

Department of Electrical Engineering and Computer Science
ENGR 210. Introduction to Circuits and Instruments (4)
 ENGR 210 - SPRING 2005

FINAL EXAMINATION-5/3/05

	Average	Grader	Possible	
1.	6.1	Kevin	10	DC circuit analysis
2.	5.4	Kevin	10	Controlled sources
3.	6.1	Mark	10	Multiple OP AMP circuits
4.	7.1	Mark	10	Computer interfacing
5.	7.2	Craig	10	Thevenin/Norton
6.	4.5	Matt	10	Capacitors and Inductors
7.	6.9	Bryan	10	Phasors
8.	4.9	Dr.M	10	Frequency response
9.	6.5	Matt	10	First order circuits
10.	7.4	?	10	Switched transients
SCORE	61.0	out of	100	possible

ACADEMIC DISHONESTY

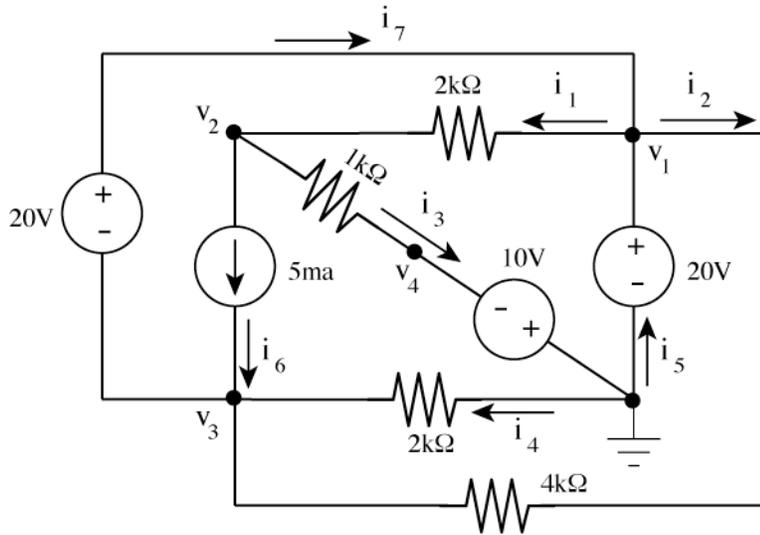
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NOTES:

- 159 students took the exam in Strosacker**
- 2 students took the exam in ESS**
- 1 student took the exam later in the day**
- 162 students total took the exam**

Problem 1 DC Circuit Analysis (10 points)

Determine the indicated node voltages (with respect to ground) and currents for the circuit shown below. All voltages are to be in volts and all currents in amperes.



$v_1 =$ _____ $i_1 =$ _____ $i_5 =$ _____
 $v_2 =$ _____ $i_2 =$ _____ $i_6 =$ _____
 $v_3 =$ _____ $i_3 =$ _____ $i_7 =$ _____
 $v_4 =$ _____ $i_4 =$ _____

SOLUTIONS:

Several of these quantities were straight forward to find. For example, simple inspection of the circuit indicates that:

$i_6 = 5\text{mA}$

$v_1 = +20\text{ Volts}$

Since these were clearly indicated on the circuit diagram they were 1/2 point each. All other voltages and currents were worth 1 point each.

The next easiest parameter to determine is $v_4 = -10\text{ volts}$. The remaining parameters require calculations.

v_3 is calculated as $v_3 = v_1 - 20 = (20) - 20 = 0\text{ volts}$

We can now calculate i_2 and i_4 using Ohm's law and paying attention to the indicated direction of these currents on the circuit diagram.

$$i_2 = \frac{v_1 - v_3}{4k} = \frac{(20) - (0)}{4k} = 5\text{ mA}$$

$$i_4 = \frac{v_5 - v_3}{2k} = \frac{(0) - (0)}{2k} = 0$$

Consider using Kirchoff's Circuit Law to find v_2 since a current source is clearly supplying one of the currents to the node.

$$\sum_{+in@2} i = +i_1 - i_3 - i_6 = +\frac{v_1 - v_2}{2k} - \frac{v_2 - v_4}{2k} - 5mA = 0$$

Substituting $v_1 = +20$ Volts and $v_4 = -10$ volts gives

$$\frac{(+20) - v_2}{2k} - \frac{v_2 - (-10)}{1k} - 5mA = 0$$

$$\frac{(+20) - v_2}{2k} - \frac{v_2 + 10}{1k} - 5mA = 0$$

Multiplying by 2000 gives

$$20 - v_2 - 2v_2 - 20 - 10 = 0$$

$$-3v_2 - 10 = 0$$

$$v_2 = -\frac{10}{3} = -3\frac{1}{3} \text{ volts}$$

This now allows us to find i_1 and i_3 using Ohm's Law and paying attention to the indicated direction of these currents on the circuit diagram.

$$i_1 = \frac{v_1 - v_2}{2k} = \frac{(20) - (-3.33)}{2k} = \frac{23.33}{2k} = 11.665 \text{ mA}$$

$$i_3 = \frac{v_2 - v_4}{1k} = \frac{\left(-\frac{11}{3}\right) - (-10)}{1k} = \frac{\left(-\frac{11}{3}\right) + 10}{1k} = 6.67 \text{ mA}$$

The remaining currents must be found using Kirchoff's Circuit Law at the appropriate nodes.

Examining node 0 (the ground)

$$\sum_{+in@0} i = +i_3 - i_4 - i_5 = +6.67mA - 0 - i_5 = 0$$

Solving this gives $i_5 = 6.67 \text{ mA}$

Examining node 3

$$\sum_{+in@1} i = +i_7 - i_1 - i_2 + i_5 = +i_7 - (11.665mA) - (5mA) + (6.67mA) = 0$$

Solving for i_7 gives $i_7 = (11.665mA) + (5mA) - (6.67mA) = 9.995mA$

In summary,

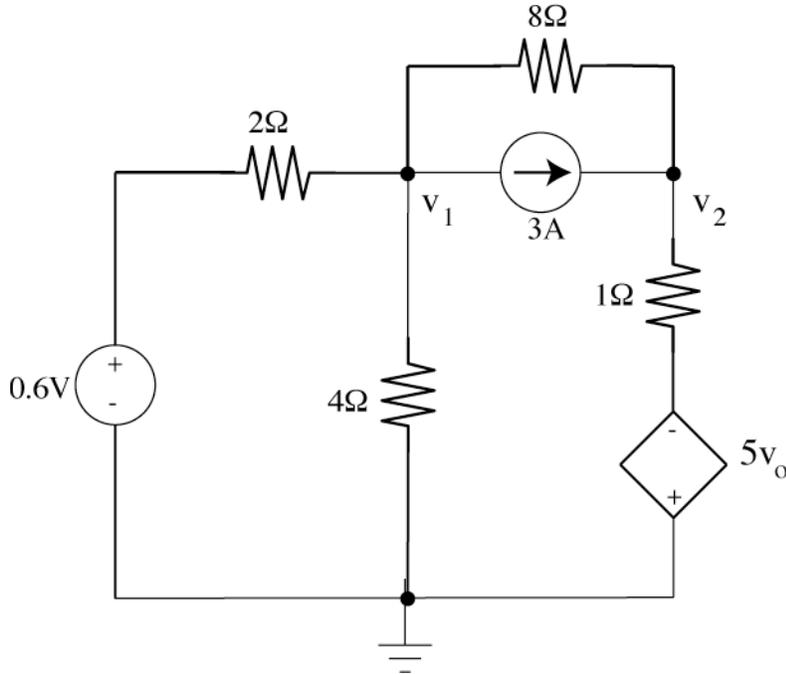
$v_1 =$	20 V	$i_1 =$	11.665mA	$i_5 =$	6.67mA
$v_2 =$	-3.33 V	$i_2 =$	5 mA	$i_6 =$	5 mA
$v_3 =$	0 V	$i_3 =$	6.67 mA	$i_7 =$	9.995 mA
$v_4 =$	-10 V	$i_4 =$	0 mA		

You received 1/2 point for v_1 and i_6 and 1 point for all other voltages and currents.

Problem 2 Circuits with controlled sources (10 points)

Find the node voltages v_1 and v_2 for the circuit shown below.

