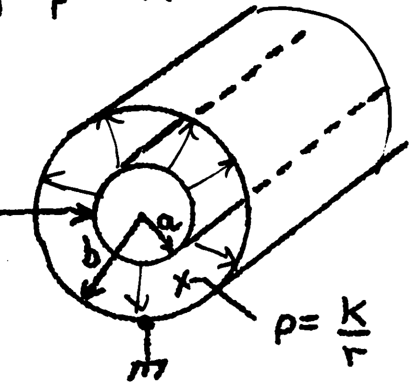


since $\rho = \underline{n} \cdot (\underline{D}_2 - \underline{D}_1)$

3. Two infinitely long conducting shells that are cyclindrical and concentric have inner and outer radii a and b, respectively. A charge is placed on the inner shell such that each length, ℓ, has a total charge Q_i coulombs. The outer shell is grounded and the space between the shells is filled with charge which distributes itself with a volume density $\rho = K/r$ coulombs$/m^3$.

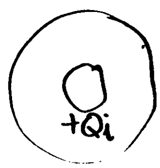
a) Using the appropriate Maxwell Equation in integral form find $\underline{E}$ in the region between the cyclinders.

b) Can you use Laplace's Equation to find Φ in this problem? Why or why not? [DON'T DO IT]



c) Find the charge Q_o that must be induced on the outer shell.

(a) $$\oint_C \underline{D} \cdot d\underline{S} = \int \rho\, dV$$



volume is a right cylinder

$$\epsilon_o E_r[2\pi r\, \ell] = Q_i + \int_a^r \rho \cdot 2\pi(r-a)\ell\, dr$$

$$\epsilon_o E\, 2\pi r \ell = Q_i + 2\pi\rho\ell\left[\frac{r^2}{2} - ar\right]_a^r$$

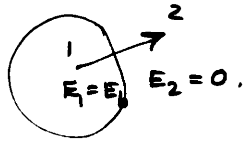
$$= Q_i + 2\pi\rho\ell\left[\left(-\frac{r^2}{2}\right) - \left(-\frac{a^2}{2}\right)\right].$$

$$E_r = \frac{Q_i + \pi\rho\ell[a^2 - r^2]}{2\pi\epsilon_o \ell \cdot r}$$

(b) No, because the general equation is Poisson's Eqn $\nabla^2\Phi = -\frac{\rho}{\epsilon}$ which only reduces to Laplace's Eqn $\nabla^2\Phi = 0$ when $\rho = 0$ in the region of interest. Obviously, $\rho \neq 0$ in this problem

(c) at $r = b$ $$E_r(b) = \frac{Q_i + \pi\rho\ell[a^2 - b^2]}{2\pi\epsilon_o\ b} \equiv E_b$$

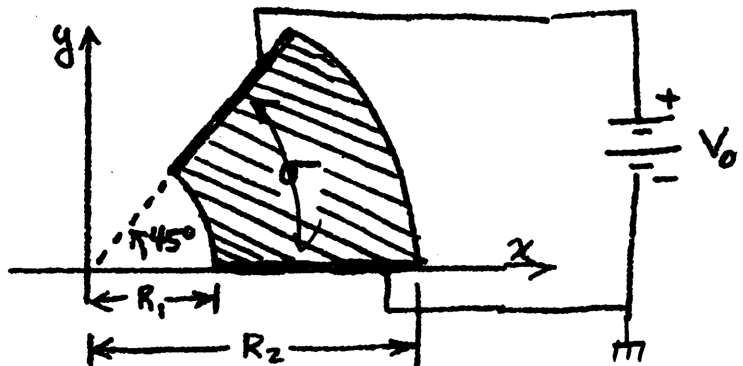
at the surface $\quad n \cdot (D_2 - D_1) = \rho_s$



$$\underline{a_r} \cdot (0 - \epsilon_o E_r(b)\underline{a_r}) = \rho_s.$$

$$\therefore\ \rho_s = -\frac{Q_i + \pi\rho\ell[a^2 - b^2]}{2\pi\epsilon_o b}$$

4. A sheet of conducting material, of uniform conductivity σ, cut in the shape shown below has a battery of potential V_0 connected to the end electrodes. The material is of thickness t. If the electrodes are perfect conductors:



(a) Find expressions for Φ and $\underline{E}$ within the material.
(b) What is the resistance of the material as connected?

(a) this is a one-dimensional Laplace's Eqn. $\nabla^2\Phi = 0$

$$\nabla^2\Phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\Phi}{\partial\phi^2} + \frac{\partial^2\Phi}{\partial z^2}$$

The only dependence must be in the ϕ direction therefore.

$$\frac{\partial^2\Phi}{\partial\phi^2} = 0$$

$$\frac{d\Phi}{d\phi} = c_1$$

$$\Phi = c_1\phi + c_2$$

at $\phi = 0$ $\Phi = 0 \therefore c_2 = 0$

$\phi = \frac{\pi}{4}$ $\Phi = V_0 = c_1\frac{\pi}{4}$.

$\therefore c_1 = \frac{4V_0}{\pi}$

$$\boxed{\Phi = \frac{4V_0}{\pi}\phi}$$

$$E = -\nabla\Phi = -\underline{a_\phi}\frac{1}{r}\frac{\partial\Phi}{\partial\phi} = \boxed{-\underline{a_\phi}\frac{1}{r}\frac{4V_0}{\pi}}$$

(b) what is R?

$$J = -\underline{a_\phi}\frac{4\sigma V_0}{\pi r}$$

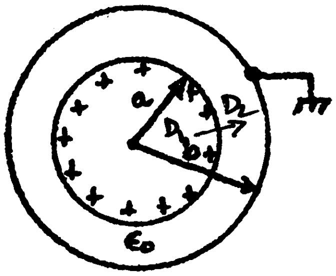
$$I = \int_{R_1}^{R_2} -\cancel{\underline{a_\phi}}\cdot\frac{4\sigma V_0}{\pi r}\cdot\cancel{\underline{a_\phi}}\,t\,dr = \frac{4\sigma V_0 t}{\pi}\int_{R_1}^{R_2}\frac{dr}{r}$$

$$= \frac{4\sigma V_0 t}{\pi}\ln(R_2/R_1)$$

$$R = \frac{\Phi}{I} = \frac{\cancel{V_0}}{\frac{4\sigma\cancel{V_0}t\ln R_2/R_1}{\pi}} = \frac{\pi}{4\sigma t\ln(R_2/R_1)}$$

5.

Consider the outer sphere of radius b grounded as shown in the drawing below. There is a charge +Q on the inner sphere.



spherical !

a) What must be the charge on the outer sphere?
b) Find E, Φ for the region $a<r<b$
c) What is the capacitance ?

(a) $+Q$

(b) $\nabla^2\Phi = 0$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \underbrace{\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}}_{\text{no } \theta \text{ or } \phi \text{ dependence.}} = 0$$

$$\therefore\quad r^2\frac{d\Phi}{dr} = c_1$$

$$d\Phi = \frac{c_1}{r^2}\,dr$$

$$\Phi(r) - \Phi(a) = -\frac{c_1}{r}\Big|_a^r = -\frac{c_1}{r} + \frac{c_1}{a}$$

$$0 = \Phi(b) - \Phi(a) = +c_1\left(-\frac{1}{b}+\frac{1}{a}\right) \qquad \therefore\quad c_1 = \frac{\Phi(b)-\Phi(a)}{\frac{1}{a}-\frac{1}{b}}$$

but $\Phi(b)=0$

$$\Phi(r) = \Phi(a) - \frac{\left(\frac{1}{a}-\frac{1}{r}\right)}{\left(\frac{1}{a}-\frac{1}{b}\right)}\Phi(a)$$

$$E = -\nabla\Phi = -\underline{a_r}\frac{\partial\Phi}{\partial r} = +\underline{a_r}\left(+\frac{1}{r^2}\right)\frac{\Phi(a)}{\left(\frac{1}{a}-\frac{1}{b}\right)}$$

$$D = \epsilon_0 E = +\underline{a_r}\,\frac{\epsilon_0}{r^2}\,\frac{\Phi(a)}{\left(\frac{1}{a}-\frac{1}{b}\right)}$$

at the inner surface

$$\rho_s = n\cdot(D_2 - D_1) = \underline{a_r}\left(+\underline{a_r}\,\frac{\epsilon_0}{r^2}\,\frac{\Phi(a)}{\left(\frac{1}{a}-\frac{1}{b}\right)}\right) = \frac{\epsilon_0}{\cancel{r^2}\ a^2}\,\frac{\Phi(a)}{\left(\frac{1}{a}-\frac{1}{b}\right)}$$

but $\rho_s = \dfrac{+Q}{4\pi a^2}$ $\qquad \therefore\quad \dfrac{Q}{4\pi \cancel{a^2}} = \dfrac{\epsilon_0}{\cancel{a^2}}\dfrac{\Phi(a)}{\left(\frac{1}{a}-\frac{1}{b}\right)}$

$$\Phi(a) = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$\therefore \quad \Phi(r) = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right) - \frac{\left(\frac{1}{a}-\frac{1}{r}\right)}{\cancel{\left(\frac{1}{a}-\frac{1}{b}\right)}} \frac{Q}{4\pi\epsilon_0}\cancel{\left(\frac{1}{a}-\frac{1}{b}\right)}$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}-\frac{1}{a}+\frac{1}{r}\right)$$

