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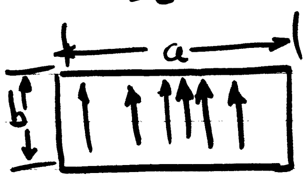
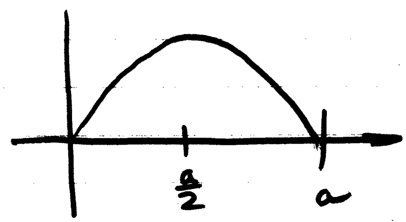
wave guides

$$\bar{E} = (V^+ e^{-j\beta z} + V^- e^{+j\beta z})\,\bar{e}(x,y)$$

$$\bar{H} = (I^+ e^{-j\beta z} - I^- e^{j\beta z})\,\bar{h}(x,y)$$

$$P = \frac{1}{2}\frac{|V^+|^2}{Z_c}$$





$$\beta = \sqrt{\omega^2\mu\epsilon - \frac{\pi^2}{a^2}}$$

for propagation $\omega > \frac{\pi}{a} c$

or $a > \frac{\lambda_0}{2}$

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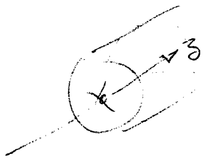
<u>TE or H modes</u> $\quad (H_z \neq 0, \; E_z = 0)$



$$\left.\begin{aligned} \bar{E} &= \bar{e}(x,y)\, e^{-j\beta z} \\ \bar{H} &= \bar{h}(x,y)\, e^{-j\beta z} + \bar{h}_z\, e^{-j\beta z} \end{aligned}\right\} e^{j\omega t}$$

$$\nabla \times \bar{E} = -j\omega\mu_0 \bar{H}$$
$$\nabla \times \bar{H} = j\omega\epsilon_0 \bar{E}$$

$$\nabla = \nabla_t + \bar{a}_z \frac{\partial}{\partial z} = \nabla_t - j\beta \bar{a}_z$$

$$(\nabla_t - j\beta\bar{a}_z) \times (\bar{h} + \bar{h}_z) = j\omega\epsilon_0 \bar{e}$$
$$(\nabla_t - j\beta\bar{a}_z) \times (\bar{e}) = -j\omega\mu_0(\bar{h} + \bar{h}_z)$$

equating z-components

1) $\nabla_t \times \bar{h} = 0$

2) $\nabla_t \times \bar{e} = -j\omega\mu_0 \bar{h}_z$

equating transverse components

3) $\nabla_t \times \bar{h}_z - j\beta\bar{a}_z \times \bar{h} = j\omega\epsilon_0 \bar{e}$

4) $-j\beta\bar{a}_z \times \bar{e} = -j\omega\mu_0 \bar{h}$

$$\bar{a}_z \times (\bar{a}_z \times e) = \frac{\omega\mu_0}{\beta}\, a_z \times \bar{h} = -\bar{e}$$

from # 3 $\quad \nabla_t \times (\nabla_t \times \bar{h}_z) = \nabla_t \cancel{\nabla_t \cdot \bar{h}_z}^{\,0} - \nabla_t^2 \bar{h}_z$

$$= j\omega\epsilon_0 \nabla_t \times \bar{e} + j\beta\left(-\frac{\beta}{\omega\mu_0} \nabla_t \times \bar{e}\right)$$

$$= \left(j\omega\epsilon_0 - \frac{j\beta^2}{\omega\mu_0}\right)(-j\omega\mu_0 h_z)$$

thus $(\nabla_t^2 - \beta^2 + k_0^2)\, \bar{h}_z = 0$

$$k_0^2 = \omega^2\mu_0\epsilon_0$$

Note:

$$\nabla_t \times \bar{a}_z h_z = -\bar{a}_z \times \nabla_t h_z$$
$$= +j\beta \bar{a}_z \times \bar{h} + j\omega\epsilon_0\left(-\frac{\omega\mu_0}{\beta}\bar{a}_z \times \bar{h}\right)$$

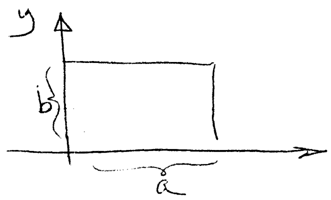
$\bar{a}_z \times [\quad] = 0 \Rightarrow [\quad]$ must add up to zero

$$(j\beta - jk_0^2)\,\bar{h} = -\nabla_t \bar{h}_z$$

$$j\left(\frac{j\beta^2 - jk_0^2}{j\beta}\right)\bar{h} = -\nabla_t h_z$$

$$h = \frac{-j\beta}{k_0^2 - \beta^2}\nabla_t h_z$$

Ex: A rectangular waveguide



boundary conditions

i) $B_{Tan} = 0$

$\frac{\partial h_z}{\partial x} = 0$ at $x = 0, a$

$\frac{\partial h_z}{\partial y} = 0$ at $y = 0, b$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2\right) h_z = 0$$

$k_c^2 = -\beta^2 + k_0^2$: $k_0 < k_c$ β is imaginary

$$h_z(x,y) = f(x)\,g(y)$$

$$= g\frac{d^2f}{dx^2} + f\frac{d^2g}{dy^2} + k_c^2 fg = 0$$

$$= \underbrace{\frac{1}{f}\frac{d^2f}{dx^2}}_{-k_x^2} + \underbrace{\frac{1}{g}\frac{d^2g}{dy^2}}_{-k_y^2} + k_c^2 = 0$$

Then $\frac{d^2f}{dx^2} + k_x^2 f = 0$

$\frac{d^2g}{dy^2} + k_y^2 g = 0$

where $k_x^2 + k_y^2 = k_c^2$

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$$\Rightarrow f = A_1 \sin k_x x + A_2 \cos k_x x$$
$$g = B_1 \sin k_y y + B_2 \cos k_y y$$

$$h_z = (A_1 \sin k_x x + A_2 \cos k_x x)(B_1 \sin k_y y + B_2 \cos k_y y)$$

from B.C.'s A_1 & $B_1 = 0$

then $h_z = A_2 B_2 \cos k_x x \cos k_y y$

$$\frac{dh_z}{dx} = -A_2 B_2 k_x \cos k_x x \cos k_y y = 0 \quad @ \; x = a$$

$\sin k_x x = 0$ for $x = a$

$$k_x a = n\pi$$
$$k_x = \frac{n\pi}{a} \qquad n = 0, 1, 2, \ldots$$
$$k_y = \frac{m\pi}{b} \qquad m = 0, 1, 2, \ldots$$

TE_{nm} mode

$$h_{z,nm} = A_{nm} \cos \frac{n\pi x}{a} \, B_{nm} \cos \frac{m\pi y}{b}$$

$$k_{c,nm}^2 = \left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2$$

$$\beta_{nm} = \sqrt{k_0^2 - k_{c,nm}^2}$$

$$\beta_{nm} = \sqrt{k_0^2 - \left(\frac{n\pi}{a}\right)^2 - \left(\frac{m\pi}{b}\right)^2}$$

TE_{nm} mode propagates only if $k_0 > k_{c,nm}$

Note: the TE_{10} mode will be the first to propagate

want to design the wave guide for a particular mode so as not to worry about matching phase of waves; i.e. design to support only one frequency

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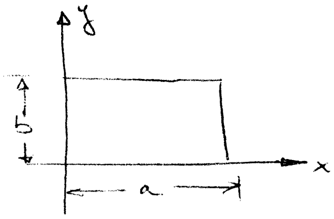


TE waves

$$\bar{E} = \bar{e}\, e^{-j\beta z}$$

$$\bar{H} = (\bar{h} + \bar{h}_z)\, e^{-j\beta z}$$

$$(\nabla_t^2 + k_c^2)\, h_z = 0 \qquad k_c^2 = k_0^2 - \beta^2$$

$$\bar{h} = -\frac{j\beta}{k_c^2} \nabla_t h_z$$

$$\bar{e} = -\frac{k_0 Z_0}{\beta}\, \bar{a}_z \times \bar{h} = -Z_w \bar{a}_z \times \bar{h}$$

$Z_w \triangleq$ wave impedance

$$\omega\mu_0 = \underbrace{\omega\sqrt{\mu_0\epsilon_0}}_{k_0}\, \underbrace{\sqrt{\frac{\mu_0}{\epsilon_0}}}_{Z_0}$$

B.C. $\dfrac{\partial h_z}{\partial n} = 0$ so that $\bar{n}\cdot\bar{e} = 0$ on boundary

nm-th solution $\quad TE_{nm}$ mode

$$h_z = A_{nm} \cos\frac{n\pi x}{a} \cos\frac{m\pi y}{b}$$

$$k_{c,mn}^2 = k_x^2 + k_y^2 = \left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2$$

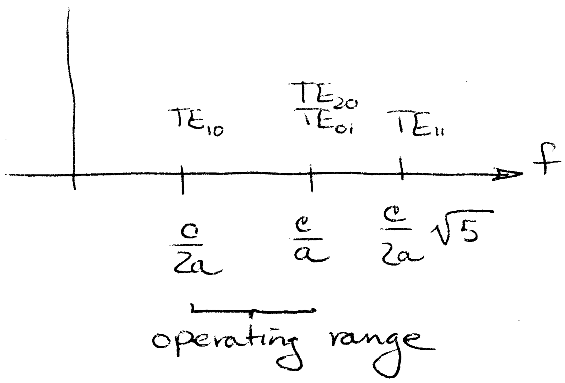
$$\beta_{nm} = \sqrt{k_0^2 - k_{c,nm}^2}$$ propagation only for $k_0 > k_{c,nm}$ or for $\mathrm{Re}[\beta_{nm}] \geq 0$

$$k_0 = \frac{\omega_c}{c} = \frac{2\pi f_0}{c} = \sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2}$$

for $a = 2b$

$$f_{c,nm} = \frac{c}{2}\sqrt{\frac{n^2}{a^2} + \frac{4m^2}{a^2}} = \frac{c}{2a}\sqrt{n^2 + 4m^2}$$

the lowes mode is the $nm = 10$



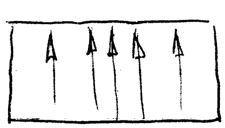
one does not operate close to the cutoff frequency in order to remain on the linear part of the graph

dominant mode (TE_{10} or H_{10} mode)

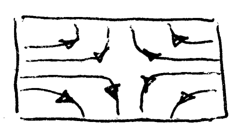
$$h_z = A \cos \frac{\pi x}{a}, \quad k_c = \frac{\pi}{a}$$

$$\bar{h} = \frac{j\beta A}{\frac{\pi}{a}} \sin \frac{\pi x}{a} \, \bar{a}_x$$

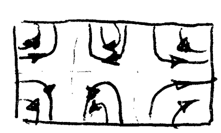
$$\bar{e} = \frac{A k_o}{\beta} Z_o \frac{j\beta}{\pi/a} \sin \frac{\pi x}{a} \, \bar{a}_y$$



TE_{10}



TE_{11}



TE_{21}

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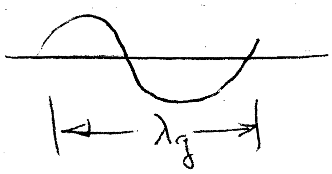
$$\bar{e} = V^+ \sin \frac{\pi x}{a} \, \bar{a_y} \, e^{-j\beta z}$$

$$\bar{h} = -\frac{V^+}{Z_w} \sin \frac{\pi x}{a} \, \bar{a_x} \, e^{-j\beta z}$$

$$\beta = \sqrt{k_o^2 - \left(\frac{\pi}{a}\right)^2} = \frac{2\pi}{\lambda_g}$$

$$k_o = \frac{2\pi}{\lambda_o} \qquad k_c = \frac{2\pi}{\lambda_c}$$

$\sin(\beta z - \omega t)$



$$\beta \lambda_g = 2\pi$$

$$\beta = \frac{2\pi}{\lambda_g}$$

then $\dfrac{2\pi}{\lambda_g} = \beta = \sqrt{\left(\dfrac{2\pi}{\lambda_o}\right)^2 - \left(\dfrac{2\pi}{\lambda_c}\right)^2}$

$$\lambda_g = \sqrt{\frac{\lambda_0^2 \, \lambda_c^2}{\lambda_c^2 - \lambda_0^2}} = \frac{\lambda_o}{\sqrt{1 - \frac{\lambda_0^2}{\lambda_c^2}}} > \lambda_o$$

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power flow

$$P = \frac{1}{2} \mathrm{Re} \int_S \bar{E} \times \bar{H}^* \cdot \bar{a}_z \, dx\,dy$$

$$= \frac{1}{2} \mathrm{Re} \int_S (E_x H_y^* - E_y H_x^*)\, dx\,dy$$

$$\frac{E_x}{H_y} = -\frac{E_y}{H_x} = Z_w$$

TE_{10}: $Z_w = \frac{k_0}{\beta} Z_0$ is real for a propagating mode

Then $P = \frac{1}{2} \mathrm{Re} \int_S \frac{1}{Z_w^*} \left(|E_x|^2 + |E_y|^2 \right) dx\,dy$

