

# ENGR 210 Lab 11

## Frequency Response of Passive RC Filters

The objective of this lab is to introduce you to the frequency-dependent nature of the impedance of a capacitor and the impact of that frequency dependence on the input-output characteristics of RC circuits. The lab consists of making a number of measurements on some simple RC circuits, then analyzing those results.

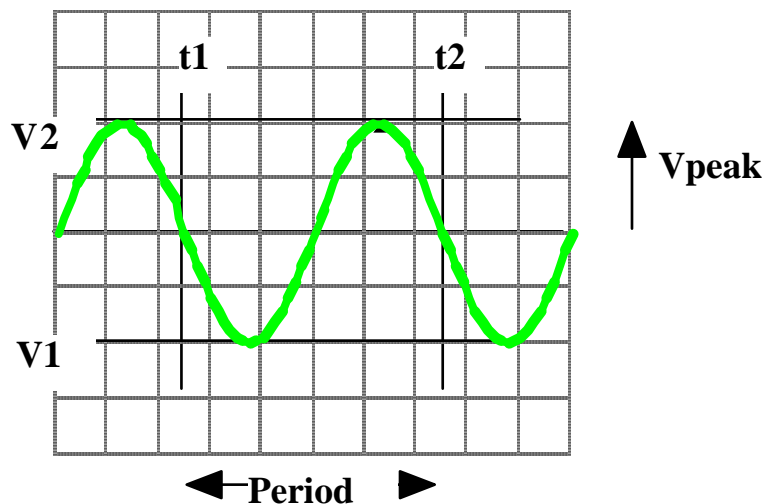
Using an oscilloscope, you will measure the amplitudes of sinusoidal input and output signals for three different RC circuits. You will also measure the phase difference between these signals. Finally, you will examine the effect of each circuit on a simulated signal — for simplicity of generation, a square wave.

**A. BACKGROUND** (Required Pre Lab calculations are indicated by a “bullet,” •)

### 1. Frequency and relative phase measurement

In this laboratory you will characterize several passive filters by measuring peak amplitudes of sinusoidal input and output signals and by indirectly measuring the phase shift of the output wave relative to the input wave. You will also indirectly measure the frequency of these signals.

In previous labs you have measured peak-to-peak voltages of sinusoidal waveforms. In this lab it is the peak amplitude, i.e., the maximum amplitude of the signal relative to 0 V, that is of interest, as shown schematically in Figure 1. To measure  $V_{\text{peak}}$ , set the voltage cursors on the oscilloscope to the signal maxima and minima as shown and measure the peak-to-peak voltage,  $V_{\text{p-p}}$ .



**Figure 1. Measurement of wave period and peak amplitude.**

The signal frequency is measured indirectly by measuring its period,  $T$ , as shown in Figure 1. To measure the period, set the time cursors to successive points where the signal crosses the time axis with a negative (or positive) slope. The frequency can then be calculated from the relationship

$$f(\text{Hz}) = \frac{1}{T(\text{sec})}, \text{ or } \omega(\text{radians/sec}) = \frac{2\pi}{T(\text{sec})} \quad (1)$$

The phase of the output wave relative to the input wave can be determined using a procedure similar to that used for measuring the period of a wave. Figure 2 shows two waveforms which represent the input to and output from a circuit displayed on an oscilloscope. Note first that the two signals have the same period and, therefore, the same frequency; however, they are shifted in time relative to each other. You will measure this time shift between the two signals, and then calculate the corresponding phase shift.

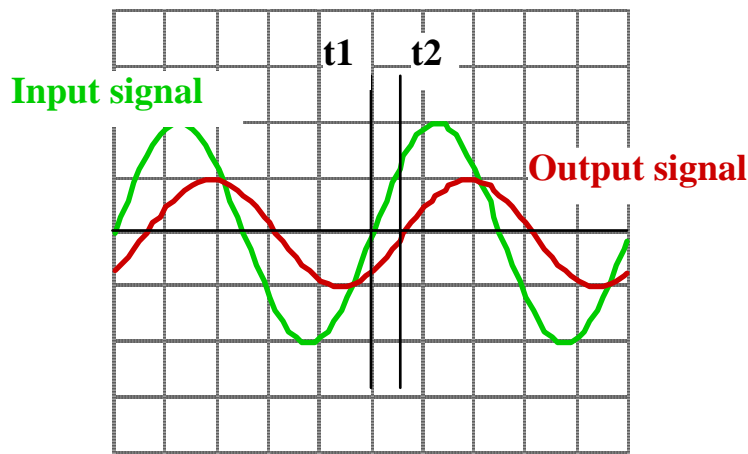


Figure 2. Measurement of phase shift.