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r}-\frac{1}{b}\right)$$

$$\underline{E}(r) = \underline{a_r}\ \frac{1}{r^2}\ \frac{\frac{Q}{4\pi\epsilon_0}\cancel{\left(\frac{1}{a}-\frac{1}{b}\right)}}{\cancel{\left(\frac{1}{a}-\frac{1}{b}\right)}}$$

$$\underline{E}(r) = \underline{a_r}\ \frac{Q}{4\pi\epsilon_0 r^2}$$

(c) $$C = \frac{Q}{\Delta\Phi} = \frac{Q}{|\Phi(b)-\Phi(a)|} = \frac{Q}{0 - \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)}$$

$$C = \frac{4\pi\epsilon_0}{\left(\frac{1}{a}-\frac{1}{b}\right)}$$

EEAP 210

LAST YEAR'S EXAM ON MAGNETOSTATICS

1. If $B = \mu_o H^2$ what is the induced magnetization M. (There is <u>no</u> permanent magnetization) and the permeability μ? Is μ independent of H?

2. What is the vector potential at the center of a loop of wire carrying a current I? What is B?

A cyclindrical conductor of radius a oriented along the z-axis carries a non-uniformly distributed current density $J = J_o r^2 \underline{a_z}$

(a) What is the total current flowing in the conductor?

(b) What is H everywhere?

4. An infinitely long solenoid of radius a has n turns/meter and carries a current I.

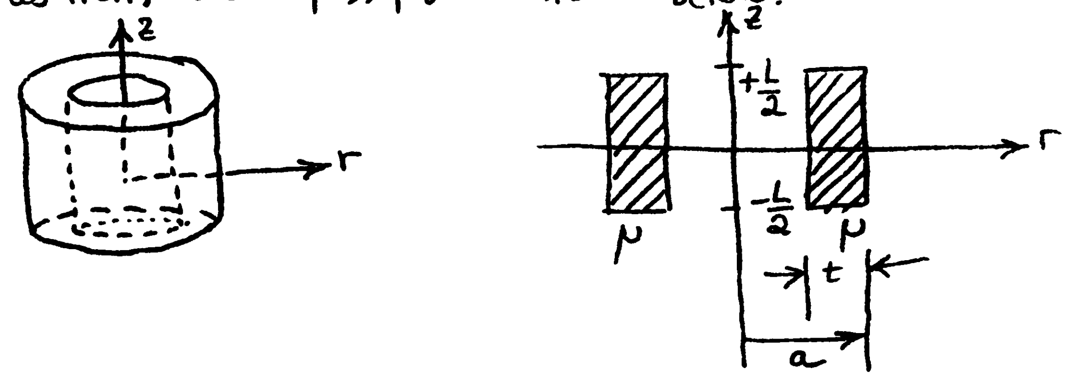
(a) What is the surface current density?

(b) what is the on-axis z-directed field?

Hint: the on-axis z-directed field for a solenoid of length ℓ is given by $H_z = \dfrac{NI}{2(\frac{\ell^2}{4} + a^2)^{1/2}}$

(c) What are the fields everywhere? State your assumptions.

5. Consider the hollow cyclinder constructed of a highly permeable material (such as iron) where $\mu \gg \mu_o$ as shown below.



(a) This hollow cyclinder is placed in a uniform magnetic field $\underline{H} = H_o \underline{a_z}$. What are B & H inside the permeable material? They may be assumed to be uniform.

(b) What is the induced magnetization $\underline{M}$? Sketch it as a function of r.

(c) What are the equivalent currents $\underline{K_m}$ for $\underline{M}$? Sketch $\underline{K_m}$ as a function of r.

(d) The z-component of the $\underline{B}$ field at the center of a cyclindrical current distribution $\underline{K} = K_o \underline{a_\phi}$ of radius a and length L is given by

$$B_z = \frac{\mu_o K_o}{2}\left[\frac{-z + \frac{L}{2}}{\sqrt{(z-\frac{L}{2})^2 + a^2}} + \frac{z + L/2}{\sqrt{(z+\frac{L}{2})^2 + a^2}}\right]$$

Using this formula what is the field at $z=0$ on the axis of the cyclinder due to the currents found in (c). Set up result - don't solve.

EEAP 210

LAST YEAR'S MAGNETOSTATIC EXAM (PART 2)

5(cont.) (e) If $a \gg t$ reduce your results of (d).

(f) Does the induced field lie in the same direction as the applied field?

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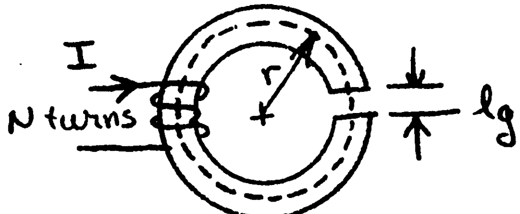
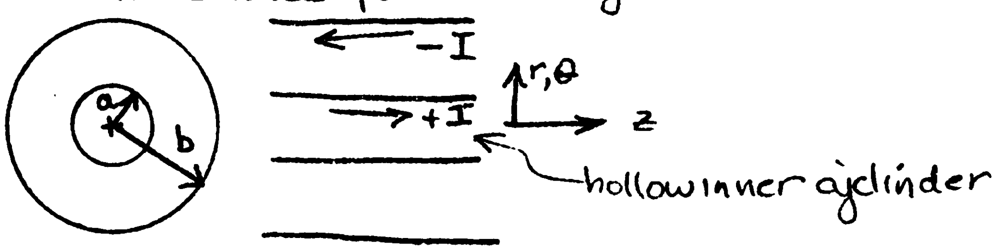
2. It is desired to produce a magnetic field $B = 0.1$ Webers$/m^2$ in the airgap of the electromagnet shown below. The cross-section of the toroid is $1cm^2$, its mean radius is $10cm$, and the gap length is [redacted] $0.1cm$, Assume the permeability of the ring is $\mu = 4000\mu_0$ and that fringing can be ignored.



(a) draw the equivalent magnetic circuit.

(b) Find the number of turns, N, required to produce this field.

3. A coaxial transmission line is made up of two thin-walled conducting cyclinders with radii a and b. A current I flows along the inner cylinder, and a return current $-I$ along the outer cyclinder. Calculate the inductance per unit length.



EEAP 210 – ELECTROMAGNETIC FIELD THEORY

EXAM #3

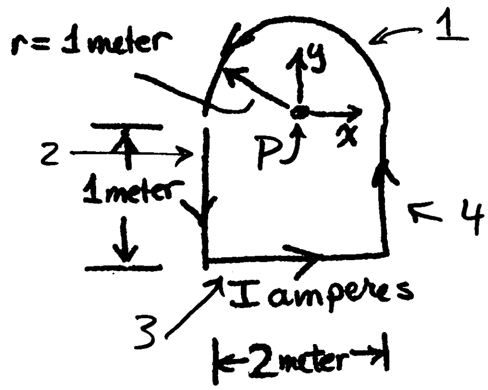
NAME: ______________________________

INSTRUCTIONS & NOTES:

1. A double sided 8.5x11 inch formula is permitted. This formula sheet will be collected with the exam.
2. Closed book & notes.
3. Problems are roughly ranked in terms of difficulty.
4. Not all problems are worth the same.

PROBLEM	SCORE	VALUE
1.		20
2.		20
3.		30
4.		30
TOTAL		100

1. A loop that is half circular and half square as shown below carries a current of I amperes. Find the magnetic field $\underline{B}$ at P. by the Biot - Savart Law. Show and evaluate all vector cross products, set up all integrals but DO NOT evaluate any integrals. The origin of the coordinate system is at P.