$\bar{J}_s = \bar{n} \times \bar{H}$: $\bar{H}$ solution in loss free case

$$\bar{E}_{tan} = Z_m \bar{J}_s$$

$$P_\ell = \text{power loss}/m = \oint \frac{1}{2} \mathrm{Re} \, |\bar{J}_s|^2 Z_m \, d\ell = \frac{1}{2} \frac{1}{\sigma \delta_s} \oint_C |\bar{H}|^2 \, d\ell$$

$$P = P_0 e^{-2\alpha z}$$

$$\frac{dP}{dz} = 2\alpha P = P_\ell \quad : \quad \alpha = \frac{P_\ell}{2P}$$

TM waves or E mode $E_z \neq 0$, $H_z = 0$

$$\bar{E}_t = -\frac{j\beta}{k_c^2} \nabla_t e_z(x,y) e^{-j\beta z}$$

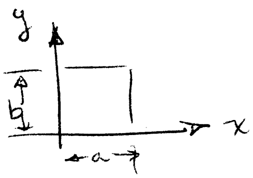
$$\bar{H}_t = Y_e \bar{a}_z \times \bar{E}_t \quad : \quad Y_e \triangleq \frac{k_0}{\beta} Y_0$$

NOTE: TE modes $Z_h = \frac{k_0}{\beta} Z_0$

$$Z_e = \frac{\beta}{k_0} Z_0$$

$$Z_e Z_h = Z_0^2$$

$$(\nabla_t^2 + k_c^2) e_z = 0 \quad ; \quad k_c^2 = k_0^2 - \beta^2$$

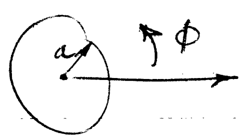


$$e_z = A_{nm} \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b}$$

$$k_{c,nm}^2 = \left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2$$

Note: no solution exists for $n = 0$ or $m = 0$

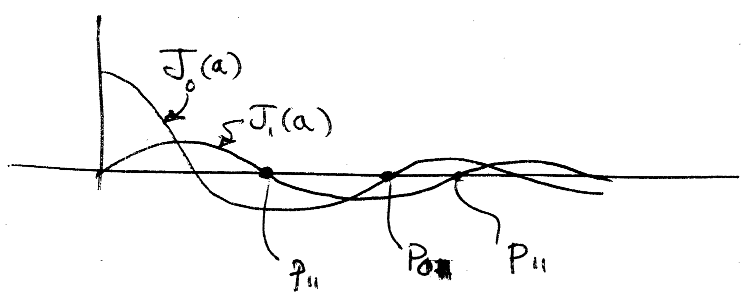
Thus TM_{11} is dominant TM mode



$$\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial e_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 e_z}{\partial \phi^2} + k_c^2 e_z\right) = 0$$

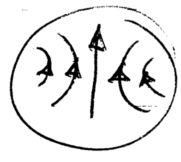
$$e_z = f(r)\, g(\phi)$$

$$e_z = J_n(k_c r) \begin{cases} \sin n\phi \\ \cos n\phi \end{cases}$$



for $n=1$ $\quad k_c a = P_{11}, P_{12}, \ldots$

$$k_c = \frac{P_{11}}{a}, \ldots$$



TE_{11}

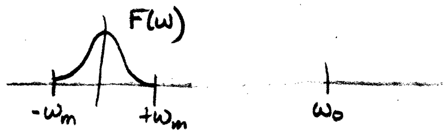
$\lambda_0 < 3.41a$ for propagation

March 3, 1972

$$\bar{E} = \mathrm{Re}\,(\bar{e} + \bar{e}_z)\, e^{-j\beta(\omega) z + j\omega t}$$

$$S_i(t) = f(t)\cos\omega_0 t$$

Spectrum of $f(t)$ is $F(\omega)$



$$S_i(\omega) = \frac{1}{2}\left[F(\omega+\omega_0) + F(\omega-\omega_0)\right]$$

$$S_o(\omega) = \frac{1}{2}\left[F(\omega+\omega_0)e^{+j\beta(\omega)z} + F(\omega-\omega_0)e^{-j\beta(\omega)z}\right]$$

note that $e^{-j\beta(\omega)z}$ for (+) frequencies

$e^{+j\beta(\omega)z}$ for (-) frequencies

$$S_o(t,z) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{j\omega t} S_o(\omega)\, d\omega = 2\cdot\frac{1}{2\pi}\,\mathrm{Re}\int_0^{\infty} e^{j\omega t} S_o(\omega)\, d\omega$$

$$= \frac{\mathrm{Re}}{2\pi}\int_0^{\infty} F(\omega-\omega_0)\, e^{-j\beta(\omega)z + j\omega t}\, d\omega$$

$$\beta(\omega) = \beta(\omega_0) + \left.\frac{d\beta}{d\omega}\right|_{\omega_0}(\omega-\omega_0) + \dots$$

$$\beta_0 \triangleq \beta(\omega_0)$$

$$\beta_0' \triangleq \left.\frac{d\beta}{d\omega}\right|_{\omega_0}$$

$$\therefore\ S_o(t,z) = \frac{\mathrm{Re}}{2\pi}\int_0^{\infty} F(\omega-\omega_0)\, e^{-j\beta_0'(\omega-\omega_0)z - j\beta_0 z + j\omega t}\, d\omega$$

$$= \frac{\mathrm{Re}}{2\pi}\int_0^{\infty} F(\omega-\omega_0)\, e^{-j\beta_0'(\omega-\omega_0)z - j\beta_0 z + j(\omega-\omega_0)t + j\omega_0 t}\, d\omega$$

let $\lambda = \omega - \omega_0$

$$S_o(t,z) = \frac{\mathrm{Re}}{2\pi}\int_{-\omega_0}^{\infty} F(\lambda)\, e^{j\lambda(t-\beta_0' z)}\, d\lambda \quad e^{j\omega_0 t - j\beta_0 z}$$

$$= \mathrm{Re}\ f(t-\beta_0' z)\, e^{j\omega_0 t - j\beta_0 z}$$

$$= f(t-\beta_0' z)\cos(\omega_0 t - \beta_0 z)$$

Signal delay $T_g = \beta_0 z$

$\frac{z}{T_g} = v_g$ = group velocity or signal ~~v~~ocity $= \frac{1}{\beta_0'} = \frac{d\omega}{d\beta} = \left(\frac{d\beta}{d\omega}\right)^{-1}$

March 3, 1972

$$T = e^{-j\psi(\omega)} \qquad \frac{d\psi}{d\omega} = \text{group delay}$$

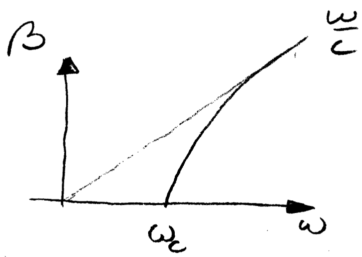
$$v_p = \frac{\omega_0}{\beta_0} = \text{phase velocity}$$

$$\int_S \left(\frac{\epsilon_0}{4} |\bar{E}|^2 + \frac{\mu_0}{4} |\bar{H}|^2 \right) dx\,dy = W = \text{energy}/m$$

energy crosssection

$$W \, v_{energy} = P$$

$$v_{energy} = \frac{P}{W} = v_g$$



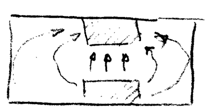
$v_p = \frac{\omega}{\beta} \to c$ as $\omega \to \infty$

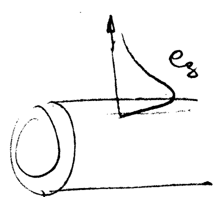
$\beta < k_0$, $v_p > c$,

$v_p \cdot v_g = c^2$

$$\left(\frac{d\beta}{d\omega} \right)^{-1} = v_g$$

ridge guide





Goubau line

dielectric coating on a conductor

3/6/72

equivalent ~~waveguide~~ transmission line model of a waveguide

$$E = C^+ (\bar{e} + \bar{e}_z) e^{-j\beta z} + C^- (\bar{e} - \bar{e}_z) e^{j\beta z}$$

$\bar{e}_z$ differs in sign because $\nabla \cdot \bar{E} = 0 = \nabla_t \cdot \bar{E} + \frac{\partial e_z}{\partial z}$

$$\bar{H} = C^+ (\bar{h} + \bar{h}_z) e^{-j\beta z} + C^- (-\bar{h} + \bar{h}_z) e^{j\beta z}$$

$$\bar{E} = C^+ \bar{e}\, e^{-jk_0 z} + C^- \bar{e}\, e^{jk_0 z}$$
$$\bar{H} = C^+ \bar{h}\, e^{-jk_0 z} - C^- \bar{h}\, e^{jk_0 z}$$

$$V = V^+ e^{-jk_0 z} + V^- e^{jk_0 z}$$
$$I = I^+ e^{-jk_0 z} - I^- e^{jk_0 z}$$
$$= \frac{V^+}{Z_c} e^{-jk_0 z} - \frac{V^-}{Z_c} e^{jk_0 z}$$

$$V^+ = \int_1^2 C^+ \bar{e} \cdot d\bar{l}$$

for waveguide

$$V = V^+ e^{-j\beta z} + V^- e^{j\beta z}$$

$$I = I^+ e^{-j\beta z} - I^- e^{j\beta z}$$

$V^+ = KC^+$ (proportional to transverse electric field)

$I^+ = k_2 C^+$ (proportional to transverse electric field)

transverse field because it is the transverse components that determine power flow

in a transmission line power flow $= \mathrm{Re}\, \frac{1}{2} V^+ (I^+)^*$

in a waveguide power flow $= \mathrm{Re}\, \frac{1}{2} \int_S C^+ (C^+)^* \bar{e} \times \bar{h}^* \cdot \bar{a}_z\, dx\, dy$

let us normalize $\bar{e}$ & $\bar{h}$ so that $\int_S \bar{e} \times \bar{h}^* \cdot \bar{a}_z \, dxdy = 1$

recall that $\bar{h} = Y_w \bar{a}_z \times \bar{e}$

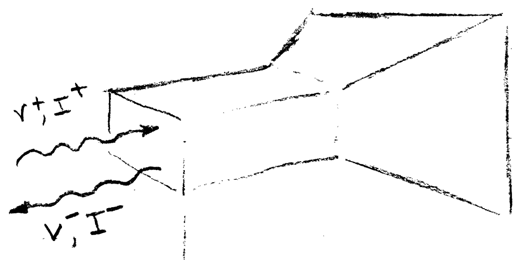
$\Rightarrow \quad \mathrm{Re}\, \frac{1}{2} V^+ (I^+)^* = \mathrm{Re}\, \frac{1}{2} C^+ (C^+)^* \implies K_1 K_2^* = 1$

$$\frac{V^+}{I^+} = \frac{k_1 C^+}{k_2 C^+} = (\text{your choice}) = \begin{cases} 1, & \text{normalized voltage \& current} \\ Z_{w}, & \text{wave impedance} \end{cases}$$

$$Z_w = \frac{k_0}{\beta} Z_0 \quad , \text{TE wave}$$

$$= \frac{\beta}{k_0} Z_0 \quad , \text{TM wave}$$

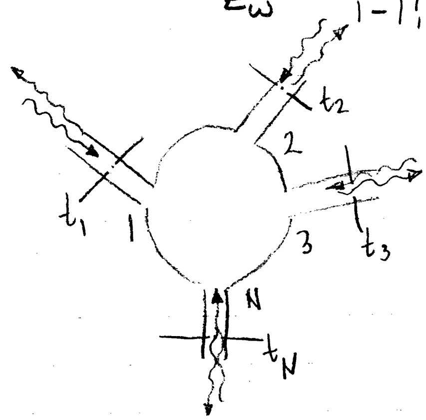
$$\bar{E} = \frac{V^+}{K_1} (\bar{e} + \bar{e}_z) e^{-j\beta z} + \frac{V^-}{K_1} (\bar{e} - \bar{e}_z) e^{+j\beta z}$$



