Fourth Edition

INSTRUCTORS MANUAL TO ACCOMPANY

THE ANALYSIS AND DESIGN OF LINEAR CIRCUITS

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April 2003

PREFACE

There are two versions of the fourth edition of *The Analysis and Design of Linear Circuits.* The *Standard* version (ISBN 0-471-27213-2) is an incremental revision and updating of the third edition. The new *Laplace-Early* version (ISBN 0-471-43299-7) retains the core features of the third edition, but uses a new chapter sequencing that emphasizes transform methods. Some users of the third edition including the authors believe that delaying phasors until after Laplace has been grasped helps students better retain what they learned and contributes to a better, more efficient understanding of phasors. Both versions are aimed at introductory circuit analysis courses for both electrical/computer engineering majors and for service courses, and both assume the same student prerequisites. Although the sequencing is different, the two versions cover the same range of topics. John Wiley & Sons, Inc offers two versions of this book as part of a continuing commitment to supplying a diversity of resources for teaching circuits courses.

This Instructors Manual solves all of the problems in the text. Both book versions have significantly revised problem sets. A full one-third of the problems are totally new, one-third are revised problems from the third edition, and the remaining third have been retained as is.

The problem solutions in this manual are worked out using Mathcad Plus version 8.0 and Mathcad Professional 2001.[1] The following is a brief discussion of Mathcad terminology for those not familiar with this computer tool. Addition, subtraction, and multiplication are indicated by "+", "−", and "·" symbols. Division is indicated as a quotient such as "$\frac{3}{4}$". The Pascal assignment operator ":=" is used to define a function or to assign a numerical value to a variable. For example, the Mathcad expressions "i:=0.02" and "R:=2500" assign the values 0.02 and 2500 to the variables "i" and "R", respectively. Similarly the expression "v(t):=2.5·exp(-10·t)" defines a function v(t) whose value is found by assigning a value to the variable "t". The "=" sign (no bold) is used to evaluate the numerical value of the expression to the left of the equal sign. For example, the Mathcad statements "$i^2 \cdot R$=2.25" and "v(0.02)=2.047" evaluate the expressions of the variables "i" and "R" and function "v(t)" defined above. The bold "**=**" sign is used in symbolic calculations and to define equality constraints in systems of equations.

The Mathcad solve block is used to solve a system of equations. The block involves four steps: (1) enter an initial guess for all of the unknowns, (2) enter the reserve word *given* which tells Mathcad that what follows is a system of equations, (3) enter the system of equations using the bold equal sign, and (4) enter any expression that involves the reserve word *find.* For example, the Mathcad statements below solve the system of equations between the *given* and the *find* statements.

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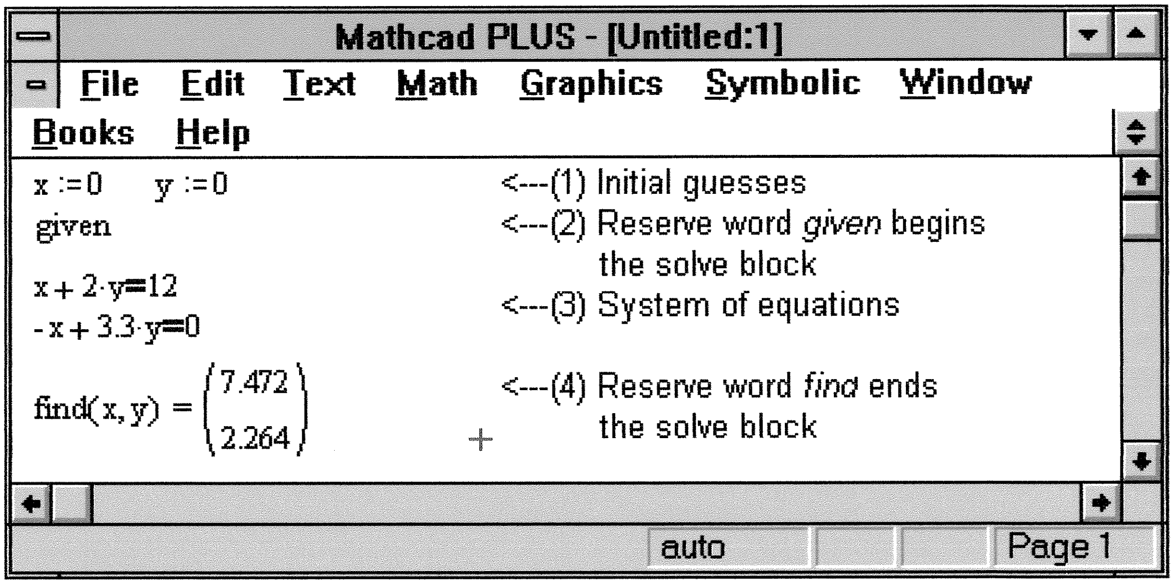
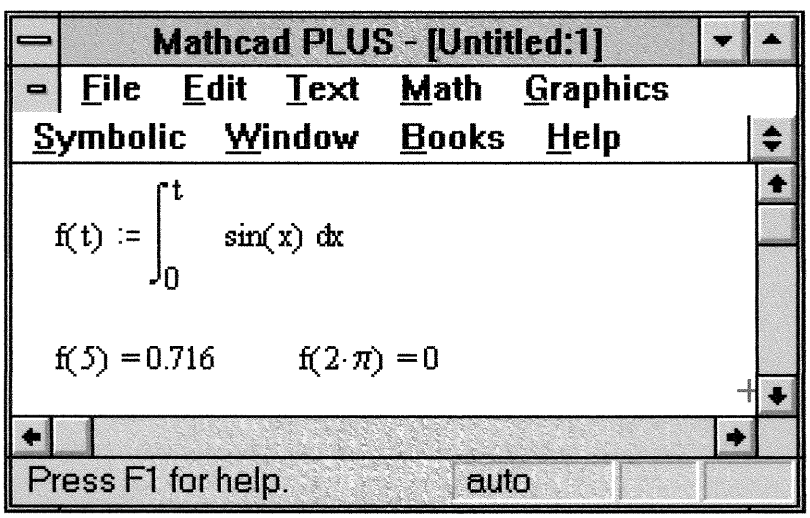
[1] Mathcad is a registered trademark of MathSoft Inc., Cambridge, Mass.

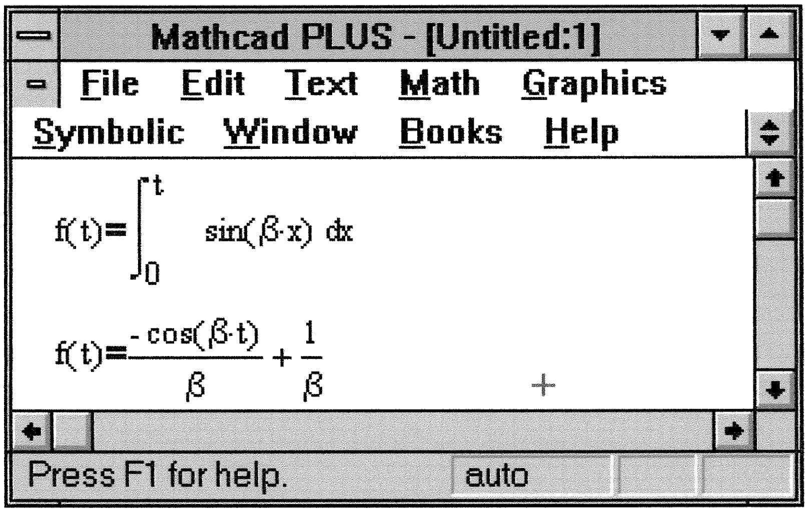


The Mathcad solve block is used in this manual to solve both linear and nonlinear equations.

Mathcad performs the calculus operations of integration and differentiations. For example, the Mathcad integral expression below uses the assignment operator to define a function f(t) that can be evaluated at specific values of "t" as indicated.



The integral expression below uses the bold equal sign. Mathcad's symbolic analysis feature can then be used to symbolically evaluate this expression as shown.



The authors gratefully acknowledge the outstanding assistance of Dr. James S. Kang of the ECE Department, Cal Poly-Pomona in the preparation of this manual. Any errors remaining in the manual are the authors' responsibility. We would appreciate hearing from users who find any errors in the manual or who have comments on the utility of Mathcad as a vehicle for creating and documenting homework problem solutions.

Roland E. Thomas
Albert J. Rosa

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SOLUTIONS FOR
ANALYSIS AND DESIGN OF LINEAR CIRCUITS

by Thomas & Rosa

CHAPTER 1, Both Versions

1-1 (a) 12 mA, (b) 455 kHz, (c) 200 ps = 0.2 ns, (d) 5 MW

1-2 (a) 22 mV, (b) 23 nF, (c) 56 kΩ, (d) 0.752 MJ. (e) 0.235 mH.

1-3 1 A = 1 C/s, 3300 Ah = 3300 C-hr/s, 1 hr = (60 min/hr)x(60 s/min) = 3600 s/hr. hence

Q = (3300 C-hr/s)x(3600 s/hr) = $3300 \cdot 3600 = 1.188 \times 10^{7}$ C

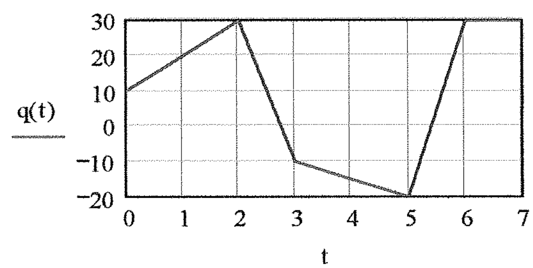
1-4 1 W = 1 J/s, 2128 kWh = 2.128 MJ-hr/s, 1 hr = (60 s/min)x(60 min/hr) = 3600 s/hr, hence
W = (2.128 MJ-hr/s)x(3.6x1000 s/hr) = 7.661 GJ.

1-5 (a) 10^{6}, (b) 10^{-3}, (c) 10^{3}, (d) 10^{-6}

1-6 $i(t) = \frac{d}{dt}q(t) = \frac{d}{dt}(3t-2)\cdot 10^{-3} = 3\cdot 10^{-3} = 3$ mA

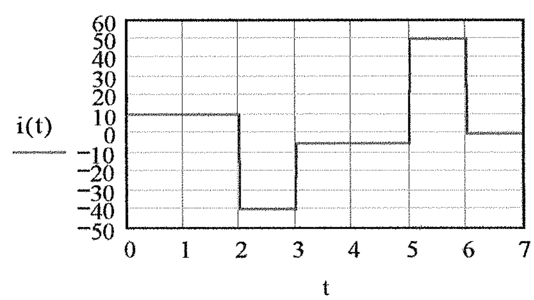
1-7 $i(t) = \frac{d}{dt}q(t) = \frac{d}{dt}\left(20\cdot e^{-3\cdot t}\right)\cdot 10^{-6} = -60\cdot e^{-3t}$ μA

1-8 $q(t) := \begin{vmatrix} 10 + 10\cdot t \ \text{ if } \ 0 \le t < 2 \\ 30 - 40\cdot(t-2) \ \text{ if } \ 2 \le t < 3 \\ -10 - 5\cdot(t-3) \ \text{ if } \ 3 \le t < 5 \\ [-20 + 50\cdot(t-5)] \ \text{ if } \ 5 \le t < 6 \\ 30 \ \text{ if } \ 6 \le t \end{vmatrix}$ $\qquad t := 0, 0.01 .. 7$



$i(t) = \frac{d}{dt}q(t)$

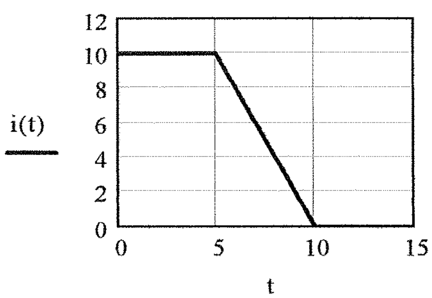
$i(t) := \begin{vmatrix} 10 \ \text{ if } \ 0 \le t < 2 \\ -40 \ \text{ if } \ 2 \le t < 3 \\ -5 \ \text{ if } \ 3 \le t < 5 \\ 50 \ \text{ if } \ 5 \le t < 6 \\ 0 \ \text{ if } \ 6 \le t \end{vmatrix}$

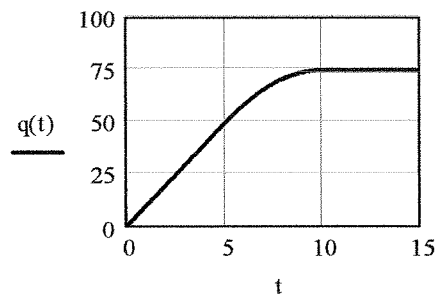


1-9 $i(t) := 3\cdot t^{2}$ A $\qquad q(t) = \int_0^t i(x)\,dx = t^{3} \qquad q(2) - q(0) = 8$ C

1-10 $i(t) := 3 \cdot e^{-2 \cdot t}$ A $\quad q(t) = \int_0^t i(x)\,dx = \frac{3}{2} - \frac{3}{2} \cdot e^{-2 \cdot t}$ $\quad q(0.1) - q(0) = 0.272$ C

1-11 $i(t) := \begin{cases} 10 & \text{if } 0 \le t < 5 \\ 20 - 2 \cdot t & \text{if } 5 \le t < 10 \\ 0 & \text{if } 10 \le t \end{cases}$ $\quad t := 0, 0.1 .. 15 \quad$ $q(t) := \begin{cases} 10 \cdot t & \text{if } 0 \le t < 5 \\ -25 + 20 \cdot t - t^2 & \text{if } 5 \le t < 10 \\ 75 & \text{if } 10 \le t \end{cases}$



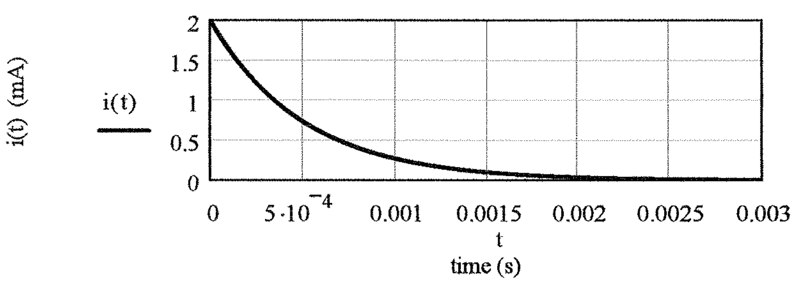


$q(10) = 75$ **<---Total charge through the device between t = 0 and t = 10 s.**

1-12 $q(t) = (1 \cdot -\exp(-2000 \cdot t)) \cdot 10^{-6}$ C

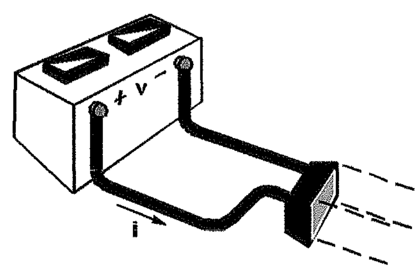
$i(t) = \frac{d}{dt}q(t) = (2000 \cdot \exp(-2000 \cdot t)) \cdot 10^{-6} = 2 \cdot e^{-2000t}$ mA

$t := 0, 10^{-6} .. 4 \cdot 10^{-3}$ $\quad i(t) := 2 \cdot \exp(-2000 \cdot t)$ mA



1-13 (a) $p := 50 \quad v := 12 \quad i := \frac{p}{v} \quad i = 4.167$ A

(b) T = (200 A-hr)/(4.167 A) = $\frac{200}{4.167} = 48$ hr



1-14 (a) $p := 75 \quad v := 120 \quad i := \frac{p}{v} \quad i = 0.625$ A

(b) w = (75 W)x(8 hr) = 600 Wh = 0.6 kWh,

Cost = (6.8 Cents/kWh)(0.6 kWh) = $4.08 cents

1-15 w = (50 A-hr)x(12 V) = 600 W-hr = (600 J-hr/s)x(3600 s/hr) = 2.16 MJ

1-16 $w(t) = \int_0^t v(x)\cdot i(x)\,dx = 75\cdot\int_0^t e^{-2\cdot x}\,dx = \frac{75}{2}\cdot\left(1 - e^{-2\cdot t}\right)$ $w(0.5) = 23.705$ W

1-17 $i(v) := \exp(v) - 10$ $p(v) := v\cdot i(v)$

$i(-3) = -9.95$ A	$p(-3) = 29.851$	absorbing
$i(1.5) = -5.518$ A	$p(1.5) = -8.277$	delivering
$i(3) = 10.086$ A	$p(3) = 30.257$	absorbing

1-18 $p(t) := 25\cdot\left(1 - 2\cdot e^{-t}\right)$

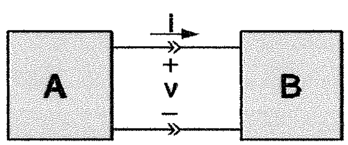
$p(0.5) = -5.327$	delivering
$p(1) = 6.606$	absorbing
$p(10) = 24.998$	absorbing

1-19 $p_{max} := 0.25$ $v := 50$ since p=vi it follows that $i_{max} := \frac{p_{max}}{v}$ $i_{max} = 5\times 10^{-3}$ A

1-20 $p(t) := \begin{cases} 5\cdot 10^3 & \text{if } 0 \le t < 20\cdot 10^{-9} \\ 0 & \text{if } 20\cdot 10^{-9} \le t \end{cases}$ Pulse period = $T_0 := \frac{1}{40}$ total energy per cycle is

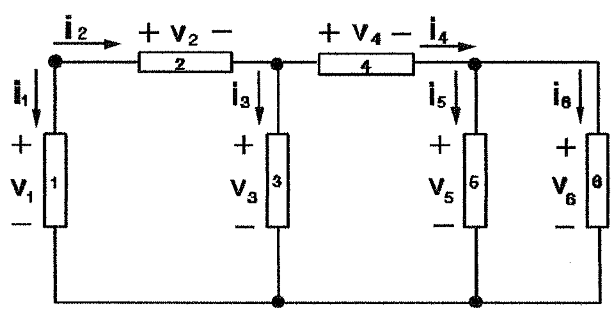
$W_{total} := 5\cdot 10^3\cdot 20\cdot 10^{-9}$, $W_{total} = 1\times 10^{-4}$ $P_{avg} := \frac{W_{total}}{T_0}$ $P_{avg} = 4\times 10^{-3}$ W

1-21



(a) p = vi = (33 V)x(-2.2 A) = - 72.6 W power transfer from B to A.
(b) p = vi = (-12 V)x(-1.2 mA) = + 14.4 mW power transfer from A to B.
(c) p = vi = (37.5 V)x(40 mA) = 1.5 W power transfer from A to B.
(d) p = vi = (-15 V)x(-43 mA) = 645 mW power transfer from A to B.

1-22



Device No. 1	p = vi = (15 V)x(-1 A) = -15 W,	delivering
Device No. 2	i = p/v = (5 W)/(5 V) = 1 A,	absorbing
Device No. 3	v = p/i = (5 W)/(0.5 A) = 10 V,	absorbing
Device No. 4	p = vi = (4 V)x(0.5 A) = 2 W,	absorbing
Device No. 5	v = p/i = (18 W)/(3 A) = 6 V,	absorbing
Device No. 6	v = p/i = (-15 W)x(-2.5 A) = 6 V,	delivering
total power =	$-15 + 5 + 5 + 2 + 18 - 15 = 0$	power balance check

1-23

Device No. 1 p = vi = (30 V)x(-2 A) = -60 W, delivering
Device No. 2 i = p/v = (20 W)/(10 V) = 2 A, absorbing
Device No. 3 i = p/v = (20 W)/(20 V) = 1 A, absorbing
Device No. 4 p = vi = (8 V)x(1 A) = 8 W, absorbing
Device No. 5 v = p/i = (-60 W)/(-5 A) = 12 V, delivering
Device No. 6 p = vi = (12 V)x(6 A) = 72 W, absorbing
total power = $-60+20+20+8-60+72=0$ power balance checks

1-24 $p(t) := 5\cdot\cos(10\cdot t)\cdot 0.5\cdot\sin(10\cdot t)$ $p(0.2) = -0.946$ Delivering $p(0.4) = 1.237$ Absorbing

1-25

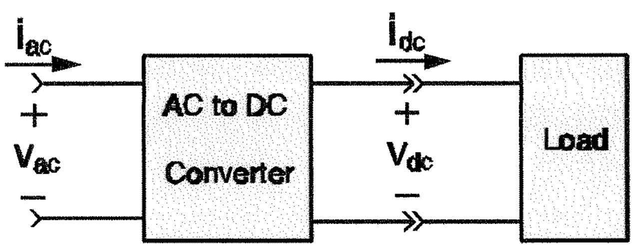
$$p(t) := 10\cdot\left(1-e^{-25\cdot t}\right)\cdot 0.5\cdot e^{-25\cdot t} \qquad w(t) = \int_0^t p(x)\,dx = 5\cdot\int_0^t \left(e^{-25\cdot x}-e^{-50\cdot x}\right)dx$$

$$w(t) := \frac{1}{10}\cdot\left(1+e^{-50\cdot t}-2\cdot e^{-25\cdot t}\right) \qquad w(0) = 0 \qquad w(1) = 0.1$$

1-26 P_{light} = (0.9 V)x(4 mA) = 3.6 mW, P_{dark}= (5.6 V)x(8 nA) = 44.8 nW

$$P_{dB} = 10\text{x}\log_{10}(P_{light}/P_{dark}) = 10\cdot\log\left(\frac{3.6\cdot 10^{-3}}{44.8\cdot 10^{-9}}\right) = 49.05\text{ dB}$$

1-27



$V_{dc} := 24$ $V_{ac} := 120$ $P_{dc} := 200$ $\eta := 0.82$ $P_{ac} := \frac{P_{dc}}{\eta}$ $P_{ac} = 243.902$

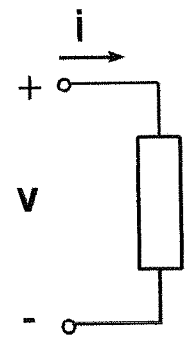
$I_{dc} := \frac{P_{dc}}{V_{dc}}$ $I_{ac} := \frac{P_{ac}}{V_{ac}}$ $I_{dc} = 8.333$ $I_{ac} = 2.033$

1-28 (a) A-h efficiency = (A-h output)/(A-h input) = $\frac{400}{75\cdot 6} = 0.889$ = **88.9%**

(b) Energy = (75x6 A-hr)x(24 V) = 10,800 W-hr = 10,800 J-hr/s

Energy = (10,800 J-hr/s)x(3600 s/hr) = 38.88 MJ

CHAPTER 2, Both Versions



2-1 (a) $R := 5000 \quad v := 50 \quad i := \frac{v}{R} \quad p := v \cdot i \quad i = 0.01 \quad p = 0.5$

(b) $i := -10^{-2} \quad p := 30 \cdot 10^{-3} \quad v := \frac{p}{i} \quad v = -3$

2-2 (a) $v := 15 \quad i := 0.005 \quad p := v \cdot i \quad p = 0.075$

(b) $i := 20 \quad v := 0 \quad p := 0$ sw closed

2-3 $v := 12 \quad i := 0.003 \quad G := \frac{i}{v} \quad G = 2.5 \times 10^{-4}$

2-4 $p := \frac{1}{4} \quad v_{max} := 12 \quad i_{max} := \frac{p}{v_{max}} \quad R_{min} := \frac{v_{max}}{i_{max}} \quad R_{min} = 576$

2-5 Given order pairs (i, v) = (7.5 mA, 15 V), (4.3 mA, 8.6 V), & (-4mA, -8 V). Assume the i-v characteristic is a str-line. Using the last two points to define the str line:

$$m := \frac{\left(4.3 \cdot 10^{-3}\right) - \left(-4 \cdot 10^{-3}\right)}{8.6 - (-8)} \qquad m = 5 \times 10^{-4}$$ <---Slope

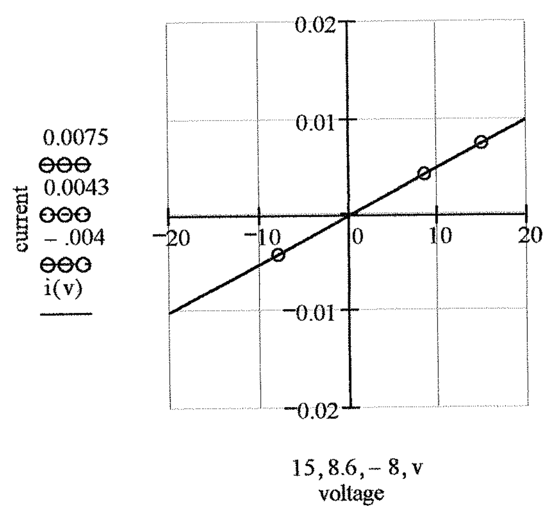
$i(v) := 7.5 \cdot 10^{-3} + m \cdot (v - 15)$ <---Str Line $\quad v := -20, 0 .. 20$

$i(15) = 7.5 \times 10^{-3}$ <---Checks

$i(8.6) = 4.3 \times 10^{-3}$ <---Checks

$i(-8) = -4 \times 10^{-3}$ <---Checks

$i(-15) = -7.5 \times 10^{-3}$ <---estimate



The i-v characteristic is a str line through the origin. The element is linear.

2-6 (a) $i_1 := -10$ $i_2 := -5$ $i_3 := -2$ $i_4 := -1$ $i_5 := -0.5$ $i_6 := 0.5$ $i_7 := 1$ $i_8 := 2$ $i_9 := 5$ $i_{10} := 10$

$n := 1,2..10$ $v_n := 75 \cdot i_n + 0.2 \cdot \left(i_n\right)^3$ $p_n := v_n \cdot i_n$

$n =$	$i_n =$	$v_n =$	$p_n =$
1	-10	-950	$9.5 \cdot 10^3$
2	-5	-400	$2 \cdot 10^3$
3	-2	-151.6	303.2
4	-1	-75.2	75.2
5	-0.5	-37.525	18.762
6	0.5	37.525	18.762
7	1	75.2	75.2
8	2	151.6	303.2
9	5	400	$2 \cdot 10^3$
10	10	950	$9.5 \cdot 10^3$

(b) nonlinear linear

$v_5 = -37.525$ $-0.5 \cdot 75 = -37.5$

$v_6 = 37.525$ $0.5 \cdot 75 = 37.5$

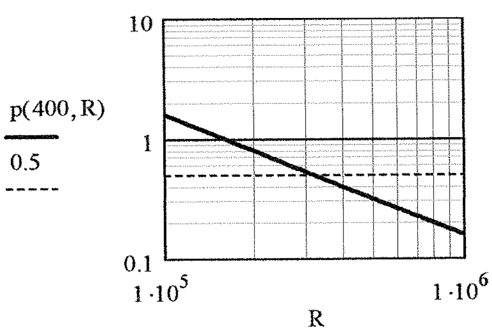
$\text{voltage\_error} := v_6 - 37.5$

$\text{voltage\_error} = 0.025$

$\text{error} := \dfrac{\text{voltage\_error}}{v_6}$ $\text{error} = 0.067\,\%$

2-7 $R := 10^4$ $p := 12 \cdot 10^{-3}$ $i := \sqrt{\dfrac{p}{R}}$ $i = 1.095 \times 10^{-3}$

2-8 $p(v,R) := \dfrac{v^2}{R}$ $R := 100000, 110000..10^6$

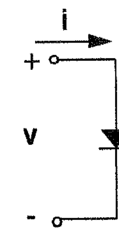


p(400,310000) = 0.516 <---Note p > 0.5
p(400,320000) = 0.5 <---Note p = 0.5
p(400,330000) = 0.485 <---Note p < 0.5

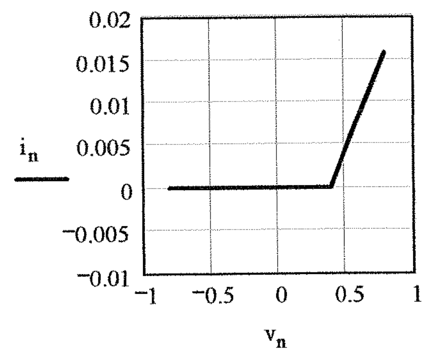
That is, if v = 400 V then p > 0.5 W for all R < 320 kW and p < 0.5 W for all R > 320 kW.
Hence, the limiting ratings are:
(1) v < 400 for all R > 320 kW.
(2) p < 0.5 W for all R < 320 kW.

2-9 $v_1 := -0.8$ $v_2 := -0.4$ $v_3 := -0.2$ $v_4 := -0.1$ $v_5 := 0$ $v_6 := 0.1$ $v_7 := 0.2$ $v_8 := 0.4$ $v_9 := 0.8$

(a) $n := 1, 2..9$ $i_n := 2 \cdot 10^{-16} \cdot \left(\exp\left(40 \cdot v_n\right) - 1\right)$ $p_n := v_n \cdot i_n$



n =	v_n =	i_n =	p_n =
1	-0.8	0	0
2	-0.4	0	0
3	-0.2	0	0
4	-0.1	0	0
5	0	0	0
6	0.1	$1.072 \cdot 10^{-14}$	$1.072 \cdot 10^{-15}$
7	0.2	$5.96 \cdot 10^{-13}$	$1.192 \cdot 10^{-13}$
8	0.4	$1.777 \cdot 10^{-9}$	$7.109 \cdot 10^{-10}$
9	0.8	0.016	0.013



(b) nonlinear, nonbilateral, and passive.

(c) For $v := 5$ $i := 2 \cdot 10^{-16} \cdot (\exp(40 \cdot v) - 1)$ $p := v \cdot i$ $i = 1.445 \times 10^{71}$ $p = 7.226 \times 10^{71}$

Model does not apply. These signal levels would vaporize the device & several nearby towns .

(d) For $v := -5$ $i := 2 \cdot 10^{-16} \cdot (\exp(40 \cdot v) - 1)$ $i = -2 \cdot 10^{-16}$ $p := v \cdot i$ $p = 1 \times 10^{-15}$

Model applies since these levels are nearly zero.

2-10 $m := 0.3$ $T_C := 400$ $i := 10^{-3}$ $R := m \cdot T_C + 100$ $R = 220$ $v := i \cdot R$ $v = 0.22$

2-11 (a) Nodes: A, B, C

Loops: 1,2; 2,3,4;1,3,4

(b) Elements in series: 3&4

Elements in parallel: 1&2

(c) KCL: Node A $i_1 + i_2 + i_3 = 0$

Node B $-i_3 + i_4 = 0$

Node C $-i_1 - i_2 - i_4 = 0$

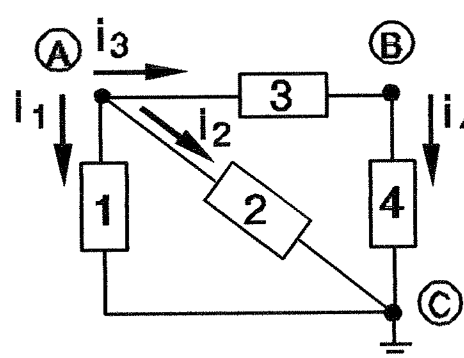
KVL: Loop 1,2 $-v_1 + v_2 = 0$

Loop 2,3,4 $-v_2 + v_3 + v_4 = 0$

Loop 1,3,4 $-v_1 + v_3 + v_4 = 0$

(d) Node A $i_3 = -i_1 - i_2 = -2$ mA

Node B $i_4 = i_3 = -2$ mA



2-12 By KCL: Node A $+i_1 + i_2 + i_3 = 0$ Node B $-i_3 + i_4 = 0$ Node C $-i_1 - i_2 - i_4 = 0$.

Hence if $i_1 := 20 \cdot 10^{-3}$ and $i_3 := -30 \cdot 10^{-3}$ then from Node A $i_2 := -i_1 - i_3$ or $i_2 = 1 \times 10^{-2}$.

From Node B $i_4 := i_3$ or $i_4 = -3 \times 10^{-2}$

2-13 **(a)** Nodes: A, B, C,D

Loops: 1,3,2; 2,4,5; 3,6,4; 1,5,6; 2,3,6,5

(b) Elements in series: none

Elements in parallel: none

(c) KCL: Node A $-i_2 - i_3 - i_4 = 0$

Node B $-i_1 + i_3 - i_6 = 0$

Node C $i_1 + i_2 + i_5 = 0$

Node D $i_4 - i_5 + i_6 = 0$

KVL: Loop 1,3,2 $-v_1 + v_2 - v_3 = 0$

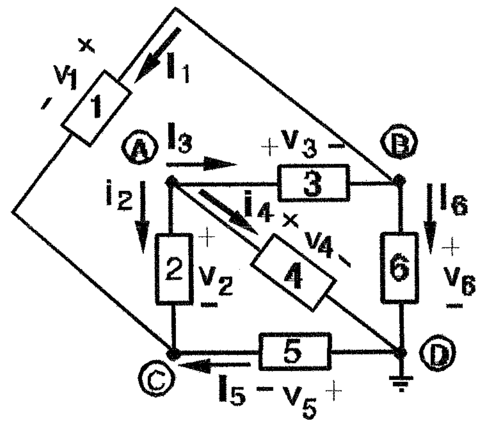
Loop 2,4,5 $-v_2 + v_4 + v_5 = 0$

Loop 3,6,4 $v_3 + v_6 - v_4 = 0$

(d) Loop 2,4,5 $v_2 = v_4 + v_5 = 1$ V

Loop 1,3,2 $v_1 = v_2 - v_3 = 9$ V

Loop 3,6,4 $v_6 = v_4 - v_3 = 0$ V



2-14 By KVL: Given Loop 1,2,3 $-v_1 + v_2 - v_3 = 0$ and Loop 2,4,5 $-v_2 + v_4 + v_5 = 0$ and Loop 3,6,4 $v_3 + v_6 - v_4 = 0$. Hence if $v_1 := -8$, $v_4 := 8$ and $v_6 := 6$ then from Loop 3,6,4 $v_3 := v_4 - v_6$ or $v_3 = 2$. From Loop 1,3,2 $v_2 := v_1 + v_3$ or $v_2 = -6$ and finally From Loop 2,4,5 $v_5 := v_2 - v_4$ or $v_5 = -14$

In summary

$v_1 = -8$ $v_2 = -6$ $v_3 = 2$ $v_4 = 8$ $v_5 = -14$ $v_6 = 6$ all in V

2-15

(a) Nodes: A, B, C,D

Loops: 1,3,2; 2,4,5; 3,6,4; 1,5,6;

(b) KCL: Node A $i_2 + i_3 + i_4 = 0$

Node B $i_1 - i_3 + i_6 = 0$

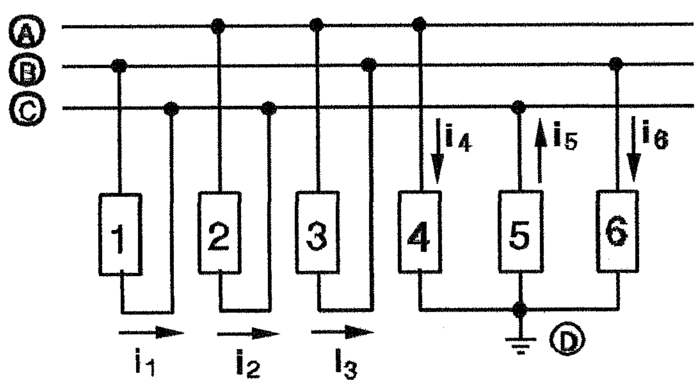
Node C $-i_1 - i_2 - i_5 = 0$

Node D $-i_4 + i_5 - i_6 = 0$

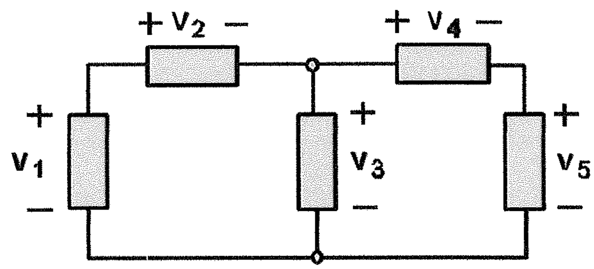
(c) Node A $i_2 = -i_3 - i_4 = -3$ mA

Node C $i_1 = -i_2 - i_5 = -2$ mA

Node B $i_6 = i_3 - i_1 = 17$ mA



2-16 $v_1 := 5 \quad v_3 := -10 \quad v_4 := 10$



By KVL $\quad -v_1 + v_2 + v_3 = 0$

$-v_3 + v_4 + v_5 = 0$

hence $\quad v_2 := v_1 - v_3 \qquad v_2 = 15$

$v_5 := v_3 - v_4 \qquad v_5 = -20$

2-17 Using KCL

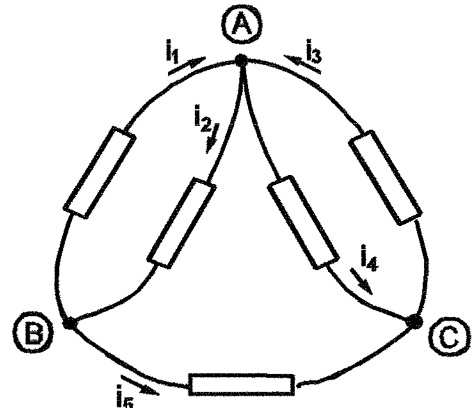
Node A: $i_1 - i_2 + i_3 - i_4 = 0$

Node C: $-i_3 + i_4 + i_5 = 0$

If: $i_1 := 2 \quad i_2 := -5 \quad i_3 := 4$

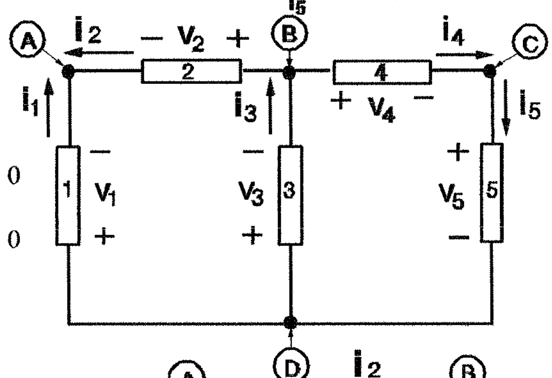
Then $i_4 := i_1 - i_2 + i_3$ and $i_5 := i_3 - i_4$

$i_4 = 11 \qquad i_5 = -7$



2-18 KVL KCL

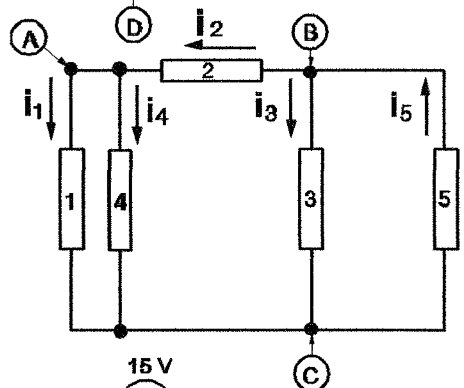
KVL	KCL
$v_1 - v_2 - v_3 = 0$	$i_1 + i_2 = 0$
$v_3 + v_4 + v_5 = 0$	$-i_2 + i_3 - i_4 = 0$
$i_4 - i_5 = 0$	$-i_1 - i_3 + i_5 = 0$



2-19

$$A = \begin{pmatrix} -1 & 1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 & 1 \\ 1 & 0 & 1 & 1 & -1 \end{pmatrix}$$

Voltage polarities follow
the passive sign convention



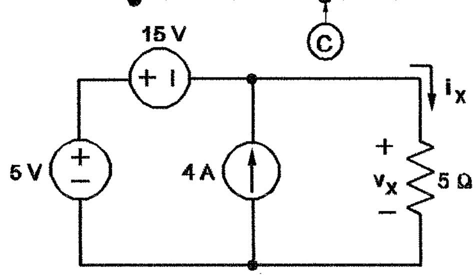
2-20

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & -1 & 0 & 1 \end{pmatrix}$$

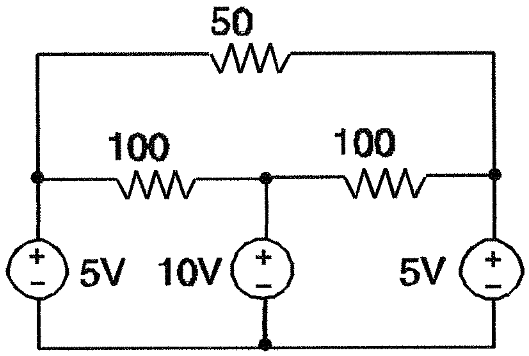
2-21 By KVL around the perimeter

$-5 + 15 + v_x = 0 \qquad v_x := 5 - 15 \qquad v_x = -10$

by Ohm's law $\qquad i_x := \dfrac{v_x}{5} \qquad i_x = -2$



2-22



KVL: $-5 + v_4 + 10 = 0$ hence $v_4 := -5$ V.

$-5 + v_6 + 5 = 0$ hence $v_6 := 0$ V.

$-10 + v_5 + 5 = 0$ hence $v_5 := 5$ V.

Element: $i_4 := \frac{v_4}{100}$ $\quad i_5 := \frac{v_5}{100}$ $\quad i_6 := \frac{v_6}{50}$

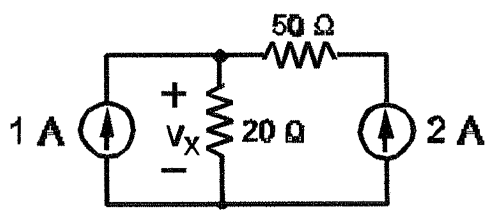
KCL: $i_1 := -i_4 - i_6 \quad i_2 := i_4 - i_5$

$i_3 := i_5 + i_6$

Summary: $i_1 = 0.05 \quad i_2 = -0.1 \quad i_3 = 0.05$

$i_4 = -0.05 \quad i_5 = 0.05 \quad i_6 = 0$

2-23

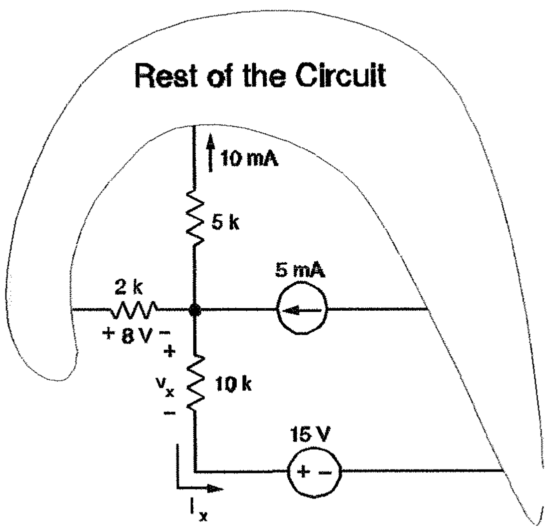


KCL: $-i_1 + 2 = 0$ hence $i_1 := 2$

$-i_x + i_1 + 1 = 0$ hence $i_x := 3$

Element: $v_x := 20 \cdot i_x$ hence $v_x = 60$

2-24



(a) $i_1 := \frac{8}{2000} \quad i_1 = 4 \times 10^{-3}$

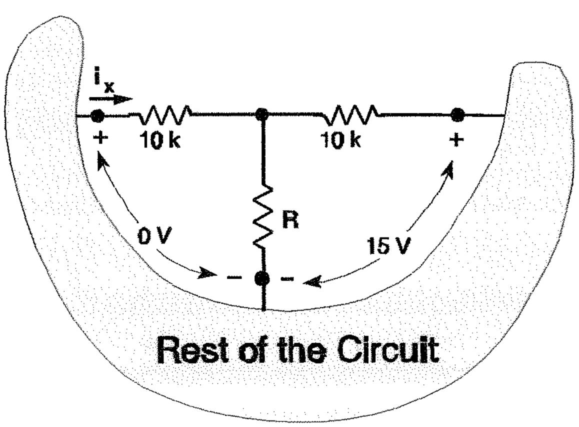
$i_x := i_1 + 0.005 - 0.01 \quad i_x = -10 \times 10^{-4}$

$v_x := 10^4 \cdot i_x \quad v_x = -10$

(b) $i_{sum} := -i_1 + 0.01 - 0.005 + i_x$

$i_{sum} = 0$ QED

2-25



(a) $i_x := 0.004 \quad v_1 := 10^4 \cdot i_x \quad v_1 := 40$

$v_R := -v_1 \quad v_2 := 15 - v_R \quad v_2 = 55$

$i_2 := \frac{v_2}{10^4} \quad i_2 = 5.5 \times 10^{-3}$

$i_R := i_x + i_2 \quad i_R = 9.5 \times 10^{-3}$

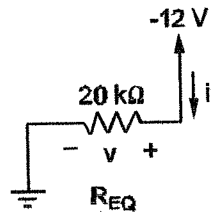
$R := \frac{v_R}{i_R} \quad R = -4.211 \times 10^3 \ \Omega$

(b) $i_{sum} := i_R - i_x - i_2$

$i_{sum} = 0$ QED

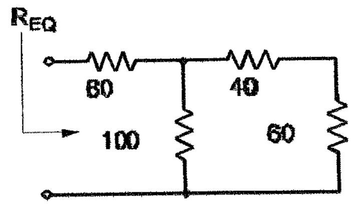
2-26 $v_x := -12 \qquad i_x := \dfrac{v_x}{20 \cdot 10^3} \qquad p_x := v_x \cdot i_x$

$v_x = -12 \qquad i_x = -6 \times 10^{-4} \qquad p_x = 7.2 \times 10^{-3}$



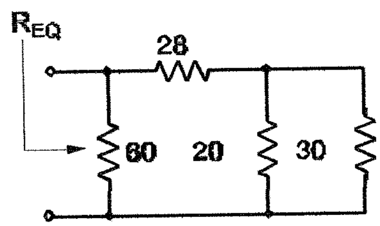
2-27 $R_{EQ} := 60 + \dfrac{1}{\dfrac{1}{100} + \dfrac{1}{40 + 60}}$

$R_{EQ} = 110$



2-28 $R_{EQ} := \dfrac{1}{\dfrac{1}{60} + \dfrac{1}{28 + \dfrac{1}{\dfrac{1}{20} + \dfrac{1}{30}}}}$

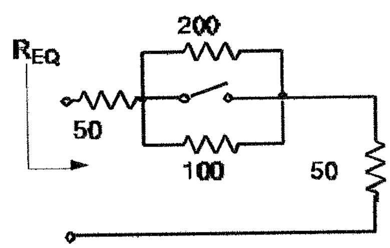
$R_{EQ} = 24$



2-29 $R_{EQ}(sw) := 50 + sw \cdot \dfrac{200 \cdot 100}{200 + 100} + 50$

Switch open $R_{EQ}(1) = 166.667$

Switch closed $R_{EQ}(0) = 100$



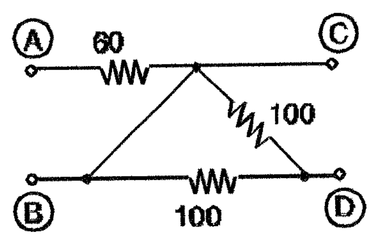
2-30 $R_{AB} = 60\ \Omega$; $R_{BC} = 0\ \Omega$;

$R_{AC} = 60\ \Omega$;

$R_{CD} = 100||100 = 50\ \Omega$;

$R_{BD} = 100||100 = 50\ \Omega$;

$R_{AD} = 60 + 100||100 = 110\ \Omega$.



2-31 $R_{AB} = 60 + 40||(40 + 80) = 90\ \Omega$

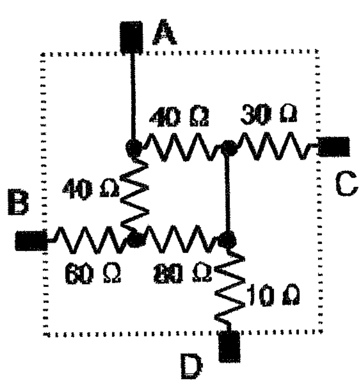
$R_{AC} = 30 + 40||(40 + 80) = 60\ \Omega$

$R_{AD} = 10 + 40||(40 + 80) = 40\ \Omega$

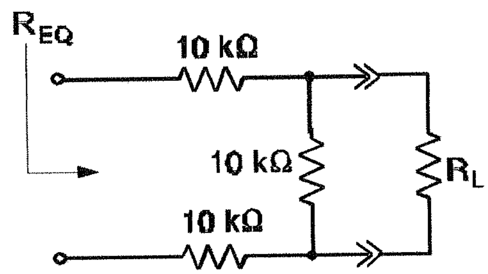
$R_{BC} = 60 + 30 + 80||80 = 130\ \Omega$

$R_{BD} = 60 + 10 + 80||80 = 110\ \Omega$

$R_{CD} = 30 + 10 = 40\ \Omega$



2-32 $R_{EQ}(R_L) := 10^4 + 10^4 + \dfrac{10^4 \cdot R_L}{10^4 + R_L}$



$R_{EQ}(10^4) = 2.5 \times 10^4 \qquad R_{EQ}(0) = 2 \times 10^4$

$R_L := 1000$ Given $\dfrac{10^4 \cdot R_L}{10^4 + R_L} + 20000 = 22000$

$\text{Find}(R_L) = 2.5 \times 10^3$

2-33 $v_S := 15 \quad R_S := 50$

$i_S := \dfrac{v_S}{R_S} \qquad i_S = 0.3$

2-34 $R := 2000 \qquad V_R := 5 \qquad i_R := \dfrac{V_R}{R}$

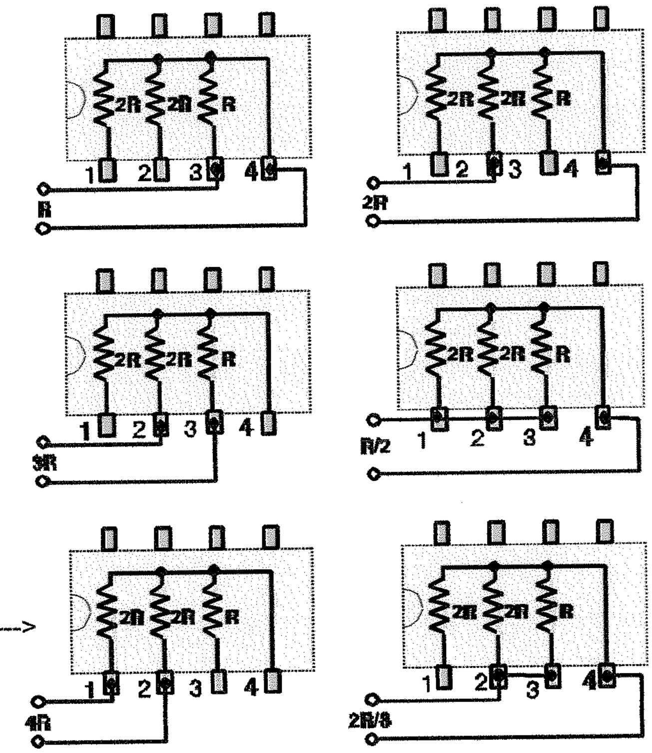
$i_R = 2.5 \times 10^{-3} \qquad i_S := 5 \cdot 10^{-3}$

$i_{Rs} := i_S - i_R \qquad i_{Rs} = 2.5 \times 10^{-3}$

$R_S := \dfrac{V_R}{i_{Rs}} \qquad R_S = 2 \times 10^3$

2-35 R_{EQ} = 8R/5 cannot be obtained

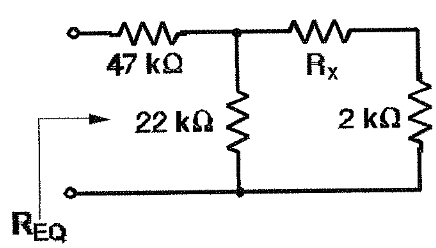
other solutions are shown ------------------------->



2-36 $R_x := 10^3$ Given

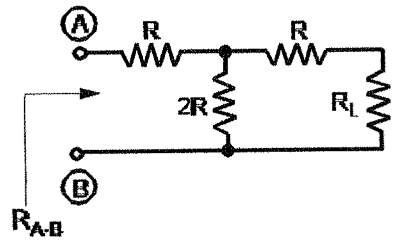
$$47 \cdot 10^3 + \frac{1}{\dfrac{1}{22 \cdot 10^3} + \dfrac{1}{R_x + 2 \cdot 10^3}} = 49 \cdot 10^3$$

$\text{Find}(R_x) = 200$ <---solution



2-37 For 1 kΩ connect four 4.3 kΩ in parallel R_{EQ} = 4.3||4.3||4.3||4.3|| = 4.3/4 = 1.075 kΩ.
For 5 kΩ connect one 4.3 kΩ in series with four in parallel R_{EQ} = 4.3 + 1.075 = 5.375 kΩ.
For 10 kΩ connect two 4.3 kΩ in series with two in parallel R_{EQ} = 2x4.2 + 4.3/2 = 10.75 kΩ.
Total number of 4.3 kΩ resistors = 4 + 5 + 4 = 13.

2-38 $$R_{AB} = R + \frac{1}{\dfrac{1}{2\cdot R} + \dfrac{1}{R + R_L}} = R_L$$



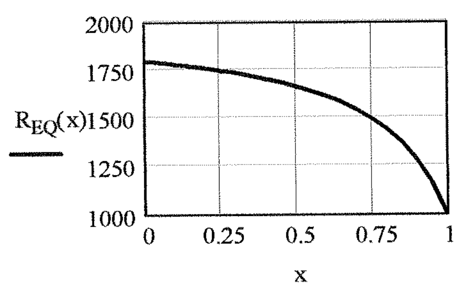
Requires $$R\cdot\frac{(5\cdot R + 3\cdot R_L)}{(3\cdot R + R_L)} = R_L$$

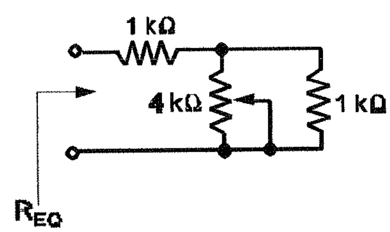
Or $$\frac{\left(5\cdot R^2 - R_L{}^2\right)}{(3\cdot R + R_L)} = 0 \qquad R = \frac{R_L}{\sqrt{5}}$$

2-39

$$R_{EQ}(x) := 1000 + \frac{1000\cdot 4000\cdot(1 - x)}{1000 + 4000\cdot(1 - x)} \qquad R_{EQ}(0) = 1.8 \times 10^3$$

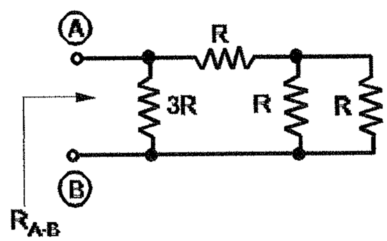
$x := 0, 0.05 .. 1 \qquad R_{EQ}(1) = 1 \times 10^3$





hence 1800 > R_{EQ} > 1000

2-40

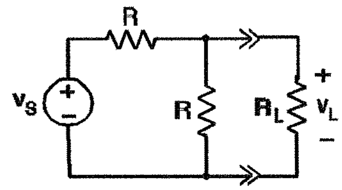


$$R_{AB} = \frac{1}{\dfrac{1}{3\cdot R} + \dfrac{1}{R + \dfrac{1}{\dfrac{1}{R} + \dfrac{1}{R}}}} = \frac{1}{\dfrac{1}{3\cdot R} + \dfrac{1}{1.5\cdot R}} = \frac{3\cdot R\cdot 1.5\cdot R}{4.5\cdot R} = R$$

2-41 By voltage division

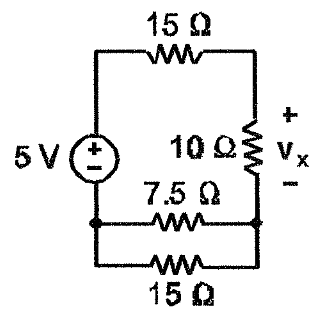
$$v_L = \frac{\dfrac{R \cdot R_L}{R+R_L}}{R + \dfrac{R \cdot R_L}{R + R_L}} \cdot v_S$$

$$v_L = v_S \cdot \frac{R_L}{(R + 2 \cdot R_L)}$$



2-42 By voltage division

$$v_x := \frac{10}{15 + 10 + \dfrac{15 \cdot 7.5}{15 + 7.5}} \cdot 5 \qquad v_x = 1.667$$

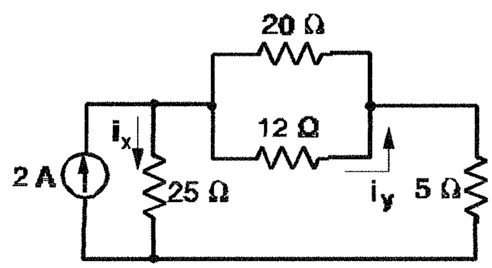


2-43 By current division

$$i_x := \frac{5 + \dfrac{20 \cdot 12}{20 + 12}}{25 + 5 + \dfrac{20 \cdot 12}{20 + 12}} \cdot 2 \qquad i_x = 0.667$$

2-44 By current division

$$i_y := \left(\frac{25}{25 + 5 + \dfrac{20 \cdot 12}{20 + 12}} \cdot 2 \right) \cdot \frac{20}{20 + 12} \quad i_y = 0.833$$



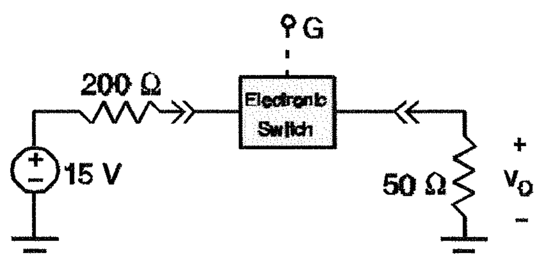
2-45 By voltage division:

$$v_{out}(R_{sw}) := \frac{50 \cdot 15}{50 + 200 + R_{sw}}$$

$R_{ON} := 100 \qquad R_{OFF} := 500 \cdot 10^{6}$

For $v_G = 5 > 2$; $v_{out}(R_{ON}) = 2.143$

For $v_G = 0.5 < 0.8$; $v_{out}(R_{OFF}) = 1.5 \times 10^{-6}$



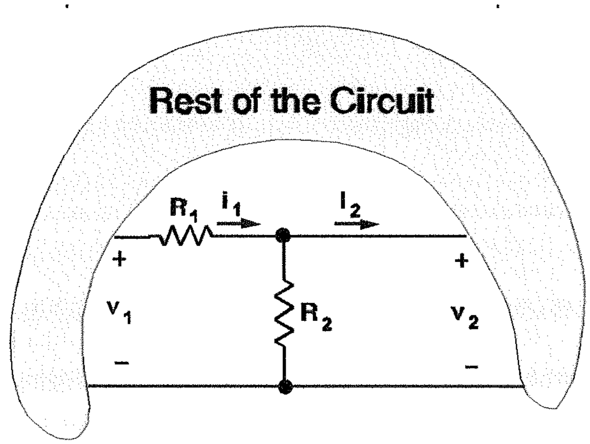
2-46

(a) If $i_1 = 0$ then $v_2 = v_1$;

(b) If $i_2 = 0$ then $v_2 = (v_1)R_2/(R_1 + R_2)$;

(c) If $v_1 = 0$ then $i_1 = (i_2)R_2/(R_1 + R_2)$;

(d) If $v_2 = 0$ then $i_1 = i_2$.



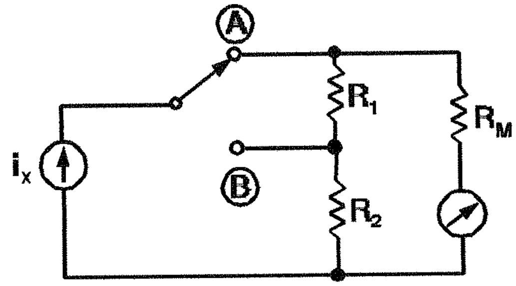
2-47 $R_M := 50 \qquad I_{FS} := 0.0005$

Position A: With $\quad i_X := 0.01$
by current division

$$\frac{(R_1 + R_2)\cdot 0.01}{R_1 + R_2 + 50} = 0.0005$$

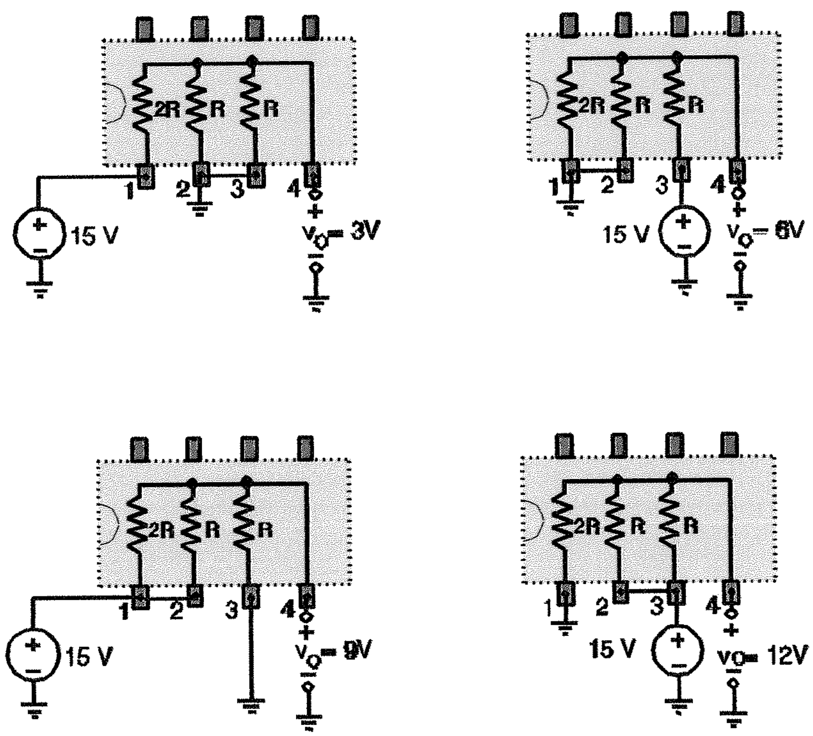
Position B: With $\quad i_X := 0.05$
by current division

$$\frac{R_2 \cdot 0.05}{R_1 + R_2 + 50} = 0.0005$$



From Pos. A: $19 \cdot R_1 + 19 \cdot R_2 = 50$ From Pos. B: $-R_1 + 99 \cdot R_2 = 50$ yields $R_1 = \frac{40}{19}\,\Omega$ & $R_2 = \frac{10}{19}\,\Omega$.

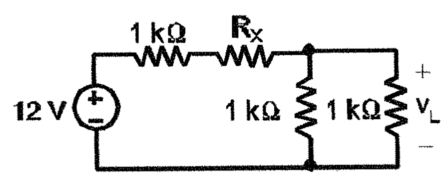
2-48 Solutions are as shown.



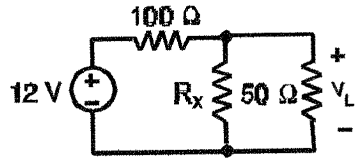
2-49 By voltage division

$$\frac{500}{1000 + R_X + 500} \cdot 12 = 2$$

yields $R_X = 1500\ \Omega$.



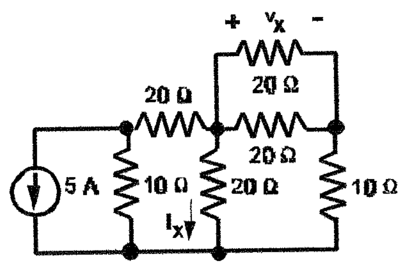
2-50 When $v_L = 3$ V the voltage across the 100 Ω resistor is 9 V. By voltage division



$$\frac{100}{100 + \dfrac{R_X \cdot 50}{50 + R_X}} \cdot 12 = 9$$

which yields $R_X = 100\ \Omega$.

2-51 By current division.



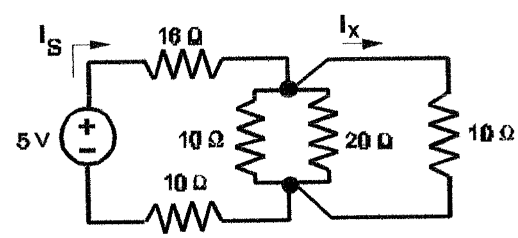
$$i_y := \frac{10}{10 + 20 + \dfrac{1}{\dfrac{1}{20} + \dfrac{1}{10 + \dfrac{1}{\dfrac{1}{20} + \dfrac{1}{20}}}}} \cdot (-5) \qquad i_y = -1.25$$

$$i_x := \frac{i_y}{2} \qquad i_x = -0.625$$

By voltage division

$$v_x := \frac{10}{10 + 10} \cdot (20 \cdot i_x) \qquad v_x = -6.25$$

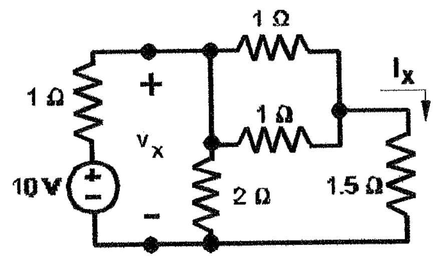
2-52 The total resistance seen by the 5-V source is $R_T := 16 + \dfrac{1}{\dfrac{1}{20} + \dfrac{2}{10}} + 10$



$R_T = 30$ hence $i_S := \dfrac{5}{R_T}$ $\quad i_S = 0.167$

$$i_x := \frac{\dfrac{1}{10}}{\dfrac{2}{10} + \dfrac{1}{20}} \cdot i_S \qquad i_x = 0.067$$

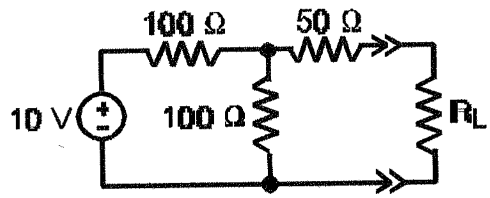
2-53 By voltage division



$$v_x := \frac{\dfrac{1}{\dfrac{1}{2} + \dfrac{1}{0.5+1.5}}}{1 + \dfrac{1}{\left(\dfrac{1}{2} + \dfrac{1}{0.5 + 1.5}\right)}} \cdot 10 \qquad v_x = 5$$

By Ohm's law $\quad i_x := \dfrac{v_x}{2} \qquad i_x = 2.5$

2-58



Using source transformations the given circuit reduces as:

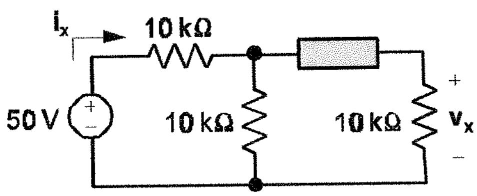
$i_L = \frac{5}{100 + R_L}$ hence $P_L = i_L^2 \cdot R_L = \frac{25 \cdot R_L}{10000 + 200 \cdot R_L + R_L^2} \geq 0.05$ or $0 \geq 10000 - 300 \cdot R_L + R_L^2$

which factors as $0 \geq (R_L - 38.1966) \cdot (R_L - 261.80)$ hence, a value in the range $38.1966 < R_L <$ $261.80\ \Omega$ will deliver at least 50 mW.

2-59 Using voltage division on the reduced circuit shown in Prob. 2-58 we obtain

$v_L = \left(\frac{R_L}{100 + R_L}\right) \cdot 5 \geq 2.5$ which requires $5 \cdot R_L \geq 250 + 2.5 \cdot R_L$ or $R_L \geq 100\ \Omega$.

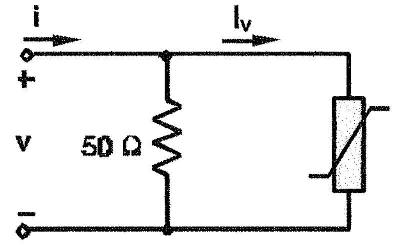
2-60



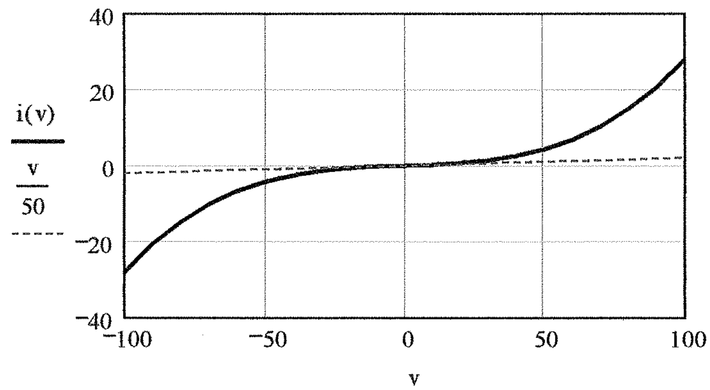
	For $R_x = 8 \cdot 10^3$	For $R_x = 80 \cdot 10^3$
$R_T = 10^4 + \frac{10^4 \cdot (10^4 + R_x)}{10^4 + 10^4 + R_x}$	$R_T = 1.643 \cdot 10^4$	$R_T = 1.9 \cdot 10^4$
$i_x = \frac{50}{R_T}$	$i_x = 3.043 \cdot 10^{-3}$	$i_x = 2.632 \cdot 10^{-3}$
$i_2 = \frac{10^4}{10^4 + 10^4 + R_x} \cdot i_x$	$i_2 = 1.087 \cdot 10^{-3}$	$i_2 = 2.632 \cdot 10^{-4}$
$v_x = 10^4 \cdot i_2$	$v_x = 10.87$	$v_x = 2.632$

2.632 mA < i_x < 3.043 mA 2.632 V < v_x < 10.87 V

2-61



(a) $i(v) := 2.6 \cdot 10^{-5} \cdot v^3 + \frac{v}{50}$ $\qquad v := -100, -90 .. 100$



(b) nonlinear (i-v curve is not a str line through the origin)
passive (i-v curve lies in the first and third quadrants so that vi >0)
bilateral (i-v curve has odd symmetry about the origin

(c) $\text{Error}(v) := 1 - \frac{v}{50} \cdot \frac{1}{i(v)}$ $\qquad v := -10$ Given $\text{Error}(v) = 0.1$ $\text{Find}(v) = -9.245$

$v := 10$ Given $\text{Error}(v) = 0.1$ $\text{Find}(v) = 9.245$

Error is 10% of less for -9.245 V < v < +9.245 V.

(d) $\text{Pvar}(v) := v \cdot \left(2.6 \cdot 10^{-5} \cdot v^3\right)$ $\qquad v := -10$ Given $\text{Pvar}(v) = 50$ $\text{Find}(v) = -37.239$

$v := 10$ Given $\text{Pvar}(v) = 50$ $\text{Find}(v) = 37.239$

Varistor power is 50 W or less for -37.239 V < V < + 37.239 V

$\text{Pres}(v) := \frac{v^2}{50}$ $\qquad v := -10$ Given $\text{Pres}(v) = 50$ $\text{Find}(v) = -50$

$v := 10$ Given $\text{Pres}(v) = 50$ $\text{Find}(v) = 50$

Resistor power is 50 W or less for v between -50 V and + 50 V

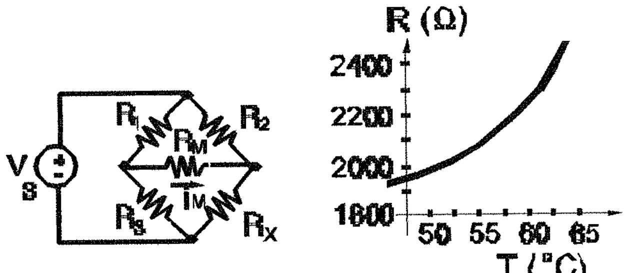
The varistor limits the safe range to -37.239 V to +37.239 V

(e) Modeled as a 50 W resistor then the voltage across the parallel combination would be 2.5 V. Modeled as a nonlinear element the actual voltage across the parallel combination is:

$v := 2.5$ $\qquad$ Given $\quad i(v) \cdot 50 + v = 5$ $\quad \text{Find}(v) = 2.48997$

The linear model predicts a voltage of 2.5 V while the nonlinear model predicts 2.48997 V. The error of about 0.4 %, making the linear model more than adequate.

2-62



(a) (b)

(a) with $i_M = 0$ then $v_M = 0$, then by voltage division $v_3 = \dfrac{R_3}{R_1 + R_3}\cdot v_s = v_x = \dfrac{R_x}{R_2 + R_x}\cdot v_s$

or $\dfrac{R_3}{R_1 + R_3} = \dfrac{R_x}{R_2 + R_x}$ therefore the balance requirement is $R_2 \cdot R_3 = R_1 \cdot R_x$.

(b) at temp = 57.5° $R_x = 2200\ \Omega$. Since $R_1 = R_2 = 2200\ \Omega$, we have $R_3 = R_x = 2200\ \Omega$

(c) with $R_1 = R_2 = 2200\ \Omega$, $R_3 = 2050\ \Omega$, and $i_M < 0$ then $R_x > 2200$ hence temp > 57.5°.

(d) with $R_1 = R_2 = 2200\ \Omega$, $R_3 = 2400\ \Omega$ and $i_M = 0$ then $R_x = 2400$ hence temp = 62.5°.

2-63 (a) Equating input resistances seen between terminals A-B, B-C, and C-A yields:

Y-Ckt Δ-Ckt

$$R_{AB} = R_1 + R_2 = \frac{R_C \cdot (R_A + R_B)}{R_C + (R_A + R_B)} = \frac{R_A \cdot R_C + R_B \cdot R_C}{R_A + R_B + R_C}$$

$$R_{BC} = R_2 + R_3 = \frac{R_A \cdot (R_B + R_C)}{R_A + (R_B + R_C)} = \frac{R_A \cdot R_B + R_A \cdot R_C}{R_A + R_B + R_C}$$

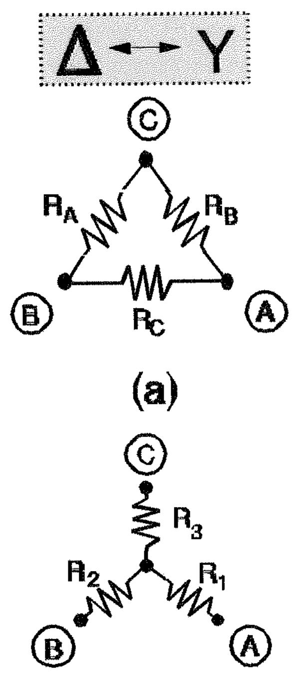
$$R_{CA} = R_1 + R_3 = \frac{R_B \cdot (R_A + R_C)}{R_B + (R_A + R_C)} = \frac{R_A \cdot R_B + R_B \cdot R_C}{R_A + R_B + R_C}$$

Solving for R_1, R_2, and R_3 yields the Δ-to-Y transformation

$$\frac{R_{AB} - R_{BC} + R_{CA}}{2} = R_1 = \frac{R_B \cdot R_C}{R_A + R_B + R_C}, \text{ QED}$$

$$\frac{R_{AB} + R_{BC} - R_{CA}}{2} = R_2 = \frac{R_C \cdot R_A}{R_A + R_B + R_C}, \text{ QED}$$

$$\frac{-R_{AB} + R_{BC} + R_{CA}}{2} = R_3 = \frac{R_A \cdot R_B}{R_A + R_B + R_C}, \text{ QED}$$



(b)

2-63 Continued

(b) Writing the Δ toY transformation as conductances yields:

$$G_1 = \frac{G_A \cdot G_B + G_B \cdot G_C + G_A \cdot G_C}{G_A} \quad G_2 = \frac{G_A \cdot G_B + G_B \cdot G_C + G_A \cdot G_C}{G_B}$$

$$G_3 = \frac{G_A \cdot G_B + G_B \cdot G_C + G_A \cdot G_C}{G_C}$$

hence we can write

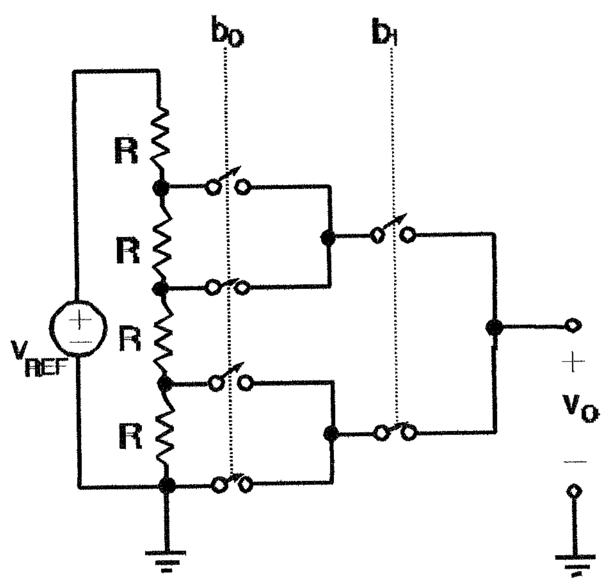
$$\frac{G_2 \cdot G_3}{G_1 + G_2 + G_3} = \frac{(G_A \cdot G_B + G_B \cdot G_C + G_A \cdot G_C)^2 \cdot \left(\frac{1}{G_B \cdot G_C}\right)}{(G_A \cdot G_B + G_B \cdot G_C + G_A \cdot G_C) \cdot \left(\frac{1}{G_A} + \frac{1}{G_B} + \frac{1}{G_C}\right)} = \frac{\left(\frac{1}{G_B \cdot G_C}\right)}{\left(\frac{1}{G_A \cdot G_B \cdot G_C}\right)} = G_A$$

$$\frac{G_1 \cdot G_3}{G_1 + G_2 + G_3} = \frac{(G_A \cdot G_B + G_B \cdot G_C + G_A \cdot G_C)^2 \cdot \left(\frac{1}{G_A \cdot G_C}\right)}{(G_A \cdot G_B + G_B \cdot G_C + G_A \cdot G_C) \cdot \left(\frac{1}{G_A} + \frac{1}{G_B} + \frac{1}{G_C}\right)} = \frac{\left(\frac{1}{G_A \cdot G_C}\right)}{\left(\frac{1}{G_A \cdot G_B \cdot G_C}\right)} = G_B$$

$$\frac{G_1 \cdot G_2}{G_1 + G_2 + G_3} = \frac{(G_A \cdot G_B + G_B \cdot G_C + G_A \cdot G_C)^2 \cdot \left(\frac{1}{G_A \cdot G_B}\right)}{(G_A \cdot G_B + G_B \cdot G_C + G_A \cdot G_C) \cdot \left(\frac{1}{G_A} + \frac{1}{G_B} + \frac{1}{G_C}\right)} = \frac{\left(\frac{1}{G_A \cdot G_B}\right)}{\left(\frac{1}{G_A \cdot G_B \cdot G_C}\right)} = G_C$$

QED

2-64



2-64 Continued By voltage division:

$(b_1\ b_0) = (0\ 0)$ produces $v_O = 0$

$(b_1\ b_0) = (0\ 1)$ produces $v_O = \frac{R}{4 \cdot R} \cdot V_{REF} = \frac{V_{REF}}{4}$

$(b_1\ b_0) = (1\ 0)$ produces $v_O = \frac{2 \cdot R}{4 \cdot R} \cdot V_{REF} = \frac{V_{REF}}{2}$

$(b_1\ b_0) = (1\ 1)$ produces $v_O = \frac{3 \cdot R}{4 \cdot R} \cdot V_{REF} = \frac{3 \cdot V_{REF}}{4}$

Evaluating the expression

$$v_O = 0.25 \cdot V_{REF} \cdot \left(b_0 \cdot 2^0 + b_1 \cdot 2^1\right)$$

$(b_1\ b_0) = (0\ 0)$ yields $v_O = 0.25 \cdot V_{REF}(0 + 0) = 0$

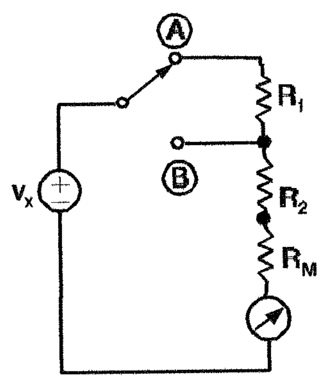
$(b_0\ b_1) = (1\ 0)$ yields $v_O = 0.25 \cdot V_{REF}(1 + 0) = \frac{V_{REF}}{4}$

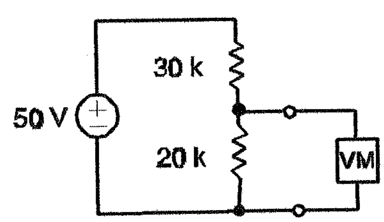
$(b_1\ b_0) = (1\ 0)$ yields $v_O = 0.25 \cdot V_{REF}(0 + 2) = \frac{V_{REF}}{2}$

$(b_1\ b_0) = (1\ 1)$ yields $v_O = 0.25 \cdot V_{REF}(1 + 2) = \frac{3 \cdot V_{REF}}{4}$

Voltage division and direct evaluation produce identical results. QED.

2-65





(a)

$R_M := 50$ and $I_{FS} := 500 \cdot 10^{-6}$. In Pos. A $\frac{50}{R_M + R_1 + R_2} = I_{FS}$, hence $R_1 + R_2 = \frac{50}{I_{FS}} - R_M$

In Pos. B $\frac{10}{R_M + R_2} = I_{FS}$, hence $R_2 := \frac{10}{I_{FS}} - R_M$ and from above $R_1 := \frac{50}{I_{FS}} - R_M - R_2$

so finally $R_1 = 8 \times 10^4$ and $R_2 = 1.995 \times 10^4$.

2-65 Continued

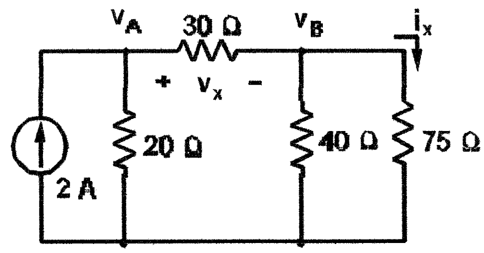
(b) The true output of the voltage divider is $\frac{20}{20+30}\cdot 50 = 20\ \mathrm{V}$. To measure 20 V the sw must be in Pos. A where the input resistance of the meter is $R_1 + R_2 + R_M = 1\times 10^5$ and the equivalent resistance of the lower leg of the divider is $\frac{100\cdot 20}{100+20} = 16.667\ \mathrm{k\Omega}$. Hence the voltmeter load reduces the divider output to $\frac{16.667\cdot 50}{16.667+30} = 17.857$ which is an error of $\frac{20-17.857}{20}\cdot 100 = 10.715$ %.

(c) At 100μA the full scale current is 5 times smaller, hence R_1 and R_2 will be nearly 5 times larger and the meter will load the circuit less thereby reducing the measurement error.