$v_1 =$ _____ $v_2 =$ _____



SOLUTION

Because you are asked for node voltages and there is an indicated current source this is a problem that can best be solved using the node voltage technique. Let node 1 be the node whose voltage is v_1 and node 2 be the node whose voltage is v_2 . There are also two other nodes whose voltages are determined by a direct connection to a voltage source. The voltage on the left hand side of the 2Ω resistor is set by the voltage source to be +12 volts. The voltage on the bottom of the 1Ω resistor is set by the dependent voltage source to be $-5v_o$ volts. Using this information we can write the node voltage equations (using KCL) at nodes 1 and 2 to be:

$$\sum_{+in@1} i = +i_1 - i_2 - i_3 - 3 = +\frac{(12) - v_1}{2} - \frac{v_1 - 0}{4} - \frac{v_1 - v_2}{8} - 3 = 0$$

$$\sum_{+in@2} i = +i_3 + 3 - i_4 = +\frac{v_1 - v_2}{8} + 3 - \frac{v_2 - (-5v_o)}{4} = 0$$

which gives two equations in two unknowns

$$+\frac{12 - v_1}{2} - \frac{v_1 - 0}{4} - \frac{v_1 - v_2}{8} - 3 = 0$$

$$+\frac{v_1 - v_2}{8} + 3 - \frac{v_2 + 5v_o}{4} = 0$$

Multiplying both equations by 8 gives

$$48 - 4v_1 - 2v_1 - v_1 + v_2 - 24 = 0 \quad (1)$$

$$+v_1 - v_2 + 24 - 8v_2 - 40v_o = 0$$

Before we can simplify we need to substitute the constraint equation $v_o = 12 - v_1$ into the second equation to get $+v_1 - v_2 + 24 - 8v_2 - 40(12 - v_1) = 0$ or

$$+v_1 - v_2 + 24 - 8v_2 - 480 + 40v_1 = 0 \quad (2)$$

We can rearrange equations (1) and (2) into more standard form to solve them:

$$+7v_1 - v_2 = 24$$

$$+41v_1 - 9v_2 = 456$$

We have two equations in two unknowns which can be solved by a variety of methods. I chose to solve the first equation for $v_2 = 7v_1 - 24$ and substitute this into the second equation giving:

$$+41v_1 - 9(7v_1 - 24) = 456$$

$$+41v_1 - 63v_1 + 216 = 456$$

$$-22v_1 = 240$$

$$v_1 = -\frac{120}{11} = -10.91$$

$$\text{Using } v_2 = 7v_1 - 24 \text{ we get } v_2 = 7v_1 - 24 = 7\left(-\frac{120}{11}\right) - 24 = -100.36$$

In conclusion,

$$v_1 = -10.91 \text{ volts}$$

$$v_2 = -110.36 \text{ volts}$$

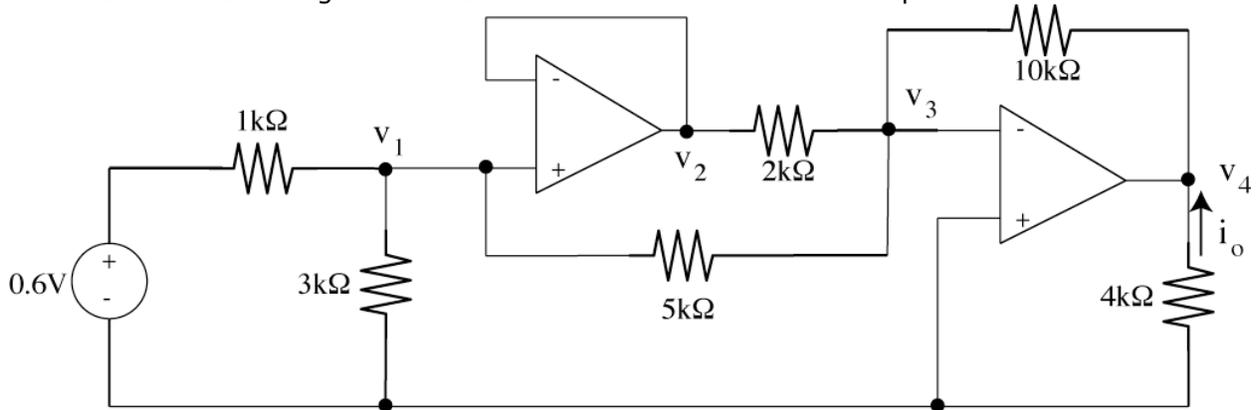
Grading: 5 points for each answer.

Partial credit: You received 3 points for the equation for v_1 and another 3 points for the equation for v_2 if they were correct allowing 4 points for the detailed solution.

Algebraically, this was the hardest problem on the exam.

Problem 3 Multiple Op Amp Circuits (10 points)

Determine the indicated node voltages (with respect to ground) and currents for the circuit shown below. All voltages are to be in volts and all currents in amperes.



$v_1 =$ _____ $v_3 =$ _____ $i_o =$ _____
 $v_2 =$ _____ $v_4 =$ _____

SOLUTION

Just as in Problem 2 this problem will reduce to two equations in two unknowns if you apply the ideal OP AMP constraints that $v_+ = v_-$ and no current goes into the OP AMP inputs.

From the circuit we see that $v_+ = v_-$ gives us $v_1 = v_2$ and $v_3 = 0$. We now apply Kirchoff's Current Law to find v_1 and v_4 . Some of you may ask why do we choose to find v_1 and v_4 . The reason is that we have no currents going into the OP AMPs (the second constraint) so all the currents at these nodes are known but we have no idea what currents are coming out of the OP AMPs. As you will recall you were told in lecture several times never to write a KCL equation at the output for an ideal OP AMP. Writing the KCL equations at nodes 1 and 3 gives:

$$\sum_{+in@1} i = +\frac{(0.6) - v_1}{1k} - \frac{v_1 - 0}{3k} - \frac{v_1 - v_3}{5k} = 0$$

$$\sum_{+in@3} i = +\frac{v_2 - v_3}{2k} + \frac{v_1 - v_3}{5k} - \frac{v_3 - v_4}{10k} = 0$$

Multiplying the first equation by 15k and the second equation by 10k gives

$$+15(0.6) - 15v_1 - 5v_1 - 3v_1 - 3v_3 = 0$$

$$+5v_2 - 5v_3 + 2v_1 - 2v_3 - v_3 + v_4 = 0$$

Using our constraints that $v_1 = v_2$ and $v_3 = 0$ allows us to simplify these equations.

$$+9 - 15v_1 - 5v_1 - 3v_1 - 3(0) = 0$$

$$+5v_2 - 5(0) + 2v_1 - 2(0) - (0) + v_4 = 0$$

Re-arranging and collecting terms gives

$$+9 - 15v_1 - 5v_1 - 3v_1 = 0$$

$$+5v_1 + 2v_1 + v_4 = 0$$

This allows us to solve for v_1 and v_4 .

$$23v_1 = 9$$

$$v_4 = -7v_1$$

$$v_1 = \frac{9}{23} = 0.391 \text{ volts and } v_4 = -7\left(\frac{9}{23}\right) = -2.739 \text{ volts.}$$

From the constraint $v_1 = v_2$ we can further write $v_2 = 0.391$ volts.

The output current i_o is simply given by Ohm's Law as

$$i_o = \frac{0 - v_4}{4k} = \frac{0 - (-2.739)}{4k} = \frac{2.739}{4k} = 0.685 \text{ milliamperes.}$$

However, you must be careful of using the passive sign convention to get the proper sign (direction) of the current.

In summary,

$$v_1 = 0.391 \text{ volts}$$

$$v_2 = 0.391 \text{ volts}$$

$$v_3 = 0 \text{ volts}$$

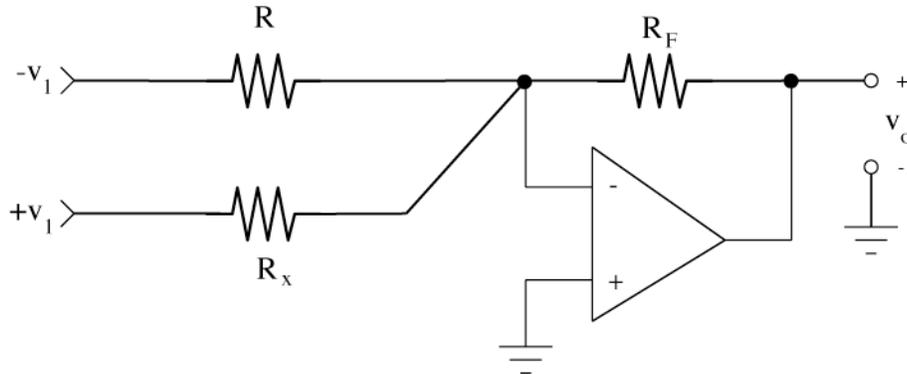
$$v_4 = -2.739 \text{ volts}$$

$$i_o = 0.685 \text{ mA}$$

Grading: Each electrical quantity was worth 2 points; we took off ½ point for the wrong sign.

Problem 4 Computer Interfacing (10 points)

The resistance R_x of a semiconductor pressure sensor varies linearly with pressure from $5k\Omega$ (0 psi) to $15k\Omega$ (100 psi). This sensor is placed in the following circuit.



(a) Determine $\frac{v_o}{v_1}$ for the above circuit.

SOLUTION:

The input/output relationship for this circuit can be derived directly using KCL and the OP AMP constraints, or it can be simply recognized from the formula sheet as a summing amplifier. We will show both solutions.

From the formula sheet we see that the circuit is exactly that of a summing amplifier:

		$K_1 = -\frac{Z_F}{Z_1}, K_2 = -\frac{Z_F}{Z_2}$
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From the block diagram we can directly write

$$v_o = K_1 v_1 + K_2 v_1 = \left(-\frac{R_F}{R}\right)(-v_1) + \left(-\frac{R_F}{R_x}\right)(v_1) = +\frac{R_F}{R} v_1 - \frac{R_F}{R_x} v_1 \text{ or } \frac{v_o}{v_1} = \frac{R_F}{R} - \frac{R_F}{R_x}$$

Alternatively, we can write KCL at the inverting input of the OP AMP:

$$\frac{-v_1 - 0}{R} + \frac{v_1 - 0}{R_x} - \frac{0 - v_o}{R_F} = 0$$

$$\frac{-v_1}{R} + \frac{v_1}{R_x} + \frac{v_o}{R_F} = 0$$

which can be re-arranged as

$$v_o = R_F \left(\frac{v_1}{R} - \frac{v_1}{R_x} \right) \text{ or } \frac{v_o}{v_1} = \frac{R_F}{R} - \frac{R_F}{R_x}$$

(b) For $v_1=15$ volts, select values of R and R_F to convert the sensor resistance to a voltage output which ranges from 0 to 5 volts.

