

Homework Solutions 5

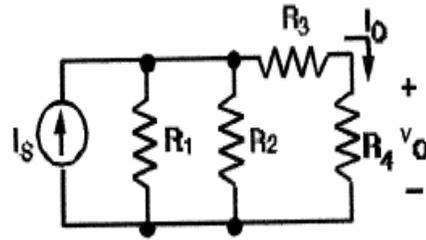
(3-22) Find the proportionality constant $K = i_o/i_s$ for the circuit in Figure P3-22

3-22 Using current division

$$i_o = \left[\frac{\frac{1}{R_3+R_4}}{\left(\frac{1}{R_3+R_4} + \frac{1}{R_1} + \frac{1}{R_2} \right)} \right] i_s$$

or $i_o = \left(\frac{R_1 \cdot R_2}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_1 \cdot R_4 + R_2 \cdot R_3 + R_2 \cdot R_4} \right) i_s$

hence $K = \frac{R_1 \cdot R_2}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_1 \cdot R_4 + R_2 \cdot R_3 + R_2 \cdot R_4}$



This problem can be tedious to solve but use your basic equations and don't forget current division, which makes this problem a lot easier.

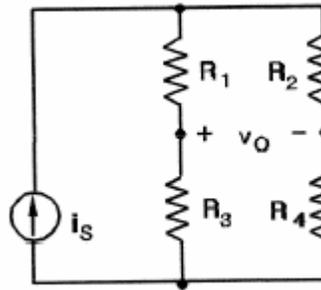
(3-23) Find the proportionality constant $K = v_o/i_s$ for the circuit in Figure P3-23.

3-23 Using current division

$$v_o = \left[\frac{(R_2 + R_4) \cdot i_s}{R_1 + R_2 + R_3 + R_4} \right] \cdot R_3 - \left[\frac{(R_1 + R_3) \cdot i_s}{R_1 + R_2 + R_3 + R_4} \right] \cdot R_4$$

or $v_o = \left(\frac{R_2 \cdot R_3 - R_1 \cdot R_4}{R_1 + R_2 + R_3 + R_4} \right) i_s$

hence $K = \frac{R_2 \cdot R_3 - R_1 \cdot R_4}{R_1 + R_2 + R_3 + R_4}$

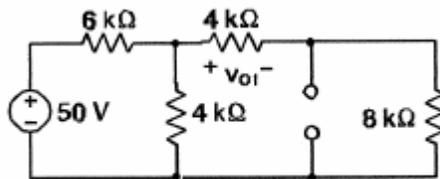


Consider the current through the resistor and the voltage drops at each resistor, an easy way to look at this problem is to consider is as dividing the current in parallel with $(R_1 + R_3) \parallel ((R_2 + R_4))$. Then, there is a voltage division between, R_1 and R_3 ; R_2 and R_4 . This gives you a better concept of the behavior of the circuit. Solve in terms of V_o/I_s .

(3-28) Use the superposition principle in the circuit of Figure P3-28 to find the V_o .

Steps to solve superposition problems

Step 1: Pick current directions through each resistor, shut off one of the sources off either current or voltage source. Label $V_o \rightarrow V_{o1}$, solve.



$$R_{IN} := 6 \cdot 10^3 + \frac{4 \cdot 10^3 (4 \cdot 10^3 + 8 \cdot 10^3)}{(4 + 4 + 8) \cdot 10^3}$$

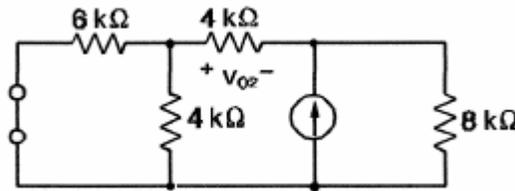
$$R_{IN} = 9 \times 10^3 \quad i_{s1} := \frac{50}{R_{IN}}$$

$$v_{o1} := \left(\frac{4 \cdot 10^3}{4 \cdot 10^3 + 12 \cdot 10^3} \right) \cdot i_{s1} \cdot 4 \cdot 10^3$$

$$v_{o1} = 5.556 \text{ V}$$

Step 2: Now shut of the other source and replace source omitted in Step 1.
Label $V_o \rightarrow V_{o2}$, solve.