Evaluate in parts

Biot - Savart Law : $\underline{B} = \dfrac{\mu_0}{4\pi} \displaystyle\int \frac{I\, d\underline{l} \times \underline{a_R}}{R^2}$

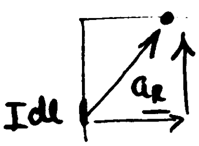
1. semicircle

$$d\underline{B}_1 = \frac{\mu_0 I\, r\, d\phi\, (-\underline{a_\phi} \times \underline{a_r})}{4\pi r^2} = \frac{\mu_0 I\, d\phi}{4\pi r}\, a_z$$

$$\underline{B}_1 = \int_0^{\pi} \frac{\mu_0 I}{4\pi r}\, d\phi\, \underline{a_z} = \frac{\mu_0 I}{4}\, \underline{a_z}$$

$r = 1$

2, side #2 :



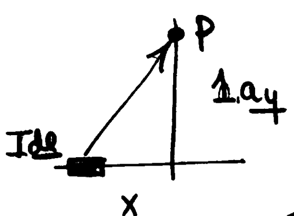
$\underline{r} = \underline{a_x} - y\, \underline{a_y}$ since $y<0$ w/above coordinates.

$$\underline{a_r} = \frac{\underline{a_x} - y\, \underline{a_y}}{\sqrt{1+y^2}}$$

$I\, d\underline{l} = -I\, dy\, \underline{a_y}$

$$\underline{B}_2 = \frac{\mu_0}{4\pi} \int_0^{-1} \frac{(-I\, dy\, \underline{a_y}) \times (\underline{a_x} - y\, \underline{a_y})}{(1+y^2)^{3/2}} = + \frac{\mu_0 I\, a_z}{4\pi} \int_0^{-1} \frac{dy}{(1+y^2)^{3/2}}$$

3. side #3 (bottom)

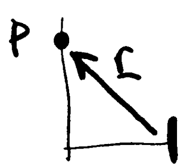


$\underline{r} = -x\, \underline{a_x} + \underline{a_y}$ $\qquad I\, d\underline{l} = I\, dx\, \underline{a_x}$

$$\underline{a_r} = \frac{-x\, \underline{a_x} + \underline{a_y}}{(x^2+1)^{1/2}}$$

$$\underline{B}_3 = \frac{\mu_0}{4\pi} \int_{-1}^{+1} \frac{I\, dx\, \underline{a_x} \times (-x\, \underline{a_x} + \frac{1}{2}\underline{a_y})}{(x^2+1)^{3/2}} = \frac{\mu_0 I\, \underline{a_z}}{4\pi} \int_{-1}^{1} \frac{dx}{(x^2+1)^{3/2}}$$

4. side #4 (essentially same as side #2)



$\underline{r} = -\underline{a_x} - y\, \underline{a_y}$ $\qquad I\, d\underline{l} = +I\, dy\, \underline{a_y}$

$$\underline{a_r} = \frac{-\underline{a_x} - y\, \underline{a_y}}{\sqrt{1+y^2}}$$

$$\underline{B}_4 = \frac{\mu_0}{4\pi} \int_{-1}^{0} \frac{(I\, dy\, \underline{a_y}) \times (-\underline{a_x} - y\, \underline{a_y})}{(1+y^2)^{3/2}} = \frac{\mu_0 I\, \underline{a_z}}{4\pi} \int_{-1}^{0} \frac{dy}{(1+y^2)^{3/2}}$$

$$\underline{B}(P) = \underline{B}_1 + \underline{B}_2 + \underline{B}_3 + \underline{B}_4$$

2. We know that certain boundary conditions require Fourier series solutions of Laplace's equation. Match the appropriate series solution to its boundary conditions

Solutions :

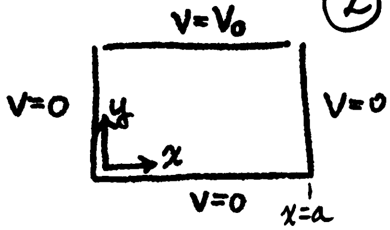
(a) $V(x,y) = \sum C_n e^{-k_n x} \sin k_n y$

(b) $V(x,y) = \sum c_n \left(e^{k_n x} - e^{-k_n x}\right) \sin k_n y$

(c) $V(x,y) = \sum c_n \sin k_n x \left(e^{k_n y} - e^{-k_n y}\right)$

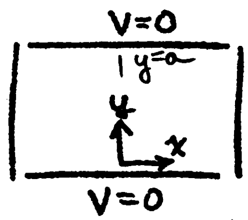
(d) $V(x,y) = \sum c_n \left(e^{k_n x} + e^{-k_n x}\right) \sin k_n y$

(e) $V(x,y) = \sum c_n \sin k_n x \left(e^{k_n y} + e^{-k_n y}\right)$



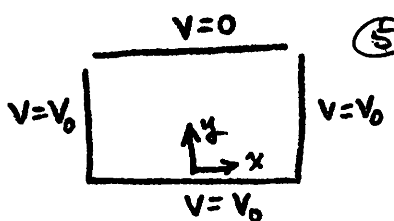
Solution: c

② Also an easy one. Zeros at $x=0$ and $x=a$ $V=V_0$ indicate sine in x direction, could be (c) or (e) but only (c) is zero at $y=0$. So, answer is (c)



Solution: b

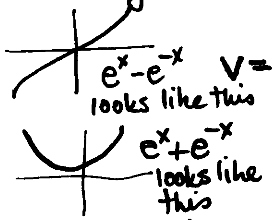
④ Note: Only remaining solutions are (b) and (c). Zero's at $y=0$, $y=a$ indicate sine in y, i.e. (b) or (d) solution. This one is harder: we must use (b) because it can match B.C. (d) cannot.



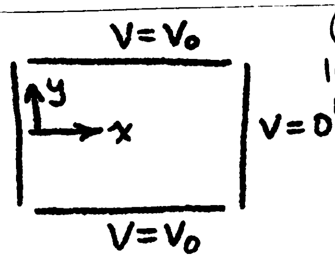
Solution: d

Pick (d) here as last choice.

⑤ continuing from ④

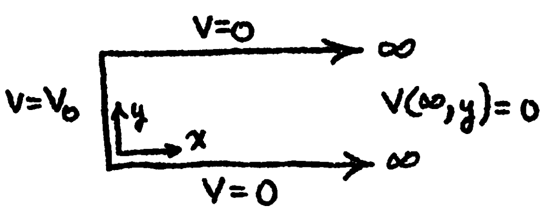


V=0 cannot match B.C's,



Solution: e

③ This is almost like ②. Requires sine in x direction. But $V(x,0) \neq 0$ this indicates (e)

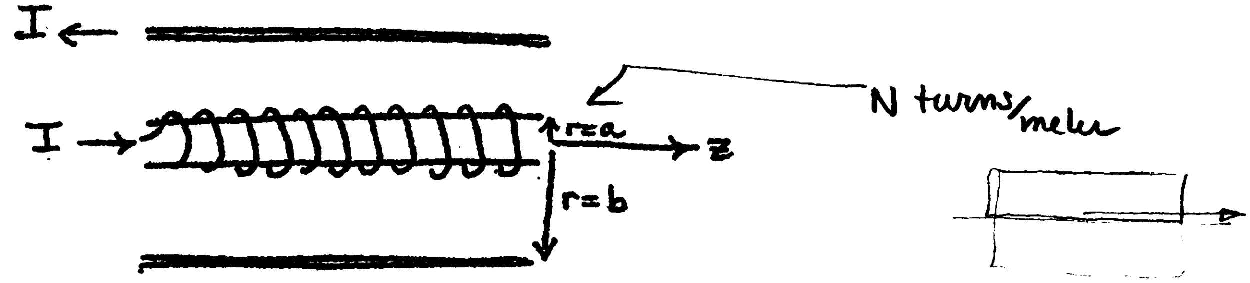


Solution: a

① First and easiest one to determine.
$V(x,y) = 0$ at $x=0$
$V(x,y) = 0$ at $x=0$.
This is obviously (a). Series solution will match V_0 at $x=0$.

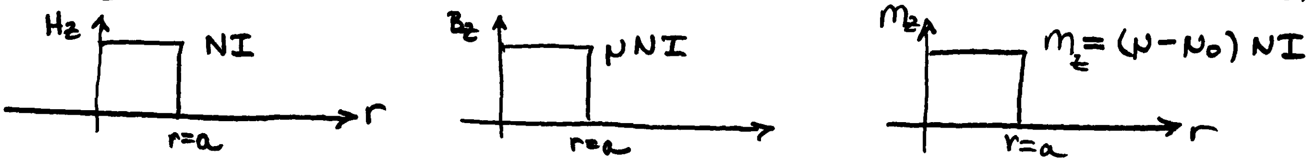
NOTE: All problems are in x-y Cartesian coordinate systems BUT not all coordinate system origins are at the same location relative to the boundary conditions.

3. A coaxial cable is constructed in which the inner conductor is a helix (i.e. a solenoid) wound over a core of permeability $\mu > \mu_0$. A current I flows through the inner conductor and returns through a thin-wall cylindrical conductor enclosing the helical core.