CHAPTER 3, Both Versions

3-1 (a)

$$\begin{pmatrix} \frac{1}{20}+\frac{1}{30} & \frac{-1}{30} \\ \frac{-1}{30} & \frac{1}{30}+\frac{1}{40}+\frac{1}{75} \end{pmatrix} \cdot \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$



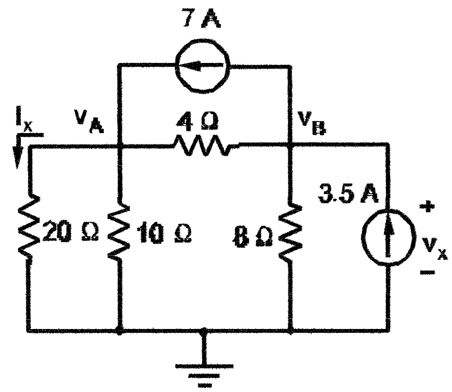
(b)

$$\begin{pmatrix} v_A \\ v_B \end{pmatrix} := \begin{pmatrix} \frac{1}{20}+\frac{1}{30} & \frac{-1}{30} \\ \frac{-1}{30} & \frac{1}{30}+\frac{1}{40}+\frac{1}{75} \end{pmatrix}^{-1} \cdot \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} 29.486 \\ 13.714 \end{pmatrix} \qquad v_x := v_A - v_B \qquad i_x := \frac{v_B}{75} \qquad v_x = 15.771 \qquad i_x = 0.183$$

3-2 (a)

$$\begin{pmatrix} \frac{1}{20}+\frac{1}{10}+\frac{1}{4} & \frac{-1}{4} \\ \frac{-1}{4} & \frac{1}{4}+\frac{1}{8} \end{pmatrix} \cdot \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} 7 \\ -7+3.5 \end{pmatrix}$$



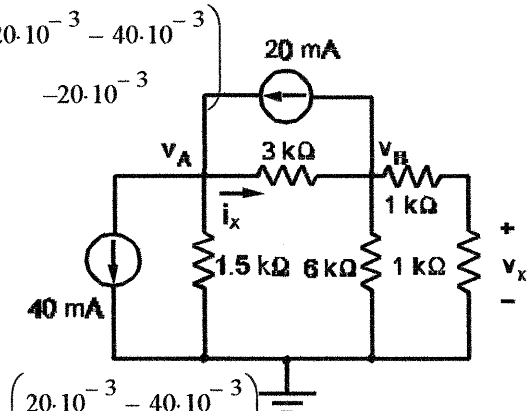
(b)

$$\begin{pmatrix} v_A \\ v_B \end{pmatrix} := \begin{pmatrix} \frac{1}{20}+\frac{1}{10}+\frac{1}{4} & \frac{-1}{4} \\ \frac{-1}{4} & \frac{1}{4}+\frac{1}{8} \end{pmatrix}^{-1} \cdot \begin{pmatrix} 7 \\ -7+3.5 \end{pmatrix}$$

$$\begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} 20 \\ 4 \end{pmatrix} \qquad v_x := v_B \qquad i_x := \frac{v_A}{20} \qquad v_x = 4 \qquad i_x = 1$$

3-3 (a)

$$\begin{pmatrix} \frac{1}{1500}+\frac{1}{3000} & \frac{-1}{3000} \\ \frac{-1}{3000} & \frac{1}{3000}+\frac{1}{6000}+\frac{1}{1000+1000} \end{pmatrix} \cdot \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} 20\cdot 10^{-3} - 40\cdot 10^{-3} \\ -20\cdot 10^{-3} \end{pmatrix}$$



(b)

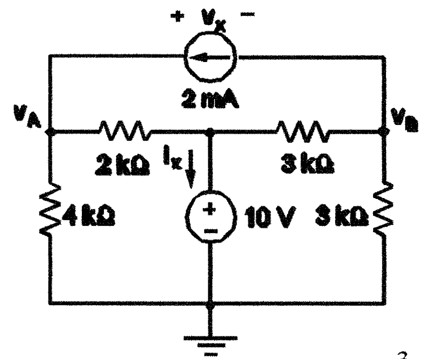
$$\begin{pmatrix} v_A \\ v_B \end{pmatrix} := \begin{pmatrix} \frac{1}{1500}+\frac{1}{3000} & \frac{-1}{3000} \\ \frac{-1}{3000} & \frac{1}{3000}+\frac{1}{6000}+\frac{1}{1000+1000} \end{pmatrix}^{-1} \cdot \begin{pmatrix} 20\cdot 10^{-3} - 40\cdot 10^{-3} \\ -20\cdot 10^{-3} \end{pmatrix}$$

$$\begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} -30 \\ -30 \end{pmatrix} \qquad v_x := \frac{v_B}{2} \qquad v_x = -15 \qquad i_x = 1 \qquad i_x := \frac{v_A - v_B}{3000}$$

3-4 (a)

$$\left(\frac{1}{2000}+\frac{1}{4000}\right)\cdot v_A - \frac{10}{2000} = 2\cdot 10^{-3}$$

$$\left(\frac{1}{3000}+\frac{1}{3000}\right)\cdot v_B - \frac{10}{3000} = -2\cdot 10^{-3}$$



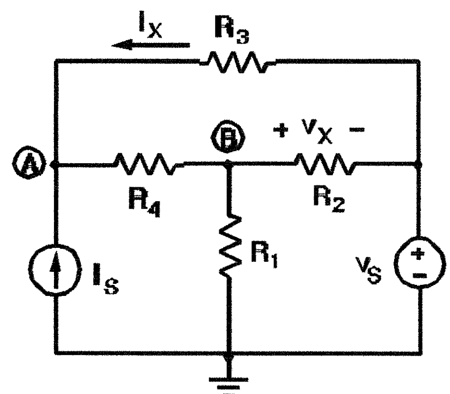
(b) Solving the 1[st] equation for v_A $\qquad v_A := \frac{28}{3}$

Solving the 2[nd] equation for v_B $\qquad v_B := 2$

$$v_x := v_A - v_B \qquad i_x := \left(\frac{v_A - 10}{2000}\right) + \frac{v_B - 10}{3000} \qquad v_x = 7.333 \qquad i_x = -3\times 10^{-3}$$

3-5 (a)

$$\begin{pmatrix} G_3+G_4 & -G_4 & -G_3 \\ -G_4 & G_1+G_2+G_4 & -G_2 \\ 0 & 0 & 1 \end{pmatrix}\cdot\begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} i_S \\ 0 \\ v_S \end{pmatrix}$$



(b) $G_1 := 10^{-4}\, G_2 := G_1 \quad G_3 := 2\cdot G_1 \quad G_4 := G_3$

$v_S := 4 \qquad i_S := 0.002$

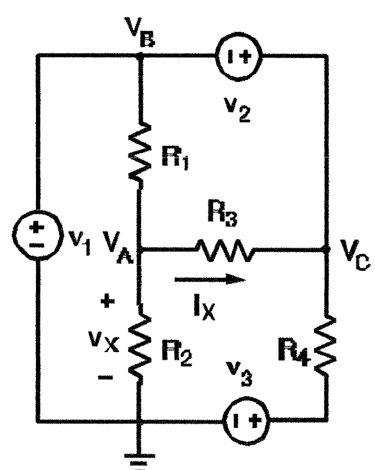
$$\begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} := \begin{pmatrix} G_3+G_4 & -G_4 & -G_3 \\ -G_4 & G_1+G_2+G_4 & -G_2 \\ 0 & 0 & 1 \end{pmatrix}^{-1}\cdot\begin{pmatrix} i_S \\ 0 \\ v_S \end{pmatrix} \qquad v_A = 10 \qquad v_B = 6 \qquad v_C = 4$$

$$v_x := v_B - v_C \quad v_x = 2 \qquad i_x := (v_C - v_A)\cdot G_3 \quad i_x = -1.2\times 10^{-3}$$

3-6 (a) Node A:

$$\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)\cdot v_A - \frac{1}{R_1}\cdot v_B - \frac{1}{R_3}\cdot v_C = 0$$

(b) Node B: $v_B = v_1$ Node C: $v_C = v_1 + v_2$



$R_1 := 10^4 \quad R_2 := 10^4 \quad R_3 := 4\cdot 10^4 \quad R_4 := 2\cdot 10^4$

$v_1 := 5 \quad v_2 := 5 \quad v_3 := 15$

$v_B := v_1 \quad v_B = 5 \quad v_C := v_1 + v_2 \quad v_C = 10$

$$v_A := \frac{\frac{1}{R_1}\cdot v_B + \frac{1}{R_3}\cdot v_C}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} \qquad \begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} 3.333 \\ 5 \\ 10 \end{pmatrix}$$

$$v_x := v_A \qquad v_x = 3.333 \text{ V} \qquad i_x := \frac{v_A - v_C}{R_3} \qquad i_x = -1.667\times 10^{-4} \text{ A}$$

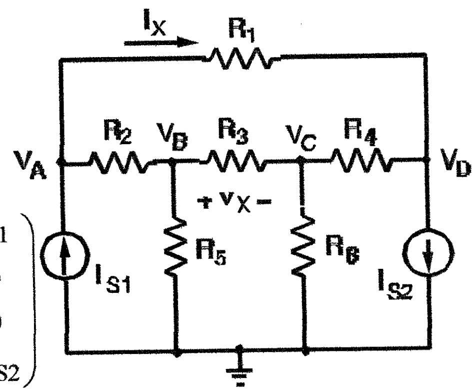
(c) $$P_{R1} := \frac{(v_B - v_A)^2}{R_1} \qquad P_{R1} = 2.778\times 10^{-4}\,\text{W}$$

3-7 (a) $R_1 := 1000 \; R_2 := R_1 \;\; R_3 := R_1 \;\; R_4 := R_1$

$R_5 := 4 \cdot R_1 \;\; R_6 := 4 \cdot R_1 \;\; i_{S1} := 0.05 \;\; i_{S2} := 0.05$

$G_1 := R_1^{-1} \;\; G_2 := R_2^{-1} \;\; G_3 := R_3^{-1} \;\; G_4 := R_4^{-1}$

$G_5 := R_5^{-1} \;\; G_6 := R_6^{-1}$



$$\begin{pmatrix} G_1 + G_2 & -G_2 & 0 & -G_1 \\ -G_2 & G_2 + G_3 + G_5 & -G_3 & 0 \\ 0 & -G_3 & G_3 + G_4 + G_6 & -G_4 \\ -G_1 & 0 & -G_4 & G_1 + G_4 \end{pmatrix} \cdot \begin{pmatrix} v_A \\ v_B \\ v_C \\ v_D \end{pmatrix} = \begin{pmatrix} i_{S1} \\ 0 \\ 0 \\ -i_{S2} \end{pmatrix}$$

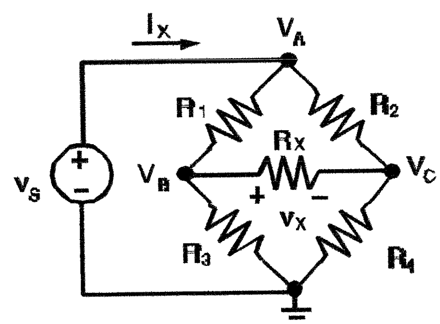
(b) $$\begin{pmatrix} v_A \\ v_B \\ v_C \\ v_D \end{pmatrix} := \begin{pmatrix} G_1 + G_2 & -G_2 & 0 & -G_1 \\ -G_2 & G_2 + G_3 + G_5 & -G_3 & 0 \\ 0 & -G_3 & G_3 + G_4 + G_6 & -G_4 \\ -G_1 & 0 & -G_4 & G_1 + G_4 \end{pmatrix}^{-1} \cdot \begin{pmatrix} i_{S1} \\ 0 \\ 0 \\ -i_{S2} \end{pmatrix} \quad \begin{pmatrix} v_A \\ v_B \\ v_C \\ v_D \end{pmatrix} = \begin{pmatrix} 18.571 \\ 5.714 \\ -5.714 \\ -18.571 \end{pmatrix}$$

$v_x := v_B - v_C \qquad v_x = 11.429 \qquad i_x := (v_A - v_D) \cdot G_1 \qquad i_x = 3.714 \times 10^{-2}$

(c) $p_{total} := v_A \cdot (-i_{S1}) + v_D \cdot i_{S2} \qquad p_{total} = -1.857$

3-8 (a)

$$\begin{pmatrix} 1 & 0 & 0 \\ -G_1 & G_1 + G_x + G_3 & -G_x \\ -G_2 & -G_x & G_2 + G_X + G_4 \end{pmatrix} \cdot \begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} v_S \\ 0 \\ 0 \end{pmatrix}$$



(b) $G_1 := (10^4)^{-1} \quad G_2 := (2 \cdot 10^4)^{-1} \quad G_3 := (6 \cdot 10^4)^{-1}$

$G_4 := (2 \cdot 10^4)^{-1} \quad G_x := (3 \cdot 10^3)^{-1} \quad v_S := 15 \quad G_x = 3.333 \times 10^{-4}$

$$\begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} := \begin{pmatrix} 1 & 0 & 0 \\ -G_1 & G_1 + G_x + G_3 & -G_x \\ -G_2 & -G_x & G_2 + G_x + G_4 \end{pmatrix}^{-1} \cdot \begin{pmatrix} v_S \\ 0 \\ 0 \end{pmatrix}$$

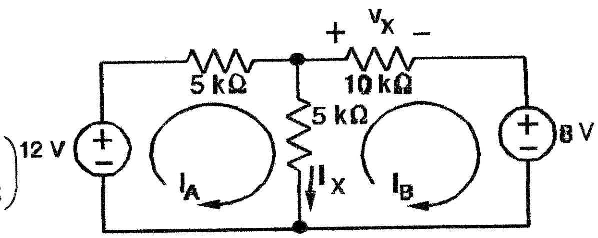
$$\begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} 15 \\ 10.728 \\ 9.983 \end{pmatrix}$$

$v_x := v_B - v_C \qquad i_x := (v_A - v_B) \cdot G_1 + (v_A - v_C) \cdot G_2$

$v_x = 0.745 \qquad i_x = 6.78 \times 10^{-4}$

(c) $P_{R2} := (v_A - v_C)^2 \cdot G_2 \qquad P_{R2} = 1.2583 \times 10^{-3}$ W

3-9 (a) $\begin{pmatrix} 5000+5000 & -5000 \\ -5000 & 5000+10000 \end{pmatrix} \cdot \begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} 12 \\ -8 \end{pmatrix}$

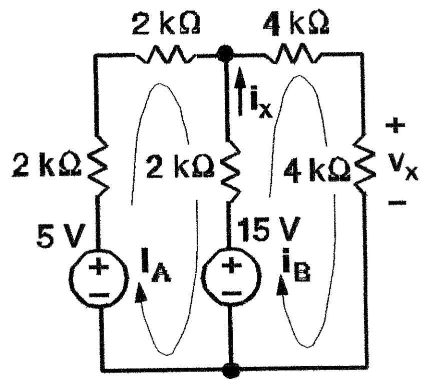


(b) $\begin{pmatrix} i_A \\ i_B \end{pmatrix} := \begin{pmatrix} 5000+5000 & -5000 \\ -5000 & 5000+10000 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 12 \\ -8 \end{pmatrix}$

$\begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} 1.12\times 10^{-3} \\ -1.6\times 10^{-4} \end{pmatrix}$ $\quad v_X := i_B \cdot 10^4 \quad i_X := i_A - i_B \quad v_X = -1.6 \quad i_X = 1.28\times 10^{-3}$

3-10 (a)

$\begin{pmatrix} 2000+2000+2000 & -2000 \\ -2000 & 2000+4000+4000 \end{pmatrix} \cdot \begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} 5-15 \\ 15 \end{pmatrix}$



(b)

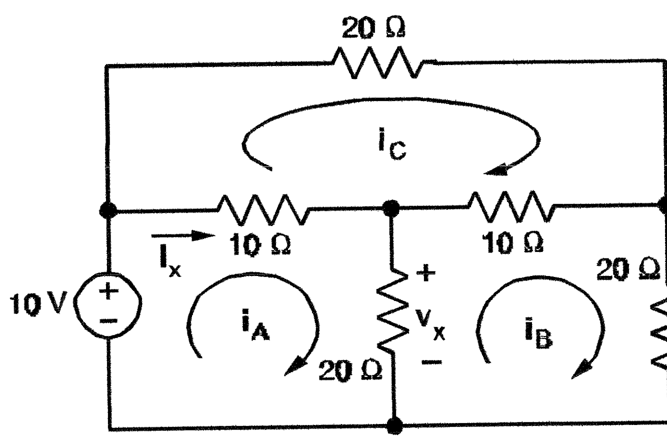
$\begin{pmatrix} i_A \\ i_B \end{pmatrix} := \begin{pmatrix} 2000+2000+2000 & -2000 \\ -2000 & 2000+4000+4000 \end{pmatrix}^{-1} \cdot \begin{pmatrix} -10 \\ 15 \end{pmatrix}$

$\begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} -1.25\times 10^{-3} \\ 1.25\times 10^{-3} \end{pmatrix}$

$v_X := i_B \cdot 4000 \quad i_X := i_B - i_A \quad v_X = 5 \quad i_X = 2.5\times 10^{-3}$

3-11 (a)

$\begin{pmatrix} 30 & -20 & -10 \\ -20 & 50 & -10 \\ -10 & -10 & 40 \end{pmatrix} \cdot \begin{pmatrix} i_A \\ i_B \\ i_C \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \\ 0 \end{pmatrix}$



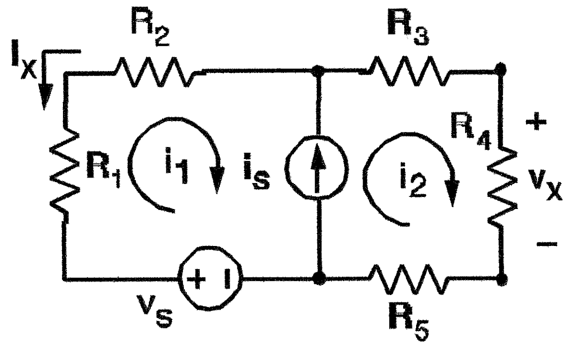
(b) $\begin{pmatrix} i_A \\ i_B \\ i_C \end{pmatrix} := \begin{pmatrix} 30 & -20 & -10 \\ -20 & 50 & -10 \\ -10 & -10 & 40 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 10 \\ 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} i_A \\ i_B \\ i_C \end{pmatrix} = \begin{pmatrix} 0.594 \\ 0.281 \\ 0.219 \end{pmatrix}$ $\quad v_X := 20\cdot(i_A - i_B) \quad v_X = 6.25 \quad i_X := i_A - i_C \quad i_X = 0.375$

3-12 Assign mesh currents as shown

(a) by KCL $i_2 - i_1 = i_S$ Super Mesh:

$(R_1 + R_2)\cdot i_1 + (R_3 + R_4 + R_5)\cdot i_2 = v_S$



(b) $R_1 := 200 \quad R_2 := 500 \quad R_3 := 60$

$R_4 := 240 \quad R_5 := 200$

$v_S := 15 \quad i_S := 0.05$

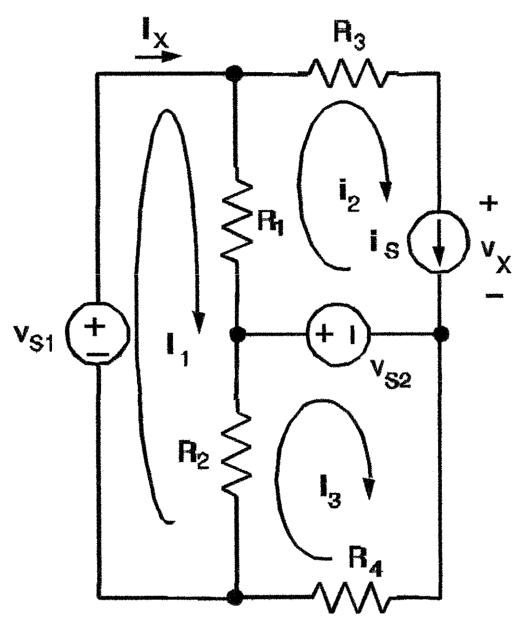
$$i_1 := \frac{-(i_S\cdot R_3 + i_S\cdot R_4 + i_S\cdot R_5 - v_S)}{(R_1 + R_2 + R_3 + R_4 + R_5)} \qquad i_2 := \frac{(i_S\cdot R_1 + i_S\cdot R_2 + v_S)}{(R_1 + R_2 + R_3 + R_4 + R_5)} \qquad \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} -8.333 \times 10^{-3} \\ 4.167 \times 10^{-2} \end{pmatrix}$$

$i_x := -i_1 \quad i_x = 8.333 \times 10^{-3} \qquad v_x := R_4\cdot(i_2) \quad v_x = 10$

(c) $p_{total} := (R_3 + R_4 + R_5)\cdot(i_2)^2 + (R_1 + R_2)\cdot i_1^{\,2} \qquad p_{total} = 0.917$ W

3-13 (a)

$$\begin{pmatrix} R_1 + R_2 & -R_1 & -R_2 \\ 0 & 1 & 0 \\ -R_2 & 0 & R_2 + R_4 \end{pmatrix} \cdot \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} v_{S1} \\ i_S \\ -v_{S2} \end{pmatrix}$$



(b)

$R_1 := 10^4 \quad R_2 := 10^4 \quad R_3 := 2000 \quad R_4 := 1000$

$v_{S1} := 12 \quad v_{S2} := 0.5 \quad i_S := 0.0025$

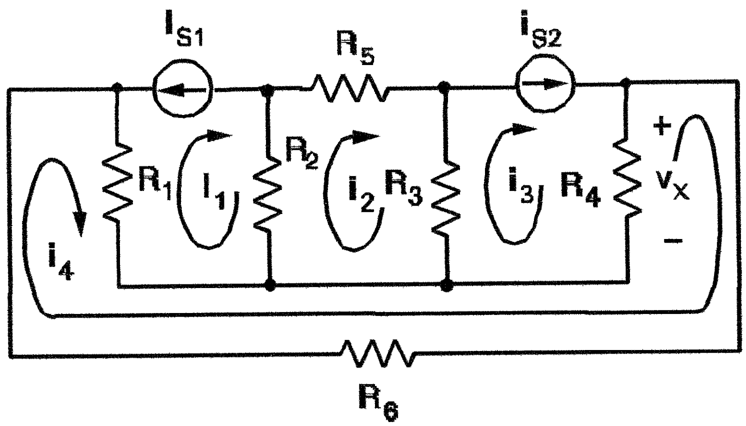
$$\begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} := \begin{pmatrix} R_1 + R_2 & -R_1 & -R_2 \\ 0 & 1 & 0 \\ -R_2 & 0 & R_2 + R_4 \end{pmatrix}^{-1} \cdot \begin{pmatrix} v_{S1} \\ i_S \\ -v_{S2} \end{pmatrix}$$

$$\begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 3.35 \times 10^{-3} \\ 2.5 \times 10^{-3} \\ 3 \times 10^{-3} \end{pmatrix}$$

$i_x := i_1 \quad v_x := v_{S2} + R_1\cdot(i_2 - i_1) + R_3\cdot i_2 \qquad i_x = 3.35 \times 10^{-3} \qquad v_x = -3$

(c) $p_{S1} := v_{S1}\cdot(-i_1) \qquad p_{S1} = -0.0402$ W

3-14 (a) Redraw R_6 as shown below



(b) $$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -R_2 & R_2+R_3+R_5 & -R_3 & 0 \\ 0 & 0 & 1 & 0 \\ -R_1 & 0 & -R_4 & R_1+R_4+R_6 \end{pmatrix} \cdot \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = \begin{pmatrix} -i_{S1} \\ 0 \\ i_{S2} \\ 0 \end{pmatrix}$$

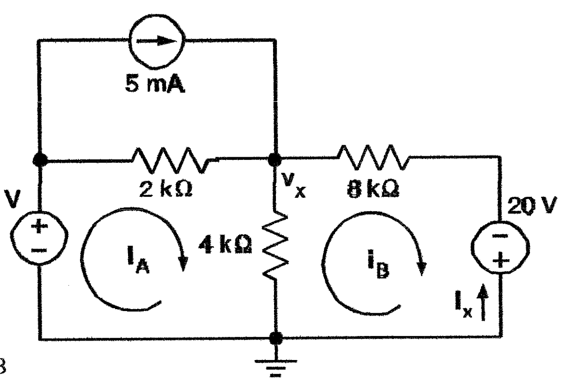
(c) $R_1 := 2000 \quad R_2 := R_1 \quad R_3 := R_1 \quad R_4 := R_1 \quad R_5 := 1000 \quad R_6 := R_5 \quad i_{S1} := 0.04 \quad i_{S2} := 0.02$

$$\begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} := \begin{pmatrix} 1 & 0 & 0 & 0 \\ -R_2 & R_2+R_3+R_5 & -R_3 & 0 \\ 0 & 0 & 1 & 0 \\ -R_1 & 0 & -R_4 & R_1+R_4+R_6 \end{pmatrix}^{-1} \cdot \begin{pmatrix} -i_{S1} \\ 0 \\ i_{S2} \\ 0 \end{pmatrix} \qquad \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = \begin{pmatrix} -4\times 10^{-2} \\ -8\times 10^{-3} \\ 2\times 10^{-2} \\ -8\times 10^{-3} \end{pmatrix}$$

$v_x := R_4 \cdot (i_3 - i_4) \qquad v_x = 56 \quad \text{V}$

3-15(a) Convert current source to a voltage source

$$\begin{pmatrix} 2000+4000 & -4000 \\ -4000 & 4000+8000 \end{pmatrix} \cdot \begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} 40+10 \\ 20 \end{pmatrix}$$



(b) $$\begin{pmatrix} i_A \\ i_B \end{pmatrix} := \begin{pmatrix} 2000+4000 & -4000 \\ -4000 & 4000+8000 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 40+10 \\ 20 \end{pmatrix}$$

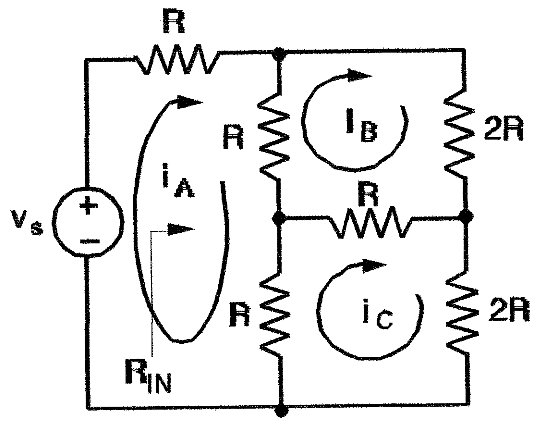
$$\begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} 1.214\times 10^{-2} \\ 5.714\times 10^{-3} \end{pmatrix} \qquad v_x := (i_A - i_B)\cdot 4000 \qquad i_x := -i_B$$

$v_x = 25.714 \qquad i_x = -5.714\times 10^{-3} \qquad i_s := .005$

(c) $p_{total} := (i_A - i_s)^2 \cdot (2000) + (i_A - i_B)^2 \cdot 4000 + i_B^{\,2} \cdot 8000$

$p_{total} = 0.529$

3-16 (a) Using mesh currents



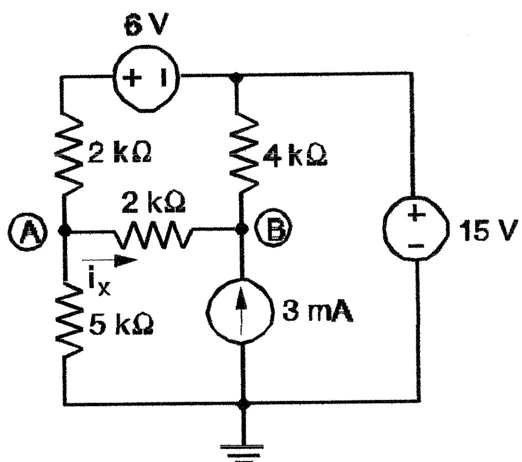
$$\begin{pmatrix} 3\cdot R & -R & -R \\ -R & 4\cdot R & -R \\ -R & -R & 4\cdot R \end{pmatrix} \cdot \begin{pmatrix} i_A \\ i_B \\ i_C \end{pmatrix} = \begin{pmatrix} v_S \\ 0 \\ 0 \end{pmatrix}$$

(b)

$$\begin{pmatrix} i_A \\ i_B \\ i_C \end{pmatrix} = \frac{1}{R} \cdot \begin{pmatrix} 3 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{pmatrix}^{-1} \cdot \begin{pmatrix} v_S \\ 0 \\ 0 \end{pmatrix} = \frac{1}{R} \cdot \begin{pmatrix} \frac{3\cdot v_S}{7} \\ \frac{v_S}{7} \\ \frac{v_S}{7} \end{pmatrix}$$

$$R_{IN} = \frac{v_S}{i_A} = \frac{v_S}{\left(\frac{3\cdot v_S}{7\cdot R}\right)} = \frac{7\cdot R}{3}$$

3-17 Using node-voltage analysis



Node A $\quad \frac{v_A - v_B}{2000} + \frac{v_A - (6+15)}{2000} + \frac{v_A}{5000} = 0$

Node B $\quad \frac{v_B - v_A}{2000} + \frac{v_B - 15}{4000} = 3\cdot 10^{-3}$

Solving the node B eq for v_B $\quad v_B = \frac{2}{3}\cdot v_A + 9$

Substituting into the node A equation

$$\frac{v_A - \left(\frac{2}{3}\cdot v_A + 9\right)}{2000} + \frac{v_A - (6+15)}{2000} + \frac{v_A}{5000} = 0$$ which yields $v_A := \frac{225}{13} \quad v_A = 17.308$

$v_B = \frac{2}{3}\cdot\frac{225}{13} + 9$ which yields $v_B := \frac{267}{13} \quad v_B = 20.538$

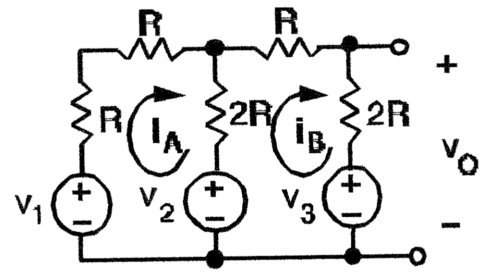
$i_x := \frac{v_A - v_B}{2000} \qquad i_x = -1.615 \times 10^{-3}$

checking

$\frac{v_A - v_B}{2000} + \frac{v_A - (6+15)}{2000} + \frac{v_A}{5000} = 0 \qquad \frac{v_B - v_A}{2000} + \frac{v_B - 15}{4000} = 3 \times 10^{-3}$ checks

3-18 Using mesh currents

$$\begin{pmatrix} 4\cdot R & -2\cdot R \\ -2\cdot R & 5\cdot R \end{pmatrix}\cdot\begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} v_1 - v_2 \\ v_2 - v_3 \end{pmatrix}$$



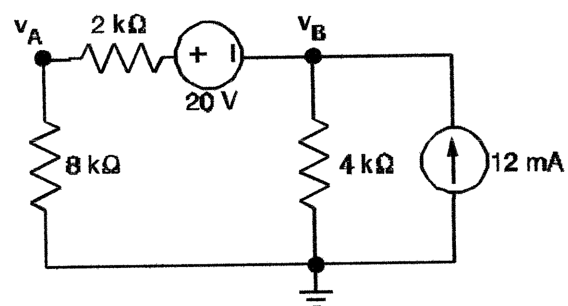
$$\begin{pmatrix} i_A \\ i_B \end{pmatrix} = \frac{1}{R}\cdot\begin{pmatrix} 4 & -2 \\ -2 & 5 \end{pmatrix}^{-1}\cdot\begin{pmatrix} v_1 - v_2 \\ v_2 - v_3 \end{pmatrix}$$

$$\begin{pmatrix} i_A \\ i_B \end{pmatrix} = \frac{1}{R}\cdot\begin{pmatrix} \dfrac{5\cdot v_1 - 3v_2}{16} + \dfrac{-v_3}{8} \\ \dfrac{v_1 + v_2}{8} + \dfrac{-v_3}{4} \end{pmatrix}$$

$$v_O = v_3 + 2\cdot R\cdot i_B = v_3 + 2\cdot R\cdot\left(\frac{v_1 + v_2}{8} + \frac{-v_3}{4}\right)$$

$$v_O = \frac{1}{4}\cdot v_1 + \frac{1}{4}\cdot v_2 + \frac{1}{2}\cdot v_3$$

3-19 Writing one mesh equation



$$(2000 + 4000 + 8000)\cdot i_A + 4000\cdot 12\cdot 10^{-3} = -20$$

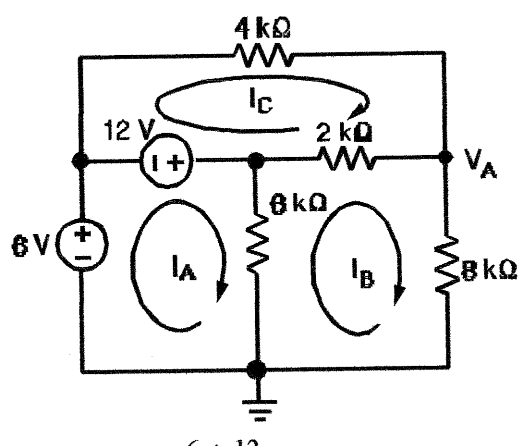
solving for i_A $\quad i_A := -4.857\cdot 10^{-3}$

$v_A := -8000\cdot i_A \qquad v_A = 38.856$

$v_B := \left(i_A + 12\cdot 10^{-3}\right)\cdot 4000 \qquad v_B = 28.572$

Checking $\quad (2000 + 4000 + 8000)\cdot i_A + 4000\cdot\left(12\cdot 10^{-3}\right) = -19.998 \quad$ <---checks the mesh equation

3-20 Writing one node equation



$$\frac{v_A - (6 + 12)}{2000} + \frac{v_A - 6}{4000} + \frac{v_A}{8000} = 0$$

solving for v_A $\quad v_A := 12$

$i_B := \dfrac{v_A}{8000} \qquad i_B = 1.5\times 10^{-3}$

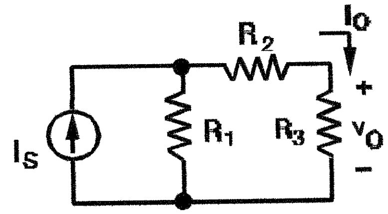
$i_C := \dfrac{6 - v_A}{4000} \qquad i_C = -1.5\times 10^{-3}$

checking $\quad -6 - 12 + 2000\cdot\left(i_B - i_C\right) + 8000\cdot i_B = 0 \quad i_A := i_B + \dfrac{6 + 12}{6000} \qquad i_A = 4.5\times 10^{-3}$

3-21 Using current division

$$i_O = \left(\frac{R_1}{R_1 + R_2 + R_3}\right) \cdot i_S$$

$v_O = R_3 \cdot i_O$ hence $K = \dfrac{R_1 \cdot R_3}{R_1 + R_2 + R_3}$

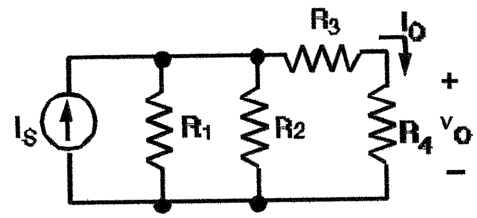


3-22 Using current division

$$i_O = \left[\frac{\frac{1}{R_3+R_4}}{\left(\frac{1}{R_3 + R_4} + \frac{1}{R_1} + \frac{1}{R_2}\right)}\right] \cdot i_S$$

$$\text{or } i_O = \left(\frac{R_1 \cdot R_2}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_1 \cdot R_4 + R_2 \cdot R_3 + R_2 \cdot R_4}\right) \cdot i_S$$

$$\text{hence } K = \frac{R_1 \cdot R_2}{R_1 \cdot R_2 + R_1 \cdot R_3 + R_1 \cdot R_4 + R_2 \cdot R_3 + R_2 \cdot R_4}$$

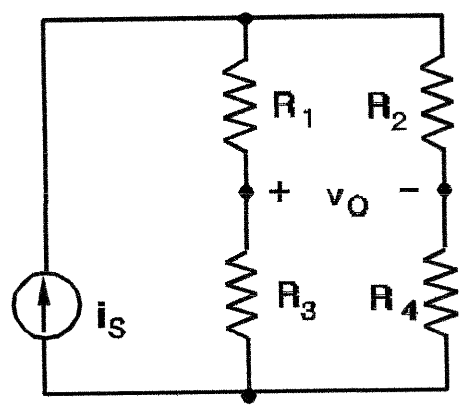


3-23 Using current division

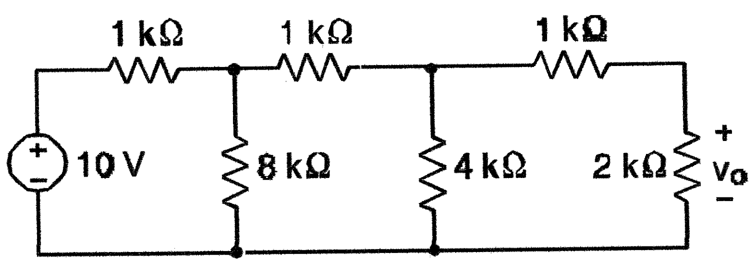
$$v_O = \left[\frac{(R_2 + R_4) \cdot i_S}{R_1 + R_2 + R_3 + R_4}\right] \cdot R_3 - \left[\frac{(R_1 + R_3) \cdot i_S}{R_1 + R_2 + R_3 + R_4}\right] \cdot R_4$$

$$\text{or } v_O = \left(\frac{R_2 \cdot R_3 - R_1 \cdot R_4}{R_1 + R_2 + R_3 + R_4}\right) \cdot i_S$$

$$\text{hence } K = \frac{R_2 \cdot R_3 - R_1 \cdot R_4}{R_1 + R_2 + R_3 + R_4}$$



3-24



Assume $v_O := 1$ $\quad i_O := \dfrac{v_O}{2000}$

$i_O = 5 \times 10^{-4}$ $\quad v_1 := 1000 \cdot i_O$

$v_2 := v_1 + v_O$ $\quad v_2 = 1.5$

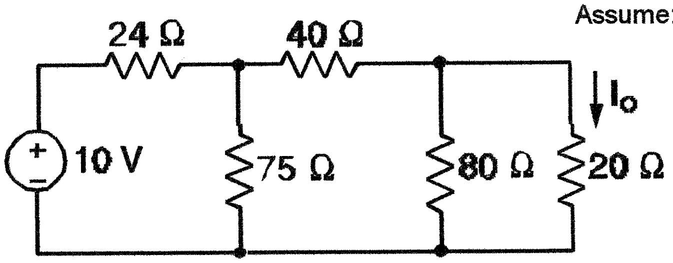
$i_2 := \dfrac{v_2}{4000}$ $\quad i_3 := i_O + i_2$

$v_3 := 1000 \cdot i_3$ $\quad v_4 := v_2 + v_3$

$v_4 = 2.375$ $\quad i_4 := \dfrac{v_4}{8000}$ $\quad i_S := i_4 + i_3$ $\quad i_S = 1.171875 \times 10^{-3}$ $\quad v_5 := 1000 \cdot i_S$ $\quad v_S := v_4 + v_5$

$v_S = 3.547$ $\quad K := \dfrac{1}{v_S}$ $\quad K = 0.281938$ $\quad v_O := K \cdot 10$ $\quad v_O = 2.819$ V

3-25



Assume: $i_O := 1$ $\quad v_O := 20 \cdot i_O$

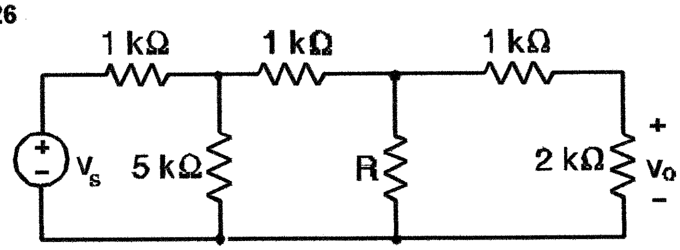
$i_1 := \frac{v_O}{80} + i_O \quad v_1 := 40 \cdot i_1$

$v_2 := v_1 + v_O \quad i_2 := \frac{v_2}{75}$

$i_3 := i_1 + i_2 \quad v_3 := 24 \cdot i_3$

$v_S := v_2 + v_3 \quad v_S = 122.4 \quad K := \frac{1}{v_S} \quad K = 8.169935 \times 10^{-3} \quad i_O := K \cdot 10 \quad i_O = 0.081699$ A

3-26



Assume $\quad v_O := 1 \quad i_O := \frac{v_O}{2000}$

$v_1 := 1000 \cdot i_O \quad v_2 := v_1 + v_O$

$i_2(R) := \frac{v_2}{R} \quad i_3(R) := i_O + i_2(R)$

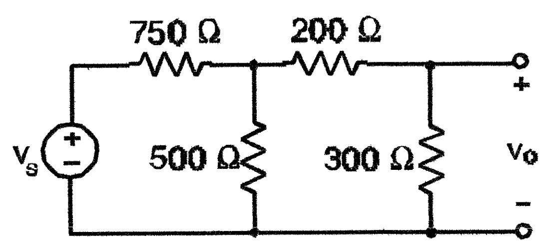
$R := 3000$

$v_3(R) := i_3(R) \cdot 1000 \quad v_4(R) := v_3(R) + v_2 \quad i_4(R) := \frac{v_4(R)}{5000} \quad i_S(R) := i_4(R) + i_3(R)$

$v_5(R) := i_S(R) \cdot 1000 \quad v_S(R) := v_5(R) + v_4(R)$

Given $\quad v_S(R) = 4 \quad R := \text{Find}(R) \quad R = 3 \times 10^3$ <---K = 1/4 requires R = 3 kΩ.

3-27 Use the Unity Output Method



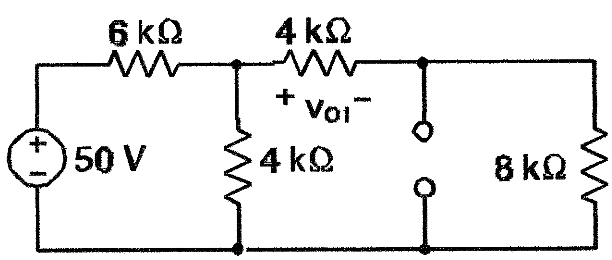
$v_O = 1 \quad i_{300} := \frac{1}{300} \quad v_{200} := i_{300} \cdot 200$

$v_{500} := v_O + v_{200} \quad v_{500} = 1.667 \quad i_{500} := \frac{v_{500}}{500}$

$v_{750} := (i_{500} + i_{300}) \cdot 750 \quad v_S := v_{750} + v_{500}$

$v_S = 6.667 \quad K := \frac{1}{v_S} \quad K = 0.15$

3-28 With the current source turned off



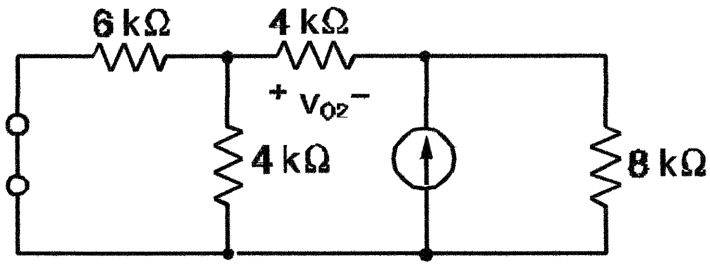
$R_{IN} := 6 \cdot 10^3 + \frac{4 \cdot 10^3 \cdot (4 \cdot 10^3 + 8 \cdot 10^3)}{(4 + 4 + 8) \cdot 10^3}$

$R_{IN} = 9 \times 10^3 \quad i_{S1} := \frac{50}{R_{IN}}$

$v_{O1} := \left(\frac{4 \cdot 10^3}{4 \cdot 10^3 + 12 \cdot 10^3} \cdot i_{S1} \right) \cdot 4 \cdot 10^3$

$v_{O1} = 5.556$ V

3-28 Continued With the voltage source turned off



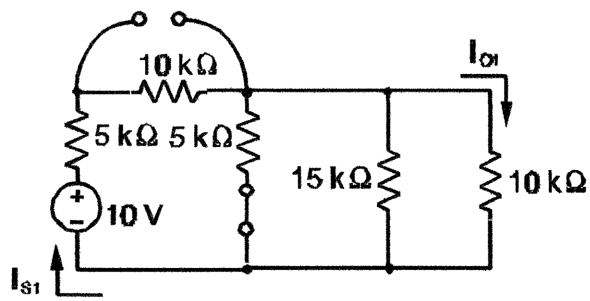
$$i_{O2} := \frac{8\cdot 10^3\cdot 0.005}{\left(8\cdot 10^3 + 4\cdot 10^3 + \frac{6\cdot 10^3\cdot 4\cdot 10^3}{6\cdot 10^3 + 4\cdot 10^3}\right)}$$

$$i_{O2} = 2.777778\times 10^{-3}$$

$$v_{O2} := 4000\cdot(-i_{O2}) \qquad v_{O2} = -11.111$$

Using superposition $\quad v_O := v_{O1} + v_{O2} \qquad v_O = -5.556$ V

3-29 With the 20 V and 1 mA sources turned off



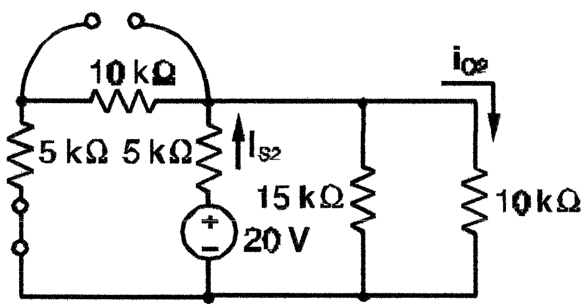
$$R_{IN1} := 15\cdot 10^3 + \frac{1}{\frac{1}{5\cdot 10^3} + \frac{1}{15\cdot 10^3} + \frac{1}{10^4}}$$

$$i_{S1} := \frac{10}{R_{IN1}} \qquad i_{S1} = 5.641\times 10^{-4}$$

$$i_{O1} := \left[\frac{10^{-4}}{\left(5\cdot 10^3\right)^{-1} + \left(15\cdot 10^3\right)^{-1} + 10^{-4}}\right]\cdot i_{S1}$$

$$i_{O1} = 1.538\times 10^{-4}$$

With the 10-V and 1 mA sources turned off



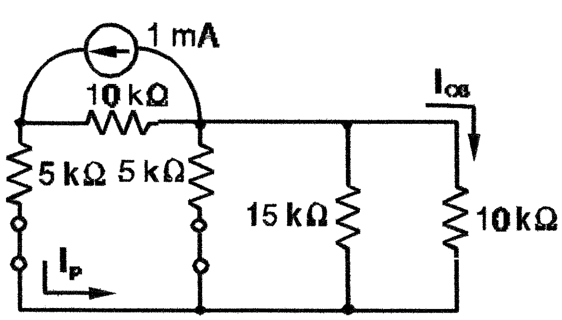
$$R_{IN2} := 5\cdot 10^3 + \frac{1}{\left(15\cdot 10^3\right)^{-1} + \left(15\cdot 10^3\right)^{-1} + 10^{-4}}$$

$$i_{S2} := \frac{20}{R_{IN2}} \qquad i_{S2} = 2.153846\times 10^{-3}$$

$$i_{O2} := \left[\frac{10^{-4}}{\left(15\cdot 10^3\right)^{-1} + \left(15\cdot 10^3\right)^{-1} + 10^{-4}}\right]\cdot i_{S2}$$

$$i_{O2} = 9.231\times 10^{-4}$$

With the 10-V and 20-V sources turned off



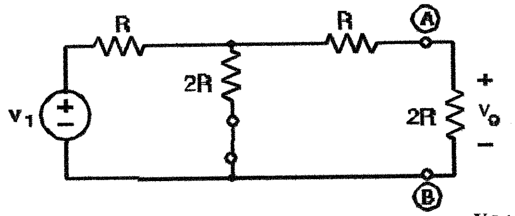
$$i_p := \frac{10^4\cdot 0.001}{10^4 + 5\cdot 10^3 + \frac{1}{\left(5\cdot 10^3\right)^{-1} + \left(15\cdot 10^3\right)^{-1} + 1\times 10^{-4}}}$$

$$i_{O3} := \left[\frac{10^{-4}}{\left[10^{-4} + \left(5\cdot 10^3\right)^{-1} + \left(15\cdot 10^3\right)^{-1}\right]}\right]\cdot(-i_p)$$

$$i_p = 5.641\times 10^{-4} \qquad i_{O3} = -1.538\times 10^{-4}$$

Using superposition $\quad i_O := i_{O1} + i_{O2} + i_{O3} \qquad i_O = 9.231\times 10^{-4}$ A

3-30 With source v_2 turned off

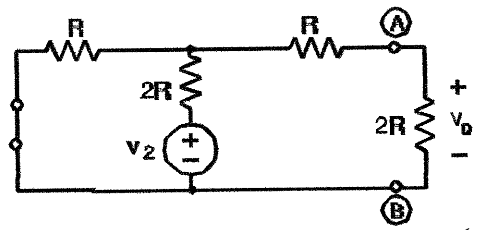


$$R_{IN1} = R + \frac{1}{\frac{1}{2 \cdot R} + \frac{1}{2 \cdot R + R}} = \frac{11}{5} \cdot R$$

$$i_{S1} = \frac{v_1}{R_{IN1}} = \frac{5 \cdot v_1}{11 \cdot R}$$

$$v_{O1} = \left[\left(\frac{2 \cdot R}{2 \cdot R + R + 2 \cdot R}\right) \cdot i_{S1}\right] \cdot 2 \cdot R = \left(\frac{2 \cdot R}{5 \cdot R}\right) \cdot \left(\frac{5 \cdot v_1}{11 \cdot R}\right) \cdot (2 \cdot R) = \frac{4 \cdot v_1}{11}$$

With source v_1 turned off



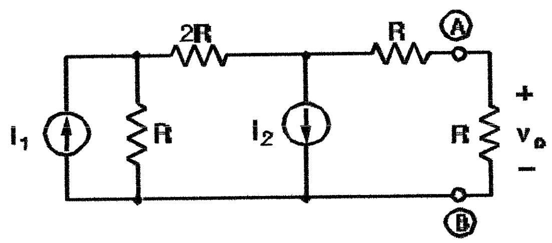
$$R_{IN2} = 2 \cdot R + \frac{1}{\frac{1}{R} + \frac{1}{2 \cdot R + R}} = \frac{11}{4} \cdot R$$

$$i_{S2} = \frac{v_2}{R_{IN2}} = \frac{4 \cdot v_2}{11 \cdot R}$$

$$v_{O2} = \left[\left(\frac{R}{2 \cdot R + R + R}\right) \cdot i_{S2}\right] \cdot 2 \cdot R = \left(\frac{R}{4 \cdot R}\right) \cdot \left(\frac{4 \cdot v_2}{11 \cdot R}\right) \cdot (2 \cdot R) = \frac{2 \cdot v_2}{11}$$

$$v_O = v_{O1} + v_{O2} = \frac{4}{11} \cdot v_1 + \frac{2}{11} \cdot v_2$$

3-31 With source i_2 turned off



$$i_{O1} = \left(\frac{R}{R + 4 \cdot R}\right) \cdot i_1 = \frac{i_1}{5}$$

$$v_{O1} = R \cdot i_{O1} = R \cdot \left(\frac{i_1}{5}\right)$$

$$v_{O2} = R \cdot i_{O2} = R \cdot \left(\frac{-3 \cdot i_2}{5}\right)$$

With source i_1 turned off $\quad i_{O2} = \left(\frac{3 \cdot R}{3 \cdot R + 2 \cdot R}\right) \cdot (-i_2) = \frac{-3 \cdot i_2}{5}$

Using superposition $\quad v_O = v_{O1} + v_{O2} = R \cdot \left(\frac{i_1}{5} - \frac{3 \cdot i_2}{5}\right)$

3-32 $p_1 = i_1^2 \cdot 100 = 0.25$ hence $i_1 := \sqrt{\frac{0.25}{100}}$ $\; i_1 = 0.05$ likewise $p_2 = i_2^2 \cdot 100 = 4$ hence $i_2 := \sqrt{\frac{4}{100}}$

$i_2 = 0.2$ Applying superposition $i := i_1 + i_2$ hence $p := \left(i^2\right) \cdot 100$ and finally $p = 6.25$ W.

Superposition applies to currents and voltages not power.

3-33 The output is of the form $v_O = K_1 \cdot v_S + K_2 \cdot i_S$ when $v_S = 10$ and $i_S = 0$ we have $2 = K_1 x10$ hence $K_1 = 1/5$. When $v_S = 10$ and $i_S = 0.01$ we have $1 = (1/5)x10 + K_2 x(0.01)$ hence $K_2 = -100$.

So the desired response is $v_O := \left(\frac{1}{5}\right) \cdot 5 + (-100) \cdot (-0.01)$ or $v_O = 2$ V.

3-34 The input-output relationship is of the form: $v_O = k_1 \cdot v_{S1} + k_2 \cdot v_{S2} + k_3 \cdot v_{S3}$
From the given data we have

Assume $k_1 := 1 \quad k_2 := 1 \quad k_3 := 1$ Given

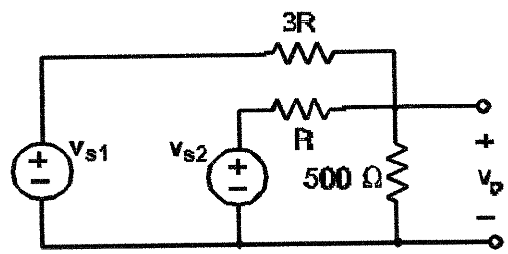
$0 = k_1 \cdot 0 + k_2 \cdot 4 + k_3 \cdot (-4) \qquad 1.5 = k_1 \cdot 2 + k_2 \cdot 0 + k_3 \cdot (2) \qquad 2 = k_1 \cdot 2 + k_2 \cdot 4 + k_3 \cdot 0$

$$\mathrm{Find}(k_1, k_2, k_3) = \begin{pmatrix} 0.5 \\ 0.25 \\ 0.25 \end{pmatrix}$$ <---Values of k's in the input-output relationship

3-35 Design requirements: (1) $v_O = K \cdot (v_{S1} + 3 \cdot v_{S2})$ (2) output across 500 Ω (3) $K > \frac{1}{20}$

The circuit below meet conditions (1) and (2).
Applying superposition with v_{S2} off

$$v_{O1} = \frac{\frac{500 \cdot R}{500+R}}{3 \cdot R + \frac{500 \cdot R}{500 + R}} \cdot v_{S1} = \frac{500}{2000 + 3 \cdot R} \cdot v_{S1}$$



Applying superposition with v_{S1} off

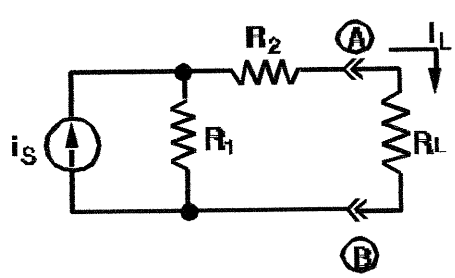
$$v_{O2} = \frac{\frac{500 \cdot 3 \cdot R}{500+3 \cdot R}}{R + \frac{500 \cdot 3 \cdot R}{500 + 3 \cdot R}} \cdot v_{S2} = \frac{1500}{2000 + 3 \cdot R} \cdot v_{S2}$$

$$v_O = \frac{500}{2000 + 3 \cdot R} \cdot (v_{S1} + 3 \cdot v_{S2}) = K \cdot (v_{S1} + 3 \cdot v_{S2})$$

$$K = \frac{500}{2000 + 3 \cdot R} > \frac{1}{20} \qquad \text{Hence} \qquad R < \frac{8000}{3}$$

3-36

(a) $v_{OC} = v_T = R_1 \cdot i_S \qquad i_{SC} = i_N = \frac{R_1}{R_1 + R_2} \cdot i_S$



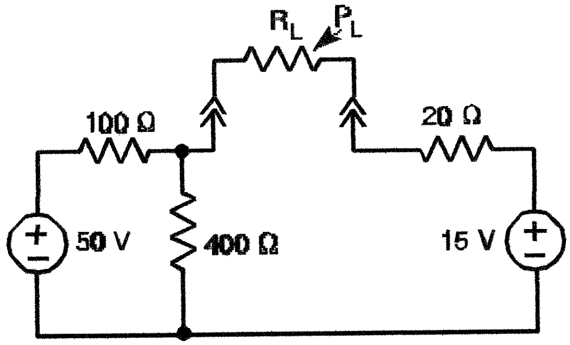
$$R_T = \frac{v_{OC}}{i_{SC}} = R_1 + R_2$$

(b)
$$i_L = \frac{v_T}{R_T + R_L} = \frac{R_1 \cdot i_S}{R_1 + R_2 + R_L}$$

(c) by current division

$$i_L = \frac{R_1 \cdot i_S}{R_1 + R_2 + R_L}$$

3-37



(a) The lookback resistance and open-ckt voltage are

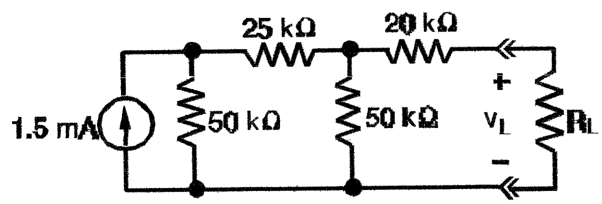
$$R_T := 20 + \frac{100 \cdot 400}{100 + 400} \qquad R_T = 100$$

$$v_T := \frac{400}{400 + 100} \cdot 50 - 15 \quad v_T = 25$$

(b) $p_L(R_L) := \left(\frac{v_T}{R_L + R_T}\right)^2 \cdot R_L$

$p_L(50) = 1.389 \qquad p_L(100) = 1.563 \qquad p_L(500) = 0.868$

3-38 Convert the 1.5-mA source to a 75-V source:



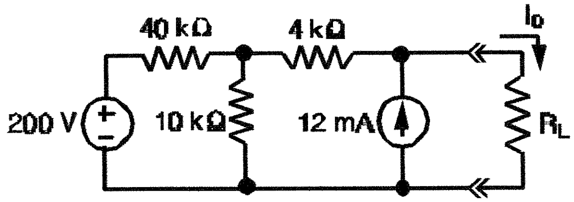
(a) $$v_{OC} = v_T = \frac{5 \cdot 10^4 \cdot 75}{5 \cdot 10^4 + 25 \cdot 10^3 + 5 \cdot 10^4} = 30$$

$$R_T = 20 \cdot 10^3 + \frac{50 \cdot 10^3 \cdot 75 \cdot 10^3}{50 \cdot 10^3 + 75 \cdot 10^3} = 50 \cdot 10^3$$

(b) $i_L(R_L) := \frac{30}{50 \cdot 10^3 + R_L} \qquad v_L(R_L) := i_L(R_L) \cdot R_L$ For R_L = 10, 25, 50, 100 kΩ load powers are

$v_L(10 \cdot 10^3) = 5 \qquad v_L(25 \cdot 10^3) = 10 \qquad v_L(50 \cdot 10^3) = 15 \qquad v_L(100 \cdot 10^3) = 20$

3-39 Convert the 200-V source to a 5-mA current source in parallel with a 40||10 = 8 kΩ resistor



(a) $i_N = i_{SC}$ $\quad i_{SC} := 0.012 + \dfrac{8\cdot 10^3}{12\cdot 10^3}\cdot 0.005$

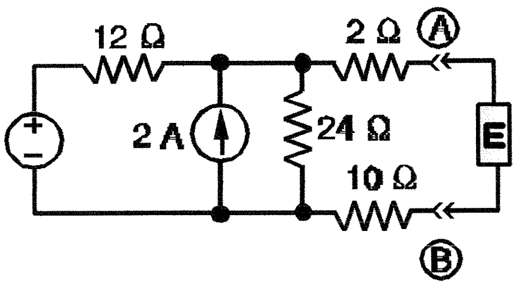
$R_T := 4\cdot 10^3 + 8\cdot 10^3 \quad R_T = 1.2\times 10^4$

$v_T := R_T\cdot i_{SC} \quad v_T = 184$

(b) For R_L = 6, 12, 24, 48 kΩ load currents are $\quad i_o(R_L) := \dfrac{v_T}{R_T + R_L}$

$i_o(6\cdot 10^3) = 1.022\times 10^{-2}$ $\quad i_o(12\cdot 10^3) = 7.667\times 10^{-3}$ $\quad i_o(24\cdot 10^3) = 5.111\times 10^{-3}$ $\quad i_o(48\cdot 10^3) = 3.067\times 10^{-3}$

3-40 Convert the 12-V source to a 1-A current source in parallel with an 12 Ω resistor



(a) $v_{OC} = v_T = (2+1)\cdot\dfrac{12\cdot 24}{12+24} = 24$

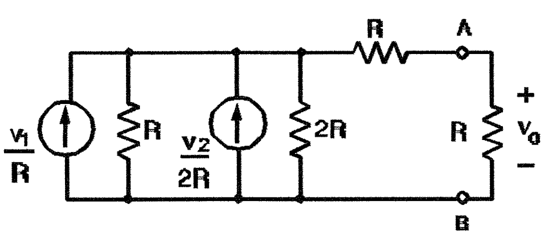
$R_T = 2 + \dfrac{12\cdot 24}{12+24} + 10 = 20$

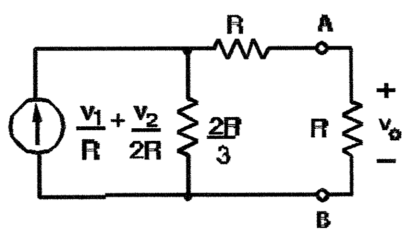
(b) For a 10-Ω load $\quad i_L := \dfrac{24}{20+10} \quad p_L := i_L^{\,2}\cdot 10 \quad p_L = 6.4$ W

(c) For a 5-V source load $\quad i_L := \dfrac{24-5}{20} \quad p_L := i_L\cdot 5 \quad p_L = 4.75$ W

3-41 **(a)** Convert the 2 voltage sources to current sources $i_1 = v_1/R$ and $i_2 = v_2/2R$.

Then combine the parallel elements as follows $i_1 + i_2$ and R||2R = 2R/3





Final Circuit

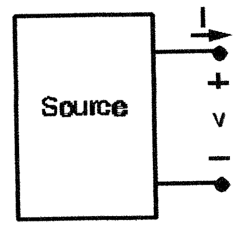
$v_T := (i_1 + i_2)\cdot 2\cdot\dfrac{R}{3}$

$v_T := \dfrac{2}{3}\cdot v_1 + \dfrac{1}{3}\cdot v_2$

$R_T := \dfrac{5}{3}\cdot R$

(b) By voltage division $v_O = \dfrac{R_L}{R_L + R_T}\cdot v_T = \dfrac{R}{R + \frac{5}{3}\cdot R}\cdot\left(\dfrac{2}{3}\cdot v_1 + \dfrac{1}{3}\cdot v_2\right) = \dfrac{1}{4}\cdot v_1 + \dfrac{1}{8}\cdot v_2$

3-42 (a) $v_T = 10 \left(\frac{2400}{R_T + 2400}\right) \cdot v_T = 6$ hence $R_T = 400 \cdot v_T - 2400 = 1600\ \Omega$



(b) $i_L(R_L) := \frac{10}{1600 + R_L}$ $\quad p_L(R_L) := \left(i_L(R_L)^2\right) \cdot R_L$

For R_L = 500, 1000, and 2000 Ω the load powers are:

$p_L(500) = 1.134 \times 10^{-2}$ $p_L(1000) = 1.479 \times 10^{-2}$ $p_L(2000) = 1.543 \times 10^{-2}$ W

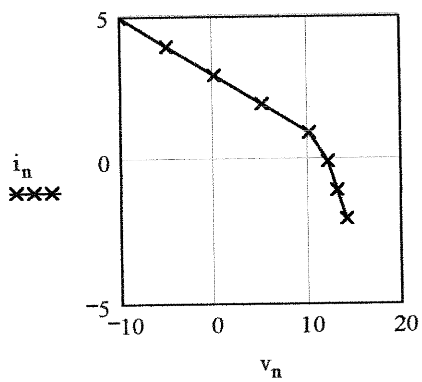
3-43 assume $v_T := 1$ $\quad R_T := 1$ $\quad$ Given $\frac{v_T}{300 + R_T} = 0.03$ $\quad \frac{v_T}{500 + R_T} = 0.02$

$\begin{pmatrix} v_T \\ R_T \end{pmatrix} := \text{Find}(v_T, R_T)$ $\quad \begin{pmatrix} v_T \\ R_T \end{pmatrix} = \begin{pmatrix} 12 \\ 100 \end{pmatrix}$ <---Thevenin Equivalent $i := \frac{v_T - 10}{R_T}$ $\quad i = 0.02$

3-44 (a) $v_1 := -10$ $\quad v_2 := -5$ $\quad v_3 := 0$ $\quad v_4 := 5$ $\quad v_5 := 10$ $\quad v_6 := 12$ $\quad v_7 := 13$ $\quad v_8 := 14$ <--- in V

$i_1 := 5$ $\quad i_2 := 4$ $\quad i_3 := 3$ $\quad i_4 := 2$ $\quad i_5 := 1$ $\quad i_6 := 0$ $\quad i_7 := -1$ $\quad i_8 := -2$ <--- in mA

$n := 1, 2 .. 8$



(b) On the range -10 < v < 10 the i-v characteristic is a straight line whose slope is $\left(\frac{i_5 - i_1}{v_5 - v_1}\right) \cdot 10^{-3} = -2 \times 10^{-4}$ & whose i-axis intercept is $i_3 = 3$ mA. The equation of this line is $i = -2 \cdot 10^{-4} \cdot v + 0.003$. The i-v characteristic of a Norton equivalent is: $i = \left(-\frac{1}{R_T}\right) \cdot v + i_N$ Hence on the range from -10 V to + 10 V parameters of the Norton equivalent are $i_N = 0.003$, $R_T = -\left(\frac{10^4}{-2}\right) = 5000$ and $v_T = i_N x R_T = 15$ V.

(c) $v_{OC} = v_T = 15$ V

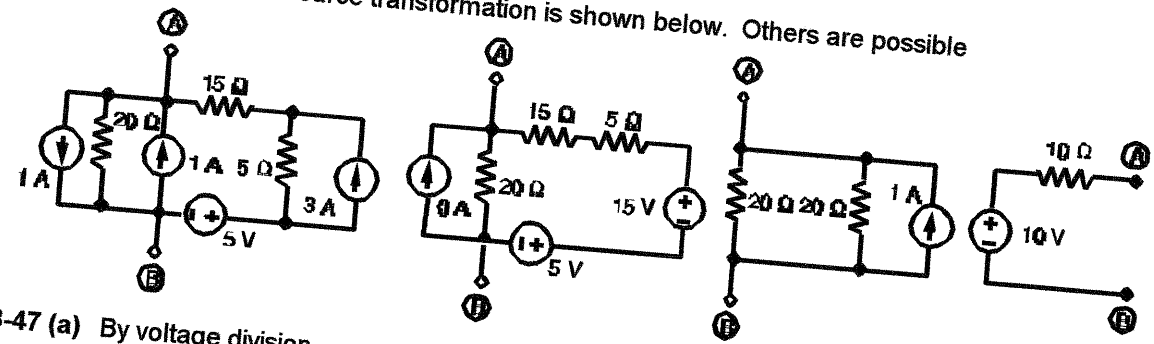
$i_{SC} = i_N = 0.003$ A

(d) The model says that v_{OC} = 15 V whereas the data (v_6) says v_{OC} = 12 V. The model only applies to the range -10 V < v < +10 V because the characteristic is nonlinear for v > 10 V

3-45 The constraint is $v_O = v_T - R_T \cdot i_L = 5 - 150 \cdot i_L = 5 - 150 \cdot \frac{5}{150 + R_L} > 3.5$

Hence $1.5 > \frac{750}{150 + R_L}$ and $R_L > \frac{750 - 1.5 \cdot 150}{1.5} = 350\ \Omega$

3-46 One sequence of source transformation is shown below. Others are possible



3-47 (a) By voltage division

$$v_{OC} = v_T = \frac{300\cdot 10^3}{300\cdot 10^3 + 100\cdot 10^3}\cdot 12 = 9$$

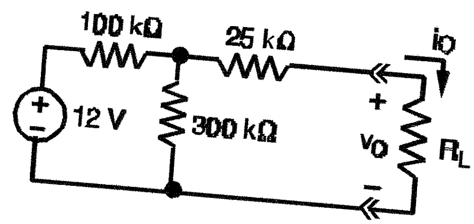
The lookback resistance is

$$R_T = 25\cdot 10^3 + \frac{300\cdot 10^3\cdot 100\cdot 10^3}{300\cdot 10^3 + 100\cdot 10^3} = 100\cdot 10^3$$

(b) The output voltage contraint is

$$v_O = \frac{R_L}{R_L + R_T}\cdot v_T = \frac{R_L}{R_L + 100\cdot 10^3}\cdot 9 = 6$$

which yields $R_L = 200\cdot 10^3\ \Omega$



3-48 See Prob. 3-47 above for circuit diagram

By voltage division

$$v_{OC} = v_T = \frac{300\cdot 10^3}{300\cdot 10^3 + 100\cdot 10^3}\cdot 12 = 9$$

The output current contraint is

$$i_O = \frac{1}{R_L + R_T}\cdot v_T = \frac{9}{R_L + 100\cdot 10^3} = 36\cdot 10^{-6}$$

The lookback resistance is $R_T = 25\cdot 10^3 + \dfrac{300\cdot 10^3\cdot 100\cdot 10^3}{300\cdot 10^3 + 100\cdot 10^3} = 100\cdot 10^3$

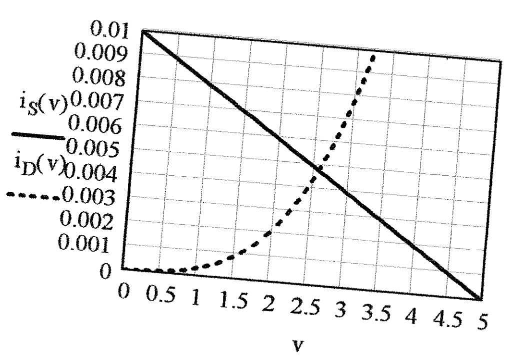
which yields $R_L = 150\cdot 10^3\ \Omega$

3-49 Source i-v characteristics: $v_T := 5 \quad R_T := 500 \quad i_S(v) := \dfrac{-1}{R_T}\cdot v + \dfrac{v_T}{R_T}$

Device i-v characteristics: $i_D(v) := 10^{-4}\cdot\left(v + 2\cdot v^{3.3}\right)$

using a Mathcad solve block: $v := 1$ Given $i_D(v) = i_S(v) \quad v := \mathrm{Find}(v) \quad i := i_S(v)$

Numerical solution: $v = 2.582 \quad i = 4.836\times 10^{-3}$



$v := 0, .1 .. 5$

<---Dashed curve is device i-v char.

<---Intersection is the graphical solution
Approx v = 2.6 V and i = 4.8 mA

<---Solid line is the source i-v char.

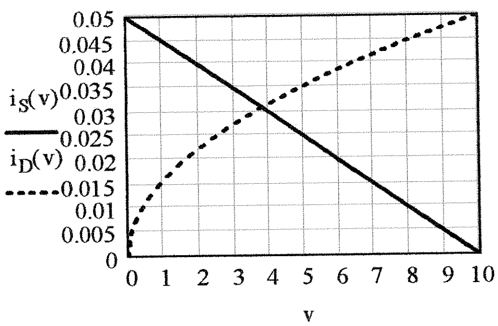
3-50 Source i-v characteristics: $v_T := 10 \quad R_T := 200 \quad i_S(v) := \frac{-1}{R_T} \cdot v + \frac{v_T}{R_T}$

Device i-v characteristics: $i_D(v) := \sqrt{2.5 \cdot 10^{-4} \cdot v}$

using a Mathcad solve block: $v := 1$ Given $i_D(v) = i_S(v) \quad v := \mathrm{Find}(v) \quad i := i_S(v)$

Numerical solution: $v = 3.82 \quad i = 3.09 \times 10^{-2}$

$v := 0, .1 .. 10$



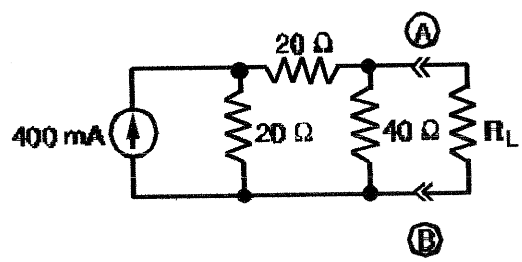
<---Dashed curve is device i-v char.

<---Intersection is the graphical solution
Approx v = 3.8 V and i = 31 mA

<---Solid line is the source i-v char.

3-51 $i_N := \frac{20 \cdot 0.4}{20 + 20} \quad R_N := \frac{40 \cdot (20 + 20)}{40 + 20 + 20}$

$i_N = 0.2 \quad R_N = 20$



(a) $R_L = R_N = 20\ \Omega$ yields max pwr

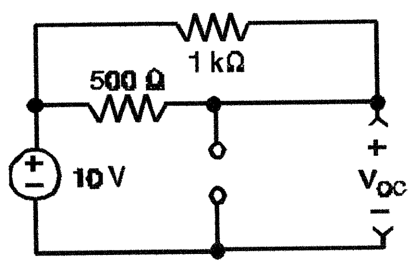
$p_{max} := 0.25 \cdot i_N^2 \cdot R_N \quad p_{max} = 2 \times 10^{-1}$

(b) $R_L = \infty\ \Omega$ yields max voltage $\quad v_{max} := i_N \cdot R_N \quad v_{max} = 4$ V

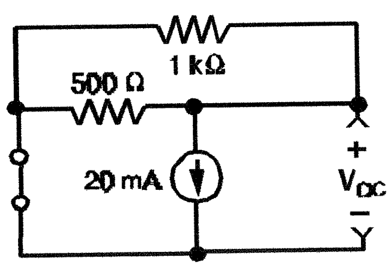
(c) $R_L = 0\ \Omega$ yields max current $\quad i_{max} := i_N \quad i_{max} = 0.2$ A

3-52 Use superposition

current source off



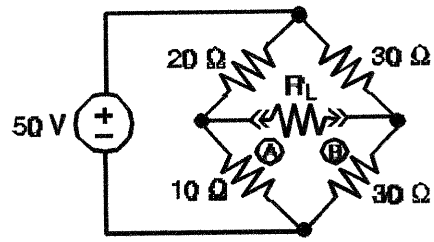
voltage source off



$i_{SC1} := \frac{10}{\left(\frac{1000 \cdot 500}{1000 + 500}\right)} \quad i_{SC1} = 0.03 \quad i_{SC2} := -0.02 \quad i_N := i_{SC1} + i_{SC2} \quad i_N = 0.01$

$R_T := \frac{1000 \cdot 500}{1000 + 500} \quad R_L := R_T \quad R_L = 333.333 \quad v_T := i_N \cdot R_T \quad p_{max} := \frac{v_T \cdot i_N}{4} \quad p_{max} = 0.2$

3-53 Using voltage division the open-ckt voltage is



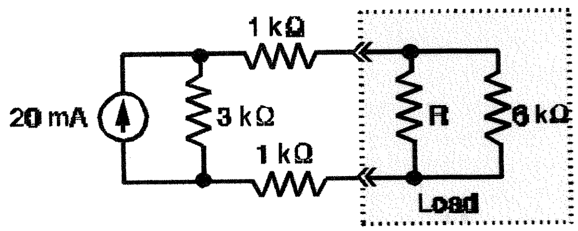
$$v_{AB} := \frac{10}{10+20}\cdot 50 - \frac{30}{30+30}\cdot 50 \qquad v_{AB} = -8.333$$

$$v_T := v_{AB} \qquad R_T := \frac{10\cdot 20}{10+20} + \frac{30\cdot 30}{30+30}$$

$$i_N := \frac{v_T}{R_T} \qquad p_{max} := 0.25\cdot i_N^{\,2}\cdot R_T \qquad p_{max} = 0.801$$

$$v_T = -8.333 \qquad R_T = 21.667 \qquad i_N = -3.846\times 10^{-1}$$

3-54



The open-circuit voltage is $v_T := 20\cdot 10^{-3}\cdot 3\cdot 10^{3}$

The look-back resistance is $R_T := (1+3+1)\cdot 10^{3}$

For max power R||6000 = R_T

$$\frac{1}{R} + \frac{1}{6000} = \frac{1}{5000} \text{ or } \qquad R := \frac{5000\cdot 6000}{1000}$$

$$v_L := \frac{v_T}{2} \qquad p_L := \frac{v_T^{\,2}}{4\cdot R_T} \qquad R = 3\times 10^{4} \quad \Omega$$

$$v_L = 30 \quad \text{V} \qquad p_L = 0.18 \quad \text{W}$$

3-55 There are two constraints

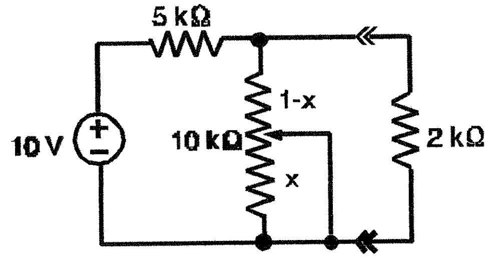
$$\text{Given} \qquad \frac{v_T}{R_T + 2\cdot 10^{3}} = 0.00075 \qquad \frac{v_T}{R_T + 10^{3}} = 0.001 \qquad \begin{pmatrix} v_T \\ R_T \end{pmatrix} := \text{Find}(v_T, R_T) \qquad \begin{pmatrix} v_T \\ R_T \end{pmatrix} = \begin{pmatrix} 3 \\ 2\times 10^{3} \end{pmatrix}$$

$$p_{max} := \frac{v_T^{\,2}}{4\cdot R_T} \qquad R_L := R_T \qquad p_{max} = 1.125\times 10^{-3} \qquad R_L = 2\times 10^{3} \quad \Omega$$

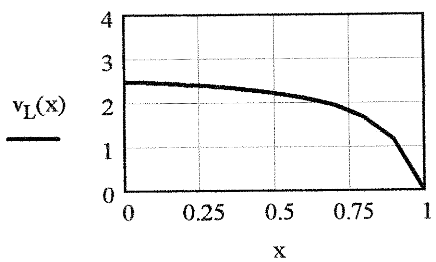
$$\text{checking} \qquad \frac{v_T}{R_T + 2\cdot 10^{3}} = 7.5\times 10^{-4} \qquad \frac{v_T}{R_T + 10^{3}} = 1\times 10^{-3}$$

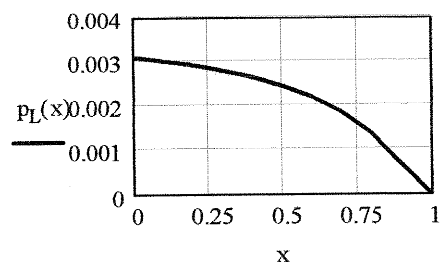
3-56 The voltage delivered to the 2 kΩ load resistor is

$$v_L(x) := \left[\frac{\dfrac{2000\cdot 10^4\cdot(1-x)}{2000+10^4\cdot(1-x)}}{5000+\dfrac{2000\cdot 10^4\cdot(1-x)}{2000+10^4\cdot(1-x)}}\right]\cdot 10$$



$x := 0, .1 .. 1$ $\quad v_L(x) := \dfrac{20\cdot(1-x)}{8-7\cdot x}$ and $p_L(x) := \dfrac{v_L(x)^2}{2000}$



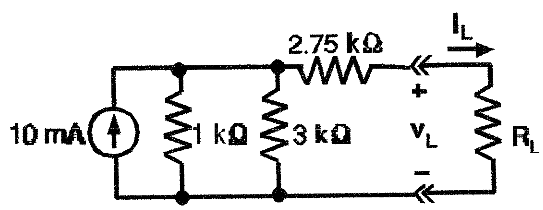


(a) Max power occurs at x = 0 $\quad p_L(0) = 3.125\times 10^{-3}$

(b) Maximum voltage occurs at x = 0 $\quad v_L(0) = 2.5$

3-57 By current division

$$i_{SC} := \frac{\dfrac{1}{2750}}{\dfrac{1}{2750}+\dfrac{1}{3000}+\dfrac{1}{1000}}\cdot 10^{-2}$$



$$i_{SC} = 2.143\times 10^{-3}$$

$$i_L = 3\cdot 10^{-3} > i_{SC} = 2.143\cdot 10^{-3}$$

The specified load current exceeds the short-circuit current. Impossible.

3-58 See Problem 3-57 above for the circuit By current division

$$i_{SC} := \frac{\dfrac{1}{2750}}{\dfrac{1}{2750}+\dfrac{1}{3000}+\dfrac{1}{1000}}\cdot 10^{-2} \quad i_{SC} = 2.143\times 10^{-3} \quad R_T := 2750 + \frac{1000\cdot 3000}{1000+3000} \quad R_T = 3.5\times 10^3$$

$$v_T := i_{SC}\cdot R_T \qquad v_T = 7.5$$

The specified load voltage is 5 V < v_T = 7.5 V. The required load resistance is

$$\frac{R_L}{R_L+3500}\cdot 7.5 = 5 \qquad R_L := \frac{3500\cdot 5}{7.5-5} \qquad R_L = 7\times 10^3$$

3-59 $v_S := 15$ $\quad \dfrac{v_S}{R_S + 100} \le 0.05$ $\quad$ ---> $\quad R_S \ge 20 \cdot v_S - 100 = 200$

$$\frac{v_S^{\,2}}{4 \cdot (R_S + 100)} \le 0.2 \quad \text{--->} \quad R_S \ge \frac{v_S^{\,2}}{0.8} - 100 = 181.25$$

Any $R_S > 200\ \Omega$ will meet both constraints

3-60 The contraints are $\quad \dfrac{v_T}{R_T + 300} = 0.05 \qquad \dfrac{120 \cdot v_T}{R_T + 120} = 12$

which yield the following two linear equations

$20 \cdot v_T - R_T + 300 = 0 \qquad 10 \cdot v_T - R_T - 120 = 0$

whose solution is

$v_T := \dfrac{300 - 120}{10} \qquad R_T := 10 \cdot v_T - 120 \qquad v_T = 18 \qquad R_T = 60$

$p_{max} := \dfrac{v_T^{\,2}}{4 \cdot R_T} \qquad p_{max} = 1.35$

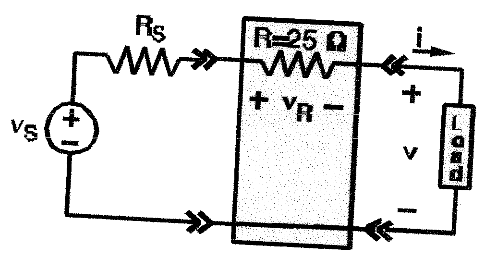
3-61 Many other designs are possible

$R_S := 50 \quad v_S := 10 \quad R_L := 50$

$v := 4 \quad i := \dfrac{v}{R_L} \quad v_R := v_S - v - i \cdot R_S$

$R := \dfrac{v_R}{i} \qquad R = 25 \quad \Omega \qquad v_R = 2$

$i = 0.08$ Source current less than 100 mA.