Reference plane

$$\Gamma_{IN} = \frac{V^+}{V^-} \qquad Z_{IN} = \frac{V^+ + V^-}{I^+ - I^-} = \frac{V^+ + V^-}{V^+ - V^-} Z_w = \frac{1 + \Gamma_{IN}}{1 - \Gamma_{IN}} Z_w$$

$$\bar{Z}_{IN} = \frac{Z_{in}}{Z_w} = \frac{1 + \Gamma_{IN}}{1 - \Gamma_{IN}}$$



know $V_i^+, V_i^-, I_i^+, I_i^-$, $i = 1, 2, \dots, N$

at i-th reference plane

$V_i = V_i^+ + V_i^-$ dependent

$I_i = I^+ - I_i^-$ = independent

$$V_i = Z_{i1} I_1 + Z_{i2} I_2 + \dots + Z_{iN} I_N$$

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & & \vdots \\ \vdots & \vdots & & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

let us normalize $\bar{e}$ & $\bar{h}$ so that $\int_s \bar{e} \times \bar{h}^* \cdot \bar{a}_z \, dx dy = 1$

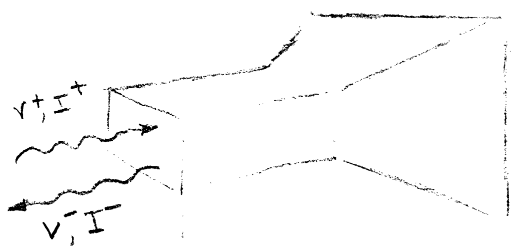
recall that $\bar{h} = Y_w \bar{a}_z \times \bar{e}$

$\Rightarrow \mathrm{Re}\, \frac{1}{2} V^+ (I^+)^* = \mathrm{Re}\, \frac{1}{2} C^+ (C^+)^* \implies K_1 K_2^* = 1$

$$\frac{V^+}{I^+} = \frac{k_1 C^+}{k_2 C^+} = (\text{your choice}) = \begin{cases} 1, & \text{normalized voltage \& current} \\ Z_w, & \text{wave impedance} \end{cases}$$

$$Z_w = \frac{k_0}{\beta} Z_0 \quad \text{, TE wave}$$
$$= \frac{\beta}{k_0} Z_0 \quad \text{, TM wave}$$

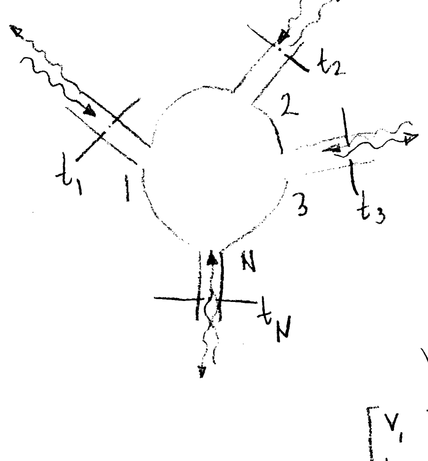
$$\bar{E} = \frac{V^+}{K_1} (\bar{e} + \bar{e}_z) e^{-j\beta z} + \frac{V^-}{K_1} (\bar{e} - \bar{e}_z) e^{+j\beta z}$$



reference plane

$$\Gamma_{IN} = \frac{V^+}{V^-} \qquad Z_{IN} = \frac{V^+ + V^-}{I^+ - I^-} = \frac{V^+ + V^-}{V^+ - V^-} Z_w = \frac{1 + \Gamma_{IN}}{1 - \Gamma_{IN}} Z_w$$

$$\bar{Z}_{IN} = \frac{Z_{in}}{Z_w} = \frac{1 + \Gamma_{IN}}{1 - \Gamma_{IN}}$$



know $V_i^+, V_i^-, I_i^+, I_i^-$, $i = 1, 2, \ldots N$

at i-th reference plane

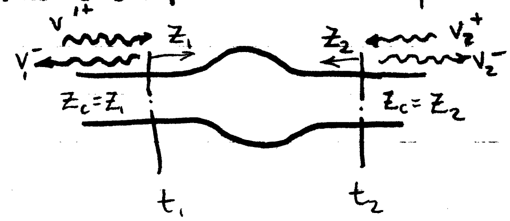
$V_i = V_i^+ + V_i^-$ dependent

$I_i = I^+ - I_i^-$ = independent

$$V_i = Z_{i1} I_1 + Z_{i2} I_2 + \cdots + Z_{iN} I_N$$

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & & \vdots \\ \vdots & \vdots & & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

impedance description of a 2-port junction



t_1 & t_2 are reference planes

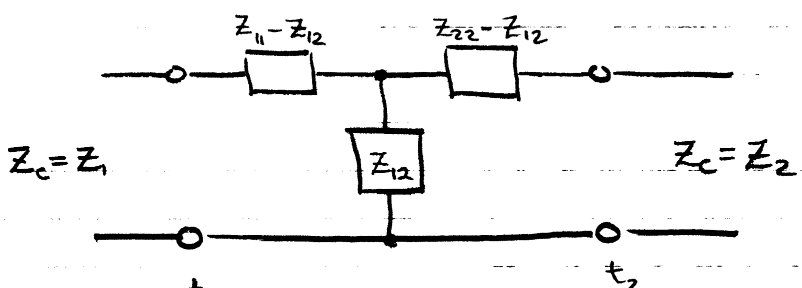
$$V_1 = V_1^+ e^{-j\beta_1 z_1} + V_1^- e^{+j\beta_1 z_1} \qquad V_2 = V_2^+ e^{-j\beta_2 z_2} + V_2^- e^{+j\beta_2 z_2}$$

$$I_1 = \frac{V_1^+}{Z_1} e^{-j\beta_1 z_1} - \frac{V_1^-}{Z_1} e^{+j\beta_1 z_1} \qquad I_2 = \frac{V_2^+}{Z_2} e^{-j\beta_2 z_2} - \frac{V_2^-}{Z_2} e^{+j\beta_2 z_2}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

in a reciprocal structure $Z_{12} = Z_{21}$

lossless case, all Z_{ij} are pure imaginary
i.e. reduces to 3 parameters for a 2-port network



reflection coefficent here is $-\rho$

$\longleftarrow \frac{\lambda_{g2}}{4} \longrightarrow$

$S = \frac{1 + |\Gamma_{IN,1}|}{1 - |\Gamma_{IN,1}|}$ $\longleftarrow d_{min} \longrightarrow$

open circuit $\longleftarrow$ transforms to short circuit

$|\Gamma_{IN,1}| = \frac{S-1}{S+1} = \rho$ but $\Gamma = -\rho$

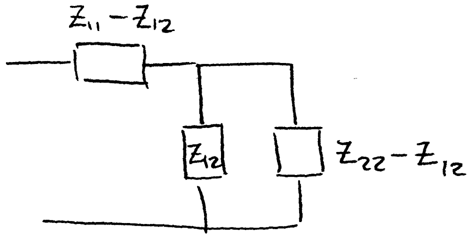
for $l = \frac{\lambda}{4}$ $\quad Z_{in} = \frac{Z_c^2}{Z_L}$

$\Gamma = \Gamma_{IN,1} e^{-2j\beta d_{min}} = -\rho$

then $Z_{in,1} = Z_{11}$

$$\Gamma_{IN,1} = \frac{Z_{11} - Z_1}{Z_{11} + Z_1} \qquad \frac{Z_{11}}{Z_1} = \frac{1 + \Gamma_{IN,1}}{1 - \Gamma_{IN,1}}$$

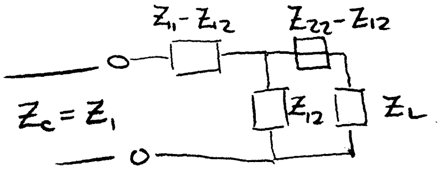
consider shorting the output



$$Z_{in} = Z_{11} - Z_{12} + \frac{Z_{12}(Z_{22} - Z_{12})}{Z_{22}}$$

$$= Z_{11} - \frac{Z_{12}^2}{Z_{22}}$$

$$Z_{12} = \sqrt{(Z_{11} - Z_{in}) Z_{22}}$$



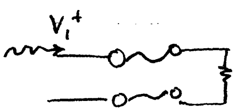
$$\Gamma_{IN} = \frac{Z_{in} - Z_1}{Z_{in} + Z_1}$$

scattering matrix description

let $Z_1 = Z_2 = 1$.

$$V_1^- = S_{11} V_1^+ + S_{12} V_2^+$$

$$V_2^- = S_{21} V_1^+ + S_{22} V_2^+$$



$Z_L = Z_c$ a matched termination

$\Gamma_{IN} = S_{11}$

a similar method may be used to determine S_{22}.

$V_2^- = S_{21} V_1^+$

Mon. Feb. 21.

Input impedance, standing wave ratio; lossy transmission lines. Sec. 3.5

Wed. Feb. 23, Equations for TE waves in waveguides, Sec's 3.1, 3.6

Fri. Feb. 25, Rectangular guide, Sec. 3.6

Mon. Feb. 28, Rectangular guide, dominant TE_{10} mode, attenuation; TM modes, Circular guides, Sec. 3.6

Wed. Feb. 30 which is March 1.

Group velocity, phase velocity, signal propagation in guides, Sec. 3.11

Fri. March 3. Other types of waveguides.

<u>Problems</u> 3.18, 3.30, 3.34. For 3.34 use a Taylor series $v_p = v_{pc} + \frac{dv_p}{d\omega}\Big|_{\omega_c}(\omega-\omega_c) + \frac{1}{2}\frac{d^2v_p}{d\omega^2}(\omega-\omega_c)^2$ where v_{pc} is v_p at ω_c = carrier frequency. Prob's due M

Mon. March 6 Equivalent voltage and current waves for waveguide modes, Impedance description of waveguide junctions. Sec's 4.1, 4.2, 4.5

Wed. Mar. 8 Impedance description of a 2-port junction, Scattering matrix. Sec.'s 4.6, 4.7

Fri. Mar. 10 Scattering matrix for a 2-port. Chain matrix description. Sec's 4.7, 4.8, 4.9

Mon. Mar. 13 Equivalent circuits for 2-port junctions, Sec. 4.6

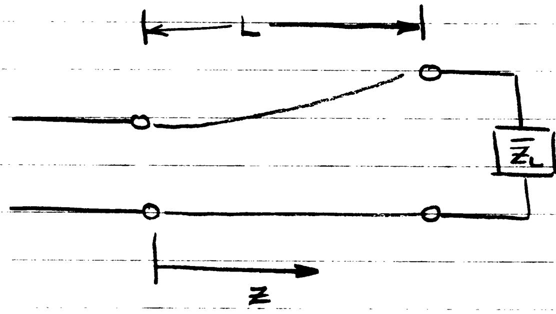
Wed. Mar. 15 Smith Chart, Stub matching, Sec's 5.1, 5.2

Fri. Mar. 17 Double stub, waveguide reactive elements, Sec's 5.3, 5.5

PROBLEMS: Due Fri. Mar. 17

4.9, 4.11, 4.12, 4.13, 4.25

Microwaves: April 3, 1972



$$d\Gamma_{IN} = \frac{1}{2}\underbrace{\frac{d(\ln \bar{Z})}{dz}}_{\frac{1}{\bar{Z}}\frac{d\bar{Z}}{dz}} e^{-2j\beta z}\, dz$$

$$\Gamma_{IN} = \frac{1}{2}\int_0^L \frac{d \ln \bar{Z}}{dz} e^{-2j\beta z}\, dz$$

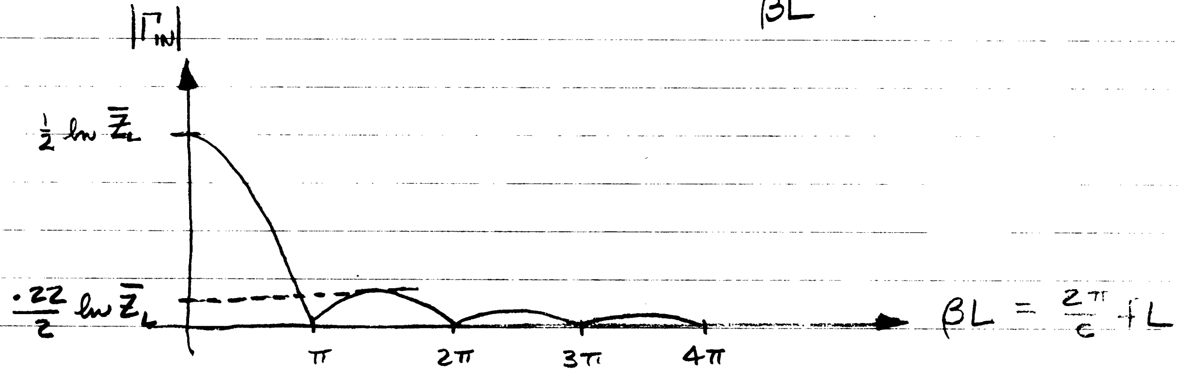
exponential taper: $\frac{d \ln \bar{Z}}{dz} = \text{constant}$

$$\ln \bar{Z} = C_1 z + C_2$$

$$= \frac{z}{L} \ln \bar{Z}_L$$

$$\bar{Z} = e^{\frac{z}{L} \ln \bar{Z}_L}$$

$$\Gamma_{IN} = \frac{1}{2} e^{-j\beta L} \ln \bar{Z}_L \frac{\sin \beta L}{\beta L}$$