(a) For the inner conductor plot $\underline{B}, \underline{H}, \underline{m}$ as functions of r. State any assumptions.

This is a simple, infinitely long solenoid. As long as the solenoid is infinitely long all the field is confined to the interior of the solenoid and it has a normal field.



(b) What is the equivalent magnetic current for $\underline{m}$? Plot this equivalent current $\underline{J_m}$ as a function of r. $\underline{J_m} = \underline{\nabla} \times \underline{m}$

differentiating $\underline{m}$

$$J_m = \underline{\nabla} \times \underline{m} = -\underline{a_\phi} \frac{\partial m_z}{\partial r} = \underline{a_\phi}\,(\mu - \mu_0)\, NI$$

The current flowing the helix may be regarded as a current I flowing in the z-direction PLUS a current I flowing in a solenoid.

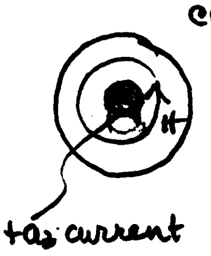
(c) what is the inductance associated with the helix? Hint, see (a).

The inductance is simply the flux through the helix divided by the current producing that flux. For any length ℓ, the total flux λ is given by

$\lambda = N\ell\, BA = N\ell(\mu N I)\pi a^2 = \mu N^2 \ell\, I \pi a^2$. $L = \lambda/I = \mu N^2 \ell \pi a^2$.

The resulting inductance per unit length is then $L/\ell = \mu N^2 \pi a^2$

(d) What is the inductance associated with the conventional (z-directed) current flow?



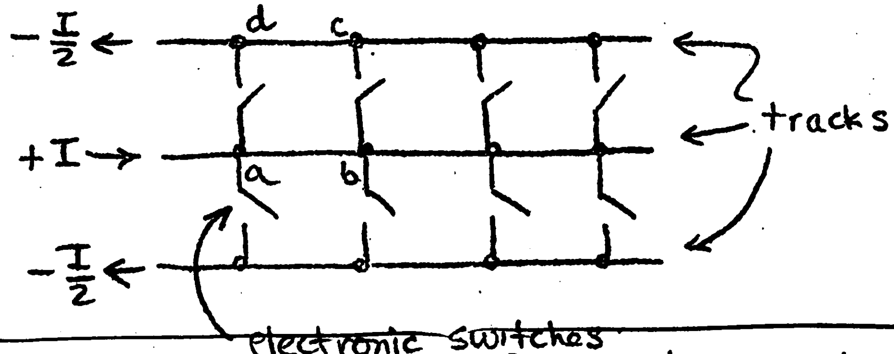
$$\oint \underline{H} \cdot \underline{d\ell} = \int \underline{J} \cdot \underline{dS} \Rightarrow H_\phi\, 2\pi r = I \qquad \therefore H_\phi = \frac{I}{2\pi r} \qquad B_\phi = \frac{\mu_0 I}{2\pi r}$$

$$\text{the flux } \lambda = \int \underline{B} \cdot \underline{dS} = \int_a^b \frac{\mu_0 I}{2\pi r}\, dr = \frac{\mu_0 I}{2\pi} \ln\left(\frac{b}{a}\right)$$

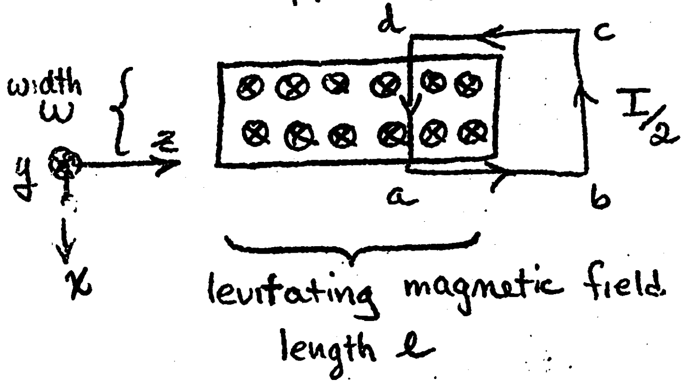
$$L = \frac{\lambda}{I} = \frac{\mu_0}{2\pi} \ln(b/a)$$

4. magnetic train (cont.)

The train is moved by magnetically produced forces. These forces are generated by electronically switching currents in conductors in the solid slab track as shown below.



In terms of a single loop formed by points a.b c d consider what happens if the train is positioned over this loop.



The train is moving in the z-direction with velocity u

(f) Using the Lorentz force law what is the force on the train?

$$\underline{dF} = dq\,\underline{v} \times \underline{B} = I\,\underline{dl} \times \underline{B}$$

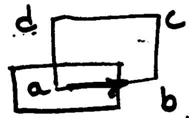
The only I which links the bottom of the train is segment ad (differents I's). For ad $I\underline{dl} = I\underline{a_x}$. Then $d\underline{F} = \frac{I}{2} dx\,\underline{a_x} \times \mu_0 \frac{NI}{2y}(\underline{a_y}) = \frac{I}{2} dx\, \frac{\mu_0 NI}{2y}\underline{a_z}$ $\therefore \underline{F} = \frac{\mu_0 w N I I}{4y}\underline{a_z}$

(g) Explain how the magnetic forces keep the train on track.

Hint: consider what happens if the train moves in the x direction

if train moves to one side as below

Since force $= \underline{I} \times \underline{B}$



side ad gives force in $\underline{a_z}$ direction

side ab gives force in $\underline{a_z} \times \underline{a_y} = -\underline{a_x}$ direction forcing train back on track.

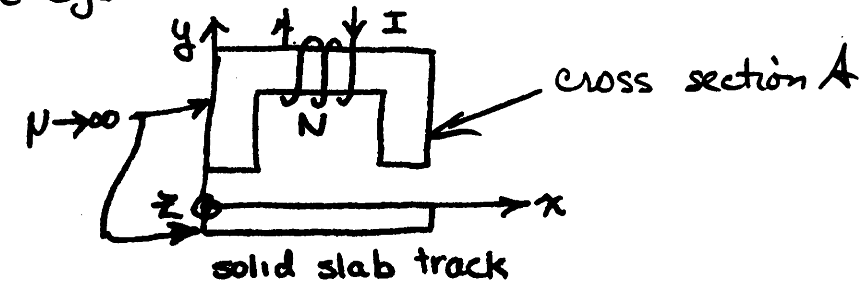
(h) If I inserted a meter in the loop abcd what voltage would it read?

$$v = -\frac{d\lambda}{dt} = -\frac{d}{dt}(l w B_{gap}) = -\frac{d}{dt}\left(l w \mu_0 \frac{NI}{2y}\right)$$

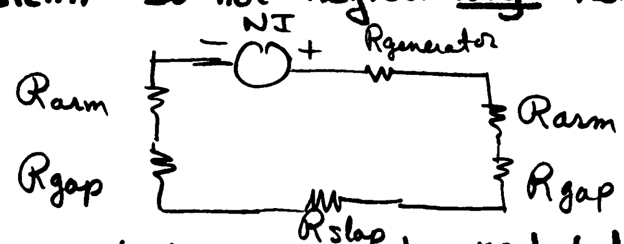
$$= -\frac{\mu_0 w N I}{2y}\frac{dl}{dt} \qquad \frac{dl}{dt} = \text{velocity } u$$

$$\therefore v = -\frac{\mu_0 w u N I}{2y}$$

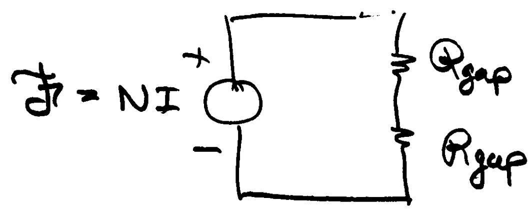
4. Magnetically levitated trains are being considered for urban transportation. The transverse cross-section of such a train can be modeled as the magnetic system shown below.



(a) Draw the equivalent magnetic circuit for the above magnetic system. Do not neglect any reluctances.



(b) What reluctances can be neglected and why? Draw the simplified circuit.



because $\mu \gg \mu_0$ so since $R = \dfrac{l}{\mu A}$
$R \to 0$ for $\mu \gg \mu_0$ materials.