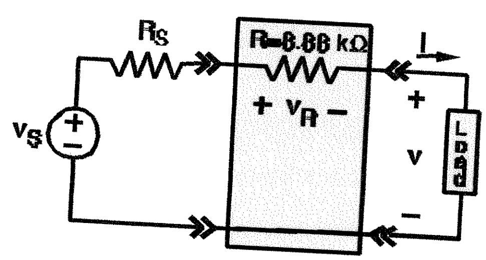
3-62 Many other designs are possible

$v_S := 15 \quad R_S := 1000 \quad v := 0.7$

$i := 10^{-15} \cdot (\exp(40 \cdot v) - 1) \qquad i = 1.446 \times 10^{-3}$

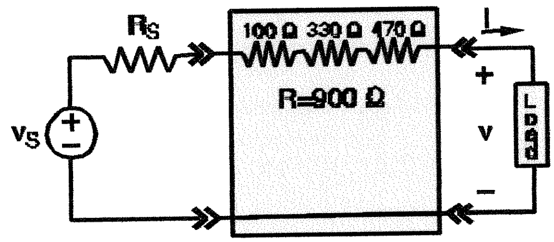
$v_R := v_S - v - i \cdot R_S \qquad v_R = 12.854 \quad R := \dfrac{v_R}{i}$

$R = 8888 \qquad i^2 \cdot R = 13.03 \cdot 10^{-3} < 50 \cdot 10^{-3}$



3-63 $v_S := 15$ $R_S := 100$ $R_L := 2000$ $v := 10 \pm 10\%$ $i := \frac{v}{R_L}$ $v_R := v_S - v - i \cdot R_S$

$R := \frac{v_R}{i}$ $R = 900$ <---Nominal value



Try 100, 330, and 470 Ω in series

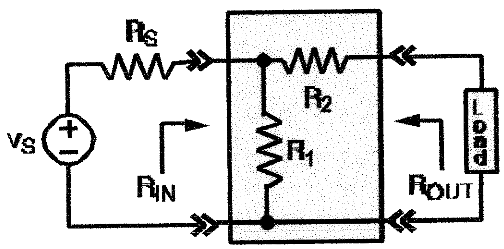
$R := 100 + 330 + 470$ $R = 900$ $\pm 5\%$

$R_{hi} := 1.05 \cdot R$ $R_{lo} := 0.95 \cdot R$

For $R = R_{hi}$ $v := \frac{R_L \cdot v_S}{R_S + R_L + R_{hi}}$ $v = 9.852$ <---Within ±10% tolerance

For $R = R_{lo}$ $v := \frac{R_L \cdot v_S}{R_S + R_L + R_{lo}}$ $v = 10.152$ <---Within ±10% tolerance

3-64 $R_S := 75$ $R_L := 500$



First guess let: $R_1 := 75$ $R_2 := 500$

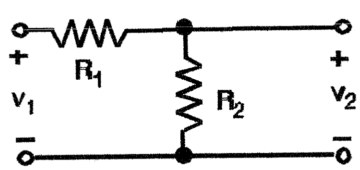
$R_{IN} := \frac{R_1 \cdot (R_2 + R_L)}{R_1 + R_2 + R_L}$ $R_{IN} = 69.767$

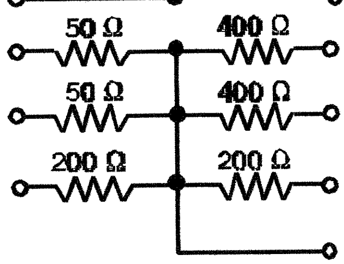
$R_{OUT} := R_2 + \frac{R_1 \cdot R_S}{R_1 + R_S}$ $R_{OUT} = 537.5$

Both answers are with ± 10%

Given $\frac{R_1 \cdot (R_2 + 500)}{R_1 + R_2 + 500} = 75$ $R_2 + \frac{R_1 \cdot 75}{R_1 + 75} = 500$ $\text{Find}(R_1, R_2) = \begin{pmatrix} 81.349 \\ 460.977 \end{pmatrix}$ <---exact solutions

3-65 Use a voltage divider





Design requirements are

$\frac{R_2}{R_1 + R_2} > 0.6$ $R_1 + R_2$ about 100 Ω

One solution is: R_1 = 50||50||400, or $R_1 := 23.53\Omega$

and R_2 = 200||200||400, or $R_2 := 80\Omega$

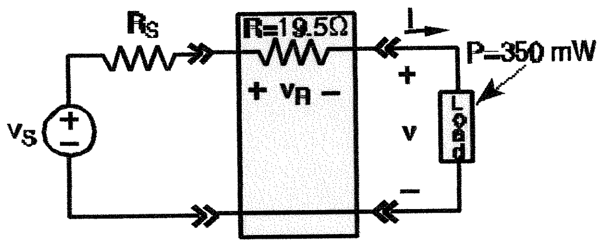
$\frac{R_2}{R_1 + R_2} = 0.773$ <---gain > 0.6

$R_1 + R_2 = 103.53$ <---about 100 Ω

Other solutions may be possible.

3-66 Design requirements are: $i > 40$ mA and $v < 2$ V. If $i > 40$ mA then $v > 50i > 2$ V
The design requirements are not consistent with a 50 Ω load. ***There is no solution to this design problem as stated.***

3-67 Many other designs are possible



$v_S := 10 \quad R_S := 50 \quad R_L := 50$

$p_{avail} := \dfrac{v_S^{\,2}}{4 \cdot R_S} \qquad p_{avail} = 0.5$

$p := 0.35 \quad i := \sqrt{p \cdot R_L^{\,-1}} \qquad i = 0.084$

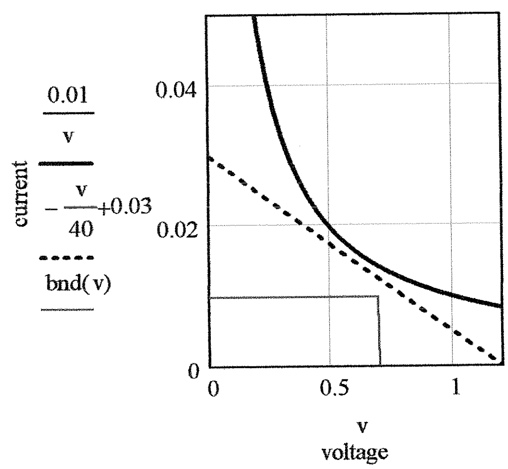
$v_R := v_S - i \cdot R_L - i \cdot R_S \qquad v_R = 1.633$

$R := \dfrac{v_R}{i} \qquad R = 19.523$

3-68

$v := 0.001, 0.005 .. 1.2$

$\mathrm{bnd}(v) := \begin{vmatrix} 0.01 & \text{if } v < 0.7 \\ 0 & \text{if } v \geq 0.7 \end{vmatrix}$ <---voltage current bound



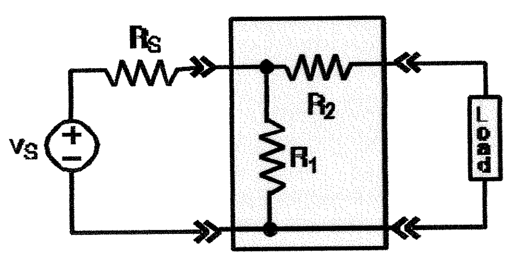
The load line $i = -\dfrac{v}{40} + 0.03$ lies entirely in the allowed region, hence the operating point will fall in the allowed ranges. The interface circuit design parameters are: $i_{SC} = 30$ mA, $v_{OC} = 1.2$ V, & $R_T = 40$ Ω
There are many circuits that meet these requirements.
For example, the L-ckt at left requires
Assume $R_1 := 1 \quad R_2 := 40$

Given $\left(\dfrac{R_1}{5 + R_1}\right) \cdot 5 = 1.2$ and

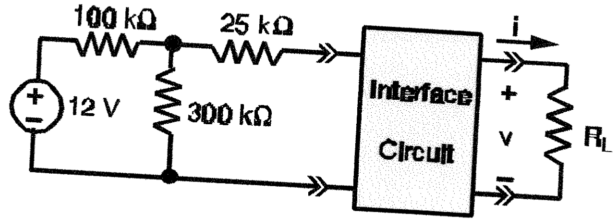
$R_2 + \dfrac{5 \cdot R_1}{5 + R_1} = 40$

$\begin{pmatrix} R_1 \\ R_2 \end{pmatrix} := \mathrm{Find}(R_1, R_2)$

$\begin{pmatrix} R_1 \\ R_2 \end{pmatrix} = \begin{pmatrix} 1.579 \\ 38.8 \end{pmatrix}$ <---One design



3-69 The Thevenin equivalent of the source circuit is



$$v_{OC} := \frac{300 \cdot 10^3}{300 \cdot 10^3 + 100 \cdot 10^3} \cdot 12$$

$$v_T := v_{OC} \qquad v_T = 9$$

$$R_T := 25 \cdot 10^3 + \frac{100 \cdot 10^3 \cdot 300 \cdot 10^3}{100 \cdot 10^3 + 300 \cdot 10^3}$$

$$R_T = 1 \times 10^5$$

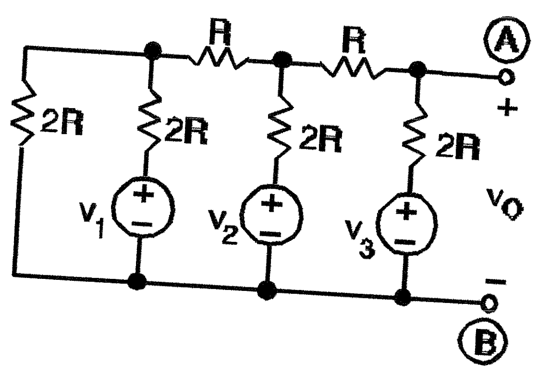
Design constraint: $\frac{R_P}{R_P + R_T} \cdot v_T < 4$ which yields $R_P < \frac{4 \cdot R_T}{v_T - 4} = 80 \cdot 10^3$ Any R_P less than 80 kΩ will work

3-70 The Thevenin equivalent of the source circuit is given in Problem 3-69 above. The maximum power available from the source is

$$p_{max} := \frac{v_T^{\,2}}{4 \cdot R_T} \qquad p_{max} = 2.025 \times 10^{-4}$$

The maximum power available is less than 1 mW. Hence no interface circuit is required.

3-71 (a) Writing node-voltage equations:



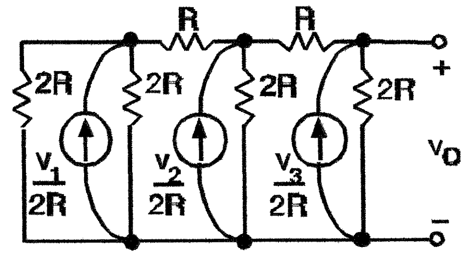
$$\begin{pmatrix} \frac{2}{R} & -\frac{1}{R} & 0 \\ -\frac{1}{R} & \frac{5}{2 \cdot R} & -\frac{1}{R} \\ 0 & -\frac{1}{R} & \frac{3}{2 \cdot R} \end{pmatrix} \cdot \begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} \frac{v_1}{2 \cdot R} \\ \frac{v_2}{2 \cdot R} \\ \frac{v_3}{2 \cdot R} \end{pmatrix}$$

Solving for the node-voltage vector

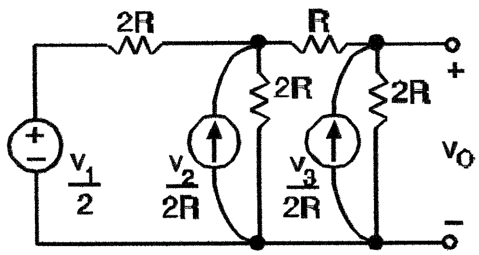
$$\begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} \frac{2}{R} & -\frac{1}{R} & 0 \\ -\frac{1}{R} & \frac{5}{2 \cdot R} & -\frac{1}{R} \\ 0 & -\frac{1}{R} & \frac{3}{2 \cdot R} \end{pmatrix}^{-1} \cdot \begin{pmatrix} \frac{v_1}{2 \cdot R} \\ \frac{v_2}{2 \cdot R} \\ \frac{v_3}{2 \cdot R} \end{pmatrix} = \begin{pmatrix} \frac{11}{16} \cdot R & \frac{3}{8} \cdot R & \frac{1}{4} \cdot R \\ \frac{3}{8} \cdot R & \frac{3}{4} \cdot R & \frac{1}{2} \cdot R \\ \frac{1}{4} \cdot R & \frac{1}{2} \cdot R & R \end{pmatrix} \cdot \begin{pmatrix} \frac{v_1}{2 \cdot R} \\ \frac{v_2}{2 \cdot R} \\ \frac{v_3}{2 \cdot R} \end{pmatrix} = \begin{pmatrix} \frac{11}{32} \cdot v_1 + \frac{3}{16} \cdot v_2 + \frac{1}{8} \cdot v_3 \\ \frac{3}{16} \cdot v_1 + \frac{3}{8} \cdot v_2 + \frac{1}{4} \cdot v_3 \\ \frac{1}{8} \cdot v_1 + \frac{1}{4} \cdot v_2 + \frac{1}{2} \cdot v_3 \end{pmatrix}$$ <---output

Hence $v_O = v_C = \frac{1}{8} \cdot v_1 + \frac{1}{4} \cdot v_2 + \frac{1}{2} \cdot v_3$

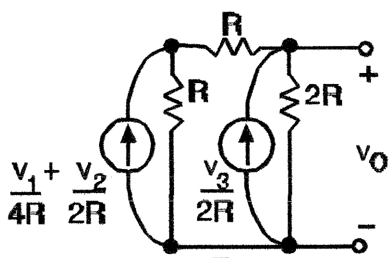
3-71 (b) Using circuit reduction methods



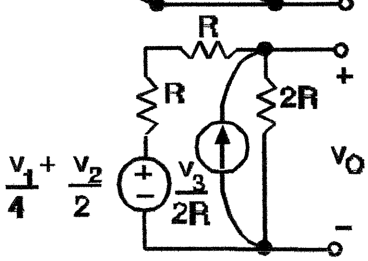
Step 1: Convert the voltage sources in series with 2R to equivalent current sources in parallel



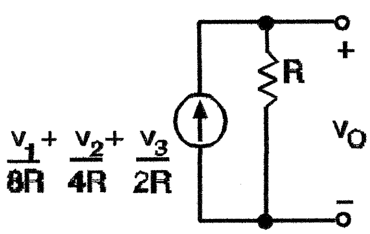
Step 2: Replace left-most 2R||2R by a single resistor R and then convert the left-most current source to an equivalent voltage sources $v_1/2$ in series with R + R = 2R.



Step 3: Convert the left-most voltage source to an equivalent current sources in parallel with 2R and then combine the two left-most current sources that are connected in parallel to an equivalent current source $\frac{v_1}{4 \cdot R} + \frac{v_2}{2 \cdot R}$. connected in parallel with 2R||2R = R



Step 4: Convert the left-most current source to an equivalent voltage sources $\frac{v_1}{4} + \frac{v_2}{2}$ in parallel with R.

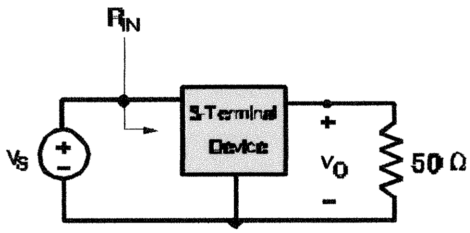


Step 5: Convert the left most voltage source in series with 2R to an equivalent current sources $\frac{v_1}{8 \cdot R} + \frac{v_2}{4 \cdot R}$ in parallel with 2R and then combine with the right most current source to produce an equivalent current source $\frac{v_1}{8 \cdot R} + \frac{v_2}{4 \cdot R} + \frac{v_3}{2 \cdot R}$. in parallel with R.

Hence $v_O = \left(\frac{v_1}{8 \cdot R} + \frac{v_2}{4 \cdot R} + \frac{v_1}{2 \cdot R} \right) \cdot R = \frac{1}{8} \cdot v_1 + \frac{1}{4} \cdot v_2 + \frac{1}{2} \cdot v_3$

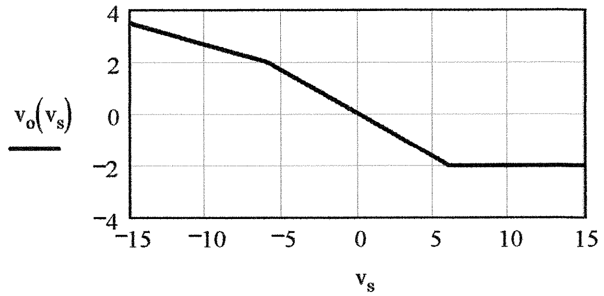
(c) Either method is acceptable. Method (a) is analytical and abstract. Method (b) is more intuitive since it works directly with the circuit model.

3-72(a)



$$v_o(v_s) := \begin{cases} 1 - \dfrac{v_s}{6} & \text{if } v_s < -6 \\ -\dfrac{v_s}{3} & \text{if } -6 \le v_s \le 6 \\ -2 & \text{if } 6 < v_s \end{cases}$$

$v_s := -15, -14.9 .. 15$



Nonlinear characteristic.
Linear on the range $-6 < v_S < 6$ V.

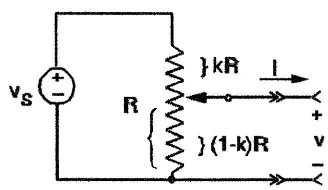
(b) In the linear range the slope is $K = -1/3$

(c) $v_S = -10$ V model predicts $v_O = 10/3 = 3.333$ V whereas the data states $v_O = 2.666$ V.
$v_S = +10$ V model predicts $v_O = -10/3 = -3.333$ V whereas the data states $v_O = -2$ V.
The 10-V inputs are outside the range for which the linear model applies.

(d) For $v_S = 1$ V input power is $p_{IN} = (v_S)^2/1000 = 1/1000$ W
For $v_S = 1$ V, $v_O = -1/3$ hence $p_O = (v_O)^2/50 = 1/450$ W
power gain $= p_O/p_{IN} = 1000/450 > 1$

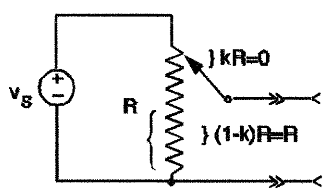
(e) Power gain greater than one--device is active.

3-73

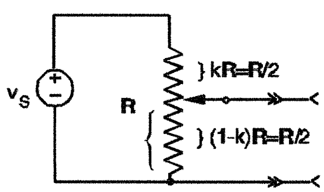


(a) $v_T = \dfrac{(1-k)\cdot R}{R}\cdot v_S = (1-k)\cdot v_S$

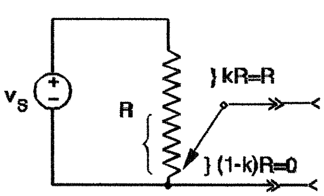
$i_N = \dfrac{v_S}{k\cdot R}$ $\qquad R_T = \dfrac{v_T}{i_N} = k\cdot(1-k)\cdot R$



(b) Wiper at the top of the pot
$k = 0$ ---> $R_T = 0$
output across an ideal v-source v_S



Wiper at the middle of the pot
$k = 0.5$---> $R_T = 0.25\cdot R$
output sees $R_T = R/2||R/2$ and $v_T = v_S/2$.



Wiper at the bottom of the pot
$k = 1$ ---> $R_T = 0$
output across short circuit, $v_T = 0$

3-73 Continued

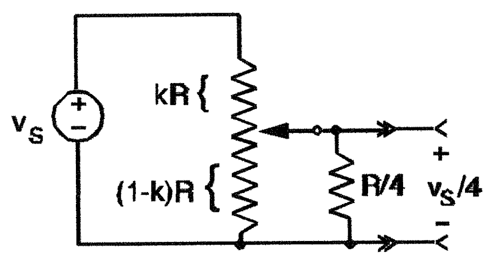
(c) $k = 0$ $\quad p_{max} = \infty$ wiper at the top of the pot so v_T is an ideal voltage source which can deliver unlimited power.

$k = 0.5$ $\quad p_{max} = \dfrac{v_T^{\,2}}{4 \cdot R_T} = \dfrac{v_S^{\,2}}{4 \cdot R}$ wiper in the middle of the pot to $R_T > 0$ and the available power is limited by max power transfer Thm.

$k = 1$ $\quad p_{max} = 0$ wiper at the bottom of the pot. output sees a short circuit hence zero available power.

(d) Maximum power available when k = 0 at the top of the pot

(e)



Applying voltage division yields two solutions for k

$$\left[\frac{\dfrac{\frac{R}{4}\cdot(1-k)\cdot R}{\left[\frac{R}{4}+(1-k)\cdot R\right]}}{\dfrac{\frac{R}{4}\cdot(1-k)\cdot R}{\left[\frac{R}{4}+(1-k)\cdot R\right]} + k \cdot R}\right] \cdot v_S = \frac{v_S}{4} \qquad k = \begin{pmatrix} \frac{3}{2} \\ \frac{1}{2} \end{pmatrix}$$

$k = \dfrac{1}{2}$ is the only physically acceptable solution

3-74 Design requirements $v_T = 36$ V and $R_T < 10\ \Omega$.

(1) First type requires 4 in series to get $V_T = 4x9 = 36$ and two series strings in parallel to get $R_T = 4x4/2 = 8 < 10\ \Omega$. Total wt = 4x2x40 = 320 grams.

(2) Second types requres 9 in series to get $V_T = 9x4 = 36$ V and one series string has $R_T = 9x0.5 = 4.5 < 10\ \Omega$. Total wt = 9x15 = 135 grams.

The second type has the minimum wt.

3-75 $R_1 := 1700 \quad R_2 := 1400 \quad R_3 := 1000 \quad v_2 := -5.2$

To find the Thevenin equi. seen by the load we write a node-voltage eq. at the interface.

$$\frac{v - v_1}{R_1} + \frac{v - v_2}{R_2} + \frac{v}{R_3} + i = 0$$

Solving for v in terms of v_1 & i

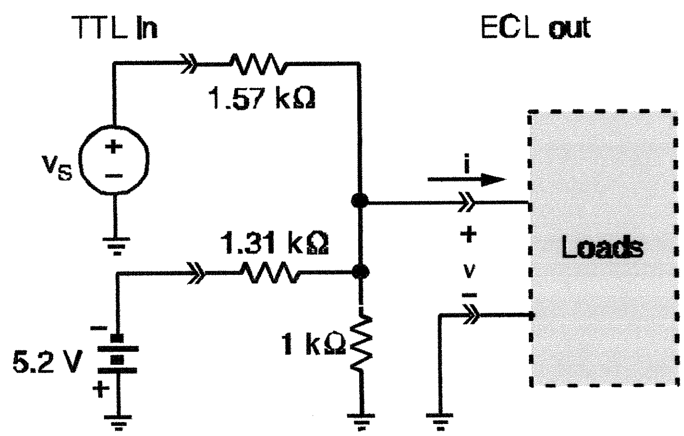
$$v(v_1, i) := \frac{v_1 \cdot R_2 \cdot R_3 + v_2 \cdot R_1 \cdot R_3 - i \cdot R_1 \cdot R_2 \cdot R_3}{R_2 \cdot R_3 + R_1 \cdot R_3 + R_1 \cdot R_2}$$

Substituting numerical values

$R_1 := 1570 \quad R_2 := 1310 \quad R_3 := 1000 \quad v_2 := 5.2$

$$v(v_1, i) := 0.25547 \cdot v_1 - 1.6131 - 434.31 \cdot i$$

This is the output voltage in terms of the TTL input v_1 and the load current i.



3-75 Continued The output voltage is $v(v_1, i) := 0.25547v_1 - 1.6131 - 434.31i$

Worst case occurs when the TTL input v_1 and the load current i are at their extreme values:

For TTL input low (v_1 = 0 to 0.4)

$v(0.0, 25\cdot 10^{-6}) = -1.624$

$v(0.4, 25\cdot 10^{-6}) = -1.522$

$v(0.0, -25\cdot 10^{-6}) = -1.602$

$v(0.4, -25\cdot 10^{-6}) = -1.5$

<---These values all fall within the ECL low range: -1.5 to -1.7

For TTL input high (v_1 = 3.0 to 3.8)

$v(3.0, 25\cdot 10^{-6}) = -0.858$

$v(3.8, 25\cdot 10^{-6}) = -0.653$

$v(3.0, -25\cdot 10^{-6}) = -0.836$

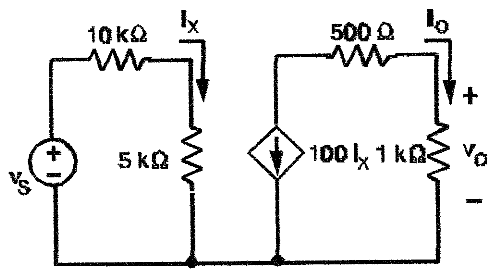
$v(3.8, -25\cdot 10^{-6}) = -0.631$

<---These values all fall within the ECL high range: -0.6 to -0.9

Ckt converts TTL inputs into ECL range output for all load currents from -0.025 to +0.025 mA.

CHAPTER 4, Both Versions

4-1



(a) $i_x = \dfrac{v_s}{5000 + 10000} = \dfrac{v_s}{15000}$

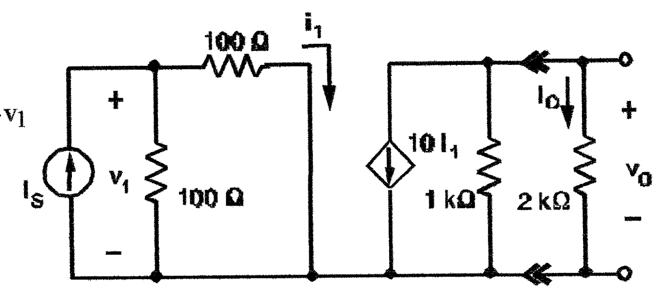
$$v_O = 1000\cdot(-100\cdot i_x) = \left(-10^5\right)\cdot\left(\frac{v_s}{15000}\right)$$

$$v_O = -\frac{20}{3}\cdot v_s \qquad K_V = \frac{v_O}{v_s} = -\frac{20}{3}$$

(b) $i_O = -100\cdot i_x \qquad K_I = \dfrac{i_O}{i_x} = -100$

4-2

(a) $v_1 = 50\cdot i_s \qquad i_1 = \dfrac{1}{2}\cdot i_s \qquad v_O = 2000\cdot i_O$



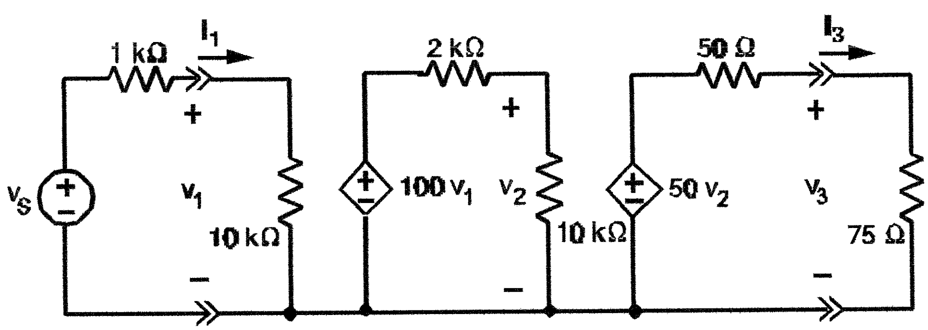
$$i_O = \left[\frac{1000\cdot(-10\cdot i_1)}{3000}\right] = \frac{-10}{3}\cdot\frac{1}{2}\cdot i_s = \frac{-10}{6}\cdot\frac{v_1}{50} = \frac{-1}{30}\cdot v_1$$

$$v_O = 2000\cdot\left(\frac{-1}{30}\cdot v_1\right) \qquad K_V = \frac{v_O}{v_1} = \frac{-200}{3}$$

(b) $K_I = \dfrac{i_O}{i_s} = -\dfrac{10}{6} = -\dfrac{5}{3}$ **(c) For** $i_s := 0.002 \qquad v_1 := 50\cdot i_s \qquad p_{IN} := v_1\cdot i_s \qquad p_{IN} = 2\times 10^{-4}$

$$i_O := -\frac{5}{3}\cdot i_s \qquad p_{OUT} := i_O^{\,2}\cdot 2000 \qquad p_{OUT} = 2.222\times 10^{-2}$$

4-3



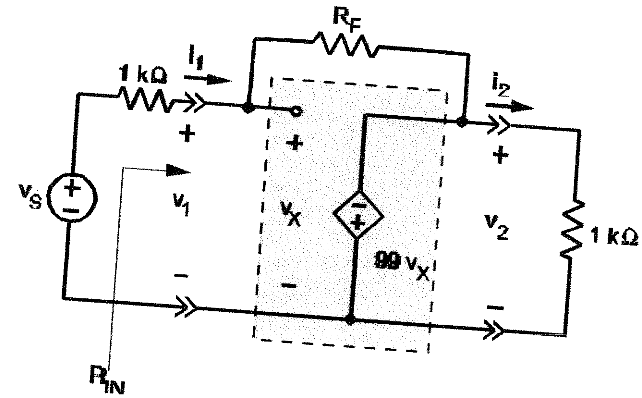
$$v_3 = \frac{75}{125}\cdot 50\cdot v_2 = 30\cdot\left(\frac{10000}{12000}\cdot 100\cdot v_1\right) = (30)\cdot\left(\frac{250}{3}\right)\cdot\left(\frac{10000}{11000}\cdot v_s\right) = \frac{25000}{11}\cdot v_s$$ **For** $v_s := 0.001$

$$v_3 := \frac{25000}{11}\cdot v_s \quad v_3 = 2.273 \qquad i_3 := \frac{v_3}{75} \qquad i_1 := \frac{v_s}{11000} \qquad K_I := \frac{i_3}{i_1} \qquad K_I = 3.333\times 10^5$$

4-4 $R_F := 10 \cdot 10^3$ $v_s := 0.01$ **(a) Writing a KCL equation at Node A yields:**

$$\frac{v_x - v_s}{1000} + \frac{v_x - (-99 \cdot v_x)}{R_F} = 0$$

solving for v_x yields $v_x := \frac{R_F \cdot v_s}{R_F + 100000}$



$v_2 := -99 \cdot v_x$ $v_2 = -9 \times 10^{-2}$ $i_2 := \frac{v_2}{1000}$

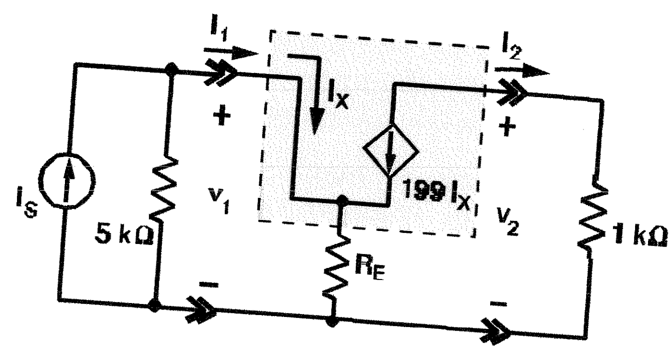
$i_1 := \frac{v_s - v_x}{1000}$ $K_I := \frac{i_2}{i_1}$ $K_I = -9.9$

(b) $R_{IN} := \frac{v_x}{i_1}$ $R_{IN} = 100$

4-5 $i_s := 25 \cdot 10^{-6}$ $R_E := 450$ **(a) Writing KVL equation around mesh x yields:**

$$5000 \cdot (i_x - i_s) + R_E \cdot (i_x + 199 \cdot i_x) = 0$$

Solving for i_x yields: $i_x := \frac{25 \cdot i_s}{25 + R_E}$



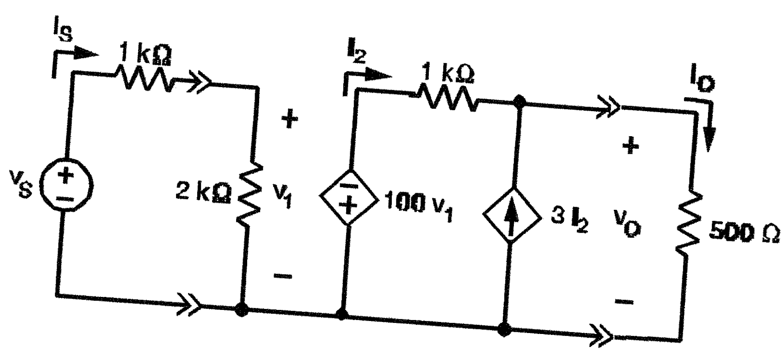
$i_2 := -199 \cdot i_x$ $i_2 = -2.618 \times 10^{-4}$

$v_2 := 1000 \cdot i_2$ $v_1 := 5000 \cdot (i_s - i_x)$

$K_V := \frac{v_2}{v_1}$ $K_V = -2.211$

(b) $R_{IN} := \frac{v_1}{i_x}$ $R_{IN} = 9 \times 10^4$

4-6 By voltage division $v_1 = \frac{2000}{1000 + 2000} \cdot v_S = \frac{2 \cdot v_S}{3}$ **(a) Writng a KCL equation at Node A yields:**



$i_2 + 3i_2 - i_O = 0$ $i_O = 4 \cdot i_2$

$$i_2 = \frac{(-100 \cdot v_1) - v_O}{1000} = \frac{-v_S}{15} - \frac{v_O}{1000}$$

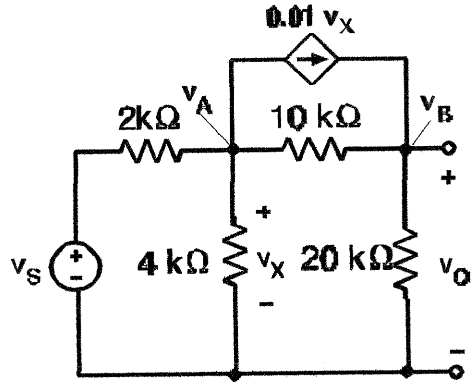
$$i_2 = \frac{i_O}{4} = \frac{v_O}{4 \cdot 500} = \frac{v_O}{2000}$$ hence,

$$\frac{-v_S}{15} - \frac{v_O}{1000} = \frac{v_O}{2200}$$ Solve for v_O:

$v_O = \frac{-400}{9} \cdot v_S$ $K_V := \frac{-400}{9}$

(b) $i_S = \frac{v_S}{3000}$ $i_O = \frac{v_O}{500} = \frac{1}{500} \cdot \left(\frac{-400}{9} \cdot v_S\right)$ $K_I := \frac{-400 \cdot 3000}{500 \cdot 9}$ $K_V = -44.444$ $K_I = -266.667$

4-7



Writing two node equations with $v_X = v_A$:

Node A:

$$\left(\frac{1}{2000}+\frac{1}{4000}+\frac{1}{10000}+10^{-2}\right)\cdot v_A-\frac{1}{10000}\cdot v_B=\frac{v_S}{2000}$$

Node B:

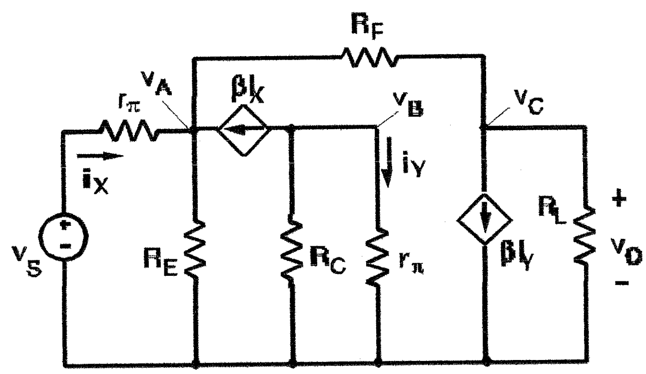
$$-\left(\frac{1}{10000}+10^{-2}\right)\cdot v_A+\left(\frac{1}{10000}+\frac{1}{20000}\right)\cdot v_B=0$$

Solving the Node B eq. for v_A: $v_A=\frac{3}{202}\cdot v_B$

Substituting into the Node A eq & solving for v_B:

$$v_B=\frac{2020}{247}\cdot v_s \quad K_V:=\frac{2020}{247} \quad K_V=8.178$$

4-8 The sum of currents leaving the nodes



Node A

$$\frac{v_A}{R_E}+\frac{v_A-v_S}{r_\pi}+\frac{v_A-v_C}{R_F}-\beta\cdot\left(\frac{v_S-v_A}{r_\pi}\right)=0$$

Node B $\left(\frac{1}{R_C}+\frac{1}{r_\pi}\right)\cdot v_B+\beta\cdot\left(\frac{v_S-v_A}{r_\pi}\right)=0$

Node C $\frac{v_C-v_A}{R_F}+\beta\cdot\left(\frac{v_B}{r_\pi}\right)+\frac{v_C}{R_L}=0$

The given numerical values are

$r_\pi := 1000 \quad R_E := 200 \quad R_C := 10000 \quad R_L := 5000 \quad R_F := 5000 \quad \beta := 100$

Subsituting these values into the node equations and using a solve block to find v_C with $v_S = 1$.

$v_A := 1 \quad v_B := 1 \quad v_C := 1 \quad v_S := 1$ <---initial guesses

Given $v_S = 1$ <---Start of solve block

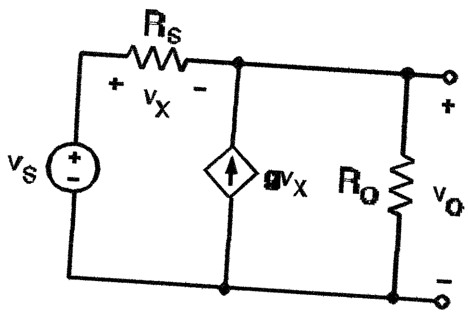
Node A: $\frac{v_A}{200}+\left(\frac{v_A-v_S}{1000}\right)+\frac{v_A-v_C}{5000}-100\cdot\left(\frac{v_S-v_A}{1000}\right)=0$

Node B: $\left(\frac{1}{10000}+\frac{1}{1000}\right)\cdot v_B+100\cdot\left(\frac{v_S-v_A}{1000}\right)=0$ Node C: $\frac{v_C-v_A}{5000}+100\cdot\left(\frac{v_B}{1000}\right)+\frac{v_C}{5000}=0$

$$\text{Find}(v_A,v_B,v_C)=\begin{pmatrix}0.999\\-0.1\\25.418\end{pmatrix}$$

<---Node voltages with $v_S = 1$, hence using proportionality the input-output relationship is $v_O = v_C = (25.418)v_s$

4-9



Applying KCL at Node A:

$$\frac{v_O - v_S}{R_S} + \frac{v_O}{R_O} - g \cdot v_X = 0$$

Applying KVL: $v_X = v_S - v_O$

Substituting into the KCL equation

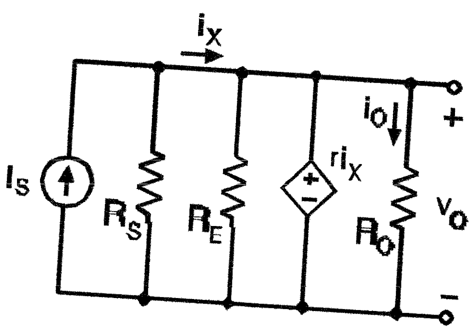
$$\frac{v_O - v_S}{R_S} + \frac{v_O}{R_O} - g \cdot (v_S - v_O) = 0$$

Solving for v_O

$$v_O = \frac{(1 + g \cdot R_S) \cdot R_O}{R_S + R_O + g \cdot R_S \cdot R_O} \cdot v_S$$

So finally $K_v = \frac{v_O}{v_S} = \frac{(g \cdot R_S + 1) \cdot R_O}{(g \cdot R_S + 1) \cdot R_O + R_S}$

4-10



Applying KCL at the input

$$i_S - \frac{v_O}{R_S} - i_X = 0$$

By KVL and Ohm's law

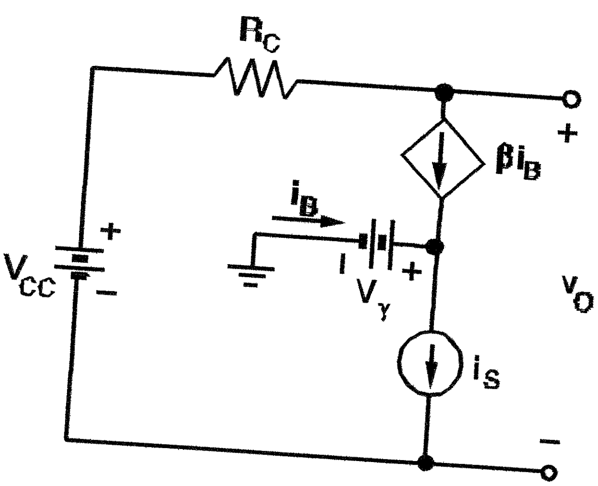
$v_O = r \cdot i_X \quad v_O = R_O \cdot i_O$ hence $i_X = \frac{R_O}{r} \cdot I_O$

and the KCL equation becomes

$$i_S - \frac{R_O \cdot i_O}{R_S} - \frac{R_O}{r} \cdot i_O = 0$$ solving for i_O

$$i_O = \left[\frac{R_S \cdot r}{[R_O \cdot (r + R_S)]} \right] \cdot i_S \quad K_I = \frac{R_S \cdot r}{[R_O \cdot (r + R_S)]}$$

4-11



By KCL

$i_B + \beta \cdot i_B = i_S$ or $i_B = \frac{i_S}{\beta + 1}$

Applying KVL around the perimeter of the ckt

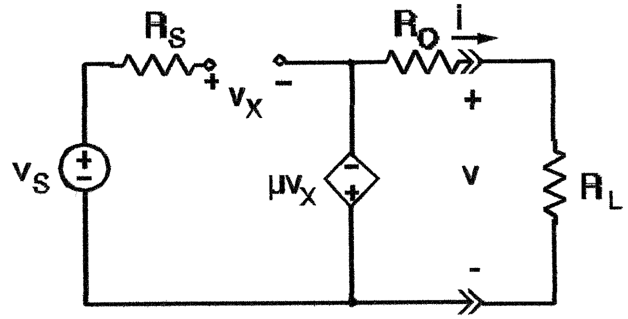
$-V_{CC} + R_C \cdot \beta \cdot i_B + v_O = 0$ using the KCL result

$$-V_{CC} + R_C \cdot \frac{\beta \cdot i_S}{\beta + 1} + v_O = 0$$ Solving for v_O

$$v_O = V_{CC} - \frac{\beta \cdot R_C}{\beta + 1} \cdot i_S$$

4-12 The input loop contains an open circuit, hence $i_S = 0$. A KVL equation around the input loop is

$$-v_S + v_X - \mu \cdot v_X = 0$$



Solving for v_x yields $v_X = \dfrac{v_S}{1-\mu}$

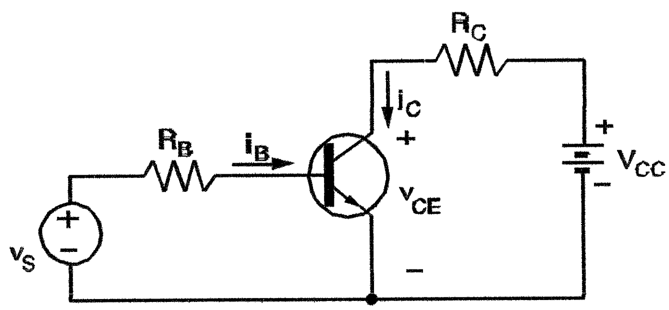
The open-ckt voltage is

$$v_T = -\mu \cdot v_X = \frac{-\mu}{1-\mu} \cdot v_S$$

The short-ckt current is $i_N = \dfrac{-\mu \cdot v_X}{R_O}$

The Thevenin resistance is $R_T = \dfrac{v_T}{i_N} = R_O$

4-13 $R_B := 50 \cdot 10^3$ $\quad R_C := 2000$ $\quad \beta := 100$ $\quad V_\gamma := 0.7$ $\quad V_{CC} := 10$



$i_{SC} := \dfrac{V_{CC}}{R_C}$ $\qquad i_{SC} = 5 \times 10^{-3}$

Assume active mode with $v_S := 2$

$i_C := \dfrac{v_S - V_\gamma}{R_B} \cdot \beta$ $\qquad i_C = 2.6 \times 10^{-3}$

$i_C < i_{SC}$ transistor is active

$v_{CE} := V_{CC} - i_C \cdot R_C$ $\qquad v_{CE} = 4.8$

For $v_S := 5$ $\quad i_C := \dfrac{v_S - V_\gamma}{R_B} \cdot \beta$ $\quad i_C = 8.6 \times 10^{-3}$ $\quad i_C > i_{SC}$ transistor is saturated

$v_{CE} := 0$ $\quad i_C := i_{SC}$ $\quad i_C = 5 \times 10^{-3}$

4-14 Same figure as Problem 4-13 $R_B := 50 \cdot 10^3$ $\quad R_C := 5000$ $\quad V_\gamma := 0.6$ $\quad \beta := 50$ $\quad V_{CC} := 15$

$i_{SC} := \dfrac{V_{CC}}{R_C}$ $\qquad i_{SC} = 3 \times 10^{-3}$ $\qquad$ To remain in the active mode $\quad 0 < i_B = \dfrac{v_S - V_\gamma}{R_B} < \dfrac{i_{SC}}{\beta}$

To avoid cutoff $\quad v_S > V_\gamma = 0.6$ $\quad$ To avoid saturation $\quad v_S < \left(V_\gamma + \dfrac{i_{SC} \cdot R_B}{\beta} \right) < 3.6$

4-15 Same figure as Prob. 4-13 $\quad R_B := 20 \cdot 10^3$ $\quad R_C := 470$ $\quad V_\gamma := 0.7$ $\quad \beta := 150$ $\quad V_{CC} := 15$

$i_{SC} := \dfrac{V_{CC}}{R_C}$ $\qquad i_{SC} = 3.191 \times 10^{-2}$ $\qquad$ To remain saturated $\quad i_B = \dfrac{v_S - V_\gamma}{R_B} > \dfrac{i_{SC}}{\beta}$ $\quad$ which requires

$v_S > V_\gamma + \dfrac{i_{SC} \cdot R_B}{\beta}$ $\qquad V_\gamma + \dfrac{i_{SC} \cdot R_B}{\beta} = 4.955$ $\qquad v_S > 4.955$

4-16 Same figure as Problem 4-13 $R_C := 1000$ $\quad V_\gamma := 0.7$ $\quad \beta := 75$ $\quad V_{CC} := 20$ $\quad v_{CE} := 10$ $\quad v_S := 2.5$

$v_{CE} = V_{CC} - R_C \cdot \beta \cdot i_B = V_{CC} - R_C \cdot \beta \cdot \left(\dfrac{v_S - V_\gamma}{R_B} \right) = \dfrac{V_{CC}}{2}$ which yields $\quad R_B := 2 \cdot R_C \cdot \beta \cdot \dfrac{(v_S - V_\gamma)}{V_{CC}}$

$$R_B = 1.35 \times 10^4$$

4-17 The Thevenin equivalent ckt to the right of the transistor is. $R_T := \dfrac{10^3}{\dfrac{1}{20}+\dfrac{1}{10}}$ $\quad v_{OC} := \dfrac{20}{10+20}\cdot 10$

$R_B := 10\cdot 10^3 \qquad V_\gamma := 0.7 \qquad \beta := 150$

$v_{OC} = 6.667 \qquad R_T = 6.667\times 10^3 \qquad i_{SC} := \dfrac{v_{OC}}{R_T} \qquad i_{SC} = 1\times 10^{-3}$ Assume active $i_B(v_S) := \dfrac{v_S - V_\gamma}{R_B}$

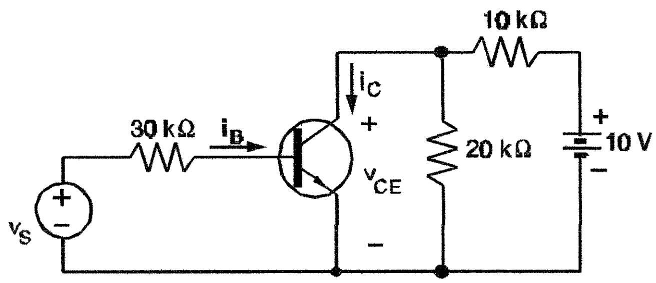
for $v_S := 0.5 \qquad i_B(v_S) = -2\times 10^{-5}$

$i_B < 0$ transistor is cutoff $\quad v_{CE} := v_{OC}$

$i_C := 0 \qquad v_{CE} = 6.667$

for $v_S := 1 \qquad i_B(v_S) = 3\times 10^{-5}$

$i_C = 0$

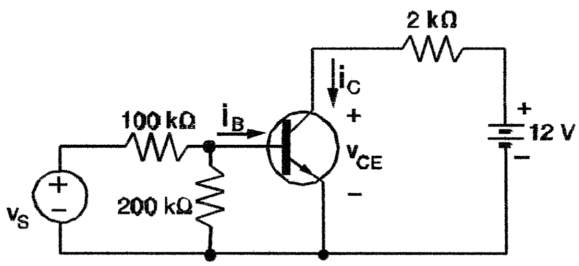


$i_C := \beta\cdot i_B(v_S) \qquad i_C = 4.5\times 10^{-3} \qquad i_C > i_{SC}$ Transistor is saturated $i_C := i_{SC} \qquad v_{CE} := 0$

4-18 The Thevenin equivalent ckt to the left of the transistor is. $R_T := \dfrac{2\cdot 10^5}{1+2} \qquad v_{OC}(v_S) := \dfrac{200}{100+200}\cdot v_S$

$V_{CC} := 12 \qquad R_C := 2000 \qquad i_{SC} := \dfrac{V_{CC}}{R_C} \qquad i_{SC} = 6\times 10^{-3}$ Assume active $i_B(v_S) := \dfrac{v_{OC}(v_S) - V_\gamma}{R_T}$

$\beta := 80 \qquad V_\gamma := 0.7$



For $v_S := 1 \qquad i_B(v_S) = -5\times 10^{-7}$

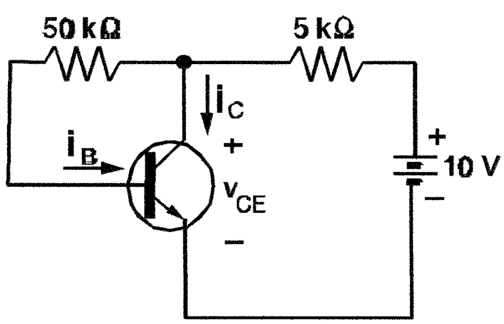
$i_B < 0$ Transistor is cutoff

$i_C := 0 \qquad v_{CE} := V_{CC} \qquad v_{CE} = 12$

For $v_S := 4 \qquad i_B(v_S) = 2.95\times 10^{-5}$

$i_C := \beta\cdot i_B(v_S) \qquad i_C = 2.36\times 10^{-3} \qquad i_C < i_{SC}$ transistor is active $\quad v_{CE} := V_{CC} - i_C\cdot R_C \qquad v_{CE} = 7.28$

4-19 $\quad R_C := 5\cdot 10^3 \qquad R_B := 50\cdot 10^3 \qquad V_{CC} := 10 \qquad V_\gamma := 0.7 \qquad \beta := 100$ Assume active



$i_B = \dfrac{v_{CE} - V_\gamma}{R_B} \qquad i_B = \dfrac{V_{CC} - (\beta+1)\cdot i_B\cdot R_C - V_\gamma}{R_B}$

$i_B := \dfrac{(V_{CC} - V_\gamma)}{(R_B + R_C\cdot\beta + R_C)} \qquad i_B = 1.676\times 10^{-5}$

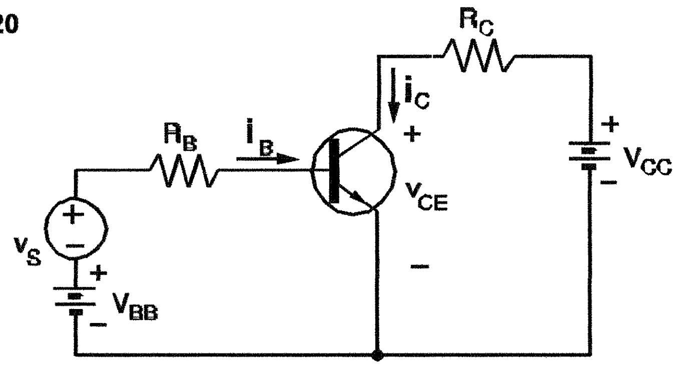
$i_C := \beta\cdot i_B \qquad i_C = 1.676\times 10^{-3}$

$i_{SC} := \dfrac{V_{CC}}{R_C} \qquad i_{SC} = 2\times 10^{-3} \qquad i_C < i_{SC}$

Transistor is in active mode

$v_{CE} := V_{CC} - (\beta+1)\cdot i_B\cdot R_C \qquad v_{CE} = 1.538$

4-20



$R_B := 500 \cdot 10^3$

$R_C := 5000$

$\beta := 100$

$V_\gamma := 0.7$

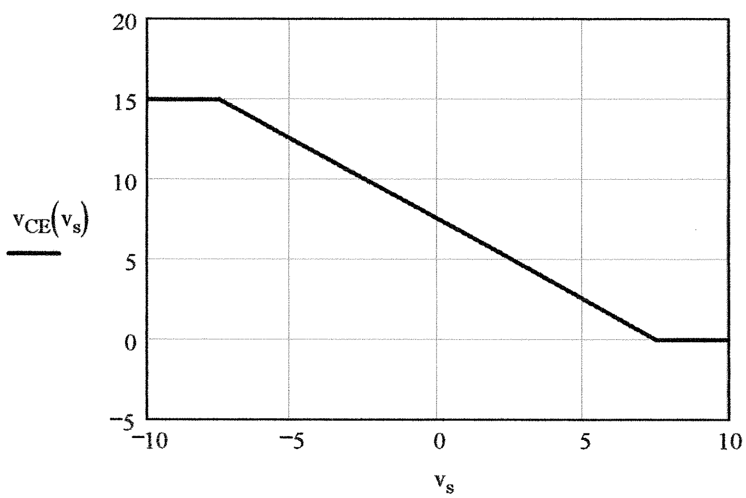
$V_{CC} := 15$

(a) $v_{CE} := \dfrac{V_{CC}}{2}$ $\quad i_C := \dfrac{V_{CC} - v_{CE}}{R_C}$ $\quad i_B := \dfrac{i_C}{\beta}$ $\quad V_{BB} := i_B \cdot R_B + V_\gamma$ $\quad V_{BB} = 8.2$ V

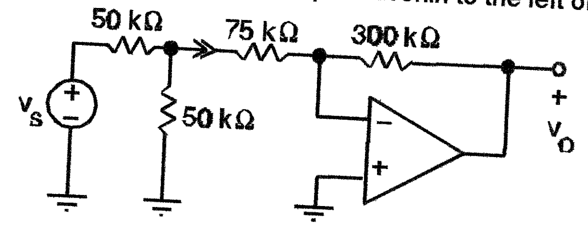
(b) $i_B = \dfrac{v_s + V_{BB} - V_\gamma}{R_B} > 0$ ---> $\quad v_s > V_\gamma - V_{BB} = -7.5$ V

$i_C = \beta \cdot \dfrac{v_s + V_{BB} - V_\gamma}{R_B} < \dfrac{V_{CC}}{R_C}$ ---> $\quad v_s < V_\gamma - V_{BB} + \dfrac{V_{CC} \cdot R_B}{\beta \cdot R_C} = 7.5$ V $\quad -7.5 < v_s < 7.5$

(c) $v_{CE}(v_s) := \begin{cases} V_{CC} \text{ if } v_s < -7.5 \\ V_{CC} - \beta \cdot \dfrac{v_s + V_{BB} - V_\gamma}{R_B} \cdot R_C \text{ if } -7.5 \le v_s \le 7.5 \\ 0 \text{ if } v_s > 7.5 \end{cases}$ $\qquad v_s := -10, -9.9 .. 10$



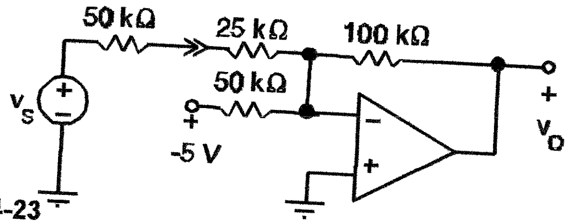
4-21 Ckt is in inverting amp. Thevenin to the left of the interface is $v_T := \frac{v_S}{2}$ $\quad R_T := \frac{50\cdot 10^3}{2}$



$$K_T := \frac{-300\cdot 10^3}{R_T + 75\cdot 10^3} \qquad K_T = -3$$

$$v_O = K_T\cdot\frac{v_S}{2} \qquad v_O = -1.5\cdot v_S$$

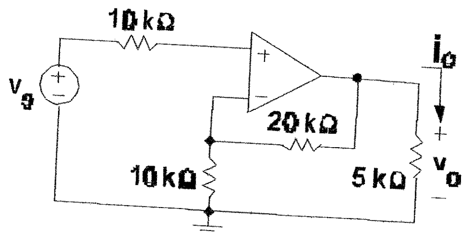
4-22 Ckt is in summing amp



$$v_O = \frac{-100\cdot 10^3}{50\cdot 10^3 + 25\cdot 10^3}\cdot v_S - \frac{-100\cdot 10^3}{50\cdot 10^3}\cdot(-$$

$$v_O = 10 - \frac{4}{3}\cdot v_S$$

4-23



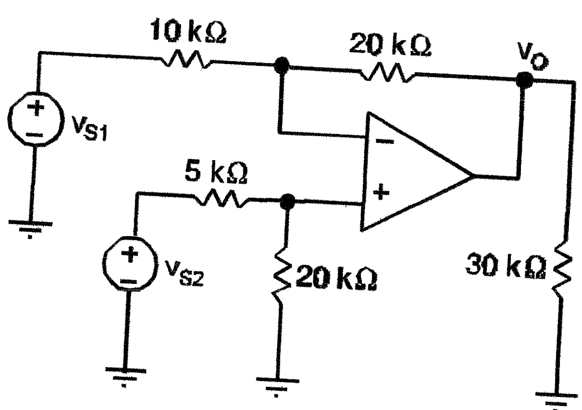
(a) $i_s := 0 \quad K = \frac{R_1 + R_2}{R_2} = \frac{10^4 + 2\cdot 10^4}{10^4} = 3$

$v_O = 3\cdot v_S$

(b) $v_s := 1.5 \quad v_O := 3\cdot v_s$

$$i_O := \frac{v_O}{5000} \qquad i_O = 9\times 10^{-4}$$

4-24



(a) $v_O = K_1\cdot v_{S1} + K_2\cdot v_{S2}$

$$K_1 = -\frac{R_2}{R_1} = -2$$

$$K_2 = \frac{R_1 + R_2}{R_1}\cdot\frac{R_4}{R_3 + R_4} = \frac{12}{5}$$

$$v_O = -2\cdot v_{S1} + 2.4\cdot v_{S2}$$

(b) $V_{CC} := 15 \quad v_{S2} := 10$

$$-V_{CC} < -2\cdot v_{S1} + 2.4\cdot v_{S2} < V_{CC}$$

$$\frac{-V_{CC} + 2.4\cdot v_{S2}}{2} < v_{S1} < \frac{V_{CC} + 2.4\cdot v_{S1}}{2}$$

$$4.5 < v_{S1} < 19.5$$

4-25 (a) For an inverting summer:

$$v_O = -v_1 - 3\cdot v_2 - 5\cdot v_3 = -\frac{R_{fb}}{R_1}\cdot v_1 - \frac{R_{fb}}{R_2}\cdot v_2 - \frac{R_{fb}}{R_3}\cdot v_3$$

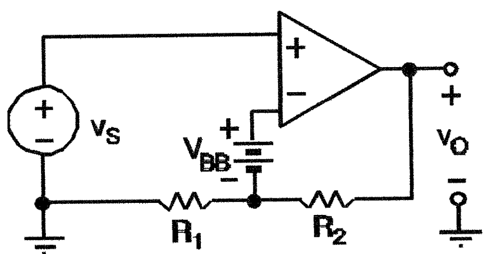
$R_{fb} := 75\cdot 10^3 \quad R_1 := 1\cdot R_{fb} \quad R_2 := 3^{-1}\cdot R_{fb} \quad R_3 := 5^{-1}\cdot R_{fb}$

$R_1 = 7.5\times 10^4 \quad R_2 = 2.5\times 10^4 \quad R_3 = 1.5\times 10^4 \ \Omega$

(b) $V_{CC} := 15 \quad -V_{CC} < -v_1 - 3\cdot(0.5) - 5\cdot(-1) < V_{CC}$ ---> $V_{CC} > v_1 - 3.5 > -V_{CC}$

$18.5 > v_1 > -11.5$ V

4-26



(a) $v_P = v_S \quad v_N = \left(\frac{R_1}{R_1 + R_2}\right)\cdot v_O + V_{BB}$

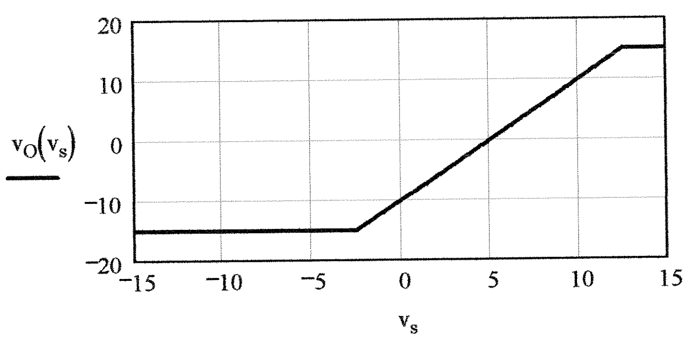
$v_p = v_N \quad \text{--->} \quad v_S = \left(\frac{R_1}{R_1 + R_2}\right)\cdot v_O + V_{BB}$

$$v_O = \frac{R_1 + R_2}{R_1}\cdot(v_s - V_{BB}) = K\cdot(v_s - V_{BB})$$

$$K = \frac{R_1 + R_2}{R_1}$$

(b) $V_{BB} := 5 \quad V_{CC} := 15 \quad R_1 = R_2 \quad K := 2 \qquad v_s := -15, -14.9 .. 15$

$$v_O(v_s) := \begin{cases} (-V_{CC}) & \text{if } v_s < \frac{-V_{CC}}{K} + V_{BB} \\ K\cdot(v_s - V_{BB}) & \text{if } \frac{-V_{CC}}{K} + V_{BB} \le v_s \le \frac{V_{CC}}{K} + V_{BB} \\ V_{CC} & \text{if } \frac{V_{CC}}{K} + V_{BB} < v_s \end{cases}$$



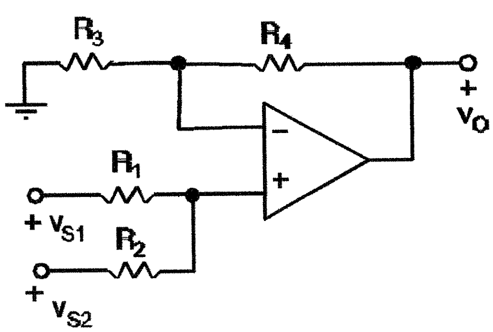
$v_O(12.5) = 15$

$v_O(5) = 0$

$v_O(0) = -10$

$v_O(-2.5) = -15$

4-27 **Circuit is a noninverting amplifer/summer. Using superposition and voltage division**



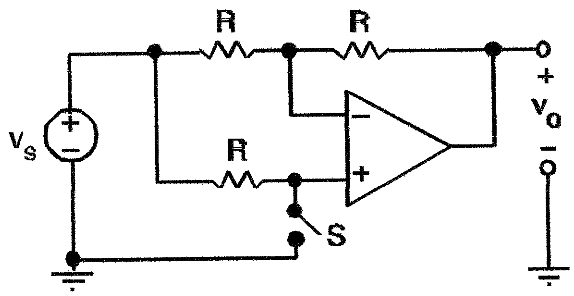
v_{S2} off $\quad v_{O1} = \frac{R_3 + R_4}{R_3}\cdot\frac{R_2}{R_1 + R_2}\cdot v_{S1}$

v_{S1} off $\quad v_{O2} = \frac{R_3 + R_4}{R_3}\cdot\frac{R_1}{R_1 + R_2}\cdot v_{S2}$

Using superposition

$$v_O = v_{O1} + v_{O2} = \frac{R_3 + R_4}{R_3\cdot(R_1 + R_2)}\cdot(R_2\cdot v_{S1} + R_1\cdot v_{S2})$$

4-28



Switch open: $i_P = 0 \quad v_P = v_s$

$$i_N = 0 \quad \frac{v_s - v_N}{R} + \frac{v_O - v_N}{R} = 0$$

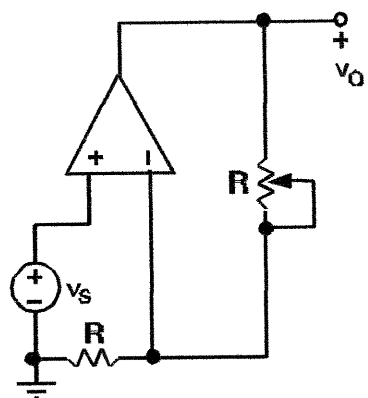
$v_O = 2\cdot v_N - v_s$ but $v_N = v_P = v_s$

yields $v_O = v_s$

Switch closed: $v_O = -\frac{R}{R}\cdot v_s = -v_s$

The circuit does the opposite of the claim, hence, the claim is false!

4-29

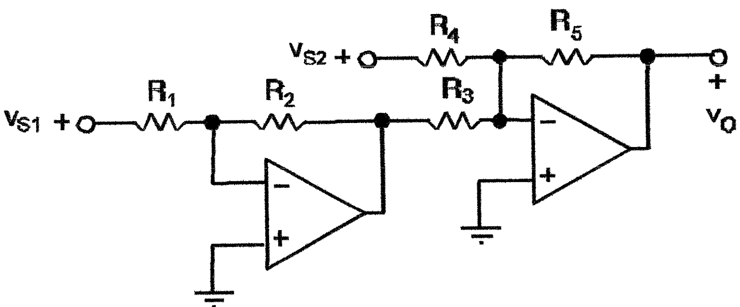


In general $K = \frac{R_{pot} + R}{R} = 1 + \frac{R_{pot}}{R}$

since $0 \le R_{pot} \le R$

the gain range is $1 \le K \le 2$

4-30



v_{S2} off $v_{O1} = \left(\frac{-R_2}{R_1}\right)\cdot\left(\frac{-R_5}{R_3}\right)\cdot v_{S1}$

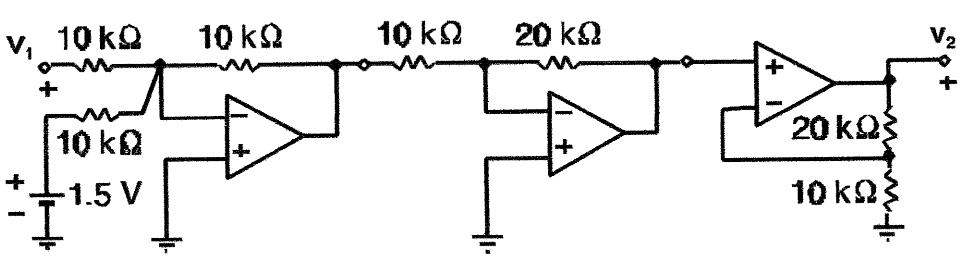
v_{S1} off $v_{O2} = \left(\frac{-R_5}{R_4}\right)\cdot v_{S2}$

Using superposition

$v_O = v_{O1} + v_{O2}$

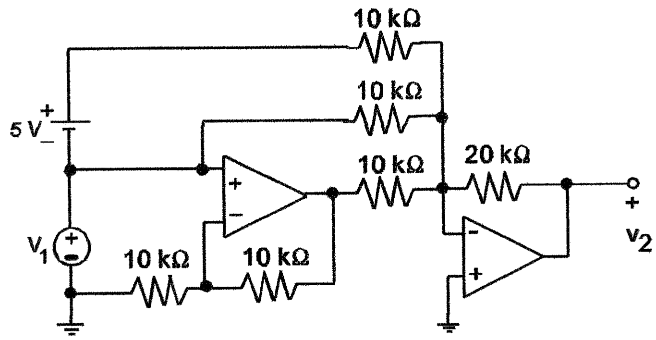
$$v_O = \frac{R_2\cdot R_5}{R_1\cdot R_3}\cdot v_{S1} - \frac{R_5}{R_4}\cdot v_{S2}$$

4-31



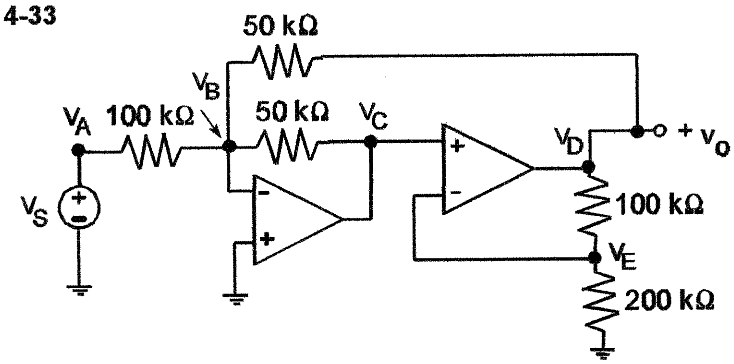
$$v_2 = \left(\frac{20 + 10}{10}\right)\cdot\left(\frac{-20}{10}\right)\cdot\left(\frac{-10}{10}\cdot v_1 - \frac{10}{10}\cdot 1.5\right) = 6.\cdot v_1 + 9.$$

4-32



$$v_2 = \left(\frac{-20}{10}\right)\cdot(v_1 + 5) + \left(\frac{-20}{10}\right)\cdot v_1 + \left(\frac{-20}{10}\right)\cdot\left(\frac{10+10}{10}\cdot v_1\right) = -8\cdot v_1 - 10$$

4-33



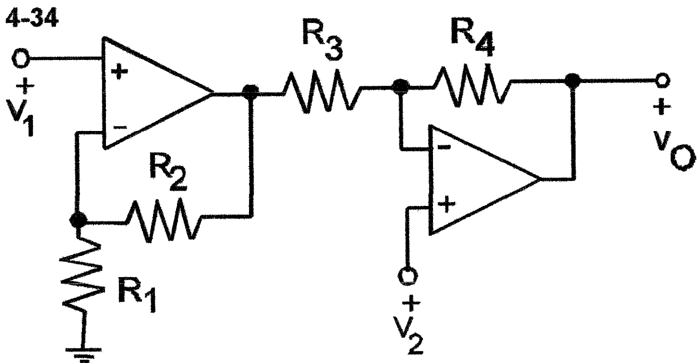
$$v_O = v_D = \left(\frac{100+200}{200}\right)\cdot v_C = \frac{3}{2}\cdot v_C$$

$$v_C = \left(\frac{-50}{100}\right)\cdot v_s + \left(\frac{-50}{50}\right)\cdot v_O = \frac{-v_S}{2} - v_O$$

$$v_O = \frac{3}{2}\cdot\left(\frac{-v_S}{2} - v_O\right) = \frac{-3}{4}\cdot v_S - \frac{3}{2}\cdot v_O$$

Solving for v_O yields $v_O = \frac{-3}{10}\cdot v_S$

4-34



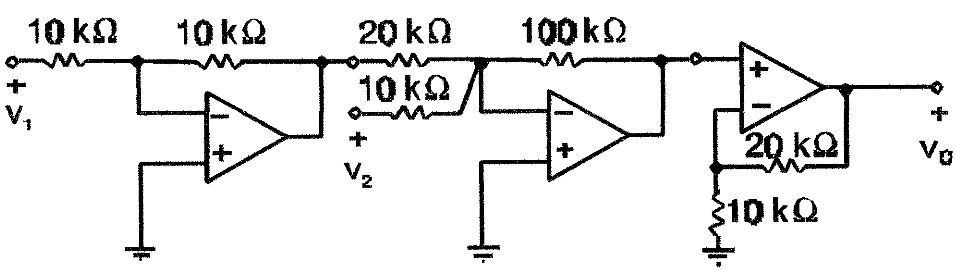
v_2 off $\quad v_{O1} = \left(\frac{R_2 + R_1}{R_1}\right)\cdot\left(\frac{-R_4}{R_3}\right)\cdot v_1$

v_1 off $\quad v_{O2} = \left(\frac{R_4 + R_3}{R_3}\right)\cdot v_2$

Using superposition

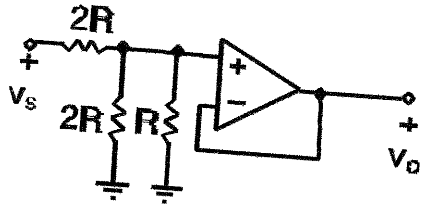
$$v_O = \left(\frac{R_4 + R_3}{R_3}\right)\cdot v_2 - \left(\frac{R_2 + R_1}{R_1}\right)\cdot\left(\frac{R_4}{R_3}\right)\cdot v_1$$

4-35

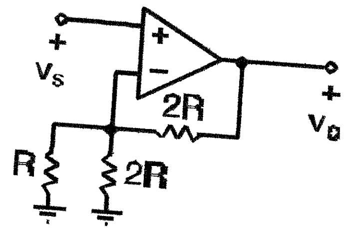


$$v_O = \left(\frac{20+10}{10}\right)\cdot\left[\frac{-100}{10}\cdot v_2 + \left(\frac{-100}{20}\right)\cdot\left(\frac{-10}{10}\right)\cdot v_1\right] = 15\cdot v_1 - 30\cdot v_2$$

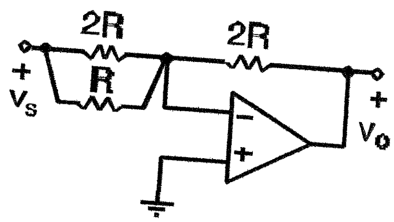
4-36



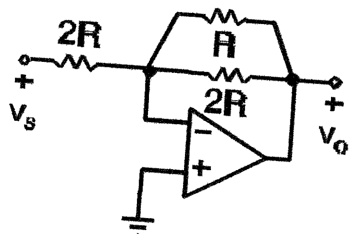
$$K=\frac{\frac{2\cdot R\cdot R}{2\cdot R+R}}{2\cdot R+\frac{2\cdot R\cdot R}{2\cdot R+R}}=\frac{1}{4}$$



$$K=\frac{\frac{2\cdot R\cdot R}{2\cdot R+R}+2\cdot R}{\left(\frac{2\cdot R\cdot R}{2\cdot R+R}\right)}=4$$

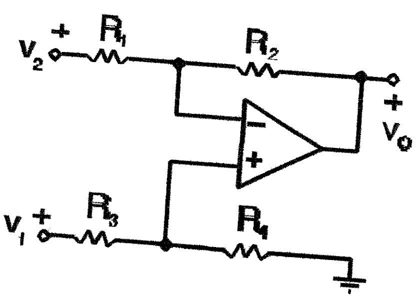


$$K=\frac{-2\cdot R}{\left(\frac{2\cdot R\cdot R}{2\cdot R+R}\right)}=-3$$



$$K=\frac{\left(\frac{-2\cdot R\cdot R}{2\cdot R+R}\right)}{2\cdot R}=\frac{-1}{3}$$

4-37



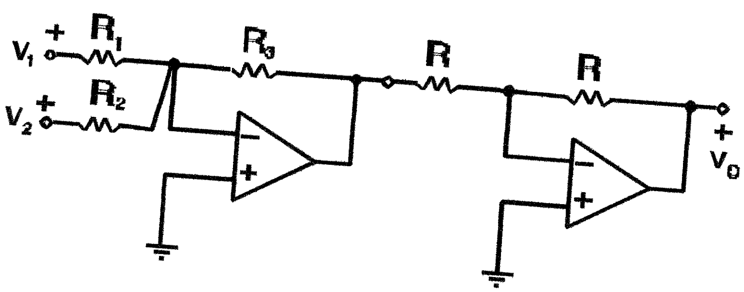
(a) $v_O=\left(\frac{R_1+R_2}{R_1}\right)\cdot\left(\frac{R_4}{R_3+R_4}\right)\cdot v_1-\left(\frac{R_2}{R_1}\right)\cdot v_2$

$\frac{R_2}{R_1}=2$ Let $R_1=10^4$ Then $R_2=2\cdot 10^4$

$\frac{R_1+R_2}{R_1}=3$ hence $\frac{R_4}{R_3+R_4}=1$

Let $R_4=10^4$ Then $R_3=0$

(b) Use an inverting summer and an inverter:



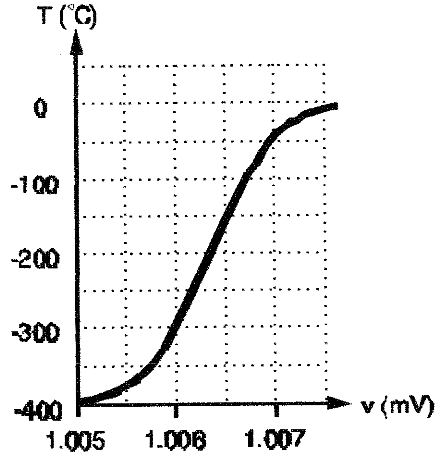
$$v_O=\left(\frac{-R_3}{R_1}\cdot v_1-\frac{R_3}{R_2}\cdot v_2\right)\cdot\left(-\frac{R}{R}\right)$$

$$v_O=\frac{R_3}{R_1}\cdot v_1+\frac{R_3}{R_2}\cdot v_2$$

$$\frac{R_3}{R_1}=2 \qquad \frac{R_3}{R_2}=1$$

Let $R_3=4\cdot 10^4$ Then $R_1=2\cdot 10^4$ $R_2=4\cdot 10^4$

4-38



At -300 °C the transducer output is 1.006 mV and the desired output is 0 V. At -100 °C the transducer output is 1.00667 mV and the desired output is +5 V.

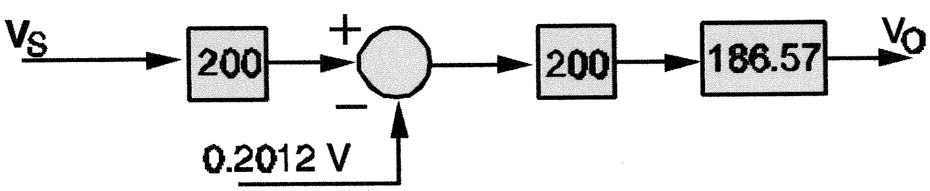
$$K = \frac{\text{output·range}}{\text{input·range}} = \frac{5 - (0)}{(1.00667 - 1.006) \cdot 10^{-3}} = 7.463 \cdot 10^{6}$$

$K = (200) \cdot (200) \cdot (186.57)$ <---Use three stages to keep stage gains below 1000.

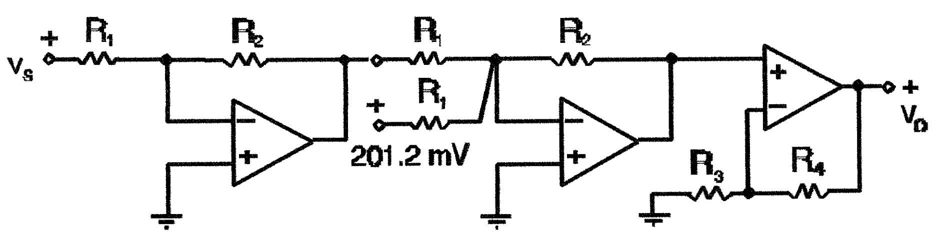
Remove bias term after one K = 200 stage

$$\text{bias} = 200 \cdot 1.006 \cdot 10^{-3} = 0.2012$$

block diagram



circuit realization



$$\frac{R_2}{R_1} = 200$$

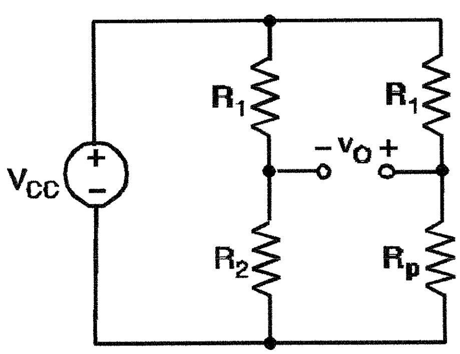
$$\frac{R_4}{R_3} + 1 = 186.57$$

Input-output relationship: $v_O(v_s) := 186.57 \cdot (-200) \cdot [(-200) \cdot v_s + 0.2012]$

Checking $v_O(1.006 \cdot 10^{-3}) = 0$ $\quad v_O(1.00667 \cdot 10^{-3}) = 5$

4-39 The output of the bridge ckt below is:

$$v_O = V_{CC} \cdot \left(\frac{R_P}{R_P + R_1} - \frac{R_2}{R_1 + R_2} \right)$$

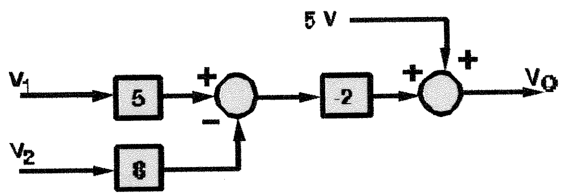


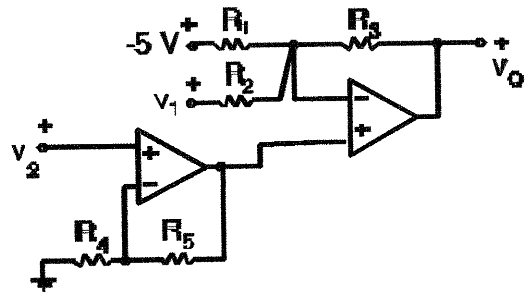
Where R_P is the transducer resistance which ranges from 5 kΩ to 15 kΩ. Let R_1 = 10 kΩ and R_2 = 5 kΩ. Then for R_P = 5 kΩ the output is $v_O = V_{CC} \cdot \left(\frac{5}{5 + 10} - \frac{5}{5 + 10} \right) = 0$ regardless of V_{CC}. When R_P = 15 kΩ the output must be

$$v_O = V_{CC} \cdot \left(\frac{15}{15 + 10} - \frac{5}{5 + 10} \right) = \frac{V_{CC} \cdot 4}{15} = 5 \text{ hence,}$$

$V_{CC} = 18.75$ Note: The problem was solved without using an Op-Amp. Engineers should not be limited in their thinking.

4-40 The output of the block diagram is $v_O = 5 + (-2)\cdot(5\cdot v_1 - 8\cdot v_2) = 5 - 10\cdot v_1 + 16\cdot v_2$





By superposition, the output of the circuit is $v_O = K_1\cdot 5 - K_2\cdot v_1 + K_3\cdot v_2$ where

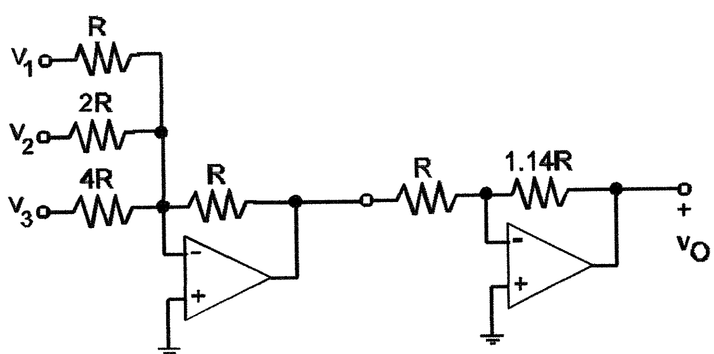
$$K_1 = \frac{R_3}{R_1} = 1 \quad K_2 = \frac{R_3}{R_2} = 10 \quad K_3 = \frac{R_4 + R_5}{R_4}\cdot\left[\frac{\dfrac{R_1\cdot R_2}{R_1 + R_2} + R_3}{\left(\dfrac{R_1\cdot R_2}{R_1 + R_2}\right)}\right] = 16$$ Let $R_2 = R_5 = 10\cdot 10^3$ then

$R_1 = R_3 = 100\cdot 10^3$ and $K_3 = \dfrac{R_4 + 10\cdot 10^3}{R_4}\cdot(12) = 16$ which yields $R_4 = 30\cdot 10^3$

10 kΩ and 100 kΩ are standard values. To get 30 kΩ use three 10 kΩ standard resistors in series.