As shown in Figure 2, set the  $t1$  time cursor at a location where the input signal crosses the time axis with a positive slope. Then set the  $t2$  time cursor to the first following point at which the output signal crosses the time axis with a positive slope. Since, in one period the phase of a periodic wave changes by  $2\pi$  radians or  $360^\circ$ , the phase shift,  $\Delta\phi$ , is given by

$$\Delta\phi(\text{radians}) = \frac{t1 - t2}{T} \times 2\pi \quad (2)$$

or

$$\Delta\phi(\text{degrees}) = \frac{t1 - t2}{T} \times 360^\circ. \quad (3)$$

*In this lab you will always measure  $t2$  for the output waveform, and  $t1$  for the input waveform.* The resulting phase shift is often called the delay of a filter.

**VERY IMPORTANT:** Be careful of the sign of the time delay in calculating delay. Note that the scope will calculate  $t2 - t1$ , which is the opposite sign of the expression in equations (2) and (3). Also, many times you will get angles between  $-270^\circ$  and  $-360^\circ$  when the actual angle should be their complement, i.e.,  $-270^\circ + 360^\circ = +90^\circ$  and  $-360^\circ + 360^\circ = 0^\circ$  respectively. When in doubt as to whether you need to take a complement look at the results you derived for these filters.

## 2. Transfer function

The circuits that you will study in this lab treat sinusoidal signals of different frequencies differently. Such circuits are said to be dispersive. Since they enable one to change the frequency characteristics of a signal by rejecting some frequencies and permitting others to pass through, they are called **filters**. A given filter has a different effect on both the magnitude and the phase of each input frequency. For example, a television antenna produces a voltage composed of signals from many transmitters. By using a filter that eliminates signals of all frequencies except those between 82 MHz and 88 MHz one is able to select the Channel 6 signal and reject all signals that would interfere with the appropriate video and audio information.

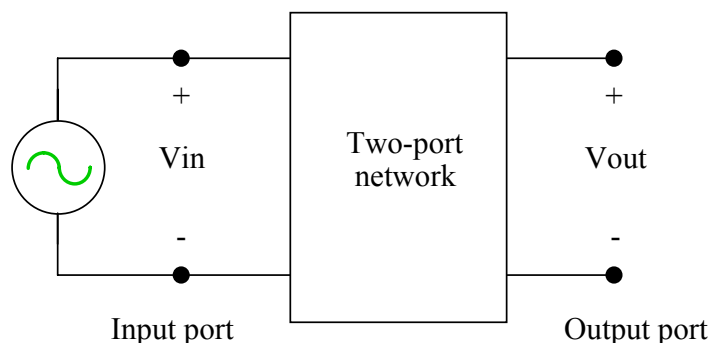


Figure 3. Two-port filter network.

Consider the filter network shown schematically in Figure 3. Networks like this are called **two-port networks** because they have an input port and an output port. Suppose that a sinusoidal signal of angular frequency  $\omega$  and phasor  $\mathbf{V}_{in}$  is input to the filter. (A “phasor” is a complex number that describes the amplitude and phase angle of a sinusoid.) In the steady state the output is sinusoidal, also with angular frequency  $\omega$ , but possibly having a different amplitude and phase angle. The output phasor is  $\mathbf{V}_{out}$ . The effect of the two-port on signals of all frequencies is described by its **transfer function**,  $H(\omega)$ .  $H(\omega)$  is defined as the ratio of the output phasor voltage to the input phasor voltage, as a function of frequency

$$H(\omega) = \frac{\hat{V}_{out}}{\hat{V}_{in}} = \frac{V_{out} e^{j\phi_{out}}}{V_{in} e^{j\phi_{in}}} \quad (4)$$

Since both phasors are complex, the transfer function is also complex and contains both phase and amplitude information. In this laboratory you will determine the transfer characteristics of several filter circuits.

