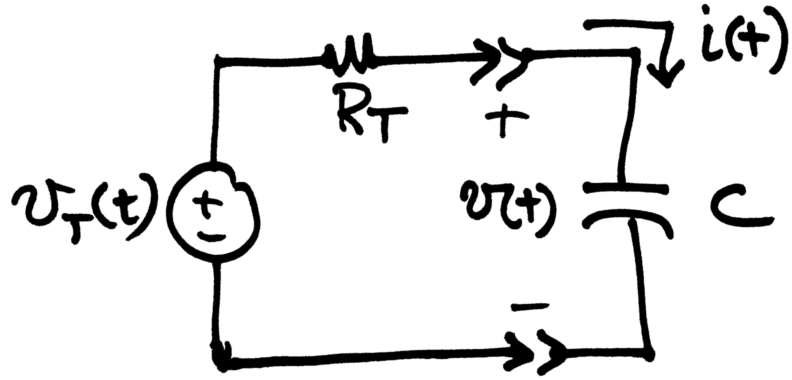
TRANSIENTS



$-v_T + i R_T + v = 0$

$\uparrow \; i = C\frac{dv}{dt}$

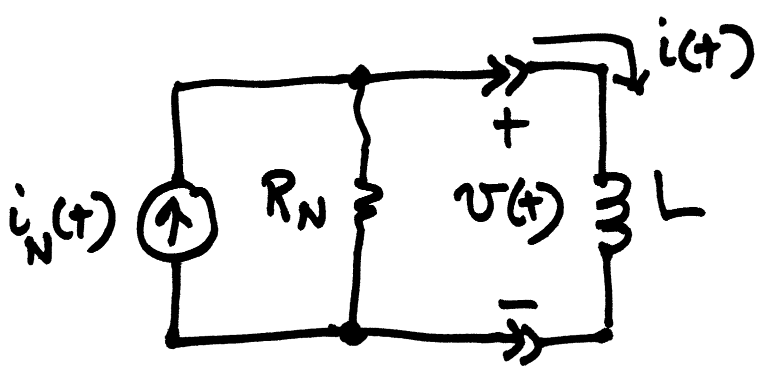
$-v_T + R_T C\frac{dv}{dt} + v = 0$

$$\underbrace{R_T C \frac{dv(t)}{dt} + v(t)}_{\substack{v(t) \\ \text{is the state variable}}} = v_T(t)$$

$\uparrow$

$A\cos(\omega t)$ sinusoidal steady state

stored energy $\frac{1}{2}Cv^2(t)$



$$\underbrace{\frac{L}{R_N}\frac{di(t)}{dt}}_{\substack{i(t)\text{ is the state} \\ \text{variable}}} + i(t) = i_N(t)$$

stored energy $\frac{1}{2}Li^2$

what does the solution depend on?

1. input $\begin{cases} \text{sinusoidal steadystate} & A\cos\omega t \\ \text{transients} & \begin{cases} \text{switch} \\ u(t) \\ \text{nothing} \end{cases} \end{cases}$

2. circuit values R_T, C

   R_N, L

3. initial conditions (switched)

---

find the solution for zero input

capacitive circuit

$$R_T C \frac{dv(t)}{dt} + v(t) = \cancelto{0}{v_T}(t)$$

$$R_T C \frac{dv}{dt} + v = 0$$

Assume a solution

Ke^{st}

method of undetermined coefficients

$$\frac{d}{dt}\left(Ke^{st}\right) = s\,Ke^{st}$$

$$R_T C\, sKe^{st} + Ke^{st} = 0$$

$$Ke^{st}\underbrace{\left(R_T C s + 1\right)}_{\text{require this to be zero}} = 0$$

$$R_T C s + 1 = 0$$

$$s = -\frac{1}{R_T C} \leftarrow \text{time constant}$$

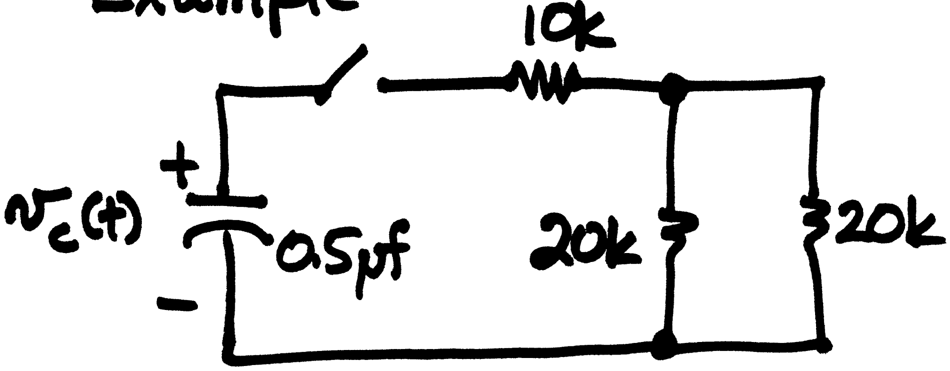
$$v(t) = Ke^{st} = Ke^{-\frac{t}{R_T C}}$$

for an inductive circuit

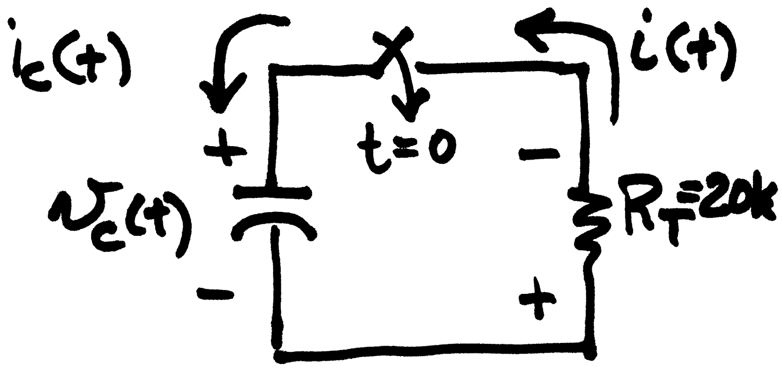
$$i(t) = Ke^{-\frac{R_N}{L}t} = Ke^{-\frac{t}{L/R_N}} \leftarrow \text{time constant}$$