(c) Find the B field in the gap.

$$R_{gap} = \frac{l}{\mu_0 A} = \frac{y}{\mu_0 A} \qquad \lambda_{gap} = B_{gap} A_{gap} = \frac{\mathcal{F}}{2R_{gap}} = \frac{NI}{2y/\mu_0 A_{gap}}$$

$$\therefore\ B_{gap} = \mu_0 \frac{NI}{2y} \qquad H_{gap} = \frac{NI}{2y}$$

(d) What is the energy stored in the B field. (in the gap)?

$$W_m = \int \frac{1}{2}\mu |H_{gap}|^2 dv = 2\,\frac{1}{2}\,\mu_0 \left(\frac{NI}{2y}\right)^2 A y = \mu_0 \left(\frac{NI}{2y}\right)^2 A y$$

↑ since 2 gaps

(e) What is the levating force on the train?

$$f_m = -\frac{\partial W_m}{\partial y} = -\mu_0 \frac{N^2 I^2}{4} A \frac{\partial}{\partial y}\left(\frac{1}{y}\right)$$

$$= +\mu_0 \frac{N^2 I^2}{4} \frac{A}{y^2}$$

EEAP 210 - ELECTROMAGNETIC FIELD THEORY

EXAM #3 MAKE-UP

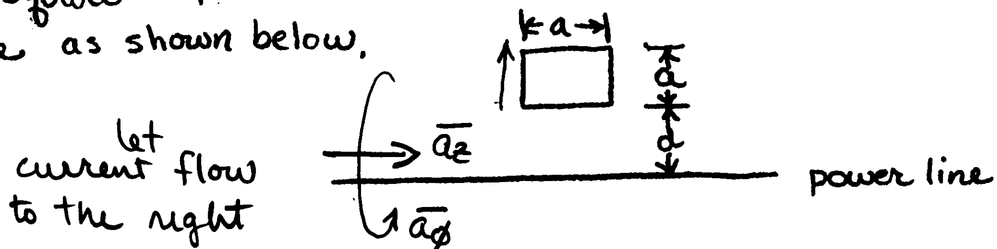
NAME: SOLUTIONS

INSTRUCTIONS & NOTES:

1. A double sided 8.5x11 inch formula sheet is permitted. This formula sheet will be collected with the exam.
2. Closed book & notes.
3. Problems are roughly ranked in terms of difficulty.
4. Not all problems are worth the same.
5. THE HIGHER GRADE (DETERMINED AS A PERCENTAGE OF THE MEAN EXAM SCORE) OF EXAM 3 OR THIS EXAM WILL BE USED IN DETERMINING YOUR FINAL COURSE GRADE.

PROBLEM	SCORE	VALUE
1		10
2		15
3		20
4		25
5		30
TOTAL		

1. A square loop of sides a is situated a distance d from a power line as shown below.



a) Find the B-field due to a current I passing thru the power line.

b) Find the flux thru the square loop.

c) If $I = I_0 \sin \omega t$ (for the power line) what voltage would a meter inserted in the square loop read?

a) $$\oint \underline{H} \cdot d\underline{l} = \int \underline{J} \cdot d\underline{s}$$

$$H_\phi \cdot 2\pi r = I$$

$$H_\phi = \frac{I}{2\pi r} \qquad B_\phi = \frac{\mu_0 I}{2\pi r}$$

b) Flux thru square loop $\triangleq \lambda$

$$\lambda = \int_d^{d+a} \int_0^a \frac{\mu_0 I}{2\pi r} \, dr \, dz = \frac{\mu_0 I}{2\pi} a \int_d^{d+a} \frac{dr}{r}$$

$$= \frac{\mu_0 I a}{2\pi} \ln\left(\frac{d+a}{d}\right)$$

(c) If $I = I_0 \sin \omega t$

$$v = -\frac{d\lambda}{dt} = -\frac{d}{dt}\left[\mu_0 \frac{I a}{2\pi} \ln\left(\frac{d+a}{d}\right)\right]$$

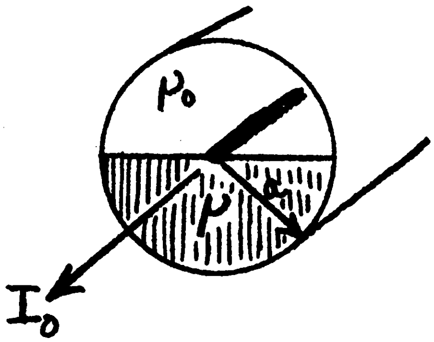
as only I is a function of time

$$v = -\frac{\mu_0 a}{2\pi} \ln\left(\frac{d+a}{d}\right) \frac{dI}{dt}$$

$$v = -\frac{\mu_0 a}{2\pi} \ln\left(\frac{d+a}{d}\right) I_0 \omega \cos \omega t$$

2. A coaxial cable is half-filled with material of permeability $\mu > \mu_0$ as shown below. Find expressions for $\underline{B}$, H, m.

Hint: what variable is continuous across the interface?



Answer to Hint: B_{normal} is continuous.

$$\oint \underline{H} \cdot \underline{dl} = \int \underline{J} \cdot \underline{ds}$$

$$H_{air}\, \pi r + H_{material}\, \pi r = I_0$$

obviously H is not continuous, but we use the hint and write H_{air} and $H_{material}$ in terms of B_n

$$\frac{B_n}{\mu_0} \pi r + \frac{B_n}{\mu} \pi r = I_0$$

$$B_n \left(\frac{1}{\mu_0} + \frac{1}{\mu}\right) \pi r = I_0$$

<u>for B</u> everywhere $B_n = \dfrac{\mu \mu_0}{\mu + \mu_0} \dfrac{I_0}{\pi r}$

<u>for H</u> $H_{air} = \dfrac{\mu I_0}{\pi r(\mu + \mu_0)}$ $\quad H_{material} = \dfrac{\mu_0 I_0}{\pi r(\mu + \mu_0)}$

<u>for m</u> since $\mu = \mu_0$ in the non-filled half of the cable $m = 0$ there.

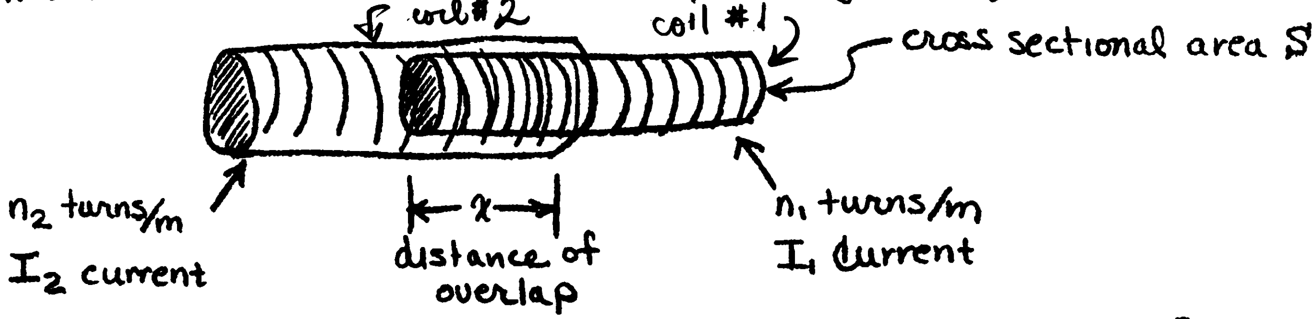
since $\mu \neq \mu_0$ in the filled half.

$$B = \mu_0 (H + m)$$

$$m = \frac{B}{\mu_0} - H_{material} = \frac{\mu \mu_0 I_0}{\mu_0 \pi r (\mu + \mu_0)} - \frac{\mu_0 I_0}{\pi r (\mu + \mu_0)}$$

$$m = \frac{\mu - \mu_0}{\mu + \mu_0} \frac{I_0}{\pi r}$$

3. Two solenoids interpenetrate as shown below. End effects and differences in solenoid diameter may be neglected.



(a) What is the mutual inductance between the solenoids?
(b) what is the energy stored in the region of overlap?
(c) what is the force between the two solenoids?

(a) H field in first coil $= n_1 l I_1$ l = length of first coil

flux in first coil $= BS = \mu_0 n_1 I_1 l S$

Φ_{21} = flux linked by second coil $= \underbrace{(n_2 x)}_{\substack{\text{\# of turns of coil \#2} \\ \text{linking flux of coil \#1}}} \; \underbrace{\mu_0 n_1 l I_1 S}_{\text{flux due to first coil}}$

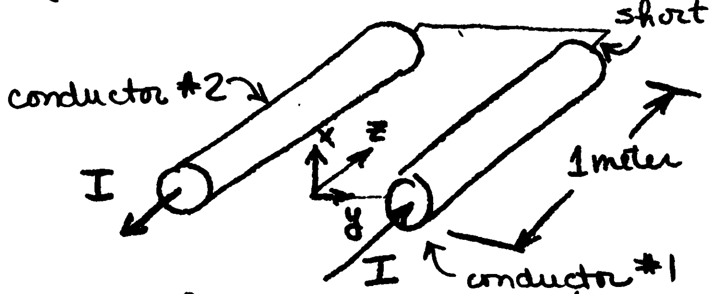
mutual inductance $L_{21} = \dfrac{\Phi_{21}}{I_1} = \dfrac{\mu_0 n_2 n_1 l I_1 S x}{I_1} = \boxed{\mu_0 n_1 n_2 S x l}$

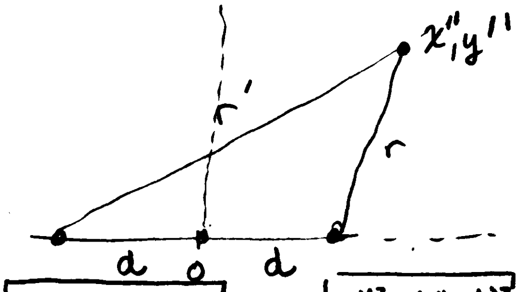
(b) $W_m = \frac{1}{2} L I^2 = \frac{1}{2} \mu_0 n_1 n_2 S x l I_1 I_2$ The actual W_m is twice this because the mutual inductances are equal.