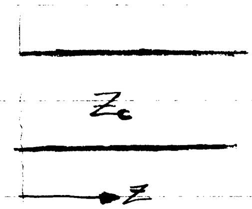


$$\frac{d \ln \bar{Z}}{dz} = \begin{cases} \frac{4z}{L^2} \ln \bar{Z}_L & 0 < z < \frac{L}{2} \\ \frac{4}{L^2}(L-z) \ln \bar{Z}_L & \frac{L}{2} < z < L \end{cases}$$

$$\bar{Z} = \begin{cases} e^{2\left(\frac{z}{L}\right)^2 \ln \bar{Z}_L} & 0 < z < \frac{L}{2} \\ e^{\left(\frac{4z}{L} - \frac{2z^2}{L^2} - 1\right) \ln \bar{Z}_L} & \frac{L}{2} < z < L \end{cases}$$

doing the integration

$$\Gamma_{IN} = \frac{1}{2} e^{-j\beta L} \ln \bar{Z}_L \left(\frac{\sin \beta L/2}{\beta L/2} \right)^2$$

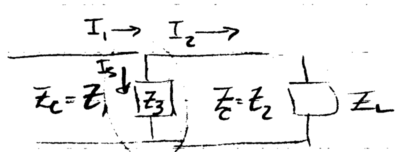


$$P = \frac{1}{2} \mathrm{Re}\, V I_g^*$$

$$V = V^+ e^{-j\beta z} + V^- e^{+j\beta z}$$

$$I = \frac{1}{Z_c} \left(V^+ e^{-j\beta z} - V^- e^{+j\beta z} \right)$$

$$\left. \frac{V_{reflected}}{V_{incident}} \right|_{z=l} = \Gamma_L = \frac{V^- e^{j\beta l}}{V^+ e^{-j\beta l}}$$



match conditions

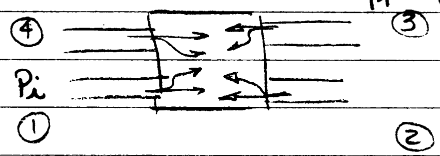
$$I_2 = I_1 - I_s$$

$$I_s = \frac{V}{Z_3} \text{ (total voltage across junction)}$$

$$= \frac{V_2^+ + V_2^-}{Z_3} = V_1^+ e^{-j\beta l_1} + V_1^- e^{j\beta l_1}$$

April 10, 1972

Directional Coupler



$C \triangleq$ coupling coefficent $= 10 \log \frac{P_i}{P_f}$

$D \triangleq$ directivity $= 10 \log \frac{P_i}{P_b}$ (typically 40db)

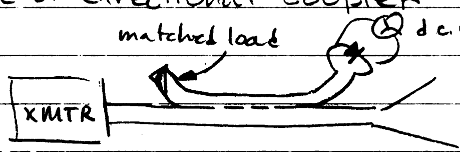
$S_{14} = 0$

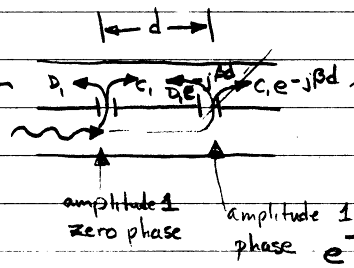
$S_{23} = 0$

$S_{41} = 0$

$S_{32} = 0$

use of directional coupler to monitor transmitter power output



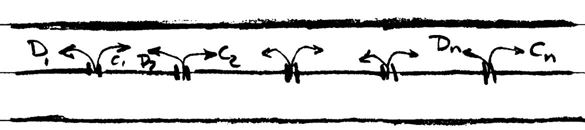


$D_1 + D_1 e^{-2j\beta d}$

$D_1 e^{-j\beta d}(e^{j\beta d} + e^{-j\beta d})$

$2D_1 e^{-j\beta d} \cos\beta d$

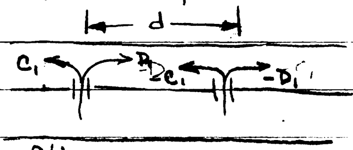
$2C_1 e^{-j\beta d}$



foreward direction $\sum_{n=1}^{N} C_n e^{-j\beta(N-1)d}$

reverse direction $\sum_{n=1}^{N} D_n e^{-j\beta(n-1)d\,2}$

Schwinger Reversed Phase Coupler

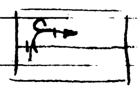
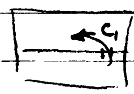


$\rightsquigarrow D_1 e^{-j\beta d} - D_1 e^{-j\beta d} = 0$

$C_1 - C_1 e^{-2j\beta d}$

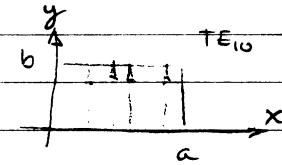
$= C_1 e^{-j\beta d}(e^{+j\beta d} - e^{-j\beta d})$

$2j C_1 e^{-j\beta d} \sin\beta d$

side views  

directivity is frequency independent
but coupling is frequency

MICROWAVES : 3/10/72



Normalization constant

$$E_y = V^+ N \sin \frac{\pi x}{a} e^{-j\beta z}$$

$$H_x = -V^+ \frac{N}{Z_w} \sin \frac{\pi x}{a} e^{-j\beta z}$$

$$Z_w = \frac{k_0}{\beta} Z_0$$

$P = \frac{1}{2}|V^+|^2$ because impedances are normalized $Z_c = 1$

$$P = \frac{1}{2} \mathrm{Re} \int_0^a \int_0^b \bar{E} \times \bar{H}^* \cdot \bar{a}_z \, dx\, dy$$

$\frac{ab}{2}$

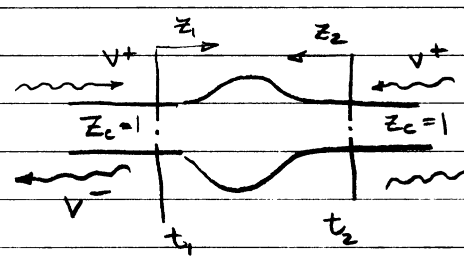
$$= \frac{1}{2} \mathrm{Re} \int_0^a \int_0^b E_y H_x^* \, dx\, dy = \frac{1}{2} \mathrm{Re} \, |V^+|^2 \frac{|N|^2}{Z_w} \int_0^a \int_0^b \sin^2 \frac{\pi x}{a} \, dx\, dy$$

if $|N|^2 = \frac{2 Z_w}{ab}$

$$P = \frac{1}{2} \mathrm{Re} \, |V^+|^2 = \frac{1}{2} |V^+|^2$$

$\bar{E} = V^+ \bar{e} \, e^{-j\beta z}$ where $\bar{e} = \sqrt{\frac{2 Z_w}{ab}} \sin \frac{\pi x}{a} \, \bar{a}_y$

$\bar{H} = \frac{V^+}{Z_c = 1} \bar{h} \, e^{-j\beta z}$ where $\bar{h} = -\sqrt{\frac{2 Z_w}{ab}} \sin \frac{\pi x}{a}$



Lossless 2-port

let matched load be placed at t_2, then $V_2^+ = 0$

$V_1^- = S_{11} V_1^+$ $\qquad P_{inc} = \frac{1}{2}|V_1^+|^2$

$V_2^- = S_{21} V_1^+$ $\qquad P_{ref} = \frac{1}{2}|V_1^-|^2 = \frac{1}{2}|S_{11}|^2 |V_1^+|^2$

$\qquad P_{tran} = \frac{1}{2}|V_2^-|^2 = \frac{1}{2}|S_{21}|^2 |V_1^+|^2$

$$P_{trans} = P_{inc} - P_{ref}$$

$$|S_{21}|^2 = 1 - |S_{11}|^2 \quad \text{or} \quad |S_{11}|^2 + |S_{12}|^2 = 1$$

for port 2 : $|S_{22}|^2 + |S_{21}|^2 = 1$

$\Rightarrow$ because $|S_{12}|^2 = |S_{21}|^2$

$|S_{11}|^2 = |S_{22}|^2$

$$|S_{12}|^2 = |S_{21}|^2 = \sqrt{1 - |S_{11}|^2}$$

let $S_{11} = \rho e^{j\theta}$

$S_{22} = \rho e^{j\theta_2}$

the scattering matrix is unitary

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

$$[S][S]^* = \underset{\text{IDENTITY MATRIX}}{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[S][S]^* = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* \\ S_{21}^* & S_{22}^* \end{bmatrix} = \begin{bmatrix} S_{11}S_{11}^* + S_{12}S_{21}^* & S_{11}S_{12}^* + S_{12}S_{22}^* \\ S_{11}^*S_{21} + S_{21}^*S_{22} & S_{21}S_{12}^* + S_{22}S_{22}^* \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$S_{11} = \rho e^{j\theta}$

$S_{22} = \rho e^{j\theta_2}$

$$|S_{12}| = |S_{21}| = \sqrt{1-\rho^2}$$

let $S_{12} = \sqrt{1-\rho^2}\, e^{j\phi}$

$$\rho e^{j\theta_1} \sqrt{1-\rho^2}\, e^{-j\phi} + \sqrt{1-\rho^2}\, e^{j\phi} \rho e^{-j\theta_2} = 0$$

$$e^{-j\theta_1 + j\phi} \left(e^{j\theta_1 - j\phi} + e^{j\phi - j\theta_2} = 0 \right)$$

$$e^{j(\theta_1+\theta_2)} + e^{j2\phi} = 0$$

$$2\phi = \theta_1 + \theta_2 \pm \pi + n2\pi$$

$$\phi = \frac{\theta_1+\theta_2}{2} \pm \frac{\pi}{2} + n\pi$$

Microwaves: 3/10/72

$$\begin{bmatrix} V_1^- \\ V_2^- \\ \vdots \\ \end{bmatrix} = [V^-] = [S][V^+]$$

$$[V] = [V^+] + [V^-]$$

$$[I] = [V^+] - [V^-]$$

$$[V] = [Z][I]$$

$$= \left(\underset{\text{UNIT MATRIX}}{[u]} + [S]\right)[V^+]$$

$$[I] = ([u] - [S])[V^+]$$

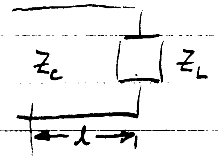
$$([u] + [S])[V^+] = [Z][I]$$

$$= [Z]([u] - [S])[V^+]$$

$$\Rightarrow \quad [u] + [S] = [Z]([u] - [S])$$

$$[Z] = ([u] + [S])([u] - [S])^{-1}$$

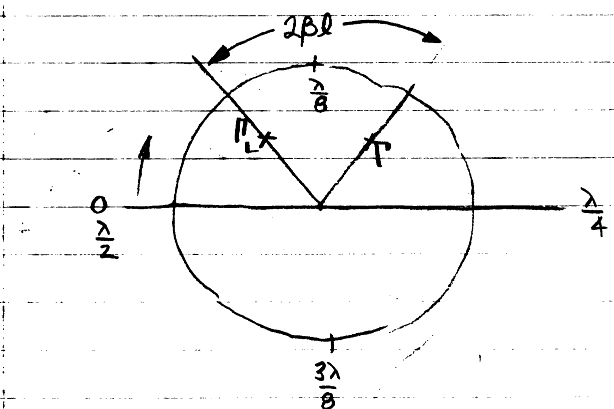
3/15/72 : Microwaves



$$\Gamma_L = \frac{\bar{Z}_L - 1}{\bar{Z}_L + 1} \qquad \bar{Z}_L = \frac{1+\Gamma_L}{1-\Gamma_L}$$

$$Z_{in} = \frac{1+\Gamma}{1-\Gamma}$$

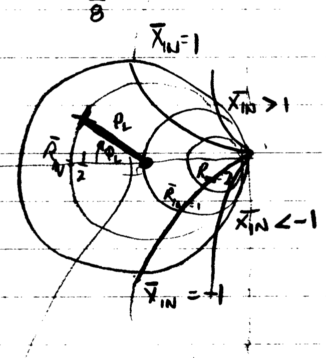
$$\Gamma = \Gamma_L e^{-2j\beta l}$$



$$\bar{Z}_{in} = \bar{R}_{in} + j\bar{X}_{in} = \frac{1+\Gamma}{1-\Gamma} = \frac{1+\rho e^{j\theta}}{1-\rho e^{j\theta}}$$

curves of constant $\bar{R}_{in}$ & $\bar{X}_{in}$

these curves are circles.