This can be done in a variety of ways ranging from guessing (not effective) to writing equations for each end of the sensor/output voltage range. We will use the latter approach and use the result from part (a) with $v_1=15$ volts. For $v_1=15$ volts the output is

$$v_o = 15 \left(\frac{R_F}{R} - \frac{R_F}{R_x} \right) \tag{1}$$

A critical part of this problem was identifying the proper output voltage corresponding to $R_x=5k\Omega$ and $R_x=15k\Omega$. First, we notice from (1) that as R_x increases the second term becomes smaller and the output voltage increases. Using this we can recognize that for $R_x=5k\Omega$ (0 psi) we want $v_o = 0$. Similarly, for $R_x=15k\Omega$ (100 psi) we want $v_o = 5$. As soon as we make this recognition we can write equation (1) for each set of parameters to solve for R and R_F .

$$\text{For } R_x=5k\Omega \text{ equation (1) becomes } 0 = 15 \left(\frac{R_F}{R} - \frac{R_F}{5000} \right) \tag{2}$$

$$\text{For } R_x=15k\Omega \text{ equation (1) becomes } 5 = 15 \left(\frac{R_F}{R} - \frac{R_F}{15000} \right) \tag{3}$$

Simplifying equation (2) gives

$$\frac{R_F}{R} - \frac{R_F}{5000} = 0 \text{ or } R = 5000 \Omega.$$

Using this result in equation (3) gives

$$5 = 15 \left(\frac{R_F}{5000} - \frac{R_F}{15000} \right)$$

which may be solved for R_F

$$\frac{5}{15} = R_F \left(\frac{1}{5000} - \frac{1}{15000} \right) = R_F \left(\frac{3-1}{15000} \right) = R_F \left(\frac{2}{15000} \right)$$

$$\text{to give } R_F = \frac{5}{15} \frac{15000}{2} = 2500 \Omega.$$

(c) Is the output voltage from your circuit a linear function of pressure?

Linearity would require that v_o (pressure) = $K R_x$.

For $R_x=5k$, $v_o=0$ volts (0 psi)

For $R_x=15k$, $v_o=5$ volts (100psi)

Name : _____ Section: _____ CWRU e-mail: _____

Since the sensor itself is a linear function of pressure $R_x=10k\Omega$ will correspond to 50 psi. If our sensor interface circuit is linear the output voltage would then be $v_o=5/2=2.5$ volts.

To check this we calculate the output voltage using $R=5000$ and $R_F=2500$ for $R_x=10k\Omega$, Using these numbers gives $v_o = 15 \left(\frac{2500}{5000} - \frac{2500}{10000} \right) = 15 \left(\frac{5000 - 2500}{10000} \right) = 15 \frac{1}{4} = 3.75$ volts. Clearly the circuit output is not a linear function of pressure (R_x).

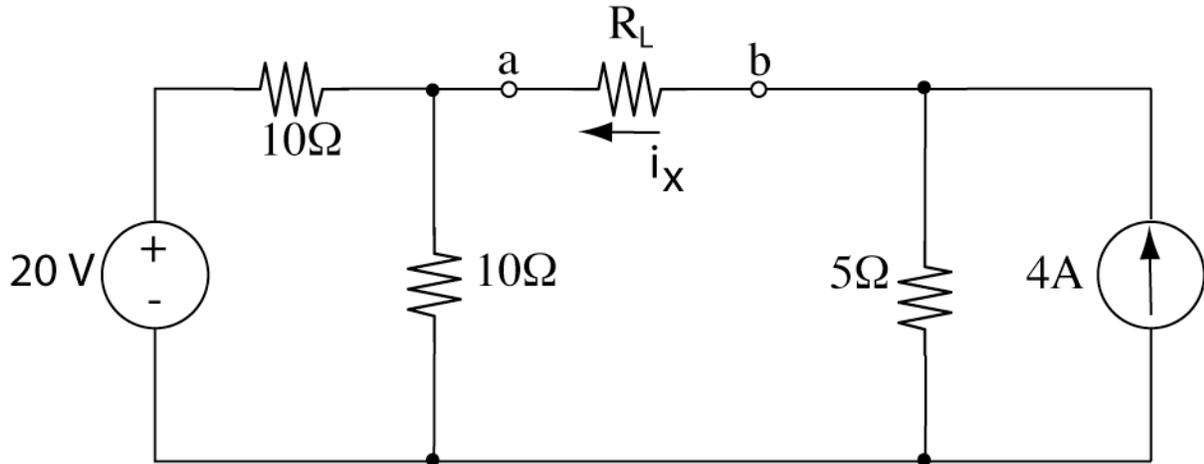
Grading:

Part (a) was worth three points.

In part (b) R and R_x were each worth three points.

Part (c) was worth one point.

Problem 5 Thevenin and Norton Equivalent Circuits (10 points)

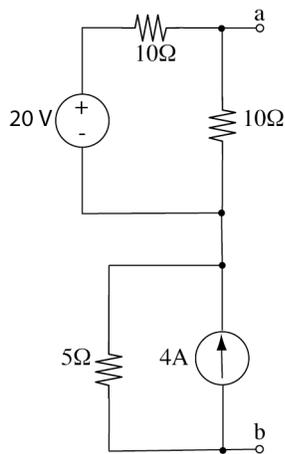


Answer the following questions about this circuit.

(a) What is the open circuit voltage V_{OC} , i.e., if $R_L = \infty$? _____

SOLUTION:

To find the open circuit voltage we remove R_L and redraw the circuit as shown below. The voltage V_{ab} is then V_{oc}

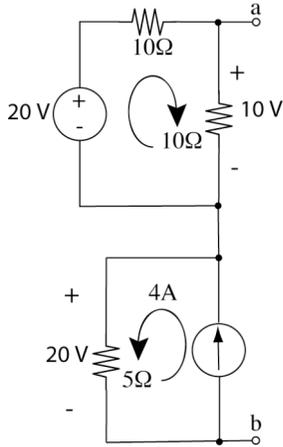


There are any number of ways to find V_{ab} for this circuit.

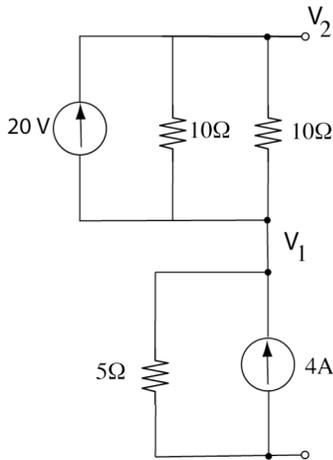
The astute student would notice that that the current from the 4A current source can only go through the 5Ω resistor creating a voltage across the 5Ω resistor. Similarly, the current from the 20V voltage source can only go through the two 10Ω resistors. The voltage across the 5Ω resistor is then $V_1 = iR = (4)(5) = 20$ volts and the voltage across the vertical 10Ω

resistor is $V_2 = \frac{10}{10+10}(20) = 10$ volts. The open circuit voltage is then

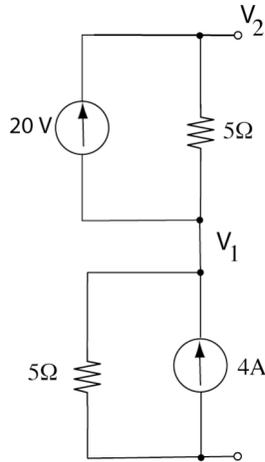
$V_{ab} = V_1 + V_2 = 20 + 10 = 30$ volts. This approach is shown in the circuit below.



Another way is to apply a source transformation to the voltage source and then use the node voltage technique to find the voltages of the resulting circuit as shown below.



We combine the two 10Ω resistors into one equivalent resistor ($R_{eq} = \frac{(10)(10)}{10+10} = \frac{100}{20} = 5\Omega$) to get the following circuit.



We can write the node voltage equations for V1 and V2 as

$$\sum_{+in@1} i = +4 - 2 - \frac{V_1 - 0}{5} + \frac{V_2 - V_1}{5} = 0$$

$$\sum_{+in@2} i = +2 - \frac{V_2 - V_1}{5} = 0$$

Simplifying these equations gives

$$10 - V_1 + V_2 - V_1 = 0$$

$$10 - V_2 + V_1 = 0$$

$$-2V_1 + V_2 = -10$$

$$V_1 - V_2 = -10$$

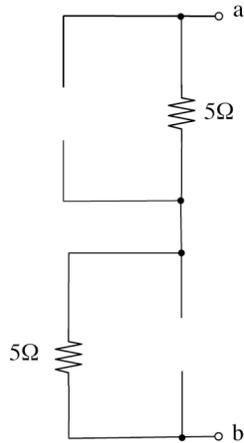
Adding these equations gives $-V_1 = -20$ or $V_1 = 20$ volts.

Using $V_1 - V_2 = -10$ we get $20 - V_2 = -10$ or $V_2 = 30$ volts which is the same result we got earlier.