### 3. Low pass filter

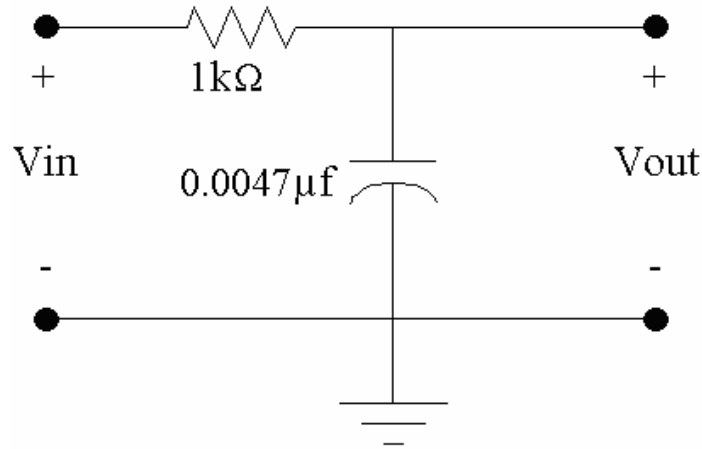


Figure 4. Low pass RC filter.

The first circuit that you will study is the low pass filter shown in Figure 4. If you think of a capacitor as being a resistor with a complex “resistance” (called an “impedance”), then this circuit is a complex voltage divider. Its (complex) transfer function is

$$H(j\omega) = \frac{1}{1 + j\omega RC} \quad (5)$$

Since  $H(\omega)$  is a complex quantity it can be described in terms of a magnitude and an angle in the complex plane. The magnitude of  $H(\omega)$ ,  $|H(\omega)|$  gives the transfer characteristic of the amplitude and the angle,  $\angle H(\omega)$  is the phase difference between the input and the output. The equations for these are

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_o}\right)^2}} \quad (6)$$

$$\angle H(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_o}\right), \quad (7)$$

where the characteristic angular frequency is  $\omega_o = 1/RC$ .

- Obtain Eqs. (6) and (7) from Eq. (5). Using the values for  $R$  and  $C$  from Figure 4, calculate the expected characteristic frequency  $\omega_o$  for the low pass filter.
- Calculate values for  $|H(\omega)|$  and  $\angle H(\omega)$  for sine waves of the following frequencies input to the low pass filter: 800 Hz, 10 kHz, 20 kHz, 35 kHz, and 80 kHz.

#### 4. High pass filter

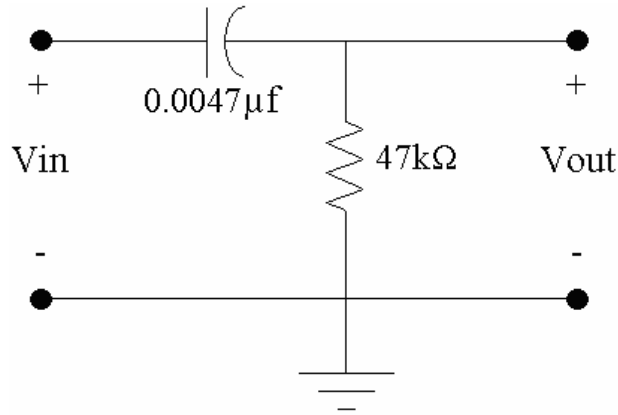


Figure 5. High pass RC filter.

The circuit for the high pass filter that you will study is shown in Figure 5. Its transfer function is

$$H(j\omega) = \frac{j\omega RC}{1 + j\omega RC} \quad (8)$$

and the magnitude and angle are

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega_o}{\omega}\right)^2}} \quad (9)$$

$$\angle H(j\omega) = \tan^{-1}\left(\frac{\omega_o}{\omega}\right). \quad (10)$$

- Obtain Eqs. (9) and (10) from Eq. (8). Using the values for R and C from Figure 5, calculate the expected characteristic frequency  $\omega_o$  for the high pass filter.
- Calculate values for  $|H(\omega)|$  and  $\angle H(\omega)$  for sine waves of the following frequencies input to the high pass filter: 200 Hz, 500 Hz, 800 Hz, 1 kHz, and 3 kHz.

#### 5. Band pass filter

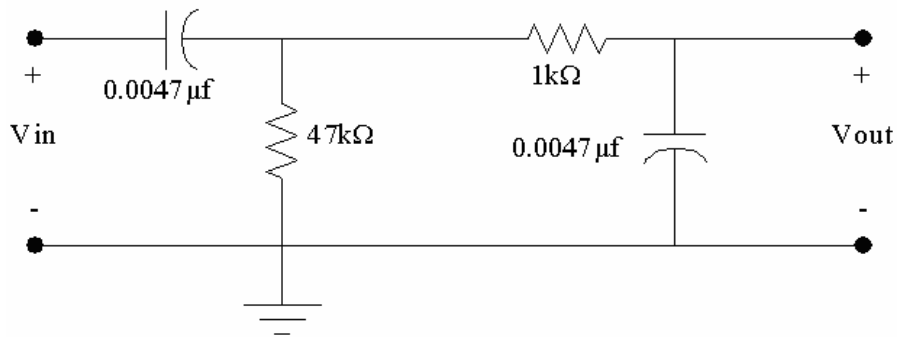


Figure 6. Band pass RC filter.