K (in both cases) comes from initial conditions

Example 7-1



$v_c(t=0) = 30$ volts



for $t \geq 0$

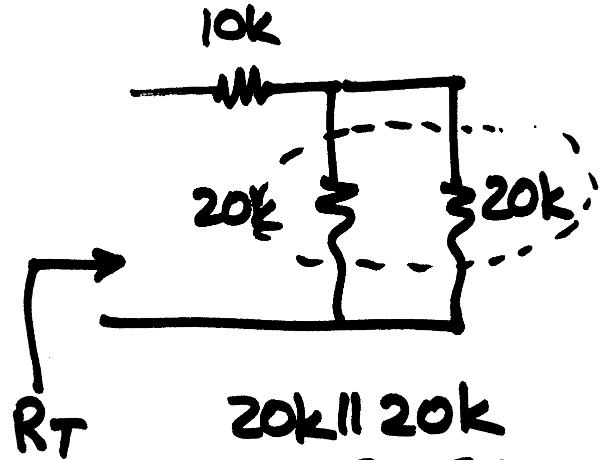
$$-v_c(t) - i(t)R_T = 0$$

$$i(t) = i_c(t) = C\frac{dv_c}{dt}$$

$$-v_c(t) - R_T C\frac{dv_c(t)}{dt} = 0$$

$$R_T C\frac{dv_c(t)}{dt} + v_c(t) = 0$$

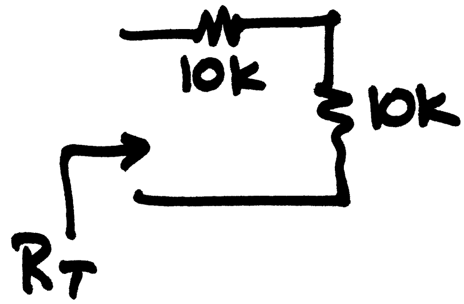
Assume $v_c(t) = Ke^{st}$

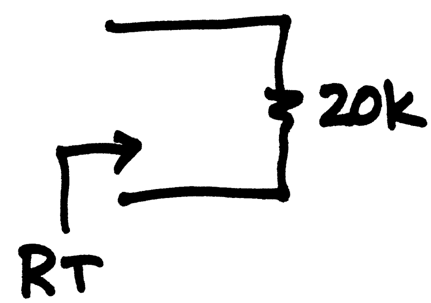


20k || 20k

$$R_{EQ} = \frac{20 \cdot 20}{20+20}$$

$$= 10k.$$





Substitute $v_c(t) = Ke^{st}$

into D.E.

$$R_T C s(Ke^{st}) + (Ke^{st}) = 0$$

$$Ke^{st}(R_T C s + 1) = 0$$

$$R_T C s + 1 = 0$$

$$s = -\frac{1}{R_T C} = -\frac{1}{(20\times10^{3})(0.5\times10^{-6})}$$

$$= -100$$

$$v_c(t) = Ke^{-100t}$$

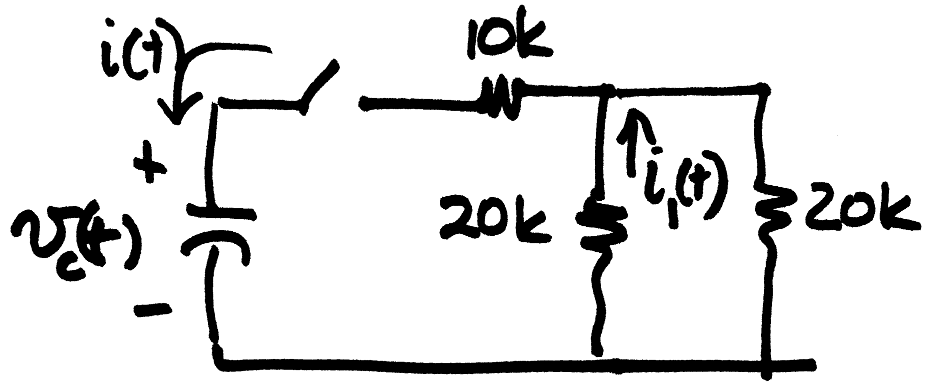
time constant $= \frac{1}{100} = 0.01$ sec.

$v_c(t=0) = 30$ volts.

$$30 = Ke^{-0} = K$$

$$v_c(t) = 30e^{-100t} \quad t \geq 0$$

$$30e^{-100t}u(t)$$



$$i(t) = i_c(t) = C\frac{dv_c}{dt}$$

$$= (0.5\times 10^{-6})\frac{d}{dt}(30e^{-100t})$$

$$i(t) = (0.5\times 10^{-6})(30)(-100)e^{-100t}$$

$$=$$

$$i_1(t) = \frac{20k}{20k+20k}\, i(t)$$