3-28 Continued With the voltage source turned off



$$i_{O2} := \frac{8 \cdot 10^3 \cdot 0.005}{\left(8 \cdot 10^3 + 4 \cdot 10^3 + \frac{6 \cdot 10^3 \cdot 4 \cdot 10^3}{6 \cdot 10^3 + 4 \cdot 10^3} \right)}$$

$$i_{O2} = 2.777778 \times 10^{-3}$$

$$v_{O2} := 4000 \cdot (-i_{O2}) \quad v_{O2} = -11.111$$

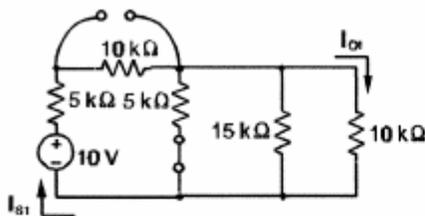
Using superposition $v_o := v_{o1} + v_{o2} \quad v_o = -5.556 \text{ V}$

Step 3: Hence, if $V_o = V_{o1} + V_{o2}$, simply added the two together, and you have found the voltage across the 4kΩ resistor. **$V_o = -5.556 \text{ V}$**

Important, remember when shutting off a voltage source it will act as a short. And, shutting off a current source it will act as a open.

(3-29) Use the superposition principle in the circuit of Figure P3-29 to **find I_o** .

3-29 With the 20 V and 1 mA sources turned off



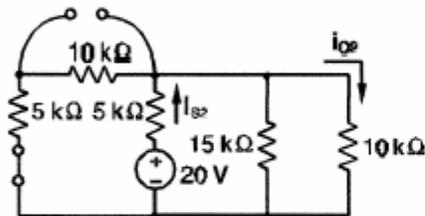
$$R_{IN1} := 15 \cdot 10^3 + \frac{1}{\frac{1}{5 \cdot 10^3} + \frac{1}{15 \cdot 10^3} + \frac{1}{10^4}}$$

$$i_{S1} := \frac{10}{R_{IN1}} \quad i_{S1} = 5.641 \times 10^{-4}$$

$$i_{O1} := \left[\frac{10^{-4}}{\left((5 \cdot 10^3)^{-1} + (15 \cdot 10^3)^{-1} + 10^{-4} \right)} \right] \cdot i_{S1}$$

$$i_{O1} = 1.538 \times 10^{-4}$$

With the 10-V and 1 mA sources turned off



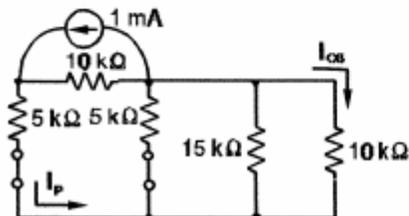
$$R_{IN2} := 5 \cdot 10^3 + \frac{1}{\left((15 \cdot 10^3)^{-1} + (15 \cdot 10^3)^{-1} + 10^{-4} \right)}$$

$$i_{S2} := \frac{20}{R_{IN2}} \quad i_{S2} = 2.153846 \times 10^{-3}$$

$$i_{O2} := \left[\frac{10^{-4}}{\left((15 \cdot 10^3)^{-1} + (15 \cdot 10^3)^{-1} + 10^{-4} \right)} \right] \cdot i_{S2}$$

$$i_{O2} = 9.231 \times 10^{-4}$$

With the 10-V and 20-V sources turned off



$$i_p := \frac{10^4 \cdot 0.001}{10^4 + 5 \cdot 10^3 + \frac{1}{\left((5 \cdot 10^3)^{-1} + (15 \cdot 10^3)^{-1} + 1 \cdot 10^{-4} \right)}}$$

$$i_{O3} := \left[\frac{10^{-4}}{\left[10^{-4} + (5 \cdot 10^3)^{-1} + (15 \cdot 10^3)^{-1} \right]} \right] \cdot (-i_p)$$

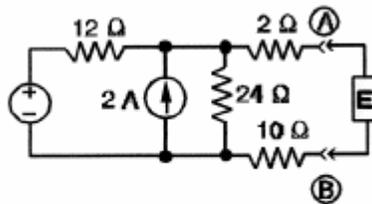
$$i_p = 5.641 \times 10^{-4} \quad i_{O3} = -1.538 \times 10^{-4}$$