The final circuit that you will study is a band pass filter that is a combination of the high pass and low pass filters examined previously, as shown in Figure 6.

- Determine the complex transfer function,  $H(\omega)$ , for this circuit. (Hint: the impedance of a capacitor is  $1/j\omega C$ . Use the impedances of the capacitors just as though they were resistances, and solve for  $V_{out}/V_{in}$ .) Is it simply an addition of the transfer functions of the low pass and high pass filters studied previously? Why or why not?

## B. LAB INSTRUCTIONS

### PART 1: A low pass filter

4. Build the low pass filter circuit shown in Figure 7. (Remember that the signal generator is connected to ground internally.) Make sure the signal generator is set to "high Z" mode!

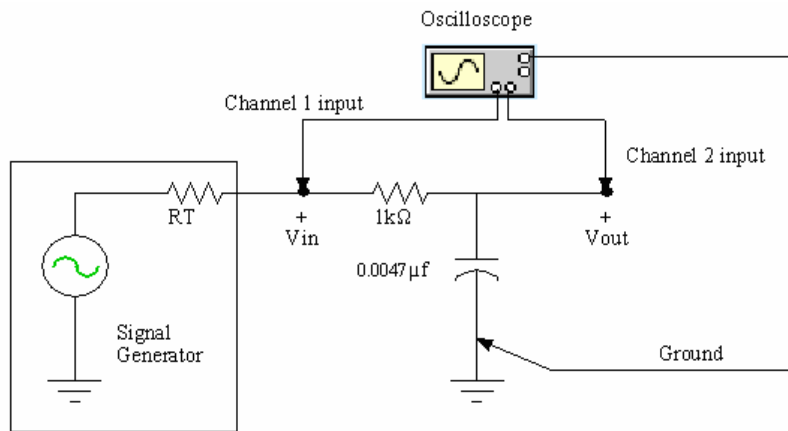


Figure 7. Low pass filter measurement circuit.

- Set up CHANNEL 1 and CHANNEL 2 of the oscilloscope for AC coupling and a 10x probe. Initially set the time base to 20  $\mu\text{sec/div}$ .
- With the function generator connected to the circuit, set  $V_{in}(t)$  to be sinusoidal at a frequency of  $\approx 9$  kHz and 10  $V_{p-p}$ , with 0 V DC offset. (Note that all specified frequencies are approximate. You will measure the actual frequency that you are using.) Using the oscilloscope measure and record the peak amplitude of  $V_{in}(t)$  and  $V_{out}(t)$  in Data Table 1.
- Measure the period of the input wave and record your result in Data Table 1. Then, as shown in Figure 2, set the t1 time cursor at a location where the input signal crosses the time axis with a positive slope. Then set the t2 time cursor to the first following point at which the output signal crosses the time axis with a positive slope.
- Complete Data Table 1 by making measurements at the remaining (approximate) frequencies. Be sure to use appropriate time base and voltage scales for each measurement.
- Set the function generator to produce a 30 kHz, 10  $V_{p-p}$  square wave with 50% duty cycle and 0 V offset voltage. Use Benchlink Scope to record this display. Be sure to record the voltage (ordinate) and time (abscissa) scales.