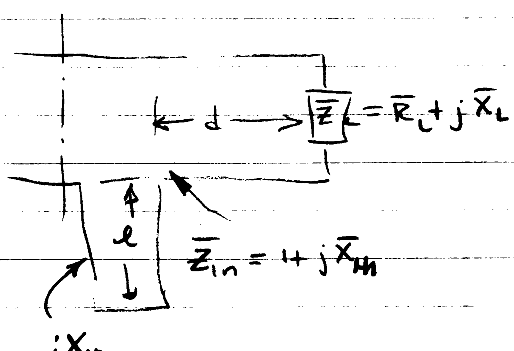


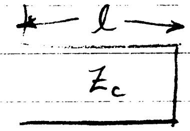
ϕ_L goes clockwise

$$\Gamma_L = \rho_L e^{-j\phi_L}$$

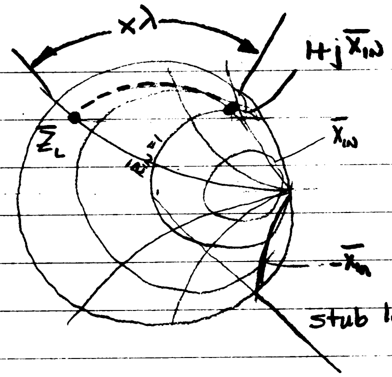
stub matching:

$$\bar{Z}_{in}' = 1 + j\bar{X}_{in} - j\bar{X}_{in} = 1$$



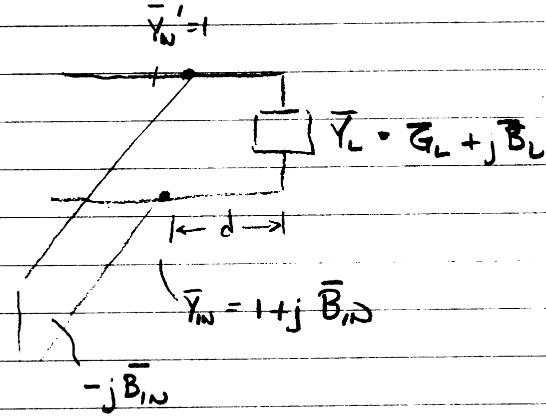


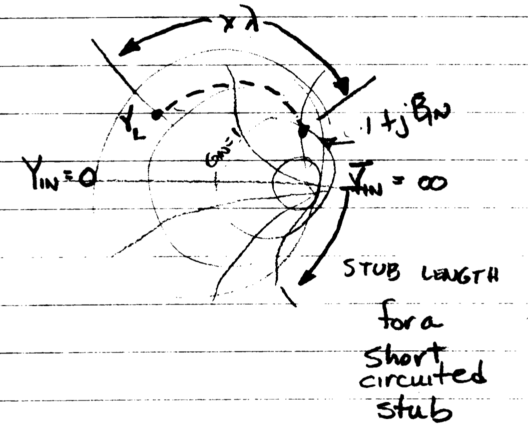
$$Z_{in} = jZ_c \tan \beta l$$



(stay on outer circumference because $\rho = 1$)

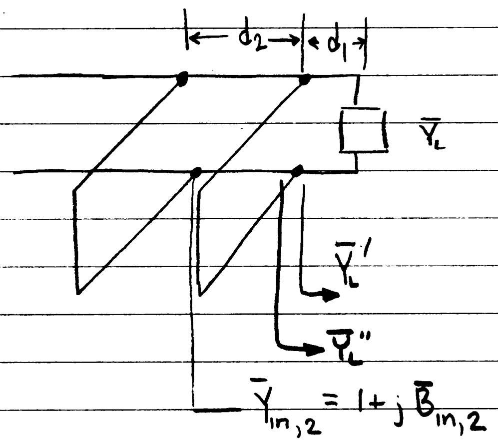
shunt stub



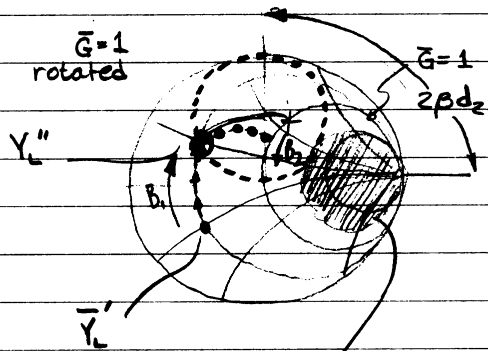


3/17/72: MICROWAVES

double stub matching



for convenience let $d_2 = \frac{\lambda}{8}$



steps: angle = distance between 2 stubs.

1. rotate $\bar{G}$ unit conductance circle
2. add susceptance to $\bar{Y}_L'$ to put it on the $\bar{G}=1$ circle rotated
3. rotate Y_L'' back onto the $\bar{G}=1$ circle
3. then add susceptance

double stub tuning cannot match loads ~~lying~~ within the ~~unit~~ conductance circle that is tangent to the rotated $\bar{G}=1$ circle.

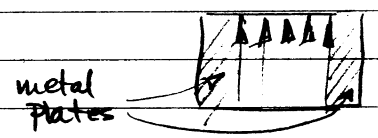
$\bar{Y}_L'' = \bar{Y}_L' + j\bar{B}_1$

$j\bar{B}_1 = \bar{Y}_L'' - \bar{Y}_L'$

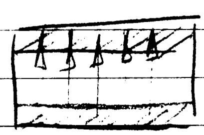
maximum value of conductance load can have is given by $\bar{G}_L \le \frac{1}{\sin^2 \beta d_2}$

diaphrams

rectangular waveguide TE$_{10}$ mode



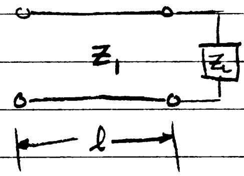
acts as an inductive element



acts as a capacitive element

March 27: Microwaves

quarter-wave transformer.

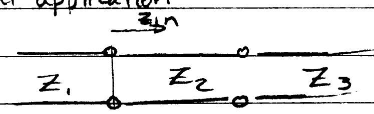


$$Z_{in} = Z_1 \frac{Z_L + j Z_1 \tan\beta l}{Z_1 + j Z_L \tan\beta l}$$

if $\beta l = \frac{\pi}{2}$, $l = \frac{\lambda}{4}$, $Z_{in} = \frac{Z_1^2}{Z_L} = Z_2$

$\therefore$ pick $Z_1 = \sqrt{Z_L Z_2}$

practical application



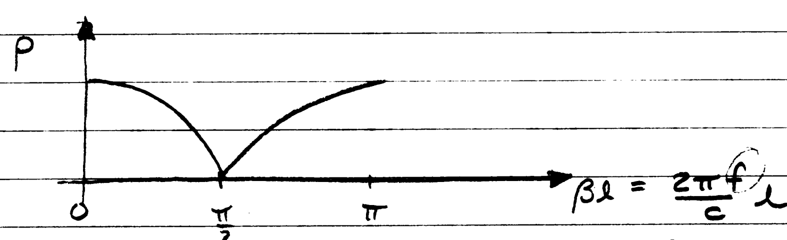
$$Z_2 = \sqrt{Z_1 Z_3}$$

$$\Gamma_{in} = \frac{Z_{in} - Z_1}{Z_{in} + Z_1} \qquad \rho = |\Gamma_{in}|$$

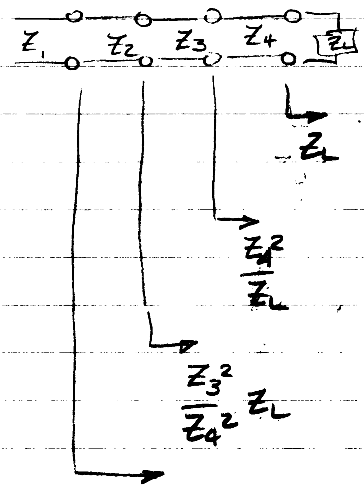
$$\rho = \frac{1}{\sqrt{1 + \left(\frac{2\sqrt{Z_2 Z_L}}{Z_L - Z_2}\right)^2 \sec^2\beta l}}$$

if βl near $\frac{\pi}{4}$ i.e. $l \approx \frac{\lambda}{4}$

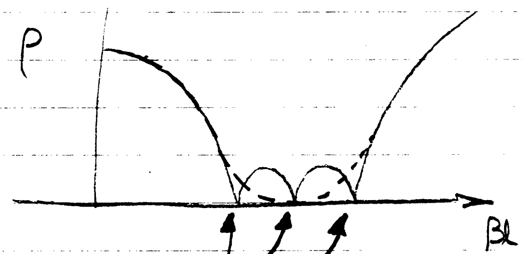
$$\rho \approx \frac{Z_L - Z_2}{2\sqrt{Z_L Z_2}} |\cos\beta l|$$



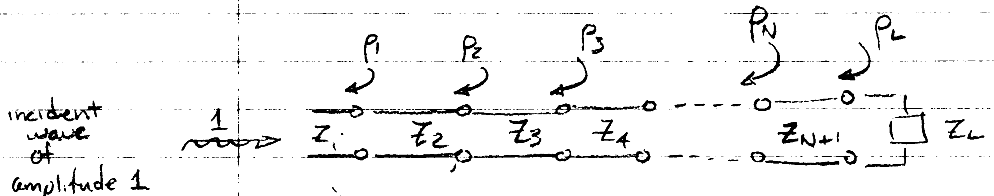
if l is constant this is ρ as a function of frequency multiple sections



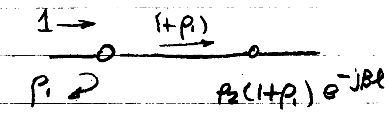
$$Z_1 = \frac{Z_2^2}{Z_3^2} \frac{Z_4^2}{Z_L}$$

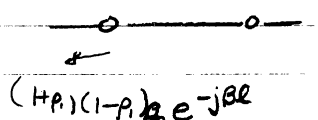


3 zeros because 3 junctions



all sections have length βl.





if one assumes that all ρ_i are so small so that all powers of ρ_i greater than 1 may be neglected

$$\Gamma_{IN} = \rho_1 + \rho_2 e^{-2j\beta l} + \rho_3 e^{-4j\beta l} + \rho_4 e^{-6j\beta l} + \ldots + \rho_N e^{-j(N-1)2\beta l} + \rho_L e^{-j2N\beta l}$$

$$\rho_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1} \approx \frac{1}{2} \ln \frac{Z_2}{Z_1} \text{ to the first order}$$

$$\Rightarrow \frac{Z_2}{Z_1} = e^{2\rho_1}$$

or in general $\frac{Z_{n+1}}{Z_n} = e^{2\rho_n}$

March 27 : Microwaves $\qquad \theta \stackrel{\Delta}{=} \beta l$

consider $\Gamma_{in} = A(1+e^{-j2\theta})^N$

$$= Ae^{-jN\theta}(e^{+j\theta}+e^{-j\theta})^N$$

$$= Ae^{-jN\theta}2^N\cos^N\theta$$

$$\Gamma_N = A\sum_{n=0}^{N} C_n^N e^{-j2n\theta}$$

$$C_n^N = C_{N-n}^N = \frac{N!}{(N-n)!\,n!}$$

$$\rho_1 = A C_0^N = A$$

$$\rho_2 = AC_1^N = NA$$

$$\rho_3 = AC_2^N = \frac{N(N-1)}{2}A$$

and in general

$$\rho_i = AC_{i-1}^N$$

FRANCIS L. MERAT

Microwaves: March 29, 1972

$$\Gamma_{IN} = \rho_0 + \rho_1 e^{-2j\theta} + \rho_2 e^{-4j\theta} + \dots + e^{-2jN\theta}$$

$$= A \sum_{n=0}^{N} C_n^N e^{-j2n\theta}, \quad \rho_n = A C_n^N$$

when $\theta = 0$

$$\Gamma_{IN} = A(1+e^{-j2\theta})^N$$

$$= A\ 2^N$$

$$\Gamma_{IN} = \frac{Z_L - Z_0}{Z_L + Z_0} = A^{2N}$$

$$A = \frac{Z_L - Z_0}{Z_L + Z_0}\ 2^{-N}$$

$$\rho_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$$

$$\frac{1}{2} \ln \frac{Z_{n+1}}{Z_n} \approx \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$$

$$\rho_n \approx \frac{1}{2} \ln \frac{Z_{n+1}}{Z_n}$$

$$\frac{Z_{n+1}}{Z_n} = e^{2\rho_n}$$

if $A = \frac{Z_L - Z_0}{Z_L + Z_0} 2^{-N} \approx 2^{-N} \frac{1}{2} \ln \frac{Z_L}{Z_0}$