(b) What is the Thevenin equivalent resistance of this circuit as seen by R_L ? _____

SOLUTION:

We simply turn off all sources (open the current sources, and short the voltage sources) in any of the above circuits. For example, we can turn off (open) the current sources in the above circuit to get



By simple inspection $R_{ab} = 5 + 5 = 10\Omega$

(c) What is the short circuit current i_{sc} , i.e., if $R_L = 0$? _____

SOLUTION

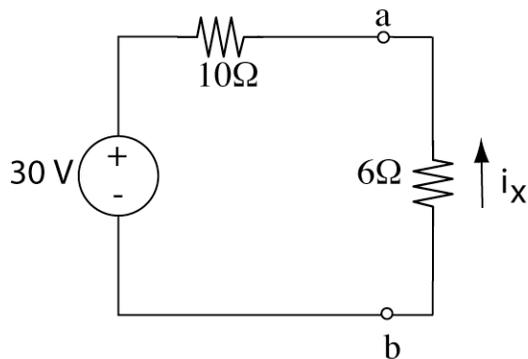
Although you could put a short between a and b and solve for i_{sc} , it is far easier to use the known relationships between the Norton and Thevenin equivalent circuits to compute i_{sc} as

$$i_{sc} = \frac{V_T}{R_T} = \frac{V_{oc}}{R_T} = \frac{+30}{5} = 6$$

(d) What is the current i_x when the load resistance R_L between a and b is 6Ω .

SOLUTION

You already have the equivalent circuit so this is simply a matter of connecting a 6Ω load resistor to the Thevenin equivalent circuit and computing the current through the resistor as shown below.

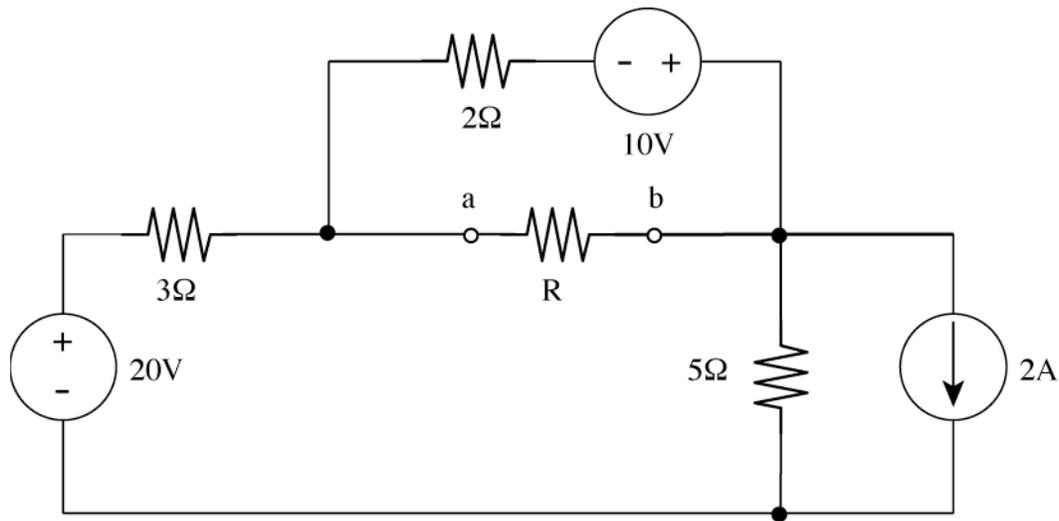


We simply calculate i_x as $i_x = -\frac{30}{10 + 6} = -\frac{30}{16} = -1.875$ amperes where we note that the - sign is introduced because the current i_x is in the opposite direction to the voltage coming from the 30 volt source.

GRADING

The Thevenin voltage was worth 4 points; the Thevenin resistance was worth 3 points. The maximum power transferred was worth 2 points. The value of R for maximum power transfer was worth one point.

Problem 5 Thevenin and Norton Equivalent Circuits (10 points)

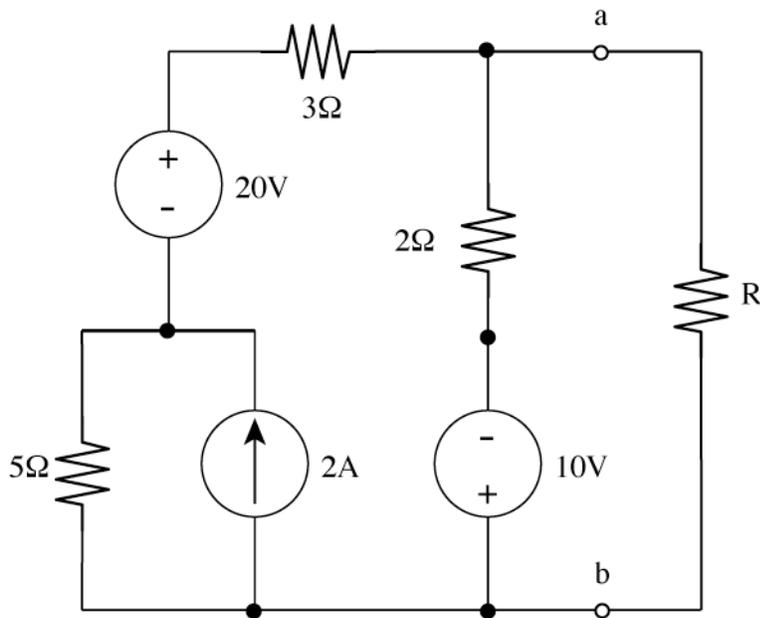


(a) What is the Thevenin equivalent of the circuit as seen by resistor R at terminals a-b.

$V_T =$ _____ $R_T =$ _____

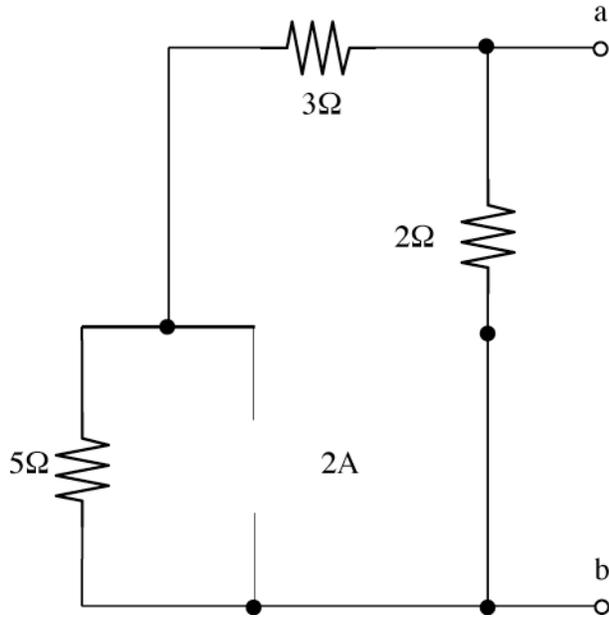
SOLUTION

As stated many times in class the solution approach often becomes obvious when you redraw the circuit. In this case we re-draw the circuit so that the terminals a-b are at the right side of the circuit as we normally draw it in our solutions or in class. The circuit will then look like the one shown below.



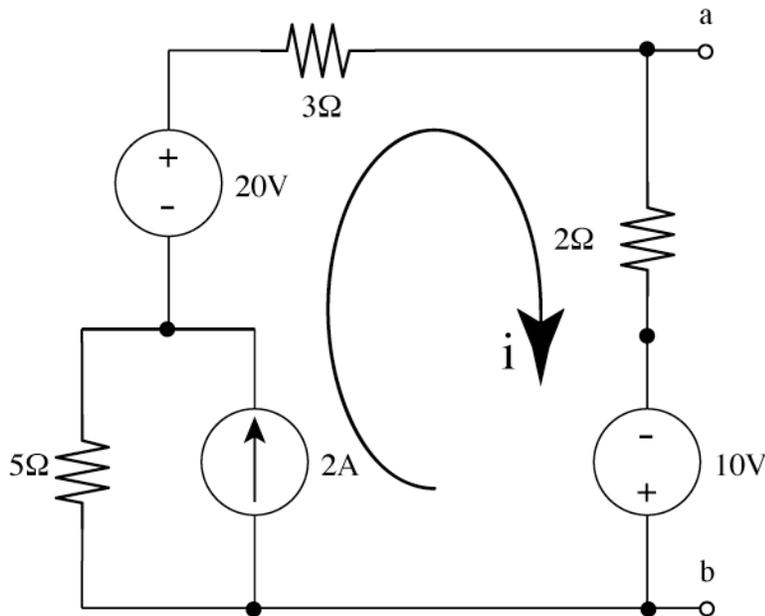
Careful examination will reveal that this is the same circuit as the original circuit.

We find R_T by simply turning off all sources (opening current sources, and shorting voltage sources) as shown below.



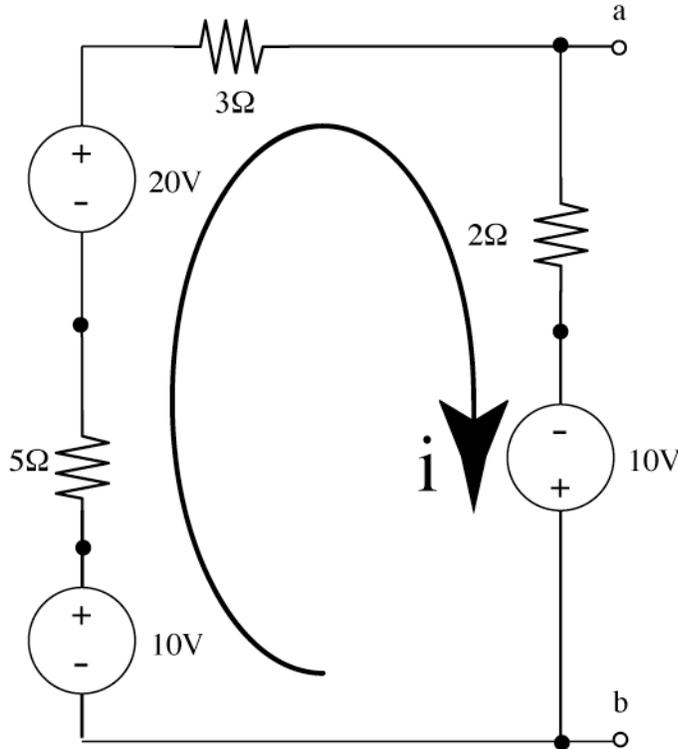
This is clearly seen to be an 8Ω ($3\Omega + 5\Omega$ in series) resistor in parallel with a 2Ω resistor. R_T is then given as $R_T = \frac{(8)(2)}{8+2} = 1.6\ \Omega$

Finding V_T is slightly more complex. We would like to use KVL in the circuit below to find the current i .