4-41 Since $v_O = K(4\cdot v_1 + 2\cdot v_2 + v_3)$ max output occurs when $v_1 = v_2 = v_3 = 5$ hence

$K(20 + 10 + 5) = 10 \quad K = \dfrac{10}{35}$ For noninverting summer below the voltage at the noninverting input is



$$v_O = \frac{R_{EQ}}{R}\cdot v_1 + \frac{R_{EQ}}{2\cdot R}\cdot v_2 + \frac{R_{EQ}}{4\cdot R}\cdot v_3$$

where R_{EQ} is

$$R_{EQ} = \frac{1}{\dfrac{1}{R} + \dfrac{1}{2\cdot R} + \dfrac{1}{4\cdot R}} = \frac{4}{7}\cdot R$$ and v_O is

$$v_O = \frac{1}{7}\cdot(4\cdot v_1 + 2\cdot v_2 + v_3)$$

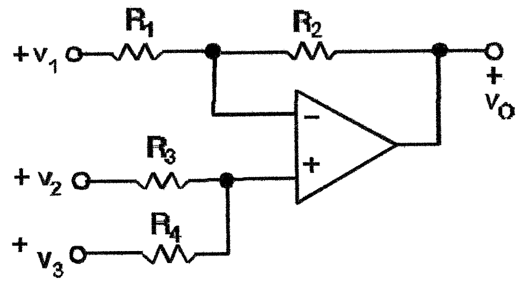
The gain G of the noninverting amp must be

$\dfrac{G}{7} = K = \dfrac{10}{35}$ hence $\quad G = 2$

4-42 By superposition, the output of the circuit below is $v_O = -K_1 \cdot v_1 + K_2 \cdot v_2 + K_3 \cdot v_3$ where

$$K_1 = \frac{R_2}{R_1} = 20 \qquad K_2 = \left(\frac{R_4}{R_3 + R_4}\right)\cdot\left(\frac{R_1 + R_2}{R_1}\right) = 3$$

$$K_3 = \left(\frac{R_3}{R_3 + R_4}\right)\cdot\left(\frac{R_1 + R_2}{R_1}\right) = 18$$



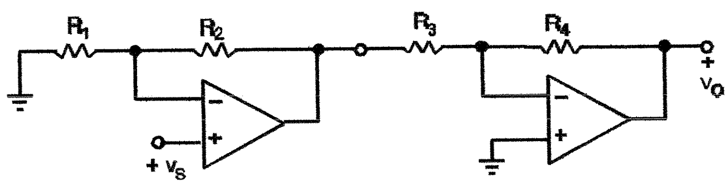
Let $R_1 := 10^4$ then $R_2 := 20 \cdot 10^4$ Since $\frac{K_2}{K_3} = \frac{R_4}{R_3} = \frac{3}{18}$ let $R_3 := 18 \cdot 10^4$ then $R_4 := 3 \cdot 10^4$

checking $\frac{R_2}{R_1} = 20$ $\quad \left(\frac{R_4}{R_3 + R_4}\right)\cdot\left(\frac{R_1 + R_2}{R_1}\right) = 3 \quad \left(\frac{R_3}{R_3 + R_4}\right)\cdot\left(\frac{R_1 + R_2}{R_1}\right) = 18$

$R_1 = 1 \times 10^4 \qquad R_2 = 2 \times 10^5 \qquad R_3 = 1.8 \times 10^5 \qquad R_4 = 3 \times 10^4$

The summer will not load the transducers (R_T = 600 Ω) since input resistance seen by each transducer is 10 kΩ or greater.

4-43



Make the closed-loop gain of each stage less than 1% of the open-loop gain A = 2×10^5, i.e.make the stage gains

$K_n < 0.01 \times 2 \times 10^5 = 2000$

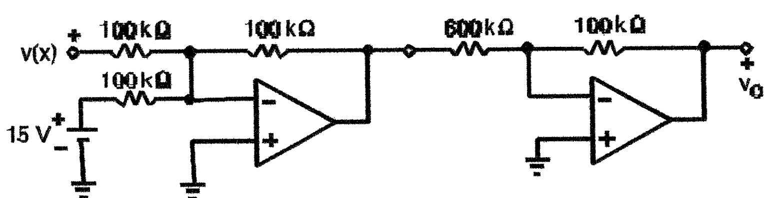
Let $K = (K_1)(K_2) = (120)(-100) = -12{,}000$.

1st Stage: $K_1 = 120 = \frac{R_1 + R_2}{R_1}$ Let $R_1 = 2000$ Then $R_2 = 238000\,\Omega$

2nd Stage: $K_2 = -100 = -\frac{R_4}{R_3}$ Let $R_3 = 2000$ Then $R_4 = 200000\,\Omega$

Input resistance of the noninverting 1st stage is much larger than 300 kΩ.

4-44 The interface ckt must implement a relationship



$v_{P.} = 15 \quad v_O = 5 \qquad 15 \cdot K_1 + K_2 = 5$

$v_{P.} = -15 \quad v_O = 0 \qquad -15 \cdot K_1 + K_2 = 0$

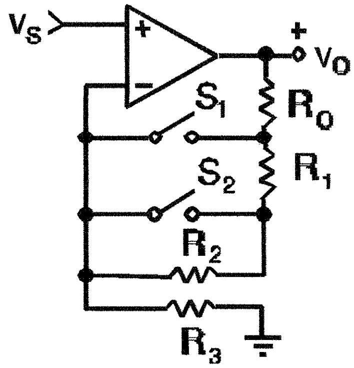
which yields $K_1 = \frac{1}{6} \qquad K_2 = \frac{15}{6}$

The circuit at left implements this relationship. Select R large enough to not load the 1k pot.

$v_O = K_1 \cdot v_P + K_2$

4-45 The gain constraints are

$$\frac{R_0 + R_1 + R_2 + R_3}{R_3} = 10 \qquad \frac{R_0 + R_1 + R_3}{R_3} = 5 \qquad \frac{R_0 + R_3}{R_3} = 2$$



Let $R_0 := 10^4$ then $\frac{10^4 + R_3}{R_3} = 2$ yields $R_3 := 10^4$

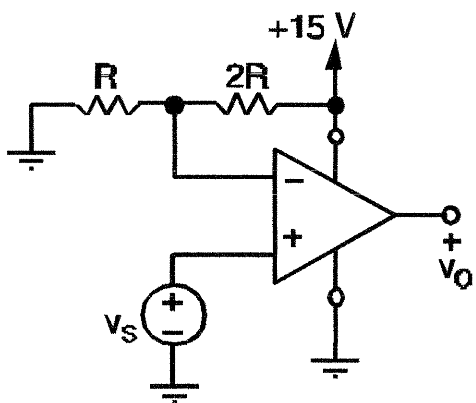
the constraint $\frac{10^4 + R_1 + 10^4}{10^4} = 5$ yields $R_1 := 3 \cdot 10^4$

the constraint $\frac{10^4 + 3 \cdot 10^4 + R_2 + 10^4}{10^4} = 10$ yields $R_2 := 5 \cdot 10^4$

checking

$$\frac{R_0 + R_1 + R_2 + R_3}{R_3} = 10 \qquad \frac{R_0 + R_1 + R_3}{R_3} = 5 \qquad \frac{R_0 + R_3}{R_3} = 2$$

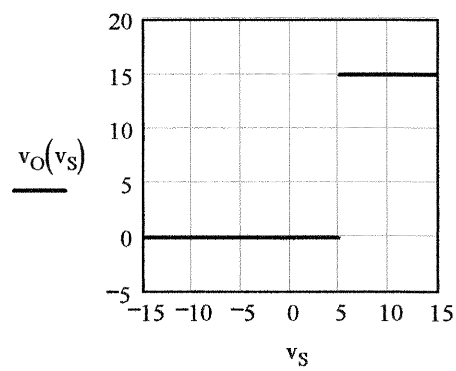
4-46 $v_S := -15, -14.9 .. 15$



(a) $v_P = v_S$ $\quad v_n := 5 \quad V_{OH} := 15 \quad V_{OL} := 0$

$$v_O(v_S) := \begin{vmatrix} V_{OH} \text{ if } v_S > v_n \\ V_{OL} \text{ if } v_S < v_n \end{vmatrix}$$

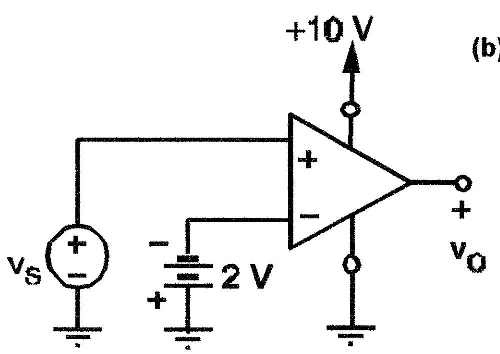
(b)



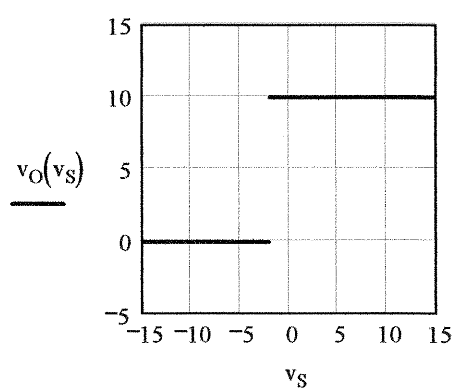
4-47 $V_{OH} := 10 \quad V_{OL} := 0$

(a) $v_P = v_S \quad v_n := -2$

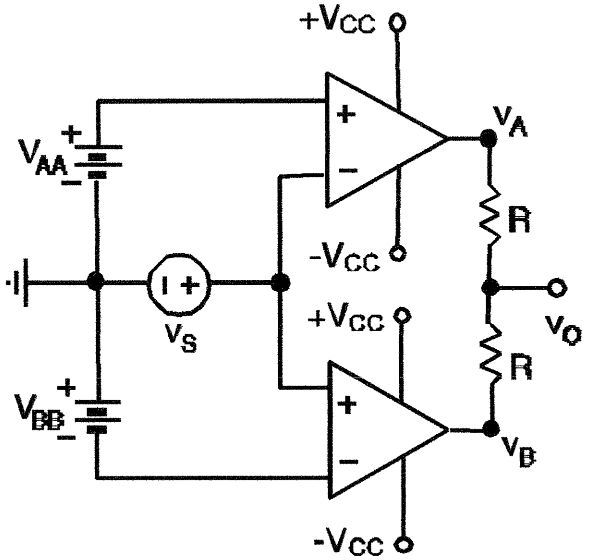
$$v_O(v_S) := \begin{vmatrix} V_{OH} \text{ if } v_S > v_n \\ V_{OL} \text{ if } v_S < v_n \end{vmatrix}$$



(b)



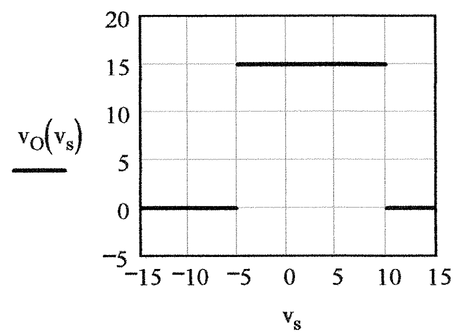
4-48 $V_{CC} := 15 \quad V_{AA} := 10 \quad V_{BB} := 5 \quad V_{OH} := V_{CC} \quad V_{OL} := -V_{CC} \qquad v_s := -15, -14.9 .. 15$



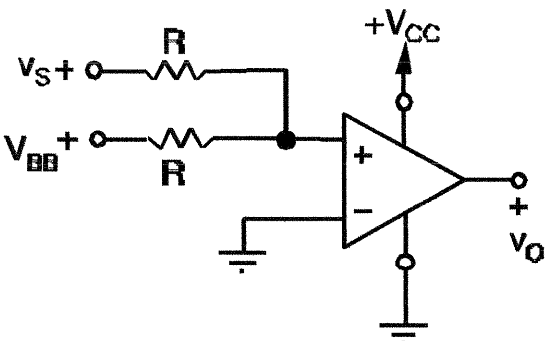
$$v_A(v_s) := \begin{vmatrix} V_{OL} & \text{if } v_s > V_{AA} \\ V_{OH} & \text{if } v_s < V_{AA} \end{vmatrix}$$

$$v_B(v_s) := \begin{vmatrix} V_{OL} & \text{if } v_s < -V_{BB} \\ V_{OH} & \text{if } v_s > -V_{BB} \end{vmatrix}$$

$$v_O(v_s) := 0.5 \cdot (v_A(v_s) + v_B(v_s))$$



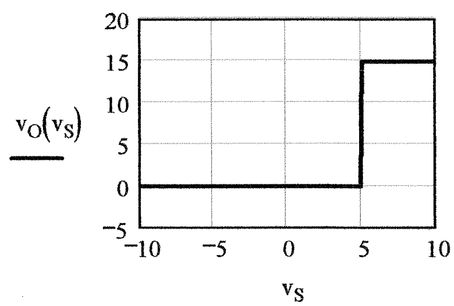
4-49 $V_{CC} := 15 \quad V_{BB} := -5 \quad V_{OH} := V_{CC} \quad V_{OL} := 0$



(a) $v_p(v_S) := \dfrac{v_S + V_{BB}}{2} \qquad v_n := 0$

$$v_O(v_S) := \begin{vmatrix} V_{OH} & \text{if } v_p(v_S) > v_n \\ V_{OL} & \text{if } v_p(v_S) < v_n \end{vmatrix}$$

(b)



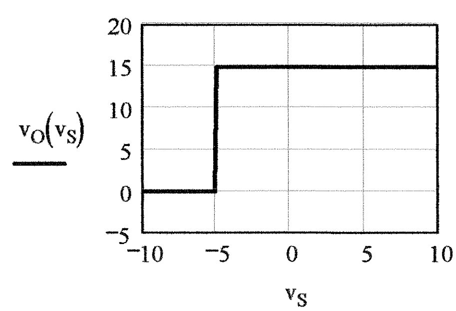
4-50 $V_{CC} := 15 \quad V_{BB} := 5 \quad V_{OH} := V_{CC} \quad V_{OL} := 0$

(a) $v_p(v_S) := \dfrac{v_S + V_{BB}}{2} \qquad v_n := 0$

$$v_O(v_S) := \begin{vmatrix} V_{OH} & \text{if } v_p(v_S) > v_n \\ V_{OL} & \text{if } v_p(v_S) < v_n \end{vmatrix}$$

Same figure as in 4-49 above

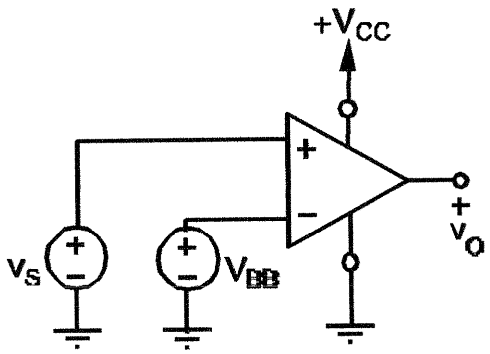
(b)

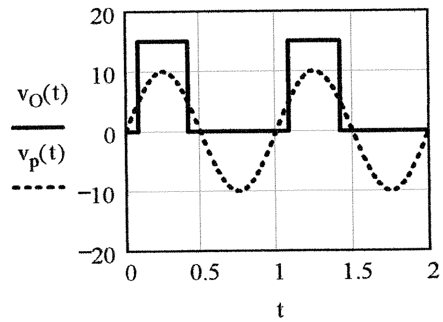


4-51 $V_{CC} := 15$ $V_{BB} := 5$ $V_{OH} := V_{CC}$ $V_{OL} := 0$ $v_p(t) := 10 \cdot \sin(2\pi \cdot t)$ $v_n := V_{BB}$

$v_O(t) := \text{if}(v_p(t) > v_n, V_{OH}, V_{OL})$

$t := 0, 0.001 .. 2$



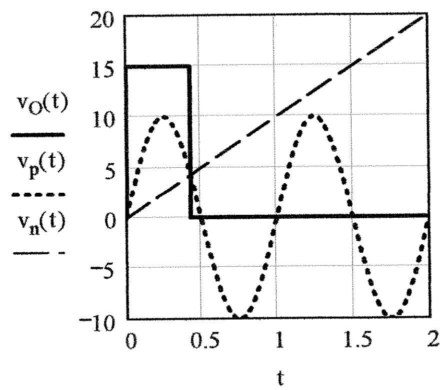


4-52 $V_{CC} := 15$ $V_{BB}(t) := 10 \cdot t$ $V_{OH} := V_{CC}$ $V_{OL} := 0$ $v_p(t) := 10 \cdot \sin(2\pi \cdot t)$ $v_n(t) := V_{BB}(t)$

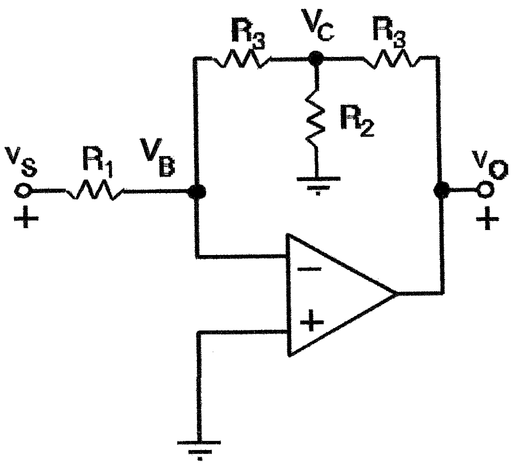
$v_O(t) := \text{if}(v_p(t) > v_n(t), V_{OH}, V_{OL})$

$t := 0, 0.001 .. 2$

Same figure as in 4-51 above



4-53



(a) Using node-voltage analysis

Node B: $\frac{v_B - v_s}{R_1} + \frac{v_B - v_C}{R_3} = 0$

Node C: $\frac{v_C - v_B}{R_3} + \frac{v_C}{R_2} + \frac{v_C - v_O}{R_3} = 0$

Since: $v_B = 0$

Node B: $\frac{v_C}{R_3} = \frac{-v_s}{R_1}$

Node C: $\left(\frac{2}{R_3} + \frac{1}{R_2}\right) \cdot v_C = \frac{v_O}{R_3}$

Solving for vo yields $v_O = \frac{-\left(2 + \frac{R_3}{R_2}\right) \cdot R_3}{R_1} \cdot v_s = -\frac{R_{FBK}}{R_1} \cdot v_s$ hence $R_{FDK} = \left(2 + \frac{R_3}{R_2}\right) \cdot R_3$ QED

(b) Let $R_1 = 20000\ \Omega$ Then $R_{FDK} = K \cdot R_1 = 8 \cdot 10^6\ \Omega$

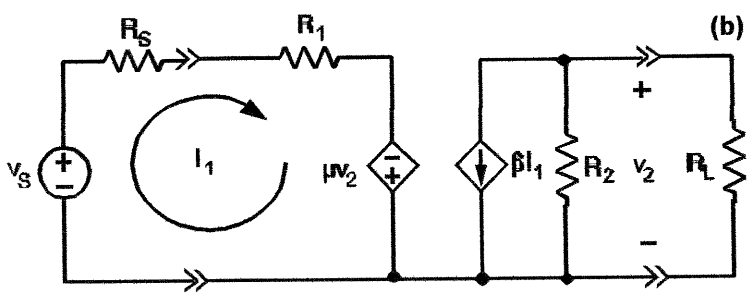
(c) With $R_{FDK} = \left(2 + \frac{R_3}{R_2}\right) \cdot R_3 = 8 \cdot 10^6$ Let $R_3 = 10^5$ Then $R_2 = \frac{R_3^{\,2}}{8000000 - 2 \cdot R_3} = 1282\,\Omega$

(d)

Circuit	No. of Resistors	Resistance Spread	Total Resistance
inverting	2	400	$8.02\text{x}10^6\,\Omega$
bridged-T	4	78	$2.213\text{x}10^5\,\Omega$

The inverting amplifier uses fewer resistors but requires a larger spread and total resistance. Resistance spread and total resistance are important design parameters in circuits that fabricate the resistors using film technology.

4-54 (a) Input mesh $(R_s + R_1) \cdot i_1 - \mu \cdot v_2 = v_s$ Output node $\left(\frac{1}{R_2} + \frac{1}{R_L}\right) \cdot v_2 + \beta \cdot i_1 = 0$



(b) with $R_s = 2000\ R_1 = 5000\ R_2 = 50000$

$R_L = 150000\ \ \mu = 0.001\ \beta = 50$

Input mesh $7000 \cdot i_1 - 0.001 \cdot v_2 = v_s$

Output node $\frac{4}{150000} \cdot v_2 + 50 \cdot i_1 = 0$

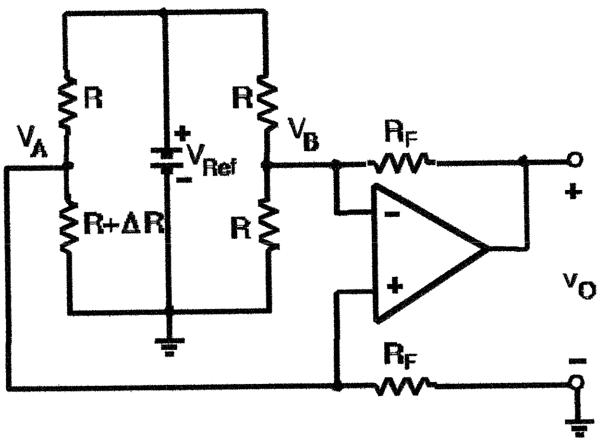
which yield $i_1 = 1.127 \cdot 10^{-4} \cdot v_s$ and $v_2 = -211.268 \cdot v_s$

(c) $R_{IN} = \frac{v_s}{i_1} - R_s = 8873 - 2000 = 687\,\Omega$ $K_V = \frac{v_2}{v_s} = -211.268$

4-54 Continued

(d) $P_{in} = v_S \cdot i_1 = 1.127 \cdot 10^{-4} \cdot v_S^2$ $\quad P_{out} = \frac{v_2^2}{R_L} = \frac{(211.268 \cdot v_S)^2}{150000} = 0.298 \cdot v_S^2$ $\quad K_P = \frac{P_{out}}{P_{in}} = 2.64 \cdot 10^3$

4-55 (a) Writing node-voltage equations



Node A: $\left(\frac{1}{R} + \frac{1}{R+\Delta R} + \frac{1}{R_F}\right) \cdot v_A - \frac{v_{Ref}}{R} = 0$

Node B: $\left(\frac{2}{R} + \frac{1}{R_F}\right) \cdot v_B - \frac{v_O}{R_F} - \frac{V_{Ref}}{R} = 0$

For an ideal OP AMP $v_A = v_B$, solving for v_O

$$v_O = \left(\frac{\Delta R \cdot R_F}{2 \cdot R \cdot R_F + R_F \cdot \Delta R + R^2 + R \cdot \Delta R}\right) \cdot \frac{R_F}{R} \cdot V_{Ref}$$

For $\Delta R/R << 1$ this reduces to

$$v_O = \left[\left(2 + \frac{R}{R_F}\right)^{-1} \cdot \frac{R_F}{R}\right] \cdot \frac{\Delta R}{R} \cdot V_{Ref} = K \cdot \frac{\Delta R}{R} \cdot V_{Ref}$$

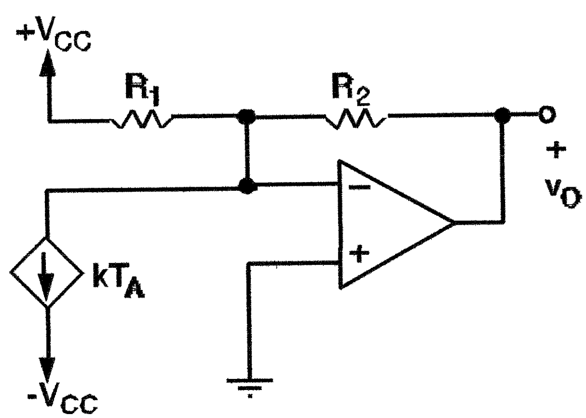
(b) For $v_O = \pm 3$ V when $\Delta R/R = \pm 0.0004$ and $V_{CC} = 15$ V requires

$$\left[\frac{R_F}{R} \cdot \left(2 + \frac{R}{R_F}\right)^{-1}\right] \cdot 4 \cdot 10^{-4} \cdot 15 = 3$$

This requires $\frac{R_F}{R} = 1000.5$ Since R = 100 Ω we have $R_F = 100.05 \cdot 10^3$

4-56 Summing currents at the inverting input $\quad V_{CC} := 10 \quad k := 10^{-6}$

$\frac{V_{CC}}{R_1} + \frac{v_O}{R_2} - k \cdot T_A = 0$ $\quad$ Solving for v_O



$$v_O = R_2 \cdot k \cdot T_A - \frac{R_2}{R_1} \cdot V_{CC}$$

$T_A = T_C + 273$ hence

$$v_O = R_2 \cdot k \cdot T_C + 273 \cdot k \cdot R_2 - \frac{R_2}{R_1} \cdot V_{CC}$$

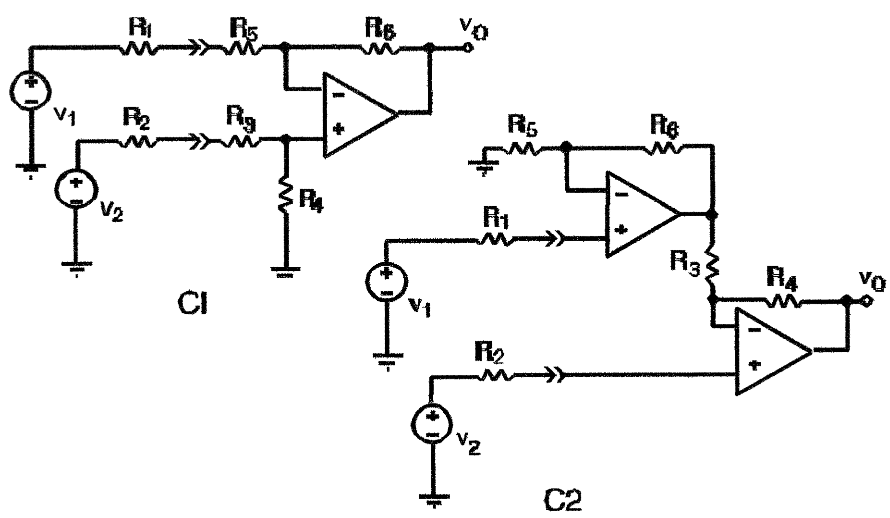
The design constraints are

$R_2 \cdot k = 10^{-1}$ $\quad 273 \cdot k \cdot R_2 - \frac{R_2}{R_1} \cdot V_{CC} = 0$

$R_2 := \frac{10^{-1}}{k}$ $\quad R_1 := \frac{V_{CC}}{273 \cdot k}$

$R_1 = 3.663 \times 10^4$ $\quad R_2 = 1 \times 10^5$

4-57



(a) Circuit C1 is a differential amplifier with

$$v_O = K_2 \cdot v_2 + K_1 \cdot v_1 = \left(\frac{R_4}{R_2 + R_3 + R_4}\right)\cdot\left(\frac{R_1 + R_5 + R_6}{R_1 + R_5}\right)\cdot v_2 + \left(\frac{-R_6}{R_1 + R_5}\right)\cdot v_1$$

Applying superposition in C2.

with v_2 off $\quad v_{O1} = K_1 \cdot v_1 = \left(\frac{R_5 + R_6}{R_5}\right)\cdot\left(\frac{-R_4}{R_3}\right)\cdot v_1$ $\quad$ with v_1 off $\quad v_{O2} = K_2 \cdot v_2 = \frac{R_3 + R_4}{R_3}\cdot v_2$

(b) For Ckt C1 with $K_1 = -K_2 = 5$, $R_1 = R_2 = 1\ \text{k}\Omega$ and $R_3 = R_5 = 10\ \text{k}\Omega$ the constraints are

$$R_4 := 1000 \quad R_6 := 1000 \quad \text{Given} \quad \left(\frac{R_4}{1000 + 10000 + R_4}\right)\cdot\left(\frac{1000 + 10000 + R_6}{1000 + 10000}\right) = 5 \quad 5\frac{R_6}{1000 + 10000} = 5$$

$$\text{Find}(R_4, R_6) = \begin{pmatrix} 5.5 \times 10^4 \\ 5.5 \times 10^4 \end{pmatrix} \quad \begin{matrix} \leftarrow R_4 \\ \leftarrow R_6 \end{matrix}$$

(c) For Ckt C2 with $K_1 = -K_2 = 5$, $R_1 = R_2 = 1\ \text{k}\Omega$ and $R_3 = R_5 = 10\ \text{k}\Omega$ the constraints are

$$R_4 := 10000 \quad R_6 := 10000 \quad \text{Given} \quad \left(\frac{10000 + R_6}{10000}\right)\cdot\left(\frac{R_4}{10000}\right) = 5 \quad \frac{10000 + R_4}{10000} = 5$$

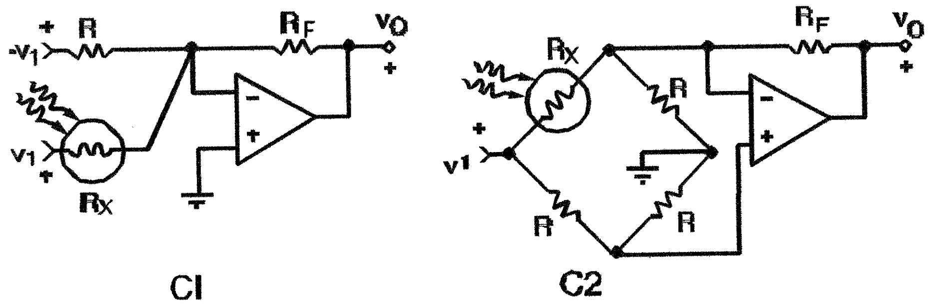
$$\text{Find}(R_4, R_6) = \begin{pmatrix} 4 \times 10^4 \\ 2.5 \times 10^3 \end{pmatrix} \quad \begin{matrix} \leftarrow R_4 \\ \leftarrow R_6 \end{matrix}$$

(d)

Circuit	Resistors	OP AMPS	Source loading
C1	6	1	R_5 loads v_1 and R_3 loads v_2
C2	6	2	None

C1 requires one less OP AMP but loads both sources.

4-58



(a) Circuit C1 is a half-bridge amplifier which operates as an inverting summer.

$$v_O = \left(\frac{-R_F}{R}\right)\cdot(-v_1) + \left(\frac{-R_F}{R_X}\right)\cdot(v_1) = R_F\cdot\left(\frac{1}{R} - \frac{1}{R_X}\right)\cdot v_1 = K\cdot v_1$$

Circuit C2 is a full bridge amplifier. Writing node equations at the OP AMP inputs:

Inverting Input: $\left(\frac{1}{R_X} + \frac{1}{R} + \frac{1}{R_F}\right)\cdot v_N - \frac{v_1}{R_X} - \frac{v_O}{R_F} = 0$ Noninverting Input $\frac{2\cdot v_P}{R} - \frac{v_1}{R} = 0$

For an ideal OP AMP $v_P = v_N$ $v_N = v_P = \frac{v_1}{2}$ Hence $v_O = \left(1 + \frac{R_F}{R} - \frac{R_F}{R_X}\right)\cdot\frac{v_1}{2}$

(b) For circuit C1 In bright sunlight $R_X = 2000$ $R_F\cdot\left(\frac{1}{R} - \frac{1}{2000}\right)\cdot 15 = -10$

In complete dark $R_X = 10000$ $R_F\cdot\left(\frac{1}{R} - \frac{1}{10000}\right)\cdot 15 = 10$ try $R_F := 1000$ $R := 1000$

Given $R_F\cdot\left(\frac{1}{R} - \frac{1}{2000}\right)\cdot 15 = -10$ $R_F\cdot\left(\frac{1}{R} - \frac{1}{10000}\right)\cdot 15 = 10$ $\text{Find}(R_F, R) = \begin{pmatrix} 3.333 \times 10^3 \\ 3.333 \times 10^3 \end{pmatrix}$ <---R_F <---R

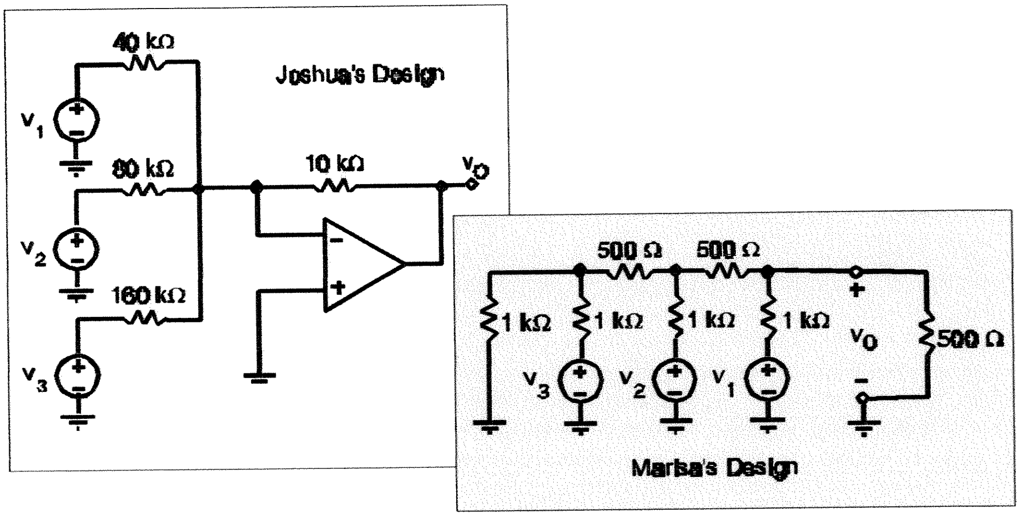
(c) For circuit C2 In bright sunlight $\left(1 + \frac{R_F}{R} - \frac{R_F}{2000}\right)\cdot\frac{15}{2} = -10$

In complete dark $\left(1 + \frac{R_F}{R} - \frac{R_F}{10000}\right)\cdot\frac{15}{2} = 1($Try $R := 1000$ $R_F := 1000$ Given

$\left(1 + \frac{R_F}{R} - \frac{R_F}{2000}\right)\cdot\frac{15}{2} = -10$ $\left(1 + \frac{R_F}{R} - \frac{R_F}{10000}\right)\cdot\frac{15}{2} = 10$ $\text{Find}(R_F, R) = \begin{pmatrix} 6.667 \times 10^3 \\ 6.667 \times 10^3 \end{pmatrix}$ <---R_F <---R

(d) Circuit C2 requires 2 more resistors than C1.
When $R_X = 2\ k\Omega$, C2 dissipates 99 mW, whereas Circuit C1 dissipates 210 mW.
When $R_X = 10\ k\Omega$, C2 dissipates 32 mW, whereas Circuit C1 dissipates 120 mW. .

4-59

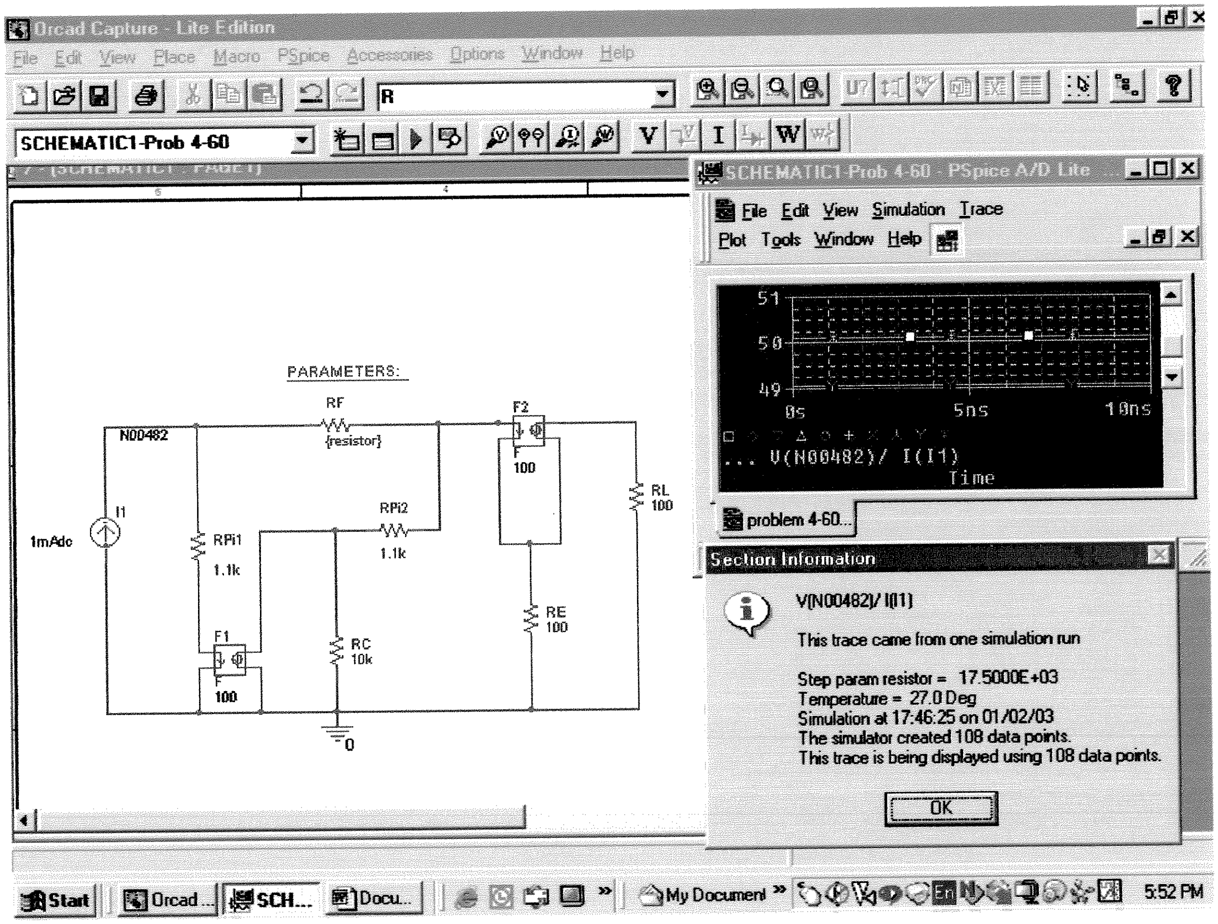


(a) Neither design is totally correct. In the required input output relationship the MSB is v_3 and the LSB is v_1. In both proposed designs the MSB is v_1 and the LSB is v_3. The connections to v_1 and v_3 must be interchanged in both designs.

(b)	Standard Resistors	Custom Resistors	OP AMPs	Parts Costs per Unit
Joshua's Design	1	3	1	$0.29
Marisa's Design	4	3	0	$0.295

Joshua's design cannot drive the required load since the OP AMP must source 15/500 = 30 mA, which is more than the 10 mA rating of the available OP AMPs. Marisa's design can drive the specified load since the 500 ohm load is built into her circuit. Choose Marisa's design since it can drive the load and the parts cost are nearly the same.

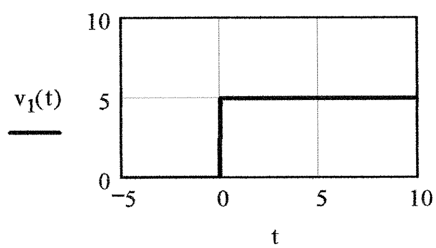
4-60 The figure below shows a circuit diagram as it appears in Orcad. It also shows the Probe output and the specific run that yielded the closest Rin to 50Ω. The run was conducted by varying RF using the PARAM function and running a Transient Analysis using a Parametric sweep. Thus, Rin = 50Ω occurs for RF about 17.5 kΩ. The input resistance can be found by setting the input current source to 1 mA and looking at the node voltage V(N00482) divided by I1 (1.0 mA) in Probe. The calculated value of Rin is found in the output file produced by PSpice's Probe . For 1 mA input and RF = 17.5 kΩ the output file lists V(N00482)/I1 = 50.1Ω.


Orcad Capture - Lite Edition
File Edit View Place Macro PSpice Accessories Options Window Help
SCHEMATIC1-Prob 4-60
SCHEMATIC1-Prob 4-60 - PSpice A/D Lite
File Edit View Simulation Trace
Plot Tools Window Help
PARAMETERS:
RF
{resistor}
N00482
F2
100
RL
100
I1
1mAdc
RPi1
1.1k
RPi2
1.1k
RE
100
F1
100
RC
10k
0
51
50
49
0s
5ns
10ns
V(N00482)/ I(I1)
Time
problem 4-60...
Section Information
V(N00482)/ I(I1)
This trace came from one simulation run
Step param resistor = 17.5000E+03
Temperature = 27.0 Deg
Simulation at 17:46:25 on 01/02/03
The simulator created 108 data points.
This trace is being displayed using 108 data points.
OK
Start
Orcad ...
SCH...
Docu...
My Document
5:52 PM

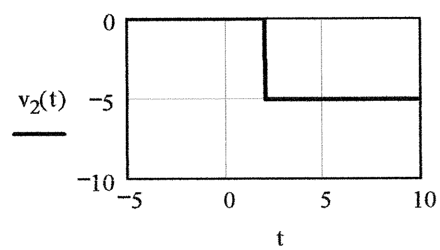
CHAPTER 5, Both Versions

$u(x) := \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \ge 0 \end{cases}$ $\delta(x) := 100 \cdot (u(x) - u(x - .01))$ $r(x) := \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } x \ge 0 \end{cases}$

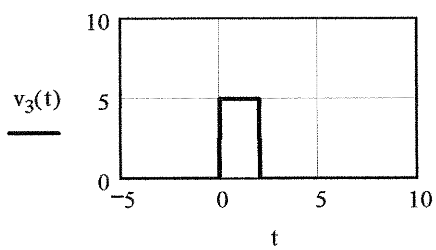
5-1 **(a)** $v_1(t) := 5 \cdot u(t)$



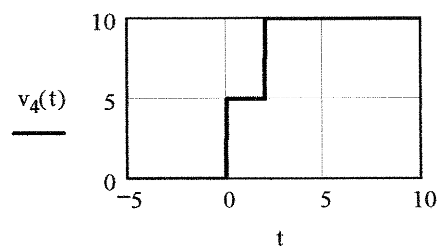
(b) $v_2(t) := -5 \cdot u(t - 2)$



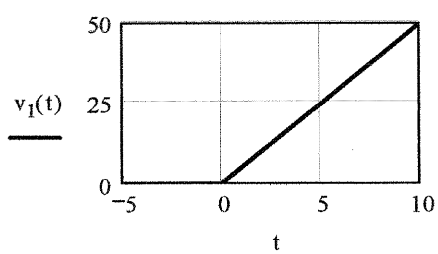
(c) $v_3(t) := v_1(t) + v_2(t)$



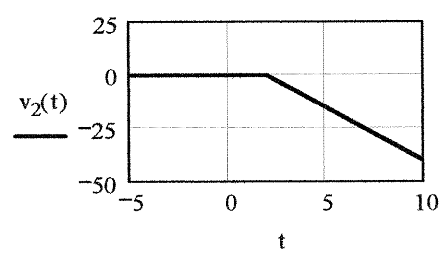
(d) $v_4(t) := v_1(t) - v_2(t)$



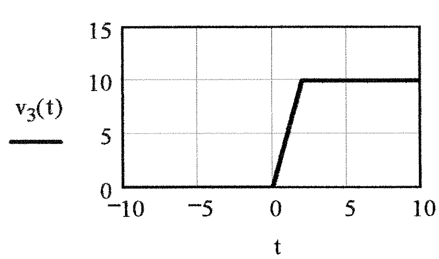
5-2 **(a)** $v_1(t) := 5 \cdot r(t)$



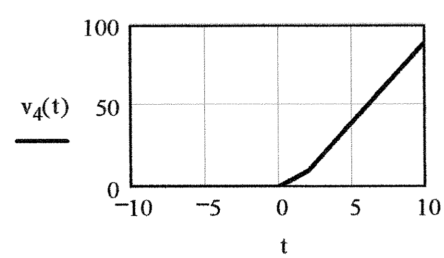
(b) $v_2(t) := -5 \cdot r(t - 2)$



(c) $v_3(t) := v_1(t) + v_2(t)$

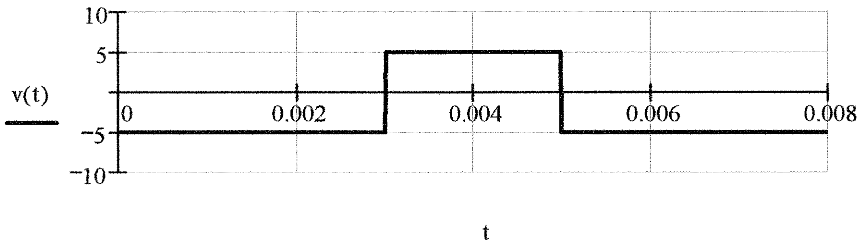


(d) $v_4(t) := v_1(t) - v_2(t)$



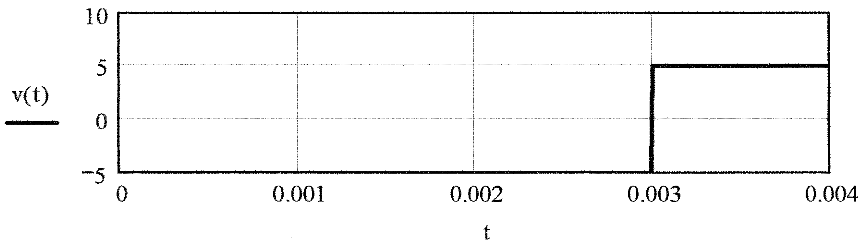
5-3 $v(t) = V_A \cdot (u(t + 0.5 \cdot T) - u(t - 0.5 \cdot T))$

5-4 $v(t) := -5 \cdot u[-(t - 0.003)] + 5 \cdot u(t - 0.003) - (10 \cdot u(t - 0.005))$



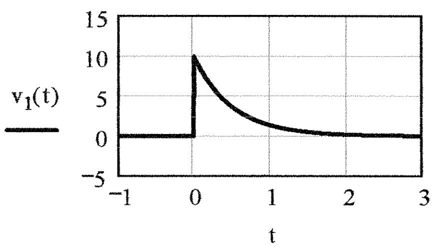
5-5

$v(t) := 2500 \cdot r(t) \cdot \left(u(t) - u\left(t - 2 \cdot 10^{-3}\right)\right)$



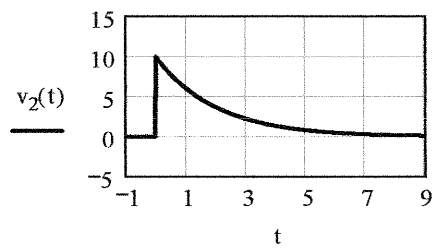
5-6 **(a)** $v_1(t) := 10 \cdot \exp((-2 \cdot t)) \cdot u(t)$

$V_A = 10 \quad T_C = 0.5$



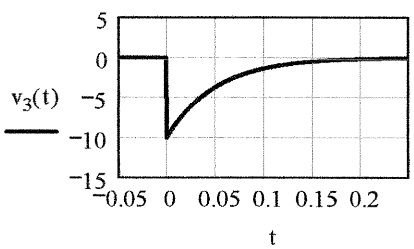
(b) $v_2(t) := 10 \cdot \exp(-0.5 \cdot t) \cdot u(t)$

$V_A = 10 \quad T_C = 2$



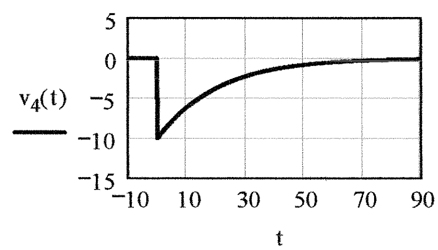
(c) $v_3(t) := -10 \cdot \exp(-20 \cdot t) \cdot u(t)$

$V_A = -10 \quad T_C = 0.05$

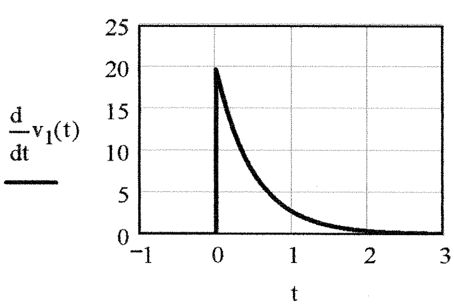


(d) $v_4(t) := -10 \cdot \exp(-0.05 \cdot t) \cdot u(t)$

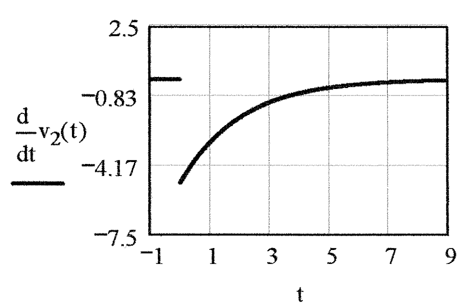
$V_A = -10 \quad T_C = 20$



5-7 **(a)** $v_1(t) := [10 \cdot -\exp((-2 \cdot t)) \cdot u(t)]$



(b) $v_2(t) := 10 \cdot \exp(-0.5 \cdot t) \cdot u(t)$



5-8 $V_A \cdot \exp\left(\frac{-0.005}{T_C}\right) = 4 \quad V_A \cdot \exp\left(\frac{-0.006}{T_C}\right) = 2 \quad \frac{\exp\left(\frac{-0.005}{T_C}\right)}{\exp\left(\frac{-0.006}{T_C}\right)} = 2 \quad \exp\left(\frac{0.006}{T_C}\right) = 2 \cdot \exp\left(\frac{0.005}{T_C}\right)$

$\frac{0.006}{T_C} = \ln(2) + \frac{0.005}{T_C} \quad T_C := \frac{1}{1000 \cdot \ln(2)} \quad T_C = 1.4427 \times 10^{-3} \quad V_A := 4 \cdot \exp\left(\frac{0.005}{T_C}\right) \quad V_A = 128$

Checking solution $\quad 128 \cdot \exp\left(\frac{-0.005}{1.4427 \cdot 10^{-3}}\right) = 4 \quad 128 \cdot \exp\left(\frac{-0.006}{1.4427 \cdot 10^{-3}}\right) = 2$ **Checks**

5-9

$V_A \cdot \exp\left(\frac{-0.005}{T_C}\right) = 5 \quad V_A \cdot \exp\left(\frac{-0.007}{T_C}\right) = 3.5 \quad \frac{\exp\left(\frac{-0.005}{T_C}\right)}{\exp\left(\frac{-0.007}{T_C}\right)} = \frac{10}{7} \exp\left(\frac{0.007}{T_C}\right) = \frac{10}{7} \cdot \exp\left(\frac{0.005}{T_C}\right)$

$\frac{0.007}{T_C} = \ln\left(\frac{10}{7}\right) + \frac{0.005}{T_C} \quad T_C := \frac{1}{500 \cdot \ln\left(\frac{10}{7}\right)} \quad T_C = 5.6073 \times 10^{-3} \quad V_A := 5 \cdot \exp\left(\frac{0.005}{T_C}\right) \quad V_A = 12.196$

$12.196 \cdot \exp\left(\frac{-0.005}{5.6073 \cdot 10^{-3}}\right) = 5 \quad 12.196 \cdot \exp\left(\frac{-0.007}{5.6073 \cdot 10^{-3}}\right) = 3.5 \quad 12.196 \cdot \exp\left(\frac{-0.002}{5.6073 \cdot 10^{-3}}\right) = 8.537$

5-10 Substituting the suggested solution into the differential equation yields

$$\frac{d}{dt} V_A \cdot e^{-\alpha \cdot t} + \alpha \cdot V_A \cdot e^{-\alpha \cdot t} = -\alpha \cdot V_A \cdot e^{-\alpha \cdot t} + \alpha \cdot V_A \cdot e^{-\alpha \cdot t}$$

The expression on the right side sums to zero--QED

5-11 (a) $v_1(t) := 10 \cdot \cos(2000 \cdot \pi \cdot t) + 10 \cdot \sin(2000 \cdot \pi \cdot t) \quad f := 1000 \quad a := 10 \quad b := 10$

$T_0 := \frac{1}{f} \quad \phi := \text{atan}\left(\frac{-b}{a}\right) \quad T_S := -\phi \cdot \frac{T_0}{2 \cdot \pi} \quad V_A := \sqrt{\left(a^2 + b^2\right)} \quad \phi := \phi \cdot \frac{180}{\pi}$

$T_0 = 1 \times 10^{-3} \quad f = 1 \times 10^3 \quad V_A = 14.142 \quad T_S = 1.25 \times 10^{-4} \quad \phi = -45$

(b) $v_2(t) := -30 \cdot \cos(2000 \cdot \pi \cdot t) + (-20) \cdot \sin(2000 \cdot \pi \cdot t) \quad f := 1000 \quad a := -30 \quad b := -20$

$T_0 := \frac{1}{f} \quad \phi := \text{atan}\left(\frac{-b}{a}\right) + \pi \quad T_S := -\phi \cdot \frac{T_0}{2 \cdot \pi} \quad V_A := \sqrt{\left(a^2 + b^2\right)} \quad \phi := \phi \cdot \frac{180}{\pi}$

$T_0 = 1 \times 10^{-3} \quad f = 1 \times 10^3 \quad V_A = 36.056 \quad T_S = -4.064 \times 10^{-4} \quad \phi = 146.31$

5-11 Continued

(c) $v3(t) := 10\cdot\cos\left(2\cdot\pi\cdot\frac{t}{10}\right) + (-10)\cdot\sin\left(2\cdot\pi\cdot\frac{t}{10}\right)$ $\quad f := \frac{1}{10}$ $\quad a := 10 \quad b := -10$

$T_0 := \frac{1}{f} \quad \phi := \operatorname{atan}\left(\frac{-b}{a}\right) \quad T_S := -\phi\cdot\frac{T_0}{2\cdot\pi} \quad V_A := \sqrt{\left(a^2 + b^2\right)} \quad \phi := \phi\cdot\frac{180}{\pi}$

$T_0 = 10 \quad f = 0.1 \quad V_A = 14.142 \quad T_S = -1.25 \quad \phi = 45$

(d) $v_4(t) := -20\cdot\cos(800\cdot\pi\cdot t) + 30\cdot\sin(800\cdot\pi\cdot t)$ $\quad f := 400$ $\quad a := -20 \quad b := 30$

$T_0 := \frac{1}{f} \quad \phi := \operatorname{atan}\left(\frac{-b}{a}\right) - \pi \quad T_S := -\phi\cdot\frac{T_0}{2\cdot\pi} \quad V_A := \sqrt{\left(a^2 + b^2\right)} \quad \phi := \phi\cdot\frac{180}{\pi}$

$T_0 = 2.5 \times 10^{-3} \quad f = 400 \quad V_A = 36.056 \quad T_S = 8.59 \times 10^{-4} \quad \phi = -123.69$

5-12 $v(t) := (10 - 30)\cdot\cos(2000\cdot\pi\cdot t) + (10 - 20)\cdot\sin(2000\cdot\pi\cdot t)$ $f := 1000 \quad a := -20 \quad b := -10$

$T_0 := \frac{1}{f} \quad \phi := \operatorname{atan}\left(\frac{-b}{a}\right) + \pi \quad T_S := -\phi\cdot\frac{T_0}{2\cdot\pi} \quad V_A := \sqrt{\left(a^2 + b^2\right)} \quad \phi := \phi\cdot\frac{180}{\pi}$

$T_0 = 1 \times 10^{-3} \quad f = 1 \times 10^{3} \quad V_A = 22.361 \quad T_S = -4.262 \times 10^{-4} \quad \phi = 153.435$

5-13 $v(t) = 15\cdot\cos(\omega_0\cdot t + \phi) \quad \frac{d}{dt}v(t) = -15\cdot\sin(\omega_0\cdot t + \phi)\cdot\omega_0$ positive slope at t = 0 requires

$-15\cdot\sin(\phi) > 0$ hence $\phi < 0 \quad v(0) = 15\cdot\cos(\phi) = 10 \quad \phi := -\operatorname{acos}\left(\frac{10}{15}\right) \quad \phi = -0.841$

$a := 15\cdot\cos(\phi) \quad a = 10 \quad b := -15\cdot\sin(\phi) \quad b = 11.18$

5-14 The waveform is of the form $v(t) = V_A\cdot\cos\left[2\cdot\pi\cdot5\cdot10^6\cdot\left(t - 125\cdot10^{-9}\right)\right] \quad v(0) = -10$

$V_A\cdot\cos\left[2\cdot\pi\cdot5\cdot10^6\cdot\left(0 - 125\cdot10^{-9}\right)\right] = -10 \quad V_A := \frac{-10}{\cos\left[2\cdot\pi\cdot5\cdot10^6\cdot\left(0 - 125\cdot10^{-9}\right)\right]} \quad V_A = 14.142$

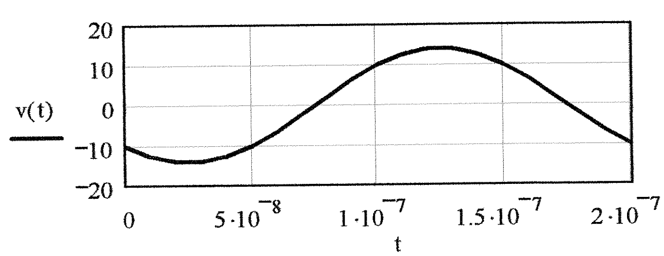
$T_0 := \frac{1}{5\cdot10^6} \quad T_S := 125\cdot10^{-9} \quad \phi := -2\cdot\pi\cdot\frac{T_S}{T_0} \quad a := V_A\cdot\cos(\phi) \quad b := -V_A\cdot\sin(\phi)$

$\phi = -3.927 \quad \phi\cdot\frac{180}{\pi} = -225 \quad a = -10 \quad b = -10$

$v(t) := a\cdot\cos\left(2\cdot\pi\cdot5\cdot10^6\cdot t\right) + b\cdot\sin\left(2\cdot\pi\cdot5\cdot10^6\cdot t\right) \quad t := 0, \frac{T_0}{20} .. T_0$

$v(0) = -10 \quad v\left(125\cdot10^{-9}\right) = 14.142$

Checks



5-15 (a) $v_1(t) = 20\cdot\cos(4000\cdot\pi\cdot t - \pi)$ $\quad f := 2000 \quad V_A := 20 \quad \phi := -\pi$

$T_0 := \frac{1}{f} \qquad a := V_A\cdot\cos(\phi) \qquad b := -V_A\cdot\sin(\phi)$

$f = 2\times 10^3 \quad T_0 = 5\times 10^{-4} \quad a = -20 \quad b = 0$

$v_2(t) = 20\cdot\cos\left(4000\cdot\pi\cdot t - \frac{\pi}{2}\right) \quad f := 2000 \quad V_A := 20 \quad \phi := \frac{-\pi}{2}$

(b) $T_0 := \frac{1}{f} \quad a := V_A\cdot\cos(\phi) \quad b := -V_A\cdot\sin(\phi)$

$f = 2\times 10^3 \quad T_0 = 5\times 10^{-4} \quad a := 0 \quad b = 20$

(c) $v_3(t) = 30\cdot\cos\left(2\cdot\pi\cdot\frac{t}{400} - \frac{\pi}{4}\right) \quad f := \frac{1}{400} \quad V_A := 30 \quad \phi := -\frac{\pi}{4}$

$T_0 := \frac{1}{f} \quad a := V_A\cdot\cos(\phi) \quad b := -V_A\cdot\sin(\phi)$

$f = 2.5\times 10^{-3} \quad T_0 = 400 \quad a = 21.213 \quad b = 21.213$

(d) $v_4(t) = 60\cdot\left(\sin\left(2000\cdot\pi\cdot t + \frac{\pi}{4}\right)\right) = 60\cdot\cos\left[\left(2000\cdot\pi\cdot t + \frac{\pi}{4}\right) - \frac{\pi}{2}\right] \quad f := 1000 \quad V_A := 60 \quad \phi := \frac{-\pi}{4}$

$T_0 := \frac{1}{f} \quad a := V_A\cdot\cos(\phi) \quad b := -V_A\cdot\sin(\phi)$

$f = 1\times 10^3 \quad T_0 = 1\times 10^{-3} \quad a = 42.426 \quad b = 42.426$

5-16 $v(t) = (20\cdot\cos(4000\cdot\pi\cdot t - \pi)) + 20\cdot\cos\left(4000\cdot\pi\cdot t - \frac{\pi}{2}\right)$

$v(t) = 20\cdot\cos(4000\cdot\pi\cdot t)\cdot\cos(\pi) + 20\cdot\sin(4000\cdot\pi\cdot t)\cdot\sin(\pi) + \left(20\cdot\cos(4000\cdot\pi\cdot t)\cdot\cos\left(\frac{\pi}{2}\right) + 20\cdot\sin(4000\cdot\pi\cdot t)\cdot\sin\left(\frac{\pi}{3}\right.\right.$

$\cos(\pi) = -1 \quad \sin(\pi) = 0 \quad \cos\left(\frac{\pi}{2}\right) = 0 \quad \sin\left(\frac{\pi}{2}\right) = 1 \quad f := 2000 \quad T_0 := \frac{1}{f}$

$v(t) = -20\cdot\cos(4000\cdot\pi\cdot t) + 20\cdot\sin(4000\cdot\pi\cdot t) \quad a := -20 \quad b := 20 \quad V_A := \sqrt{a^2 + b^2} \quad \phi := \text{atan}\left(\frac{-b}{a}\right) - \pi$

$f = 2\times 10^3 \quad T_0 = 5\times 10^{-4} \quad V_A = 28.284 \quad \phi\cdot\frac{180}{\pi} = -135$

checking $a := V_A\cdot\cos(\phi) \quad b := -V_A\cdot\sin(\phi) \quad a = -20 \quad b = 20$ **checks**

5-17 Waveform is of the form $v(t) = 75\cdot\cos\left[200\cdot 10^3\cdot\pi(t - T_S)\right] \quad \cos\left[200\cdot 10^3\cdot\pi\left(5\cdot 10^{-6} - T_S\right)\right] = 0$

$\left[200\cdot 10^3\cdot\pi\left(5\cdot 10^{-6} - T_S\right)\right] = \frac{3\cdot\pi}{2} \quad T_S := 5\cdot 10^{-6} - \frac{3\cdot\pi}{2}\cdot\frac{1}{\left(200\cdot 10^3\cdot\pi\right)} \quad T_S = -2.5\times 10^{-6} \quad T_0 := \frac{1}{10^5}$

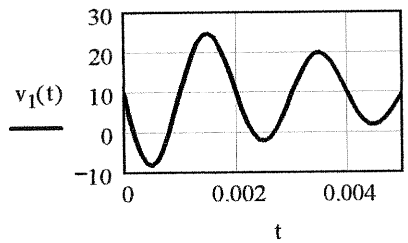
positive slope requires $-V_A\cdot 200\cdot 10^3\cdot\pi\cdot\sin\left[200\cdot 10^3\cdot\pi\left(5\cdot 10^{-6} - T_S\right)\right] > 0$

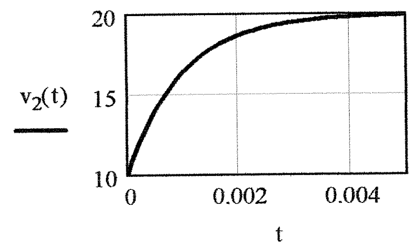
$-\sin\left[200\cdot 10^3\cdot\pi\left(5\cdot 10^{-6} - T_S\right)\right] = 1$ **slope is positive** $\quad \phi := -2\cdot\pi\cdot\frac{T_S}{T_0} \quad a := 75\cdot\cos(\phi) \quad b := -75\cdot\sin(\phi)$

$a := 0 \quad b = -75 \quad \frac{180}{\pi}\cdot\phi = 90 \quad T_S = -2.5\times 10^{-6}$

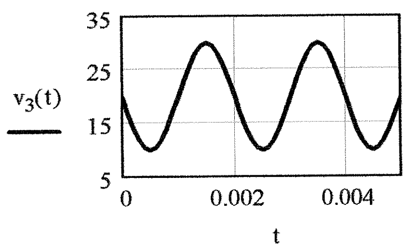
5-18 **(a)** $v_1(t) := 10\cdot\left(1 - 2\cdot e^{-200\cdot t}\cdot\sin(1000\cdot\pi\cdot t)\right)\cdot u(t)$ **(b)** $v_2(t) := \left(20 - 10\cdot e^{-1000\cdot t}\right)\cdot u(t)$

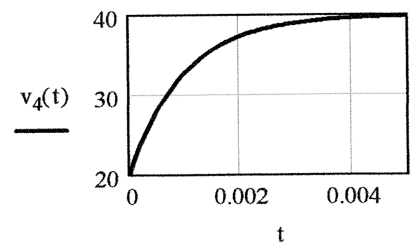
$t := 0, 10^{-4} .. 5\cdot 10^{-3}$





(c) $v_3(t) := 10\cdot(2 - \sin(1000\cdot\pi\cdot t))\cdot u(t)$ **(d)** $v_4(t) := 10\cdot\left(4 - 2\cdot e^{-1000\cdot t}\right)\cdot u(t)$





5-19 $v(t) = V_A\cdot\alpha\cdot t\cdot\exp(-\alpha\cdot t)$ $\frac{d}{dt}V_A\cdot\alpha\cdot t\cdot\exp(-\alpha\cdot t) = V_A\cdot\alpha\cdot\exp(-\alpha\cdot t) - V_A\cdot\alpha^2\cdot t\cdot\exp(-\alpha\cdot t)$

$V_A\cdot\alpha\cdot\exp(-\alpha\cdot t) - V_A\cdot\alpha^2\cdot t\cdot\exp(-\alpha\cdot t) = 0$ **at** $t = \frac{1}{\alpha}$ **hence** $v_{MAX} = V_A\cdot\exp(-1) = 0.368\cdot V_A$

5-20 **from the final condition** $V_A := 10$ **from t = 0 condition** $V_A - V_B = 8$**, hence** $V_B := 2$

from the t = 5ms condition we solve for α

$\alpha := 1000$ Given $V_A - V_B\cdot e^{-\alpha\cdot 5\cdot 10^{-6}} = 9$ $\alpha := \text{Find}(\alpha)$ $\alpha = 1.386\times 10^5$ $v(t) := V_A - V_B\cdot e^{-\alpha\cdot t}$

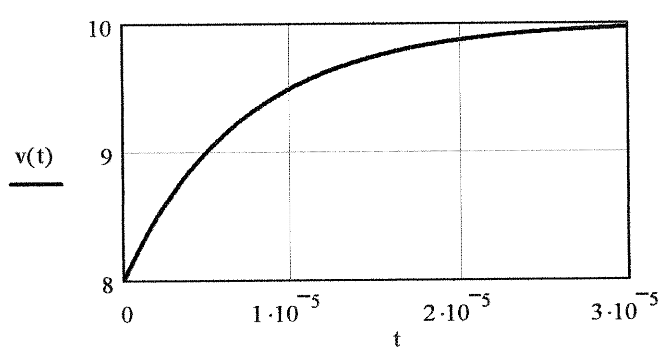
$t := 0, 10^{-7} .. 30\cdot 10^{-6}$

Checking

$v(0) = 8$

$v\left(5\cdot 10^{-6}\right) = 9$

$v(1) = 10$



5-21 $T_0 := 2 \cdot 2.5 \cdot 10^{-3}$ $f := \frac{1}{T_0}$ $f = 200$ $\beta := 2 \cdot \pi \cdot f$ $\beta = 1.257 \times 10^3$

$V_A \cdot e^{-\alpha \cdot 0.001} \cdot \sin(1257 \cdot 0.001) = 3.5$ $V_A \cdot e^{-\alpha \cdot 0.002} \cdot \sin(1257 \cdot 0.002) = 0.8$

$\frac{V_A \cdot e^{-\alpha \cdot 0.001} \cdot \sin(1257 \cdot 0.001)}{V_A \cdot e^{-\alpha \cdot 0.002} \cdot \sin(1257 \cdot 0.002)} = \frac{3.5}{0.8}$ yields $\alpha := 993.577$ $V_A := \frac{3.5}{\left(e^{-\alpha \cdot 0.001} \cdot \sin(1257 \cdot 0.001)\right)}$ $V_A = 9.938$

$t := 0, .00025 .. 0.004$ $v(t) := V_A \cdot e^{-\alpha \cdot t} \cdot \sin(\beta \cdot t)$

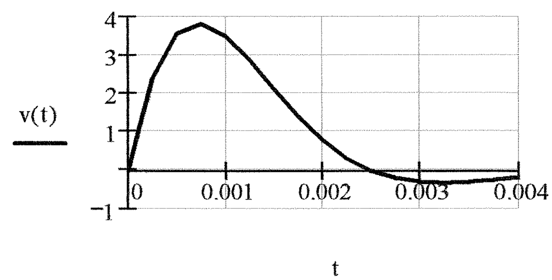
Checking

$v(0.001) = 3.5$

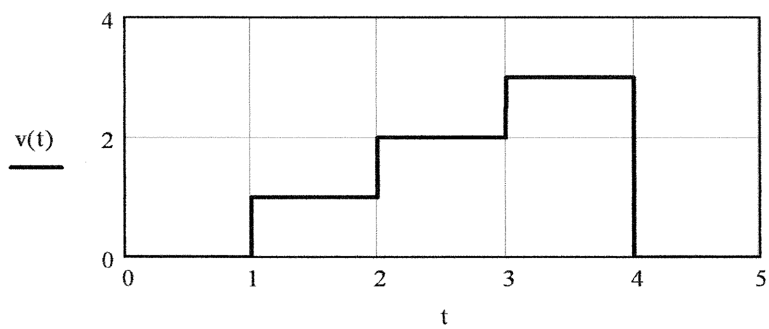
$v(0.002) = 0.801$

$v(0.0025) = 0$

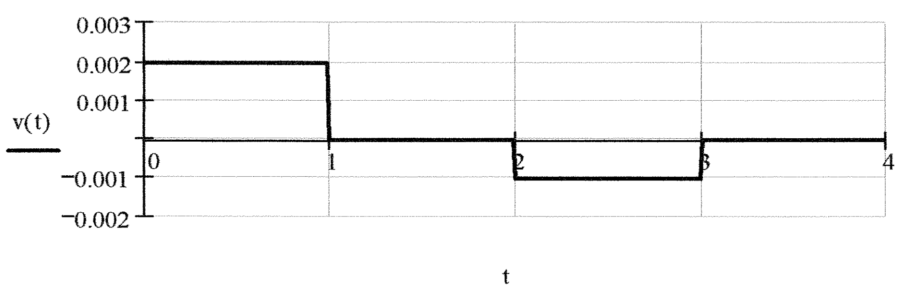
$v(0.005) = 0$



5-22 $v(t) := u(t-1) + u(t-2) + u(t-3) - 3 \cdot u(t-4)$ $t := 0, 0.002 .. 5$



5-23 $v(t) := 2 \cdot 10^{-3} u(t) - 2 \cdot 10^{-3} \cdot u(t-1) - 10^{-3} u(t-2) + 10^{-3} u(t-3)$ $t := 0, 0.01 .. 6$



5-24 from condition at t = 0 $V_A := 5$ from the minimum value condition $V_B := 7$

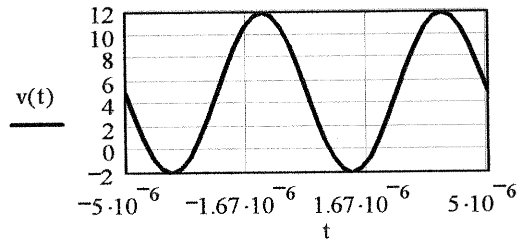
from the periodic condtion $T_0 := 5 \cdot 10^{-6}$ $f := \frac{1}{T_0}$ $\beta := 2 \cdot \pi \cdot f$ $\beta = 1.257 \times 10^6$

$v(t) := V_A - V_B \cdot \sin(\beta \cdot t)$ $t := -T_0, -T_0 + 0.05 \cdot T_0 .. T_0$

checking

$v(0) = 5$ $v(T_0) = 5$

$v\left(\frac{T_0}{4}\right) = -2$ $v\left(\frac{T_0}{4} + T_0\right) = -2$



5-25 The waveform is of the form $v(t) = V_A + V_B \cdot \cos\left(2 \cdot \pi \cdot \frac{t}{T_0}\right)$ with $T_0 := 2 \cdot 10^{-3}$ $\frac{2}{T_0} = 1 \times 10^3$

$V_A + V_B \cdot \cos(0) = -5$ hence $V_A = -5 - V_B$

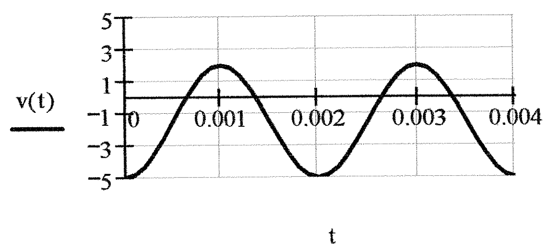
$V_A + V_B \cdot \cos(\pi) = 2$ hence $V_A = 2 + V_B$ $2 + V_B = -5 - V_B$ or $V_B := -3.5$ $V_A := -1.5$

$v(t) := -1.5 - 3.5 \cdot \cos(1000 \cdot \pi \cdot t)$ $t := 0, 0.05 \cdot T_0 .. 2 \cdot T_0$

Checking $T_0 = 2 \times 10^{-3}$

$v(0) = -5$ $v(T_0) = -5$

$v\left(\frac{T_0}{2}\right) = 2$ $v\left(\frac{T_0}{2} + T_0\right) = 2$

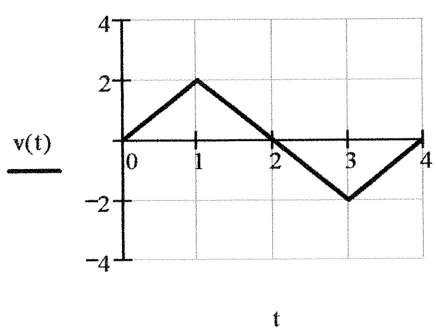


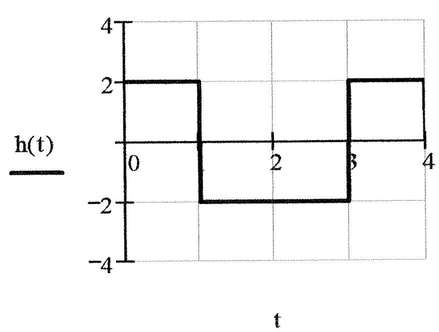
5-26 From Prob. 5-25 above $v(t) = -1.5 - 3.5 \cdot \cos(1000 \cdot \pi \cdot t)$

$\frac{d}{dt} v(t) = -3.5 \cdot 1000 \cdot \pi \cdot (-\sin(1000 \cdot \pi \cdot t)) = 1.1 \cdot 10^4 \cdot \sin(1000 \cdot \pi \cdot t)$

5-27 $v(t) := 2 \cdot r(t) - (4 \cdot r(t-1)) + 4 \cdot r(t-3) - 2 \cdot r(t-4)$

$h(t) := 2 \cdot u(t) - 4 \cdot u(t-1) + 4 \cdot u(t-3) - 2 \cdot u(t-4)$ $t := 0, 0.004 .. 4$





5-28 The waveform is $v(t) = V_A \cdot (1 - \exp(-\alpha \cdot t))$ $\quad v(0) = 0 \quad v(\infty) = V_A$ <---Final Value

50% rise means $V_A \cdot (1 - \exp(-\alpha \cdot T_1)) = \frac{V_A}{2}$ or $\exp(\alpha \cdot T_1) = 2$ or $T_1 = \left(\frac{\ln(2)}{\alpha}\right) = \frac{0.693}{\alpha}$

5-29 $T_0 := 0.005 \quad f := T_0^{-1} \quad \beta := 2 \cdot \pi \cdot f \quad \beta = 1.257 \times 10^3 \quad \frac{T_0}{2} = 2.5 \times 10^{-3}$

$$V_A \cdot \exp(-\alpha \cdot 1.3 \cdot 10^{-3}) \cdot \sin\left[1257 \cdot (1.3 \cdot 10^{-3})\right] = 18$$

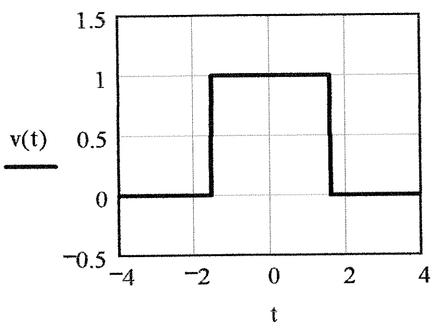
$$V_A \cdot \exp\left[-\alpha \cdot \left(1.3 \cdot 10^{-3} + \frac{T_0}{2}\right)\right] \cdot \sin\left[1257 \cdot \left(1.3 \cdot 10^{-3} + \frac{T_0}{2}\right)\right] = -10$$

$$\frac{V_A \cdot \exp(-\alpha \cdot 1.3 \cdot 10^{-3}) \cdot \sin\left[1257 \cdot (1.3 \cdot 10^{-3})\right]}{V_A \cdot \exp\left[-\alpha \cdot (1.3 \cdot 10^{-3} + 2.5 \cdot 10^{-3})\right] \cdot \sin\left[1257 \cdot (1.3 \cdot 10^{-3} + 2.5 \cdot 10^{-3})\right]} = \left(-1 \cdot \exp(2.5 \cdot 10^{-3} \cdot \alpha)\right) = \frac{18}{-10}$$

$\alpha := \frac{\ln\left(\frac{18}{10}\right)}{2.5 \cdot 10^{-3}} \quad \alpha = 235.115 \quad t := 1.3 \cdot 10^{-3} \quad V_A := \frac{18}{e^{-\alpha \cdot t} \cdot \sin(\beta \cdot t)} \quad V_A = 24.483 \quad \beta = 1.257 \times 10^3$

$v(t) := V_A \cdot \exp(-\alpha \cdot t) \cdot \sin(\beta \cdot t) \quad v(1.3 \cdot 10^{-3}) = 18 \quad v(1.3 \cdot 10^{-3} + 0.5 \cdot T_0) = -10$ <---checking

5-30 $v(t) := u(\cos(t)) \quad t := -4, -3.99 .. 4$ The period of v(t) is equal to the period of sin(t), hence $T_0 := 2\pi$



Checking $t := 0.5 \quad v(t) = 1 \quad v(t + T_0) = 1$

$t := 1.5 \quad v(t) = 1 \quad v(t + T_0) = 1$

v(t) is not zero for t < 0 because cos(t) has both positive and negative value for t < 0.

Changing the period of the sinusoidal argument will change the period of v(t).

Changing to $v(t) = V_A u(\cos(t))$ will change the amplitude.