Using superposition $i_o := i_{O1} + i_{O2} + i_{O3}$

$$i_o = 9.231 \times 10^{-4} \quad \text{A}$$

- (3-40)** (a) Find the Thevenin or Norton equivalent at terminals A and B in the Figure P3-40.
 (b) Use the equivalent circuit to find interface power when a 10Ω load is connected between terminals A and B.
 (c) Repeat (b) when a $5V$ source is connected between terminals A and B with the plus terminal at terminal A.

3-40 Convert the 12-V source to a 1-A current source in parallel with an 12Ω resistor



$$(a) v_{OC} = v_T = (2 + 1) \cdot \frac{12 \cdot 24}{12 + 24} = 24$$

$$R_T = 2 + \frac{12 \cdot 24}{12 + 24} + 10 = 20$$

$$(b) \text{ For a } 10\text{-}\Omega \text{ load } i_L := \frac{24}{20 + 10} \quad p_L := i_L^2 \cdot 10 \quad p_L = 6.4 \quad \text{W}$$

$$(c) \text{ For a } 5\text{-V source load } i_L := \frac{24 - 5}{20} \quad p_L := i_L \cdot 5 \quad p_L = 4.75 \quad \text{W}$$

Remember to open the circuit at A and B, and use superposition to solve for V_{oc}

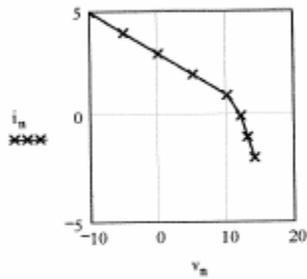
$$(a) V_{oc} = 12 \cdot 24 / (12 + 24) + 2 \cdot (12 \cdot 24) / (24 + 12) = (2 + 1) \cdot (12 \cdot 24) / (12 + 24) = \underline{\underline{24V}}$$

R_T is called the look back resistance or Thevenin resistance, this is found by looking backwards in the circuit from terminal A and finding the equivalent resistance.

- (b) Place a 10Ω resistor in for your load, find I_L through the load and solve for power.
 (c) To find the current through the load remember that subtract the $5V$ voltage drop through the load from the V_{oc} , then divide by the Thevenin resistance.

(3-44) Figure P3-42 shows the sources circuit with two accessible terminals. Some voltage and current measurements are at the accessible terminals are

3-44 (a) $v_1 := -10$ $v_2 := -5$ $v_3 := 0$ $v_4 := 5$ $v_5 := 10$ $v_6 := 12$ $v_7 := 13$ $v_8 := 14$ ← in V
 $i_1 := 5$ $i_2 := 4$ $i_3 := 3$ $i_4 := 2$ $i_5 := 1$ $i_6 := 0$ $i_7 := -1$ $i_8 := -2$ ← in mA
 $n := 1, 2, \dots, 8$



(b) On the range $-10 < v < 10$ the i - v characteristic is a straight line whose slope is $\left(\frac{i_5 - i_1}{v_5 - v_1}\right) \cdot 10^{-3} = -2 \times 10^{-4}$ & whose i -axis intercept is $i_3 = 3$ mA. The equation of this line is $i = -2 \cdot 10^{-4} \cdot v + 0.003$. The i - v characteristic of a Norton equivalent is: $i = \left(\frac{1}{R_T}\right) \cdot v + i_N$. Hence on the range from -10 V to $+10$ V parameters of the Norton equivalent are $i_N = 0.003$, $R_T = -\left(\frac{10^4}{-2}\right) = 5000$ and $v_T = i_N \times R_T = 15$ V.

(c) $v_{OC} = v_T = 15$ V
 $i_{SC} = i_N = 0.003$ A

(d) The model says that $v_{OC} = 15$ V whereas the data (v_8) says $v_{OC} = 12$ V. The model only applies to the range -10 V $< v < +10$ V because the characteristic is nonlinear for $v > 10$ V

In between the ranges of -10 and 10 the i - v curve acts like a resistor's slope but >10 V the curve becomes non-linear.