However, we can't use KVL immediately because of the current source so we use a source transformation to convert the 2A current source and parallel 5Ω resistor into a series 5Ω

resistor and a $V_{source} = i_{source} R_{parallel} = (2)(5) = 10$ volts. This transformed circuit is drawn below.



This is clearly a circuit that can be solved using Kirchoff's Voltage Law. Going around the loop clockwise from the lower left and summing voltages gives

$$-10 + 5i - 2 = +3i + 2i - 10 = 0$$

Solving for i gives

$$10i = 40$$

$$i = 4 \text{ amperes.}$$

Using the b terminal as our zero voltage reference we can now calculate the voltage at terminal a as $V_T = V_{ab} = V_{10\text{voltsource}} + i(2\Omega) = -10 + 4(2) = -2$ volts. A common mistake was not noticing that the polarity of the 10 volt source was -10 volts and getting +18 volts instead of -2 volts.

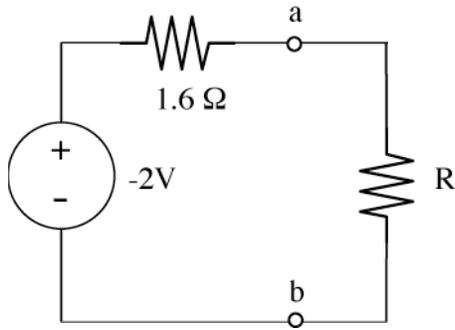
In summary, $V_T = -2$ volts and $R_T = 1.6\Omega$

(b) What is the maximum power that can be dissipated in R ?

Power = _____

SOLUTION

Consider the Thevenin equivalent circuit shown below which was just calculated in (a).



The maximum power in R occurs when $R=R_T=1.6\Omega$. When $R=R_T$ the voltage at point a will

be given by $V_a = \frac{1.6}{1.6+1.6}(-2) = -1$ volt. The maximum power dissipated in R is then

given by $P_{R,\max} = \frac{(V_a)^2}{R} = \frac{(-1)^2}{1.6} = 0.625$ watts.

(c) What is the value of R which gives this maximum power dissipation?

R= _____

SOLUTION

From (b) the maximum power dissipation occurs when $R=1.6\Omega$.

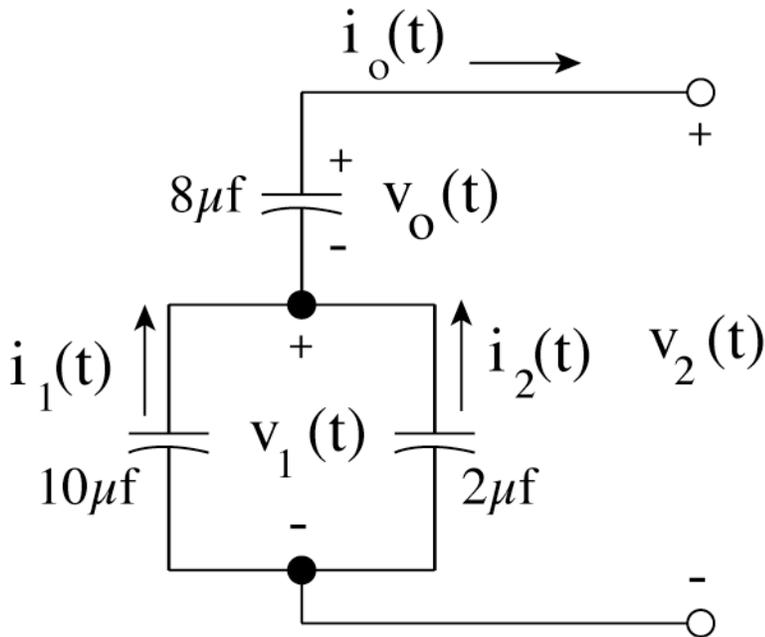
GRADING

The bulk of the problem was calculating the Thevenin equivalent circuit. V_T was worth 4 points; R_T was worth 3 points since it was easier to calculate.

The resistance for maximum power transfer was worth 1 point. The actual value of maximum power was worth 2 points.

Problem 6 Capacitors and Inductors (10 points)

Consider the capacitor circuit shown below. The current is $i_o(t) = 1.92e^{-20t}$ milliamperes for $t \geq 0$. The initial conditions are $v_o(0) = -5$ volts and $v_1(0) = +25$ volts.



a) What is $v_o(t)$ for $t \geq 0$?

SOLUTION

This was clearly the most difficult problem on the exam and required that you understand the relationship between voltage and current in a capacitor.

We start with $-i_o = C \frac{dv_o}{dt}$ where the minus sign comes from the given direction of i_o .

We then re-arrange the expression as $dv_o = \frac{1}{C} i_o dt$ so we can integrate it. Integrating gives

$$-\int_0^t dv_o = \frac{1}{C} \int_0^t i_o dt$$

$$v_o(t) - v_o(0) = -\frac{1}{C} \int_0^t i_o dt = -\frac{1}{8 \times 10^{-6}} \int_0^t 1.92 \times 10^{-3} e^{-20t} dt = -\frac{1.92 \times 10^{-3}}{8 \times 10^{-6}} \int_0^t e^{-20t} dt$$

$$v_o(t) - v_o(0) = -240 \frac{e^{-20t}}{-20} \Big|_0^t = 12[e^{-20t} - 1]$$

$$v_o(t) = 12[e^{-20t} - 1] + v_o(0) = 12e^{-20t} - 12 + (-5) = 12e^{-20t} - 17 \text{ for } t \geq 0$$

This was the crux of the problem. All the other answers are similarly calculated.

b) What is $v_1(t)$ for $t \geq 0$?

SOLUTION

The secret to this part of the problem is understanding how two capacitors in parallel operate. Consider that for capacitors C_1 and C_2 we can write $-i_1 = C_1 \frac{dv_1}{dt}$ and

$-i_2 = C_2 \frac{dv_1}{dt}$. We can then write the currents at the node between the capacitors as

$$i_o = i_1 + i_2 = -C_1 \frac{dv_1}{dt} - C_2 \frac{dv_1}{dt} = -(C_1 + C_2) \frac{dv_1}{dt}.$$

We then proceed with this equation in effectively the same manner as part (a).

$$i_o = -(C_1 + C_2) \frac{dv_1}{dt}$$

Rearranging gives

$$-\int_0^t dv_1 = \frac{1}{C_1 + C_2} \int_0^t i_o dt$$

$$v_1(t) - v_1(0) = -\frac{1}{C_1 + C_2} \int_0^t i_o dt = -\frac{1}{10 \times 10^{-6} + 2 \times 10^{-6}} \int_0^t 1.92 \times 10^{-3} e^{-20t} dt = -\frac{1.92 \times 10^{-3}}{12 \times 10^{-6}} \int_0^t e^{-20t} dt$$

$$v_1(t) - v_1(0) = -160 \frac{e^{-20t}}{-20} \Big|_0^t = 8[e^{-20t} - 1]$$

$$v_1(t) = 8[e^{-20t} - 1] + v_1(0) = 8e^{-20t} - 8 + (25) = 8e^{-20t} + 17 \text{ for } t \geq 0$$

c) What is $i_1(t)$ for $t \geq 0$?

SOLUTION

This is the easiest part of the problem. For C_1 we use the capacitor relationship

$-i_1 = C_1 \frac{dv_1}{dt}$ where the negative sign appears again because of the direction of the indicated current.

$$i_1 = -C_1 \frac{dv_1}{dt} = -(10 \times 10^{-6}) \frac{d(8e^{-20t} + 17)}{dt} = -(10 \times 10^{-6}) [(-20)8e^{-20t}] = (1600 \times 10^{-6})e^{-20t}$$
$$i_1 = 1.6e^{-20t} \text{ milliamperes}$$