Example: $N = 2$

$C_0^2 = 1, \quad C_1^2 = 2 \quad C_2^2 = 1$

$$\rho_0 = \frac{1}{8} \ln \frac{Z_L}{Z_0}$$

$$\rho_1 = \left(\frac{1}{8} \ln \frac{Z_L}{Z_0}\right) 2$$

$$\rho_2 = \rho_0$$

$$\rho_0 = \rho_2 = \frac{1}{2}\ln\frac{Z_L}{Z_2} = \frac{1}{2}\ln\frac{Z_1}{Z_0} \qquad \rho_1 = \frac{1}{2}\ln\frac{Z_2}{Z_1} = 2\rho_0$$

$$\frac{Z_L}{Z_2} = \frac{Z_1}{Z_0} \qquad \frac{Z_2}{Z_1} = \left(\frac{Z_1}{Z_0}\right)^2$$

$$Z_1 = \frac{Z_L Z_0}{Z_2} \qquad Z_1^{\,3} = Z_2 Z_0^{\,2}$$

$$\left(\frac{Z_L Z_0}{Z_2}\right)^3 = Z_2 Z_0^{\,2}$$

$$Z_L^{\,3} Z_0 = Z_2^{\,4}$$

$$Z_2 = Z_L^{\frac{3}{4}} Z_0^{\frac{1}{4}}$$

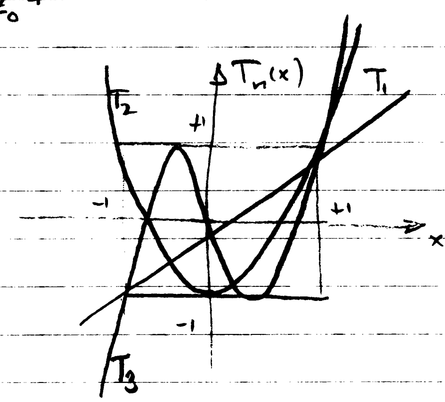
$$Z_1 = \frac{Z_L Z_0}{Z_L^{\frac{3}{4}} Z_0^{\frac{1}{4}}} = Z_L^{\frac{1}{4}} Z_0^{\frac{3}{4}}$$

Chebyshev Polynomial

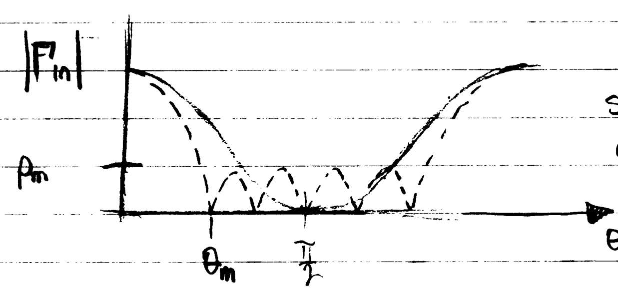
$T_1(x) = x$

$T_2(x) = 2x^2 - 1$

$T_3(x) = 4x^3 - 3x$



if $x = \cos\theta \quad T_n(\cos\theta) = \cos n\theta$



same number of sections as the number of zeros of $|\Gamma_{in}|$

let $\rho_0 = \rho_N$, $\rho_1 = \rho_{N-1}$ etc.

$$\Gamma_{in} = \rho_0 + \rho_1 e^{-2j\theta} + \rho_2 e^{-4j\theta} + \dots + \rho_N e^{-2jN\theta}$$

$$= e^{-jN\theta}\left[\left(\rho_0 e^{jN\theta} + \rho_N e^{-jN\theta}\right) + \left(\rho_1 e^{j(N-1)\theta} + \rho_{N-1} e^{-j(N-1)\theta}\right) + \dots\right]$$

$$= 2e^{-jN\theta}\left[\rho_N \cos N\theta + \rho_{N-1}\cos(N-1)\theta + \dots\right]$$

want Γ_{in} to be of form $T_N\left(\frac{\cos\theta}{\cos\theta_m}\right)$

Microwaves: March 29, 1972

$$\Gamma_{IN} = 2\cancel{e^{-jN\theta}} \left[\rho_N \cos N\theta + \rho_{N-1} \cos(N-1)\theta + \ldots \right] = A\cancel{e^{-jN\theta}} T_N \left(\frac{\cos\theta}{\cos\theta_m} \right)$$

by inspection $\rho_m = A$

at $\theta = 0$

$$\Gamma_{IN} = \frac{Z_L - Z_0}{Z_L + Z_0} = A T_N (\sec\theta_m) = \rho_m T_N (\sec\theta_m)$$

where θ_m is the only unknown.

Mon. March 27, Quarter-wave transformers, Sec's 5.6 - 5.9

Wed. March 29, Quarter-wave transformers, Sec's 5.9, 5.10

Fri. March 31, Tapered lines, Sec. 5.12

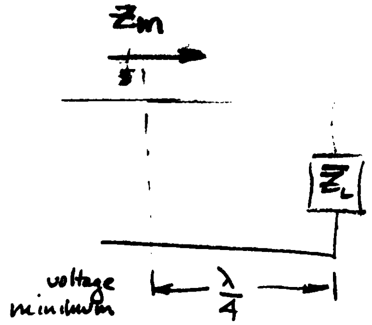
Problems: $\boxed{5.3}$, $\boxed{5.6}$, $\boxed{5.12}$, $\boxed{5.15}$, $\boxed{5.16}$?

A week or so from now

Quiz: Mon. April 3

The quiz will be based on Sections 3.3, 3.5, 4.6, 4.7, 4.8, 5.1, 5.2, 5.3, 5.5. There will be a problem on stub matching using the Smith chart (which will be provided). The quiz will be open book and the problems similar to those assigned as homework.

5.3



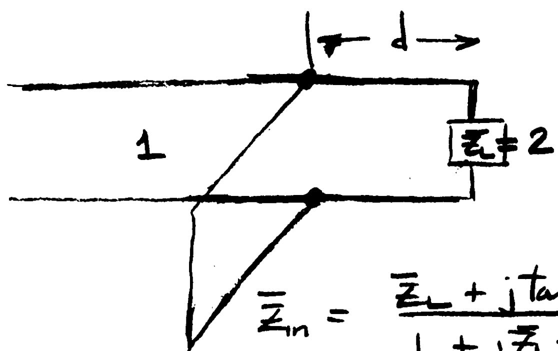
at a voltage minimum the reflection coefficent is negative and real; hence,

if $-\rho = |\Gamma|$ $\quad \bar{Z}_{in} = \dfrac{1+\Gamma}{1-\Gamma} \qquad \bar{Z}_{in} = \dfrac{1-\rho}{1+\rho} = \dfrac{1}{S} = \dfrac{1}{2}$

transforming back to the load we can say that

$$\bar{Z}_{in} = \frac{\bar{Z}_L + j\tan\beta\frac{\lambda}{4}}{1 + j\bar{Z}_L\tan\beta\frac{\lambda}{4}} = \frac{1}{\bar{Z}_L}$$

$$\Rightarrow \quad \bar{Z}_L = \frac{1}{\bar{Z}_{in}} = \frac{1}{\frac{1}{2}} = 2$$



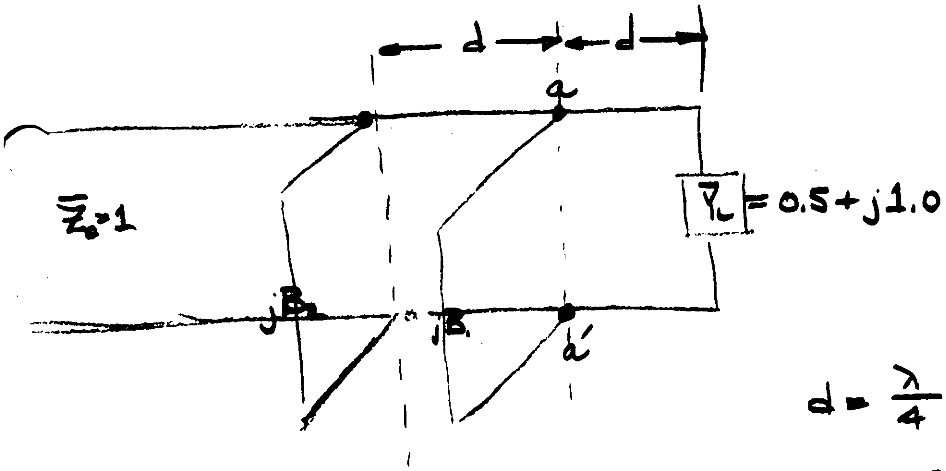
$$\bar{Z}_{in} = \frac{\bar{Z}_L + j\tan\beta d}{1 + j\bar{Z}_L\tan\beta d}$$

to match $\bar{Z}_{in} = 1 + j\bar{B}$

$$1 + j\bar{B} = \frac{\bar{Z}_L + j\tan\beta d}{1 + j\bar{Z}_L\tan\beta d} = \frac{\bar{Z}_L + j\tan\beta d + j\bar{Z}_L^2\tan\beta d - \bar{Z}_L\tan^2\beta d}{1 + Z_L^2\tan^2\beta d}$$

equating real parts

$$1 = \frac{\bar{Z}_L - \bar{Z}_L\tan^2\beta d}{1 + \bar{Z}_L^2\tan^2\beta d}$$



$$d = \frac{\lambda}{4}$$

$$\beta d = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

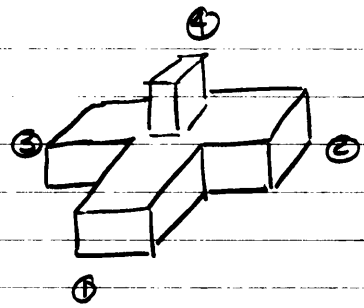
$$\text{at } aa' \quad \bar{Y}_L' = \frac{\bar{Y}_L + j \tan \beta d}{1 + j \bar{Y}_L \tan \beta d} = \frac{1}{\bar{Y}_L} = \frac{1}{0.5 + j1.0} \; \frac{0.5 - j1.0}{.25 + 1.0}$$

$$= \frac{0.5 - j1.0}{1.25}$$

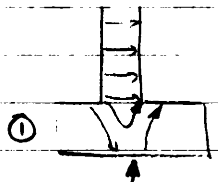
$$= 0.4 - j0.8$$

$$= \bar{G}' + j\bar{B}'$$

April 12, 1972

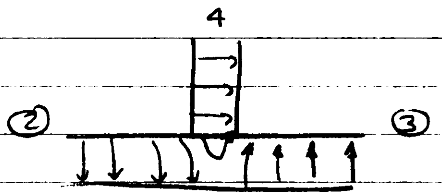


① wave entering



antisymmetric ⇒ no coupling into port ①

$S_{14} = S_{41} = 0$



$S_{42} = S_{24} = -S_{34} = -S_{43}$

it is possible to choose matching elements to make
$S_{44} = 0$, $S_{11} = 0$

$$\begin{bmatrix} 0 & S_{12} & S_{12} & 0 \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{12} & S_{23} & S_{33} & -S_{24} \\ 0 & S_{24} & -S_{24} & 0 \end{bmatrix}$$

$S_{13} = S_{12}$ (symmetry)

From conservation of energy dot product of any row with its complex conjugate is 1, the dot product of a row with the complex conjugate of any other row is 0.

$$|S_{12}|^2 + |S_{12}|^2 = 1$$

$$|S_{12}| = \frac{1}{\sqrt{2}}$$

$$|S_{22}|^2 + |S_{23}|^2 + |S_{24}|^2 = \frac{1}{2}$$

$$|S_{23}|^2 + |S_{33}|^2 + |S_{24}|^2 = \frac{1}{2}$$

$\Rightarrow |S_{22}| = |S_{33}|$

$$|S_{24}|^2 = \frac{1}{2} = |S_{12}|^2$$

this means that

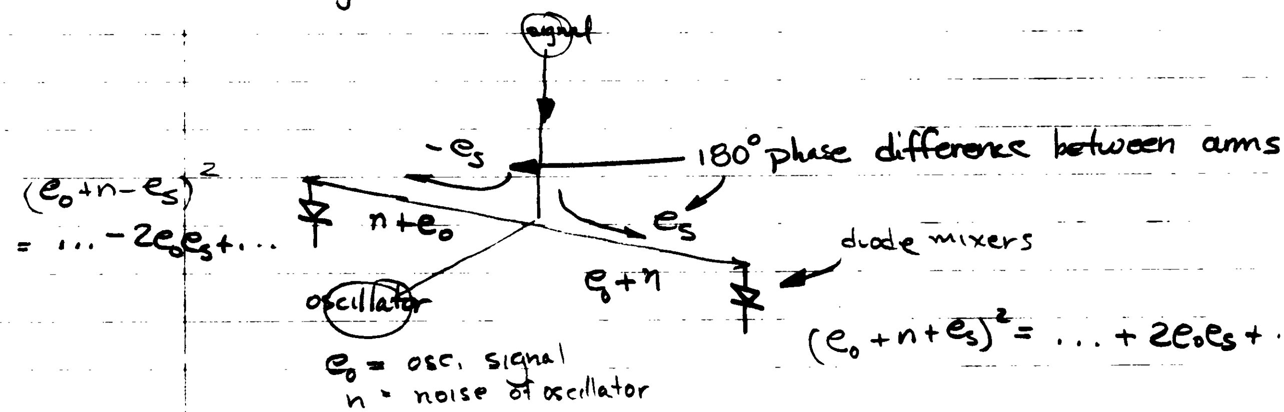
$$|S_{23}|^2 + |S_{33}|^2 = 0$$

which is true only for $|S_{23}| = |S_{33}| = 0$

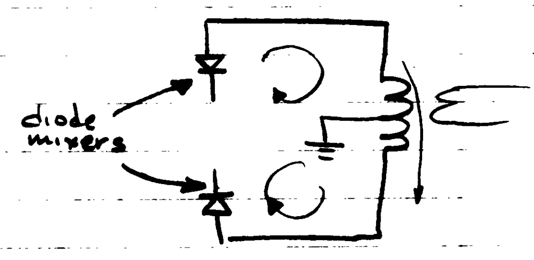
the same arguement shows that $|S_{22}| = 0$

$$\begin{bmatrix} 0 & S_{12} & S_{12} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{12} & 0 & 0 & -S_{24} \\ 0 & S_{24} & -S_{24} & 0 \end{bmatrix}$$