5-31 (a) $v_1(t) = 150 - 80 \cdot \sin(2000 \cdot \pi \cdot t) \cdot u(t)$ aperiodic, causal $V_{max} := 150 + 80 \quad V_{min} := 150 - 80$

$V_p := V_{max} \quad V_{pp} := V_{max} - V_{min} \quad V_p = 230 \quad V_{pp} = 160$

(b) $v_2(t) = 40 \cdot \sin(2000 \cdot \pi \cdot t) \cdot (u(t) - u(t - 1))$ aperiodic, causal $V_p = 40 \quad V_{pp} = 80$

(c) $v_3(t) = 15 \cdot \cos(2000 \cdot \pi \cdot t) + 10 \cdot \sin(2000 \cdot \pi \cdot t)$ periodic, noncausal $V_p := \sqrt{10^2 + 15^2}$

$V_p = 18.028 \quad V_{pp} := 2 \cdot V_p \quad V_{pp} = 36.056 \quad V_{rms} := \frac{V_p}{\sqrt{2}} \quad V_{rms} = 12.748 \quad V_{avg} = 0$

(d) $v_4(t) = (10 - 5 \cdot \exp(-400 \cdot t) \cdot u(t))$ aperiodic, causal $V_p = 10 \quad V_{pp} = 5$

5-32 (a) $v_1(t) := 10\cdot\cos(2000\cdot\pi\cdot t) + 10\cdot\sin(2000\cdot\pi\cdot t)$ $\quad f := 1000 \quad a := 10 \quad b := 10$

$V_A := \sqrt{(a^2 + b^2)} \quad V_p := V_A \quad V_{pp} := 2\cdot V_A \quad V_{rms} := V_A\cdot(\sqrt{2})^{-1} \quad V_{avg} := 0$

$V_p = 14.142 \quad V_{pp} = 28.284 \quad V_{rms} = 10 \quad V_{avg} = 0$

(b) $v_2(t) := -30\cdot\cos(2000\cdot\pi\cdot t) + (-20)\cdot\sin(2000\cdot\pi\cdot t)$ $\quad f := 1000 \quad a := -30 \quad b := -20$

$V_A := \sqrt{(a^2 + b^2)} \quad V_p := V_A \quad V_{pp} := 2\cdot V_A \quad V_{rms} := V_A\cdot(\sqrt{2})^{-1} \quad V_{avg} := 0$

$V_p = 36.056 \quad V_{pp} = 72.111 \quad V_{rms} = 25.495 \quad V_{avg} = 0$

(c) $v_3(t) := 10\cdot\cos\left(2\cdot\pi\cdot\frac{t}{10}\right) + (-10)\cdot\sin\left(2\cdot\pi\cdot\frac{t}{10}\right)$ $\quad f := \frac{1}{10} \quad a := 10 \quad b := -10$

$V_A := \sqrt{(a^2 + b^2)} \quad V_p := V_A \quad V_{pp} := 2\cdot V_A \quad V_{rms} := V_A\cdot(\sqrt{2})^{-1} \quad V_{avg} := 0$

$V_p = 14.142 \quad V_{pp} = 28.284 \quad V_{rms} = 10 \quad V_{avg} = 0$

(d) $v_4(t) := -20\cdot\cos(800\cdot\pi\cdot t) + 30\cdot\sin(800\cdot\pi\cdot t)$ $\quad f := 400 \quad a := -20 \quad b := 30$

$V_A := \sqrt{(a^2 + b^2)} \quad V_p := V_A \quad V_{pp} := 2\cdot V_A \quad V_{rms} := V_A\cdot(\sqrt{2})^{-1} \quad V_{avg} := 0$

$V_p = 36.056 \quad V_{pp} = 72.111 \quad V_{rms} = 25.495 \quad V_{avg} = 0$

5-33 (a) $v_1(t) = 20\cdot\cos(4000\cdot\pi\cdot t - \pi) \quad V_A := 20 \quad V_p := V_A \quad V_{pp} := 2\cdot V_A \quad V_{rms} := V_A\cdot(\sqrt{2})^{-1}$

$V_{avg} := 0 \quad V_p = 20 \quad V_{pp} = 40 \quad V_{rms} = 14.142 \quad V_{avg} = 0$

(b) $v_2(t) = 20\cdot\cos\left(4000\cdot\pi\cdot t - \frac{\pi}{2}\right) \quad V_A := 20 \quad V_p := V_A \quad V_{pp} := 2\cdot V_A \quad V_{rms} := V_A\cdot(\sqrt{2})^{-1}$

$V_{avg} := 0 \quad V_p = 20 \quad V_{pp} = 40 \quad V_{rms} = 14.142 \quad V_{avg} = 0$

(c) $v_3(t) = 30\cdot\cos\left(2\cdot\pi\cdot\frac{t}{400} - \frac{\pi}{4}\right) \quad V_A := 30 \quad V_p := V_A \quad V_{pp} := 2\cdot V_A \quad V_{rms} := V_A\cdot(\sqrt{2})^{-1}$

$V_{avg} := 0 \quad V_p = 30 \quad V_{pp} = 60 \quad V_{rms} = 21.213 \quad V_{avg} = 0$

(d) $v_4(t) = 60\cdot\sin\left(2000\cdot\pi\cdot t + \frac{\pi}{4}\right) \quad V_A := 60 \quad V_p := V_A \quad V_{pp} := 2\cdot V_A \quad V_{rms} := V_A\cdot(\sqrt{2})^{-1}$

$V_{avg} := 0 \quad V_p = 60 \quad V_{pp} = 120 \quad V_{rms} = 42.426 \quad V_{avg} = 0$

5-34 $v(t) := u(t-1) + u(t-2) + u(t-3) - 3\cdot u(t-4) \quad V_p := 3 \quad V_{pp} := 3$

$V_{avg} := \frac{1}{4}\cdot\int_0^4 v(t)\,dt \quad V_{avg} = 1.5 \quad V_{rms} := \sqrt{\frac{1}{4}\cdot\int_0^4 v(t)^2\,dt} \quad V_{rms} = 1.871$

5-35 $v(t) := (2\cdot u(t) - 2\cdot u(t-1) - u(t-2) + u(t-3))\cdot 10^{-3}$ $\quad V_p := 0.002 \quad V_{pp} := 0.003$

$$V_{avg} = \frac{1}{4}\cdot\int_0^4 v(t)\,dt = \frac{10^{-3}}{4} = 2.5\cdot 10^{-4} \qquad V_{rms} = \sqrt{\frac{1}{4}\cdot\int_0^4 v(t)^2\,dt} = \frac{\sqrt{5}}{2}\cdot 10^{-3} = 1.118\cdot 10^{-3}$$

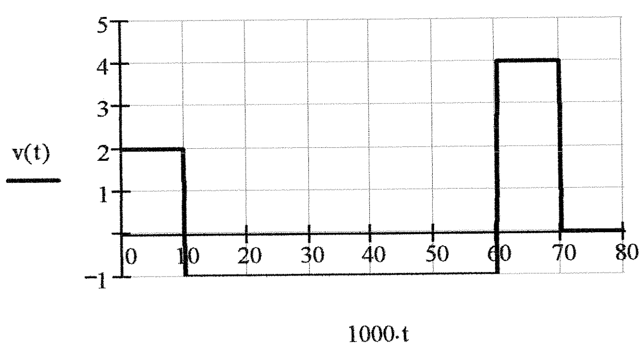
5-36 $v(t) = V_A\cdot\exp\left(\frac{-t}{T_C}\right) \qquad v(T_1) = V_A\cdot\exp\left(\frac{-T_1}{T_C}\right) = 0.707\cdot V_A \quad T_1 = -T_C\cdot\ln(0.707)$

$$v(T_2) = V_A\cdot\exp\left(\frac{-T_2}{T_C}\right) = 0.26\cdot V_A \quad T_2 = -T_C\cdot\ln(0.26)$$

$$T_{fall} = T_2 - T_1 = T_C\cdot(\ln(0.707) - \ln(0.26)) \quad \ln(0.707) - \ln(0.26) = 1 \qquad T_{fall} = T_C$$

5-37 The cosine term has a max of +10 and a min of -10, hence $V_0 + 10 < 15$ and $V_0 - 10 > -15$ which yields $5 > V_0 > -5$

5-38 $v(t) := 2\cdot u(t) - 3\cdot u(t-0.01) + 5\cdot u(t-0.06) - 4\,u(t-0.07) \quad t := 0, 0.0002 .. 0.08$



$V_{max} := 4 \quad V_{min} := -1 \qquad V_p := V_{max}$

$V_{pp} := |V_{max}| + |V_{min}| \quad V_{pp} = 5$

$$v_{avg} := \frac{1}{0.07}\cdot\int_0^{0.07} v(x)\,dx$$

$v_{avg} = 0.143$

5-39 Given $v(t) = 100 - 200\cdot\cos(2\cdot\pi\cdot 10000) - 75\cdot\sin(4\cdot\pi\cdot 10000\cdot t) + 35\cdot\cos(8\cdot\pi\cdot 10000\,mV$

$f_0 := 10^4 \quad T_0 := \frac{1}{f_0} \qquad T_0 = 1\times 10^{-4} \quad V_{avg} = 0.1\ V \qquad V_1 := 200 \qquad f_{max} := 40000$

5-40 Given $v(t) = V_A\cdot(u(t) - u(t-T)) \qquad V_{avg} = \frac{1}{T_0}\cdot\int_0^T V_A\,dt = V_A\cdot\frac{T}{T_0} = V_A\cdot D$

5-41
$$v(t) = \begin{cases} V_0 + (V_1 - V_0)\cdot\left(1 - \exp\left(\frac{-t}{T_{C1}}\right)\right) & \text{if } 0 \le t \le T_D \\ \left[V_0 + (V_1 - V_0)\cdot\left(1 - \exp\left(\frac{-T_D}{T_{C1}}\right)\right)\cdot\exp\left[\frac{-(t - T_D)}{T_{C2}}\right]\right] & \text{if } T_D \le t \end{cases}$$

$$v(0) = V_0 + (V_1 - V_0)\cdot(1 - 1) = V_0 \qquad v(\infty) = V_0 + (V_1 - V_0)\cdot\left(1 - \exp\left(\frac{-T_D}{T_{C1}}\right)\right)\cdot\exp(-\infty) = V_0$$

If $V_1 > V_0$ then $\quad V_{max} = v(T_D) = V_0 + (V_1 - V_0)\cdot\left(1 - \exp\left(\frac{-T_D}{T_{C1}}\right)\right)$ If $V_1 < V_0$ then $V_{max} = V_0$

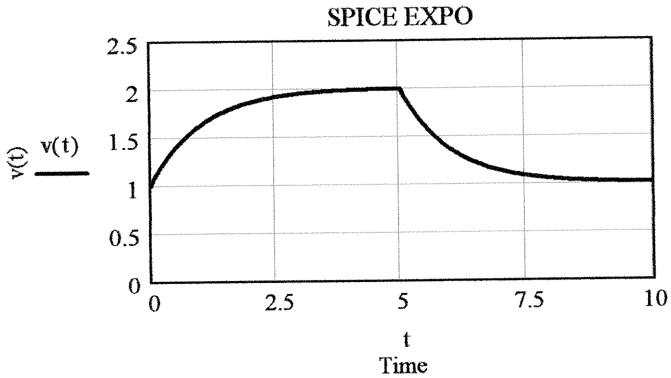
5-41 Continued

$V_0 := 1 \quad V_1 := 2 \quad T_{C1} := 1 \quad T_{C2} := 1 \quad T_D := 5 \cdot T_{C1}$ <---For graphing purposes only

$$v(t) := \begin{cases} V_0 + (V_1 - V_0)\cdot\left(1 - \exp\left(\frac{-t}{T_{C1}}\right)\right) & \text{if } 0 \le t \le T_D \\ \left[V_0 + (V_1 - V_0)\cdot\left(1 - \exp\left(\frac{-T_D}{T_{C1}}\right)\right)\cdot\exp\left[\frac{-(t - T_D)}{T_{C2}}\right]\right] & \text{if } T_D \le t \end{cases}$$

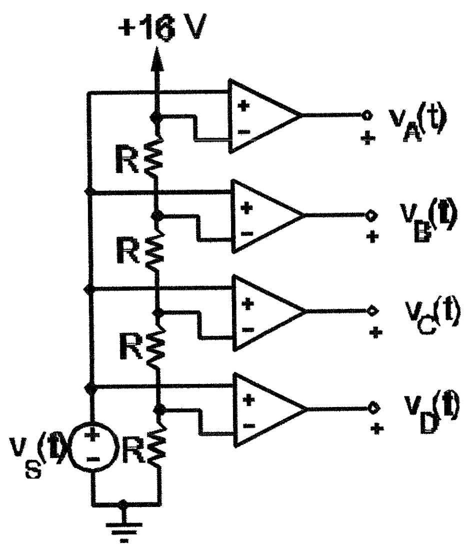
$t := 0, 0.01 \cdot T_D .. T_D + 5 \cdot T_{C2}$

$$v(T_D) = V_0 + (V_1 - V_0)\left(1 - e^{\frac{T_D}{T_{C1}}}\right)$$



Duration = $T_D + 5T_{C2}$

5-42 $v_S(t) := 25 \cdot e^{-2000 \cdot t} \quad V_{OH} := 10 \text{ V} \quad V_{OL} := 0 \text{ V} \qquad t := 0, 0.000001 .. .001$



$$v_A(t) := \begin{cases} V_{OH} & \text{if } v_S(t) > 16 \\ 0 & \text{otherwise} \end{cases}$$

$$v_B(t) := \begin{cases} V_{OH} & \text{if } v_S(t) > 12 \\ 0 & \text{otherwise} \end{cases}$$

$$v_C(t) := \begin{cases} V_{OH} & \text{if } v_S(t) > 8 \\ 0 & \text{otherwise} \end{cases}$$

$$v_D(t) := \begin{cases} V_{OH} & \text{if } v_S(t) > 4 \\ 0 & \text{otherwise} \end{cases}$$

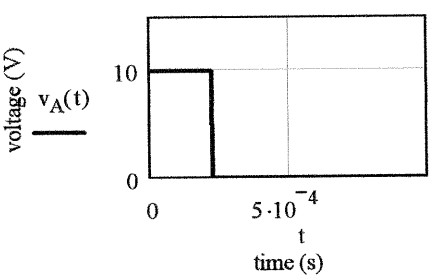
5-42 Continued

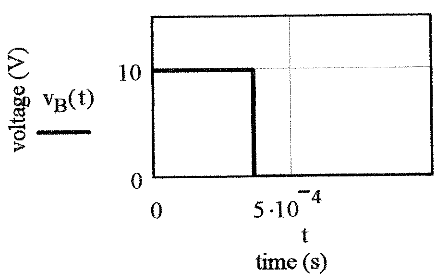
$v_S(T_A) = 16 \quad T_A := \ln\left(\frac{25}{16}\right)\cdot\frac{1}{2000} \quad 1000\cdot T_A = 0.223 \quad v_A(t) := V_{OH}\cdot u(T_A - t)$

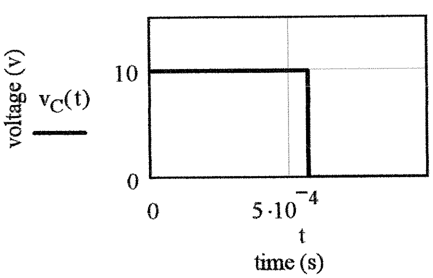
$v_S(T_B) = 12 \quad T_B := \ln\left(\frac{25}{12}\right)\cdot\frac{1}{2000} \quad 1000\cdot T_B = 0.367 \quad v_B(t) := V_{OH}\cdot u(T_B - t)$

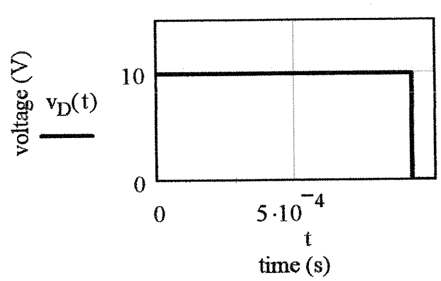
$v_S(T_C) = 8 \quad T_C := \ln\left(\frac{25}{8}\right)\cdot\frac{1}{2000} \quad 1000\cdot T_C = 0.57 \quad v_C(t) := V_{OH}\cdot u(T_C - t)$

$v_S(T_D) = 4 \quad T_D := \ln\left(\frac{25}{4}\right)\cdot\frac{1}{2000} \quad 1000\cdot T_D = 0.916 \quad v_D(t) := V_{OH}\cdot u(T_D - t)$

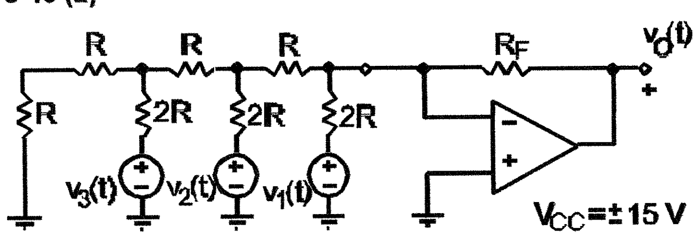








5-43 (a)



KCL at Node A $\quad t := 0, 0.01..10$

$$\frac{v_A}{2\cdot R} + \frac{v_A - v_3}{2R} + \frac{v_A - v_B}{R} = 0$$

KCL at Node B with Vc=0

$$\frac{v_B - v_A}{R} + \frac{v_B - v_2}{2\cdot R} + \frac{v_B}{R} = 0$$

KCL at Node C with Vc=0

$$\frac{v_B}{R} + \frac{v_1}{2R} + \frac{v_O}{R_F} = 0$$

Multiply by R & placed in matrix form

$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2.5 & 0 \\ 0 & 1 & \frac{R}{R_F} \end{pmatrix}\cdot\begin{pmatrix} v_A \\ V_B \\ v_O \end{pmatrix} = \begin{pmatrix} 0.5\cdot v_3 \\ 0.5\cdot v_2 \\ -0.5\cdot v_1 \end{pmatrix}$$

$$\Delta_O = -0.5\cdot v_3 - v_2 - 2\cdot v_1$$

$$\Delta = 4\cdot\frac{R}{R_F}$$

$$v_O = \frac{\Delta_O}{\Delta} = -\left(\frac{R_F}{8\cdot R}\right)\cdot(v_3 + 2\cdot v_2 + 4\cdot v_1)$$

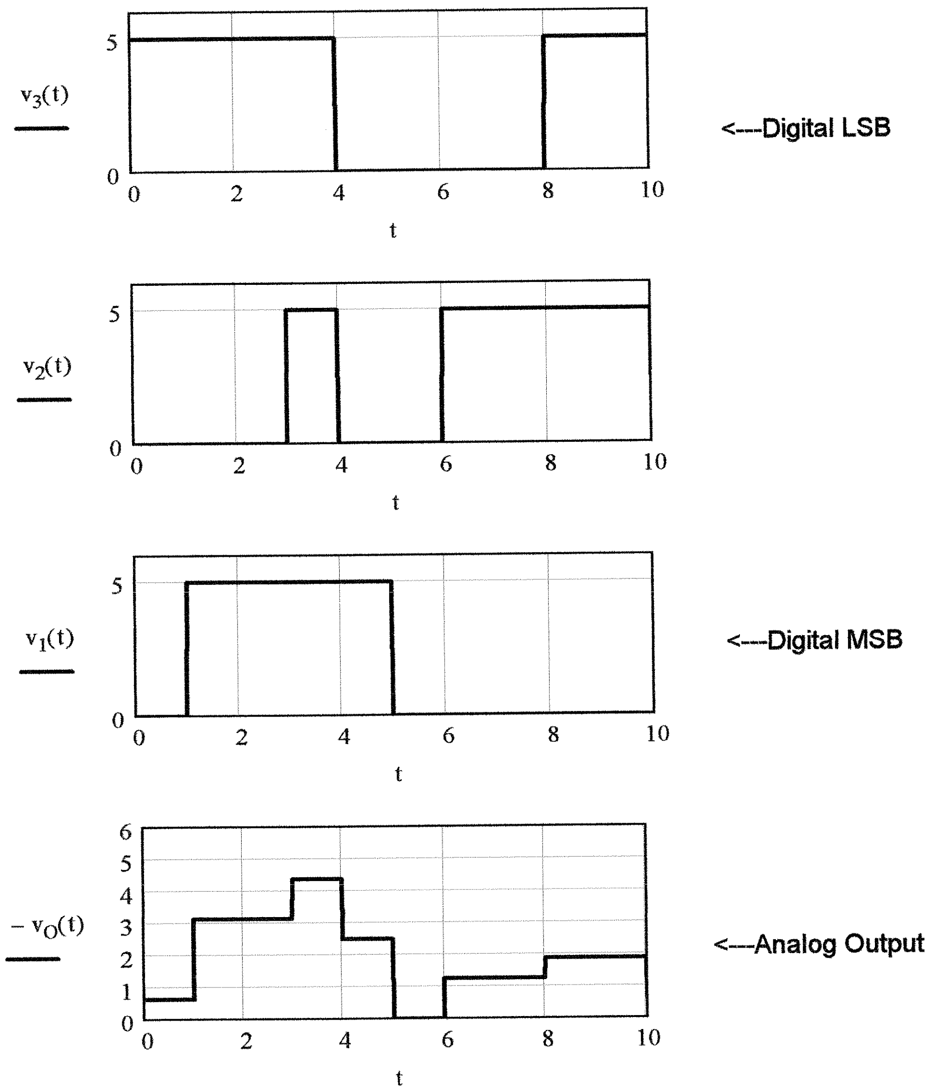
QED

5-43 Continued

(b) $v_1(t) := 5 \cdot u(t-1) - 5 \cdot u(t-5)$ $\quad v_2(t) := 5 \cdot u(t-3) - 5 \cdot u(t-4) + 5 \cdot u(t-6) - 5 \cdot u(t-10)$

$v_3(t) := 5 \cdot u(t) - 5 \cdot u(t-4) + 5 \cdot u(t-8)$ $\quad v_O(t) := -\frac{1}{8} \cdot \left(v_3(t) + 2 \cdot v_2(t) + 4 \cdot v_1(t)\right)$

(c)



5-44

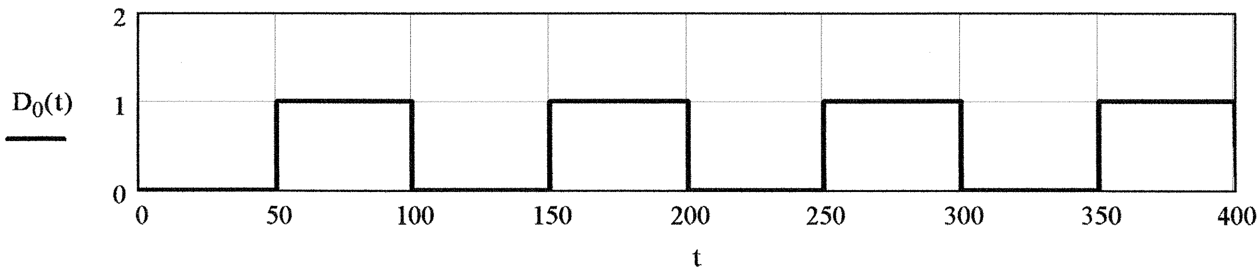
$D_0(t) := u(t-50) - u(t-100) + u(t-150) - u(t-200) + u(t-250) - u(t-300) + u(t-350) - u(t-400)$

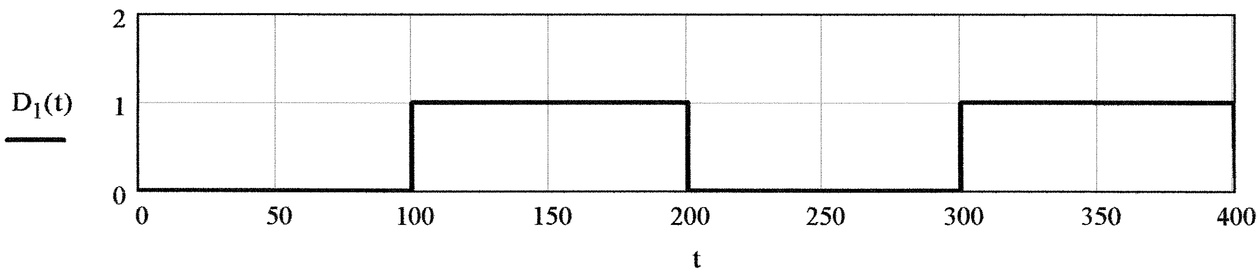
$D_1(t) := u(t-100) - u(t-200) + u(t-300) - u(t-400)$

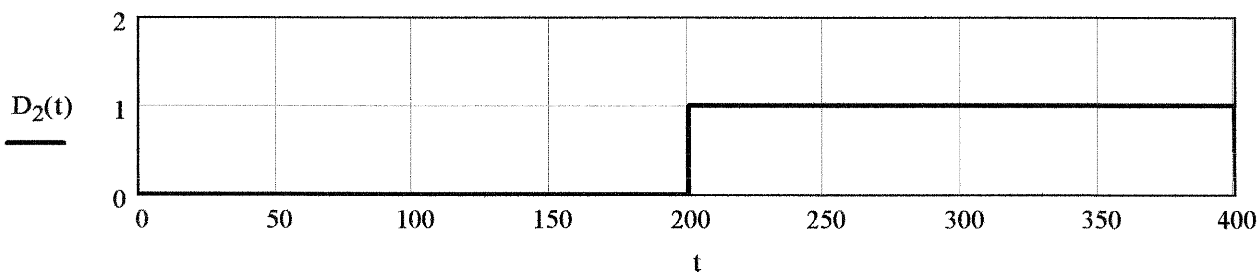
$D_2(t) := u(t-200) - u(t-400)$ $\quad D_4(t) := \left(D_2(t) \wedge D_0(t)\right)$

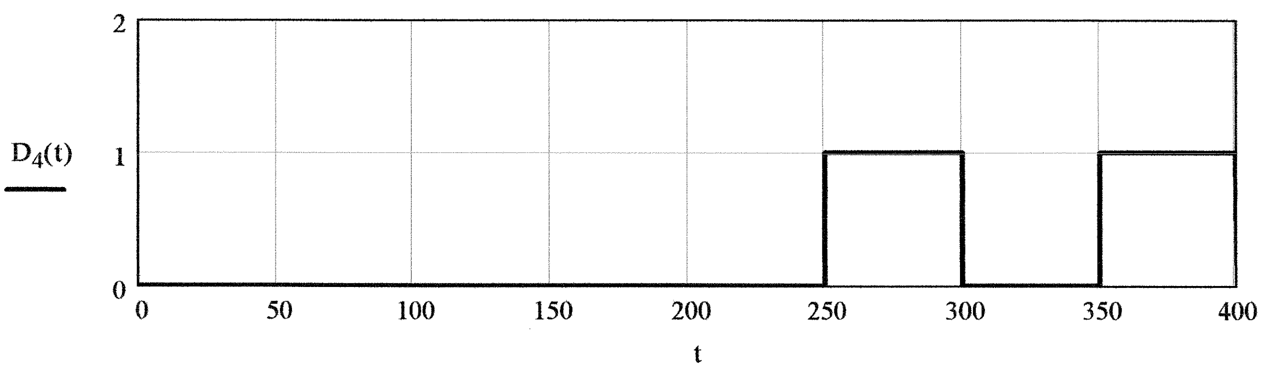
$v_O(t) := D_0(t) + 2 \cdot D_1(t) + 4 \cdot D_2(t)$

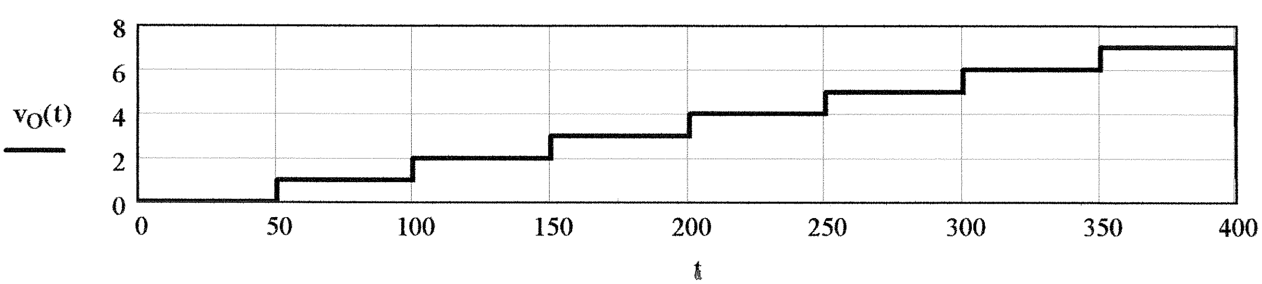
5-44 Continued $t := 0,.1..400$







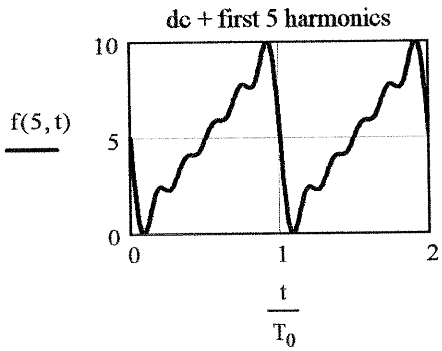


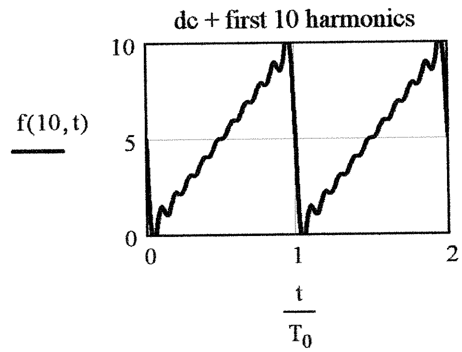


5-45 $V_A := 10$ $f_0 := 50\cdot 10^3$ $n := 1, 2 .. 10$ $V_0 := 5$ $V_n := \frac{V_A}{n\cdot\pi}$ $T_0 := \frac{1}{f_0}$

$$f(k,t) := V_0 - \sum_{i=1}^{k} V_i \cdot \sin((2\cdot\pi\cdot i\cdot f_0\cdot t))$$

$$t := 0, \frac{T_0}{200} .. 2\cdot T_0$$





Answer: Sawtooth wave

5-46

$$Vabs_{avg} = \frac{1}{T_0}\cdot\left(\int_0^{\frac{T_0}{2}} V_A\cdot\sin\left(2\cdot\pi\cdot\frac{x}{T_0}\right)dx - \int_{\frac{T_0}{2}}^{T_0} V_A\cdot\sin\left(2\cdot\pi\cdot\frac{x}{T_0}\right)dx\right)$$

$$Vabs_{avg} = \frac{1}{T_0}\cdot\left[\frac{1}{\pi}\cdot T_0\cdot V_A - \left(\frac{-1}{\pi}\cdot T_0\cdot V_A\right)\right] = \frac{2\cdot V_A}{\pi}$$

$$V_{rms} = \frac{V_A}{\sqrt{2}} \qquad K = \frac{V_{rms}}{Vabs_{avg}} = \frac{V_A}{\sqrt{2}}\cdot\frac{\pi}{2\cdot V_A} = \frac{\pi}{2\cdot\sqrt{2}} \quad K := \frac{\pi}{2\cdot\sqrt{2}} \qquad K = 1.111$$

K applies only to a sinewave. A different factor would be required for other waveforms.

CHAPTER 6, Both Versions

$u(x) := \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \ge 0 \end{cases}$ $\delta(x) := 100\cdot(u(x) - u(x - .01))$ $r(x) := \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } x \ge 0 \end{cases}$

6-1 $C := 2\cdot 10^{-6}$ $v_C(t) = 3\cdot\exp(-4000\cdot t)\cdot u(t)$

$i_C(t) = C\cdot\left(\frac{d}{dC}v_C(t)\right) = 2\cdot 10^{-6}\cdot(-4000)\cdot 3\cdot\exp(-4000\cdot t) = -24\cdot 10^{-3}\cdot\exp(-4000\cdot t)$

$p_C(t) = 3\cdot(\exp(-4000\cdot t))\cdot\left(-24\cdot 10^{-3}\cdot\exp(-4000\cdot t)\right) = -72\cdot 10^{-3}\cdot\exp(-8000\cdot t)$ $P_C < 0$ **delivering**

$w_C = 0.5\cdot C\cdot v_C(t)^2 = 0.5\cdot 2\cdot 10^{-6}\cdot(3\cdot\exp(-4000\cdot t))^2 = 9\cdot 10^{-6}\cdot\exp(-8000\cdot t)$

6-2 $C := 3.3\cdot 10^{-6}$ $v_C(t) := 15\cdot\cos(2\cdot\pi\cdot 1000\cdot t)$ $w_C(t) := 0.5\cdot C\cdot v_C(t)^2$

$w_C\left(0.5\cdot 10^{-3}\right) = 3.712\times 10^{-4}$ $w_C\left(0.75\cdot 10^{-3}\right) = 0$ $w_C\left(10^{-3}\right) = 3.712\times 10^{-4}$

6-3 $L := 100\cdot 10^{-3}$ $i_L(t) = 30\cdot 10^{-3}\cdot\exp(-4000\cdot t)\cdot u(t)$

$v_L(t) = L\cdot\left(\frac{d}{dC}i_L(t)\right) = 100\cdot 10^{-3}\cdot(-4000)\cdot 30\cdot 10^{-3}\cdot\exp(-4000\cdot t) = -12\cdot\exp(-4000\cdot t)$

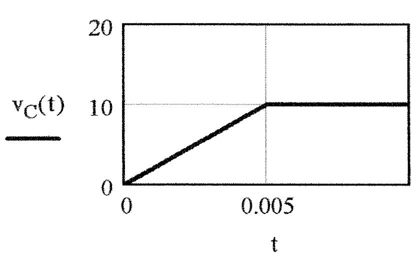
$p_L(t) = v_L(t)\cdot i_L(t) = \left(30\cdot 10^{-3}\cdot\exp(-4000\cdot t)\right)\cdot(-12\cdot\exp(-4000\cdot t)) = -0.36\cdot\exp(-8000\cdot t)$ $p_L(t) < 0$ **delivering**

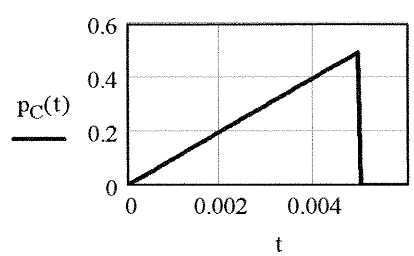
$w_L(t) = 0.5\cdot L\cdot I_L(t)^2 = 0.5\cdot 100\cdot 10^{-3}\cdot\left(30\cdot 10^{-3}\cdot\exp(-4000\cdot t)\right) = 45\cdot 10^{-6}\cdot\exp(-8000\cdot t)$

6-4 $I_A := 50\cdot 10^{-3}$ $T_S := 5\cdot 10^{-3}$ $C := 25\cdot 10^{-6}$ $i_C(t) := I_A\cdot\left(u(t) - u\left(t - T_S\right)\right)$

$v_C(t) := \frac{1}{C}\cdot\int_0^t i_C(x)\,dx$ $v_C(t) := \frac{I_A}{C}\cdot\left(r(t) - r\left(t - T_S\right)\right)$

$p_C(t) := v_C(t)\cdot i_C(t)$ $t := 0, 0.00005 .. 0.01$





<--- $p_C(t) > 0$

absorbing power

6-5 $C := 200\cdot 10^{-9}$ $v_C(0) = 30$ $i_C(t) = 0.4\cdot\cos\left(10^5\cdot t\right)$

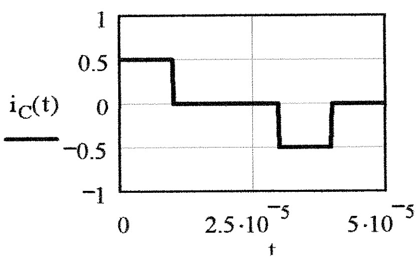
$$v_C(t) = \frac{1}{C}\cdot\left(\int_0^t i_C(x)\,dx\right) + v_C(0) = 5\cdot 10^6\cdot\int_0^t 0.4\cdot\cos\left(10^5\cdot x\right)dx + 30 = 20\cdot\sin\left(10^5\cdot t\right) + 30$$

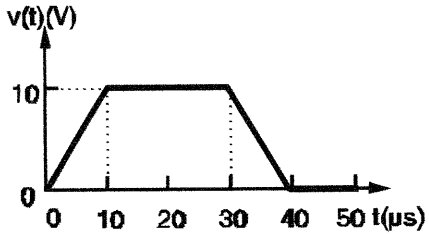
6-6 $C := 0.5\cdot 10^{-6}$ $T_1 := 10\cdot 10^{-6}$ $T_2 := 30\cdot 10^{-6}$ $T_3 := 40\cdot 10^{-6}$ $V_A := 10$ $B := \frac{V_A}{T_1}$

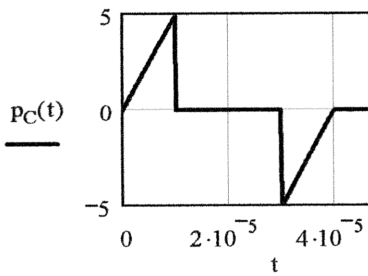
$v_C(t) := B\cdot\left(r(t) - r(t - T_1) - r(t - T_2) + r(t - T_3)\right)$

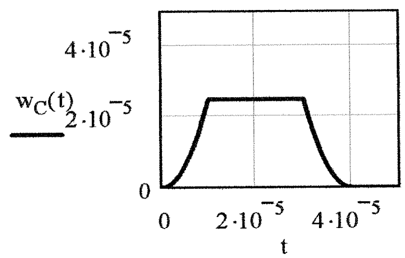
$i_C(t) := C\cdot B\cdot\left(u(t) - u(t - T_1) - u(t - T_2) + u(t - T_3)\right)$ $B\cdot C = 0.5$ $t := 0, 0.0000002 .. 0.00005$

$p_C(t) := v_C(t)\cdot i_C(t)$ $w_C(t) := 0.5\cdot C\cdot v_C(t)^2$









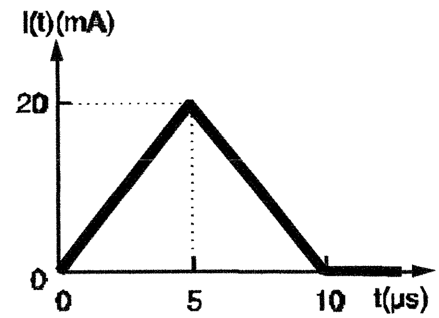
<--- $P_C(t)$ **is positive and negative delivering and absrobing power.**

6-7 $C := 10^{-8}$ $T_1 := 5\cdot 10^{-6}$ $T_2 := 10\cdot 10^{-6}$ $I_A := 0.02$ $B := I_A\cdot T_1^{-1}$ $B = 4\times 10^3$ $v_C(0) = -5$

$i_C(t) := \left(B\cdot r(t) - 2\cdot B\cdot r(t - T_1)\right) + B\cdot r(t - T_2)$ $v_C(t) = \frac{1}{C}\cdot\int_0^t i_C(x)\,dx + v_C(0)$

$$v_C(t) := \frac{B}{C}\cdot\left[\frac{t^2}{2}\cdot u(t) - (t - T_1)^2\cdot u(t - T_1) + \frac{(t - T_2)^2}{2}\cdot u(t - T_2)\right] - 5$$

$v_C(5\cdot 10^{-6}) = 0$ $v_C(10\cdot 10^{-6}) = 5$ $v_C(20\cdot 10^{-6}) = 5$



6-8 $L := 10\cdot 10^{-3}$ $i_L(0) = 10^{-3}$ $v_L(t) := 50\cdot 10^{-3}\cdot\exp(-5000\cdot t)$

$$i_L(t) := \frac{1}{10\cdot 10^{-3}}\cdot\int_0^t 50\cdot 10^{-3}\cdot\exp(-5000\cdot x)\,dx + 10^{-} \quad i_L(t) := \frac{-1}{1000}\cdot\exp(-5000\cdot t) + \frac{1}{500} \quad t := 0, 10^{-5} .. 10^{-3}$$

$$p_L(t) = \left(50\cdot 10^{-3}\cdot\exp(-5000\cdot t)\right)\cdot\left(\frac{-1}{1000}\cdot\exp(-5000\cdot t) + \frac{1}{500}\right)$$

$p_L(t) := -5\cdot 10^{-5}\cdot\exp(-10000\cdot t) + 10^{-4}\cdot\exp(-5000\cdot t)$

The dominant exponential in $p_L(t)$ is positive, hence the inductor is absorbing power

6-9 $L := 0.5 \quad i_L(t) := 10^{-2} \cdot e^{-2000 \cdot t} \cdot \sin(1000 \cdot t) \quad v_L(t) := L \cdot \frac{d}{dt} i_L(t)$

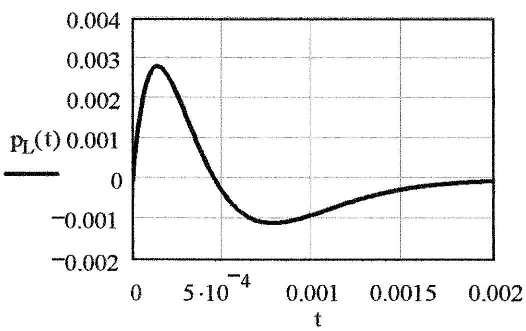
$$v_L(t) = 0.5 \cdot \frac{d}{dt} 10^{-2} \cdot e^{-2000 \cdot t} \cdot \sin(1000 \cdot t) = (-10 \cdot \sin(1000 \cdot t) + 5 \cdot \cos(1000 \cdot t)) \cdot \exp(-2000 \cdot t)$$

$$v_L(t) := 5 \cdot e^{-2000 \cdot t} \cdot (-2 \cdot \sin(1000 \cdot t) + \cos(1000 \cdot t)) \qquad t := 0, 10^{-5} .. 0.002$$

$$p_L(t) = v_L(t) \cdot i_L(t) = 5 \cdot e^{-2000 \cdot t} \cdot (-2 \cdot \sin(1000 \cdot t) + \cos(1000 \cdot t)) \cdot \left(10^{-2} \cdot e^{-2000 \cdot t} \cdot \sin(1000 \cdot t)\right)$$

$$p_L(t) := .05 \cdot e^{-4000 \cdot t} \cdot \left(-2 \cdot \sin(1000 \cdot t)^2 + \sin(1000 \cdot t) \cdot \cos(1000 \cdot t)\right)$$

$$w_L(t) = \left(0.5 \cdot L \cdot i_L(t)^2\right) = 0.25 \cdot \left(10^{-2} \cdot e^{-2000 \cdot t} \cdot \sin(1000 \cdot t)\right)^2 \quad w_L(t) := \left(25 \cdot 10^{-6} \cdot \exp(-4000. \cdot t)\right) \cdot \sin(1000 \cdot t)^2$$



<--- $p_L(t)$ **is positive and negative**
Inductor absorbs and delivers power.

6-10 $L = 0.05 \quad v_L(t) = 5 \cdot \cos(1000 \cdot t) - 2 \cdot \sin(3000 \cdot t) \quad i_L(t) = \frac{1}{L} \cdot \int_0^t v_L(x)\, dx$

$$i_L(t) = \frac{1}{0.05} \cdot \int_0^t (5 \cdot \cos(1000 \cdot x) - 2 \cdot \sin(3000 \cdot x))\, dx = 20\left(\frac{5}{1000} \cdot \sin(1000 \cdot t) + \frac{2}{3000} \cdot \cos(3000 \cdot t) - \frac{2}{3000}\right)$$

$$i_L(t) := -1.333 \cdot 10^{-2} + 10^{-1} \cdot \sin(1000 \cdot t) + 1.333 \cdot 10^{-2} \cdot (\cos(3000 \cdot t))$$

In $v_L(t)$ the amplitude ratio of the 1000 to 3000 ac components is 2.5:1 whereas in $i_L(t)$ the ratio is 7.5:1. The integration produces a dc component and reduces the high frequency relative to the the low frequency ac component.

6-11 $C := 50 \cdot 10^{-9} \quad v_C(t) := -100 \cdot \exp(-1000 \cdot t) \quad i_C(t) = C \cdot \frac{d}{dt} v_C(t)$

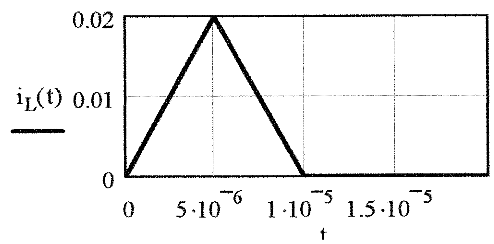
$$i_C(t) = 50 \cdot 10^{-9} \cdot (-100)(-1000) \cdot \exp(-1000 \cdot t) = 5 \cdot 10^{-3} \cdot \exp(-1000 \cdot t)$$

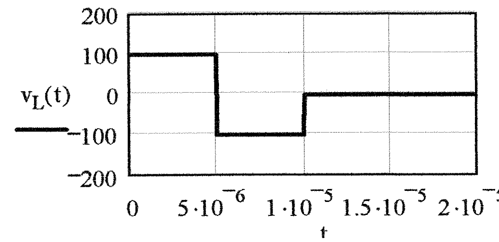
$p_C(t) = v_C(t) \cdot i_C(t) = -0.5 \cdot \exp(-2000 \cdot t) \qquad p_C < 0$ **delivering**

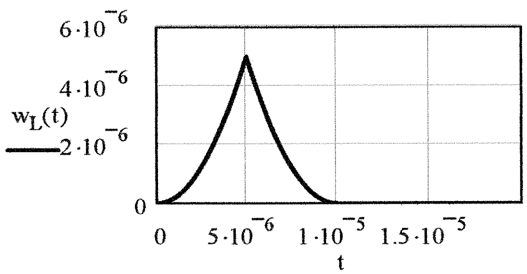
6-12 $L := 0.025 \quad T_1 := 5\cdot 10^{-6} \quad T_2 := 10\cdot 10^{-6} \quad I_A := 20\cdot 10^{-3} \quad B := I_A\cdot T_1^{-1} \qquad B = 4\times 10^{3}$

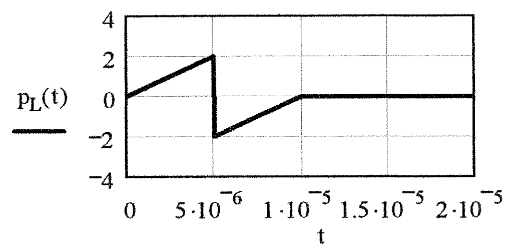
$i_L(t) := (B\cdot r(t) - 2\cdot B\cdot r(t - T_1)) + B\cdot r(t - T_2) \qquad v_L(t) := L\cdot B\cdot (u(t) - 2\cdot u(t - T_1) + u(t - T_2))$

$p_L(t) := v_L(t)\cdot i_L(t) \qquad w_L(t) := 0.5\cdot L\cdot i_L(t)^2 \qquad t := 0, 10^{-8} .. 2\cdot 10^{-5} \qquad L\cdot B = 100$





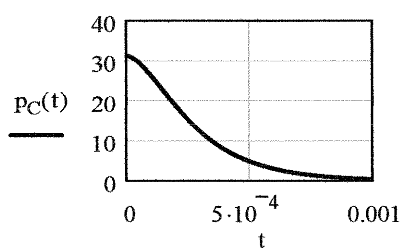




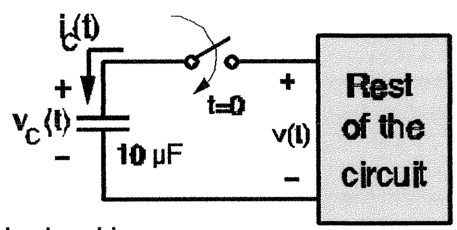
6-13 $L := 100\cdot 10^{-6} \quad i_L(5\cdot 10^{-6}) = 0 = \frac{1}{10^{-4}}\cdot \int_0^{5\cdot 10^{-6}} 10^{6}\cdot t\, dt + i_L(0) = \frac{1}{8} + i_L(0) \qquad i_L(0) = \frac{-1}{8} = -0.125$

6-14 $C := 10\cdot 10^{-6} \qquad v_C(t) = 50 - 25\cdot \exp(-5000\cdot t) \qquad i_C(t) = C\cdot \frac{d}{dt} v_C(t) = 10^{-5}\cdot 25\cdot 5000\cdot \exp(-5000\cdot t)$

$i_C = 1.25\cdot \exp(-5000\cdot t) \; p_C(t) = v_C(t)\cdot i_C(t) \quad t := 0, 10^{-6} .. 10^{-3} \quad p_C(t) := 62.5\cdot \exp(-5000\cdot t) - 31.25\cdot \exp(-10000\cdot$



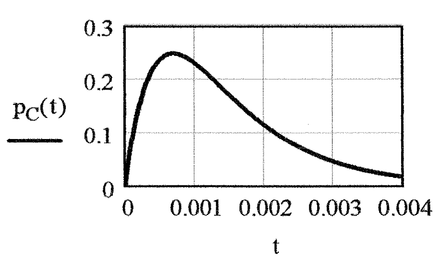
$p_C(t) > 0$



For t>0 the capacitor is absorbing power

6-15 $C := 10^{-5} \quad v_C(t) := 10\cdot (1 - e^{-1000\cdot t}) \quad i_C = C\cdot \left(\frac{d}{dt} v_C(t)\right) = 10^{-5}\cdot 10\cdot (1000\cdot e^{-1000\cdot t}) = 0.1\cdot e^{-1000\cdot t}$

$p_C(t) = v_C(t)\cdot i_C(t) \qquad p_C(t) := e^{-1000\cdot t} - e^{-2000\cdot t} \qquad t := 0, 0.00001 .. 0.004$

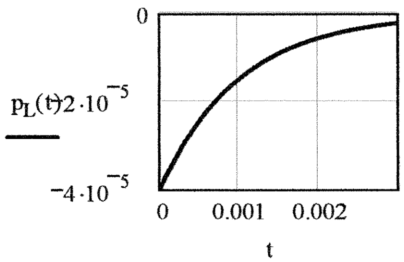


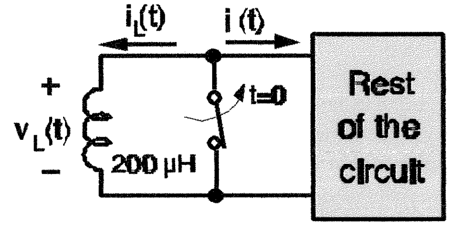
$p_C(t) > 0$ **For t>0 the capacitor is absorbing power**

6-16 $L := 200\cdot10^{-6}$ $i(t) := -20\cdot10^{-3}\cdot\exp(-500\cdot t)$ $i_L(t) = -i(t) = 20\cdot10^{-3}\cdot\exp(-500\cdot t)$

$v_L(t) = L\cdot\frac{d}{dt}i_L(t) = 200\cdot10^{-6}\cdot20\cdot10^{-3}\cdot(-500)\cdot\exp(-500\cdot t) = -2\cdot10^{-3}\cdot\exp(-500\cdot t)$ $p_L(t) = v_L(t)\cdot i_L(t)$

$v_L(t) := -2\cdot10^{-3}\cdot\exp(-500\cdot t)$ $p_L(t) := -40\cdot10^{-6}\cdot\exp(-1000\cdot t)$ $t := 0, 0.00001 .. 0.003$ $p_L(t) < 0$



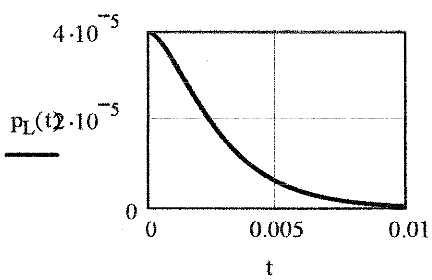


For t>0 the inductor is delivering power

6-17 $L := 200\cdot10^{-6}$ $i(t) = 40\cdot10^{-3} - 20\cdot10^{-3}\cdot\exp(-500\cdot t$ $i_L(t) = -i(t) = -(40\cdot10^{-3} - 20\cdot10^{-3}\cdot\exp(-500\cdot t)$

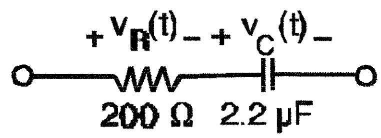
$v_L(t) = L\cdot\frac{d}{dt}i_L(t) = 200\cdot10^{-6}\cdot20\cdot10^{-3}\cdot(-500)\cdot\exp(-500\cdot t) = -2\cdot10^{-3}\cdot\exp(-500\cdot t)$ $p_L(t) = v_L(t)\cdot i_L(t)$

$v_L(t) := -2\cdot10^{-3}\cdot\exp(-500\cdot t)$ $p_L(t) := 80\cdot10^{-6}\cdot\exp(-500\cdot t) - 40\cdot10^{-6}\cdot\exp(-1000\cdot t)$ $t := 0, 0.00001 .. 0.01$



$p_L(t) > 0$ For t>0 the inductor is absorbing power

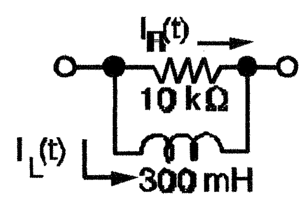
6-18 $C = 2.2\cdot10^{-6}$ $R = 200$ $v_C(t) = 10\cdot\cos(2000\cdot t)$



$v_R(t) = R\cdot i(t) = R\cdot C\cdot\frac{d}{dt}v_C(t) = 2.2\cdot10^{-6}\cdot200\cdot\frac{d}{dt}(10\cdot\cos(2000\cdot t)$

$v_R(t) = -8.8\cdot\sin(2000\cdot t)$

6-19 $L = 0.3$ $R = 10^4$ $i_L(t) = 10^{-2}\cdot\exp(-1000\cdot t)$



$i_R(t) = \frac{v(t)}{R} = \frac{L}{R}\cdot\frac{d}{dt}i_L(t) = \frac{0.3}{10000}\cdot\left[\frac{d}{dt}\left(10^{-2}\cdot\exp(-1000\cdot t)\right)\right]$

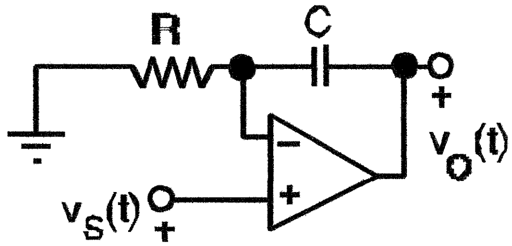
$i_R = -3\cdot10^{-4}\cdot\exp(-1000\cdot t)$

6-20 $L = 0.002$ $i_L(t) = 100\cdot t\cdot\exp(-1000\cdot t)$ $v_L(t) = L\cdot\frac{d}{dt}i_L(t) = 0.2\cdot\exp(-1000\cdot t) - 200\cdot t\cdot\exp(-1000\cdot t)$

$p_L(t) := (0.2\cdot\exp(-1000\cdot t) - 200\cdot t\cdot\exp(-1000\cdot t))\cdot(100\cdot t\cdot\exp(-1000\cdot t$ $p_L\left(0.5\cdot10^{-3}\right) = 1.839\times10^{-3}$ absorbing

$p_L\left(1\cdot10^{-3}\right) = 0$ Neither $p_L\left(2\cdot10^{-3}\right) = -7.326\times10^{-4}$ delivering

6-21 Summing currents at the inverting input



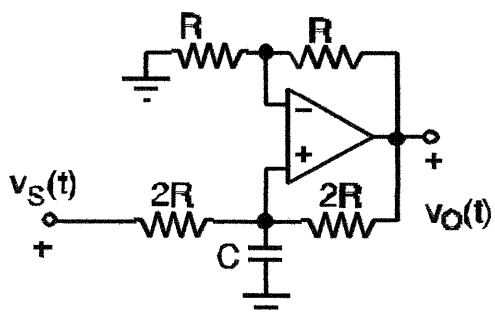
$$\frac{v_S(t)}{R} + C\cdot\frac{d}{dt}\left(v_S(t) - v_O(t)\right) = 0$$

$$\frac{d}{dt}v_O(t) = \frac{d}{dt}v_S(t) + \frac{v_S(t)}{R\cdot C}$$

Integrating both sides

$$v_O(t) - v_O(0) = v_S(t) - v_S(0) + \frac{1}{R\cdot C}\cdot\int_0^t v_S(x)\,dx$$

6-22 Voltage at the inverting input is $v_N = v_O/2$. Sum of currents at the noninverting. input is:



$$\frac{v_S - v_P}{2\cdot R} + \frac{v_O - v_P}{2\cdot R} - C\cdot\frac{d}{dt}v_P = 0 \quad \text{but} \quad v_P = v_N = \frac{v_O}{2}$$

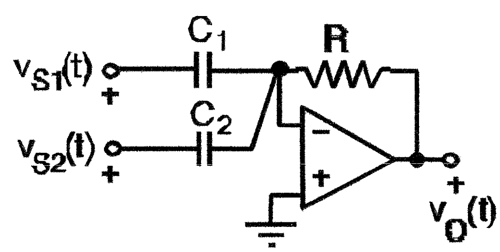
$$\frac{v_S - \frac{v_O}{2}}{2\cdot R} + \frac{v_O - \frac{v_O}{2}}{2\cdot R} - C\cdot\frac{d}{dt}\frac{v_O}{2} = \frac{1}{2}\cdot\frac{v_S}{R} - C\cdot\frac{d}{dt}\frac{v_O}{2} = 0$$

$$\text{hence} \quad \frac{d}{dt}v_O = \frac{v_S}{R\cdot C} \quad \text{or} \quad v_O = \frac{1}{R\cdot C}\cdot\int_0^t v_S(x)\,dx \quad \text{QED}$$

6-23 Summing currents entering the inverting input

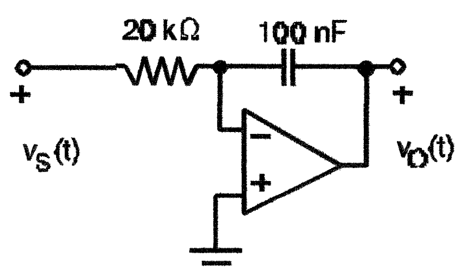
$$C_1\cdot\left[\frac{d}{dt}\left(v_{S1}(t) - v_N(t)\right)\right] + C_2\cdot\left[\frac{d}{dt}\left(v_{S2}(t) - v_N(t)\right)\right] + \frac{v_O(t) - v_N(t)}{R} = 0$$

for and ideal OP AMP $v_N(t) = 0$ hence



$$v_O(t) = -R\cdot C_1\cdot\frac{d}{dt}v_{S1}(t) - R\cdot C_2\cdot\frac{d}{dt}v_{S2}(t)$$

6-24 The circuit is an inverting integrator with $R := 20\cdot 10^3$ $C := 100\cdot 10^{-9}$ $R\cdot C = 2\times 10^{-3}$



$$v_O(t) = -10 - 500\cdot\int_0^t 5\,dx = -10 - 2500\cdot t$$

OP AMP saturates with v_O = -15 V

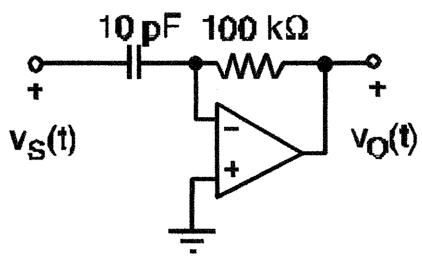
$$v_O = -15 \qquad t := \frac{-15 + 10}{-2500} \qquad t = 2\times 10^{-3}$$

6-25 $\frac{1}{R \cdot C} = 500$ $v_O(t) = \frac{-1}{RC} \cdot \int_0^T 5\,dx + 0 = -500 \cdot 5 \cdot T = -15$ $T := \frac{3}{500}$ $T = 6 \times 10^{-3}$

6-26 $\frac{1}{R \cdot C} = 500$ $v_O(t) = \frac{-1}{RC} \cdot \int_0^t 5 \cdot \sin(\omega \cdot x)\,dx + 0 = -500\left[\frac{5}{\omega} \cdot (1 - \cos(\omega \cdot t))\right] = -15$

worst case occurs when $\cos(\omega \cdot t) = -1$ $-500 \cdot \frac{10}{\omega} = -15$ or $\omega = \frac{1000}{3}$

6-27 The circuit is an inverting differentiator with $R := 100 \cdot 10^3$ $C := 10 \cdot 10^{-12}$ $R \cdot C = 1 \times 10^{-6}$



For $v_S = V_A \cdot \sin(10^6 \cdot t) \cdot u(t)$ the output is

$$|v_O(t)| = \left|-10^{-6} \cdot \left(\frac{d}{dt} V_A \cdot \sin(10^6 \cdot t)\right)\right|$$

$$|v_O(t)| = \left|-V_A \cdot \cos(10^6 \cdot t)\right| < 15$$

To avoid saturation $|V_A| < 15$.

6-28 The circuit is an inverting differentiator with $R := 100 \cdot 10^3$ $C := 10 \cdot 10^{-12}$ $R \cdot C = 1 \times 10^{-6}$

For $v_S = 5 \cdot \sin(\omega \cdot t) \cdot u(t)$ the output is

$$|v_O(t)| = \left|-10^{-6} \cdot \left(\frac{d}{dt} 5 \cdot \sin(\omega \cdot t)\right)\right|$$

$$|v_O(t)| = \left|-5 \cdot 10^{-6} \cdot \omega \cdot \cos(\omega \cdot t)\right| < 15$$

To avoid saturation $\omega < \frac{15}{5 \cdot 10^{-6}} = 3 \cdot 10^6$.

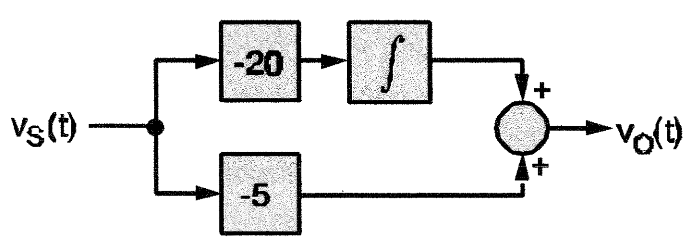
6-29 The circuit is an inverting differentiator with $R := 100 \cdot 10^3$ $C := 10 \cdot 10^{-12}$ $R \cdot C = 1 \times 10^{-6}$

For $v_S(t) = 5 \cdot \exp(-\alpha \cdot t) \cdot u(t)$ in the linear range the output is

$|v_O(t)| = \left|-10^{-6} \frac{d}{dt} 5 \cdot \exp(-\alpha \cdot t)\right| = \left|5 \cdot \alpha \cdot 10^{-6} \cdot \exp(-\alpha \cdot t)\right| < 15$ hence $|\alpha| < \frac{15}{5 \cdot 10^{-6}} = 3 \cdot 10^6$

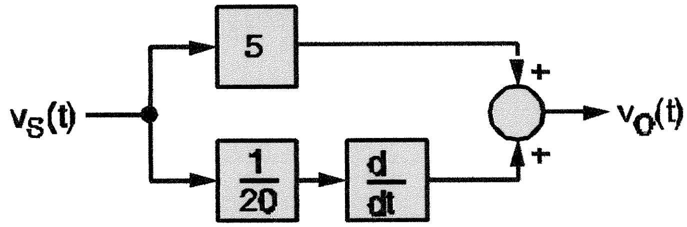
6-30

$$v_O(t) = -5 \cdot v_S(t) - 20 \cdot \int_0^t v_s(x)\,dx$$



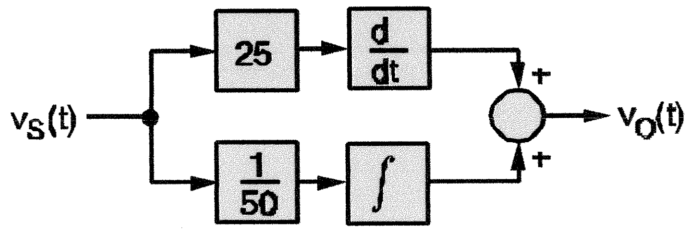
6-31

$$v_O(t) = 5 \cdot v_S(t) + \frac{1}{20} \cdot \frac{d}{dt} v_S(t)$$



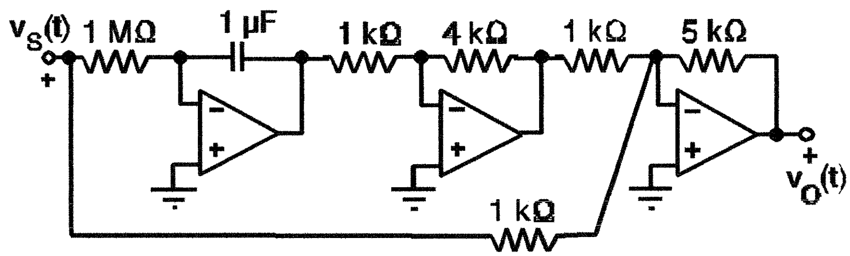
6-32

$$v_O(t) = \frac{1}{50} \cdot \int_0^t v_S(x)\,dx + 25 \cdot \frac{d}{dt} v_S(t)$$



6-33

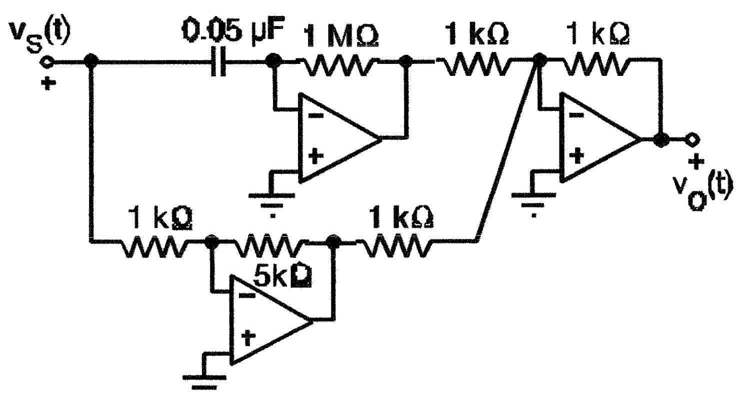
$$v_O(t) = -5 \cdot v_S(t) - 20 \cdot \int_0^t v_S(x)\,dx$$



One possible solution

6-34

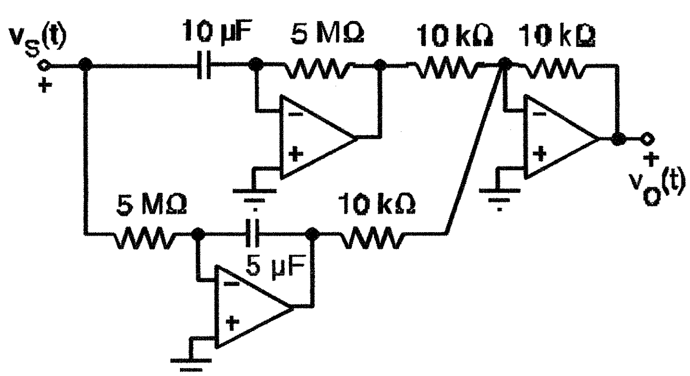
$$v_O(t) = 5 \cdot v_S(t) + \frac{1}{20} \cdot \frac{d}{dt} v_S(t)$$



One possible solution

6-35

$$v_O(t) = \frac{1}{50} \cdot \int_0^t v_S(x)\,dx + 25 \cdot \frac{d}{dt} v_S(t)$$

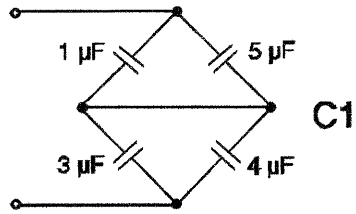


One possible solution

6-36 For C!:

$$C_{EQ} := \left(\frac{1}{10^{-6} + 5 \cdot 10^{-6}} + \frac{1}{2 \cdot 10^{-6} + 4 \cdot 10^{-6}} \right)^{-1}$$

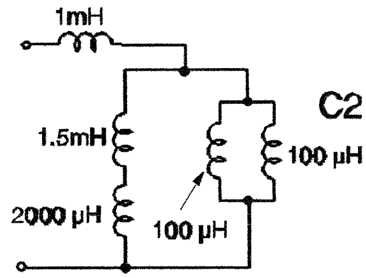
$$C_{EQ} = 3 \times 10^{-6} \quad \text{F}$$



For C2:

$$L_{EQ} := 10^{-3} + \left(\frac{2}{100 \cdot 10^{-6}} + \frac{1}{1.5 \cdot 10^{-3} + 2000 \cdot 10^{-6}} \right)^{-1}$$

$$L_{EQ} = 1.0493 \times 10^{-3} \quad \text{H}$$



6-37 Eq. (6-30) Given a series connection of N inductors, by KVL the voltage across the connection is

$$v(t) \;=\; v_1(t) + v_2(t) + .. + v_N(t)$$

Using the derivative i-v relationship for each inductor

$$v(t) \;=\; L_1 \cdot \frac{d}{dt} i_1(t) + L_2 \cdot \frac{d}{dt} i_2(t) + .. + L_N \cdot \frac{d}{dt} i_N(t)$$

In a series connection KCL requires

$$i(t) \;=\; i_1(t) \;=\; i_2(t) \;= ... =\; i_N(t)$$

Hence the i-v relationship can be written as

$$v(t) \;=\; L_1 \cdot \frac{d}{dt} i(t) + L_2 \cdot \frac{d}{dt} i(t) + .. + L_N \cdot \frac{d}{dt} i(t) \;=\; (L_1 + L_2 + .. + L_N) \cdot \frac{d}{dt} i(t)$$

Therefore $L_{EQ} \;=\; L_1 + L_2 + .. + L_N$ and $i_{EQ}(0) = i(0)$

6-37 Eq. (6-31) Given a parallel connection of N inductors, by KCL the current into the connection is

$$i(t) \;=\; i_1(t) + i_2(t) + .. \quad i_N(t)$$

Using the integral i-v relationship for each inductor

$$i(t) \;=\; i_1(0) + \frac{1}{L_1} \cdot \int_0^t v_1(x)\,dx + i_2(0) + \frac{1}{L_2} \cdot \int_0^t v_2(x)\,dx + .. + \frac{1}{L_N} \cdot \int_0^t v_N(x)\,dx$$

In a parallel connection KVL requires

$$v(t) \;=\; v_1(t) \;=\; v_2(t) \;= ... =\; v_N(t)$$

Hence the i-v relationship can be written as

$$i(t) \;=\; i_1(0) + i_2(0) + .. + i_N(0) + \left(\frac{1}{L_1} + \frac{1}{L_2} + .. + \frac{1}{L_N} \right) \cdot \int_0^t v(x)\,dx$$

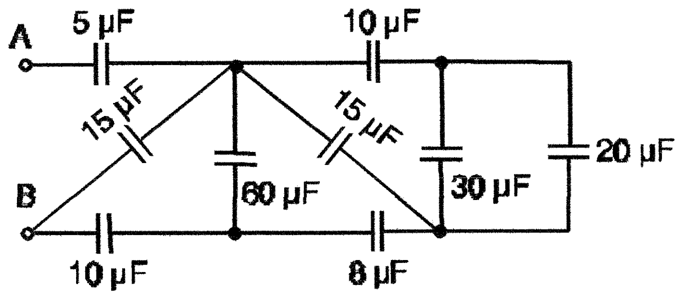
Therefore

$$L_{EQ} \;=\; \left(\frac{1}{L_1} + \frac{1}{L_2} + .. + \frac{1}{L_N} \right)^{-1} \quad \text{and} \quad i_{EQ}(0) \;=\; i_1(0) + i_2(0) + .. + i_N(0)$$

6-38 $L_{EQ} := \left(\frac{1}{3 \cdot 10^{-3} + 100 \cdot 10^{-6}} + \frac{1}{12 \cdot 10^{-3}} \right)^{-1}$ $L_{EQ} = 2.464 \times 10^{-3}$ H

6-39 $C_{EQ} := \left(\frac{1}{6.8 \cdot 10^{-6}} + \frac{1}{6.8 \cdot 10^{-6}} \right)^{-1} + \left(\frac{1}{3.3 \cdot 10^{-6}} + \frac{1}{4.7 \cdot 10^{-6}} \right)^{-1}$ $C_{EQ} = 5.339 \times 10^{-6}$ F

6-40

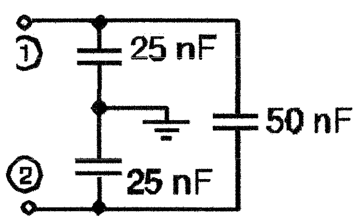


all capacitance in µF

$$C_{AB} := \left[\left[\left[\left[\left[\left[\left[(30+20)^{-1} + \frac{1}{10}\right]^{-1} + 15\right]^{-1} + \frac{1}{8}\right]^{-1} + 60\right]^{-1} + \frac{1}{10}\right]^{-1} + 15\right]^{-1} + \frac{1}{5}\right]^{-1} \qquad C_{AB} = 4.128 \ \mu F$$

6-41 **(a)** $C_{EQ} := 25 \cdot 10^{-9} + \left(\frac{1}{25 \cdot 10^{-9}} + \frac{1}{50 \cdot 10^{-9}} \right)^{-1}$ $C_{EQ} = 4.167 \times 10^{-8}$

(b) $C_{EQ} := 25 \cdot 10^{-9} + 50 \cdot 10^{-9}$ $C_{EQ} = 7.5 \times 10^{-8}$



6-42 $V_{charge} := 5000$ $W_{stored} := 250$ $C_{total} := \frac{2 \cdot W_{stored}}{V_{charge}^{2}}$ hence $C_{total} = 2 \times 10^{-5}$

minimum number in series = $\frac{5}{1.5} = 3.333$ Therefore, connect 4 in series

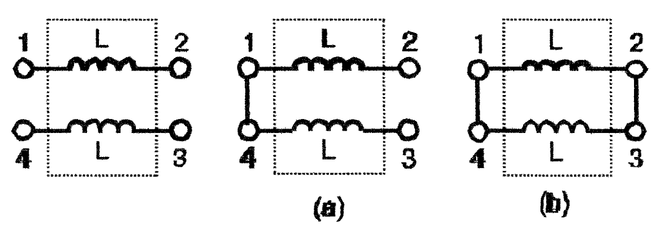
$C_{series} := \frac{20 \cdot 10^{-6}}{4}$ number of series strings = $\frac{C_{total}}{C_{series}} = 4$ total of 16-20 µF capacitors

4 strings each consisting of four 20 µF capacitors connected in series, total of 16 capacitors

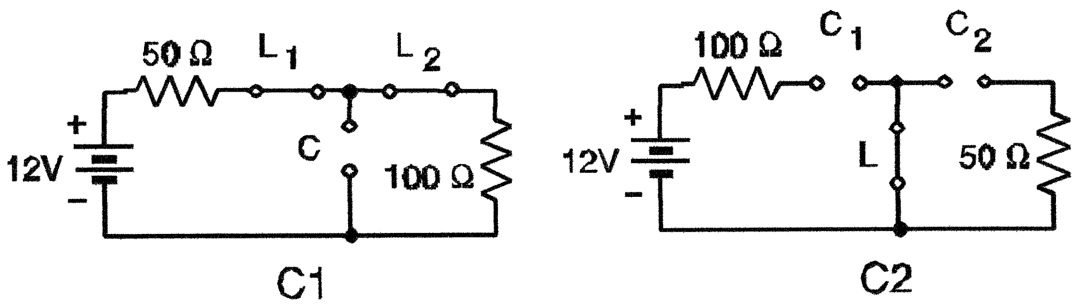
6-43 $2 \cdot L = 260 \cdot 10^{-6}$ therefore $L := 130 \cdot 10^{-6}$

(a) $L_{EQ} := L$ $L_{EQ} = 1.3 \times 10^{-4}$

(b) $L_{EQ} := \frac{L}{2}$ $L_{EQ} = 6.5 \times 10^{-5}$



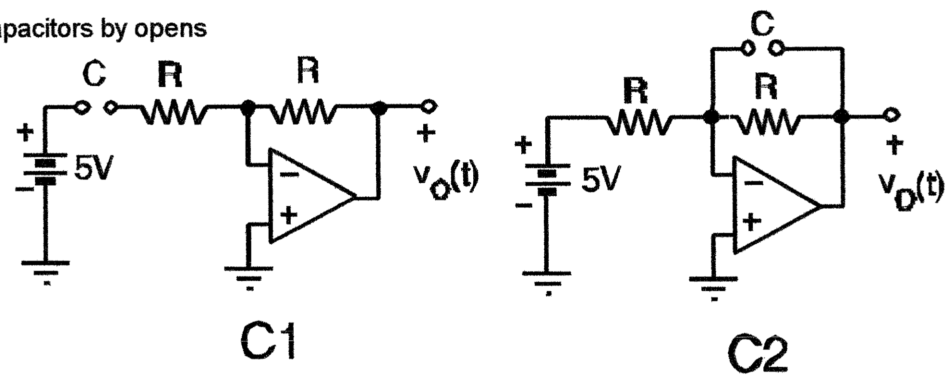
6-44 Replace inductors by shorts and capacitors by opens



For C1 $i_{L2} := \dfrac{12}{50 + 100}$ $\quad i_{L1} := i_{L2}$ $\quad v_C := i_{L2} \cdot 100$ $\quad i_{L1} = 0.08$ $\quad i_{L2} = 0.08$ $\quad v_C = 8$

For C2 when C_1 is open $i_L := 0$ hence $v_{C2} := 0$ and $v_{C1} := 12$ V

6-45 Replace capacitors by opens



For C1 when C is open the source and OP AMP are disconnected hence $v_O := 0$ V

For C2 when C is open the OP AMP circuit is an inverter hence $v_O := -5$ V

6-46 (a) $C_{EQ} := 11 \cdot \left(\dfrac{1.5 \cdot 10^{-3}}{16} \right)$ $\quad C_{EQ} = 1.031 \times 10^{-3}$ F

(b) $W_{total} := 0.5 \cdot C_{EQ} \cdot (16 \cdot 300)^2$ $\quad W_{total} = 1.188 \times 10^4$ J

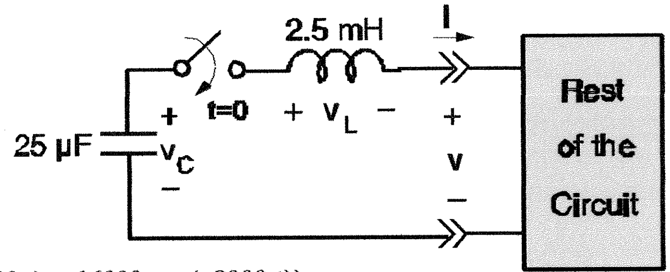
(c) $v(t) := 4800 \cdot e^{-500 \cdot t}$ $\quad i(t) = C_{EQ} \cdot \dfrac{d}{dt} v(t) = C_{EQ} \cdot 4800 \cdot (-500) \cdot e^{-500 \cdot t} = -2475 \cdot e^{-500t}$

the power delivered by the capacitor is $p(t) = v(t) \cdot (-i(t)) = 11.88 \cdot e^{-1000 \cdot t}$ MW
The peak power occurs at t = 0 with $p_{max} = p(0) = 11.88$ MW

(d) $T := \dfrac{5}{500}$ $\quad P_{avg} := \dfrac{1}{T} \cdot \int_0^T 11.88 \cdot \exp(-1000 \cdot t)\, dt$ $\quad P_{avg} = 1.188$ MW

6-47 $C := 25 \cdot 10^{-6}$ $L := 2.5 \cdot 10^{-3}$ $V_0 := 30$

$i(t) = 2 \cdot (\exp(-2000 \cdot t) - \exp(-8000 \cdot t))$



(a) $v_L(t) = L \cdot \frac{d}{dt} i(t) = 2.5 \cdot 10^{-3} \cdot (-4000 \cdot \exp(-2000 \cdot t) + 16000 \cdot \exp(-8000 \cdot t))$

$v_L(t) := 40 \cdot \exp(-8000 \cdot t) - 10 \cdot \exp(-2000 \cdot t) \qquad v_L(0) = 30$

(b) $v_C = \frac{-1}{C} \cdot \int_0^t i(x)\, dx + V_0$ <---Note minus sign due to reference directions for i(t) and $v_C(t)$

$v_C(t) = -4 \cdot 10^4 \cdot \int_0^t 2 \cdot (\exp(-2000 \cdot x) - \exp(-8000 \cdot x))\, dx + 30$

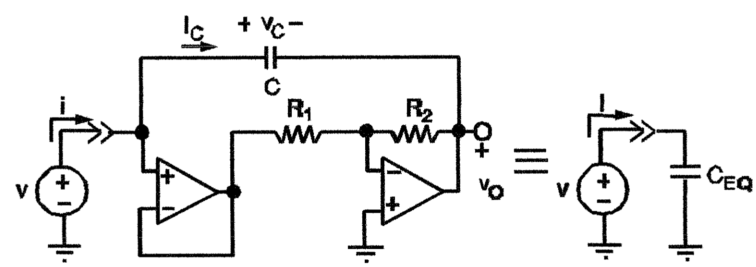
$v_C(t) := 40 \cdot \exp(-2000 \cdot t) - 10 \cdot \exp(-8000 \cdot t) \qquad v_C(0) = 30$

(c) $v(t) = v_C(t) - v_L(t) = 40 \cdot \exp(-2000 \cdot t) - 10 \cdot \exp(-8000 \cdot t) - (40 \cdot \exp(-8000 \cdot t) - 10 \cdot \exp(-2000 \cdot t))$

$v(t) := 50 \cdot \exp(-2000 \cdot t) - 50 \cdot \exp(-8000 \cdot t) \qquad v(0) = 0$

(d) $R = \frac{v(t)}{i(t)} = \frac{50 \cdot \exp(-2000 \cdot t) - 50 \cdot \exp(-8000 \cdot t)}{2 \cdot (\exp(-2000 \cdot t) - \exp(-8000 \cdot t))} = 25$

6-48



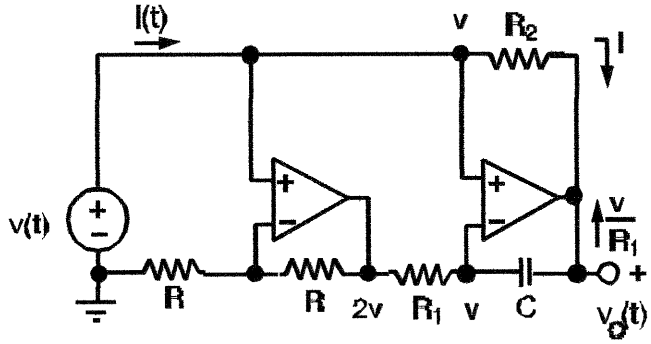
(a) The voltage follower output is v(t), hence $v_O(t) = \frac{-R_2}{R_1} \cdot v(t)$ and the voltage across the capacitor is $v_C(t) = v(t) - v_O(t)$ or $v_C(t) = \left(1 + \frac{R_2}{R_1}\right) \cdot v(t)$. The voltage follower draws no current, hence

$i(t) = i_C(t) = C \cdot \frac{d}{dt} v_C(t) = C \cdot \left(1 + \frac{R_2}{R_1}\right) \cdot \frac{d}{dt} v(t)$ so finally $C_{EQ} = \left(1 + \frac{R_2}{R_1}\right) \cdot C$ QED

(b) Since $v_C(t) = \left(1 + \frac{R_2}{R_1}\right) \cdot v(t) \qquad v(0) = \frac{R_1}{R_1 + R_2} \cdot V_0$

(c) $\frac{1}{2} \cdot C_{EQ} \cdot v(0)^2 = \frac{1}{2} \cdot \left(\frac{R_1 + R_2}{R_1}\right) \cdot C \cdot \left(\frac{R_1}{R_1 + R_2} \cdot v_C(0)\right)^2 = \frac{R_1}{R_1 + R_2} \cdot \left(\frac{1}{2} \cdot C \cdot v_C(0)^2\right) = \left(\frac{R_1}{R_1 + R_2}\right) \cdot W_0$

6-49



(a) The 1st OP AMP is a noninverting amplifier with K=2, hence its output is 2 V. A voltage v appears at both inputs to the 2nd OP AMP, therefore the voltage across R_1 is v and the currents through R_1 and C are both v/R_1 . By KVL the output voltage is $v_O = v - v_C$. By Ohm's law $i = (v - v_O)/R_2 = v_C/R_2$.

Using the *i-v* relationship for C $\quad v_C(t) = \frac{1}{C}\cdot\int_0^t \frac{v(x)}{R_1}\,dx + v_C(0)$

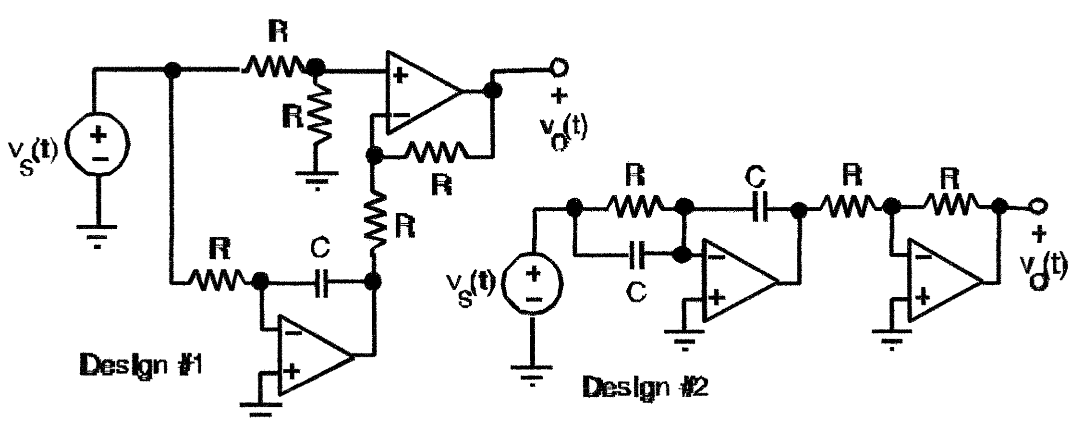
But $\quad i(t) = \frac{v_C(t)}{R_2} = \frac{1}{R_1\cdot R_2\cdot C}\cdot\int_0^t v(x)\,dx + \frac{v_C(0)}{R_2}$

so finally $\quad L_{EQ} = R_1\cdot R_2\cdot C \quad$ QED with $\quad i_{LEQ}(0) = \frac{v_C(0)}{R_2}$

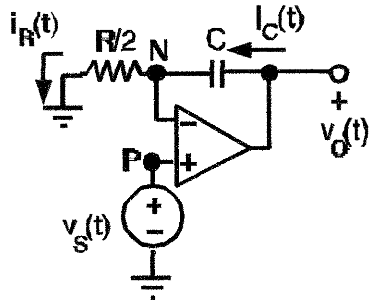
(b) if $\quad v_C = V_0 \quad$ then $\quad i_{LEQ}(0) = \frac{v_C(0)}{R_2}$

(c) $\quad w_L = \frac{1}{2}\cdot L_{EQ}\cdot i(t)^2 = \frac{1}{2}\cdot R_1\cdot R_2\cdot C\cdot\left(\frac{v_C(t)}{R_2}\right)^2 = \frac{R_1}{R_2}\cdot\left(\frac{1}{2}\cdot C\cdot v_C^{\,2}\right) = \frac{R_1}{R_2}\cdot w_C = \frac{R_1}{R_2}\cdot w_0$

6-50 (a) two possible designs are shown below, both with RC = 1/50



(b) In the RonAl Circuit $v_P = v_S$ For an ideal OP AMP $v_P = v_N = v_S$ hence the current i_R is



$$i_R = \frac{v_S}{\left(\frac{R}{2}\right)} = \frac{2 \cdot v_S}{R}$$ by KCL at the inverting input

$$i_R = i_C = \frac{2 \cdot v_S}{R}$$ by KVL $v_C = v_O - v_S$ Using the i-v relationship for the capacitor

$$i_C = C \cdot \frac{d}{dt} v_C = C \cdot \frac{d}{dt}(v_O - v_S) = C \cdot \frac{d}{dt} v_O - C \cdot \frac{d}{dt} v_S = \frac{2 \cdot v_S}{R}$$

Solving for the output voltage $v_O = v_S + \frac{2}{R \cdot C} \cdot \int_0^t v_S(x)\, dx$ For R = 10 kΩ and C = 4 μF

$\frac{2}{R \cdot C} = 50$ and the RonAL design will work.