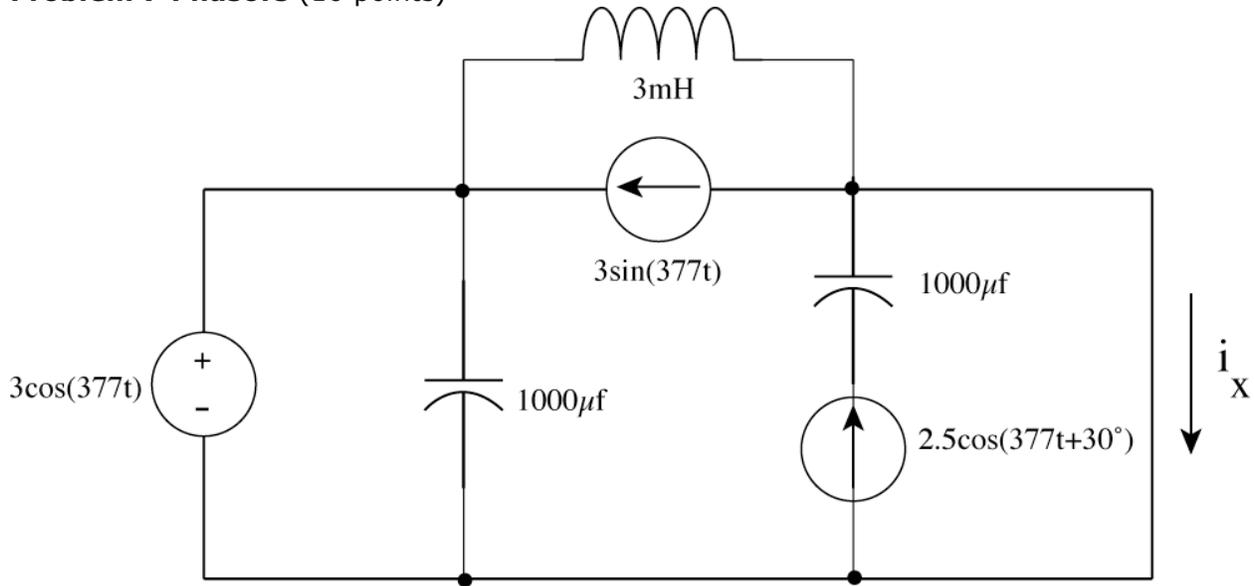
GRADING

Part (a) was 5 points since it was the most substantive part of the problem.

Part (b) was worth 3 points since it derived from (a)

Part (c) was only worth 2 points since it was effectively a straightforward application of a formula.

Problem 7 Phasors (10 points)



(a) What is the frequency f in Hertz at which this circuit is operating? $f =$ _____

SOLUTION:

From the sources we identify $\omega = 377$ as the angular frequency of the source. Converting it

to linear frequency we get $f = \frac{\omega}{2\pi} = \frac{377}{2\pi} = 60$

(b) Transform each source to its phasor equivalent

$3\cos(377t) \rightarrow$ _____

$3\sin(377t) \rightarrow$ _____

$2.5\cos(377t+30^\circ) \rightarrow$ _____

SOLUTION:

$3\cos(377t) \rightarrow 3\angle 0^\circ$

$3\sin(377t) = 3\cos(377t - 90^\circ) \rightarrow 3\angle -90^\circ = -j3$

$2.5\cos(377t + 30^\circ) \rightarrow 2.5\angle 30^\circ = 2.165 + j1.25$

(c) Transform each component to its equivalent impedance.

$3 \text{ mH} \rightarrow$ _____

$1000 \mu\text{f} \rightarrow$ _____

SOLUTION

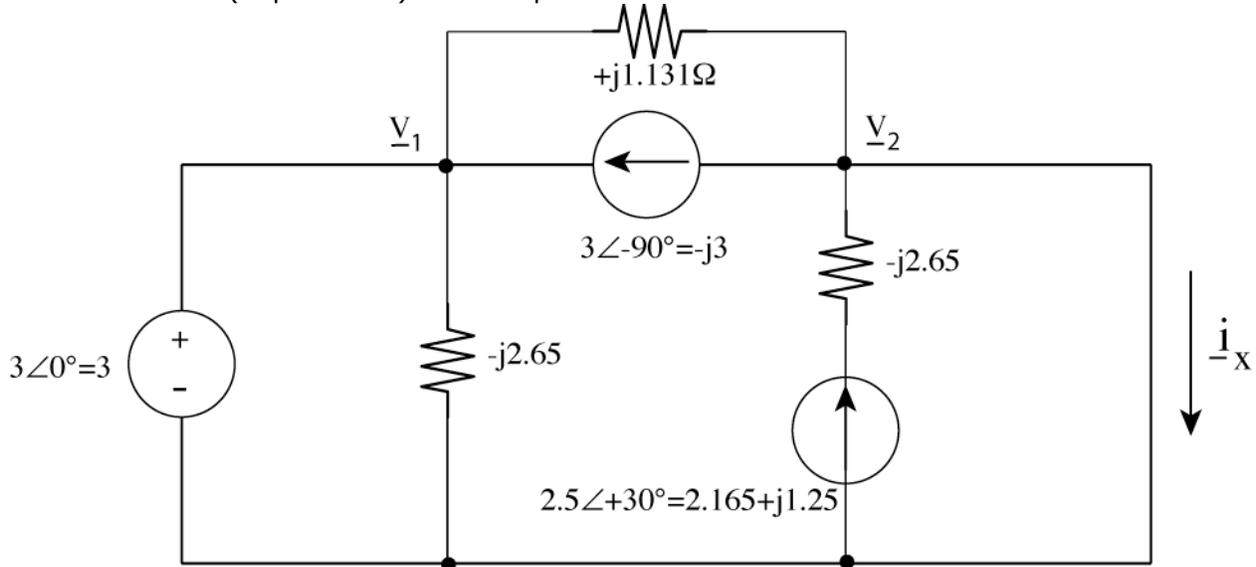
$Z_L = j\omega L = j(377)(0.003) = +j1.131$

$Z_C = \frac{1}{j\omega C} = \frac{1}{j(377)(1000 \times 10^{-6})} = -j2.65$

(d) Redraw the above circuit as an equivalent phasor circuit.

SOLUTION

This is simply re-drawing the original circuit replacing all the elements with complex resistances (impedances) and complex sources.



(e) Find the phasor current \underline{i}_x .

SOLUTION

This is a simple application of KCL. Using KCL at node 2 we can write

$$\sum_{+in@2} i = +\frac{V_1 - V_2}{j1.131} - (-j3) + (2.165 + j1.25) - \underline{I}_x = 0$$

Substituting the node voltages $V_1 = 3$ (since it is connected directly to the voltage source) and $V_2 = 0$ (since it is connected directly to ground) gives

$$+\frac{3-0}{j1.131} - (-j3) + (2.165 + j1.25) - \underline{I}_x = 0$$

Simplifying and solving

$$-j2.653 + j3 + 2.165 + j1.25 - \underline{I}_x = 0$$

$$\underline{I}_x = 2.165 + j1.597 = 2.69 \angle 36.4^\circ$$

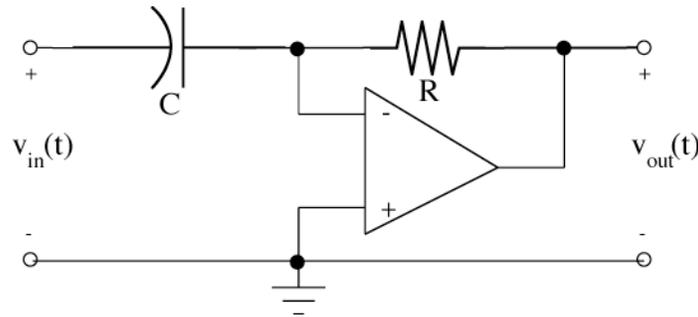
(f) What is $i_x(t)$?

SOLUTION

$$\underline{I}_x = 2.69 \angle 36.4^\circ \rightarrow i_x(t) = 2.69 \cos(377t + 36.4^\circ)$$

Problem 8 Frequency response (10 points)

Consider the circuit shown below where $v_{in}(t) = 5 \cos 1000t$ volts, $R=10k\Omega$, and $C=0.1\mu f$.



(a) Write the expression for the output in the time domain. $v_{out}(t) =$ _____

SOLUTION:

You can solve this problem in several ways. The simplest to is identify this circuit as a differentiator from the formula sheet provided, i.e.,

		$K = -RC$
--	--	-----------

From this information you can write

$$v_{out}(t) = K \frac{dv_{in}(t)}{dx} = -RC \frac{dv_{in}(t)}{dx} = -(10000)(0.1 \times 10^{-6}) \frac{d(5 \cos 1000t)}{dx}$$

$$v_{out}(t) = -(0.001)(5)(-1000 \sin 1000t) = 5 \sin 1000t$$

You can also solve this problem directly from the OP AMP constraints and using KCL. From the OP AMP constraints the voltage at the inverting (-) input is zero volts since the non-inverting (+) input is grounded. Writing KCL at the inverting input gives

$$\sum_{+in@-input} i = +i_C - i_R = C \frac{d(v_{in} - 0)}{dt} - \frac{0 - v_{out}}{R} = 0$$

Rearranging gives $C \frac{dv_{in}}{dt} + \frac{v_{out}}{R} = 0$ or $v_{out} = -RC \frac{dv_{in}}{dt}$ which is the same expression as above and can be evaluated the same way to give $v_{out}(t) = 5 \sin 1000t$

(b) Convert the circuit elements to phasors and evaluate the expression

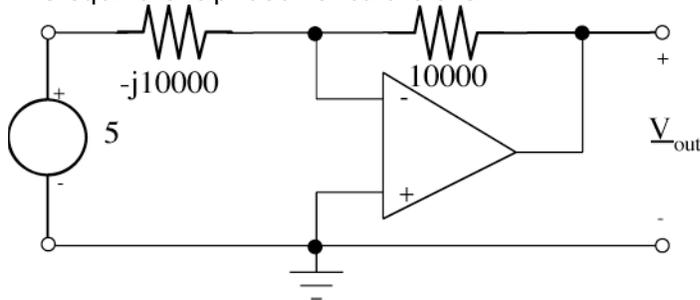
$$T(j\omega) = \frac{v_{out}(j\omega)}{v_{in}(j\omega)} = \text{_____}$$