(c) $f_x = -\dfrac{\partial W_m}{\partial x} = - \mu_0 n_1 n_2 l S I_1 I_2$

There are many ways to get (c) depending upon which fields you calculated.

4. Two parallel cylindrical conductors of radius a are separated by a distance 2d as shown below.





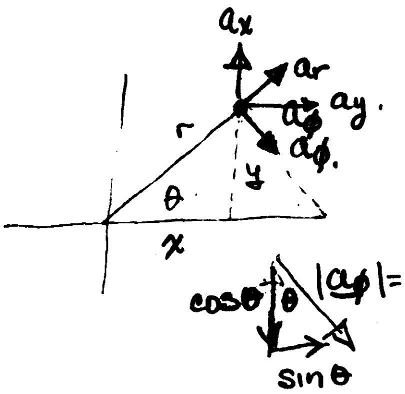
$r' = \sqrt{x''^2 + (y''+d)^2}$ $\quad r = \sqrt{x''^2 + (y''-d)^2}$

(a) Find the H field in the x, y plane. You may assume no z-dependence. Hint: use superposition

(b) What is the self-inductance between the conductors?

from problem #1 for a conductor $H_\phi = \dfrac{I}{2\pi r}$

$$\underline{H}_{total} = \frac{I}{2\pi} \underbrace{\frac{1}{r}\, \underline{a_\phi}}_{\text{coordinates of conductor \#1}} - \frac{I}{2\pi} \underbrace{\frac{1}{r'}\, \underline{a_\phi'}}_{\text{coordinates of conductor \#2}} \qquad (*)$$



$$a_\phi = \sin\theta\, \underline{a_y} - \cos\theta\, \underline{a_x}$$

where $\cos\theta = \dfrac{y}{r} = \overbrace{\dfrac{y''-d}{r}}^{\text{cond. \#1}} \qquad \overbrace{\dfrac{y''+d}{r'}}^{\text{cond \#2}}$

$\sin\theta = \dfrac{x}{r} = \dfrac{x''}{r} \qquad \dfrac{x''}{r'}$

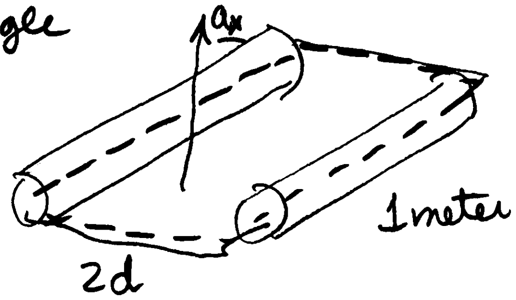
$$H_{total} = \frac{I}{2\pi} \underbrace{\frac{\sin\theta\, \underline{a_y} - \cos\theta\, \underline{a_x}}{r}}_{\text{cond \#1}} - \frac{I}{2\pi} \underbrace{\frac{\sin\theta\, \underline{a_y} - \cos\theta\, \underline{a_x}}{r'}}_{\text{cond \#2.}}$$

$$= \frac{I}{2\pi}\left[\frac{x''\, \underline{a_y} - (y''-d)\underline{a_x}}{r^2} - \frac{x''\, \underline{a_y} - (y''+d)\underline{a_x}}{r'^2}\right]$$

$$= \frac{I}{2\pi}\left[\underline{a_x}\left\{\frac{(y''+d)}{r^2} - \frac{(y''-d)\, a_y}{r'^2}\right\} + \underline{a_y}\left\{\frac{x''}{r'^2} - \frac{x''}{r^2}\right\}\right]$$

The answer labeled (*) and any after are acceptable.

(b) for self-inductance we are concerned by the flux intercepted by a contour enclosing the circuit, i.e. a rectangle



from (a) this lies in the y-z plane, i.e. $x''=0$.

then $r' = y''+d \qquad r = y''-d$

and $$H_{total} = \frac{I}{2\pi}\,\underline{a}_x\left[\frac{1}{y''+d} - \frac{1}{y''-d}\right]$$

and is entirely y directed.

By definition $L = \dfrac{\Phi}{I}$.

$$\Phi = \int_{-(d+a)}^{+(d+a)} \mu_0 H_x\,(1\,\text{meter})\,dy'' = \frac{\mu_0 I}{2\pi}\int_{-(d+a)}^{+(d+a)}\left(\frac{1}{y''+d} - \frac{1}{y''-d}\right)dy''$$

$$= \frac{\mu_0 I}{2\pi}\Big[\ln(y+d) - \ln(y-d)\Big]_{-d+a}^{d+a}$$

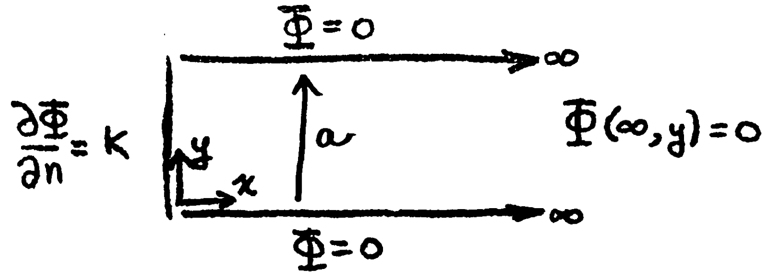
$$= \frac{\mu_0 I}{2\pi}\Big[\ln(2d+a) - \ln(a) - \ln(-a) + \ln(-2d-a)\Big]$$

$$= \frac{\mu_0 I}{2\pi}\ln\frac{2d+a}{a}\cdot\frac{-2d-a}{-a} = \frac{\mu_0 I}{2\pi}\ln\left(\frac{2d+a}{a}\right)^2$$

$$= \frac{\mu_0 I}{\pi}\ln\left(\frac{2d+a}{a}\right)$$

$$L = \frac{\mu_0}{\pi}\ln\left(\frac{2d+a}{a}\right)$$

5. Find $\Phi(x,y)$ for the boundary conditions shown below.



$$\nabla^2 \Phi = 0$$

by separation of variables $k_x^2 + k_y^2 = 0$. where $\Phi(x,y) = f(x)g(y)$ which leaves us with a choice of cosines or exponentials for solutions along each axis.

pick exponentials in x sines & cosines in y.

$$f(x) = c_1 e^{-kx} + c_2 e^{+kx} \qquad g(y) = c_3 \cos ky + c_4 \sin ky$$

note that the signs of k_x^2 and k_y^2 are included in these solutions as separation of variables requires $k_x^2 = -k_y^2$. so we can drop the subscript.