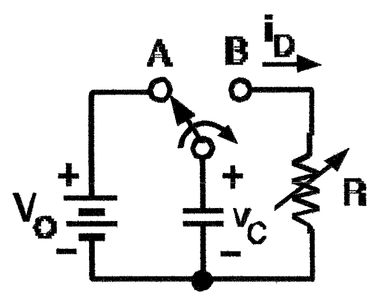
(c) Relative part costs for the in-house and RonAl designs

RonAl Design		Design 1		Design 2	
1C	1500	5R	2000	3R	1200
DA-10	5000	1C	1500	2C	3000
Total	6500	2 OAs	5000	2 OAs	5000
		Total	8500	Total	9200

RonAl < Design 1 < Design 2

6-51

(a) $$\frac{d}{dt} v_C(t) = \frac{V_0 - V_1}{T_1} = \frac{5.5 - 3}{3000} = 8.333 \cdot 10^{-4}$$



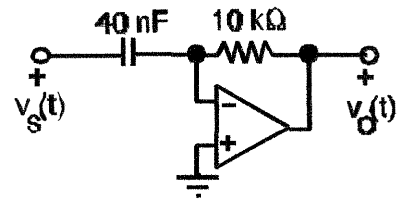
(b) $I_0 = C \cdot \left(\frac{d}{dt} v_C(t)\right)$ hence

$$C = \left[\frac{I_0}{\left(\frac{d}{dt} v_C(t)\right)}\right] = \left[\frac{10^{-3}}{\left(8.333 \cdot 10^{-4}\right)}\right] = 1.2 \quad F$$

(c) $W_R = w_C(0) - w_C(T_1)$ $\qquad W_R := \left(0.5 \cdot 1.2 \cdot 5.5^2\right) - 0.5 \cdot 1.2 \cdot 3^2 \quad W_R = 12.75 \quad J$

(d) $$\frac{d}{dt} v_C(t) = \frac{V_1 - V_2}{T_2 - T_1} = \frac{3 - 2}{1200} = 8.333 \cdot 10^{-4}$$ Yes, they are consistent

6-52 $v_S(t) = 5\cdot\sin(2\cdot\pi\cdot f\cdot t)$



$v_O(t) = -R\cdot C\cdot\left(\frac{d}{dt}v_S(t)\right) = -5\cdot R\cdot C\cdot 2\cdot\pi\cdot f\cdot\cos(2\cdot\pi\cdot f\cdot t)$

to avoid saturation $5\cdot R\cdot C\cdot 2\cdot\pi\cdot f \le 15$

hence $R\cdot C \le \frac{3}{2\cdot\pi\cdot f}$ since $f \le 1000$ the least upper bound occurs when f = 1000, hence

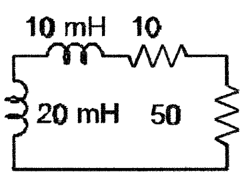
$R\cdot C \le 4.775\cdot 10^{-4}$ Let $R := 10^4$ Then $C < 4.775\cdot 10^{-8}$ let C = 40 nF

CHAPTER 7 Standard Version (7-1 thru 7-30)
CHAPTER 7 Laplace-Early Version (7-1 thru 7-33)

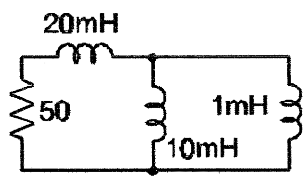
7-1 Char Eq: $s + 1500 = 0$, natural response, $v_N(t) = Ke^{-1500t}$, forced response $v_F(t) = 0$.
$v(0) = -15$ requires $K = -15$, hence $v(t) = -15e^{-1500t}$ is the solution.

7-2 Char Eq: $10^{-4}s + 10^{-1} = 0$, natural response, $i_N(t) = Ke^{-1000t}$, forced response $i_F(t) = 0$.
$i(0) = -20$ mA requires $K = -20$, hence $i(t) = -20\,e^{-1000t}$ mA is the solution.

7-3



C1



C2

C1: $L_{EQ} := 20 \cdot 10^{-3} + 10 \cdot 10^{-3}$ $\quad R_{EQ} := 10 + 50$

$T_C := \frac{L_{EQ}}{R_{EQ}}$ $\quad T_C = 5 \times 10^{-4}$

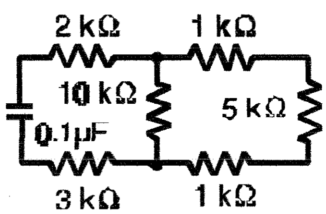
C2:

$L_{EQ} := 20 \cdot 10^{-3} + \left[\left(1 \cdot 10^{-3}\right)^{-1} + \left(10 \cdot 10^{-3}\right)^{-1}\right]^{-1}$

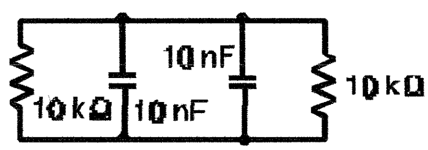
$R_{EQ} := 50$ $\quad T_C := \frac{L_{EQ}}{R_{EQ}}$ $\quad T_C = 4.182 \times 10^{-4}$

7-4

C1



C2



C1:

$R_{EQ} := 2 \cdot 10^{3} + 3 \cdot 10^{3} + \left[\left(10 \cdot 10^{3}\right)^{-1} + \left(7 \cdot 10^{3}\right)^{-1}\right]^{-1}$

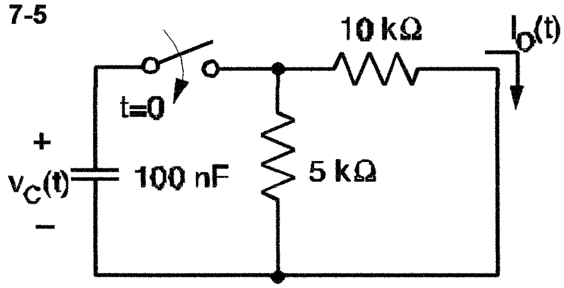
$C_{EQ} := 10^{-7}$ $\quad T_C := R_{EQ} \cdot C_{EQ}$ $\quad T_C = 9.118 \times 10^{-4}$

C2: $R_{EQ} := \left[\left(10 \cdot 10^{3}\right)^{-1} + \left(10 \cdot 10^{3}\right)^{-1}\right]^{-1}$

$C_{EQ} := 10 \cdot 10^{-9} + 10 \cdot 10^{-9}$ $\quad T_C := R_{EQ} \cdot C_{EQ}$

$T_C = 1 \times 10^{-4}$

7-5



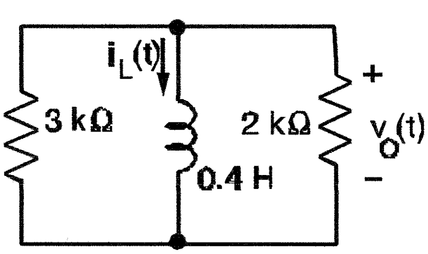
$R_{EQ} := \left[\left(5 \cdot 10^{3}\right)^{-1} + \left(10 \cdot 10^{3}\right)^{-1}\right]^{-1}$ $\quad C := 100 \cdot 10^{-9}$

$T_C := R_{EQ} \cdot C$ $\quad T_C = 3.333 \times 10^{-4}$ $\quad T_C^{-1} = 3 \times 10^{3}$

$v_C(t) := 15 \cdot \exp(-3000 \cdot t)$

$i_O = \frac{v_C(t)}{10^4}$ $\quad i_O(t) := 1.5 \cdot 10^{-3} \cdot \exp(-3000 \cdot t)$

7-6



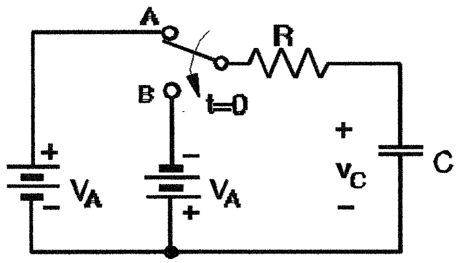
$R_{EQ} := \left[\left(2 \cdot 10^{3}\right)^{-1} + \left(3 \cdot 10^{3}\right)^{-1}\right]^{-1}$ $\quad L := 400 \cdot 10^{-3}$

$T_C := \frac{L}{R_{EQ}}$ $\quad T_C = 3.333 \times 10^{-4}$ $\quad T_C^{-1} = 3 \times 10^{3}$

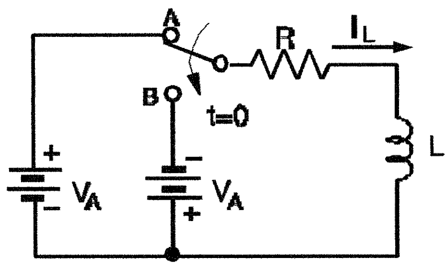
$i_L(t) := 25 \cdot 10^{-3} \cdot \exp(-3000 \cdot t)$ $\quad R_{EQ} = 1.2 \times 10^{3}$

$v_O = -i_L(t) \cdot R_{EQ}$ $\quad v_O(t) := -30 \cdot \exp(-3000 \cdot t)$

7-7



C1



C2

(a) SW in Pos. A the initial conditions are: C1: $IC = v_C(0) = V_A$ C2: $IC = i_L(0) = V_A \cdot R^{-1}$

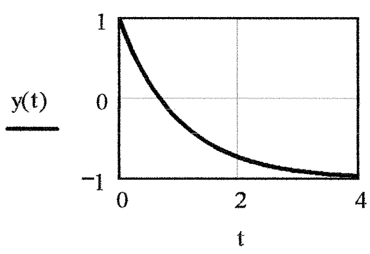
SW in Pos. B the final conditions are: C1: $FC = v_C(\infty) = -V_A$ C2: $FC = i_L(\infty) = -V_A \cdot R^{-1}$

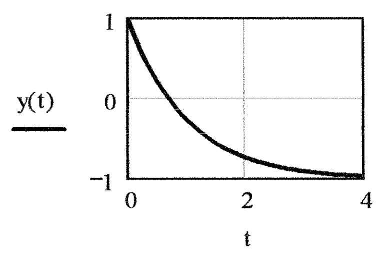
(b) SW in pos. B then $R_T = R$ and the time constants are C1: $T_C = R \cdot C$ C2: $T_C = \frac{L}{R}$

(c) For both circuits $y(t) = FC + (IC - FC)\exp(-t/T_C)$, hence

C1: $y(t) = v_C(t) = V_A \cdot \left(-1 + 2 \cdot \exp\left(-\frac{t}{R \cdot C}\right)\right)$ C2: $y(t) = i_L(t) = \frac{V_A}{R} \cdot \left(-1 + 2 \cdot \exp\left(-\frac{R \cdot t}{L}\right)\right)$

$T_C := 1$ $t := 0, 0.1 .. 4$ $y(t) := -1 + 2 \cdot \exp(-t)$





7-8 See circuits with Problem 7-7 above

(a) SW in Pos. B the initial conditions are: C1: $IC = v_C(0) = -V_A$ C2: $IC = i_L(0) = -V_A \cdot R^{-1}$

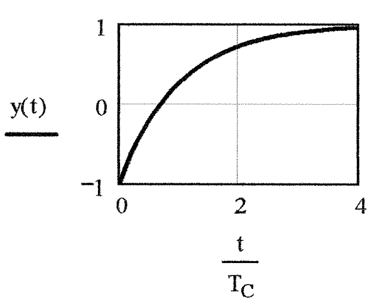
SW in Pos. A the final conditions are C1: $FC = v_C(\infty) = V_A$ C2: $FC = i_L(\infty) = V_A \cdot R^{-1}$

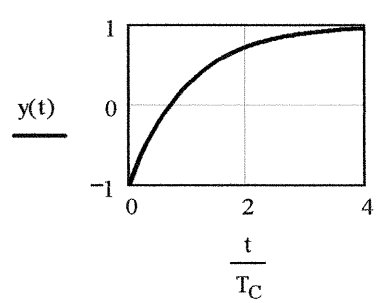
(b) SW in pos. A then $R_T = R$ and the time constants are C1: $T_C = R \cdot C$ C2: $T_C = \frac{L}{R}$

(c) For both circuits $y(t) = FC + (IC - FC)\exp(-t/T_C)$, hence

C1: $y(t) = v_C(t) = V_A \cdot \left(1 - 2 \cdot \exp\left(-\frac{t}{R \cdot C}\right)\right)$ C2: $y(t) = i_L(t) = \frac{V_A}{R} \cdot \left(1 - 2 \cdot \exp\left(-\frac{R \cdot t}{L}\right)\right)$

$T_C := 1$ $t := 0, 0.1 .. 4$ $y(t) := 1 - 2 \cdot \exp(-t)$





7-9 The given initial condition is $IC = v_C(0) = 0$ By inspection the final condition is $FC = v_C(\infty) = I_A \cdot R$

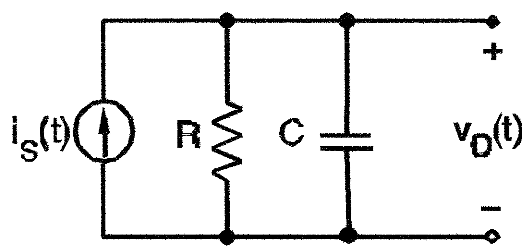
and the circuit time constant is $T_C = R \cdot C$

The state variable response is of the form

y(t) = FC + (IC - FC)exp(-t/T_C)

hence $v_C(t) = I_A \cdot R \cdot \left(1 - \exp\left(\frac{t}{RC}\right)\right)$ <---forced respomse $I_A R$

The output is $v_O(t) = v_C(t)$ <--- natural response is -I_AR exp(-t/RC)



7-10 The given initial cond. is $IC = i_L(0) = 0$ By inspection the final condition is $FC = i_L(\infty) = \frac{V_A}{R}$

and the circuit time constant is $T_C = \frac{L}{R}$

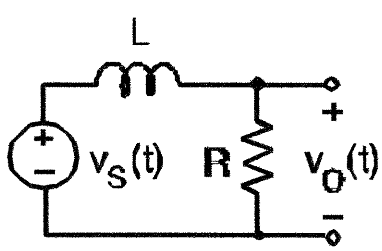
The state variable response is of the form

y(t) = FC + (IC - FC)*exp(-t/T_C)

hence $i_L(t) = \frac{V_A}{R} \cdot \left(1 - \exp\left(\frac{-R \cdot t}{L}\right)\right)$ The output is

$v_O(t) = R \cdot i_L(t) = V_A \cdot \left(1 - \exp\left(\frac{-R \cdot t}{L}\right)\right)$ <--- forced response V_A

<--- natural response is -V_A exp(-Lt/R)



7-11 By voltage division with sw in postion A:

$$IC = \left(\frac{10^4}{10^4 + 10^4 + 2\cdot 10^4}\right)\cdot 12 = 3$$

In Pos. B $FC = \left(\frac{10^4}{10^4 + 10^4}\right)\cdot 12 = 6$

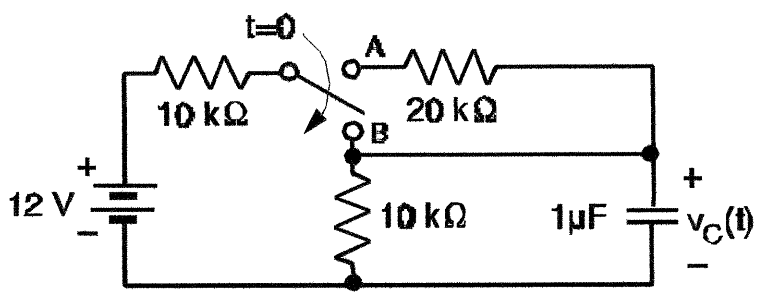
$v_{CF}(t) := 6$ <---Forced Response

$$R_T = \left[\left(10^4\right)^{-1} + \left(10^4\right)^{-1}\right]^{-1} = 5000$$

$v_{CN}(t) := -3\cdot\exp(-200\cdot t)$ <---Natural Response

$T_C := 5000\cdot 10^{-6}$ $\quad T_C = 5\times 10^{-3}$

$v_C(t) = 6 - 3\,\exp(-200\cdot t)$



7-12 See circuit in Prob. 7-11 above. By voltage division with sw in position B: $IC = \frac{10\cdot 10^3\cdot 12}{20\cdot 10^3} = 6$

With sw in position A: $FC = \frac{10\cdot 10^3\cdot 12}{40\cdot 10^3} = 3$ $\quad R_T = \left[\left(10\cdot 10^3\right)^{-1} + \left(30\cdot 10^3\right)^{-1}\right]^{-1} = 7500$

$T_C := 7500\cdot 10^{-6}$ $\quad T_C = 7.5\times 10^{-3}$ $\quad v_C(t) := 3 + 3\cdot\exp[-(133.3\cdot t)]$

$v_{CF}(t) = 3$ <---Forced Response

$v_{CN}(t) = 3\cdot\exp(-133.3\cdot t)$ <---Natural Response

7-13 by inspection: $IC := 0$

$R_T := \left[(2000)^{-1} + (600)^{-1}\right]^{-1} + 500$ $\quad FC := \frac{2000}{2000 + 600}\cdot 15$ $\quad FC = 11.538$

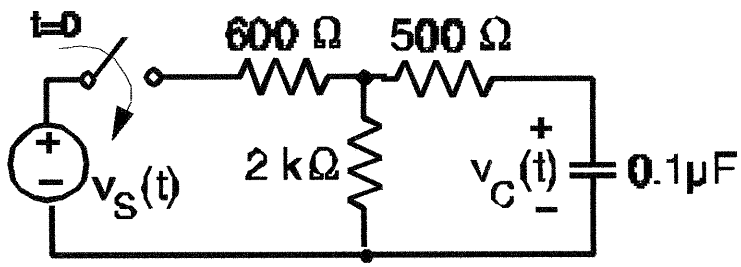
$R_T = 961.538$

$T_C := R_T\cdot C$ $\quad T_C = 9.615\times 10^{-5}$ $\quad \frac{1}{T_C} = 10400$

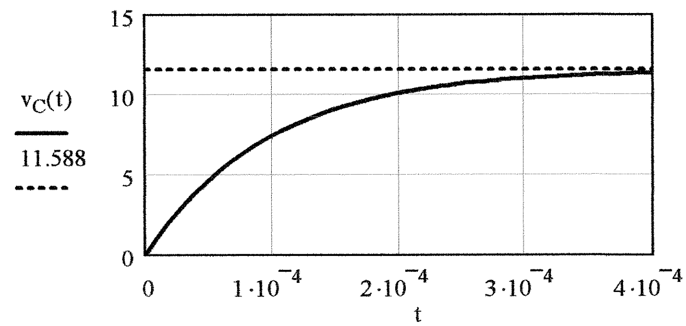
$v_C(t) := 11.538\cdot\left(1 - \exp\left(-\frac{t}{T_C}\right)\right)$ $\quad t := 0, 0.1\cdot T_C .. 5\cdot T_C$

$v_{CF}(t) := 11.538$ $\quad C := 10^{-7}$

7-13 Continued



$$v_{CN}(t) := -11.538 \cdot \exp\left(-\frac{t}{T_C}\right)$$



7-14 See Figure with Problem 7-13 above

for t <200 us : $IC := 0$ $\quad FC := \frac{2000}{2000 + 600} \cdot 15$ $\quad FC = 11.538$ $\quad C := 10^{-7}$

$R_T := \left[\left(2 \cdot 10^3\right)^{-1} + (600)^{-1}\right]^{-1} + 500$ $\quad R_T = 961.538$ $\quad T_C := R_T \cdot C$ $\quad T_C = 9.615 \times 10^{-5}$ $\quad \alpha_1 := \frac{1}{T_C}$

$v_{C1}(t) := FC + (IC - FC) \cdot \exp(-\alpha_1 \cdot t)$

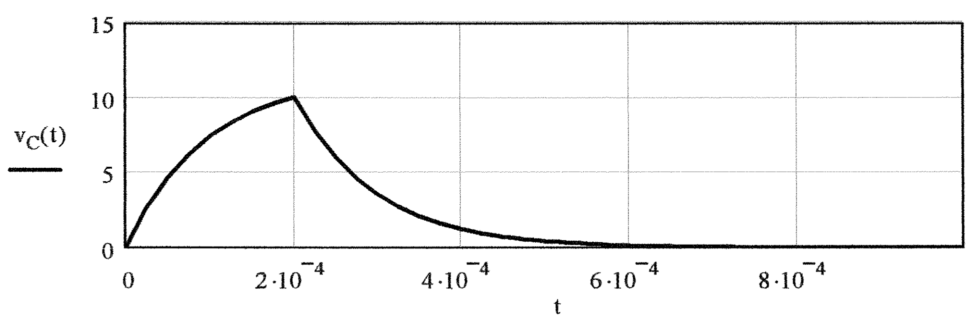
for t > 200 us: $IC := v_{C1}\left(200 \cdot 10^{-6}\right)$ $\quad IC = 10.097$ $\quad FC := 0$

$R_T := 500 + 2000$ $\quad R_T = 2.5 \times 10^3$ $\quad T_C := R_T \cdot C$ $\quad T_C = 2.5 \times 10^{-4}$ $\quad \alpha_2 := \frac{1}{T_C}$

$v_{C2}(t) := FC + (IC - FC) \cdot \exp\left[-\alpha_2 \cdot \left(t - 200 \cdot 10^{-6}\right)\right]$

$$v_C(t) := \begin{cases} v_{C1}(t) & \text{if } t \le 200 \cdot 10^{-6} \\ v_{C2}(t) & \text{if } t > 200 \cdot 10^{-6} \end{cases}$$

$t := 0, 0.1 \cdot T_C .. 5 \cdot T_C$



7-15 $\frac{d}{dt}v(t) + 200\cdot v(t) = 25\cdot\cos(100\cdot t)$ $\quad v_N(t) = K\cdot\exp(-200\cdot t)$ $\quad v_F(t) = A\cdot\cos(100\cdot t) + B\cdot\sin(100\cdot t)$

$$\frac{d}{dt}v_F(t) + 200\cdot V_F(t) = -100\cdot A\cdot\sin(100\cdot t) + 100\cdot B\cdot\cos(100\cdot t) + 200\cdot(A\cdot\cos(100\cdot t) + B\cdot\sin(100\cdot t)) = 25\cdot\cos(100\cdot t)$$

hence $\quad -100\cdot A + 200\cdot B = 0 \quad 100\cdot B + 200\cdot A = 25 \quad A = 2\cdot B \quad 500\cdot B = 25 \quad B := \frac{1}{20} \quad A := \frac{1}{10}$

$$v(t) = \frac{1}{10}\cdot\cos(100\cdot t) + \frac{1}{20}\cdot\sin(100\cdot t) + K\cdot\exp(-200\cdot t) \quad v(0) = \frac{1}{10} + K = 0 \quad K := \frac{-1}{10}$$

$$v(t) = \frac{1}{10}\cdot\cos(100\cdot t) + \frac{1}{20}\cdot\sin(100\cdot t) - \frac{1}{10}\cdot\exp(-200\cdot t)$$ checking in Mathcad

$$\frac{d}{dt}\left(\frac{1}{10}\cdot\cos(100\cdot t) + \frac{1}{20}\cdot\sin(100\cdot t) - \frac{1}{10}\cdot\exp(-200\cdot t)\right) + 200\cdot\left(\frac{1}{10}\cdot\cos(100\cdot t) + \frac{1}{20}\cdot\sin(100\cdot t) - \frac{1}{10}\cdot\exp(-200\cdot t)\right)$$

using Mathcad symbolic evaluation yields $\quad 25\cdot\cos(100\cdot t)$

7-16 $\frac{d}{dt}v(t) + 200\cdot v(t) = 25\cdot\sin(100\cdot t)$ $\quad v_N(t) = K\cdot\exp(-200\cdot t)$ $\quad v_F(t) = A\cdot\cos(100\cdot t) + B\cdot\sin(100\cdot t)$

$$\frac{d}{dt}v_F(t) + 200\cdot V_F(t) = -100\cdot A\cdot\sin(100\cdot t) + 100\cdot B\cdot\cos(100\cdot t) + 200\cdot(A\cdot\cos(100\cdot t) + B\cdot\sin(100\cdot t)) = 25\cdot\sin(100\cdot t)$$

hence $\quad -100\cdot A + 200\cdot B = 25 \quad 100\cdot B + 200\cdot A = 0 \quad B = -2\cdot A \quad -500\cdot A = 25 \quad A := \frac{-1}{20} \quad B := \frac{1}{10}$

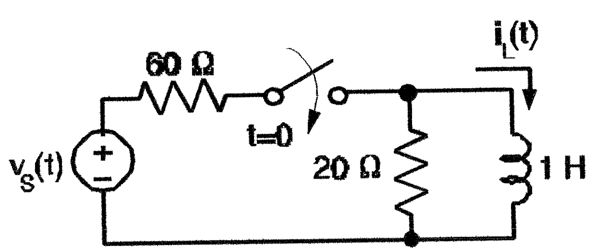
$$v(t) = \frac{-1}{20}\cdot\cos(100\cdot t) + \frac{1}{10}\cdot\sin(100\cdot t) + K\cdot\exp(-200\cdot t) \quad v(0) = \frac{-1}{20} + K = 0 \quad K := \frac{1}{20}$$

$$v(t) = \frac{-1}{20}\cdot\cos(100\cdot t) + \frac{1}{10}\cdot\sin(100\cdot t) + \frac{1}{20}\cdot\exp(-200\cdot t)$$ checking in Mathcad

$$\frac{d}{dt}\left(\frac{-1}{20}\cdot\cos(100\cdot t) + \frac{1}{10}\cdot\sin(100\cdot t) + \frac{1}{20}\cdot\exp(-200\cdot t)\right) + 200\cdot\left(\frac{-1}{20}\cdot\cos(100\cdot t) + \frac{1}{10}\cdot\sin(100\cdot t) + \frac{1}{20}\cdot\exp(-200\cdot t)\right)$$

using Mathcad symbolic evaluation yields $\quad 25\cdot\sin(100\cdot t)$

7-17



$$R_T := \frac{60\cdot 20}{80} \quad R_T = 15 \quad G_N = \frac{1}{15}$$

$$v_T = \frac{20}{80}\cdot v_S = \frac{v_S}{4} \quad i_N = \frac{v_T}{R_T} = \frac{v_S}{60} \quad v_S = 20\cdot\cos(5\cdot t)$$

$$G_N\cdot L\cdot\frac{d}{dt}i_L(t) + i_L(t) = i_N(t)$$

$$\frac{1}{15}\cdot\frac{d}{dt}i_L(t) + i_L(t) = \frac{20}{60}\cdot\cos(5\cdot t)$$

7-17(cont)

$$\frac{d}{dt}i_L(t) + 15\cdot i_L(t) = 5\cdot\cos(5\cdot t) \qquad i_N(t) = K\cdot\exp(-15\cdot t) \qquad i_F(t) = A\cdot\cos(5\cdot t) + B\cdot\sin(5\cdot t)$$

$$\frac{d}{dt}i_F(t) + 15\cdot i_F(t) = -5\cdot A\cdot\sin(5\cdot t) + 5\cdot B\cdot\cos(5\cdot t) + 15\cdot(A\cdot\cos(5\cdot t) + B\cdot\sin(5\cdot t)) = 5\cdot\cos(5\cdot t)$$

hence $\quad -5\cdot A + 15\cdot B = 0 \qquad 5\cdot B + 15\cdot A = 3 \qquad A = 3\cdot B \qquad 50\cdot B = 5 \qquad B := \frac{1}{10} \qquad A := \frac{3}{10}$

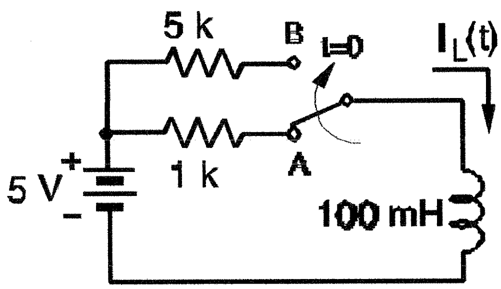
$$i_L(t) = \frac{3}{10}\cdot\cos(5\cdot t) + \frac{1}{10}\cdot\sin(5\cdot t) + K\cdot\exp(-15\cdot t) \qquad i_L(0) = \frac{3}{10} + K = 0 \qquad K := \frac{-3}{10}$$

$$i_L(t) = \frac{3}{10}\cdot\cos(5\cdot t) + \frac{1}{10}\cdot\sin(5\cdot t) + \frac{-3}{10}\cdot\exp(-15\cdot t)$$ checking in Mathcad

$$\frac{d}{dt}\left(\frac{3}{10}\cdot\cos(5\cdot t) + \frac{1}{10}\cdot\sin(5\cdot t) + \frac{-3}{10}\cdot\exp(-15\cdot t)\right) + 15\cdot\left(\frac{3}{10}\cdot\cos(5\cdot t) + \frac{1}{10}\cdot\sin(5\cdot t) + \frac{-3}{10}\cdot\exp(-15\cdot t)\right)$$

using Mathcad symbolic evaluation yields $\quad 5\cdot\cos(5\cdot t)$

7-18 Sw in pos A: $i_L(0) = \frac{5}{1000} \qquad IC := 5\cdot 10^{-3}$ Sw in pos B: $G_N := 5000^{-1} \quad L := 0.1$

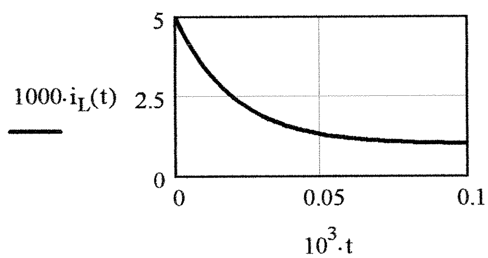


$T_C := G_N\cdot L \qquad T_C = 2\times 10^{-5} \qquad \alpha := T_C^{-1}$

$\alpha = 5\times 10^{4} \qquad FC := \frac{5}{5000} \qquad FC = 1\times 10^{-3}$

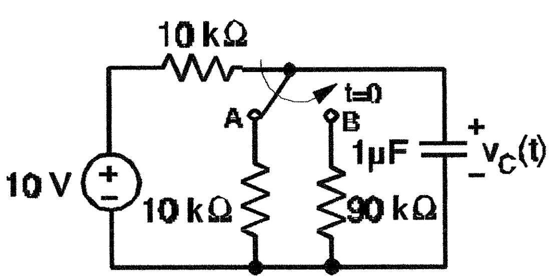
$i_L(t) := FC + (IC - FC)\cdot\exp(-\alpha\cdot t)$

$i_L(t) := 0.001 + 0.004\exp(-\alpha\cdot t) \qquad t := 0, 0.1\cdot T_C .. 5\cdot T_C$



7-19 sw in pos A: $IC := \frac{10\cdot 10}{20} \qquad IC = 5 \qquad C := 1\cdot 10^{-6}$

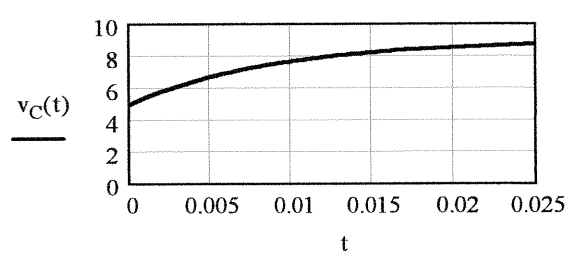
Sw in pos B: $R_T := \left[\left(10^4\right)^{-1} + \left(9\cdot 10^4\right)^{-1}\right]^{-1}$



$R_T = 9\times 10^{3} \qquad T_C := R_T\cdot C \qquad \alpha := T_C^{-1}$

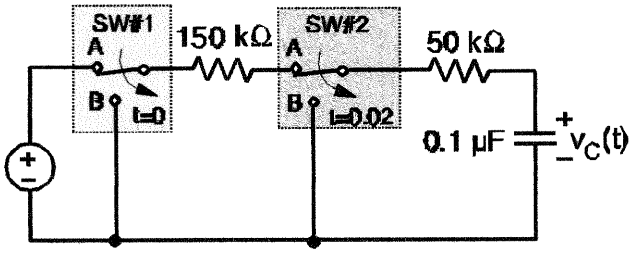
$\alpha = 111.111 \qquad FC := \frac{90\cdot 10}{100} \qquad FC = 9$

$v_C(t) := 9 + (5 - 9)\cdot\exp(-\alpha\cdot t) \qquad t := 0, 0.1\cdot T_C .. 5\cdot T_C$



7-20 for t < 0 sw#1 in Pos A and sw#2 in Pos A: $C := 10^{-7}$ $IC := 10$

for 0 < t < 20 ms, sw#1 in Pos B sw#2 in Pos A: $FC = 0$ $R_T := 200 \cdot 10^3$ $T_C := R_T \cdot C$



$T_C = 0.02$ $\alpha_1 := T_C^{-1}$

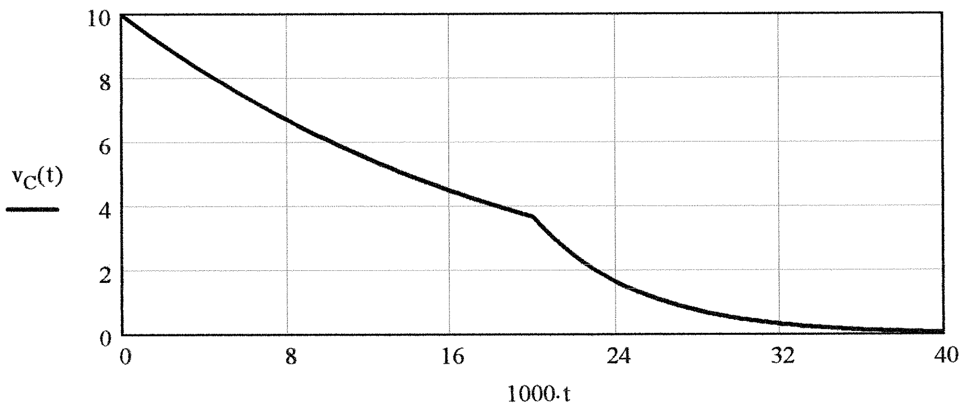
$\alpha_1 = 50$ $v_{C1}(t) := IC \cdot \exp(-\alpha_1 \cdot t)$

for t > 20 ms,
sw#1 in Pos B sw#2 in Pos B:

$IC := v_{C1}(0.02)$ $IC = 3.679$

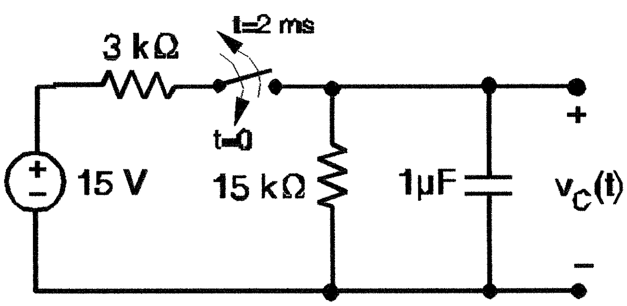
$FC := 0$ $R_T := 50 \cdot 10^3$ $T_C := R_T \cdot C$ $T_C = 5 \times 10^{-3}$ $\alpha_2 := T_C^{-1}$ $\alpha_2 = 200$

$v_{C2}(t) := IC \cdot \exp[-\alpha_2 \cdot (t - 0.02)]$ $v_C(t) := \begin{vmatrix} (v_{C1}(t)) \text{ if } t \le 0.02 \\ (v_{C2}(t)) \text{ if } t > 0.02 \end{vmatrix}$ $t := 0, 0.1 \cdot T_C .. 5 \cdot T_C + 0.02$



7-21 For t < 0 sw is open IC = 0.
For 0 < t < 2 ms the switch is closed and

$R_T := \left[(3 \cdot 10^3)^{-1} + (15 \cdot 10^3)^{-1}\right]^{-1}$ $R_T = 2.5 \times 10^3$



$C := 10^{-6}$ $T_C := R_T \cdot C$ $\alpha_1 := (T_C)^{-1}$

$\alpha_1 = 4 \times 10^3$ $FV := \frac{15}{18} \cdot 15$ $FV = 12.5$

$v_{C1}(t) := FV \cdot (1 - \exp(-\alpha_1 \cdot t))$

$v_{C1}(t) = 12.5 \cdot (1 - \exp(-400 \cdot t\ )$ 0 < t < 2ms

For t > 2 ms the switch is open again $IV := v_{C1}(2 \cdot 10^{-3})$ $IV = 12.496$

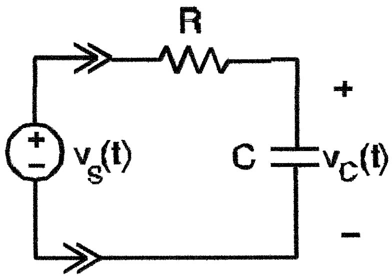
$R_T := 15 \cdot 10^3$ $T_C := R_T \cdot C$ $\alpha_2 := (T_C)^{-1}$

$FV := 0$ $\alpha_2 = 66.667$

$v_{C2}(t) := IV \cdot \exp(-\alpha_2 \cdot t)$

$v_{C2}(t) = 6.883 \cdot \exp(-66.667 \cdot t)$ t > 2 ms

7-22 For no initial capacitor voltage the zero-state response for $0 < t < T$ is



$$v_C(t) = V_A \cdot \left(1 - \exp\left(\frac{t}{RC}\right)\right) = 5 \cdot \left(1 - \exp\left(\frac{-t}{RC}\right)\right)$$

This response reaches 4 V when

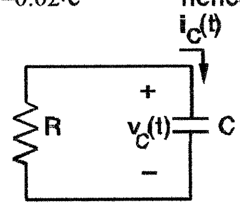
$$5 \cdot \left(1 - \exp\left(\frac{-t}{RC}\right)\right) = 4 \quad \text{or} \quad \exp\left(\frac{-t}{RC}\right) = \left(1 - \frac{4}{5}\right) = 0.2$$

hence $t = -RC \ln(0.2) = 32.2$ ns.

To be detected the pulse duration T must exceed this time. Hence the minimum duration is 32.2 ns

7-23 zero input respose $v_C(t) := 10 \cdot e^{-1000 \cdot t}$ and $i_C(t) := -0.02 \cdot e^{-1000 \cdot t}$ **(a)** $T_C := 1000^{-1}$

(b) $v_C(0) = 10$ **(c)** $i_C(t) = C \cdot \frac{d}{dt} v_C(t) = 10 \cdot (-1000) \cdot C \cdot e^{-1000 \cdot t} = -0.02 \cdot e^{-1000 \cdot t}$ hence $C := \frac{0.02}{10 \cdot 1000}$



$C = 2 \times 10^{-6}$ and $R := \frac{T_C}{C}$ $R = 500$

(d) $W_C := 0.5 C \cdot \left(v_C\left(10^{-3}\right)\right)^2$ $W_C = 1.353 \times 10^{-5}$

7-24 zero-input resps $i_L(t) = 0.005 \cdot e^{-5000 \cdot t}$ & $v_L(t) := -10 \cdot e^{-5000 \cdot t}$ **(a)** $T_C := 5000^{-1}$

(b) $i_L(0) = 5 \times 10^{-3}$

(c) $v_L(t) = L \cdot \frac{d}{dt} i_L(t) = 0.005 \cdot (-5000) \cdot L \cdot e^{-5000 \cdot t} = -10 \cdot e^{-5000 \cdot t}$ $L := \frac{10}{0.005 \cdot 5000}$ $L = 0.4$ $R := \frac{L}{T_C}$

$R = 2000$

(d) $W_L := 0.5 L \cdot \left(i_L(0)\right)^2$ $W_L = 5 \times 10^{-6}$ $v_C(t) := 15 - 10 \cdot e^{-2000 \cdot t}$ $i_C(t) := 0.01 \cdot e^{-2000 \cdot t}$

7-25

(a) $T_C := 2000^{-1}$ $T_C = 5 \times 10^{-4}$ **(b)** $v_C(0) = 5$ $C = \frac{i_C(t)}{\frac{d}{dt} v_C(t)} = \frac{0.01 e^{-2000 \cdot t}}{10 \cdot 2000 \cdot e^{-2000 \cdot t}}$

$C := \frac{0.01}{20000}$ $C = 5 \times 10^{-7}$ $R := \frac{T_C}{C}$ $R = 1 \times 10^3$

$v_S = v_C(\infty) = 15$ **(d)** $W_C := 0.5 \cdot C \cdot \left(v_C\left(10^{-3}\right)\right)^2$ $W_C = 4.656 \times 10^{-5}$

7-26 $v_C(t) = 10 - 10 \cdot e^{-500 \cdot t} + \left[15 \cdot e^{-500 \cdot (t - 0.005)} - 15\right] \cdot u(t - 0.005)$ $C := 100 \cdot 10^{-9}$

(a) $T_C := 500^{-1}$ $T_C = 2 \times 10^{-3}$ **(b)** $v_C(0) = 0$ $v_C(\infty) = -5$ **(c)** $R := \frac{T_C}{C}$ $R = 2 \times 10^4$

$v_S = 10 \cdot u(t) - 15 \cdot u(t - 0.005)$

7-27 $i_L(t) = 0.005 - 0.01 \cdot e^{-1000 \cdot t}$ $v_L(t) := e^{-1000 \cdot t}$ **(a)** $T_C := 1000^{-1}$ **(b)** $i_L(0) = -0.005$

$i_L(\infty) = 0.005$ **(c)** $L = \frac{v_L(t)}{\frac{d}{dt} i_L(t)} = \frac{e^{-1000 \cdot t}}{10 \cdot e^{-1000 \cdot t}}$ $L := 0.1$ $R := \frac{L}{T_C}$ $R = 100$ $v_S = i_L(\infty) \cdot R = 0.5$

(d) $i_L(\infty) = \left|i_L(0)\right| = 0.005$ hence $W_L(0) = W_L(\infty)$ $W_L := 0.5 \cdot L \cdot (0.005)^2$ $W_L = 1.25 \times 10^{-6}$

7-28 zero-input respose $v_L(t) := -10 \cdot e^{-500 \cdot t}$ **(a)** $T_C := 500^{-1}$

(b) state variable zero-input response is

$i_L(t) = 0.01 \cdot e^{-5000 \cdot t}$ and therefore $v_L(t) = L \cdot \frac{d}{dt} i_L(t) = 0.01 \cdot (-500) \cdot L \cdot e^{-500 \cdot t} = -10 \cdot e^{-500 \cdot t}$

hence, $L := \frac{10}{0.01 \cdot 500}$ $L = 2$ and $R := \frac{L}{T_C}$ $R = 1000$

7-29

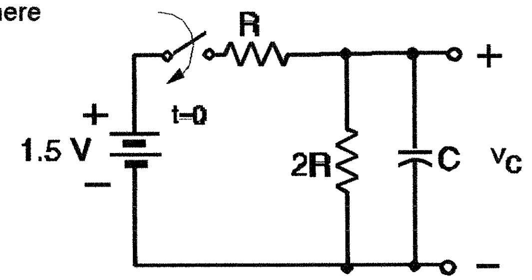
$v_C(t) = 10 - 10 \cdot e^{-100 \cdot t}$ $v_S = v_C(\infty) = 10$ $R \cdot C = \frac{1}{100}$ Let $C := 10^{-6}$ $R := \frac{1}{100 \cdot C}$ $R = 1 \times 10^{4}$

7-30 $v_S(t) = 5 \cdot u(t)$ hence $v_C(t) = 5 - 5 \cdot \exp\left(\frac{-t}{RC}\right)$ the requirement $v_C(.005) = 2.5$ means

$5 - 5 \cdot \exp\left(\frac{-0.005}{RC}\right) = 2.5$ or $RC = 7.214 \cdot 10^{-3}$ let $C := 10^{-6}$ $R := \frac{7.214 \cdot 10^{-3}}{C}$ $R = 7.214 \times 10^{3}$

7-31 (Laplace Early) For the RC circuit below the step response is $v_C(t) = V_T \cdot \left(1 - \exp\left(-\frac{t}{R_T \cdot C}\right)\right)$

where



$V_T = \frac{2 \cdot R}{2 \cdot R + R} \cdot 1.5 = 1.0$ $R_T = \frac{2 \cdot R \cdot R}{2 \cdot R + R} = \frac{2 \cdot R}{3}$

To meet the design requirement the time at which the reponse reaches one-half of its final value (T_{half}) must be in the range

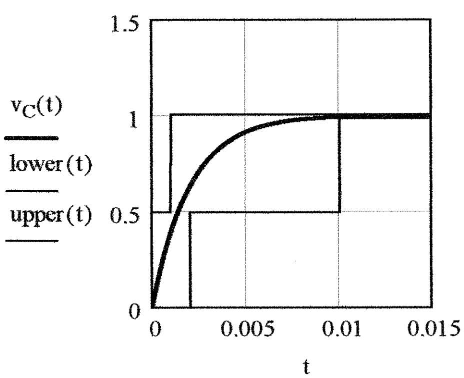
$0.001 < T_{half} < 0.002$ s. For a first order step response $T_{half} = T_C x \ln(2)$, hence

$1.443 \cdot 10^{-3} = \frac{0.001}{\ln(2)} < T_C < \frac{0.002}{\ln(2)} = 2.885 \cdot 10^{-3}$

Let $T_C := 0.002$ and $R := 10^{4}$ then $R_T := 2 \cdot \frac{R}{3}$ and $C := \frac{T_C}{R_T}$ $C = 3 \times 10^{-7}$

$v_C(t) := \left(1 - \exp\left(-\frac{t}{T_C}\right)\right)$ $\text{upper}(t) := \begin{cases} 0.5 & \text{if } 0 \le t < 0.001 \\ 1.01 & \text{if } 0.001 \le t \end{cases}$ $\text{lower}(t) := \begin{cases} 0 & \text{if } 0 \le t < 0.002 \\ 0.5 & \text{if } 0.002 \le t < 0.010 \\ 0.99 & \text{if } 0.01 \le t \end{cases}$

$t := 0, 10^{-5} .. 0.015$



Testing design

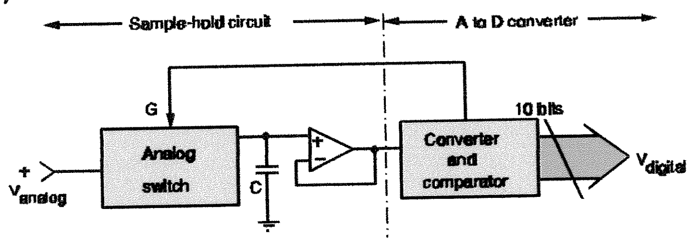
$v_C(0.001) = 0.3935 < 0.5$

$v_C(0.002) = 0.6321 > 0.5$

$v_C(0.01) = 0.9933 > 0.99$

Circuit meets design requirements

7-32 (Laplace Early)



Switch $R_{on} = 50$, $R_{off} = 10^8$ and the capacitor $C = 20 \cdot 10^{-12}$

(a) sample mode: $T_C = 50 \cdot 20 \cdot 10^{-12} = 10^{-9}$; hold mode: $T_C = 10^8 \cdot 20 \cdot 10^{-12} = 2 \cdot 10^{-3}$.

(b) Let N=number of sample-hold cycles/second, for f = 1000 Hertz,

N>2f = 2000 sample-hold cycles/sec.

(c) Let $N = 10N_{min}$=20,000; T_{sm}=sample mode time, T_{hm}=Hold mode time, T_s=cycle time;

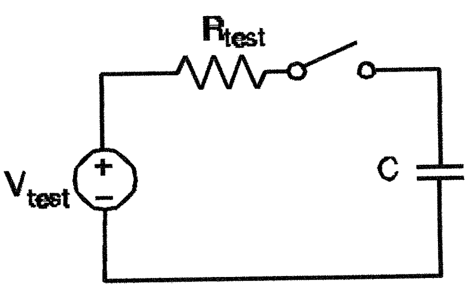
$T_s=1/N=1/20000$, $T_s=T_{sm}+T_{hm}=T_{sm}+9T_{sm}=10T_{sm}$, $Tsm=T_s/10=1/200000=5\ \mu s$.

(d) $T_{sm}=5\ \mu s$, in the sample mode $5T_c$=5 ns, i.e. $T_{sm}>>T_c$, hence capacitor will be charged.

(e) $T_{hm}=9T_{sm}=45$ us, in the hold mode T_c=2 ms, exp(-45 μs/2 ms)=0.9778, or about

(1 - 0.9778)x100 = 2.225% will be lost during the hold mode.

7-33 (Laplace Early) $R_{test} := 200$ $V_{test} := 10$ $V_{rated} := 11$ $i_{30} < 1.5 \cdot 10^{-3}$ $C_{nom} := 1.4$ **+80% -20%**



After the switch closes the capacitor voltage is

$$v_C(t) = V_{test} \cdot \left(1 - \exp\left(\frac{-t}{R_{test} \cdot C}\right)\right)$$

The test voltage is consistent since

$v_C(t) \le V_{test} < V_{rated}$

The current is

$$i(t) = C \cdot \frac{d}{dt} v_C(t) = \frac{V_{test}}{R_{test}} \cdot \exp\left(\frac{-t}{R_{test} \cdot C}\right)$$

the 30 minute current depends of the value of C

$$i_{30}(C) := \frac{V_{test}}{R_{test}} \cdot \exp\left(\frac{-30 \cdot 60}{R_{test} \cdot C}\right)$$

$C_{max} := 1.8 \cdot C_{nom}$ $i_{30}(C_{max}) = 1.406 \times 10^{-3}$ $C_{min} := 0.8 \cdot C_{nom}$ $i_{30}(C_{min}) = 1.618 \times 10^{-5}$

The 30 minute current is less than 1.5 mA for C within the specified range.
The specification is internally consistent.

CHAPTER 7 (Cont) Standard Version (use 7-XX) and CHAPTER 8 Laplace-Early Version (use 8-XX).

7-31, 8-1 Char. eq: $s^2 + 16 \cdot s + 64 = 0$ roots are $\text{polyroots}\left((64\ \ 16\ \ 1)^T\right) = \begin{pmatrix} -8 \\ -8 \end{pmatrix}$ critically damped

$v_N(t) = K_1 \cdot e^{-8t} + K_2 \cdot t \cdot e^{-8 \cdot t}$ $\quad v_F(t) = 0 \quad v(t) = v_N(t) + v_F(t)$

$v(0) = 0$ implies $K_1 = 0$ $\quad \frac{d}{dt}v(0) = 12$ implies $K_2 = 12$ Solution is $v(t) = 12 \cdot t \cdot e^{-8 \cdot t}$

7-32, 8-2 Char. eq: $s^2 + 20 \cdot s + 500 = 0$ roots are $\text{polyroots}\left((500\ \ 20\ \ 1)^T\right) = \begin{pmatrix} -10 - 20i \\ -10 + 20i \end{pmatrix}$

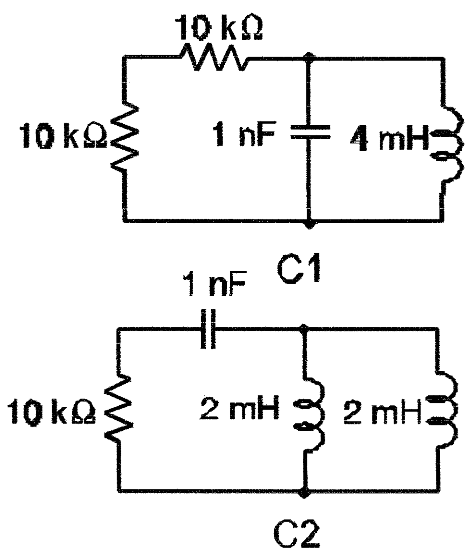
underdamped

$v_N(t) = e^{-10 \cdot t} \cdot \left(K_1 \cdot \cos(20 \cdot t) + K_2 \cdot \sin(20 \cdot t)\right) \quad v_F(t) = 0 \quad v(t) = v_N(t) + v_F(t)$

$v(0) = 5$ implies $K_1 = 5$ $\quad \frac{d}{dt}v(0) = 30$ implies $-10 \cdot K_1 + 20 \cdot K_2 = 30$ hence $K_2 = 4$

Solution is $v(t) = e^{-10 \cdot t} \cdot (5 \cdot \cos(10 \cdot t) + 4 \cdot \sin(10 \cdot t))$

7-33, 8-3



C1: parallel RLC with

$L := 4 \cdot 10^{-3} \quad C := 10^{-9} \quad G_N := \left(20 \cdot 10^3\right)^{-1}$

char eq: $L \cdot C \cdot s^2 + G_N \cdot L \cdot s + 1$

$\text{polyroots}\left(\begin{pmatrix} 1 \\ G_N \cdot L \\ L \cdot C \end{pmatrix}\right) = \begin{pmatrix} -2.5 \times 10^4 + 4.994i \times 10^5 \\ -2.5 \times 10^4 - 4.994i \times 10^5 \end{pmatrix}$

roots of are complex--C1 is underdamped

C2: series RLC with

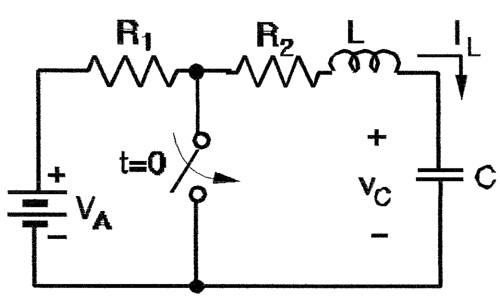
$L := 1 \cdot 10^{-3} \quad C := 10^{-9} \quad R_T := 10 \cdot 10^3$

char eq: $L \cdot C \cdot s^2 + R_T \cdot C \cdot s + 1$

$\text{polyroots}\left(\begin{pmatrix} 1 \\ R_T \cdot C \\ L \cdot C \end{pmatrix}\right) = \begin{pmatrix} -9.899 \times 10^6 \\ -1.01 \times 10^5 \end{pmatrix}$

roots are real & distinct--C2 is overdamped

7-34, 8-4 Series RLC ckt with: $R_1 := 2000 \quad R_2 := 3000 \quad L := 1 \quad C := 0.5 \cdot 10^{-6} \quad V_A := 10$



for t < 0 sw open $v_C(0) = V_A \quad i_L(0) = 0$

for t > 0 sw closed $v_C(\infty) = 0 \quad i_L(\infty) = 0$

and $v_T := 0 \quad R_T := R_2$

$$L \cdot C \cdot \frac{d^2}{dt^2}v_C(t) + R_T \cdot C \cdot \frac{d}{dt}v_C(t) + v_C(t) = 0$$

Char. eq. $L \cdot C \cdot s^2 + R_T \cdot C \cdot s + 1 = 0$ roots are

$\text{polyroots}\left(\left(1\ \ R_T\cdot C\ \ L\cdot C\right)^T\right) = \begin{pmatrix} -2\times 10^3 \\ -1\times 10^3 \end{pmatrix}$ **overdamped** $\alpha_1 := 1000 \quad \alpha_2 := 2000$

$v_C(t) = K_1\cdot\exp(-\alpha_1\cdot t) + K_2\cdot\exp(-\alpha_2\cdot t)$ $\quad v_C(0) = V_A$ **implies** $K_1 + K_2 = V_A$ $\quad \frac{d}{dt}v_C(0) = 0$ **implies**

$-\alpha_1\cdot K_1 - \alpha_2\cdot K_2 = 0 \quad K_1 := 1 \quad K_2 := 1$ **<---Initial guesses for solve block**

Given $K_1 + K_2 = V_A \quad -\alpha_1\cdot K_1 - \alpha_2\cdot K_2 = 0 \quad \text{Find}(K_1,K_2) = \begin{pmatrix} 20 \\ -10 \end{pmatrix}$

$v_C(t) := 20\cdot\exp(-1000\cdot t) - 10\cdot\exp(-2000\cdot t)$

evaluating $i_L(t) = C\cdot\frac{d}{dt}v_C(t) = 0.5\cdot 10^{-6}\cdot\frac{d}{dt}(20\cdot\exp(-1000\cdot t) - 10\cdot\exp(-2000\cdot t))$ **yields**

$i_L(t) := -10\cdot 10^{-3}\cdot\exp(-1000\cdot t) + 10\cdot 10^{-3}\cdot\exp(-2000\cdot t)$

7-35, 8-5 Series RLC ckt with: $R_1 := 3000 \quad R_2 := 2000 \quad L := 1 \quad C := 2\cdot 10^{-7} \quad V_A := 10$

for t < 0 sw open $v_C(0) = V_A \quad i_L(0) = 0$ **See Figure P7-34 above**

for t > 0 sw closed $v_C(\infty) = 0 \quad i_L(\infty) = 0$ **and** $v_T := 0 \quad R_T := R_2$

$$L\cdot C\cdot\frac{d^2}{dt^2}v_C(t) + R_T\cdot C\cdot\frac{d}{dt}v_C(t) + v_C(t) = 0$$

Char. eq. $L\cdot C\cdot s^2 + R_T\cdot C\cdot s + 1 = 0$ **roots are**

$\text{polyroots}\left(\left(1\ \ R_T\cdot C\ \ L\cdot C\right)^T\right) = \begin{pmatrix} -1\times 10^3 - 2i\times 10^3 \\ -1\times 10^3 + 2i\times 10^3 \end{pmatrix}$ **underdamped** $\alpha := 1000 \quad \beta := 2000$

(c) $v_C(t) = \exp(-\alpha\cdot t)(K_1\cdot\cos(\beta\cdot t) + K_2\cdot\sin(\beta\cdot t))$

$v_C(0) = V_A$ **implies** $K_1 = V_A = 10 \quad \frac{d}{dt}v_C(0) = 0$ **implies** $-\alpha\cdot K_1 + \beta\cdot K_2 = 0 \quad K_2 = \frac{\alpha}{\beta}\cdot K_1 = 5$

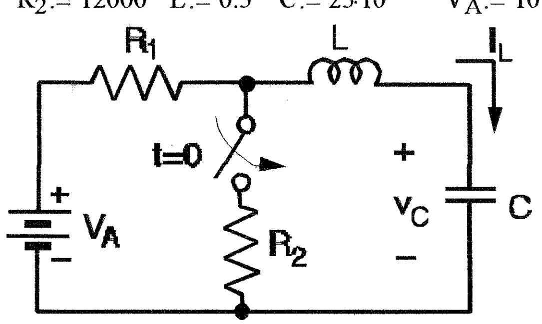
$v_C(t) = \exp(-1000\cdot t)(10\cdot\cos(2000\cdot t) + 5\cdot\sin(2000\cdot t))$

evaluating $i_L(t) = C\cdot\frac{d}{dt}v_C(t) = 2\cdot 10^{-7}\cdot\frac{d}{dt}[\exp(-1000\cdot t)(10\cdot\cos(2000\cdot t) + 5\cdot\sin(2000\cdot t))]$

yields $i_L(t) = \frac{-1}{200}\cdot\exp(-1000\cdot t)\cdot\sin(2000\cdot t)$

7-36, 8-6 Series RLC ckt with: $R_1 := 5000$

$R_2 := 12000 \quad L := 0.5 \quad C := 25\cdot 10^{-9} \quad V_A := 10$



(a) for t < 0 sw open $v_C(0) = V_A \quad i_L(0) = 0$

for t > 0 sw closed $v_C(\infty) = V_A\cdot\frac{R_2}{R_1 + R_2} = v_T$

$i_L(\infty) = 0$ **and** $v_T := 7.059 \quad R_T := \frac{R_2\cdot R_1}{R_1 + R_2}$

(b) $L\cdot C\cdot\frac{d^2}{dt^2}v_C(t) + R_T\cdot C\cdot\frac{d}{dt}v_C(t) + v_C(t) = 0$

Char. eq. $L\cdot C\cdot s^2 + R_T\cdot C\cdot s + 1 = 0$ **roots are**

$\text{polyroots}\left(\left(1 \;\; R_T\cdot C \;\; L\cdot C\right)^T\right) = \begin{pmatrix} -3.529\times 10^3 - 8.218i\times 10^3 \\ -3.529\times 10^3 + 8.218i\times 10^3 \end{pmatrix}$ **underdamped** $\alpha := 3529 \quad \beta := 8218$

(c) $v_C(t) = K_1\cdot\exp(-\alpha\cdot t)\cdot\cos(\beta\cdot t) + K_2\cdot\exp(-\alpha\cdot t)\cdot\sin(\beta\cdot t) + \frac{v_T}{L\cdot C}$ $\quad v_C(0) = V_A$ **implies** $K_1 = V_A - v_T$

$\frac{d}{dt}v_C(0) = 0$ **implies** $-\alpha\cdot K_1 + \beta\cdot K_2 = 0$

$K_1 := 1 \quad K_2 := 1$ **<---initial guesses for solve block**

Given $K_1 = V_A - v_T \quad -\alpha\cdot K_1 + \beta\cdot K_2 = 0 \quad \text{Find}(K_1, K_2) = \begin{pmatrix} 2.941 \\ 1.263 \end{pmatrix}$

$v_C(t) := \exp(-3529\cdot t)(2.941\cdot\cos(8218\cdot t) + 1.263\cdot\sin(8218\cdot t)) + 7.059$

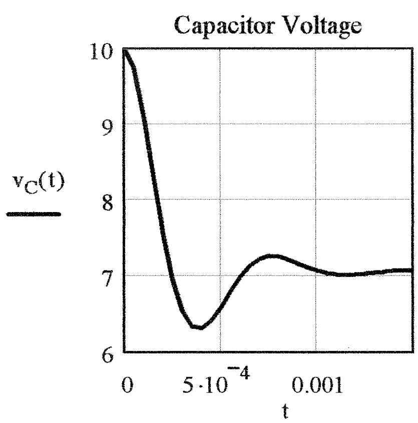
<-----------------------------natural----------------------------><---forced--->

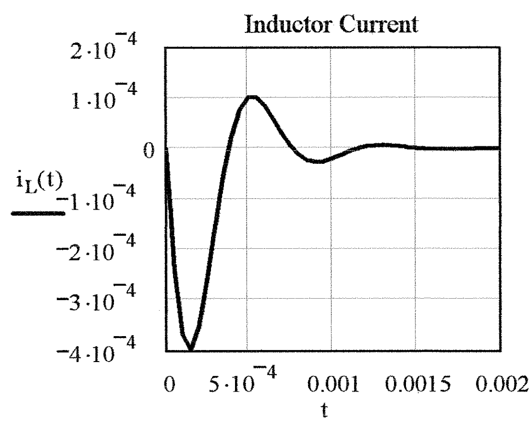
evaluating $i_L(t) = C\cdot\frac{d}{dt}v_C(t) = 25\cdot 10^{-9}\cdot\frac{d}{dt}[\exp(-3529\cdot t)(2.941\cdot\cos(8218\cdot t) + 1.263\cdot\sin(8218\cdot t)) + 7.059]$

yields

$$i_L(t) := 25\cdot 10^{-9}\left[\begin{array}{l}\exp(-3529\cdot t)(2.941\cdot -8218\cdot\sin(8218\cdot t) + 1.263\cdot 8218\cdot\cos(8218\cdot t)) \;... \\ + (-3529)\cdot\exp(-3529\cdot t)\cdot(2.941\cdot\cos(8218\cdot t) + 1.263\cdot\sin(8218\cdot t))\end{array}\right]$$

$t := 0, 0.1\cdot\alpha_2^{-1} .. 5\cdot\alpha_1^{-1}$





Circuit is underdamped

7-37, 8-7 Series RLC ckt with: $R_1 := 1000$

$R_2 := 2000 \;\; L := 0.5 \;\; C := 400\cdot 10^{-9} \quad V_A := 15$

See circuit in 7-36, 8-6 above.

(a) for t < 0 sw closed $v_C(0) = \frac{R_2\cdot V_A}{R_1 + R_2} = 10$

$i_L(0) = 0$

for t > 0 sw open $v_C(\infty) = V_A = 15 \quad i_L(\infty) = 0$

and $v_T := V_A \quad R_T := R_1$

(b) $L\cdot C\cdot\frac{d^2}{dt^2}v_C(t) + R_T\cdot C\cdot\frac{d}{dt}v_C(t) + v_C(t) = v_T$

Char. eq. $L\cdot C\cdot s^2 + R_T\cdot C\cdot s + 1 = 0$ roots are

$\text{polyroots}\left(\begin{pmatrix}1 & R_T\cdot C & L\cdot C\end{pmatrix}^T\right) = \begin{pmatrix}-1\times 10^3 - 2i\times 10^3 \\ -1\times 10^3 + 2i\times 10^3\end{pmatrix}$ **underdamped** $\alpha := 1000 \quad \beta := 2000$

(c) $v_C(t) = \exp(-\alpha\cdot t)\cdot(K_1\cdot\cos(\beta\cdot t) + K_2\cdot\sin(\beta\cdot t)) + v_T$ $\quad v_C(0) = 10$ **implies** $K_1 + v_T = 10$

$\frac{d}{dt}v_C(0) = 0$ **implies** $-\alpha\cdot K_1 + \beta\cdot K_2 = 0$

$K_1 := 1 \quad K_2 := 1$ **<---Initial guesses for solve block**

Given $K_1 + v_T = 10 \quad -\alpha\cdot K_1 + \beta\cdot K_2 = 0 \quad \text{Find}(K_1, K_2) = \begin{pmatrix}-5 \\ -2.5\end{pmatrix}$

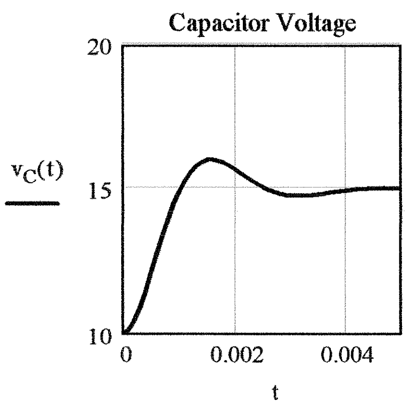
$v_C(t) := \exp(-1000\cdot t)\cdot(-5\cdot\cos(2000\cdot t) - 2.5\cdot\sin(2000\cdot t)) + 15$

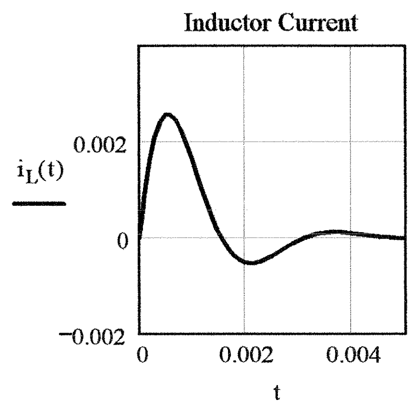
<---------------------------natural------------------------><-forced->

evaluating $i_L(t) = C\cdot\frac{d}{dt}v_C(t) = 400\cdot 10^{-9}\cdot\frac{d}{dt}[\exp(-1000\cdot t)\cdot(-5\cdot\cos(2000\cdot t) - 2.5\cdot\sin(2000\cdot t)) + 15]$

yields $i_L(t) := 5\cdot 10^{-3}\cdot\exp(-1000.\cdot t)\cdot\sin(2000.\cdot t) \quad t := 0, 0.1\cdot\alpha^{-1} .. 5\cdot\alpha^{-1}$

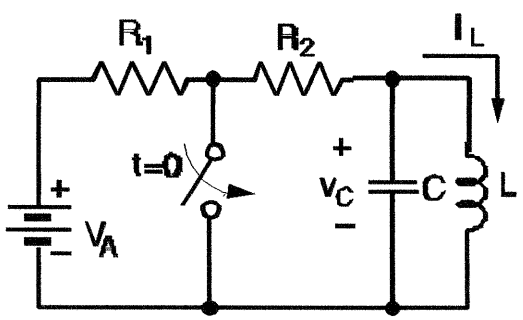
<---------------natural-------------->





Circuit is underdamped

7-38, 8-8 **Parallel RLC ckt with:** $R_1 := 3000 \quad R_2 := 2000 \quad L := 0.4 \quad C := 0.25\cdot 10^{-6} \quad V_A := 15$



for t < 0 sw open $v_C(0) = 0$

$i_L(0) = \frac{V_A}{R_1 + R_2} = 3\cdot 10^{-3}$

for t > 0 sw closed $v_C(\infty) = 0 \quad i_L(\infty) = 0$ **and**

$i_N := 0 \quad G_N := R_2^{-1}$

$L\cdot C\cdot\frac{d^2}{dt^2}i_L(t) + G_N\cdot L\cdot\frac{d}{dt}i_L(t) + i_L(t) = 0$

Char. eq. $L\cdot C\cdot s^2 + G_N\cdot L\cdot s + 1 = 0$ **roots are**

$$\text{polyroots}\left[\left(1 \quad G_N \cdot L \quad L \cdot C\right)^T\right] = \begin{pmatrix} -1 \times 10^3 + 3i \times 10^3 \\ -1 \times 10^3 - 3i \times 10^3 \end{pmatrix}$$ underdamped $\alpha := 1000 \quad \beta := 3000$

$i_L(t) = \exp(-\alpha \cdot t) \cdot (K_1 \cdot \cos(\beta \cdot t) + K_2 \cdot \sin(\beta \cdot t))$ $\quad i_L(0) = 3 \cdot 10^{-3}$ implies $K_1 = 3 \cdot 10^{-3}$ $\quad \frac{d}{dt} i_L(0) = 0$

implies $-\alpha \cdot K_1 + \beta \cdot K_2 = 0 \quad K_2 = \frac{\alpha}{\beta} \cdot K_1 = 10^{-3}$

$i_L(t) := \exp(-1000 \cdot t) \cdot \left(3 \cdot 10^{-3} \cdot \cos(3000 \cdot t) + 10^{-3} \cdot \sin(3000 \cdot t)\right)$

evaluating $v_C(t) = L \cdot \frac{d}{dt} i_L(t) = 0.4 \cdot \frac{d}{dt}\left[\exp(-1000 \cdot t) \cdot \left(3 \cdot 10^{-3} \cdot \cos(3000 \cdot t) + 10^{-3} \cdot \sin(3000 \cdot t)\right)\right]$, yields

$v_C(t) := -(4 \cdot \exp(-1000. \cdot t) \cdot \sin(3000 \cdot t))$

7-39, 8-9 Parallel RLC ckt with: $R_1 := 3000 \quad R_2 := 2000 \quad L := 0.08 \quad C := 0.25 \cdot 10^{-6} \quad V_A := 15$

See circuit in Prob 7-38, 8-8 above

for t < 0 sw open $v_C(0) = 0$ for t > 0 sw closed $v_C(\infty) = 0 \quad i_L(\infty) = 0 \quad i_L(0) = \frac{V_A}{R_1 + R_2} = 3 \cdot 10^{-3}$

and $i_N := 0 \quad G_N := R_2^{-1} \quad L \cdot C \cdot \frac{d^2}{dt^2} i_L(t) + G_N \cdot L \cdot \frac{d}{dt} i_L(t) + i_L(t) = 0$ Char. eq. $L \cdot C \cdot s^2 + G_N \cdot L \cdot s + 1 = 0$

$$\text{polyroots}\left[\left(1 \quad G_N \cdot L \quad L \cdot C\right)^T\right] = \begin{pmatrix} -1 \times 10^3 + 7i \times 10^3 \\ -1 \times 10^3 - 7i \times 10^3 \end{pmatrix}$$ underdamped $\alpha := 1000 \quad \beta := 7000$

$i_L(t) = \exp(-\alpha \cdot t) \cdot (K_1 \cdot \cos(\beta \cdot t) + K_2 \cdot \sin(\beta \cdot t))$

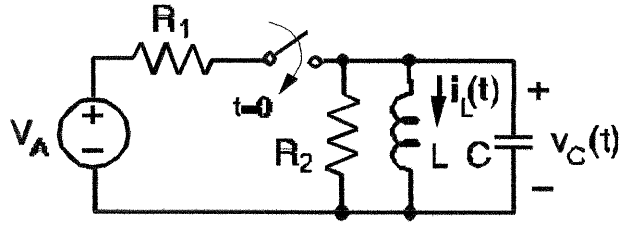
$i_L(0) = 3 \cdot 10^{-3}$ implies $K_1 := 3 \cdot 10^{-3} \quad \frac{d}{dt} i_L(0) = 0$ implies $-\alpha \cdot K_1 + \beta \cdot K_2 = 0$

$K_2 = \frac{\alpha}{\beta} \cdot K_1 \quad K_2 := \frac{K_1}{7} \quad K_2 = 4.286 \times 10^{-4}$

$i_L(t) := \exp(-1000 \cdot t) \cdot \left(3 \cdot 10^{-3} \cdot \cos(7000 \cdot t) + 4.286 \cdot 10^{-3} \cdot \sin(7000 \cdot t)\right) \quad v_C(t) = L \cdot \frac{d}{dt} i_L(t)$

$$v_C(t) = 0.08 \cdot \left[\frac{d}{dt}\left[\exp(-1000 \cdot t) \cdot \left(3 \cdot 10^{-3} \cdot \cos(7000 \cdot t) + 4.286 \cdot 10^{-3} \cdot \sin(7000 \cdot t)\right)\right]\right] = -1.714 \cdot \exp(-1000 \cdot t) \cdot \sin(7000t)$$

7-40, 8-10 Parallel RLC ckt with: $R_1 := 4000 \quad R_2 := 4000 \quad L := 0.8 \quad C := 50 \cdot 10^{-9} \quad V_A := 20$



for t < 0 sw pen $v_C(0) = 0 \quad i_L(0) = 0$

for t > 0 sw closed $v_C(\infty) = 0$

$i_L(\infty) = \frac{V_A}{R_1} = 5 \cdot 10^{-3}$ and

$i_N := 5 \cdot 10^{-3} \quad G_N := \left(R_1^{-1} + R_2^{-1}\right)$

$L \cdot C \cdot \frac{d^2}{dt^2} i_L(t) + G_N \cdot L \cdot \frac{d}{dt} i_L(t) + i_L(t) = i_N$

Char. eq. $L \cdot C \cdot s^2 + G_N \cdot L \cdot s + 1 = 0$ roots are

$\text{polyroots}\left(\begin{pmatrix}1 & G_N \cdot L & L \cdot C\end{pmatrix}^T\right) = \begin{pmatrix}-5 \times 10^3 \\ -5 \times 10^3\end{pmatrix}$ critically damped $\alpha := 5000$

$i_L(t) = K_1 \cdot \exp(-\alpha \cdot t) + K_2 \cdot t \cdot \exp(-\alpha \cdot t) + i_N$ $\quad i_L(0) = 0$ implies $K_1 + i_N = 0$ $\quad \frac{d}{dt} i_L(0) = 0$ implies

$-\alpha \cdot K_1 + K_2 = 0$ which yields $K_1 = -5 \cdot 10^{-3}$ $\quad K_2 = -25$

$i_L(t) = -5 \cdot 10^{-3} \cdot \exp(-5000 \cdot t) - 25 \cdot t \cdot \exp(-5000 \cdot t) + 5 \cdot 10^{-3}$

evaluating $v_C(t) = L \cdot \frac{d}{dt} i_L(t) = 0.8 \cdot \frac{d}{dt}\left(-5 \cdot 10^{-3} \cdot \exp(-5000 \cdot t) - 25 \cdot t \cdot \exp(-5000 \cdot t) + 5 \cdot 10^{-3}\right)$ yields

$v_C(t) = 100000. \cdot t \cdot \exp(-5000. \cdot t)$ V

7-41, 8-11 Parallel RLC ckt with: $R_1 := 4000 \quad R_2 := 4000 \quad L := 1.25 \quad C := 50 \cdot 10^{-9} \quad V_A := 20$

for t < 0 sw open $v_C(0) = 0 \quad i_L(0) = 0$ See Figure in problem 7-40, 8-10 above

for t > 0 sw closed $v_C(\infty) = 0 \quad i_L(\infty) = \frac{V_A}{R_1} = 5 \cdot 10^{-3}$ and $i_N := 5 \cdot 10^{-3} \quad G_N := \left(R_1^{-1} + R_2^{-1}\right)$

$$L \cdot C \cdot \frac{d^2}{dt^2} i_L(t) + G_N \cdot L \cdot \frac{d}{dt} i_L(t) + i_L(t) = i_N$$

Char. eq. $L \cdot C \cdot s^2 + G_N \cdot L \cdot s + 1 = 0$ roots are

$\text{polyroots}\left(\begin{pmatrix}1 & G_N \cdot L & L \cdot C\end{pmatrix}^T\right) = \begin{pmatrix}-8 \times 10^3 \\ -2 \times 10^3\end{pmatrix}$ overdamped $\alpha_1 := 2000 \quad \alpha_2 := 8000$

$i_L(t) = K_1 \cdot \exp(-\alpha_1 \cdot t) + K_2 \cdot \exp(-\alpha_2 \cdot t) + i_N$ $\quad i_L(0) = 0$ implies $K_1 + K_2 + i_N = 0$ $\quad \frac{d}{dt} i_L(0) = 0$

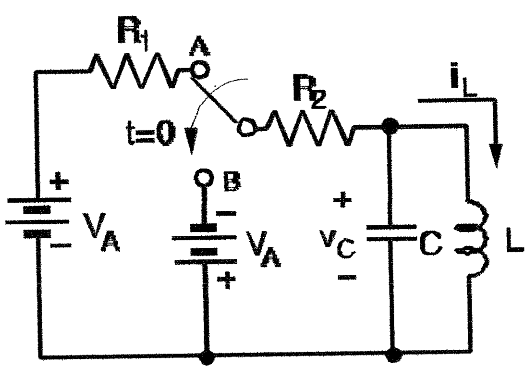
implies $-\alpha_1 \cdot K_1 - \alpha_2 \cdot K_2 = 0$ which yields $K_1 = -6.667 \cdot 10^{-3}$ $\quad K_2 = 1.667 \cdot 10^{-3}$

$i_L(t) = -6.667 \cdot 10^{-3} \cdot \exp(-2000 \cdot t) + \left(1.667 \cdot 10^{-3} \cdot \exp(-8000 \cdot t) + 5 \cdot 10^{-3}\right)$

evaluating $v_C(t) = L \cdot \frac{d}{dt} i_L(t) = 1.25 \cdot \frac{d}{dt}\left[-6.667 \cdot 10^{-3} \cdot \exp(-2000 \cdot t) + \left(1.667 \cdot 10^{-3} \cdot \exp(-8000 \cdot t) + 5 \cdot 10^{-3}\right)\right]$

yields $v_C(t) = 16.67 \cdot \exp(-2000. \cdot t) - 16.67 \cdot \exp(-8000. \cdot t)$ V

7-42, 8-12 Parallel RLC ckt with: $R_1 := 1000 \quad R_2 := 4000 \quad L := 0.625 \quad C := 6.25 \cdot 10^{-9} \quad V_A := 15$



(a) for t < 0 sw Pos A: $v_C(0) = 0$

$i_L(0) = \frac{V_A}{R_1 + R_2} = 3 \cdot 10^{-3}$ for t > 0 Pos. B:

$v_C(\infty) = 0 \quad i_L(\infty) = \frac{V_A}{R_2} = -3.75 \cdot 10^{-3}$ and

$i_N := -3.75 \cdot 10^{-3} \quad G_N := (R_2)^{-1}$

(b) $L \cdot C \cdot \frac{d^2}{dt^2} i_L(t) + G_N \cdot L \cdot \frac{d}{dt} i_L(t) + i_L(t) = i_N$

Char. eq. $L \cdot C \cdot s^2 + G_N \cdot L \cdot s + 1 = 0$ roots are

$\text{polyroots}\left(\left(1 \;\; G_N \cdot L \;\; L \cdot C\right)^T\right) = \begin{pmatrix} -3.2 \times 10^4 \\ -8 \times 10^3 \end{pmatrix}$ **overdamped** $\alpha_1 := 8000 \quad \alpha_2 := 32000$

(c) $i_L(t) = K_1 \cdot \exp(-\alpha_1 \cdot t) + K_2 \cdot \exp(-\alpha_2 \cdot t) + i_N$

$i_L(0) = 3 \cdot 10^{-3}$ **implies** $K_1 + K_2 + i_N = 3 \cdot 10^{-3}$ **or** $K_1 + K_2 = 6.75 \cdot 10^{-3}$ $\frac{d}{dt} i_L(0) = 0$ **implies**

$-\alpha_1 \cdot K_1 - \alpha_2 \cdot K_2 = 0$ **which yields** $K_1 := \dfrac{-6.75 \cdot 10^{-3} \cdot \alpha_2}{\alpha_1 - \alpha_2}$ $K_2 := \dfrac{6.75 \cdot 10^{-3} \cdot \alpha_1}{\alpha_1 - \alpha_2}$ $K_1 = 9 \times 10^{-3}$

$K_2 = -2.25 \times 10^{-3}$ $\quad i_L(t) := 9 \cdot 10^{-3} \cdot \exp((-8000 \cdot t)) - 2.25 \cdot 10^{-3} \cdot \exp((-32000 \cdot t)) - 3.75 \cdot 10^{-3}$

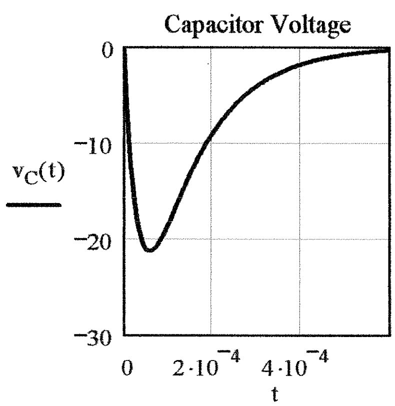
<-------------------------natural-------------------------> <--forced-->

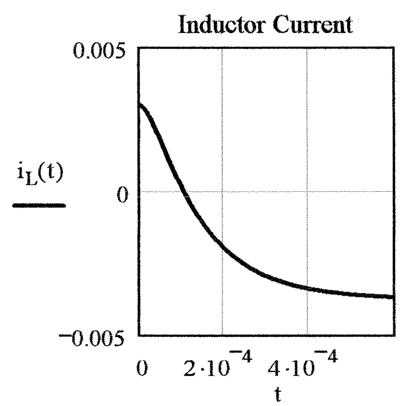
evaluating

$v_C(t) = L \cdot \frac{d}{dt} i_L(t) = 0.625 \cdot \frac{d}{dt}\left[9 \cdot 10^{-3} \cdot \exp((-8000 \cdot t)) - 2.25 \cdot 10^{-3} \cdot \exp((-32000 \cdot t)) - 3.75 \cdot 10^{-3}\right]$ **yields**

$v_C(t) := -45. \cdot \exp(-8000. \cdot t) + 45. \cdot \exp(-32000. \cdot t) \quad t := 0, 0.1 \cdot \alpha_2^{-1} .. 5 \cdot \alpha_1^{-1}$

<------------------natural------------------>





Circuit is overdamped

7-43, 8-13 **Parallel RLC ckt with:** $R_1 := 10 \quad R_2 := 40 \quad L := 1 \quad C := 100 \cdot 10^{-6} \quad V_A := 12$

See Figure 7-42, 8-12 above

(a) for t < 0 sw Pos B: $v_C(0) = 0$

$i_L(0) = \dfrac{V_A}{R_2} = -300 \cdot 10^{-3}$ **for t > 0 Pos. A:**

$v_C(\infty) = 0 \quad i_L(\infty) = \dfrac{V_A}{R_1 + R_2} = 240 \cdot 10^{-3}$ **and**

$i_N := 240 \cdot 10^{-3} \quad G_N := (R_1 + R_2)^{-1}$

(b) $L \cdot C \cdot \frac{d^2}{dt^2} i_L(t) + G_N \cdot L \cdot \frac{d}{dt} i_L(t) + i_L(t) = i_N$

Char. eq. $L \cdot C \cdot s^2 + G_N \cdot L \cdot s + 1 = 0$ **roots are**

$\text{polyroots}\left(\left(1 \quad G_N \cdot L \quad L \cdot C\right)^T\right) = \begin{pmatrix} -100 \\ -100 \end{pmatrix}$ **critically damped** $\alpha := 100$

(c) $i_L(t) = K_1 \cdot \exp(-\alpha \cdot t) + K_2 \cdot t \cdot \exp(-\alpha \cdot t) + i_N$