To convert this circuit to its phasor equivalent we identify $\omega = 1000$ from $v_{in}(t) = 5 \cos 1000t$. Using this we convert the capacitor to an equivalent phasor

$$\text{impedance } Z_C = \frac{1}{j\omega C} = \frac{1}{j(1000)(0.1 \times 10^{-6})} = -j10000$$

The source transforms as $5 \cos 1000t \rightarrow 5 \angle 0^\circ$

The equivalent phasor circuit is then



Determining the transfer function is nothing more than evaluating KCL at the inverting (-) input in the phasor domain.

$$\sum_{+in@-input} i = +i_C - i_R = \frac{v_{in} - 0}{\frac{1}{j\omega C}} - \frac{0 - v_{out}}{R} = j\omega C v_{in} + \frac{v_{out}}{R} = 0$$

$$j\omega C v_{in} + \frac{v_{out}}{R} = 0$$

$$v_{out} = -j\omega RC v_{in}$$

Putting this in the request form gives

$$T(j\omega) = \frac{v_{out}}{v_{in}} = -j\omega RC = -j\omega(10000)(0.1 \times 10^{-6}) = -j0.001\omega = -\frac{j\omega}{1000}$$

Several students evaluated this expression for $\omega = 1000$ whereas the question asked for the result as a function of ω

(c) Sketch $|T(j\omega)|$ for $10 < \omega < 10000$ radians/second. Label your axes.

SOLUTION

The expression from (b) is very similar to many which we plotted in class, on homework, and on quizzes. We can slightly modify the expression to make it easier to plot as

$$T(j\omega) = \frac{v_{out}}{v_{in}} = -j0.001\omega = -j \frac{\omega}{1000}$$

Converting this to decibels gives

$$20 \log_{10} |T(j\omega)| = 20 \log_{10} \left| -j \frac{\omega}{1000} \right| = 20 \log_{10} \left| \frac{\omega}{1000} \right|$$

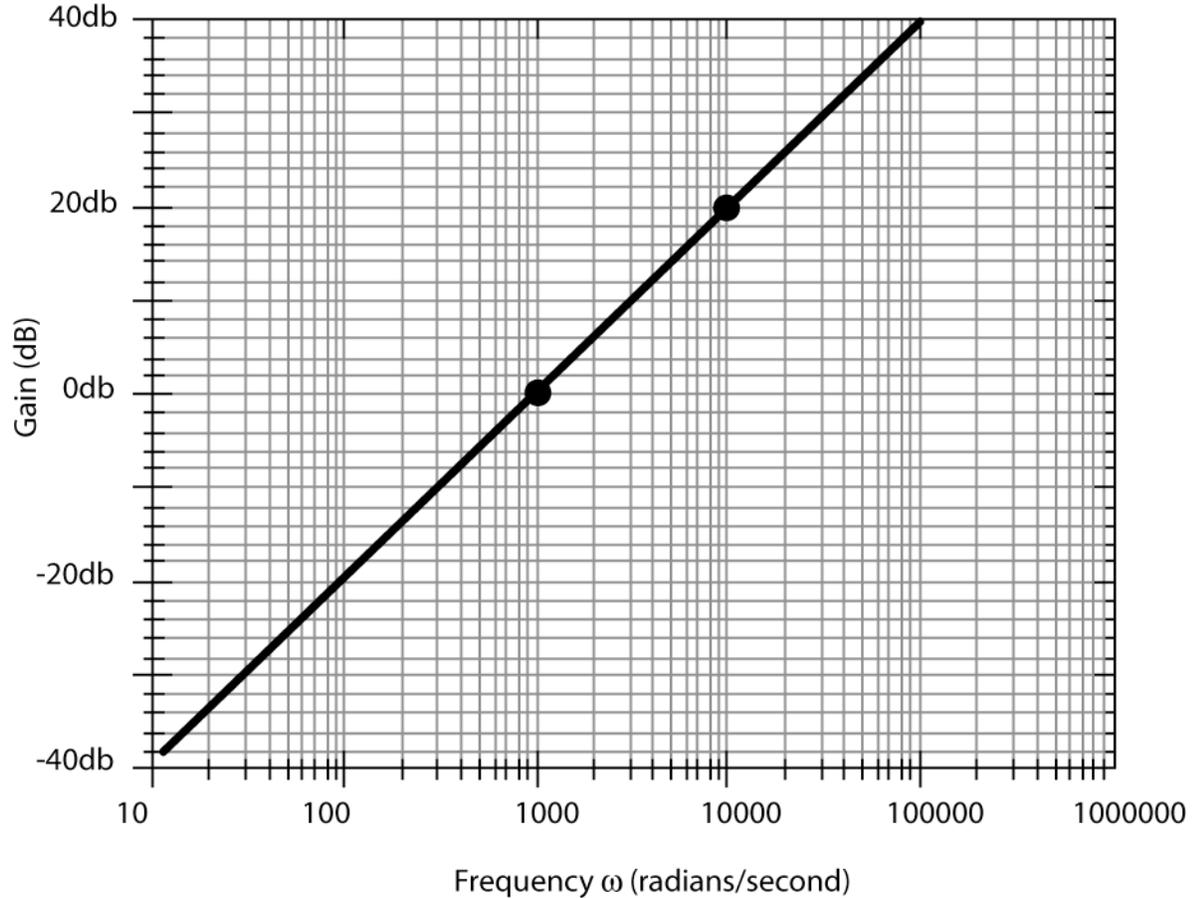
This can be quickly evaluated for a few points and plotted. At $\omega = 1000$ we have

$$20 \log_{10} |T(j\omega)| = 20 \log_{10} \left| \frac{1000}{1000} \right| = 20 \log_{10} |1| = 0 \text{ dB}$$

Now let's consider $\omega = 10000$ which gives

$$20 \log_{10} |T(j\omega)| = 20 \log_{10} \left| \frac{10000}{1000} \right| = 20 \log_{10} |10| = 20 \text{ dB}$$

These points form a straight line and are plotted on the chart below.



(d) What kind of filter is this?

SOLUTION

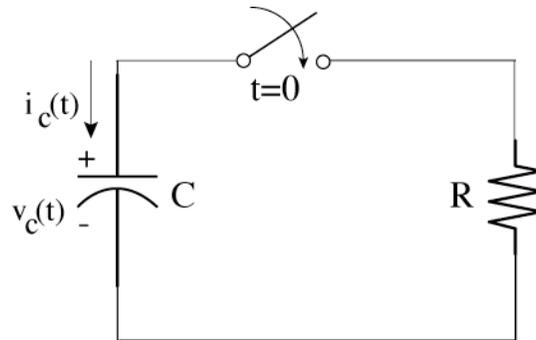
Since the filter amplifies high frequencies (gain > 1, i.e., greater than 0db) and attenuates low frequencies (gain < 1, i.e., less than 0db) this is a high-pass filter.

GRADING:

Parts (a), (b) and (c) were worth three points each. We took of two points in part (c) if you indicated the presence of a passband. Part (d) was worth one point.

Problem 9 Electrical Transients (10 points)

An airbag deploys when a pendulum switch detects a sudden deceleration greater than 10g and closes a switch. A simple equivalent circuit of the airbag deployment device is shown below. You are given that $v_c(0) = 24$ volts, $R = 100\Omega$, and $C = 0.1\text{mf}$.



a) Determine $v_c(t)$ for $t \geq 0$.

SOLUTION:

We solve the problem by using KVL for $t \geq 0$.

For $t \geq 0$ the switch is closed and summing the voltages in the circuit gives

$$-v_c(t) - i_c(t)R = 0$$

Note that the minus sign for the voltage across R comes about because of the direction of the capacitor current.

Using the capacitor constraint gives

$$-v_c(t) - C \frac{dv_c(t)}{dt} R = 0$$

which after rearranging gives the differential equation which describes this circuit as

$$RC \frac{dv_c(t)}{dt} + v_c(t) = 0$$

We can evaluate RC as $RC = (100)(0.1 \times 10^{-3}) = 0.01$ seconds.

By inspection the steady-state (DC) solution occurs when the derivative goes to zero,

i.e., $\frac{dv_c(t)}{dt} \rightarrow 0$, giving $v_{c,DC}(t) = 0$

The natural or transient solution occurs when the right hand side (the sources) are set to zero. Since there are no sources in this problem the equation for the natural solution

$$\text{becomes } RC \frac{dv_{C,NAT}(t)}{dt} - v_{C,NAT}(t) = 0$$

We have seen this equation many times and know it has a solution of the form

$$v_{C,NAT}(t) = Ke^{-\frac{t}{RC}} \text{ which can be evaluated for this problem as } v_{C,NAT}(t) = Ke^{-\frac{t}{0.01}} = Ke^{-100t}$$

The total solution is then the sum of these solutions

$$v_C(t) = v_{C,DC}(t) + v_{C,NAT}(t) = 0 + Ke^{-100t} = Ke^{-100t}$$

To determine K we must evaluate this expression for the initial condition which is given as $v_C(t=0) = +24$ volts.

Equating these expressions gives $K=24$

$$v_C(t=0) = Ke^{-100(0)} = K = 24$$

The final expression is then $v_C(t) = 24e^{-100t}$ volts for $t \geq 0$.

b) Determine an equation for the energy dissipated in the resistor as a function of time.