## Part 2: A high pass filter

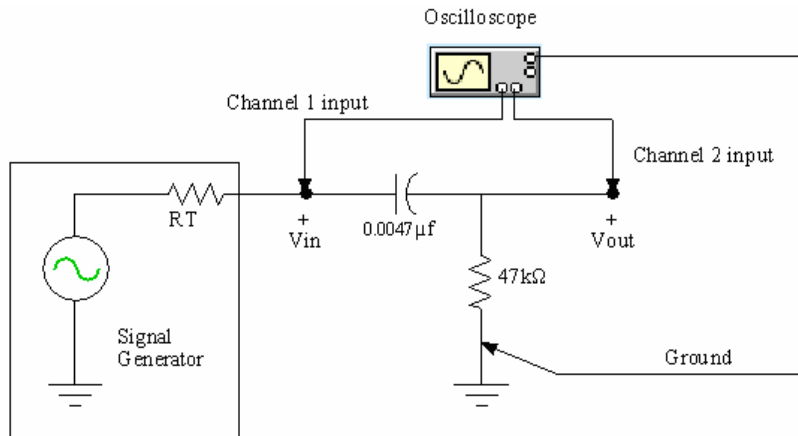
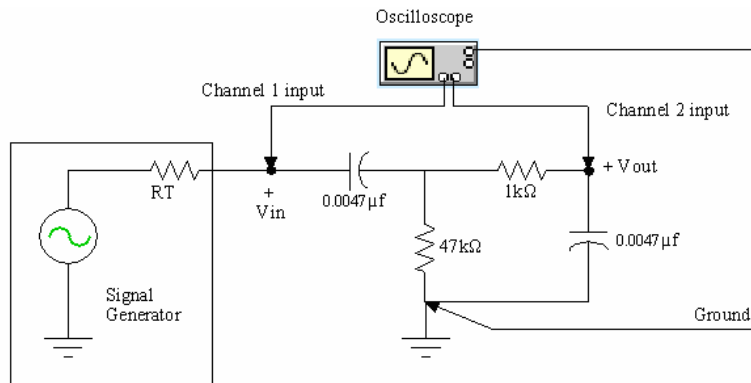


Figure 8. High pass filter measurement circuit.

- Build the high pass filter circuit shown in Figure 8.
- Set the time base of the oscilloscope to 2 ms/div initially. (Keep the CHANNEL 1 and CHANNEL 2 settings as before.) With the function generator connected to the circuit, set  $v_{in}(t)$  to be sinusoidal at a frequency of  $\approx 100$  Hz and 10 V<sub>p-p</sub>, with 0 V DC offset.
- Measure the peak amplitude of  $V_{in}(t)$ ,  $V_{out}(t)$ , the period  $T$ , and the time shift  $t_2 - t_1$  between input and output signals. Record these measurements in Data Table 2.
- Complete Data Table 2 by making the measurements at the remaining (approximate) frequencies. Be sure to use appropriate time base and voltage scales for each measurement.
- Set the function generator to produce a 500 Hz, 10 V<sub>p-p</sub> square wave with 0 V offset voltage. Adjust the oscilloscope to display both the input and output waveforms. Use Benchlink Scope to record this display. Be sure to record the voltage (ordinate) and time (abscissa) scales.

### Part 3: A band pass filter



**Figure 9. Band pass filter measurement circuit.**

- Build the band pass filter circuit shown in Figure 9. With the function generator connected to the circuit, set  $V_{in}(t)$  to be sinusoidal at a frequency of  $\approx 200$  Hz and  $10 V_{p-p}$ , with 0 V DC offset.
- Measure the peak amplitude of  $V_{in}(t)$  and  $V_{out}(t)$ , the period  $T$ , and the time shift  $t_2 - t_1$  between the input and output signals. Record these measurements in Data Table 3.
- Complete Data Table 3 by making measurements at the remaining (approximate) frequencies. Be sure to use appropriate time base and voltage scales for each measurement.
- Set the function generator to produce a 20 kHz,  $10 V_{p-p}$  square wave with 0 V offset voltage. Adjust the oscilloscope to display both the input and output waveforms. Use Benchlink Scope to record this display. Be sure to record the voltage (ordinate) and time (abscissa) scales. Repeat this procedure with a 1 kHz square wave.



# DATA AND REPORT SHEET FOR LAB 11

Student Name (Print): \_\_\_\_\_ Student ID: \_\_\_\_\_

Student Signature: \_\_\_\_\_ Date: \_\_\_\_\_

Student Name (Print): \_\_\_\_\_ Student ID: \_\_\_\_\_

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Student Name (Print): \_\_\_\_\_ Student ID: \_\_\_\_\_

Student Signature: \_\_\_\_\_ Date: \_\_\_\_\_

Lab Group: \_\_\_\_\_

Make sure you have the following oscilloscope displays before leaving the lab.