Applying the boundary conditions in each direction:

$\Phi(\infty, y) = 0$ requires $C_2 = 0$ $\Phi(x,0) = 0$ requires $C_3 = 0$

$\Phi(x,a) = 0$ requires $k = \frac{n\pi}{a}$

At this point $\Phi_n(x,y) = f(x)g(y) = C_1 e^{-kx} C_4 \sin ky$ where $k = \frac{n\pi}{a}$

since any n is as good as another our final solution must be a sum of all such solutions

$$\Phi(x,y) = \sum_{n=1}^{\infty} c_n e^{-k_n x} \sin k_n y \qquad k_n \triangleq \frac{n\pi}{a}$$

to meet the boundary condition at $x=0$

$$K = \left. \frac{\partial \Phi(x,y)}{\partial x} \right|_{x=0} = \left| \sum_{n=1}^{\infty} c_n(-k_n) e^{-k_n x} \sin k_n y \right|_{x=0}$$

$$\therefore \quad K = \sum_{n=1}^{\infty} c_n(-k_n) \sin k_n y$$

multiplying both sides by $\sin k_m y$ and integrating over y.

$$\int_0^a K \sin k_m y \, dy = \sum -c_n k_n \underbrace{\int_0^a \sin k_n y \sin k_m y \, dy}_{= \frac{1}{2} \text{ if } m=n, \text{ 0 otherwise}}$$

$$\therefore \quad C_m = -\frac{2\int_0^a K \sin k_m y \, dy}{k_m} \quad \text{where } k_m = \frac{m\pi}{a}.$$

67 took exam at scheduled time

EEAP 210 – ELECTROMAGNETIC FIELD THEORY
FINAL EXAM
MAY 8, 1984

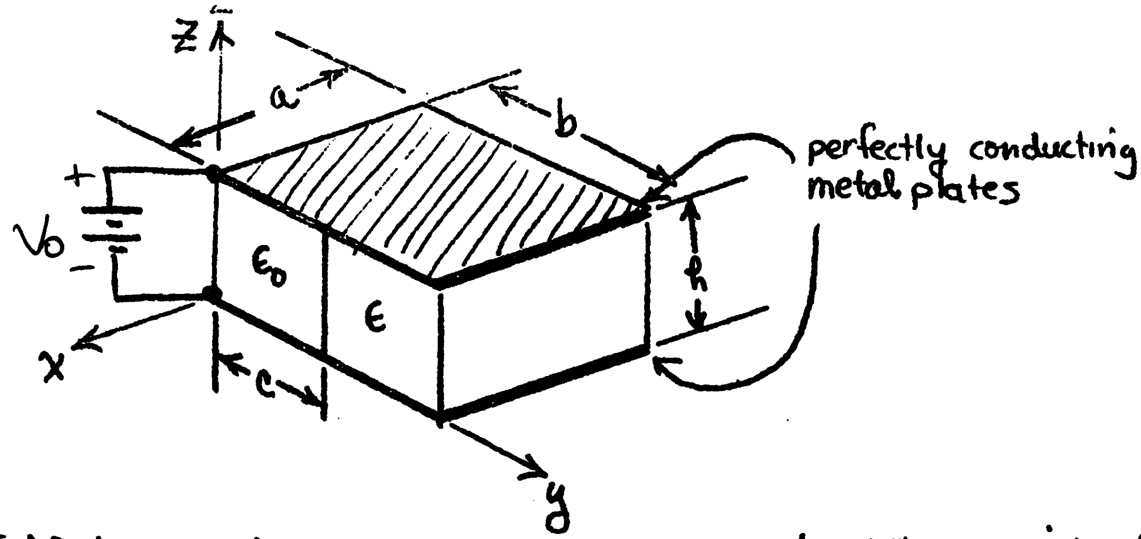
NAME:

INSTRUCTIONS & NOTES

1. You are allowed 3 8.5×11 formula sheets, i.e. one sheet double sided plus one side of another. These sheets will be collected with the exam.
2. Closed book and notes.

PROBLEM	SCORE	VALUE
1.		50
2.		50
3.		50
4.		50
5.		50
6.		50
TOTAL		300

1.

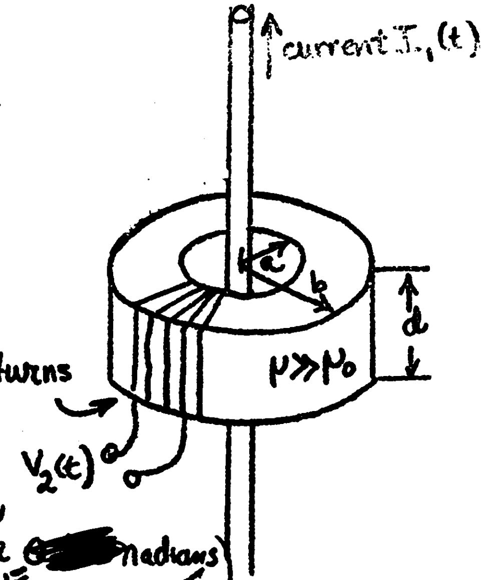


(a) Determine the capacitance of the parallel-plate capacitor (with two dielectrics) shown above.

(b) What is the total energy stored in the field?

(c) What is the force upon the section of dielectric where $\epsilon \neq \epsilon_0$. Specify its direction.

. The device shown on the right consists of a toroidal coil with a current carrying wire thru its center. An n- turn coil is wrapped around the toroid.

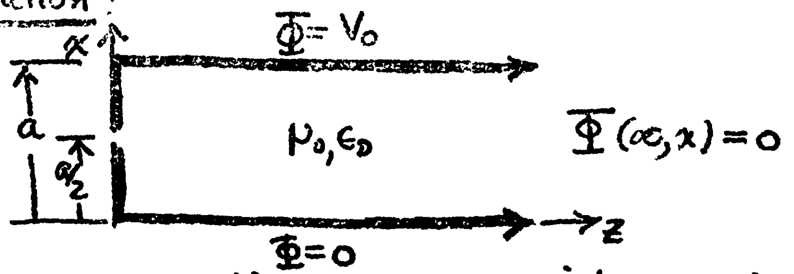


(a) write an expression for the mutual inductance M between the wire and the toroidal coil if the current in the wire is $I(t) = I_0 \sin \omega t$

(b) If $\mu = 5000\mu_0$, $a = 1\text{cm}$, $b = 2\text{cm}$, $d = 2\text{cm}$, $n = 100$ and $I_0 = 10$ amperes, ($f = 60\text{Hz}$. or $\omega =$ [redacted] 377 radians/sec)

what is $V_2(t)$?

3. Laplace's Equation:



(a) Write Laplace's Equation in the appropriate coordinate system. Indicate the general solution for this problem.

(b) Applying the given boundary conditions solve for all constants. You may leave integrals as such.

4. <u>Propagation</u> An electric field is of the form

$$\underline{E} = 100\, e^{j(2\pi \times 10^{6} t + 2\pi \times 10^{-2} z)}\ \underline{a_x} \text{ volts/meter.}$$

actually $\mu \neq 4\pi \times 10^{-7}$

(a) What is the (linear) frequency and wavelength of the field (wave)?

(b) If the medium is a <u>lossless</u> dielectric with $\boxed{\mu = 4\pi \times 10^{-7}}$ H/m and $\epsilon = 4\epsilon_0$ $\left(\epsilon_0 = \frac{1}{36\pi} \times 10^{-9} \text{ F/m}\right)$, what is the wave impedance η and the magnetic field H?

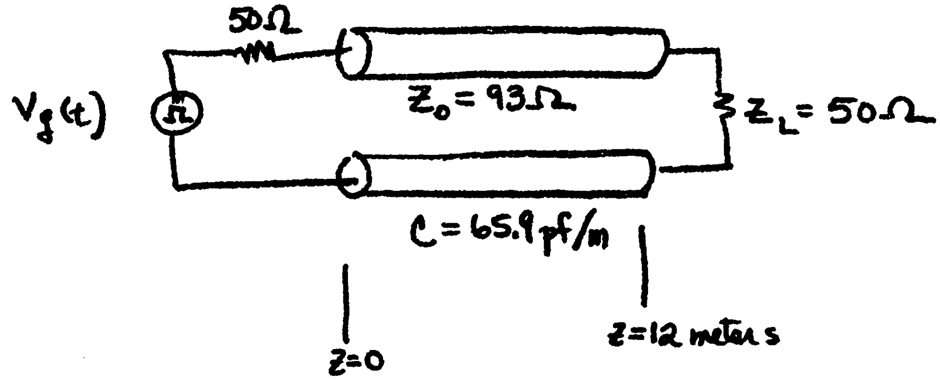
(c) What is the time averaged power/unit area carried by the wave, i.e. the Poynting vector $\mathcal{S}$?

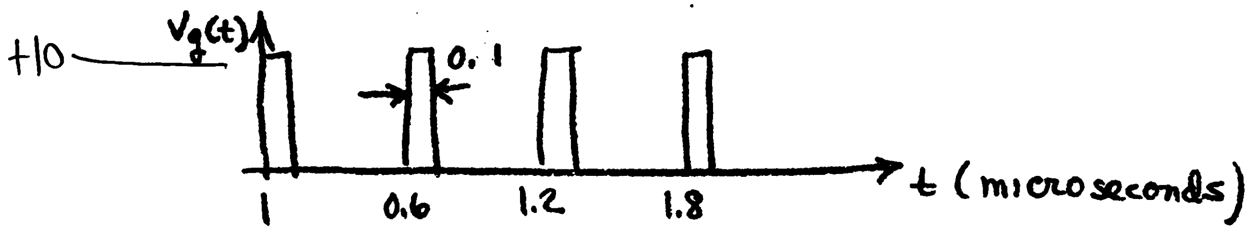
(d) If the medium becomes lossy with $\sigma = 1/\Omega\text{-m}$ and $\mu_0, 4\epsilon_0$ as before write the expression for γ^2. Identify the conduction and displacement current terms. What are α and β (numerically)? Is the material a conductor or insulator?