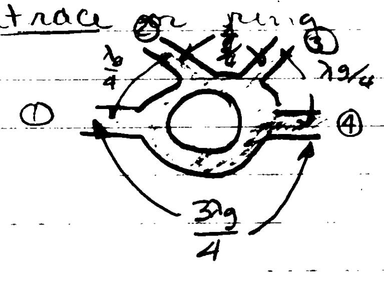
magic-T used as a balanced modulator to cancel local oscillator noise



go into this type of circuit



Katrace or ring circuit



$S_{13} = 0$

$S_{24} = 0$

MICROWAVES: April 17, 1972

circularly polarized wave

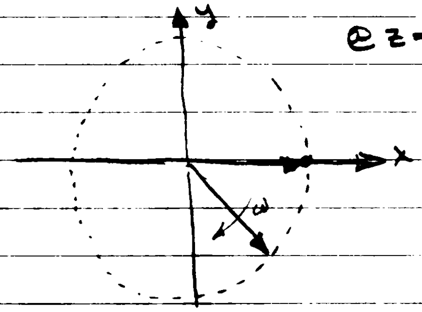
$\bar{E} = E_0 \bar{a}_x e^{-j\beta z}$ linearly polarized wave

$\bar{E} = E_0(\bar{a}_x + j\bar{a}_y) e^{-j\beta z}$ circularly polarized wave

$\mathcal{E}_x = E_0 \,\mathrm{Re}\, e^{j\omega t - j\beta z} = E_0 \cos(\omega t - \beta z)$

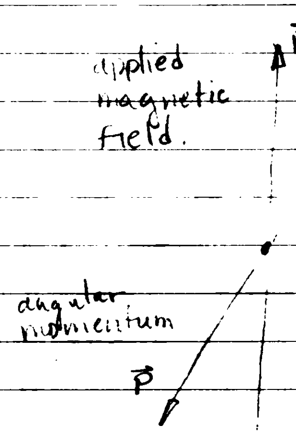
$\mathcal{E}_y = \mathrm{Re}\, jE_0 e^{j\omega t - j\beta z} = -E_0 \sin(\omega t - \beta z)$

@ z = 0



if $\bar{E} = E_0(\bar{a}_x + j\bar{a}_y)e^{-j\beta z}$ negative circularly polarized wave

if $\bar{E} = E_0(\bar{a}_x - j\bar{a}_y)e^{-j\beta z}$ positive circularly polarized wave.



$\vec{m}$ magnetic dipole moment

$\vec{T} = \vec{m} \times \vec{B}_0$

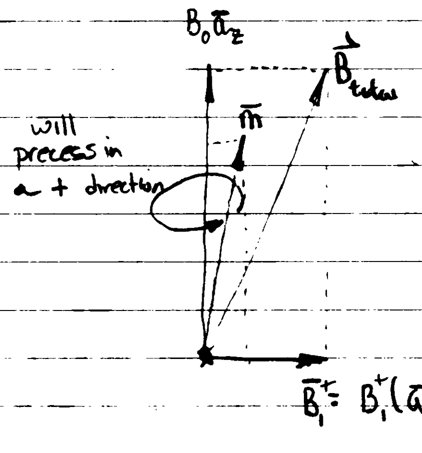
$\gamma = \dfrac{m}{P}$ = gyromagnetic ratio

direction of $\vec{P}$ is opposite that of m

$\vec{m} \times B_0 = \vec{T} = \dfrac{d\vec{P}}{dt}$

$T = \vec{m} \times \vec{B}_0 = -\gamma \vec{P} \times \vec{B}_0 = \vec{\omega}_0 \times \vec{P} = \gamma \vec{B}_0 \times \vec{P}$

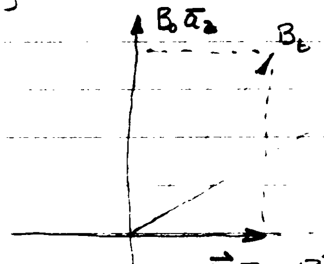
$\omega_0 = \gamma B_0$



proportional to H_1^+

$m_1^+ ; \; B_1^+ = \mu_0(\vec{H}_1^+ + \vec{M}_1^+) = \mu^+ \vec{H}_1^+$

apply an a.c. field which is negatively circularly polarized.



$$\vec{B}_1^- = \mu^- \bar{H}_1^-$$

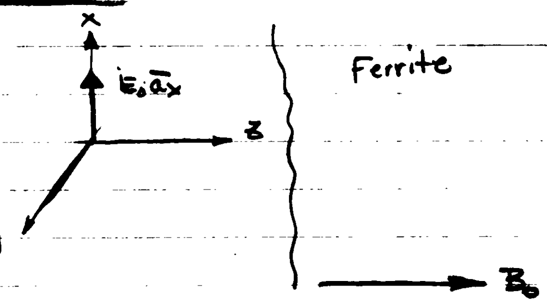
$$\vec{B}_1^- = B^-(\bar{a}_x + j\bar{a}_y)$$

$$\mu^+ = \mu_0\left(1 + \frac{\gamma\mu_0 M_s}{\omega_0 - \omega}\right)$$

M_s = saturation magnetization

$$\mu^- = \mu_0\left(1 + \frac{\gamma\mu_0 M_s}{\omega_0 + \omega}\right)$$

Faraday rotation



$$\frac{E_0}{2}(\bar{a}_x + j\bar{a}_y)e^{-j\beta_- z} + \frac{E_0}{2}(\bar{a}_x - j\bar{a}_y)e^{-j\beta_+ z}$$

$$E_0\bar{a}_x = \frac{E_0}{2}(\bar{a}_x + j\bar{a}_y) + \frac{E_0}{2}(\bar{a}_x - j\bar{a}_y)$$

−circular polariz. $\qquad$ +circular polariz.

$\beta_- = \omega\sqrt{\epsilon\mu_-}$ $\qquad$ $\beta_+ = \omega\sqrt{\epsilon\mu_+}$

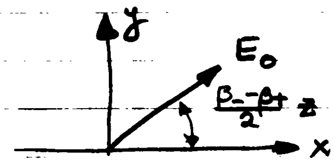
$$\frac{E_0}{2}\left(e^{-j\beta_- z} + e^{-j\beta_+ z}\right) + j\frac{E_0}{2}\left(e^{-j\beta_- z} - e^{-j\beta_+ z}\right)$$

$$= \frac{E_0}{2}e^{-j\left(\frac{\beta_- + \beta_+}{2}\right)z}\left(e^{-j\frac{\beta_- - \beta_+}{2}z} + e^{j\frac{\beta_- - \beta_+}{2}z}\right)$$

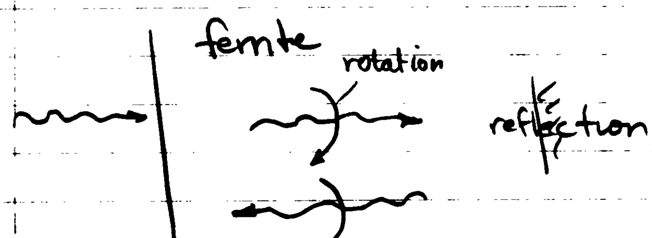
$$+ j\frac{E_0}{2}e^{-j\frac{\beta_- + \beta_+}{2}z}\left(e^{-j\frac{\beta_- - \beta_+}{2}z} - e^{+j\frac{\beta_- - \beta_+}{2}z}\right)$$

$$= \frac{E_0}{2}e^{-j\left(\frac{\beta_- + \beta_+}{2}\right)z}\left(\bar{a}_x \cos\left(\frac{\beta_- - \beta_+}{2}\right)z + \bar{a}_y \sin\left(\frac{\beta_- - \beta_+}{2}\right)z\right)$$

x & y components are in time phase but polarization is no longer in the x-direction.



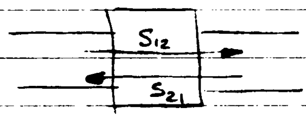
rotation is a non-reciprocal effect



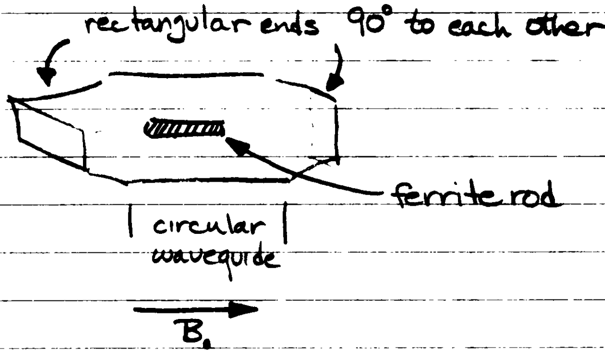
rotation continues in same direction irregardless of the direction of propagation of the wave.

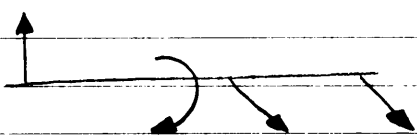
Microwaves: April 18, 1972

gyrator: (2 port, 180° differential phase shift)

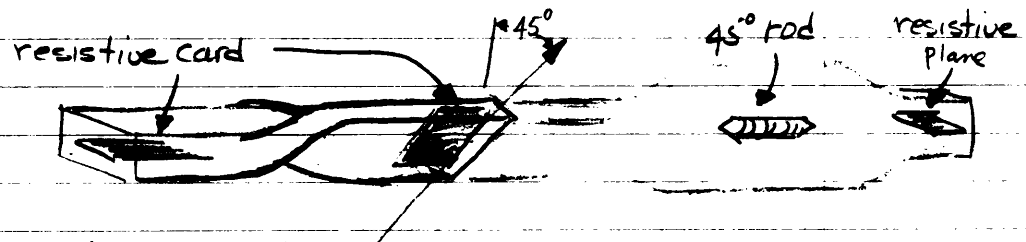


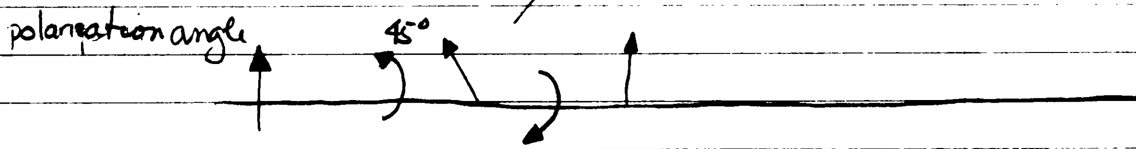
$$S_{12} = -S_{21}$$





isolators isolate an oscillator from a load



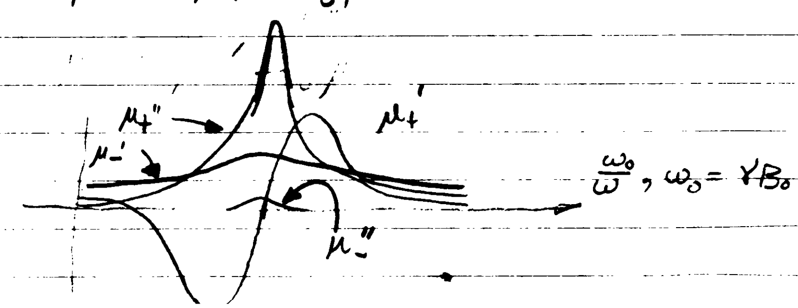


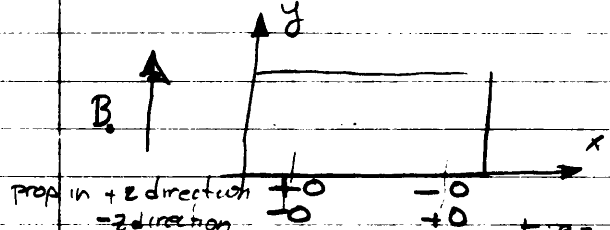
1 db foreward direction

20-30 db reverse direction

for a ferrite material losses are present

$$\mu_+ = \mu_+' - j\mu_+'' \qquad \mu_- = \mu_-' - j\mu_-''$$





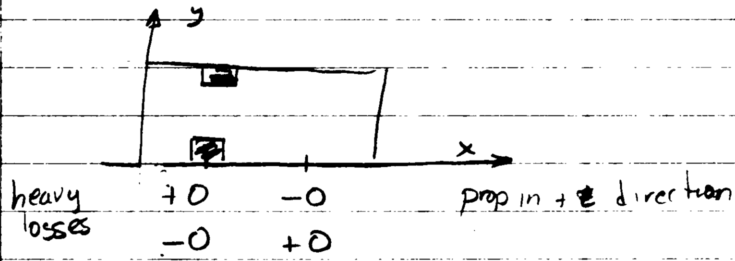
o denotes circular polarization

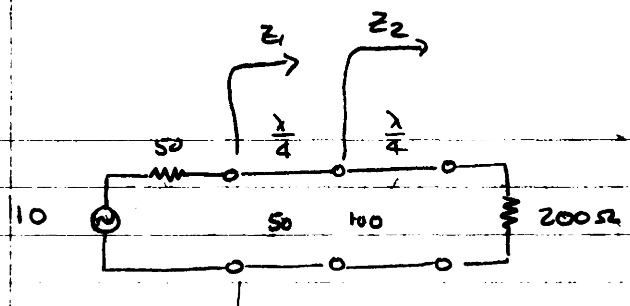
$$H_x = \sin\frac{\pi x}{a}\, e^{\pm j\beta z}$$