$i_L(0) = -300 \cdot 10^{-3}$ **implies** $K_1 + i_N = -300 \cdot 10^{-3}$ $\quad K_1 := -300 \cdot 10^{-3} - i_N \quad K_1 = -0.54$

$\frac{d}{dt} i_L(0) = 0$ **implies** $-\alpha \cdot K_1 + K_2 = 0 \quad K_2 := \alpha \cdot K_1 \quad K_2 = -54$

$i_L(t) := -0.54 \cdot \exp(-100 \cdot t) - 54 \cdot t \cdot \exp(-100 \cdot t) + 240 \cdot 10^{-3}$

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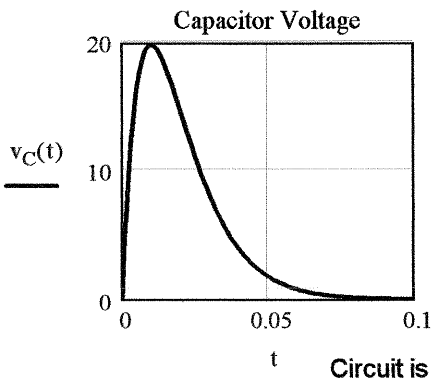
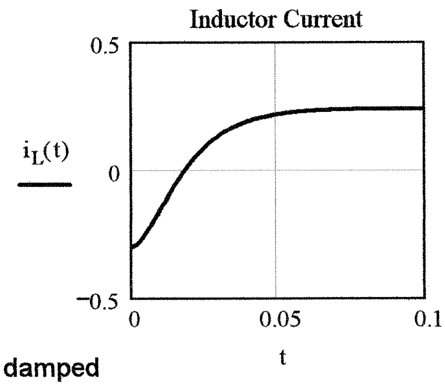
evaluating $v_C(t) = L \cdot \frac{d}{dt} i_L(t) = 1 \cdot \frac{d}{dt}\left(-0.54 \cdot \exp(-100 \cdot t) - 54 \cdot t \cdot \exp(-100 \cdot t) + 240 \cdot 10^{-3}\right)$

yields $v_C(t) := 5400. \cdot t \cdot \exp(-100. \cdot t)$ V $\quad t := 0, 0.1 \cdot \alpha^{-1} .. 10 \cdot \alpha^{-1}$

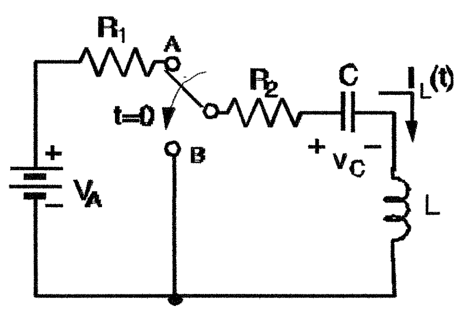
<-----------natural--------->





Circuit is critically damped

7-44, 8-14 Series RLC ckt with: $R_1 := 500 \quad R_2 := 500 \quad L := 0.25 \quad C := 3.2 \cdot 10^{-6} \quad V_A := 5$



for t < 0 sw Pos A: $v_C(0) = V_A \quad i_L(0) = 0$ **for t > 0 Pos. B:** $v_C(\infty) = 0 \quad i_L(\infty) = 0$

and $v_T := 0 \quad R_T := R_2$

(b) $L \cdot C \cdot \frac{d^2}{dt^2} v_C(t) + R_T \cdot C \cdot \frac{d}{dt} v_C(t) + v_C(t) = 0$

Char. eq. $L \cdot C \cdot s^2 + R_T \cdot C \cdot s + 1 = 0$ **roots are**

$\text{polyroots}\left(\left(1 \quad R_T \cdot C \quad L \cdot C\right)^T\right) = \begin{pmatrix} -1 \times 10^3 - 500i \\ -1 \times 10^3 + 500i \end{pmatrix}$ **underdamped** $\alpha := 1000 \quad \beta := 500$

(c) $v_C(t) = \exp(-\alpha \cdot t) \cdot (K_1 \cdot \cos(\beta \cdot t) + K_2 \cdot \sin(\beta \cdot t))$ $\quad v_C(0) = V_A$ **implies** $K_1 := V_A \quad K_1 = 5$

$\frac{d}{dt} v_C(0) = 0$ **implies** $-\alpha \cdot K_1 + \beta \cdot K_2 = 0 \quad K_2 := \frac{\alpha}{\beta} \cdot K_1 \quad K_2 = 10$

$v_C(t) := \exp(-1000 \cdot t) \cdot (5 \cdot \cos(500 \cdot t) + 10 \cdot \sin(500 \cdot t))$

evaluating $i_L(t) = C \cdot \frac{d}{dt} v_C(t) = 3.2 \cdot 10^{-6} \cdot \frac{d}{dt}[\exp(-1000 \cdot t) \cdot (5 \cdot \cos(500 \cdot t) + 10 \cdot \sin(500 \cdot t))]$

yields $i_L(t) := -\left(4 \cdot 10^{-2}\right) \cdot \exp(-1000. \cdot t) \cdot \sin(500. \cdot t)$ A

7-45, 8-15 Series RLC ckt with: $R_1 := 500 \quad R_2 := 500 \quad L := 0.25 \quad C := 10^{-6} \quad V_A := 5$

See Figure 7-44, 8-14 above

(a) for t < 0 sw Pos B: $v_C(0) = 0 \quad i_L(0) = 0$ for t > 0 Pos. A: $v_C(\infty) = V_A \quad i_L(\infty) = 0$ and $v_T := V_A \quad R_T := R_1 + R_2$

(b) $L \cdot C \cdot \frac{d^2}{dt^2} v_C(t) + R_T \cdot C \cdot \frac{d}{dt} v_C(t) + v_C(t) = v_T$

Char. eq. $L \cdot C \cdot s^2 + R_T \cdot C \cdot s + 1 = 0$ roots are

$$\text{polyroots}\left[\left(1 \quad R_T \cdot C \quad L \cdot C\right)^T\right] = \begin{pmatrix} -2 \times 10^3 \\ -2 \times 10^3 \end{pmatrix}$$ Critically damped $\alpha := 2000$

(c) $v_C(t) = K_1 \cdot \exp(-\alpha \cdot t) + K_2 \cdot t \cdot \exp(-\alpha \cdot t) + v_T \quad v_C(0) = 0$ implies $K_1 := -v_T \quad K_1 = -5$

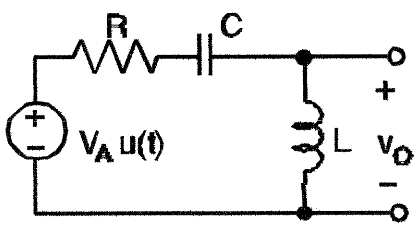
$\frac{d}{dt} v_C(0) = 0$ implies $-\alpha \cdot K_1 + K_2 = 0 \quad K_2 := \alpha \cdot K_1 \quad K_2 = -1 \times 10^4$

$v_C(t) := -5 \cdot \exp(-2000 \cdot t) - 10^4 \cdot t \cdot \exp(-2000 \cdot t) + 5$ V

evaluating $i_L(t) = C \cdot \frac{d}{dt} v_C(t) = \left(10^{-6}\right) \cdot \frac{d}{dt}\left(-5 \cdot \exp(-2000 \cdot t) - 10^4 \cdot t \cdot \exp(-2000 \cdot t) + 5\right)$

yields $i_L(t) := 20 \cdot t \cdot \exp(-2000 \cdot t)$ A

7-46, 8-16 Series RLC ckt with: $R := 3000 \quad L := 2.5 \quad C := 2 \cdot 10^{-6} \quad V_A := 100$



$v_C(0) = 0 \quad i_L(0) = 0 \quad R_T := R$

$L \cdot C \cdot \frac{d^2}{dt^2} v_C(t) + R_T \cdot C \cdot \frac{d}{dt} v_C(t) + v_C(t) = 100$

Char. eq. $L \cdot C \cdot s^2 + R_T \cdot C \cdot s + 1 = 0$ roots are

$$\text{polyroots}\left[\left(1 \quad R_T \cdot C \quad L \cdot C\right)^T\right] = \begin{pmatrix} -1 \times 10^3 \\ -200 \end{pmatrix}$$ overdamped $\alpha_1 := 200 \quad \alpha_2 := 1000$

$v_C(t) = K_1 \cdot \exp(-\alpha_1 \cdot t) + K_2 \cdot \exp(-\alpha_2 \cdot t) + 100 \quad v_C[(0) = 0$ implies $K_1 + K_2 + 100 = 0$

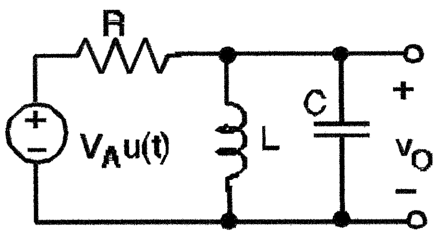
$\frac{d}{dt} v_C(0) = 0$ implies $-\alpha_1 \cdot K_1 - \alpha_2 \cdot K_2 = 0 \quad K_1 := \frac{-100 \cdot \alpha_2}{\alpha_2 - \alpha_1} \quad K_1 = -125 \quad K_2 := \frac{100 \cdot \alpha_1}{\alpha_2 - \alpha_1} \quad K_2 = 25$

$v_C(t) = -125 \cdot \exp(-200 \cdot t) + 25 \cdot \exp(-1000 \cdot t) + 100 \quad v_O(t) = L \cdot \frac{d}{dt} i_L(t) = L \cdot \frac{d}{dt}\left(C \cdot \frac{d}{dt} v_C(t)\right)$

$v_O(t) = L \cdot C \cdot \left(\frac{d^2}{dt^2} v_C(t)\right) = 5 \cdot 10^{-6} \cdot \frac{d^2}{dt^2}(-125 \cdot \exp(-200 \cdot t) + 25 \cdot \exp(-1000 \cdot t) + 100)$

$v_O(t) = -25 \cdot \exp(-200. \cdot t) + 125 \cdot \exp(-1000. \cdot t)$ V

7-47, 8-17 Parallel RLC ckt with: $R := 4000 \quad L := 0.16 \quad C := 25 \cdot 10^{-9} \quad V_A := 100$



$v_C(0) = 0 \quad i_L(0) = 0 \quad i_N = 0.025 \quad G_N := R^{-1}$

$$L \cdot C \cdot \frac{d^2}{dt^2} i_L(t) + G_N \cdot L \cdot \frac{d}{dt} i_L(t) + i_L(t) = 0.025$$

Char. eq. $L \cdot C \cdot s^2 + G_N \cdot L \cdot s + 1 = 0$ roots are

$$\text{polyroots}\left(\left(1 \quad G_N \cdot L \quad L \cdot C\right)^T\right) = \begin{pmatrix} -5 \times 10^3 - 1.5i \times 10^4 \\ -5 \times 10^3 + 1.5i \times 10^4 \end{pmatrix}$$ underdamped $\alpha := 5000 \quad \beta := 15000$

$i_L(t) = \exp(-\alpha \cdot t) \cdot \left(K_1 \cdot \cos(\beta \cdot t) + K_2 \cdot \sin(\beta \cdot t)\right) + 0.025 \quad i_L[(0) = 0$ implies $K_1 + 0.025 = 0$

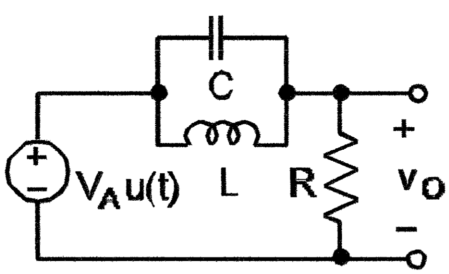
$\frac{d}{dt} i_L(0) = 0$ implies $-\alpha \cdot K_1 + \beta \cdot K_2 = 0 \quad K_1 := -0.025 \quad K_2 := -8.333 \cdot 10^{-3}$

$$i_L(t) := \exp(-5000 \cdot t) \cdot \left(-0.025 \cdot \cos(15000 \cdot t) - 8.333 \cdot 10^{-3} \cdot \sin(15000 \cdot t)\right) + 0.025$$

$$v_O(t) = L \cdot \frac{d}{dt} i_L(t) = 0.16 \cdot \frac{d}{dt}\left[\exp(-5000 \cdot t) \cdot \left(-0.025 \cdot \cos(15000 \cdot t) - 8.333 \cdot 10^{-3} \cdot \sin(15000 \cdot t)\right)\right]$$

$$v_O(t) = 66.67 \cdot \exp(-5000. \cdot t) \cdot \sin(15000. \cdot t) \text{ V}$$

7-48, 8-18 Parallel RLC ckt with: $R := 4000 \quad L := 0.4 \quad C := 12.5 \cdot 10^{-9} \quad V_A := 100$



$v_C(0) = 0 \quad i_L(0) = 0 \quad i_N = 0.025 \quad G_N := R^{-1}$

$$L \cdot C \cdot \frac{d^2}{dt^2} i_L(t) + G_N \cdot L \cdot \frac{d}{dt} i_L(t) + i_L(t) = 0.025$$

Char. eq. $L \cdot C \cdot s^2 + G_N \cdot L \cdot s + 1 = 0$ roots are

$$\text{polyroots}\left(\left(1 \quad G_N \cdot L \quad L \cdot C\right)^T\right) = \begin{pmatrix} -1 \times 10^4 - i \times 10^4 \\ -1 \times 10^4 + i \times 10^4 \end{pmatrix}$$ underdamped $\alpha := 10000 \quad \beta := 10000$

$i_L(t) = \exp(-\alpha \cdot t) \cdot \left(K_1 \cdot \cos(\beta \cdot t) + K_2 \cdot \sin(\beta \cdot t)\right) + 0.025 \quad i_L[(0) = 0$ implies $K_1 + 0.025 = 0$

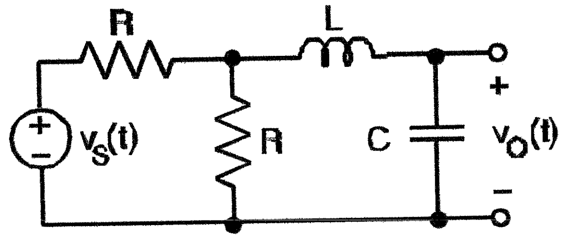
$\frac{d}{dt} i_L(0) = 0$ implies $-\alpha \cdot K_1 + \beta \cdot K_2 = 0 \quad K_1 := -0.025 \quad K_2 := -0.025$

$$i_L(t) := \exp(-10000 \cdot t) \cdot (-0.025 \cdot \cos(10000 \cdot t) - 0.025 \cdot \sin(10000 \cdot t)) + 0.025$$

$$v_O(t) = V_A - L \cdot \frac{d}{dt} i_L(t) = 100 - 0.4 \cdot \frac{d}{dt}\left[\exp\left(-10^4 \cdot t\right) \cdot \left(-0.025 \cdot \cos\left(10^4 \cdot t\right) - 0.025 \cdot \sin\left(10^4 \cdot t\right)\right) + 0.025\right]$$

$$v_O(t) := 100 - 200 \cdot \exp(-10000 \cdot t) \cdot \sin(10000 \cdot t) \text{ V}$$

7-49, 8-19 Series RLC circuit with $v_O = v_C$ $R_T = \frac{R}{2}$ and $v_T = \frac{v_S}{2}$ in general the ckt diff. equation is



$$L \cdot C \cdot \frac{d^2}{dt^2} v_C(t) + R_T \cdot C \cdot \frac{d}{dt} v_C(t) + v_C(t) = v_T$$

which becomes

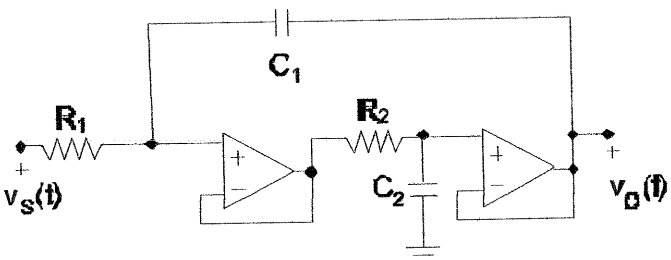
$$L \cdot C \cdot \frac{d^2}{dt^2} v_O(t) + \frac{R \cdot C}{2} \cdot \frac{d}{dt} v_O(t) + v_O(t) = \frac{v_S}{2}$$

the standard form is

$$\frac{1}{\omega_0^{\;2}} \cdot \frac{d^t}{dt^t}\left(y(t) + \frac{2\zeta}{\omega_0} \cdot \frac{d}{dt} y(t) + y(t) = x(t) \right)$$

hence $\omega_0 = \frac{1}{\sqrt{L \cdot C}}$ $\zeta = \frac{R}{4} \cdot \sqrt{\frac{C}{L}}$

7-50, 8-20



Applying KCL
Node 1:

$$C_1 \cdot \frac{d}{dt}(v_1 - v_O) = \frac{v_S - v_1}{R_1}$$

Node 2:

$$C_2 \cdot \left(\frac{d}{dt} v_O \right) = \frac{v_1 - v_O}{R_2}$$

solving Node 2 for v_1 $v_1 = R_2 \cdot C_2 \cdot \frac{d}{dt} v_O + v_O$

Substituting into the Node 1 equation yields

$$C_1 \cdot \frac{d}{dt}\left(R_2 \cdot C_2 \cdot \frac{d}{dt} v_O + v_O - v_O \right) = \frac{v_S - \left(R_2 \cdot C_2 \cdot \frac{d}{dt} v_O + v_O \right)}{R_1}$$

which reduces to

$$R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot \frac{d^2}{dt^2}\left(v_O + R_2 \cdot C_2 \cdot \frac{d}{dt} v_O + v_O = v_S \right)$$

$$\omega_0 = \frac{1}{\sqrt{R_1 \cdot R_2 \cdot C_1 \cdot C_2}} \qquad 2 \cdot \zeta = \sqrt{\frac{R_2 \cdot C_2}{R_1 \cdot C_1}}$$

7-51, 8-21 Series RLC circuit: $v_C(t) := 10 - 50 \cdot e^{-4000 \cdot t} + 40 \cdot e^{-5000 \cdot t}$ $C := 10^{-6}$

the natural frequencies are s=-4000 and s=-5000 so the characteristic eq is

$$(s+4000)(s+5000) = s^2 + 9000 \cdot s + 20 \cdot 10^6 = s^2 + \frac{R_T}{L} \cdot s + \frac{1}{L \cdot C}$$

(a) $L := \frac{1}{20 \cdot 10^6 \cdot C}$ $L = 0.05$ $R_T := 9000 \cdot L$ $R_T = 450$ $v_T = v_C(\infty) = 10$

(b) $i_L(t) = C \cdot \frac{d}{dt} v_C = 10^{-6} \cdot \frac{d}{dt}\left(10 - 50 \cdot e^{-4000 \cdot t} + 40 \cdot e^{-5000 \cdot t} \right) = \frac{1}{5} \cdot \exp(-4000 \cdot t) - \frac{1}{5} \cdot \exp(-5000) A$

7-52, 8-22 Series RLC ckt: $i_L(t) := 5\cdot 10^{-3}\cdot[\exp(-2000\cdot t)\cdot(\sin(1000\cdot t) - \cos(1000\cdot t))]$

the natural frequencies are s=-2000-j1000 and s=-2000+j1000 so the characteristic eq is

$$(s+2000+j1000)(s+2000-j1000) = s^2 + 4000\cdot s + 5\cdot 10^6 = s^2 + \frac{R}{L}\cdot s + \frac{1}{L\cdot C}$$

(a) $L := 0.1 \qquad C := \frac{1}{5\cdot 10^6\cdot L} \qquad C = 2\times 10^{-6} \quad R := 4000\cdot L \quad R = 400$

(b) With zero input $\quad v_C = -v_L - v_R \qquad v_L = L\cdot\frac{d}{dt}i_L \qquad v_R = R\cdot i_L$

$$v_L(t) = 0.1\cdot\left[\frac{d}{dt}5\cdot 10^{-3}\cdot(\exp(-2000\cdot t)\cdot\sin(1000\cdot t) - \exp(-2000\cdot t)\cdot\cos(1000\cdot t))\right]$$

$$v_L(t) = -0.5\cdot\exp(-2000\cdot t)\cdot\sin(1000\cdot t) + 1.5\cdot\exp(-2000\cdot t)\cdot\cos(1000\cdot t\ V$$

$$v_R(t) = 2\cdot\exp(-2000\cdot t)\cdot\sin(1000\cdot t) - 2\cdot\exp(-2000\cdot t)\cdot\cos(1000\cdot t\ V$$

$$v_C(t) = -1.5\cdot\exp(-2000\cdot t)\cdot\sin(1000\cdot t) + 0.5\cdot(2\cdot\exp(-2000\cdot t)\cdot\cos(1000\cdot t)\ V$$

7-53, 8-23 Parallel RLC circuit:

$v_C(t) := e^{-100\cdot t}\cdot(5\cdot\cos(500\cdot t) + 25\cdot\sin(500\cdot t)) \qquad i_L(t) := 0.02 - 0.025\cdot e^{-100\cdot t}\cdot\cos(500\cdot t)$

the natural frequencies are s=-100+j500 and s=-100-j500 so the characteristic eq is

$$(s+100-j500)(s+100+j500) = s^2 + 200\cdot s + 26\cdot 10^4 = s^2 + \frac{1}{R\cdot C}\cdot s + \frac{1}{L\cdot C}$$

$v_L = L\cdot\frac{d}{dt}i_L = v_C$ hence $\quad L = \frac{v_C}{\frac{d}{dt}i_L} = \frac{e^{-100\cdot t}\cdot(5\cdot\cos(500\cdot t) + 25\cdot\sin(500\cdot t))}{e^{-100\cdot t}\cdot(2.5\cdot\cos(500\cdot t) + 12.5\cdot\sin(500\cdot t))} = 2$

$L := 2\ H \quad C := \frac{1}{26\cdot 10^4\cdot L} \quad C = 1.923\times 10^{-6}\ F \quad R := \frac{1}{200\cdot C} \quad R = 2.6\times 10^3\ \Omega$

7-54, 8-24 An RLC circuit:

$v_C(t) := 2\cdot e^{-2000\cdot t}\cdot\cos(1000\cdot t) - 4\cdot e^{-2000\cdot t}\cdot\sin(1000\cdot t) \quad i_L(t) := e^{-2000\cdot t}\cdot(-0.08\cdot\cos(1000\cdot t) + 0.06\cdot\sin(1000\cdot t))$

(a) In a parallel circuit $L = (v_C(t))\cdot\left(\frac{d}{dt}i_L(t)\right)^{-1}$ In the present case

$$\frac{2\cdot e^{-2000\cdot t}\cdot\cos(1000\cdot t) - 4\cdot e^{-2000\cdot t}\cdot\sin(1000\cdot t)}{\frac{d}{dt}e^{-2000\cdot t}\cdot(-0.08\cdot\cos(1000\cdot t) + 0.06\cdot\sin(1000\cdot t))} = \frac{1}{10}\cdot\frac{(\cos(1000.\cdot t) - 2.\cdot\sin(1000.\cdot t))}{(11.\cdot\cos(1000.\cdot t) - 2.\cdot\sin(1000.\cdot t))}$$

The ratio is not constant, hence this is <u>not a parallel circuit</u>

In a series circuit $C = i_L(t)\cdot\left(\frac{d}{dt}v_C(t)\right)^{-1}$ In the present case

$$C = \frac{\left[e^{-2000\cdot t}\cdot(-0.08\cdot\cos(1000\cdot t) + 0.06\cdot\sin(1000\cdot t))\right]}{\frac{d}{dt}\left(2\cdot e^{-2000\cdot t}\cdot\cos(1000\cdot t) - 4\cdot e^{-2000\cdot t}\cdot\sin(1000\cdot t)\right)} = 10^{-5}$$

<u>Is a series circuit with</u> $\quad C := 10^{-5}\ F$

(b) the characteristic equation is $(s+2000)^2 + 1000^2 = s^2 + 4000\cdot s + 5000000 = s^2 + \frac{R}{L}\cdot s + \frac{1}{L\cdot C}$

$C := 10^{-5}$ $\quad L := \frac{1}{C\cdot 5000000}$ $\quad L = 0.02$ H $\quad R := 4000\cdot L$ $\quad R = 80$ Ω

7-55, 8-25 Parallel RLC circuit with $i_L(t) := 0.01\cdot e^{-10\cdot t}\cdot \cos(20\cdot t)$ $\qquad v_C(0) = -10$

$$v_C(t) = L\cdot\frac{d}{dt}i_L(t) = L\cdot\frac{d}{dt}0.01\cdot e^{-10\cdot t}\cdot\cos(20\cdot t) = -0.1\cdot L\cdot e^{-10\cdot t}\cdot[\cos(20\cdot t) + 2\cdot[\sin\cdot(20\cdot t)]]$$

$v_C(0) = -0.1\cdot L = -10$ $\quad L := 100\,H$ $\quad v_C(t) = 10\cdot\left[e^{-10\cdot t}\cdot[\cos(20\cdot t) + 2\cdot\sin\cdot(20\cdot t)]\right]$ V

7-56, 8-26 In a series RLC circuit $L := 400\cdot 10^{-3}$ $\quad v_L(t) := K_1\cdot e^{-5000\cdot t} + K_2\cdot e^{-10000\cdot t}$

the characteristic equation is $(s+5000)\cdot(s+10000) = s^2 + 15000\cdot s + 5\cdot 10^7 = s^2 + \frac{R}{L}\cdot s + \frac{1}{L\cdot C} = 0$

$R := 15000\cdot L$ $\quad R = 6\times 10^3$ Ω $\quad C := \frac{1}{5\cdot 10^7\cdot L}$ $\quad C = 5\times 10^{-8}$ F

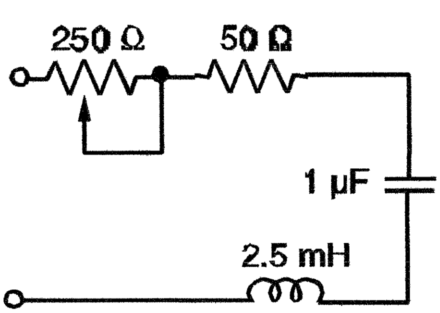
7-57, 8-27 In a series RLC circuit $\zeta = \frac{R}{2}\cdot\sqrt{\frac{C}{L}} = 0.5$ $\quad \omega_0 = \frac{1}{\sqrt{L\cdot C}} = 4\cdot 10^6$

Let $C := 10^{-9}$ F $\quad L := \frac{1}{C\cdot\left(4\cdot 10^6\right)^2}$ $\quad L = 6.25\times 10^{-5}$ H $\quad R := \sqrt{\frac{L}{C}}$ $\quad R = 250$ Ω

7-58, 8-28 for a series RLC: $s^2 + \frac{R}{L}\cdot s + \frac{1}{L\cdot C} = (s+200)\cdot(s+800) = s^2 + 1000\cdot s + 16\cdot 10^4$ hence the design constraints are $\frac{R}{L} = 1000$ and $L\cdot C = \frac{10^{-4}}{16}$. Let R = 500 ohms, then L = 0.5 H, and C =12.5 μF

7-59, 8-29 for a parallel RLC: $s^2 + \frac{1}{RC}\cdot s + \frac{1}{L\cdot C} = (s+500)\cdot(s+500) = s^2 + 1000\cdot s + 25\cdot 10^4$ hence the design constraints are $\frac{1}{RC} = 1000$ and $L\cdot C = \frac{10^{-4}}{25}$. Let C = 10 μF, then L = 0.4 H, and R =100 Ω.

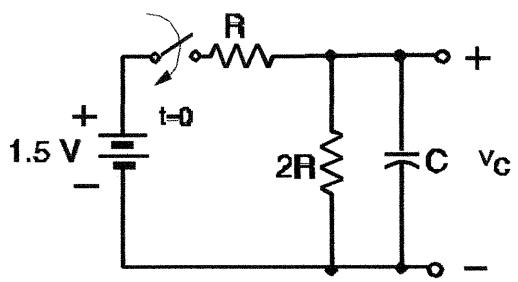
7-60, 8-30 In a series RLC circuit $\zeta = \frac{R}{2}\cdot\sqrt{\frac{C}{L}}$ $\quad C := 10^{-6}$ $\quad L := 2.5\cdot 10^{-3}$ $\quad R_{max} := 300$ $\quad R_{min} := 50$



$\zeta_{min} := \frac{R_{min}}{2}\cdot\sqrt{\frac{C}{L}}$ $\qquad \zeta_{max} := \frac{R_{max}}{2}\cdot\sqrt{\frac{C}{L}}$

$\zeta_{min} = 0.5$ $\qquad \zeta_{max} = 3$

7-61 For the RC circuit below the step response is $v_C(t) = V_T \cdot \left(1 - \exp\left(-\frac{t}{R_T \cdot C}\right)\right)$ where



$$V_T = \frac{2 \cdot R}{2 \cdot R + R} \cdot 1.5 = 1.0 \quad R_T = \frac{2 \cdot R \cdot R}{2 \cdot R + R} = \frac{2 \cdot R}{3}$$

To meet the design requirement the time at which the reponse reaches one-half of its final value (T_{half}) must be in the range

$0.001 < T_{half} < 0.002$ s. For a first order step response $T_{half} = T_C x \ln(2)$, hence

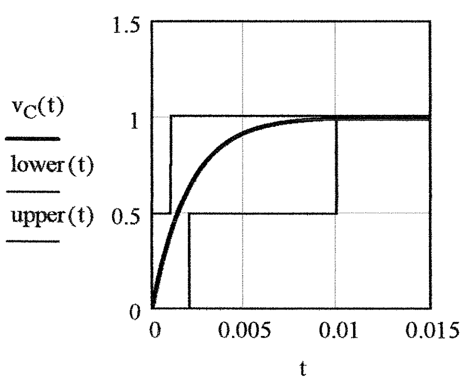
$$1.443 \cdot 10^{-3} = \frac{0.001}{\ln(2)} < T_C < \frac{0.002}{\ln(2)} = 2.885 \cdot 10^{-3} \text{ s}$$

Let $T_C := 0.002$ and $R := 10^4$ then $R_T := 2 \cdot \frac{R}{3}$ and $C := \frac{T_C}{R_T}$ $\quad C = 3 \times 10^{-7}$

$$v_C(t) := \left(1 - \exp\left(-\frac{t}{T_C}\right)\right)$$

$t := 0, 10^{-5} .. 0.015$

$$\text{upper}(t) := \begin{vmatrix} 0.5 \text{ if } 0 \le t < 0.001 \\ 1.01 \text{ if } 0.001 \le t \end{vmatrix} \quad \text{ower}(t) := \begin{vmatrix} 0 \text{ if } 0 \le t < 0.002 \\ 0.5 \text{ if } 0.002 \le t < 0.010 \\ 0.99 \text{ if } 0.01 \le t \end{vmatrix}$$



Testing design

$v_C(0.001) = 0.3935 < 0.5$

$v_C(0.002) = 0.6321 > 0.5$

$v_C(0.01) = 0.9933 > 0.99$

Circuit meets design requirements

7-62, 8-31 for a parallel RLC the characteristic equation is

$$s^2 + \frac{1}{RC} \cdot s + \frac{1}{L \cdot C} = \left(s + 2 \cdot 10^5\right)^2 + \left(10^7\right)^2 = \left(s^2 + 4 \cdot 10^5 \cdot s + 1.0004 \cdot 10^{14}\right)$$

hence the design constraints are

$\frac{1}{RC} = 4 \cdot 10^5$ and $L \cdot C = 10^{-14}$.

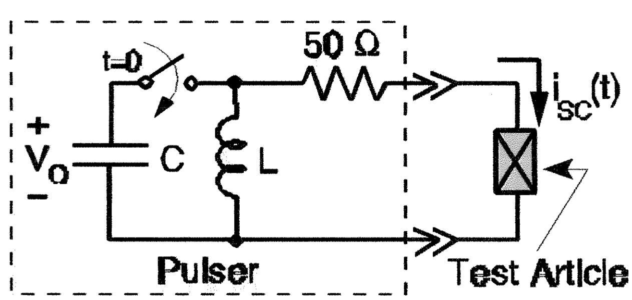
Since R = 50, then C = 50 nF

and L =0.2 μH. At t = 0 the

current through the resistor is

$\frac{V_0}{R} = 2000$ A, hence

$V_0 = 2000 \cdot 50 = 100$ kV.



7-63, 8-32 (a) Waveform is of the form $v(t) := V_A \cdot e^{-\alpha \cdot t} \cdot \sin(\beta \cdot t)$ with t=0 at the left edge of the screen.

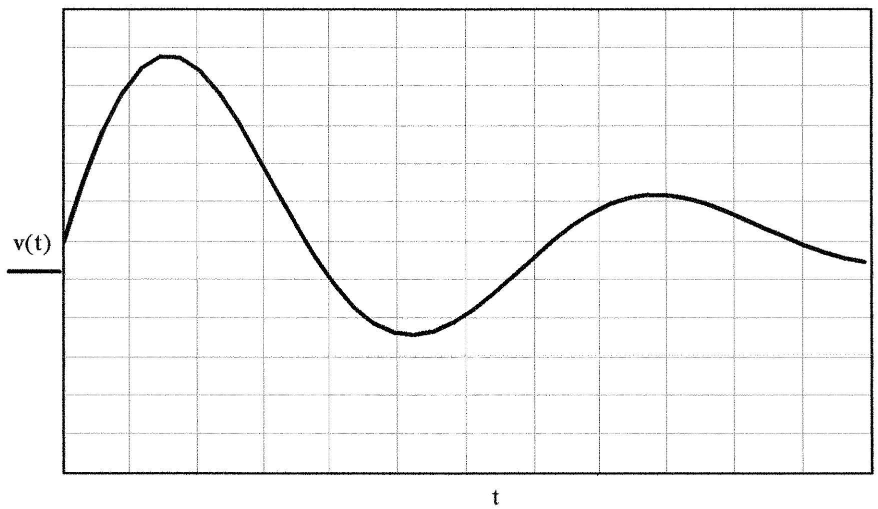
The period is about 7.25 division. $T_0 := 7.25 \cdot 10^{-6}$ $\quad \beta := 2 \cdot \frac{\pi}{T_0}$ $\quad \beta = 8.666 \times 10^5$

at $t := 1.6 \cdot 10^{-6}$ $\quad v(t) = 4.8 \cdot 0.5 = 2.4$ $\quad$ at $t := 5.3 \cdot 10^{-6}$ $\quad v(t) = -2.4 \cdot 0.5 = -1.2$

Initial guesses for solve block $V_A := 2.4$ $\quad \alpha := 10^5$ $\quad$ Given

$$V_A \cdot e^{-\alpha \cdot 1.6 \cdot 10^{-6}} \cdot \sin\left(8.666 \cdot 10^5 \cdot 1.6 \cdot 10^{-6}\right) = 2.4 \qquad V_A \cdot e^{-\alpha \cdot 5.3 \cdot 10^{-6}} \cdot \sin\left(8.666 \cdot 10^5 \cdot 5.3 \cdot 10^{-6}\right) = -1.2$$

$$\text{Find}(V_A, \alpha) = \begin{pmatrix} 3.309 \\ 1.9 \times 10^5 \end{pmatrix} \quad \text{hence} \quad v(t) := 3.309 \cdot e^{-1.9 \cdot 10^5 \cdot t} \cdot \sin\left(8.666 \cdot 10^5 \cdot t\right) \quad t := 0, 0.04 \cdot T_0 .. 12 \cdot 10^{-6}$$



(b) Characteristic Equation $(s + \alpha + j \cdot \beta) \cdot (s + \alpha - j \cdot \beta) = s^2 + 2 \cdot \alpha \cdot s + \alpha^2 + \beta^2$

$\beta = 8.666 \times 10^5$ $\quad \alpha := 1.9 \cdot 10^5$ $\quad s^2 + 2 \cdot \alpha \cdot s + \alpha^2 + \beta^2 = s^2 + 3.8 \cdot 10^5 \cdot s + 7.872 \cdot 10^{11}$

(c) for a series RLC ckt $s^2 + \frac{R}{L} \cdot s + \frac{1}{LC} = s^2 + 3.8 \cdot 10^5 \cdot s + 7.872 \cdot 10^{11}$ hence for $R := 2200$

$L := R \cdot \left(3.8 \cdot 10^5\right)^{-1}$ $\quad C := \left(L \cdot 7.872 \cdot 10^{11}\right)^{-1}$ $\quad L = 5.789 \times 10^{-3}$ $\quad C = 2.194 \times 10^{-10}$

(d) $i_L(t) = i_R(t) = \frac{v(t)}{2200} = \left(1.504 \cdot 10^{-3}\right) \cdot \left(\exp\left(-1.9 \cdot 10^5\right) \cdot \sin\left(8.666 \cdot 10^5 \cdot t\right)\right) i_L(0) = 0$

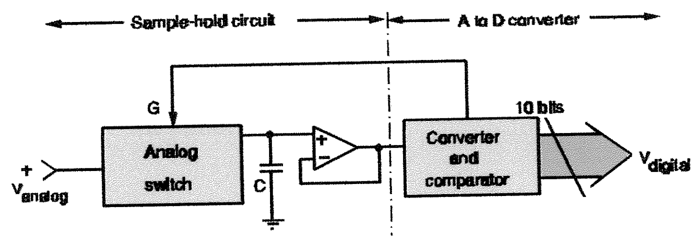
$v_L(t) = L \cdot \frac{d}{dt} i_L(t) = \left(5.789 \cdot 10^{-3}\right) \cdot \frac{d}{dt} 1.504 \cdot 10^{-3} \cdot \exp\left(-1.9 \cdot 10^5 \cdot t\right) \cdot \sin\left(8.666 \cdot 10^5 \cdot t\right)$

$v_L(t) = -1.654 \cdot \exp\left(-1.9 \cdot 10^5 \cdot t\right) \cdot \sin\left(8.666 \cdot 10^5 \cdot t\right) + 7.545 \cdot \exp\left(-1.9 \cdot 10^4 \cdot t\right) \cdot \cos\left(8.666 \cdot 10^5 \cdot t\right)$

$v_C(t) = -\left(v_R + v_L\right) = -1.655 \cdot \exp\left(-1.9 \cdot 10^5 \cdot t\right) \cdot \sin\left(8.666 \cdot 10^5 \cdot t\right) - 7.545 \cdot \exp\left(-1.9 \cdot 10^5 \cdot t\right) \cdot \cos\left(8.666 \cdot 10^5 \cdot t\right)$

$v_C(0) = -7.545\,\text{V}$

7-64



Switch $R_{on} = 50$, $R_{off} = 10^8$ and the capacitor $C = 20 \cdot 10^{-12}$

(a) sample mode: $T_C = 50 \cdot 20 \cdot 10^{-12} = 10^{-9}$; hold mode: $T_C = 10^8 \cdot 20 \cdot 10^{-12} = 2 \cdot 10^{-3}$.

(b) Let N=number of sample-hold cycles/second, for f = 1000 Hertz,

N>2f = 2000 sample-hold cycles/sec.

(c) Let N = $10N_{min}$=20,000; T_{sm}=sample mode time, T_{hm}=Hold mode time, T_s=cycle time;

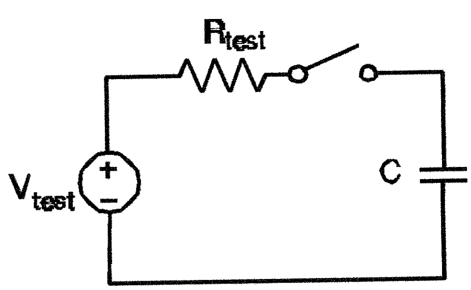
T_s=1/N=1/20000, $T_s=T_{sm}+T_{hm}=T_{sm}+9T_{sm}=10T_{sm}$, Tsm=$T_s$/10=1/200000=5 μs.

(d) T_{sm}=5 μs, in the sample mode $5T_c$=5 ns, i.e. $T_{sm} >> T_c$, hence capacitor will be charged.

(e) $T_{hm}=9T_{sm}$=45 us, in the hold mode T_c=2 ms, exp(-45 μs/2 ms)=0.9778, or about

(1 - 0.9778)x100 = 2.225% will be lost during the hold mode.

7-65 $R_{test} := 200$ $V_{test} := 10$ $V_{rated} := 11$ $i_{30} < 1.5 \cdot 10^{-3}$ $C_{nom} := 1.4$ **+80% -20%**



After the switch closes the capacitor voltage is

$$v_C(t) = V_{test} \cdot \left(1 - \exp\left(\frac{-t}{R_{test} \cdot C}\right)\right)$$

The test voltage is consistent since

$$v_C(t) \le V_{test} < V_{rated}$$

The current is

$$i(t) = C \cdot \frac{d}{dt} v_C(t) = \frac{V_{test}}{R_{test}} \cdot \exp\left(\frac{-t}{R_{test} \cdot C}\right)$$

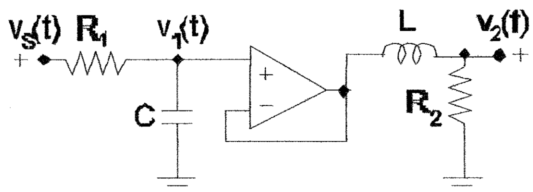
the 30 minute current depends of the value of C

$$i_{30}(C) := \frac{V_{test}}{R_{test}} \cdot \exp\left(\frac{-30 \cdot 60}{R_{test} \cdot C}\right)$$

$C_{max} := 1.8 \cdot C_{nom}$ $i_{30}(C_{max}) = 1.406 \times 10^{-3}$ $C_{min} := 0.8 \cdot C_{nom}$ $i_{30}(C_{min}) = 1.618 \times 10^{-5}$

The 30 minute current is less than 1.5 mA for C within the specified range.
The specification is internally consistent.

7-66, 8-33 The output of the RC circuit is governed by the differential equation



$$R_1 \cdot C_1 \cdot \frac{d}{dt} v_1 + v_1 = v_S \qquad (1)$$

v_1 is the input to the RL circuit which is governed by the diff. Eq

$$L \cdot \frac{d}{dt} i_L + R_2 \cdot i_L = v_1$$

but by Ohm's law $v_2 = R_2 \cdot i_L$ hence this equation can be written as

$$\frac{L}{R_2} \cdot \frac{d}{dt} v_2 + v_2 = v_1 \qquad (2)$$

differentiating eq. (2) yields $\frac{L}{R_2} \cdot \frac{d^2}{dt^2}\left(v_2 + \frac{d}{dt} v_2 = \frac{d}{dt} v_1\right)$

using eq. (1) to eliminate $\frac{d}{dt} v_1$ from this equation yields $\frac{L}{R_2} \cdot \frac{d^2}{dt^2}\left(v_2 + \frac{d}{dt} v_2 = \frac{v_S - v_1}{R_1 \cdot C_1}\right)$

using eq. (2) to eliminate v_1 from this equation yields

$$\frac{L}{R_2} \cdot \frac{d^2}{dt^2}\left[v_2 + \frac{d}{dt} v_2 = \frac{v_S - \left(\frac{L}{R_2} \cdot \frac{d}{dt} v_2 + v_2\right)}{R_1 \cdot C_1}\right]$$

which can be rearranged as

$$R, 1 \cdot C_1 \cdot \frac{L}{R_2} \cdot \frac{d^2}{dt^2}\left[v_2 + \left(R_1 \cdot C_1 + \frac{L}{R_2}\right) \cdot \frac{d}{dt} v_2 + v_2 = v_S\right]$$

This is a linear 2nd order differential equation relating the input vs to the output v_2.
the characteristic equation is

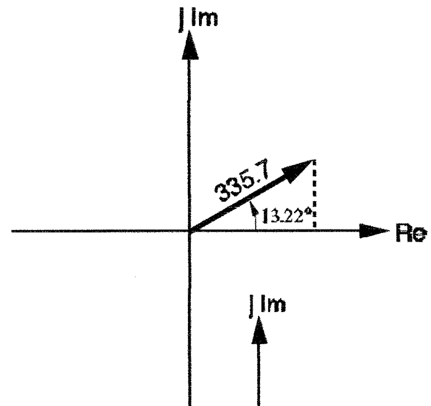
$$\left(R, 1 \cdot C_1 \cdot \frac{L}{R_2}\right) \cdot s^2 + \left(R_1 \cdot C_1 + \frac{L}{R_2}\right) \cdot s + 1 = (R_1 \cdot C_1 \cdot s + 1) \cdot \left(\frac{L}{R_2} \cdot s + 1\right) = 0$$

whose roots are $s = \frac{-1}{R_1 \cdot C_1}$ and $s = \frac{-R_2}{L}$ QED

CHAPTER 8 Standard Version (use 8-xx) and CHAPTER 15 Laplace-Early Version (use 15-xx)

8-1, 15-1 $V_1 := 250\cdot\exp\left(j\cdot\frac{\pi}{4}\right)$

$V_2 := 150\cdot\exp(j\cdot 0) + 100\cdot\exp\left[j\cdot\left(\frac{-\pi}{2}\right)\right]$



$V_1 = 176.777 + 176.777j \quad V_2 = 150 - 100j$

$V_1 + V_2 = 326.777 + 76.777j$

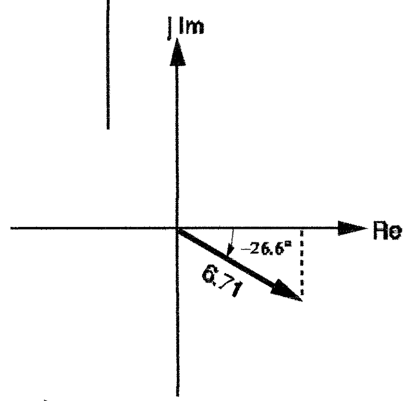
$\left|V_1 + V_2\right| = 335.675$

$\frac{180}{\pi}\cdot\arg(V_1 + V_2) = 13.222$

$v_1(t) + v_2(t) = 335.675\cdot\cos(\omega\cdot t + 13.222)$

8-2, 15-2 $I_1 := 6\cdot\exp(j\cdot 0) \quad I_1 = 6$

$I_2 := 3\cdot\exp\left(j\cdot\frac{-\pi}{2}\right) \quad I_2 = -3j$



$\left|I_1 + I_2\right| = 6.708 \quad \frac{180}{\pi}\cdot\arg(I_1 + I_2) = -26.565$

$i_1(t) + i_2(t) = 6.708\cdot\cos(\omega\cdot t - 26.565)$

8-3, 15-3 (a) $v_1(t) = \text{Re}\left[10\cdot\exp\left[j\cdot\left(10^4\cdot t - \frac{\pi}{6}\right)\right]\right] = 10\cos\left(10^4\cdot t - \frac{\pi}{6}\right)$

(b) $v_2(t) = \text{Re}\left[60\cdot\exp\left[j\cdot\left(10^4\cdot t - 220\cdot\frac{\pi}{180}\right)\right]\right] = 60\cdot\cos\left(10^4\cdot t - 220\cdot\frac{\pi}{180}\right)$

(c) $i_1(t) := \text{Re}\left[5\cdot\exp\left[j\cdot\left(10^4\cdot t + \frac{\pi}{2}\right)\right]\right] = 5\cdot\cos\left(10^4\cdot t + \frac{\pi}{2}\right)$

(d) $i_2(t) = \text{Re}\left[2\cdot\exp\left[j\cdot\left(10^4\cdot t + 3\cdot\frac{\pi}{2}\right)\right]\right] = 2\cdot\cos\left(10^4\cdot t + 3\cdot\frac{\pi}{2}\right)$

8-4, 15-4 $V_1 := 10\cdot\exp\left(-j\cdot\frac{\pi}{6}\right) \quad V_1 = 8.66 - 5j$

$V_2 := 60\cdot\exp\left(\frac{-j\cdot 11\cdot\pi}{9}\right) \quad V_2 = -45.963 + 38.567j$

$2\cdot V_1 + V_2 = -28.6 + 28.6j \quad \left|2\cdot V_1 + V_2\right| = 40.5$

$\frac{180}{\pi}\cdot\arg(2\cdot V_1 + V_2) = 135.075$

$2\cdot v_1(t) + v_2 = 40.5\cdot\cos\left(10^4\cdot t + 135.075^\circ\right)$

$I_1 := 5\cdot\exp\left(j\cdot\frac{\pi}{2}\right) \quad I_1 = 5j$

$I_2 := 2\cdot\exp\left(j\cdot 3\cdot\frac{\pi}{2}\right) \quad I_2 = -2j$

$I_1 + 3\cdot I_2 = -j \quad \left|I_1 + 3\cdot I_2\right| = 1$

$\frac{180}{\pi}\cdot\arg(I_1 + 3\cdot I_2) = -90$

$i_1(t) + 3\cdot i_2 = \cos\left(200\cdot t - 90^\circ\right)$

8-5, 15-5 $V := 20 + j\cdot 5 \quad \omega := 20 \quad j\cdot\omega\cdot V = -100 + 400j$

$|j\cdot\omega\cdot V| = 412 \quad \frac{180}{\pi}\cdot\arg(j\cdot\omega\cdot V) = 104 \quad \frac{d}{dt}v(t) = 412\cdot\cos(\omega\cdot t + 104^o)$

8-6, 15-5 (a) $V_1 := (10 + j\cdot 40) \quad |V_1| = 41.2 \quad \frac{180}{\pi}\cdot\arg(V_1) = 75.96 \quad v_1(t) = 41.2\cdot\cos(10\cdot t + 75.96^o)$ V

(b) $V_2 := (8 - j\cdot 3)\cdot 5\cdot\exp\left(-j\cdot\frac{\pi}{3}\right) \quad |V_2| = 42.7 \quad \frac{180}{\pi}\cdot\arg(V_2) = -80.6 \quad v_2(t) = 42.7\cdot\cos(20\cdot t - 80.6^o)$ V

(c) $I_1 := 8 - j\cdot 3 + \frac{3}{j} \quad |I_1| = 10 \quad \frac{180}{\pi}\cdot\arg(I_1) = -36.87 \quad i_1(t) = 10\cdot\cos(300\cdot t - 36.87)$ A

(d) $I_2 := \frac{3 + j\cdot 1}{1 - j\cdot 3} \quad |I_2| = 1 \quad \frac{180}{\pi}\cdot\arg(I_2) = 90 \quad i_2(t) = \cos(50\cdot t + 90)$ A

8-7, 15-7 $V_1 := 10\cdot\exp\left(j\cdot\frac{\pi}{4}\right) \quad V_2 := j\cdot 500\cdot\frac{1}{200}\cdot V_1 + V_1 \quad V_2 = -10.607 + 24.749j$

$|V_2| = 26.93 \quad \frac{180}{\pi}\cdot\arg(V_2) = 113.2$

$v_2(t) = 26.93\cdot\cos(500\cdot t + 113.2^o)$ V

8-8, 15-8 $V_1 := 50\cdot\exp\left(-j\cdot\frac{\pi}{4}\right) \quad V_2 := 25\cdot\exp\left(-j\cdot\frac{\pi}{2}\right) \quad V_3 := -(V_1 + V_2) \quad |V_3| = 69.948$

$\frac{180}{\pi}\cdot\arg(V_3) = 120.361$ **so finally** $v_3(t) = 69.95\cdot\cos(\omega\cdot t + 120.4^o)$ V

8-9, 15-9

$v(t) = 12\cdot\sqrt{2}\cdot\cos\left[1000\cdot\pi\cdot\left(t - 2.5\cdot 10^{-4}\right)\right] = 12\cdot\sqrt{2}\cdot\cos\left(1000\cdot\pi\cdot t - \frac{\pi}{4}\right) \quad V := 12\cdot\sqrt{2}\cdot\exp\left(-j\cdot\frac{\pi}{4}\right) \quad V = 12 - 12j$

8-10, 15-10 $V_1 := -3 + j\cdot 4 \quad \frac{180}{\pi}\cdot\arg(V_1) + 90 = 216.87 \quad V_2 := 10\cdot\exp\left[j\cdot\left(216.87\cdot\frac{\pi}{180}\right)\right]$

$|V_2| = 10 \quad V_2 = -8 - 6j \quad v_2(t) = \mathrm{Re}(V_2\cdot xp(j\cdot\omega\cdot t)) = 10\cdot\cos(\omega\cdot t + 216.87^o)$ V

8-11, 15-11 $Z_{EQ1}(\omega) := 25 + j\cdot\omega\cdot 0.02 \quad Z_{EQ2}(\omega) := \left(25^{-1} + j\cdot\omega\cdot 20\cdot 10^{-6}\right)^{-1}$

(a) $Z_{EQ1}(1000) = 25 + 20j \quad |Z_{EQ1}(1000)| = 32.016 \quad 180\cdot\pi^{-1}\cdot\arg(Z_{EQ1}(1000)) = 38.66$

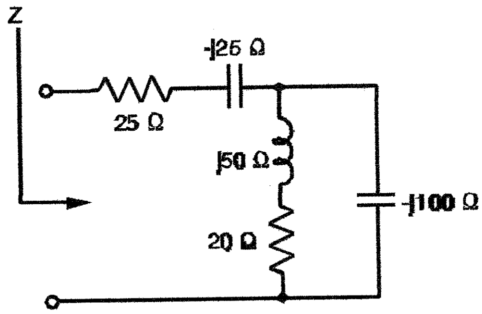
(b) $Z_{EQ2}(1000) = 20 - 10j \quad |Z_{EQ2}(1000)| = 22.361 \quad 180\cdot\pi^{-1}\cdot\arg(Z_{EQ2}(1000)) = -26.565$

$Z_{EQ3}(\omega) := \left(Z_{EQ1}(\omega)^{-1} + Z_{EQ2}(\omega)^{-1}\right)^{-1}$

(c) $Z_{EQ3}(1000) = 15.529 - 0.118j \quad |Z_{EQ3}(1000)| = 15.53 \quad 180\cdot\pi^{-1}\cdot\arg(Z_{EQ3}(1000)) = -0.434$

(d) $Z_{EQ3}(4000) = 6.595 - 10.388j \quad |Z_{EQ3}(4000)| = 12.305 \quad 180\cdot\pi^{-1}\cdot\arg(Z_{EQ3}(4000)) = -57.59$

8-12, 15-12 $Z := 25 - j\cdot 25 + \dfrac{1}{\left(\dfrac{1}{-j\cdot 100} + \dfrac{1}{20 + j\cdot 50}\right)}$



$Z = 93.966 + 47.414j$

$|Z| = 105.25 \quad \dfrac{180}{\pi}\cdot\arg(Z) = 26.775$

8-13, 15-13 $Z_{EQ1}(\omega) := j\cdot\omega\cdot 0.1 + \dfrac{1}{j\cdot\omega\cdot 10^{-5}} \quad Z_{EQ2}(\omega) := \left(\dfrac{1}{60} + \dfrac{1}{j\cdot\omega\cdot 0.03}\right)^{-1}$

(a) $Z_{EQ1}(2000) = 150j \quad |Z_{EQ1}(2000)| = 150 \quad 180\cdot\pi^{-1}\cdot\arg(Z_{EQ1}(2000)) = 90$

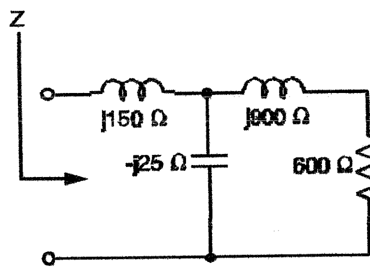
(b) $Z_{EQ2}(2000) = 30 + 30j \quad |Z_{EQ2}(2000)| = 42.426 \quad 180\cdot\pi^{-1}\cdot\arg(Z_{EQ2}(2000)) = 45$

$Z_{EQ3}(\omega) := Z_{EQ1}(\omega) + Z_{EQ2}(\omega)$

(c) $Z_{EQ3}(2000) = 30 + 180j \quad |Z_{EQ3}(2000)| = 182.483 \quad 180\cdot\pi^{-1}\cdot\arg(Z_{EQ3}(2000)) = 80.538$

(d) $Z_{EQ3}(1000) = 12 + 24j \quad |Z_{EQ3}(1000)| = 26.833 \quad 180\cdot\pi^{-1}\cdot\arg(Z_{EQ3}(1000)) = 63.435$

8-14, 15-14 $Z := j\cdot 150 + \dfrac{1}{\left(\dfrac{1}{-j\cdot 25} + \dfrac{1}{600 + j\cdot 900}\right)}$



$Z = 0.333 + 124.514j$

$|Z| = 124.515 \quad 180\cdot\pi^{-1}\cdot\arg(Z) = 89.847$

8-15, 15-15 $V := 200\cdot\exp\left(-j\cdot\dfrac{\pi}{3}\right) \quad I := 20\cdot 10^{-3} + j\cdot 0$

(a) $Z := \dfrac{V}{I} \quad Z = 5000 - 8660j \quad \Omega$

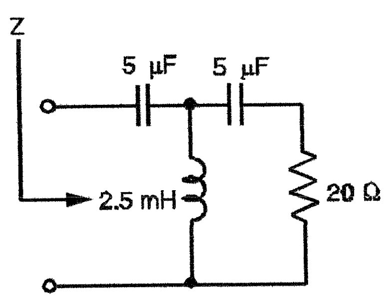
(b) $V := 150\cdot\exp\left(-j\cdot 3\cdot\dfrac{\pi}{2}\right) \quad I := \dfrac{V}{Z} \quad I = -1.299\times 10^{-2} + 7.5j\times 10^{-3}$

$|I| = 1.5\times 10^{-2} \quad \dfrac{180}{\pi}\cdot\arg(I) = 150$

$i(t) = 1.5\cdot 10^{-2}\cdot\cos\left(1000\cdot t + 150^{o}\right)$ A

8-16, 15-16

$Z := \dfrac{1}{j\cdot 20\cdot 10^{3}\cdot 5\cdot 10^{-6}} + \dfrac{1}{\left[\dfrac{1}{j\cdot\left(20\cdot 10^{3}\right)\cdot 2.5\cdot 10^{-3}} + \dfrac{1}{20 + \dfrac{1}{j\cdot\left(20\cdot 10^{3}\right)\cdot 5\cdot 10^{-6}}}\right]}$



$Z := \dfrac{1}{j\cdot 0.1} + \dfrac{1}{-j\cdot 0.02 + 0.04 + j\cdot 0.02} \quad Z = 25 - 10j \;\; \Omega$

8-17, 15-17 $R := 600 \quad L := 10^{-3} \quad C := 10^{-9} \quad Z_L(L) := j \cdot 10^5 \cdot L \quad Z_C(C) := \frac{1}{j \cdot 10^5 \cdot C}$

Given $\quad \mathrm{Re}\left[\left[R^{-1} + Z_L(L)^{(-1)}\right]^{-1} + Z_C(C)\right] = 100 \qquad \mathrm{Im}\left[\left[R^{-1} + Z_L(L)^{(-1)}\right]^{-1} + Z_C(C)\right] = 0$

$$\begin{pmatrix} L \\ C \end{pmatrix} := \mathrm{Find}(L, C) \quad \begin{pmatrix} L \\ C \end{pmatrix} = \begin{pmatrix} 2.683 \times 10^{-3} \\ 4.472 \times 10^{-8} \end{pmatrix}$$

Checking solution: $\left[R^{-1} + \left(j \cdot 10^5 \cdot L\right)^{-1}\right]^{-1} + \left(j \cdot 10^5 \cdot C\right)^{-1} = 100$ **as required**

8-18, 15-18 $i(t) = 10^{-2} \cdot \cos(1000 \cdot t) \quad v(t) = 2 \cdot \sqrt{2} \cdot \cos\left(1000 \cdot t - 45^o\right) \quad R := 200$

$I := 10^{-2} \cdot \exp(j \cdot 0) \quad V := 2 \cdot \sqrt{2} \cdot \exp\left(-j \cdot \frac{\pi}{4}\right) \quad Z_{EQ} := \frac{V}{I} \quad Z_{EQ} = 200 - 200j \quad Z_C := -j \cdot 200$

$C := \frac{1}{1000 \cdot 200} \quad C = 5 \times 10^{-6}$

8-19, 15-19 $\quad Y := \frac{1}{R} + j \cdot (\omega \cdot C) = \frac{1}{600 - j \cdot 600} = \frac{600}{2 \cdot 600^2} + j \cdot \frac{600}{2 \cdot 600^2} = \frac{1}{1200} + \frac{j}{1200}$

$R := 1200 \quad \omega := 10^6 \quad C := \frac{1}{1200 \cdot \omega} \quad C = 8.333 \times 10^{-10}$ **F**

checking $\quad \frac{1}{\left[\frac{1}{R} + j \cdot (\omega \cdot C)\right]} = 600 - 600j$

8-20, 15-20 $V := 110 \quad \omega_1 := 2 \cdot \pi \cdot 60 \quad \omega_2 := 2 \cdot \pi \cdot 400 \quad I_1 := 0.25 \quad I_2 := 0.12$

$\left(\left|\frac{V}{I_1}\right|\right)^2 = \left(\left|R + j \cdot \omega_1 \cdot L\right|\right)^2 = R^2 + \omega_1^{\,2} \cdot L^2 \qquad \left(\left|\frac{V}{I_2}\right|\right)^2 = \left(\left|R + j \cdot \omega_2 \cdot L\right|\right)^2 = R^2 + \omega_2^{\,2} \cdot L^2$

$\omega_2^{\,2} \cdot L^2 - (\omega_1)^2 \cdot L^2 = \left(\left|\frac{V}{I_2}\right|\right)^2 - \left(\left|\frac{V}{I_1}\right|\right)^2 \qquad L := \sqrt{\frac{\left(\left|\frac{V}{I_2}\right|\right)^2 - \left(\left|\frac{V}{I_1}\right|\right)^2}{\omega_2^{\,2} - \omega_1^{\,2}}} \qquad R := \sqrt{\left(\left|\frac{V}{I_2}\right|\right)^2 - \omega_2^{\,2} \cdot L^2}$

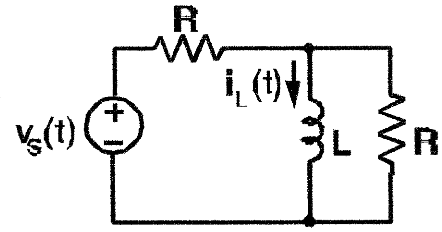
$L = 0.324 \qquad R = 422.747$

8-21, 15-21 $V := 10 + j \cdot 0 \quad R := 100 \quad Z_L := j \cdot 500 \cdot 0.04 \quad Z := R + Z_L \quad Z = 100 + 20j$

$I := \frac{V}{Z} \quad I = 9.615 \times 10^{-2} - 1.923j \times 10^{-2} \quad |I| = 9.806 \times 10^{-2} \quad 180 \cdot \pi^{-1} \cdot \arg(I) = -11.31$

$i(t) = 98.06 \cdot \cos(500 \cdot t - 11.31)$ mA

8-22, 15-22 by voltage division



$$V_O = \frac{\frac{1}{R}}{\frac{1}{R} + \frac{1}{j\cdot\omega\cdot L} + \frac{1}{R}}\cdot V_A\cdot \exp(j\cdot 0)$$

$$V_O = \frac{\omega\cdot L\cdot V_A}{2\cdot\omega\cdot L - j\cdot R}$$

$$I = \frac{V_O}{j\cdot\omega\cdot L} = \frac{V_A}{R + j\cdot 2\cdot\omega\cdot L} = \frac{V_A}{R^2 + (2\cdot\omega\cdot L)^2}\cdot(R - j\cdot 2\cdot\omega\cdot L)$$

8-23, 15-23 $I := 0.3 + j\cdot 0 \quad R := 10^4 \qquad Z_C := \frac{1}{j\cdot 2000\cdot 50\cdot 10^{-9}} \qquad Y_{EQ} := \frac{1}{R} + \frac{1}{Z_C}$

$I_R := \frac{1}{R\cdot Y_{EQ}}\cdot I \qquad I_C := \frac{1}{Z_C\cdot Y_{EQ}}\cdot I$

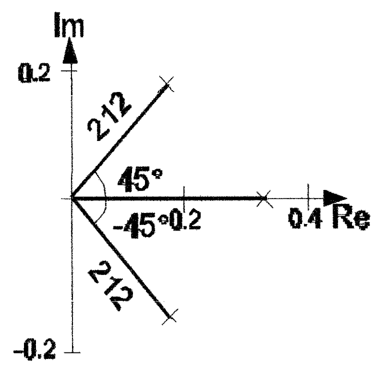
$I_R = 0.15 - 0.15j \qquad I_C = 0.15 + 0.15j$

$|I_R| = 0.212 \qquad \frac{180}{\pi}\cdot\arg(I_R) = -45$

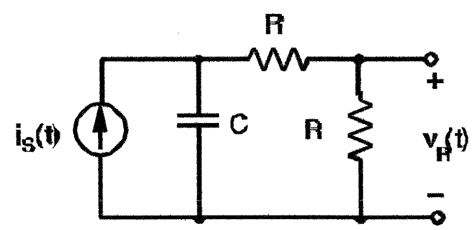
$|I_C| = 0.212 \qquad \frac{180}{\pi}\cdot\arg(I_C) = 45$

$i_R(t) = 212\cdot\cos(2000\cdot t - 45^o)$ mA

$i_C(t) = 212\cdot\cos(2000\cdot t + 45^o)$ mA



8-24, 15-24 by current division



$$I_R = \frac{\frac{1}{j\cdot\omega\cdot C}}{2\cdot R + \frac{1}{j\cdot\omega\cdot C}}\cdot I_A\cdot\exp(-j\cdot 0) = \frac{I_A}{1 + j\cdot\omega\cdot 2\cdot R\cdot C}$$

$$V_R = R\cdot I_R = \frac{R\cdot I_A}{1 + j\cdot\omega\cdot 2\cdot R\cdot C} = \frac{R\cdot I_A}{1 + (\omega\cdot 2\cdot R\cdot C)^2}\cdot(1 - j\cdot\omega\cdot 2\cdot R\cdot C)$$

8-25, 15-25 $V := 50\cdot\exp\left(j\cdot\frac{\pi}{4}\right) \quad V = 35.355 + 35.355j \quad R := 5000 \quad Z_C := \frac{1}{j\cdot 1000\cdot 200\cdot 10^{-9}} \quad Z_C = -5j\times 10^3$

$I_R := \frac{V}{R} \qquad I_C := \frac{V}{Z_C}$

$I_R = 7.071\times 10^{-3} + 7.071j\times 10^{-3}$

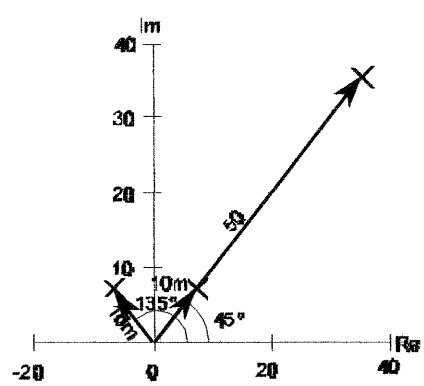
$I_C = -7.071\times 10^{-3} + 7.071i\times 10^{-3}$

$|I_R|\cdot 1000 = 10 \quad 180\cdot\pi^{-1}\cdot\arg(I_R) = 45$

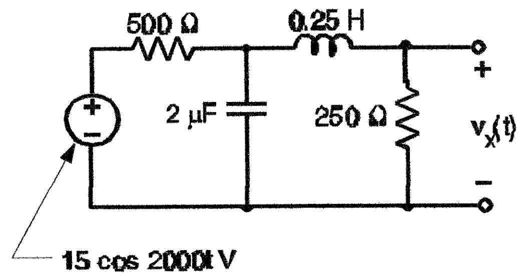
$|I_C|\cdot 1000 = 10 \quad 180\cdot\pi^{-1}\cdot\arg(I_C) = 135$

$i_R(t) = 10\cdot\cos(1000\cdot t + 45)$ mA

$i_C(t) = 10\cdot\cos(1000\cdot t + 135)$ mA



8-26, 15-26 $Z := \dfrac{1}{j\cdot 2000\cdot 2\cdot 10^{-6} + \dfrac{1}{250 + j\cdot 2000\cdot 0.25}}$



$Z_{IN} := 500 + Z \quad Z_{IN} = 625 - 375j$ <-Input Imp.

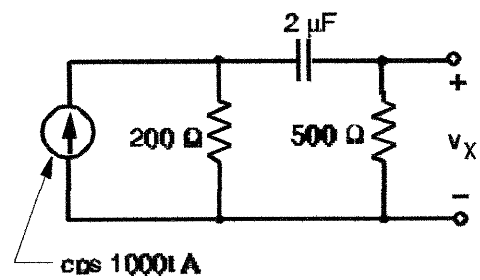
By double voltage division

$$V_x := \left(\frac{Z}{Z_{IN}}\cdot 15\right)\cdot\frac{250}{250 + j\cdot 2000\cdot 0.25}$$

$V_x = -0.882 - 3.529j \quad |V_x| = 3.638$

$\dfrac{180}{\pi}\cdot\arg(V_x) = -104 \quad v_x(t) = 3.638\cdot\cos\left(2000\cdot t - 104^{o}\right)$
V

8-27, 15-27 $I := 1 + j\cdot 0 \quad Z := \dfrac{1}{\left(\dfrac{1}{200} + \dfrac{1}{500 + \dfrac{1}{j\cdot 10^{3}\cdot 2\cdot 10^{-6}}}\right)}$



$Z = 162.162 - 27.027j$ <--- Input Impedance

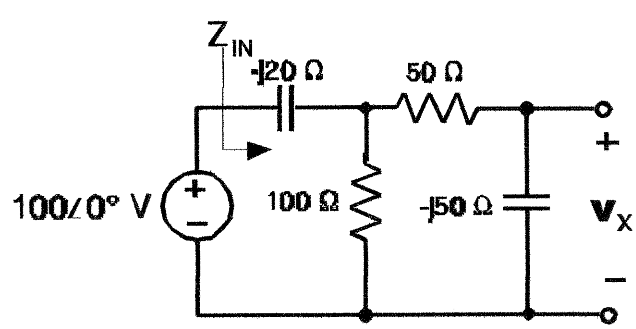
by current division $I_X := \dfrac{200}{200 + \left(500 + \dfrac{1}{j\cdot 10^{3}\cdot 2\cdot 10^{-6}}\right)}\cdot I$

$I_X = 0.189 + 0.135j \quad V_X := 500\cdot I_X$

$|V_X| = 116.248 \quad 180\cdot\pi^{-1}\cdot\arg(V_X) = 35.538$

$v_X(t) = 116.3\cdot\cos(1000\cdot t + 35.54)$ V

8-28, 15-28 $Z := \dfrac{1}{\dfrac{1}{100} + \dfrac{1}{50 - j\cdot 50}}$



$Z = 40 - 20j \quad Z_{IN} := -j\cdot 20 + Z$

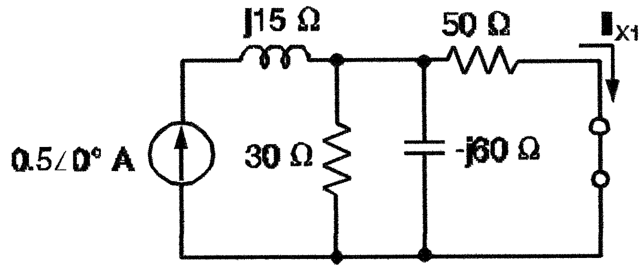
$Z_{IN} = 40 - 40j$ <---input impedance

by double voltage division

$$V_X := \left(\frac{Z}{Z_{IN}}\cdot 100\right)\cdot\left(\frac{-j\cdot 50}{50 - j\cdot 50}\right)$$

$V_X = 50 - 25j$ V

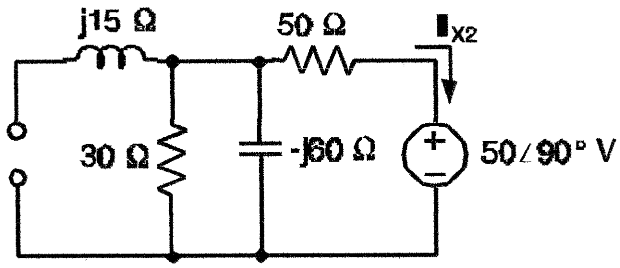
8-29, 15-29 With the right source off use current division



$$I_{x1} := \frac{\frac{1}{50}}{\frac{1}{30}+\frac{1}{-j\cdot 60}+\frac{1}{50}}\cdot 0.5$$

$$I_{x1} = 0.171 - 0.053j$$

With the left source off

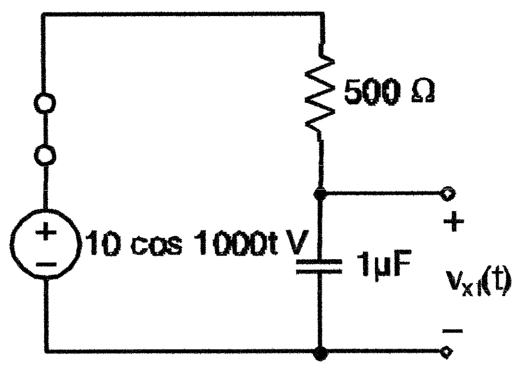


$$Z_{IN} := 50 + \frac{1}{\frac{1}{30}+\frac{1}{-j\cdot 60}} \qquad Z_{IN} = 74 - 12j$$

$$I_{x2} := \frac{-50\cdot \exp\left(j\cdot\frac{\pi}{2}\right)}{Z_{IN}} \qquad I_{x2} = 0.107 - 0.658j$$

$$I_{x1} + I_{x2} = 0.278 - 0.712j \text{ A}$$

8-30, 15-30 With the upper source off



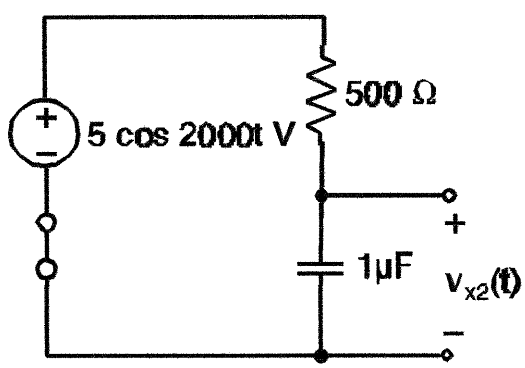
$$V_{x1} := \frac{\frac{1}{j\cdot 1000\cdot 10^{-6}}}{500+\frac{1}{j\cdot 1000\cdot 10^{-6}}}\cdot 10\cdot \exp(j\cdot 0)$$

$$V_{x1} = 8 - 4j \qquad |V_{x1}| = 8.944$$

$$\frac{180}{\pi}\cdot \arg(V_{x1}) = -26.57$$

$$v_{x1}(t) = 8.944\cdot\cos\left(1000\cdot t - 26.57^{o}\right)$$

with the lower source off



$$V_{x2} := \frac{\frac{1}{j\cdot 2000\cdot 10^{-6}}}{500+\frac{1}{j\cdot 2000\cdot 10^{-6}}}\cdot 5\cdot \exp(j\cdot 0)$$

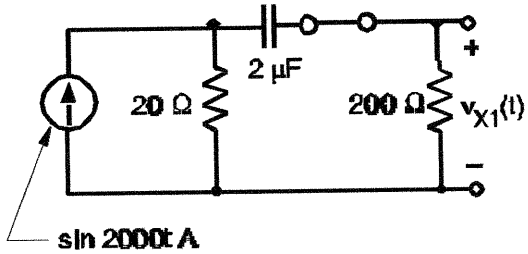
$$V_{x2} = 2.5 - 2.5j \qquad |V_{x2}| = 3.536$$

$$\frac{180}{\pi}\cdot \arg(V_{x2}) = -45$$

$$v_{x2}(t) = 3.536\cdot\cos\left(2000\cdot t - 45^{o}\right)$$

$$v_x(t) = v_{x1}(t) + v_{x2}(t) = 8.944\cdot\cos\left(1000\cdot t - 26.57^{o}\right) + 3.536\cdot\cos\left(2000\cdot t - 45^{o}\right) V$$

8-31, 15-31 With the right source off $I := 0 - j\cdot 1$



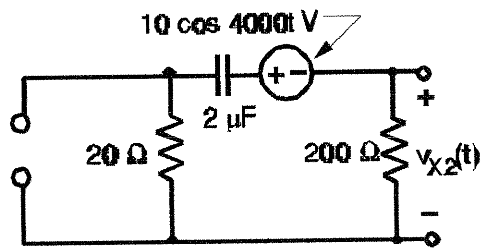
$$I_{x1} := \left(\frac{20}{20 + 200 + \dfrac{1}{j\cdot 2000\cdot 2\cdot 10^{-6}}} \right)\cdot I$$

$V_{x1} := I_{x1}\cdot 200$ ▪ $V_{x1} = 9.017 - 7.935j$

$\left|V_{x1}\right| = 12.011$ ▪ $\frac{180}{\pi}\cdot \arg\left(V_{x1}\right) = -41.348$

$v_{x1}(t) = 12.011\cdot \cos\left(2000\cdot t - 41.384^{o}\right)$

With the left source off $V := 10 + j\cdot 0$



$$V_{x2} := \left(\frac{200}{20 + \dfrac{1}{j\cdot 4000\cdot 2\cdot 10^{-6}} + 200} \right)\cdot (-V)$$

$V_{x2} = -6.872 - 3.905j$

$\left|V_{x2}\right| = 7.904$ $\quad \frac{180}{\pi}\cdot \arg\left(V_{x2}\right) = -150.396$

$v_{x2}(t) = 7.904\cdot \cos\left(2000\cdot t - 150.396^{o}\right)$

$v_x(t) = v_{x1}(t) + v_{x2}(t) = 12.011\cdot \cos\left(2000\cdot t - 41.384^{o}\right) + 7.904\cdot \cos\left(4000\cdot t - 150.396^{o}\right)$ V

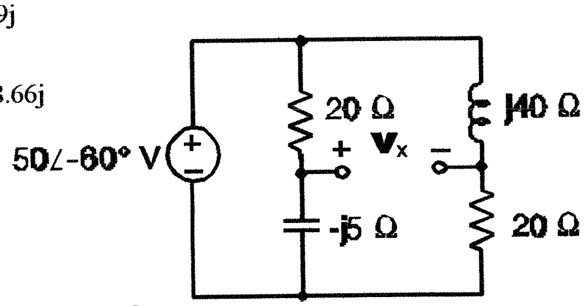
8-32, 15-32 Use voltage division on the left and right dividers.

$V_{left} := \frac{-j\cdot 5}{20 - j\cdot 5}\cdot 50\exp\left(-j\cdot \frac{\pi}{3}\right)$ $\quad V_{left} = -8.718 - 8.429j$

$V_{right} := \frac{20}{20 + j\cdot 40}\cdot 50\cdot \exp\left(-j\cdot \frac{\pi}{3}\right)$ $\quad V_{right} = -12.321 - 18.66j$

$V_x := V_{left} - V_{right}$ $\quad V_x = 3.603 + 10.231j$

$\left|V_x\right| = 10.847$ $\quad \frac{180}{\pi}\cdot \arg\left(V_x\right) = 70.601$

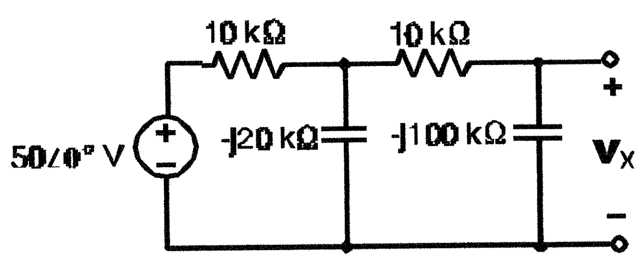


$Z_{IN} := \left[(20 - j\cdot 5)^{-1} + (20 + j\cdot 40)^{-1}\right]^{-1}$ $\quad Z_{IN} = 17.168 + 2.478j$ Ω

8-33, 15-33 $V_X := 1$ $\quad I_1 := \frac{V_X}{-j\cdot 10^5}$ $\quad V_1 := I_1\cdot 10^4$

$V_2 := 1 + V_1$ $\quad I_2 := \frac{V_2}{-j\cdot 2\cdot 10^4}$ $\quad I_3 := I_1 + I_2$

$V_3 := I_3\cdot 10^4$ $\quad V_S := V_2 + V_3$ $\quad K := \frac{1}{V_S}$

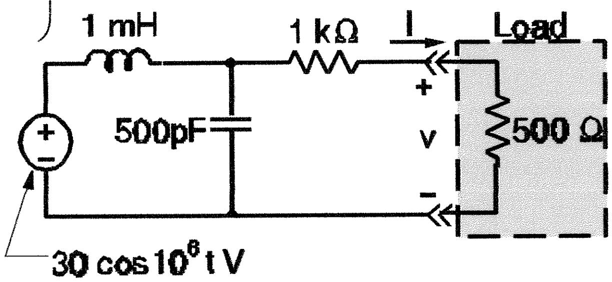


$K = 0.682 - 0.503j$ $\quad Z_{IN} := \frac{V_S}{I_3}$

$Z_{IN} = 1.028\times 10^4 - 1.669j\times 10^4$ $\quad$ For $V_S := 50 + j\cdot 0$ $\quad V_X := K\cdot V_S$

$V_X = 34.111 - 25.135j$ $\quad \left|V_X\right| = 42.371$ $\quad 180\pi^{-1}\cdot \arg\left(V_X\right) = -36.384$

8-34, 15-34 $Z_T := 1000 + \dfrac{1}{\left(\dfrac{1}{j\cdot 10^6\cdot 10^{-3}} + j\cdot 10^6\cdot 0.5\cdot 10^{-9}\right)}$



$Z_T = 1\times 10^3 + 2j\times 10^3$

$V_T := \dfrac{-j\cdot 2000}{j\cdot 1000 - j\cdot 2000}\cdot 30\cdot\exp\left(-j\cdot\dfrac{\pi}{2}\right)$ $V_T = -60j$

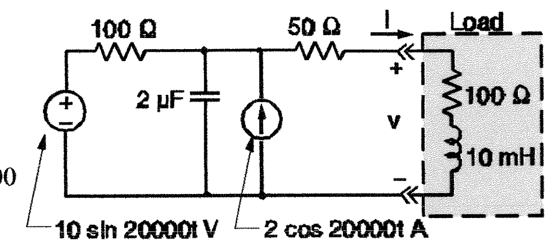
$I := \dfrac{V_T}{Z_T + 500}$ $V := 500\cdot I$ $|I| = 2.4\times 10^{-2}$

$\dfrac{180}{\pi}\cdot\arg(I) = -143.13$ $|V| = 12$ $180\cdot\pi^{-1}\cdot\arg(V) = -143.13$

$i(t) = 24\cdot\cos\left(10^6\cdot t - 143.13^o\right)$ mA $v(t) = 12\cdot\cos\left(10^6\cdot t - 143.13^o\right)$ V

8-35, 15-35 $Z_C := \dfrac{1}{j\cdot 2\cdot 10^4\cdot 2\cdot 10^{-6}}$ $Z_T := 50 + \left[(100)^{-1} + (Z_C)^{-1}\right]^{-1}$ $Z_T = 55.882 - 23.529j$

Using superposition, voltage division, & current division yields the Thevenin voltage as



$V_T := \left(\dfrac{Z_C}{100 + Z_C}\right)\cdot 10\cdot\exp\left(-j\cdot\dfrac{\pi}{2}\right) + \left[\left(\dfrac{100^{-1}}{100^{-1} + Z_C^{-1}}\right)\cdot 2\right]100$

<-----due to V source---------><-----due to I source----->

$V_T = 9.412 - 47.647j$ $Z_L := 100 + j\cdot 2\cdot 10^4\cdot 10^{-2}$ $Z_L = 100 + 200j$ $I := \dfrac{V_T}{Z_T + Z_L}$

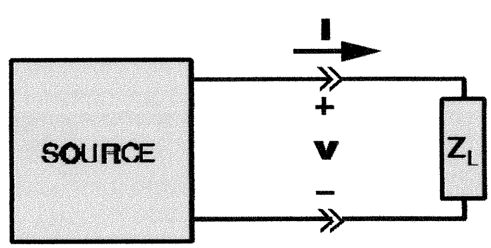
$I = -0.125 - 0.164j$ $V := I\cdot Z_L$ $V = 20.265 - 41.432j$

$|I| = 2.063\times 10^{-1}$ $180\cdot\pi^{-1}\cdot\arg(I) = -127.371$ $|V| = 46.123$ $\dfrac{180}{\pi}\cdot\arg(V) = -63.936$

$i(t) = 206\cdot\cos\left(2\cdot 10^4\cdot t - 127.4^o\right)$ mA $v(t) = 46.1\cdot\cos\left(2\cdot 10^4\cdot t - 63.9^o\right)$ V

8-36, 15-36 $I_N := 0.0048 - j\cdot 0.0036$ $Z_L := -j\cdot 2\cdot 10^4$

$\dfrac{Z_T}{Z_T + Z_L}\cdot I_N = 10^{-2} + j\cdot 0$ $Z_T := \dfrac{Z_L}{100\cdot I_N - 1}$



$Z_T = 1.8\times 10^4 + 2.6j\times 10^4$

$V_T := I_N\cdot Z_T$ $V_T = 180 + 60j$

8-37, 15-37 $I_{SC} := 0.8 - j\cdot 0.4$ $Z_L := \left(j\cdot 2\cdot 10^6\cdot 10^{-9}\right)^{-1}$ $Z_L = -500j$ $I_L := 3 + j\cdot 0$

Given $\dfrac{V_T}{Z_T} = I_{SC}$ $\dfrac{V_T}{Z_T + Z_L} = I_L$ $\begin{pmatrix} V_T \\ Z_T \end{pmatrix} := \mathrm{Find}(V_T, Z_T)$ $\begin{pmatrix} V_T \\ Z_T \end{pmatrix} = \begin{pmatrix} 360 + 480j \\ 120 + 660j \end{pmatrix}$

The capacitive reactance in Z_L cancels part of the inductive reactance in Z_T so that $|Z_T + Z_L| = 200$ which is less than $|Z_T| = 670.82$ that is, $|Z_L + Z_T| < |Z_T|$.

hence, the load current is greater than the short-circuit current.