(e) For the wave of (d), write expressions for $\underline{E}$ and $\underline{H}$ if the wave propagates at an angle of 30° relative to the Z-axis.

Note: sine 30° = 0.5, cos 30° = 0.866, tan 30° = 0.577) is this specific enough.

5. Pulses on transmission line:





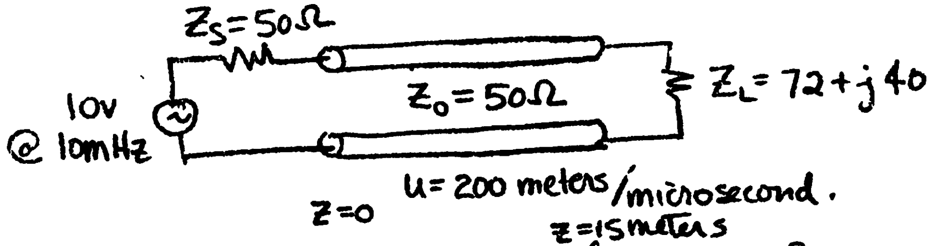
(a) What is the input voltage to the cable, i.e. $V(0, t=0+)$, at the beginning of the first pulse?

(b) When does the first pulse arrive at the load and what is the resulting voltage across the load?

(c) The generator sends out four pulses as shown above. Sketch $V(z=12m, t)$. You may ignore a pulse after its <u>second</u> reflection from the load. Indicate both time and voltage in your drawing.

(d) If $Z_0 = 50\Omega$, sketch $V(z=12m, t)$

5. An antenna with an impedance of $72+j40\,\Omega$ is connected to a 10 MHz generator via a 15-meter section of RG-58A/U coaxial cable as shown below. The cable is driven by a generator with an open circuit voltage output of 10-volts and an internal impedance of 50Ω.



(a) What is the input impedance of the cable?

(b) What is the voltage measured at the load?

(c) What is the power delivered to the antenna?

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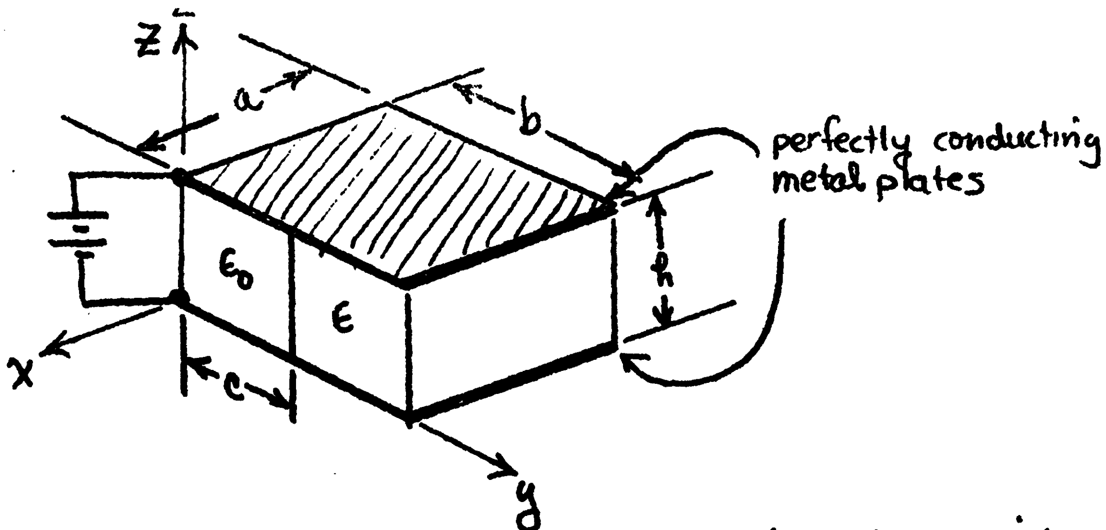

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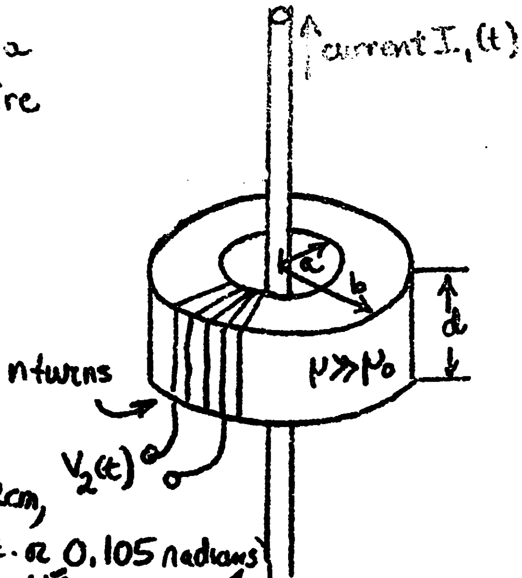

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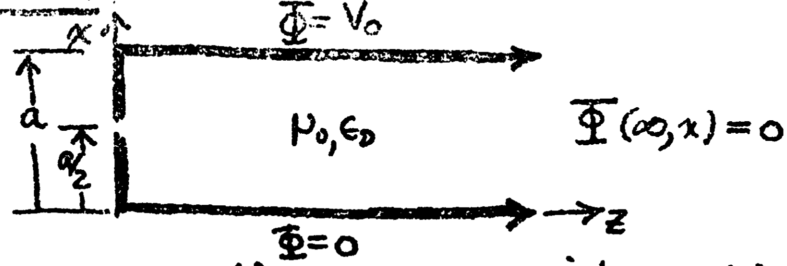
3. Laplace's Equation



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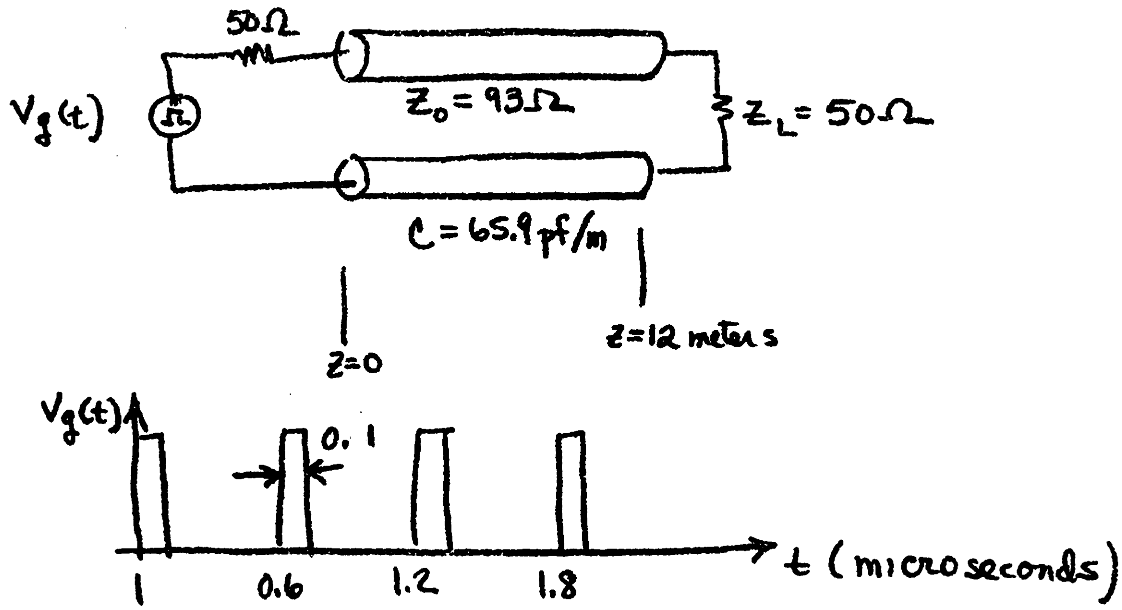

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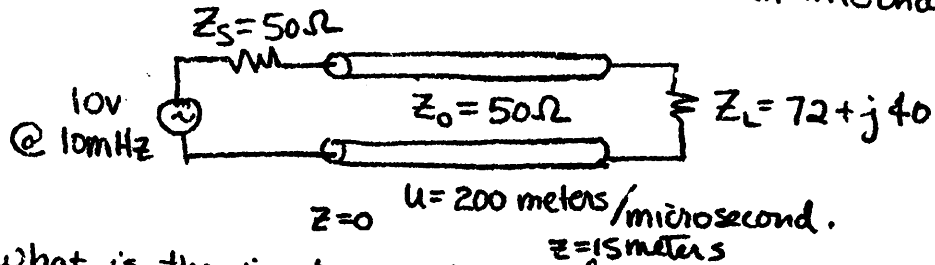

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