$$\nabla \cdot \bar{H} = 0 \qquad \frac{\partial H_x}{\partial x} + \frac{\partial H_z}{\partial z} = 0$$

$$H_z = \frac{\frac{\pi}{a}\cos\frac{\pi x}{a}\, e^{\pm j\beta z}}{\mp j\beta}$$

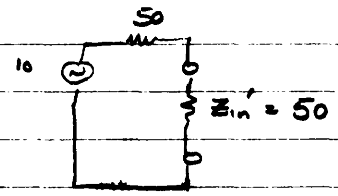
add ferrite





$$Z_{in} = \frac{100^2}{200} = 50\Omega$$

$$Z_{in}' = \frac{50^2}{50} = 50$$



$$I_g = \frac{10}{100} = 0.1$$

$$P_L = \frac{1}{2} I_g^2 Z_{in}' = \frac{1}{2}\left(\frac{1}{100}\right)(50) = \frac{1}{4} \text{ watt}$$

on line 1

$$V_1 = V_1^+ e^{-j\beta z} + V_1^- e^{j\beta z_1}$$

$$I_1 = \frac{V_1^+}{50} e^{-j\beta z_1} - \frac{V_1^-}{50} e^{j\beta z_1}$$

$$\cancel{\frac{V_1^+ e^{-j\beta\frac{\lambda}{4}}}{V_1^- e^{+j\beta\frac{\lambda}{4}}} = 0}$$

at $z_1 = \frac{\lambda}{4}$ $\quad V_{inc} = V_1^+ e^{-j\beta\frac{\lambda}{4}}$

$$V_{ref} = V_1^- e^{j\beta\frac{\lambda}{4}}$$

$$\cancel{\frac{V_1^+}{V_1^-} e^{-j\beta\frac{\lambda}{2}} = 0}$$

$$0 = \frac{V_{ref}}{V_{inc}} = \frac{V_1^-}{V_1^+} e^{+j\beta\frac{\lambda}{2}} = -\frac{V_1^-}{V_1^+}$$

$$e^{+j\beta\frac{\lambda}{2}} = e^{j\pi} = \cos\pi + j\sin\pi = -1 \qquad \frac{V_{ref}}{V_{inc}} = \frac{Z_{in} - Z_c}{Z_{in} + Z_c} = 0$$

$$\cancel{e^{-j\pi} = e^{-j\beta\frac{\lambda}{2}} = \cos\pi - j\sin\pi = -1 \Rightarrow V_1^+}$$

$\Rightarrow V_1^- = 0$

or $V_1^+ = +5$ at $z_1 = 0$ $\quad V_1 = V_1^+ + V_1^- = \frac{Z_{in}'}{Z_{in}' + Z_g} V_g = 5$ volts

$$\Rightarrow V_1^+ = 5;\ V_1^- = 0$$

on line 2

$$V_2 = V_2^+ e^{-j\beta z} + V_2^- e^{j\beta z_1}$$

$$I_2 = \frac{V_2^+}{100} e^{-j\beta z} - \frac{V_2^-}{100} e^{j\beta z_2}$$

at $z_2 = \frac{\lambda}{4}$

$$V_{inc} = V_2^+ e^{-j\beta\frac{\lambda}{4}}$$

$$V_{ref} = V_2^- e^{j\beta\frac{\lambda}{4}}$$

$$\frac{V_{ref}}{V_{inc}} = \frac{Z_L - Z_c}{Z_L + Z_c} = \frac{1}{3} = \frac{V_2^-}{V_2^+} e^{-j\beta\frac{\lambda}{2}} = -\frac{V_2^-}{V_2^+}$$

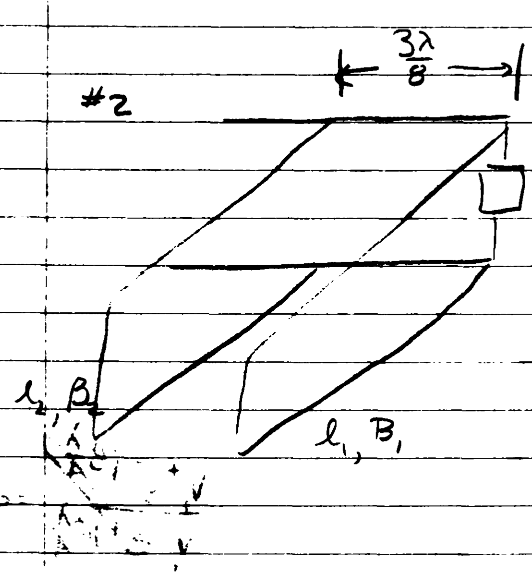
$$\Rightarrow V_2^- = \frac{1}{3} V_2^+$$

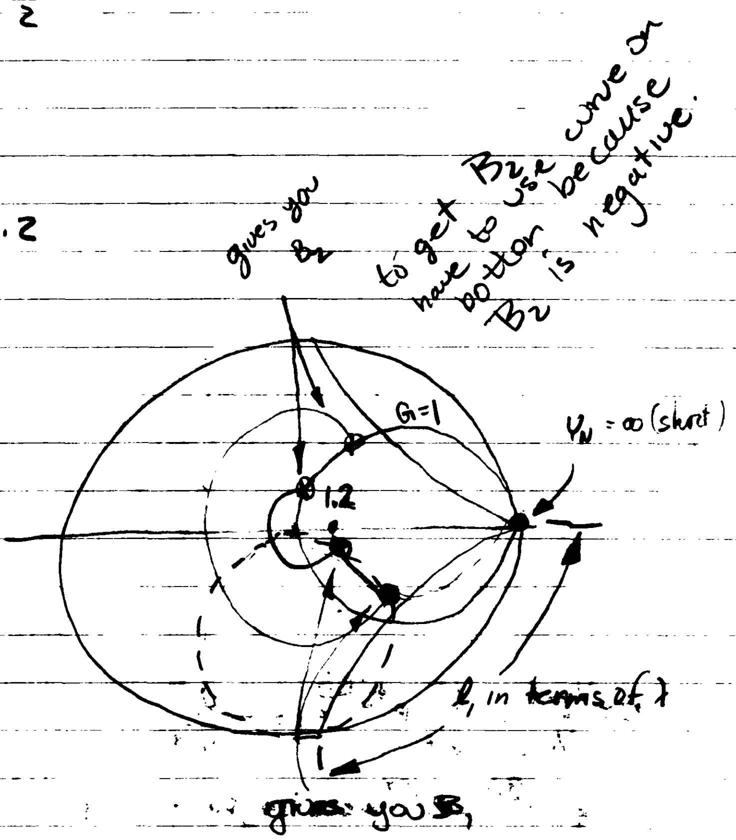
at $z_2 = 0$ $\quad V_2 = V_1\left(z_1 = \frac{\lambda}{4}\right) = V_2(z_2 = 0) = V_2^+ + V_2^-$

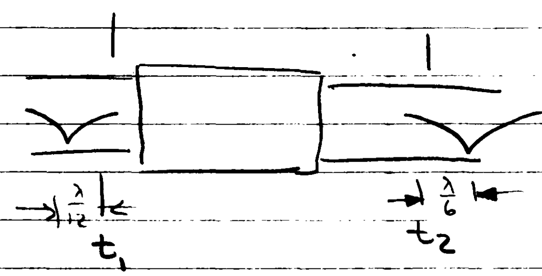
$$\Rightarrow V_2^+ + V_2^- = V_2^+\left(1 - \frac{1}{3}\right) = \frac{2}{3} V_2^+ = V_1^+ e^{-j\beta\frac{\lambda}{4}} = 5e^{-j\frac{\pi}{2}} = -j5$$

$$\therefore V_2^+ = -j\frac{15}{2}$$

#2







VSWR = 3

$$|S_{11}| = \frac{3-1}{3+1} = \frac{1}{2} = |S_{22}|$$

$$|S_{12}| = \sqrt{1 - |S_{11}|^2} = \sqrt{\frac{3}{4}}$$

S

$\Gamma = S_{11} e^{-2j\beta d} = S_{11} e^{-j\frac{\pi}{3}} = \Gamma_1 = -\frac{1}{2}$

$S_{11} = -\frac{1}{2} e^{j\frac{\pi}{3}} = \frac{1}{2} e^{j\frac{4\pi}{3}}$

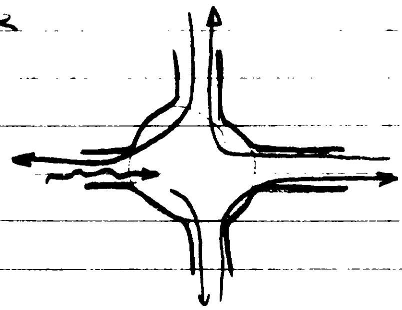
$S_{22} = \frac{1}{2} e^{-j\frac{7\pi}{6}} = \frac{1}{2} e^{j\frac{5\pi}{6}}$

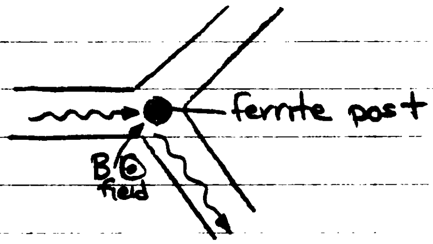
$S_{12} = |S_{12}| e^{j\phi}$ $\quad \phi = \frac{\theta_1 + \theta_2}{2} + \frac{\pi}{2} \pm n\pi$

$= \frac{4\pi/3 - 7\pi/6}{2} + \frac{\pi}{2} \pm n\pi$

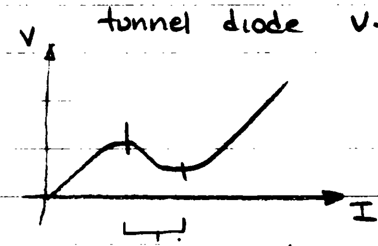
MICROWAVES: APRIL 24, 1972

CIRCULATOR

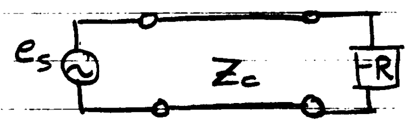




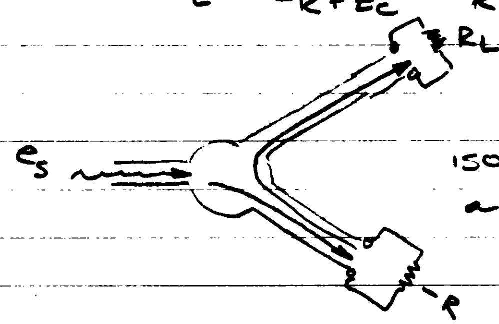
tunnel diode V-I curve



model a negative resistance amplifier



$$\Gamma_L = \frac{-R - Z_c}{-R + Z_c} = \frac{R + Z_c}{R - Z_c}, \quad |\Gamma_L| > 1$$



a circulator isolates input from output for a negative resistance amplifier