SOLUTION:

The energy is the integral of the power dissipated by the resistor. Expressed mathematically

$$w_{resistor} = \int_0^t p_{resistor}(t) dt = \int_0^t \frac{v_{resistor}^2}{R} dt = \int_0^t \frac{(24e^{-100t})^2}{100} dt$$

Evaluating this expression gives

$$w_{resistor} = 5.76 \frac{e^{-200t}}{-200} \Big|_0^t = -0.0288 [e^{-200t} - 1] = 0.0288 [1 - e^{-200t}]$$

c) The airbag will deploy when the energy absorbed by the resistor (in reality a fuse) reaches 25 mJ. How long it will take for the air bag to deploy after the switch closes?

SOLUTION

This is now a numerical problem where we simply set the expression from (b) equal to 0.025 joules (25 millijoules) and solve for t.

Name : _____ Section: _____ CWRU e-mail: _____

$$w_{resistor} = 0.0288[1 - e^{-200t}] = 0.025$$

$$1 - e^{-200t} = \frac{0.025}{0.0288} = 0.868$$

Rearranging gives

$$e^{-200t} = 1 - 0.868 = 0.13194$$

Taking the natural logarithm gives

$$-200t = \ln(0.13194) = -2.025$$

$$t = \frac{-2.025}{-200} = 0.010 \text{ seconds.}$$

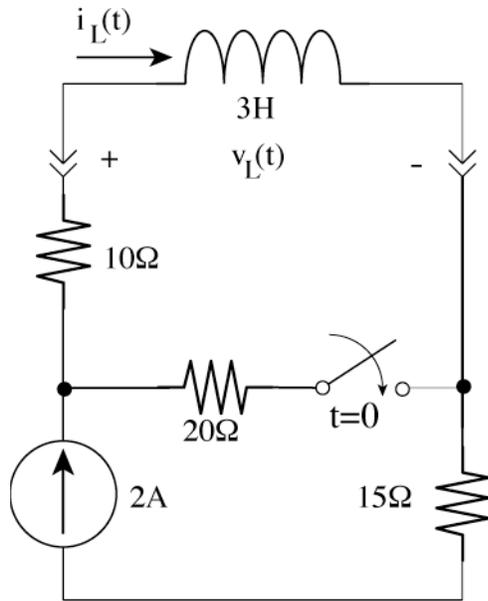
The air bag will deploy in about 10 milliseconds.

GRADING

Parts (a) and (b) were worth 4 points each. Part (c) was worth 2 points.

Problem 10 Switched circuits (10 points)

The switch in the circuit shown below has been open for a long time and the circuit is in steady state. At $t=0$ the switch closes.



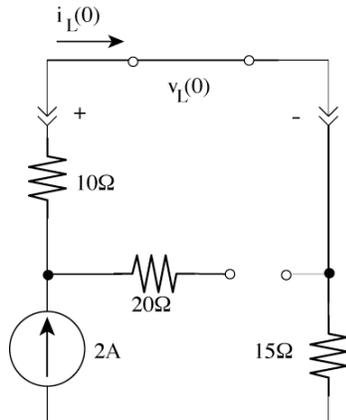
(a) What is the initial value of i_L , i.e., $i_L(0)$?

SOLUTION

As a comment, this problem is almost identical to an example done with a capacitor in class. To find the initial value we recognize that the circuit is in steady state which means that the current through the inductor is not changing and the switch is in its open state. Since the current through the inductor is constant this means that the inductor is behaving like

a short, i.e., $v_L = L \frac{di_L(t)}{dt} = 0$. We can then replace the inductor with an equivalent

short and determine i_L for the following circuit.

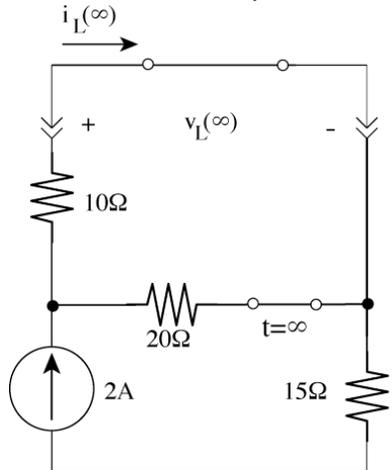


Although the circuit looks complicated the only current is flowing through the series combination of the 10Ω and the 15Ω resistor and must come from the $2A$ current course. By inspection $i_L(0) = 2$ amperes.

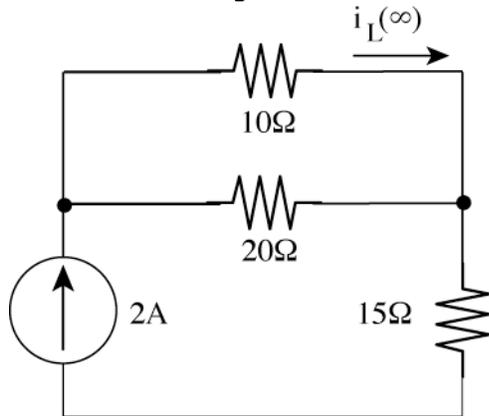
(b) What is the final value of i_L , i.e., $i_L(\infty)$?

SOLUTION

The final value is harder to compute as the switch is now closed. Because the time is long after the switch closed the inductor again behaves like a short resulting in the following circuit for analysis.



In this form the circuit looks complicated to analyze. However we can get a better understanding of the circuit and its analysis by re-arranging it as shown below.



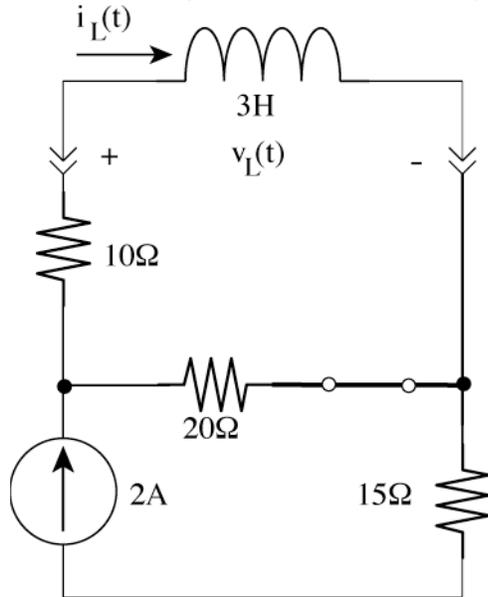
I removed all the confusing notation and kept only the arrow identifying the current which we want to identify. This is now seen to be a current divider where a 2A incoming current splits between the 10Ω and the 20Ω resistors allowing us to use the current divider relationship.

$$i_L(\infty) = \frac{20}{20 + 10} 2 = \frac{4}{3} \text{ ampere.}$$

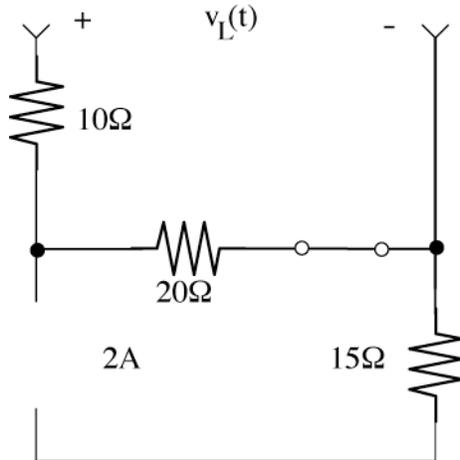
(c) What is the time constant of this circuit for $t \geq 0$?

SOLUTION

The time constant come from the circuit with the inductor in it for $t \geq 0$, i.e., with the switch closed. L is clearly know but we will need to compute the Thevenin equivalent resistance as seen by the inductor to compute the time constant.



We don't need the Thevenin voltage or the Norton current so all we need to do is turn off the current source and find the R_T of the following circuit as seen by the inductor.



Clearly this is simply two resistors in series so $R_T = 10 + 20 = 30$ ohms. Note that there is no current through the 15Ω resistor because it is not connected to anything.

We can now evaluate the time constant as $T_L = \frac{L}{R_T} = \frac{3}{30} = 0.1$

(d) Write the expression for $i_L(t)$ for $t \geq 0$.

SOLUTION

We can use the initial-final value theorem to write this since we now know the initial value, the final value and the time constant.

Write the initial-final value theorem

$$i_L(t) = (IV - FV)e^{-\frac{t}{T_L}} + FV = (i_L(0) - i_L(\infty))e^{-\frac{t}{L/R}} + i_L(\infty)$$

and substitute the appropriate circuit values to get

$$i_L(t) = \left(2 - \frac{4}{3}\right)e^{-\frac{t}{0.1}} + \frac{4}{3}$$

or

$$i_L(t) = \frac{2}{3}e^{-10t} + \frac{4}{3} \text{ amperes for } t \geq 0.$$

(e) What part of (d) is the forced response?

SOLUTION

The forced response is the DC (steady-state) response. This is $i_{L,DC}(t) = \frac{4}{3}$ ampere.

(f) What part of (d) is the natural response?

SOLUTION

The natural response is the transient response. This is $i_{L,NAT}(t) = \frac{2}{3}e^{-10t}$ ampere.

(g) What is the state variable for this circuit?

SOLUTION

This was explicitly covered many times in class. The state variable is the circuit parameter which describes the energy of the reactive element. Since $w_L = \frac{1}{2}Li^2$ the state variable is the inductive current i_L

GRADING

The major part of the problem was determining the initial value, the final value and the time constant. Consequently, parts (a), (b) and (c) were worth 2 points each. The remaining four parts (d) to (g) were worth one point each.