**Part 1(f). Low Pass Filter Square Wave Response**

**Part 2(e). High Pass Filter Square Wave Response**

**Part 3(d). Band Pass Filter Square Wave Response (20kHz)**

**Part 3(d). Band Pass Filter Square Wave Response (1kHz)**

**Data Table 1.** Measured Low Pass Filter Response

Nominal Frequency	V <sub>in</sub>	V <sub>out</sub>	V <sub>out</sub> /V <sub>in</sub>	Period	Calculated Frequency	t <sub>2</sub> - t <sub>1</sub>	Δφ (deg) (calculated)
1 kHz							
9							
17							
30							
50							
150							

**Data Table 2.** Measured High pass filter Response

Nominal Frequency	V <sub>in</sub>	V <sub>out</sub>	V <sub>out</sub> /V <sub>in</sub>	Period	Calculated Frequency	t <sub>2</sub> - t <sub>1</sub>	Δφ (deg) (calculated)
100 Hz							
400							
700							
1200							
3500							
11 kHz							

**Data Table 3.** Measured Band pass filter Response.

Nominal Frequency	V <sub>in</sub>	V <sub>out</sub>	V <sub>out</sub> /V <sub>in</sub>	Period	Calculated Frequency	t <sub>2</sub> - t <sub>1</sub>	Δφ (deg) (calculated)
200 Hz							
500							
1 kHz							
4							
20							
40							
70							
200							

**Data Table 4.** Calculated Low Pass Filter Response

Calculated characteristic frequency: \_\_\_\_\_

Nominal Frequency	$ H(\omega) $	$\angle H(\omega)$
800 Hz		
10 kHz		
20		
35		
80		

**Data Table 5.** Calculated High pass filter Response

Calculated characteristic frequency: \_\_\_\_\_

Nominal Frequency	$ H(\omega) $	$\angle H(\omega)$
200 Hz		
500		
800		
1 kHz		
3		

**QUESTIONS, Part I****1. Low pass filter**

- Obtain Eqs. (6) and (7) from Eq. (5). Using the values for R and C from Figure 4, calculate the expected characteristic frequency for the low pass filter
- Calculate values for  $|H(\omega)|$  and  $\angle H(\omega)$  for sine waves of the following frequencies input to the low pass filter: 800 Hz, 10 kHz, 20 kHz, 35 kHz, and 80 kHz.
- Record your results in Data Table 4

## 2. High pass filter

- Obtain Eqs. (9) and (10) from Eq. (8). Using the values for R and C from Figure 5, calculate the expected characteristic frequency for the high pass filter.
- Calculate values for  $|H(\omega)|$  and  $\angle H(\omega)$  for sine waves of the following frequencies input to the high pass filter: 200 Hz, 500 Hz, 800 Hz, 1 kHz, and 3 kHz. You may use EWB for this.
- Record your results in Data Table 5

## 3. Band pass filter

- Determine the complex transfer function,  $H(\omega)$ , for the circuit of Figure 6. Is it simply an addition of the transfer functions of the low pass and high pass filters studied previously? Why or why not?

## QUESTIONS, Part II

4. Plot your experimental results for voltage gain,  $|H(w)|$ , as a function of  $\log_{10}(\text{linear frequency})$  for each of the three filters — low pass, high pass and band pass. Plot each filter on a separate graph. In a separate set of three graphs, plot your experimental results for phase shift (in degrees) as a function of  $\log_{10}(\text{frequency})$ . In all six graphs draw a **smooth** curve that fits your data points. Do **not** simply connect the points in a “dot-to-dot” fashion. These plots must be computer-generated. You can do all of this in Microsoft Excel; however, you will need to use the “Scatter plot” format since the frequencies at which the measurements were made are not uniformly spaced.
5. Plot the data points that you calculated in Part I for the low pass (Data Table 4) and high pass (Data Table 5) filters on the appropriate plots. You can either plot these points by HAND on the plots from 1, or you can computer generate combined plots. However, you generate this data the calculated responses should be simply unconnected data points. Do these points fall reasonably close to the smooth curves that you generated in 1, above? (“Reasonably” should take into consideration the tolerances on the components only.) If not, why not?
6. The cutoff frequency,  $\omega_o$ , of a filter is defined as the frequency at which the voltage gain is equal to 0.707 of its maximum value. For each filter, what is the maximum voltage gain and what is the measured cutoff frequency? How does this compare with the predicted value of  $\omega_o = 1/RC$ ? Is your measured value within the tolerances of the components? (Note: A bandpass filter should have two cutoff frequencies.)
7. Discuss the differences between the input and output square waves for each filter. Can you give a simple explanation of what causes these differences.
8. Explain why equations (2) and (3) must use  $t_1-t_2$  to get the proper sign of the phase response. Also explain why you occasionally need to take the complement of the angle when computing the phase response of a filter.