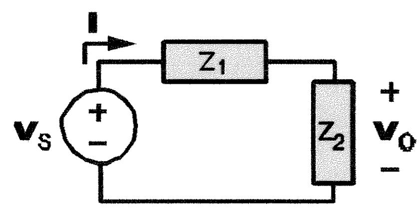
8-38, 15-38 Treating the -j100 kΩ capacitive reactance as a load

$$Z_T := 10^4 + \left[\left(10^4\right)^{-1} + \left(-j\cdot 2\cdot 10^4\right)^{-1}\right]^{-1} \quad Z_T = 1.8\times 10^4 - 4j\times 10^3 \quad V_T := \frac{-j\cdot 2\cdot 10^4}{10^4 - j\cdot 2\cdot 10^4}\cdot 50 \quad V_T = 40 - 20j$$

$$V_X := \frac{-j\cdot 10^5\cdot V_T}{Z_T - j\cdot 10^5} \qquad V_X = 34.111 - 25.135j$$

8-39, 15-39 $V_S := 100 + j\cdot 0 \qquad V_O := 50\cdot 0.01\cdot \exp\left[j\cdot\left(\frac{-35}{180}\cdot\pi\right)\right] \qquad \omega := 10^4$ Using a voltage divider

$\frac{Z_2}{Z_1 + Z_2} = \frac{V_O}{V_S}$ Let $Z_2 := 50$



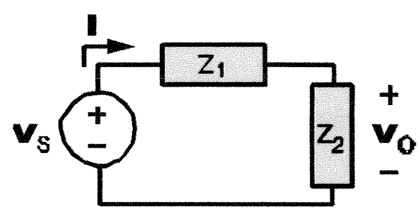
Solving for Z_1 $\quad Z_1 := \left(\frac{V_S}{V_O} - 1\right)\cdot Z_2$

$Z_1 = 8.142\times 10^3 + 5.736j\times 10^3$

$R_1 := \mathrm{Re}(Z_1) \qquad R_1 = 8.142\times 10^3\ \Omega \qquad L_1 := \frac{\mathrm{Im}(Z_1)}{\omega} \qquad L_1 = 0.574$ H

8-40, 15-40 $V_S := 100\cdot \exp\left(-j\cdot\frac{\pi}{2}\right) \qquad V_O := 50\cdot \exp\left[j\cdot\left(-\frac{\pi}{4}\right)\right] \qquad \omega := 10^3$

Using a voltage divider $\quad \frac{Z_2}{Z_1 + Z_2} = \frac{V_O}{V_S}$ Let $\quad Z_2 := 10^4$

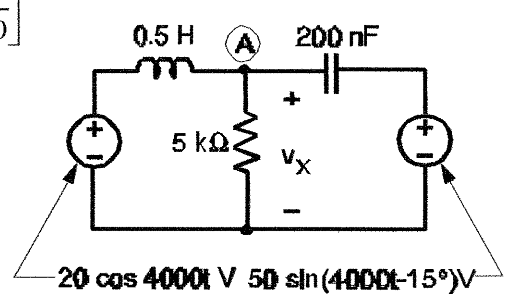


Solving for Z_1 $\quad Z_1 := \left(\frac{V_S}{V_O} - 1\right)\cdot Z_2$

$Z_1 = 4.142\times 10^3 - 1.414j\times 10^4 \quad R_1 := \mathrm{Re}(Z_1) \quad R_1 = 4.142\times 10^3$

$C_1 := \frac{1}{\omega\cdot\left(\left|\mathrm{Im}(Z_1)\right|\right)} \quad C_1 = 7.071\times 10^{-8}$ F

8-41, 15-41 $V_{S1} := 20 + j\cdot 0 \quad V_{S2} := 50\cdot \exp\left[-j\cdot(15+90)\cdot\frac{\pi}{180}\right]$



$\omega := 4000 \quad L := 0.5 \quad R := 5000 \quad C := 200\cdot 10^{-9}$

$Z_L := j\cdot\omega\cdot L \quad Z_C := \frac{1}{j\cdot\omega\cdot C}$

KCL ar Node A $\quad \frac{V_A - V_{S1}}{Z_L} + \frac{V_A - V_{S2}}{Z_C} + \frac{V_A}{R} = 0$

which yields $\quad V_A := \frac{\left(V_{S1}\cdot Z_C + V_{S2}\cdot Z_L\right)}{\left(Z_C\cdot R + Z_L\cdot R + Z_L\cdot Z_C\right)}\cdot R$

$V_X := V_A \quad V_X = 12.474 - 120.474j \quad \left|V_X\right| = 121.118$

$180\cdot\pi^{-1}\cdot \arg\left(V_X\right) = -84.089$

$v_X(t) = 121.118\cdot\cos\left(4000\cdot t - 84.089^{\circ}\right)$ V

8-42, 15-42

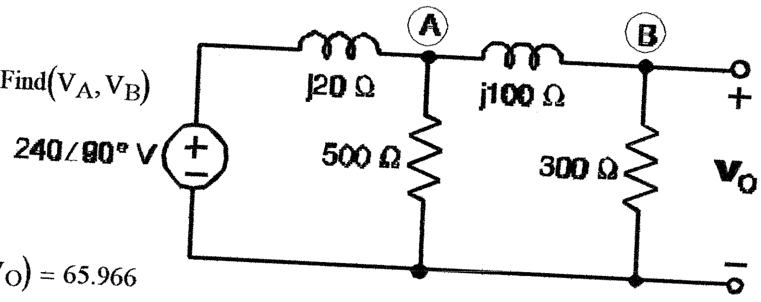
$V_S := 0 + j\cdot 240 \quad Z_{L1} := j\cdot 20 \quad Z_{L2} := j\cdot 100 \quad R_1 := 500 \quad R_2 := 300$

$V_A := 1 + j\cdot 1 \quad V_B := 1 + j\cdot 1$ Given

$$\frac{V_A - V_S}{Z_{L1}} + \frac{V_A - V_B}{Z_{L2}} + \frac{V_A}{R_1} = 0$$

$$\frac{V_B - V_A}{Z_{L2}} + \frac{V_B}{R_2} = 0 \quad \begin{pmatrix} V_A \\ V_B \end{pmatrix} := \mathrm{Find}(V_A, V_B)$$

$V_O := V_B \quad V_O = 90 + 203j$



$|V_O| = 222.155 \quad 180\cdot\pi^{-1}\cdot\arg(V_O) = 65.966$

8-43, 15-43 $R_1 := 10^4 \quad R_2 := 10^4 \quad \mu := 100$

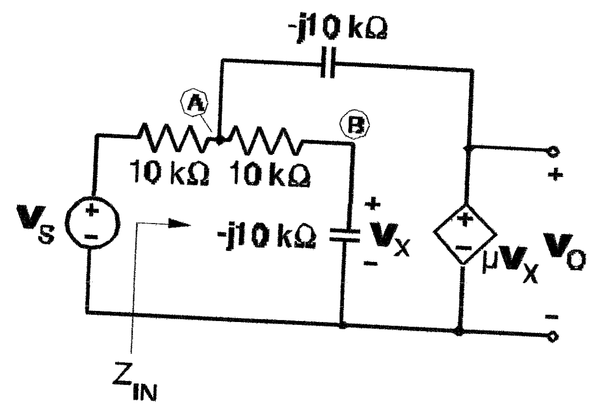
$Z_{C1} := -j\cdot 10^4 \quad Z_{C2} := -j\cdot 10^4$ Assume $V_S := 1 + j\cdot 0$

$V_A := 1 + j \quad V_B := 1 + j \quad V_O := 1 + j$

Given

$$\frac{V_A - V_S}{R_1} + \frac{V_A - V_O}{Z_{C1}} + \frac{V_A - V_B}{R_2} = 0$$

$$\frac{V_B - V_A}{R_2} + \frac{V_B}{Z_{C2}} = 0 \qquad V_O = \mu\cdot V_B$$



$$\begin{pmatrix} V_A \\ V_B \\ V_O \end{pmatrix} := \mathrm{Find}(V_A, V_B, V_O) \qquad \begin{pmatrix} V_A \\ V_B \\ V_O \end{pmatrix} = \begin{pmatrix} -1.031\times 10^{-2} + 1.031j\times 10^{-2} \\ 1.031j\times 10^{-2} \\ 1.031j \end{pmatrix}$$

$K := \dfrac{V_O}{V_S} \quad K = 1.031j \qquad I_{IN} := \dfrac{V_S - V_A}{R_1} \qquad Z_{IN} := \dfrac{V_S}{I_{IN}} \qquad Z_{IN} = 9.897\times 10^3 + 100.989j \ \Omega$

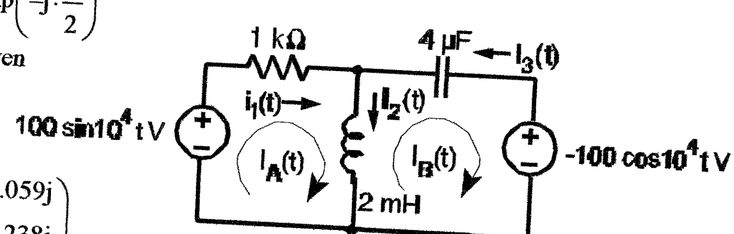
8-44, 15-44 $R := 1000 \quad C := 4\cdot 10^{-6} \quad L := 2\cdot 10^{-3} \quad \omega := 10^4$

$Z_C := \dfrac{1}{j\cdot\omega\cdot C} \quad Z_L := j\cdot\omega\cdot L \quad V_{S1} := 100\cdot\exp\left(-j\cdot\dfrac{\pi}{2}\right)$

$V_{S2} := -100 \quad I_A := 1 + j \quad I_B := 1 + j$ Given

$I_A\cdot R + (I_A - I_B)\cdot Z_L - V_{S1} = 0$

$I_B\cdot Z_C + (I_B - I_A)\cdot Z_L + V_{S2} = 0$



$$\begin{pmatrix} I_A \\ I_B \end{pmatrix} := \mathrm{Find}(I_A, I_B) \quad \begin{pmatrix} I_A \\ I_B \end{pmatrix} = \begin{pmatrix} -0.406 - 0.059j \\ 1.624 + 20.238j \end{pmatrix}$$

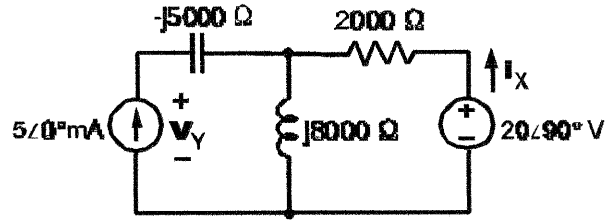
$I_1 := I_A \quad I_3 := -I_B \quad I_2 := I_1 + I_3$

$I_1 = -0.406 - 0.059j$ A $\quad I_2 = -2.03 - 20.297j$ A $\quad I_3 = -1.624 - 20.238j$ A

8-45, 15-45 $I_S := 5\cdot10^{-3} + j\cdot0 \quad V_S := 0 + j\cdot20$

$(2000 + j\cdot8000)\cdot I_A - j\cdot8000\cdot I_S = -V_S$

$$I_A := \frac{-V_S + j\cdot8000\cdot I_S}{(2000 + j\cdot8000)}$$

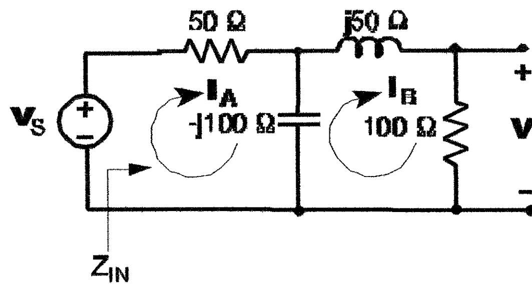


$I_A = 2.353\times10^{-3} + 5.882j\times10^{-4} \quad I_x := -I_A$

$I_x = -2.353\times10^{-3} - 5.882j\times10^{-4}$ A $\qquad V_y := -j\cdot5000\cdot I_S + j\cdot8000\cdot(I_S - I_A) \quad V_y = 4.706 - 3.824j$ V

8-46, 15-46 $R_1 := 50 \qquad R_2 := 100 \qquad Z_C := -j\cdot100 \qquad Z_L := j\cdot50$

Assume $\quad V_S := 1 + j\cdot0$



Given

$(R_1 + Z_C)\cdot I_A - Z_C\cdot I_B = V_S$

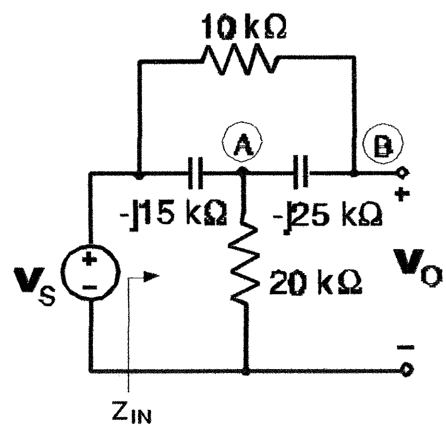
$-Z_C\cdot I_A + (Z_L + Z_C + R_2)\cdot I_B = 0$

$$\begin{pmatrix} I_A \\ I_B \end{pmatrix} := \text{Find}(I_A, I_B) \qquad \begin{pmatrix} I_A \\ I_B \end{pmatrix} = \begin{pmatrix} 6.341\times10^{-3} + 2.927j\times10^{-3} \\ 4.878\times10^{-3} - 3.902j\times10^{-3} \end{pmatrix}$$

$$K := \frac{I_B\cdot R_2}{V_S} \quad K = 0.488 - 0.39j \qquad Z_{IN} := \frac{V_S}{I_A} \qquad Z_{IN} = 130 - 60j \ \Omega$$

8-47, 15-47 $\qquad$ Assume: $V_S := 1 + j\cdot0$

$V_A := 1 + j\cdot1 \qquad V_B := 1 + j\cdot1 \qquad$ Given



$$\frac{V_A - V_S}{-j\cdot15\cdot10^3} + \frac{V_A}{20\cdot10^3} + \frac{V_A - V_B}{-j\cdot25\cdot10^3} = 0$$

$$\frac{V_B - V_S}{10\cdot10^3} + \frac{V_B - V_A}{(-j\cdot25\cdot10^3)} = 0$$

$$\begin{pmatrix} V_A \\ V_B \end{pmatrix} := \text{Find}(V_A, V_B) \qquad \begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{pmatrix} 0.777 + 0.354\cdot j \\ 0.847 - 0.028\cdot j \end{pmatrix}$$

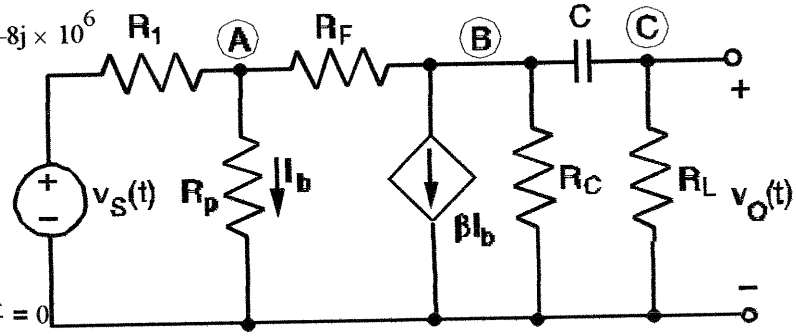
$$K := \frac{V_B}{V_S} \qquad I_S := \frac{V_S - V_A}{-j\cdot15\cdot10^3} + \frac{V_S - V_B}{10\cdot10^3} \qquad Z_{IN} := \frac{V_S}{I_S}$$

$K = 0.84729 - 0.02813j \qquad Z_{IN} = 2.132\times10^4 - 9.706j\times10^3 \ \Omega$

8-48, 15-48 $V_S := 80 \cdot 10^{-3}$ $R_1 := 10^4$ $R_P := 5 \cdot 10^3$

$R_F := 10^6$ $R_C := 10 \cdot 10^3$ $R_L := 10^5$ $C := 2.5 \cdot 10^{-10}$

$\beta := 50$ $\omega := 500$ $Z_C := \dfrac{1}{j \cdot \omega \cdot C}$ $Z_C = -8j \times 10^6$



$V_A := 1 + j$ $V_B := 1 + j$ $V_C := 1 + j$

Given

node-voltage eqs.

$$\frac{V_A - V_S}{R_1} + \frac{V_A}{R_P} + \frac{V_A - V_B}{R_F} = 0$$

$$\frac{V_B - V_A}{R_F} + \frac{V_B}{R_C} + \frac{V_B - V_C}{Z_C} + \beta \cdot \frac{V_A}{R_P} = 0$$

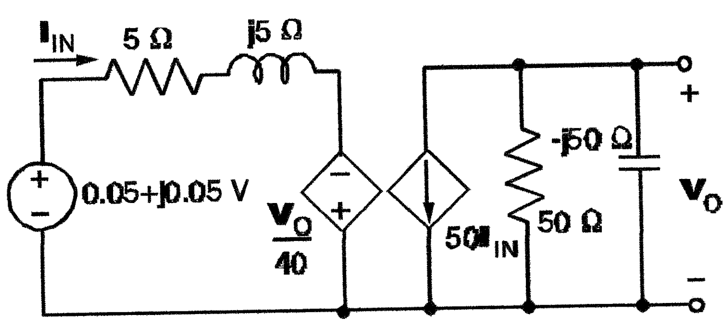
$$\frac{V_C - V_B}{Z_C} + \frac{V_C}{R_L} = 0$$

$$\begin{pmatrix} V_A \\ V_B \\ V_C \end{pmatrix} := \text{Find}(V_A, V_B, V_C) \quad \begin{pmatrix} V_A \\ V_B \\ V_C \end{pmatrix} = \begin{pmatrix} 0.02 + 6.125j \times 10^{-6} \\ -1.98 + 1.844j \times 10^{-3} \\ -3.324 \times 10^{-4} - 0.025j \end{pmatrix} \quad V_O := V_C$$

$|V_O| = 2.475 \times 10^{-2}$ V $\quad 180 \cdot \pi^{-1} \cdot \arg(V_O) = -90.77 \quad v_O(t) = 2.475 \cdot 10^{-2} \cdot \cos(500 \cdot t - 90.77^o)$

8-49, 15-49 $V_S := 0.05 + j \cdot 0.05$ $I_{IN} := \dfrac{V_S}{50 + j \cdot 25}$

$V_O := 1000 \cdot I_{IN}$ Given



Input mesh equation:

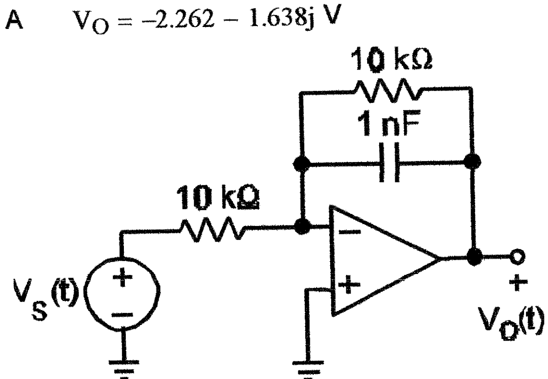
$$[(5 + j \cdot 5) \cdot I_{IN} - V_S] - \frac{V_O}{40} = 0$$

Output node equation:

$$\left(\frac{1}{50} + \frac{1}{-j \cdot 50}\right) \cdot V_O + 50 \cdot I_{IN} = 0$$

$$\begin{pmatrix} I_{IN} \\ V_O \end{pmatrix} := \text{Find}(I_{IN}, V_O) \quad I_{IN} = 2.496 \times 10^{-4} + 1.56j \times 10^{-3} \text{ A} \quad V_O = -2.262 - 1.638j \text{ V}$$

8-50, 15-50 $V_O := 10 + j \cdot 0$ $\omega := 10^5$ $Z_1 := 10^4$



$$Z_2 := \left[\left(10^4\right)^{-1} + j \cdot \omega \cdot 10^{-9}\right]^{-1}$$

$K := \dfrac{-Z_2}{Z_1}$ $V_S := \dfrac{V_O}{K}$

$V_S = -10 - 10j$ $\quad |V_S| = 14.142 \quad \dfrac{180}{\pi} \cdot \arg(V_S) = -135$

8-51, 15-51 $V_S := 35 + j\cdot 0$ $R := 50$ $Z_L := j\cdot 500\,0.1$ $I_L := V_S(Z_L + R)^{-1}$ $V_L := V_S$

$P_L := \frac{1}{2}\cdot(|I_L|)^2\cdot R$ $V_L = 35$ V $I_L = 0.35 - 0.35j$ A $P_L = 6.125$ W

8-52, 15-52 $V_S := 50 + j\cdot 0$ $Y_C := j\cdot 2500\,8\cdot 10^{-6}$ $V_L := V_S$ $I_L := V_S\left[(Y_C)^{-1} + R\right]^{-1}$

$P_L := \frac{1}{2}\cdot(|I_L|)^2\cdot R$ $V_L = 50$ V $I_L = 0.5 + 0.5j$ A $P_L = 12.5$ W

8-53, 15-53 $V_S := 15 + j\cdot 0$ $\omega := 2500$ $P_L := 0.1$ $R := \frac{1}{2}\cdot\frac{(|V_S|)^2}{P_L}$ $R = 1.125\times 10^3$

Assume $C := 10^{-9}$

Given $\left|\frac{1}{R} + j\cdot\omega\cdot C\right|\cdot|V_S| = 35\cdot 10^{-3}$ $C := \text{Find}(C)$ $C = 8.63\times 10^{-7}$

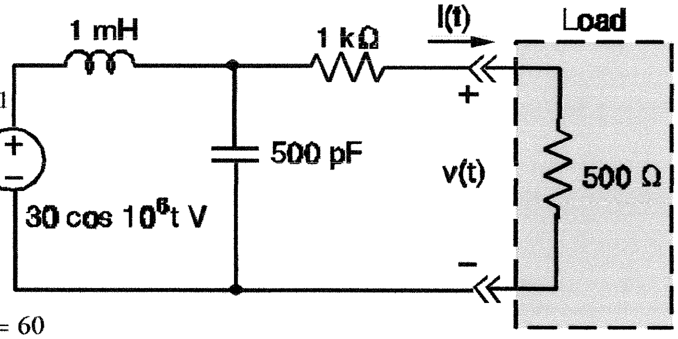
8-54, 15-54

$R := 800$ $X_L := 400$ $I_S := 0.2$ Using two path current division $I_R := \frac{j\cdot X_L}{R + j\cdot X_L}\cdot I_S$

$I_R = 0.04 + 0.08j$ $P_R := \frac{1}{2}\cdot(|I_R|)^2\cdot R$ $P_R = 3.2$ W

8-55, 15-55

First find the Thevenin equivalent



$Z_T := 1000 + \left(j\cdot 10^6\cdot 0.5\cdot 10^{-9} + \frac{1}{j\cdot 10^6\cdot 10^{-3}}\right)^{-1}$

$Z_T = 1\times 10^3 + 2j\times 10^3$

$V_T := \frac{(j\cdot 10^6\cdot 0.5\cdot 10^{-9})^{-1}}{(j\cdot 10^6\cdot 0.5\cdot 10^{-9})^{-1} + j\cdot 10^6\cdot 10^{-3}}\cdot 30$ $V_T = 60$

(a) for $Z_L := 500$ $I_L := \frac{V_T}{Z_T + Z_L}$ $P_L := 0.5\cdot(|I_L|)^2\cdot \text{Re}(Z_L)$ $P_L = 0.144$ W

(b) $P_{MAX} := \frac{(|V_T|)^2}{8\cdot \text{Re}(Z_T)}$ $P_{MAX} = 0.45$ W

(c) For max power $Z_L := \overline{Z_T}$ hence $Z_L = 1\times 10^3 - 2j\times 10^3$ Ω

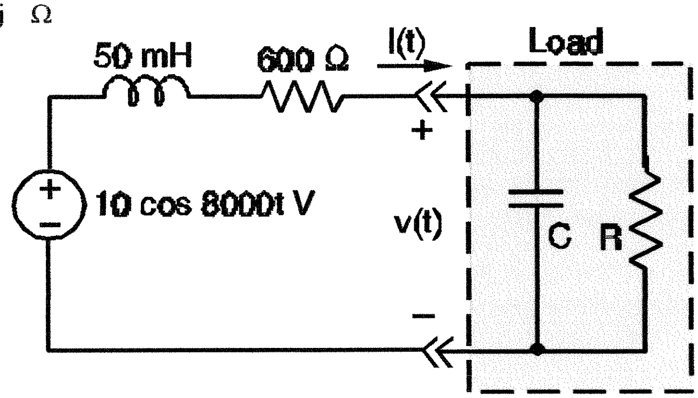
8-56, 15-56 $V := 200$ rms $P := 100$ $R := 8$ $\omega := 2\cdot\pi\cdot 60$ $I := \sqrt{\frac{P}{R}}$ $I = 3.536$ rms

$Z := \frac{V}{I}$ $Z = 56.569$ $L := \frac{\sqrt{(|Z|)^2 - R^2}}{\omega}$ $L = 0.14854$ H

$Z_L := R + j\cdot\omega\cdot L$ $Z_L = 8 + 56j$ Ω

8-57, 15-57 $V_T := 10 + j\cdot 0 \quad \omega := 8000$

$Z_T := 600 + j\cdot\omega\cdot 50\cdot 10^{-3} \quad Z_T = 600 + 400j \quad \Omega$



(a) $P_{MAX} := \dfrac{(|V_T|)^2}{8\cdot Re(Z_T)}$

$P_{MAX} = 2.083 \times 10^{-2}$

$Z_L(R, C) := \dfrac{1}{\left(\dfrac{1}{R} + j\cdot\omega\cdot C\right)}$

(b) $R := 100 \quad C := 10^{-6}$ <----Initial guesses

Given $Re(Z_L(R,C)) = 600 \quad Im(Z_L(R,C)) = -400$ <----Conjugate match condition

$Find(R, C) = \begin{pmatrix} 866.667 \\ 9.615 \times 10^{-8} \end{pmatrix}$ <-----Values of R and C for conjugate match

8-58, 15-58 $V_T := 10 + j\cdot 0$ <--No load condition $V := 4\cdot\exp\left[-j\cdot\left(\dfrac{\pi}{180}\right)\cdot 60\right]$ <--100 Ω load condition

$Z_T := 1 + j$ Given $\dfrac{100\cdot V_T}{100 + Z_T} = V \quad Z_T := Find(Z_T) \quad Z_T = 25 + 216.506j \quad P_{MAX} := \dfrac{(|V_T|)^2}{8\cdot Re(Z_T)}$

$P_{MAX} = 0.5$ W For max power $Z_L := \overline{Z_T}$ hence $Z_L = 25 - 216.506j \ \Omega$

8-59, 15-59

(a) $V_T := 10 \quad Z_T := 100 + j\cdot 50 \quad P_{MAX} := \dfrac{(|V_T|)^2}{8\cdot Re(Z_T)} \quad P_{MAX} = 0.125$ W

(b) $I_L(R_L) := \dfrac{V_T}{R_L + Z_T} \qquad P_L(R_L) := 0.5\cdot(|I_L(R_L)|)^2\cdot R_L$

Values of load power for standard resistance values are:

$P_L(10) = 0.034 \quad P_L(15) = 0.048 \quad P_L(22) = 0.063 \quad P_L(33) = 0.082$

$P_L(47) = 0.097 \quad P_L(68) = 0.111 \quad P_L(100) = 0.118 \quad P_L(150) = 0.115$

$P_L(220) = 0.105 \quad P_L(330) = 0.088 \quad P_L(470) = 0.072 \quad P_L(680) = 0.056$

$P_L(100)$ is the largest hence $R_L = 100$ draws the most power

$\dfrac{P_L(100)}{P_{MAX}} = 0.941$ hence $P_L(100)$ is about 94% of the maximum available power

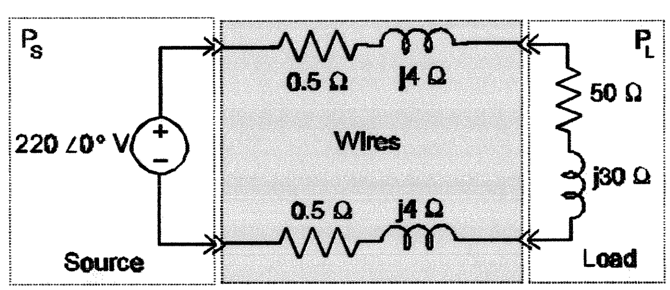
8-60, 16-60 $V := 220$

$Z_W := 0.5 + j\cdot 4 \quad Z_L := 50 + j\cdot 30$

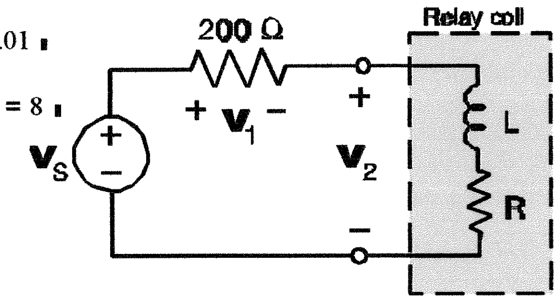
$Z_{total} := 2\cdot Z_W + Z_L \quad I := \dfrac{V}{Z_{total}}$

$P_L := (|I|)^2\cdot Re(Z_L) \quad P_S := (|I|)^2\cdot Re(Z_{total})$

$P_L = 598.269$ W $P_S = 610.235$ W



8-61, 15-61 $\omega := 2\cdot\pi\cdot 1000$ ▪ $V_S := 10$ ▪ $R := 100$ ▪ $L := 0.01$ ▪



(a) Given ▪ $\dfrac{\left|\left|V_S\right|\right|\cdot 200}{\left|R+200+j\cdot\omega\cdot L\right|} = 4$ ▪ $\dfrac{\left|V_S\right|\cdot\left|R+j\cdot\omega\cdot L\right|}{\left|R+200+j\cdot\omega\cdot L\right|} = 8$ ▪

$\begin{pmatrix} R \\ L \end{pmatrix} := \text{Find}(R,L)$ ▪ $R = 125\ \Omega$ ▪ $L = 6.047\times 10^{-2}$ H ▪

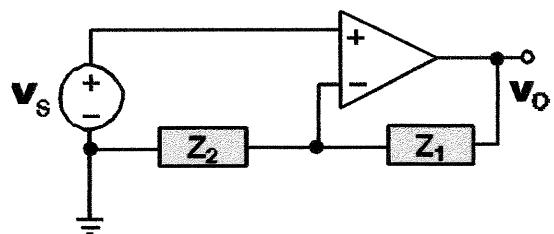
$500\cdot\sqrt{1-.8^2} = 300$

(b) $V_1 := \dfrac{V_S\cdot 200}{R+200+j\cdot\omega\cdot L}$ $V_1 = 2.6 - 3.04j$ $V_2 := \dfrac{V_S\cdot(R+j\cdot\omega\cdot L)}{R+200+j\cdot\omega\cdot L}$ $V_2 = 7.4 + 3.04j$

KVL yields $V_S - V_1 - V_2 = 0$ ▪ <---sums to zero as required

8-62, 15-62

(a) writing a KCL equation at the inverting input



$\dfrac{V_n}{Z_2} + \left(\dfrac{V_n - V_O}{Z_1}\right) = 0$ but $V_n = V_p = V_S$ hence

$\dfrac{V_S}{Z_2} + \left(\dfrac{V_S - V_O}{Z_1}\right) = 0$ solving for $V_O = \dfrac{Z_1+Z_2}{Z_2}\cdot V_S$

$K = \dfrac{Z_1+Z_2}{Z_2} = 1 + \dfrac{Z_1}{Z_2}$

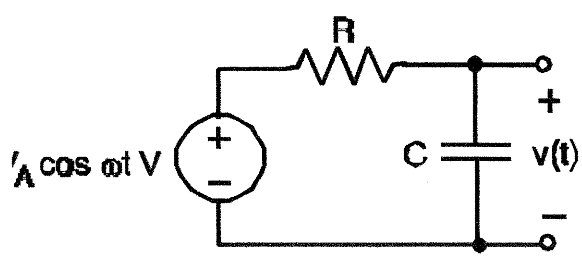
(b) $Z_1 := 10^5$ $\dfrac{Z_1+Z_2}{Z_2} = 11 + 10j$ $Z_2 := 10^4 - j\cdot 2\cdot 10^4$

$\left|\dfrac{Z_1+Z_2}{Z_2}\right| = 5$ $\dfrac{180}{\pi}\cdot\arg\left(\dfrac{Z_1+Z_2}{Z_2}\right) = 53.13$ Mag = 5 Angle = 53.13°

(c) $K := 2\cdot\exp\left(j\cdot\dfrac{-\pi}{3}\right)$ Let $Z_2 := 10^4$ then $Z_1 := K\cdot Z_2 - Z_2$ $Z_1 = -1.732j\times 10^4$

Z_2 is a 10 kΩ resistor and Z_1 is a capacitor

8-63, 15-63 (a) By voltage division



$$V = \frac{V_A\cdot\dfrac{1}{j\cdot\omega\cdot C}}{R+\dfrac{1}{j\cdot\omega\cdot C}} = \frac{V_A}{1+j\cdot\omega\cdot R\cdot C}\cdot\frac{1-j\cdot\omega\cdot R\cdot C}{1-j\cdot\omega\cdot R\cdot C}$$

$$V = \frac{V_A}{1+(\omega\cdot R\cdot C)^2} - j\cdot\frac{V_A\cdot\omega\cdot R\cdot C}{1+(\omega\cdot R\cdot C)^2}$$

(b) $v(t) = \text{Re}\left(V\cdot e^{j\cdot\omega\cdot t}\right) = \text{Re}\left[\dfrac{V_A\cdot e^{j\cdot\omega\cdot t}}{1+(\omega\cdot R\cdot C)^2}\right] + \text{Re}\left[\dfrac{-j\cdot V_A\cdot\omega\cdot R\cdot C\cdot e^{j\cdot\omega\cdot t}}{1+(\omega\cdot R\cdot C)^2}\right]$

$$v(t) = \frac{V_A}{1+(\omega\cdot R\cdot C)^2}\cdot\cos(\omega\cdot t) + \frac{\omega\cdot R\cdot C\cdot V_A}{1+(\omega\cdot R\cdot C)^2}\cdot\sin(\omega\cdot t)\ \text{V}$$

8-63, 15-63 Continued

(c) $R\cdot C\cdot\frac{d}{dt}v(t) + v(t) = V_A\cdot\cos(\omega\cdot t)$

(d) Substituting the v(t) from (b) into this equation yields

$$R\cdot C\cdot\frac{d}{dt}v(t) = \frac{-\omega\cdot R\cdot C\cdot V_A}{1+(\omega\cdot R\cdot C)^2}\cdot\sin(\omega\cdot t) + \frac{(\omega\cdot R\cdot C)^2\cdot V_A}{1+(\omega\cdot R\cdot C)^2}\cdot\cos(\omega\cdot t)$$

$$v(t) = \frac{V_A}{1+(\omega\cdot R\cdot C)^2}\cdot\cos(\omega\cdot t) + \frac{\omega\cdot R\cdot C\cdot V_A}{1+(\omega\cdot R\cdot C)^2}\cdot\sin(\omega\cdot t)$$

The right sides of sum to $\frac{(\omega\cdot R\cdot C)^2\cdot V_A}{1+(\omega\cdot R\cdot C)^2}\cdot\cos(\omega\cdot t) + \frac{V_A}{1+(\omega\cdot R\cdot C)^2}\cdot\cos(\omega\cdot t) = V_A\cdot\cos(\omega\cdot t)$ QED

8-64, 15-64 $V_C = V_A + j\cdot 0$ $\quad v_C(t) = V_A\cdot\cos\left(2\cdot\pi\cdot\frac{t}{T_0}\right)$

$$w_C(t) = \left(\frac{1}{2}\cdot C\cdot v_C(t)^2\right) = \frac{1}{2}\cdot C\cdot {V_A}^2\cdot\left(\cos\left(2\cdot\pi\cdot\frac{t}{T_0}\right)\right)^2$$

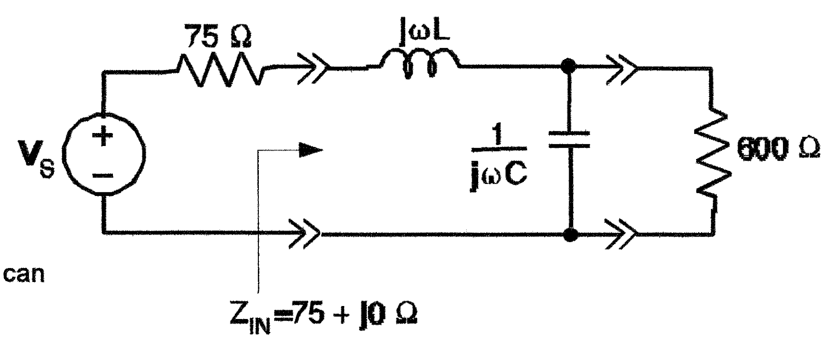
$$W_{avg} = \frac{1}{T_0}\cdot\int_0^{T_0} w_C(t)\,dt = \frac{1}{2}\cdot C\cdot {V_A}^2\cdot\left[\frac{1}{T_0}\cdot\int_0^{T_0}\left(\cos\left(2\cdot\pi\cdot\frac{t}{T_0}\right)\right)^2 dt\right]$$

$$\left[\frac{1}{T_0}\cdot\int_0^{T_0}\left(\cos\left(2\cdot\pi\cdot\frac{t}{T_0}\right)\right)^2 dt\right] = \frac{1}{T_0}\cdot\left[\frac{T_0}{2} + \left(\sin\left(4\cdot\pi\cdot\frac{T_0}{T_0}\right)\right)\cdot\frac{T_0}{8\cdot\pi}\right] + 0 = \frac{1}{2}$$

$W_{avg} = \frac{1}{4}\cdot C\cdot {V_A}^2$ J

8-65, 15-65

$\omega := 10^6$



The input impecance of the circuit can be written as

$$Z_{IN} = j\cdot 10^6\cdot L + \frac{1}{j\cdot 10^6\cdot C + \frac{1}{600}} = j\cdot 10^6\cdot L + \frac{600}{1 + j\cdot 6\cdot 10^8\cdot C}$$

$$Z_{IN} = j\cdot 10^6\cdot L + \frac{600}{1+(6\cdot 10^8\cdot C)^2} - j\cdot\frac{600\cdot(6\cdot 10^8\cdot C)}{\left[1+(6\cdot 10^8\cdot C)^2\right]}\ \Omega$$

8-65, 15-65 Continued

The design requirement is that $Z_{IN} = 75 + j0$. The design constraints for the resistance is

$$\frac{600}{1+\left(6\cdot 10^{8}\cdot C\right)^{2}} = 75 \quad \text{and the reactance} \quad 10^{6}\cdot L = \frac{600\cdot\left(6\cdot 10^{8}\cdot C\right)}{\left[1+\left(6\cdot 10^{8}\cdot C\right)^{2}\right]}$$

Solving the resistance constrain for C yields $C := \frac{\sqrt{7}}{6}\cdot 10^{-8}$ inserting this in the reactance constraint

and solving for L yields $L := 10^{-4}\cdot\frac{3}{4}\cdot\sqrt{7}$ checking

$L = 1.984\times 10^{-4}$ $C = 4.41\times 10^{-9}$ $j\cdot 10^{6}\cdot L + \dfrac{1}{j\cdot 10^{6}\cdot C + \dfrac{1}{600}} = 75$ <---$Z_{\cdot IN} = 75 + j0$

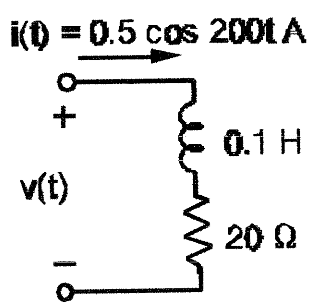
$$Z_{Th} := \frac{1}{j\cdot 10^{6}\cdot C + \dfrac{1}{j\cdot 10^{6}\cdot L + 75}} \qquad Z_{Th} = 600 \quad \Omega$$

8-66, 15-66 The phasor current is $I = 0.5 + j\cdot 0$; the impedance is $Z = 20 + j\cdot 20$ The phasor voltage is

$$V = I\cdot Z = (0.5 + j\cdot 0)\cdot(20 + j\cdot 20) = 10 + j\cdot 10 = 10\cdot\sqrt{2}\cdot\exp\left(j\cdot\frac{\pi}{4}\right)$$

hence the steady-state voltage is

$$v(t) = \text{Re}\left[10\cdot\sqrt{2}\cdot\exp\left[j\cdot\left(200\cdot t + \frac{\pi}{4}\right)\right]\right] = 10\cdot\sqrt{2}\cdot\cos\left(200\cdot t + \frac{\pi}{4}\right)\ \text{V}$$



Comment: Your answer is correct, but the method of solution uses an invalid mixture of time and phasor domain concepts. You can not mix waveforms and phasors. Impedance relates phasor voltage and current, not waveforms. Thus, your starting place $v(t) = (R + j\cdot\omega\cdot L)\times i(t)$ is not a valid equation.

CHAPTER 9, Both Versions

9-1 $F(s) = L(f(t)) = L\left[A\cdot\left(e^{-\alpha\cdot t} - 2\right)\cdot u(t)\right]$

$$F(s) = \frac{A}{s+\alpha} - \frac{2\cdot A}{s} = \frac{-A\cdot(s+2\cdot\alpha)}{s\cdot(s+\alpha)}$$

<---zero at s = -2α

<---Poles at s = 0 and at s = -α

9-2 $F(s) = L(f(t)) = L\left[A\cdot\left[(1+\alpha\cdot t)\cdot e^{-\alpha\cdot t}\right]\cdot u(t)\right]$

$$F(s) = \frac{A}{s+\alpha} + \frac{A\cdot\alpha}{(s+\alpha)^2} = A\cdot\frac{(s+2\cdot\alpha)}{\left[(s+\alpha)^2\right]}$$

<---zero at s = -2α

<---Double pole at s = -α

9-3 $F(s) = L(f(t)) = L\left[A\cdot(2\cdot\sin(\beta\cdot t) + 1)\cdot u(t)\right]$

$$F(s) = \frac{2\cdot A\cdot\beta}{s^2+\beta^2} + \frac{A}{s} = \frac{A\cdot\left(s^2 + 2\cdot\beta\cdot s + \beta^2\right)}{s\cdot\left(s^2+\beta^2\right)} = \frac{A\cdot(s+\beta)^2}{s\cdot\left(s^2+\beta^2\right)}$$

<---Double zero at s = -β

<---Poles at s = 0 and s = ± jβ

9-4 $F(s) = L(f(t)) = L\left[A\cdot(\cos(\beta\cdot t) - \sin(\beta\cdot t))\cdot u(t)\right]$

$$F(s) = \frac{A\cdot s}{s^2+\beta^2} - \frac{A\cdot\beta}{s^2+\beta^2} = \frac{A\cdot(s-\beta)}{\left(s^2+\beta^2\right)}$$

<---zero at s = β

<---Poles at s = ± jβ

9-5 $F(s) = L(f(t)) = L\left(A\cdot\delta(t) - A\cdot\beta\cdot e^{-\alpha\cdot t}\cdot\sin(\beta\cdot t)\cdot u(t)\right)$

$$F(s) = A - \frac{A\cdot\beta^2}{(s+\alpha)^2+\beta^2} = \frac{A\cdot(s+\alpha)^2}{(s+\alpha)^2+\beta^2}$$

<---Double zero at s = -α

<---Poles at s = -α ± jβ

9-6 $F(s) = L(f(t)) = L\left[A\cdot\left(2\cdot\alpha\cdot t - 1 + e^{-\alpha\cdot t}\right)\cdot u(t)\right]$

$$F(s) = \frac{2\cdot\alpha\cdot A}{s^2} - \frac{A}{s} + \frac{A}{s+\alpha} = \frac{A\cdot\alpha\cdot(s+2\cdot\alpha)}{s^2\cdot(s+\alpha)}$$

<---Zero at s = -2α

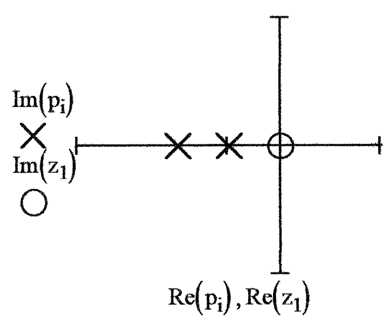
<---Poles at s = -α and double poles at s = 0

9-7 (a) $F(s) = L\left(5\cdot e^{-5\cdot t} - 10e^{-10\cdot t}\right)\cdot u(t)$

$$F(s) = \frac{5}{s+5} - \frac{10}{s+10}$$

$$F(s) = \frac{-5\cdot s}{(s+5)\cdot(s+10)}$$

$i := 1, 2..2 \quad p_1 := -5 \quad p_2 := -10 \quad z_1 := 0$



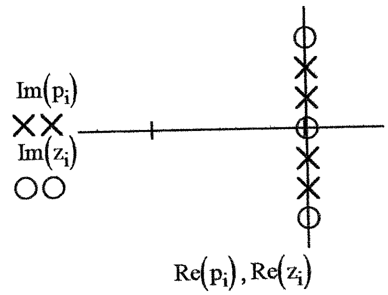
9-7 Continued

(b) $F(s) = L[(10\cdot\cos(20\cdot t) - 16\cdot\cos(10\cdot t))\cdot u(t)$ $\quad i := 1,2..4 \quad p_1 := j\cdot 10 \quad p_2 := -j\cdot 10 \quad p_3 := j\cdot 20$

$p_4 := -j\cdot 20 \quad z_1 := 0 \quad z_2 := -j\cdot 30 \quad z_3 := j\cdot 30$

$$F(s) = \frac{10\cdot s}{s^2 + 20^2} - \frac{16\cdot s}{s^2 + 10^2}$$

$$F(s) = \frac{-6\cdot s\cdot\left(s^2 + 30^2\right)}{\left(s^2 + 10^2\right)\cdot\left(s^2 + 20^2\right)}$$

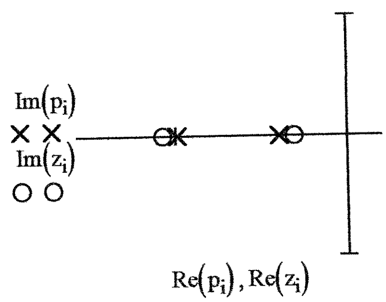


9-8 (a) $F(s) = L\cdot\left[\delta(t) + \left(5\cdot e^{-50\cdot t} - 5\cdot e^{-20\cdot t}\right)\cdot u(t)\right]$ $\quad i := 1,2..2 \quad p_1 := -20 \quad p_2 := -50 \quad z_1 := -15.63\!: z_2 := -54.36$

$$F(s) = 1 + \frac{5}{s + 50} - \frac{5}{s + 20}$$

$$F(s) = \frac{\left(s^2 + 70\cdot s + 850\right)}{(s + 50)\cdot(s + 20)}$$

$$F(s) = \frac{(s + 15.635)\cdot(s + 54.365)}{(s + 50)\cdot(s + 20)}$$

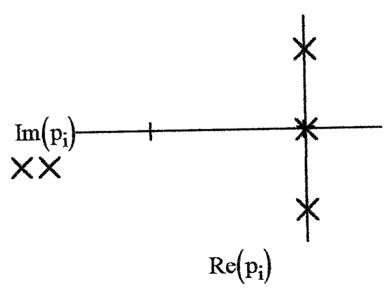


(b) $F(s) = L[(25 - 25\cdot\cos(500\cdot t))\cdot u(t)]$ $\quad i := 1,2..3 \quad p_1 := 0 \quad p_2 := j\cdot 500 \quad p_3 := -j\cdot 500$

$$F(s) = \frac{25}{s} - \frac{25\cdot s}{s^2 + 500^2}$$

$$F(s) = \frac{2500^2}{s\cdot\left(s^2 + 500^2\right)}$$

$$F(s) = \frac{2500^2}{s\cdot(s + j\cdot 500)\cdot(s - j\cdot 500)}$$



9-9 $F(s) = L\cdot\left[\delta(t) + \left(25\cdot e^{-50\cdot t} - 1250\cdot t\cdot e^{-50\cdot t}\right)\cdot u(t)\right]$

$$F(s) = 1 + \frac{25}{s + 50} - \frac{1250}{(s + 50)^2} = \frac{\left(s^2 + 125\cdot s + 2500\right)}{(s + 50)^2}$$

$$F(s) = \frac{(s + 25)\cdot(s + 100)}{(s + 50)^2}$$

<---Zeros at s = -25 and s = -100

<---Double pole at s = -50

9-10 $F(s) = L(f(t)) = L\left[\left(5 - 2\cdot e^{-5\cdot t} - 3\cdot\cos(5\cdot t) - 2\cdot\sin(5\cdot t)\right)\cdot u(t)\right]$

$$F(s) = \frac{5}{s} - \frac{2}{s+5} - \frac{3s}{s+5^2} - \frac{10}{s+5^2} = \frac{25\cdot(s+25)}{s\cdot(s+5)\cdot\left(s^2+5^2\right)}$$ **<---Zero at s = -25** **<---Poles at s = 0, s = -5 and s = ± j5**

9-11 $F(s) = L\cdot\left[A\cdot\exp\left[-\alpha\cdot(t-T)\right]\cdot\sin\left[\beta\cdot(t-T)\right]\cdot u(t)\right] = \exp(-T\cdot s)\cdot L\left(A\cdot\exp(-\alpha\cdot t)\sin(\beta\cdot t)\right) = \dfrac{A\cdot\beta\cdot\exp(-T\cdot s)}{(s+\alpha)^2+\beta^2}$

9-12 $f(t) = 5\cdot\left(e^{-t}\cdot\sin(4\cdot t)\right)\cdot u(t)$ $\qquad F(s) = \dfrac{20}{\left[(s+1)^2+4^2\right]}$

(a) $G(s) = L\left(\dfrac{d}{dt}f(t)\right) = s\cdot F(s) - f(0) = s\left[\dfrac{20}{\left[(s+1)^2+4^2\right]}\right] - 0 = \dfrac{20\cdot s}{\left[(s+1)^2+4^2\right]}$

(b) $g(t) = \dfrac{d}{dt}f(t) = \dfrac{d}{dt}\left[5\cdot\left(e^{-t}\cdot\sin(4\cdot t)\right)\right] = -5\cdot\exp(-t)\cdot\sin(4\cdot t) + 20\cdot\exp(-t)\cdot\cos(4\cdot t)$

$$G(s) = L(g(t)) = \frac{-20}{(s+1)^2+4^2} + \frac{20\cdot(s+1)}{(s+1)^2+4^2} = \frac{20\cdot s}{\left[(s+1)^2+4^2\right]}$$ **<---QED**

(c) Poles are the same. G(s) has a zero at s = 0 due to t-domain differentiations

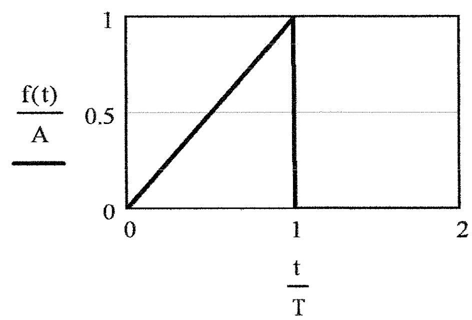
9-13 $A := 1 \quad T := 1 \quad u(t) := \Phi(t) \qquad\qquad t := 0, 0.005\cdot T .. 2\cdot T$

(a) $f(t) := \dfrac{A}{T}\cdot t\cdot u(t) - \dfrac{A}{T}\cdot(t-T)\cdot u(t-T) - A\cdot u(t-T)$

(b) $F(s) = L\left[\dfrac{A}{T}\cdot t\cdot u(t) - \dfrac{A}{T}\cdot(t-T)\cdot u(t-T) - A\cdot u(t-T)\right]$

$$F(s) = \frac{A}{T\cdot s^2} - e^{-T\cdot s}\cdot\frac{A}{T\cdot s^2} - e^{-T\cdot s}\cdot\frac{A}{s}$$

$$F(s) = \frac{A}{T\cdot s^2}\cdot\left(1 - e^{-T\cdot s} - T\cdot s\cdot e^{-T\cdot s}\right)$$



(c) $F(s) = \displaystyle\int_0^T \frac{A\cdot t}{T}\cdot e^{-s\cdot t}\,dt = -\frac{A}{T}\cdot e^{-s\cdot t}\cdot\frac{s\cdot t+1}{s^2}\Bigg|_0^T \quad -\frac{A}{T}\cdot e^{-s\cdot T}\cdot\left(\frac{s\cdot T+1}{s^2}\right) + \frac{A}{T\cdot s^2}$

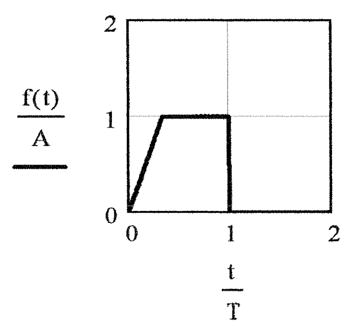
$$F(s) = \frac{A}{T\cdot s^2} - e^{-T\cdot s}\cdot\frac{A}{T\cdot s^2} - e^{-T\cdot s}\cdot\frac{A}{s}$$ **QED**

9-14 (a) $f(t) := \dfrac{3\cdot A}{T}\cdot t\cdot u(t) - \dfrac{3\cdot A}{T}\cdot\left(t - \dfrac{T}{3}\right)\cdot u\left(t - \dfrac{T}{3}\right) - A\cdot u(t-T)$

$t := 0, 0.005\cdot T .. 2\cdot T$

(b) $F(s) = L\left[\dfrac{3\cdot A}{T}\cdot t\cdot u(t) - \dfrac{3\cdot A}{T}\cdot\left(t - \dfrac{T}{3}\right)\cdot u\left(t - \dfrac{T}{3}\right) - A\cdot u(t-T)\right]$

$$F(s) = \frac{3\cdot A}{T\cdot s^2} - e^{-\frac{T}{3}\cdot s}\cdot\frac{3\cdot A}{T\cdot s^2} - e^{-T\cdot s}\cdot\frac{A}{s}$$



9-14 Continued

(c) $$F(s) = \int_0^{\frac{T}{3}} \frac{3\cdot A\cdot t}{T}\cdot e^{-s\cdot t}\,dt + \int_{\frac{T}{3}}^{T} A\cdot e^{-s\cdot t}\,dt = \frac{3\cdot A}{T}\cdot e^{-s\cdot t}\cdot\left(\frac{s\cdot t+1}{s^2}\right)\Bigg|_0^{\frac{T}{3}} + \frac{-A\cdot e^{-s\cdot t}}{s}\Bigg|_{\frac{T}{3}}^{T}$$

$$F(s) = \left(\frac{-3\cdot A}{T}\cdot e^{-s\cdot\frac{T}{3}}\cdot\frac{s\cdot\frac{T}{3}+1}{s^2}\right) + \frac{3\cdot A}{T\cdot s^2} + \frac{-A\cdot e^{-s\cdot T}}{s} + \frac{A\cdot e^{-\frac{T}{3}\cdot s}}{s} = \frac{3\cdot A}{T\cdot s^2} - e^{-\frac{T}{3}\cdot s}\cdot\frac{3\cdot A}{T\cdot s^2} - e^{-T\cdot s}\cdot\frac{A}{s}$$ **QED**

9-15 $f(t) = 1600\cdot t + 8 + 75\cdot\exp(-10\cdot t) + 17\cdot\exp(-50\cdot t)$

$$F(s) = \frac{1600}{s^2} + \frac{8}{s} + \frac{75}{s+10} + \frac{17}{s+50} = 100\cdot\frac{s^3 + 60\cdot s^2 + 1000\cdot s + 8000}{s^2\cdot(s+10)\cdot(s+50)}$$

Double pole at s = 0 and simple poles at s = -10 and s = -50

$$\text{polyroots}\left(\begin{pmatrix}8000\\1000\\60\\1\end{pmatrix}\right) = \begin{pmatrix}-40\\-10+10i\\-10-10i\end{pmatrix}$$ **<---zeros**

9-16 (a) $F_1(s) = \frac{s-20}{s\cdot(s+10)} = \frac{-2}{s} + \frac{3}{(s+10)}$ $f_1(t) = (-2 + 3\cdot\exp(-10\cdot t))\cdot u(t)$

(b) $F_2(s) = \frac{s^2+100}{s\cdot(s+10)} = 1 + \frac{10}{s} - \frac{20}{(s+10)}$ $f_2(t) = \delta(t) + (10 - 20\cdot\exp(-10\cdot t))\cdot u(t)$

9-17 (a) $F_1(s) = \frac{s+10}{s\cdot(s+5)} = \frac{2}{s} - \frac{1}{(s+5)}$ $f_1(t) = (2 - \exp(-5\cdot t)\cdot u(t)$

(b) $F_2(s) = \frac{s}{(s+5)\cdot(s+10)} = \frac{-1}{(s+5)} + \frac{2}{(s+10)}$ $f_2(t) = (2\cdot\exp(-10\cdot t) - \exp(-5\cdot t))\cdot u(t)$

9-18 (a) $F_1(s) = \frac{20\cdot(s+20)}{(s+10)^2+20^2} = \frac{10-j\cdot 5}{s+10-j\cdot 20} + \frac{10+j\cdot 5}{s+10+j\cdot 20}$ $|10-j\cdot 5| = 11.18$ $\frac{180}{\pi}\cdot\arg(10-j\cdot 5) = -26.565$

$f_1(t) = 22.36\cdot e^{-10\cdot t}\cdot\cos\left(20\cdot t - 26.6^o\right)\cdot u(t)$ or

$f_1(t) = (20\cdot\exp(-10\cdot t)\cdot\cos(20\cdot t) + 10\cdot\exp(-10\cdot t)\cdot\sin(20\cdot t))\cdot u(t)$

(b) $F_2(s) = \frac{20\cdot(s-20)}{(s+10)^2+20^2} = \frac{10+j\cdot 15}{s+10-j\cdot 20} + \frac{10-j\cdot 15}{s+10+j\cdot 20}$ $|10+j\cdot 15| = 18.028$ $\frac{180}{\pi}\cdot\arg(10+j\cdot 15) = 56.31$

$f_2(t) = 36.06\cdot e^{-10\cdot t}\cdot\cos\left(20\cdot t + 56.3^o\right)\cdot u(t)$

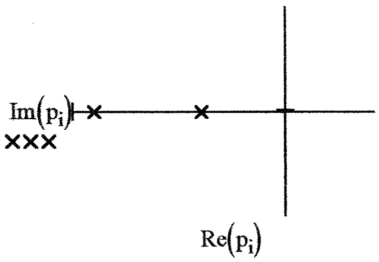
9-19 $F(s) = \frac{s\cdot(s-\beta)}{s^2+\beta^2} = 1 - \frac{\beta\cdot s}{s^2+\beta^2} - \beta\cdot\frac{\beta}{s^2+\beta^2}$ $f(t) = \delta(t) - \beta\cdot(\cos(\beta\cdot t) + \sin(\beta\cdot t))\cdot u(t)$

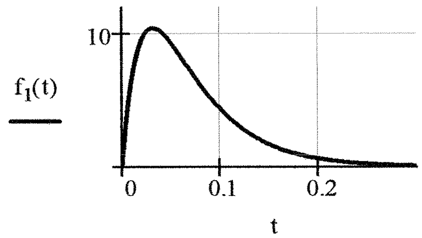
$\cos(\beta\cdot t) + \sin(\beta\cdot t) = \sqrt{2}\cdot\cos\left(\beta\cdot t - \frac{\pi}{4}\right)$

9-20 $F(s) = \dfrac{\alpha^2}{s^2\cdot(s+\alpha)} = \dfrac{-1}{s} + \dfrac{\alpha}{s^2} + \dfrac{1}{s+\alpha}$ $\quad f_1(t) = \left(-1 + \alpha\cdot t + e^{-\alpha\cdot t}\right)\cdot u(t)$

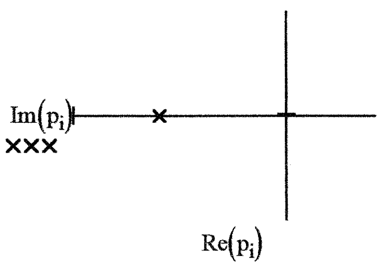
9-21 (a) $F_1(s) = \dfrac{900}{s^2 + 65\cdot s + 900} = \dfrac{900}{(s+20)\cdot(s+45)} = \dfrac{36}{(s+20)} - \dfrac{36}{(s+45)}$ $\quad f_1(t) := 36\cdot\left(e^{-20\cdot t} - e^{-45\cdot t}\right)\cdot u(t)$

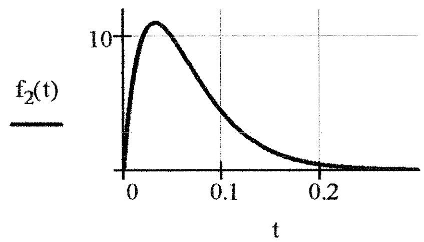
$i := 1,2..2 \quad t := 0, 0.002..0.3 \qquad p_1 := -20 \quad p_2 := -45$





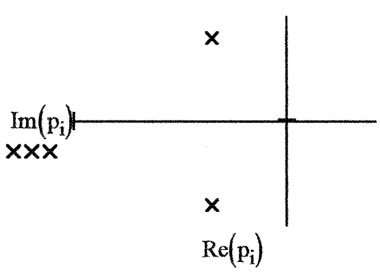
(b) $F_2(s) = \dfrac{900}{s^2 + 60\cdot s + 900} = \dfrac{900}{(s+30)^2}$ $\quad p_1 := -30 \quad p_2 := -30 \qquad f_2(t) := 900\cdot\left(t\cdot e^{-30\cdot t}\right)\cdot u(t)$

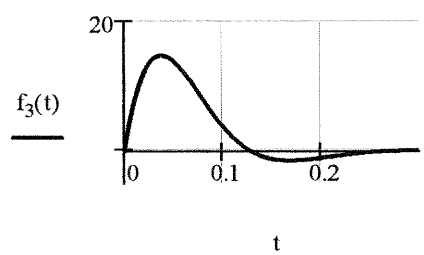




(c) $F_3(s) = \dfrac{900}{s^2 + 36\cdot s + 900} = \dfrac{900}{(s+18)^2 + 24^2} = \dfrac{900}{24}\cdot\dfrac{24}{(s+18)^2 + 24^2}$ $\quad f_3(t) := \dfrac{900}{24}\cdot\left(e^{-18\cdot t}\cdot\sin(24\cdot t)\right)\cdot u(t)$

$i := 1,2..2 \quad t := 0, 0.002..0.3 \quad p_1 := -18 + j\cdot 24 \qquad p_2 := \overline{p_1} \qquad \dfrac{900}{24} = 37.5$





9-22 (a) $F_1(s) = \dfrac{16}{(s+2)\cdot(s+4)\cdot(s+6)} = \dfrac{2}{(s+2)} - \dfrac{4}{(s+4)} + \dfrac{2}{(s+6)}$

$f_1(t) = (2\cdot\exp(-2\cdot t) - 4\cdot\exp(-4\cdot t) + 2\cdot\exp(-6\cdot t))\cdot u(t)$

(b) $F_2(s) = \dfrac{\left(s^2 + 4\cdot s + 16\right)}{\left(s^2 + 16\right)\cdot s} = \dfrac{1}{s} + \dfrac{4}{\left(s^2 + 4^2\right)}$ $\quad f_2(t) = (1 + \sin(4\cdot t))\cdot u(t)$

9-23 (a) $F_1(s) = \dfrac{(s+2)\cdot(s+6)}{s\cdot(s+4)\cdot(s+8)} = \dfrac{3}{(8\cdot s)} + \dfrac{1}{[4\cdot(s+4)]} + \dfrac{3}{[8\cdot(s+8)]}$

$$f_1(t) = \left(\frac{3}{8} + \frac{1}{4}\cdot\exp(-4\cdot t) + \frac{3}{8}\cdot\exp(-8\cdot t)\right)\cdot u(t)$$

(b) $F_2(s) = \dfrac{\left(s^2+45\right)\cdot\left(s^2+80\right)}{s\cdot\left(s^2+20\right)\cdot\left(s^2+60\right)} = \dfrac{3}{s} - \dfrac{15}{8}\cdot\dfrac{s}{\left(s^2+20\right)} - \dfrac{1}{8}\cdot\dfrac{s}{\left(s^2+60\right)}$

$$f_2(t) = \left(3 - \frac{15}{8}\cdot\cos\left(\sqrt{20}\cdot t\right) - \frac{1}{8}\cdot\cos\left(\sqrt{60}\cdot t\right)\right)\cdot u(t)$$

9-24(a) $F_1(s) = \dfrac{4\cdot(s+4)}{s\cdot\left(s^2+4\cdot s+8\right)}$

$$F_1(s) = \frac{2}{s} - 2\cdot\frac{(s+2)}{\left[(s+2)^2+2\right]}$$

$$f_1(t) = (2 - 2\cdot\exp(-2\cdot t)\cdot\cos(2\cdot t))\cdot u(t)$$

9-24(b) $F_2(s) = \dfrac{(s-1)^2}{(s+4)\cdot\left(s^2+2\cdot s+17\right)}$

$$F_2(s) = \frac{1}{(s+4)} - \frac{4}{(s+1)^2+4^2}$$

$$f_2(t) = (\exp(-4\cdot t) - \exp(-t)\cdot\sin(4\cdot t))\cdot u(t)$$

9-25(a) $F_1(s) = \dfrac{2\cdot\left(s^2+4\cdot s+16\right)}{s\cdot\left(s^2+8\cdot s+32\right)}$

$$F_1(s) = \frac{1}{s} + \frac{(s+4)}{\left[(s+4)^2+16\right]} - \frac{4}{\left[(s+4)^2+16\right]}$$

$$f_1(t) = (1 + \exp(-4\cdot t)\cdot\cos(4\cdot t) - \exp(-4\cdot t)\cdot\sin(4\cdot t))\cdot u(t)$$

9-25(b) $F_2(s) = \dfrac{\left(s^2+20\cdot s+400\right)}{s\cdot\left(s^2+50\cdot s+400\right)}$

$$F_2(s) = \frac{1}{s} + \frac{1}{(s+40)} - \frac{1}{(s+10)}$$

$$f_2(t) = (1 + \exp(-40\cdot t) - \exp(-10\cdot t))\cdot u(t)$$

9-26(a) $F(s) := \dfrac{(s+50)^2}{(s+10)\cdot(s+100)}$

$$F(s) := 1 + \frac{160}{[9\cdot(s+10)]} - \frac{250}{[9\cdot(s+100)]}$$

$$f(t) = \delta(t) + \left(\frac{160}{9}\cdot\exp(-10\cdot t) - \frac{250}{9}\cdot\exp(-100\cdot t)\right)\cdot u(t)$$

9-26(b) $F(s) := \dfrac{s+1}{\left(s^2-2\cdot s+1\right)}$

$$F(s) := \frac{2}{(s-1)^2} + \frac{1}{(s-1)}$$

$$f(t) := (2\cdot t\cdot\exp(t) + \exp(t))\cdot u(t)$$

9-27 $F(s) = \dfrac{s+\gamma}{s+20}$

(a) $f(t) = \delta(t) - 5\cdot e^{-20\cdot t}\cdot u(t)$ ---> $F(s) = 1 - \dfrac{5}{s+20} = \dfrac{s+15}{s+20}$ ---> $\gamma = 15$

(b) $f(t) = \delta(t)$ ---> $F(s) = 1$ ---> $\gamma = 20$

(c) $f(t) = \delta(t) + 5\cdot e^{-20\cdot t}\cdot u(t)$ ---> $F(s) = 1 + \dfrac{5}{s+20} = \dfrac{s+25}{s+20}$ ---> $\gamma = 25$

9-28 $F(s) = \dfrac{\left(s^2 + b_1 \cdot s + b_0\right) \cdot K}{(s+5)^2 \cdot \left(s^2 + 20^2\right)}$

(a) $f(t) = 20 \cdot t \cdot e^{-5 \cdot t}$ ---> $F(s) = \dfrac{20}{(s+5)^2}$ ---> $b_0 = 20^2$ $b_1 = 0$ $K = 20$

(b) $f(t) = 5 \cdot \sin(20 \cdot t)$ ---> $F(s) = \dfrac{100}{\left(s^2 + 20^2\right)}$ ---> $b_0 = 5^2$ $b_1 = 10$ $K = 100$

(c) $f(t) = 4 \cdot \sin(20 \cdot t) + \cos(20 \cdot t) - e^{-5 \cdot t}$ ---> $F(s) = \dfrac{85 \cdot s}{\left(s^2 + 400\right) \cdot (s+5)}$ ---> $b_1 = 5$ $b_0 = 0$ $K = 85$

9-29 $F(s) = \dfrac{\left(s^2 + 3 \cdot s + 4\right) \cdot s}{\left(s^3 + 6 \cdot s^2 + 16 \cdot s + 16\right) \cdot (s+2)} = \dfrac{\left(s^2 + 3 \cdot s + 4\right) \cdot s}{(s+2)^2 \cdot \left[(s+2)^2 + 2^2\right]}$

$F(s) = \dfrac{-1}{(s+2)^2} + \dfrac{1}{(s+2)} - \dfrac{2}{\left[(s+2)^2 + 2^2\right]}$

$f(t) = (-t \cdot \exp(-2 \cdot t) + \exp(-2 \cdot t) - \exp(-2 \cdot t) \cdot \sin(2 \cdot t)) \cdot u(t)$

9-30 $F(s) = \dfrac{\left(s^3 + 2 \cdot s^2 + s + 2\right) \cdot 40}{\left(s^3 + 4 \cdot s^2 + 4 \cdot s + 16\right) \cdot s} = \dfrac{(s+2) \cdot \left(s^2 + 1\right) \cdot 40}{s \cdot (s+4) \cdot \left(s^2 + 2^2\right)}$

$F(s) = \dfrac{17}{(s+4)} + \dfrac{5}{s} - 6 \cdot \left(\dfrac{2}{s^2 + 2^2}\right) + 18 \cdot \left(\dfrac{s}{s^2 + 2^2}\right)$

$f(t) = (5 + 17 \cdot \exp(-4 \cdot t) - 6 \cdot \sin(2 \cdot t) + 18 \cdot \cos(2 \cdot t)) \cdot u(t)$

9-31(a) $y(0) = 5$ $L\left(\dfrac{d}{dt}y + 10 \cdot y\right) = s \cdot Y(s) - 5 + 20 \cdot Y(s) = 0$ ---> $Y(s) = \dfrac{5}{s+20}$ ---> $y(t) = \left(5 \cdot e^{-20 \cdot t}\right) \cdot u(t)$

(b) $y(0) = -10$ $L\left[10^{-2} \cdot \left(\dfrac{d}{dt}y\right) + y\right] = L(10 \cdot u(t))$ ---> $10^{-2} \cdot (s \cdot Y(s) + 10) + Y(s) = \dfrac{10}{s}$

$Y(s) = \dfrac{10 \cdot (100 - s)}{s \cdot (s+100)} = \dfrac{10}{s} - \dfrac{20}{s+100}$ ---> $y(t) = (10 - 20 \cdot \exp(-100 \cdot t)) \cdot u(t)$

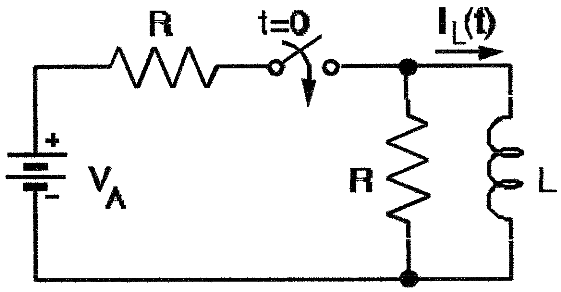
9-32 $y(0) = 0$ $L\left(\dfrac{d}{dt}y + 500y\right) = L(2500 \exp(-1000 \cdot t))$ ---> $(s+500) \cdot Y(s) = \dfrac{2500}{s+1000}$

$Y(s) = \dfrac{2500}{(s+500) \cdot (s+1000)} = \dfrac{5}{(s+500)} - \dfrac{5}{(s+1000)}$ $y(t) = \left(5 \cdot e^{-500 \cdot t} - 5 \cdot e^{-1000 \cdot t}\right) \cdot u(t)$

9-33 (a) For t < 0 the inductor acts like a short circuit and $i_L(0) = 0$.
For t > 0 the inductor sees a Norton ckt $R_N = R/2 = 50$ and $I_N = V_A/R = 0.1A$.

The ckt diff eq is: $\frac{L}{R_N}\cdot\frac{d}{dt}i_L + i_L = i_N$ or $2\cdot 10^{-3}\cdot\frac{d}{dt}i_L + i_L = 0.1\cdot u(t)$

(b) $\mathcal{L}\left(2\cdot 10^{-3}\cdot\frac{d}{dt}i_L(t) + i_L(t)\right) = \mathcal{L}(0.1\cdot u(t))$



$$2\cdot 10^{-3}\cdot s\cdot I_L(s) + I_L(s) = \frac{0.1}{s}$$

$$I_L(s) = \frac{50}{s\cdot(s+500)} = \frac{0.1}{s} - \frac{0.1}{s+500}$$

$$i_L(t) = 0.1\cdot(1 - \exp(-500\cdot t))\cdot u(t)$$

9-34 (a) For t < 0 the inductor acts like a short circuit and $i_L(0) = V_A/R = 2$ A.
For t > 0 the inductor sees a Norton ckt $R_N = R = 50$ and $I_N = 0$.

The ckt diff eq is: $\frac{L}{R_N}\cdot\frac{d}{dt}i_L + i_L = i_N$ or $10^{-3}\cdot\frac{d}{dt}i_L + i_L = 0$

(b) $\mathcal{L}\left(10^{-3}\cdot\frac{d}{dt}i_L + i_L\right) = 10^{-3}\cdot(s\cdot I_L(s) - 2) + I_L(s) = 0$

$$I_L(s) = \frac{2}{(s+1000)}$$

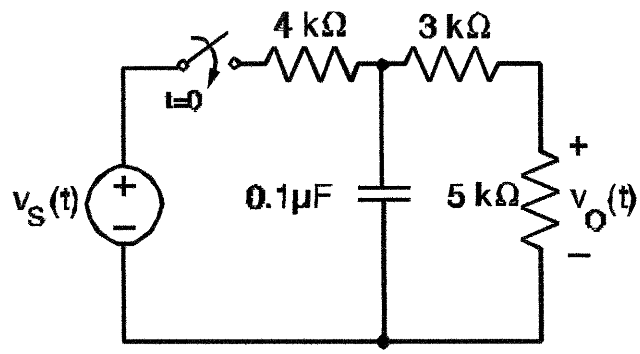
$$i_L(t) = 2\cdot\exp(-1000\cdot t)\cdot u(t)$$

9-35 (a) For t < 0 the capacitor acts like an open ckt and $v_C(0) = 0$.
For t > 0 the Thevenin ckt is $R_T = 8000||4000 = 2667$ & $v_T = 2v_S/3$.

The ckt diff eq is: $R_T\cdot C\cdot\frac{d}{dt}v_C + v_C = v_T$ or $\frac{1}{3750}\cdot\frac{d}{dt}v_C + v_C = 10\cdot e^{-2500\cdot t}$

(b) $\mathcal{L}\left(\frac{1}{3750}\cdot\frac{d}{dt}v_C + v_C\right) = \mathcal{L}\left(10\cdot e^{-2500\cdot t}\right)$

$$\frac{s\cdot V_C(s)}{3750} + V_C(s) = \frac{10}{s+2500}$$



$$V_C(s) = \frac{37500}{(s+2500)\cdot(s+3750)}$$

$$V_C(s) = \frac{30}{(s+2500)} - \frac{30}{(s+3750)}$$

$$v_C(t) = 30\cdot\left(e^{-2500\cdot t} - e^{-3750\cdot t}\right)\cdot u(t)$$

$$v_O(t) = \frac{5}{8}\cdot v_C(t) = 18.75\cdot\left[\left(e^{-2500\cdot t} - e^{-3750\cdot t}\right)\cdot u(t)\right]$$

9-36 (a) See Problem 9-35 above for circuit diagram and development of the circuit diff. eq.

The ckt diff eq is: $R_T \cdot C \cdot \frac{d}{dt} v_C + v_C = v_T$ or $\frac{1}{3750} \cdot \frac{d}{dt} v_C + v_C = 6.67 \cdot \sin(1000 \cdot t) \cdot u(t)$

(b) $L\left(\frac{1}{3750} \cdot \frac{d}{dt} v_C + v_C\right) = L(6.67 \cdot \sin(1000 \cdot t) \cdot u(t))$ $\qquad \frac{s \cdot V_C(s)}{3750} + V_C(s) = \frac{6.67 \cdot 1000}{s^2 + 1000^2}$

Solve for $V_C(s)$ $\qquad V_C(s) = \frac{25 \cdot 10^6}{\left(s^2 + 1000^2\right) \cdot (s + 3750)} = \frac{400}{241 \cdot (s + 3750)} - \frac{400}{241} \cdot \frac{(-3750 + s)}{\left(s^2 + 1000^2\right)}$

$$V_C(s) = \frac{400}{241 \cdot (s + 3750)} - \frac{400}{241} \cdot \frac{s}{s^2 + 1000^2} + \frac{400 \cdot 3750}{241 \cdot 1000} \cdot \frac{1000}{s^2 + 1000^2} \qquad \frac{400 \cdot 3750}{241 \cdot 1000} = \frac{1500}{241}$$

$$v_C(t) = \frac{400}{241} \cdot \exp(-3750 \cdot t) + \frac{1500}{241} \cdot \sin(1000 \cdot t) - \frac{400}{241} \cdot \cos(1000 \cdot t) \quad v_O(t) = v_C(t) \cdot \frac{5}{8} \qquad \frac{1500}{241} \cdot \frac{5}{8} = 3.89$$

$$v_O(t) = 1.037 \cdot \exp(-3750 \cdot t) + 3.89 \cdot \sin(1000 \cdot t) - 1.037 \cdot \cos(1000 \cdot t) \qquad \frac{400}{241} \cdot \frac{5}{8} = 1.037$$

9-37 y(0) = 10 and y'(0) = 0 $\quad L\left[\frac{d^2}{dt^2}\left(y + 20 \cdot \frac{d}{dt} y + 75 \cdot y\right)\right] = s^2 \cdot Y(s) - 10 \cdot s + 20 \cdot (s \cdot Y(s) - 10) + 75 \cdot Y(s) = 0$

$$Y(s) = \frac{10 \cdot (s + 20)}{s^2 + 20 \cdot s + 75} = \frac{10 \cdot (s + 20)}{(s + 5) \cdot (s + 15)} = \frac{-5}{(s + 15)} + \frac{15}{(s + 5)} \qquad y(t) = (-5 \cdot \exp(-15 \cdot t) + 15 \cdot \exp(-5 \cdot t)) \cdot u(t)$$

9-38 y(0) = 0 and y'(0) = 0 $\qquad L\left[\frac{d^2}{dt^2}\left(y + 7 \cdot \frac{d}{dt} y + 10 \cdot y\right)\right] = L(30 \cdot u(t))$

$$s^2 \cdot Y(s) + 7 \cdot s \cdot Y(s) + 10 \cdot Y(s) = \frac{30}{s} \quad \text{--->} \quad Y(s) = \frac{30}{s \cdot \left(s^2 + 7 \cdot s + 10\right)}$$

$$Y(s) = \frac{30}{s \cdot (s + 2) \cdot (s + 5)} = \frac{3}{s} - \frac{5}{(s + 2)} + \frac{2}{(s + 5)} \qquad \text{--->} \qquad y(t) = \left(3 - 5 \cdot e^{-2 \cdot t} + 2 \cdot e^{-5 \cdot t}\right) \cdot u(t)$$

9-39 (a) For t < 0 the capacitor acts like an open ckt and the inductor like a short ckt. Hence:

$$v_C(0) = 0 \qquad i_L(0) = \frac{V_A}{2 \cdot R} = \frac{10}{2000} = 5 \cdot 10^{-3} \qquad \frac{d}{dt} i_L(0) = \frac{1}{L} \cdot v_C(0) = 0$$

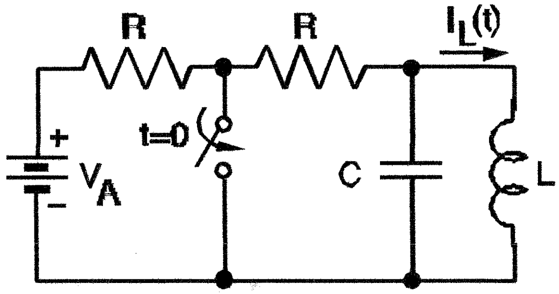
For t > 0 the Norton ckt seen by the L & C in parallel is $i_N = 0$ and $R_N = R$. The ckt diff eq. is

$$L \cdot C \cdot \frac{d^2}{dt^2} i_L + \frac{L}{R_N} \cdot \frac{d}{dt} i_L + i_L = i_N$$

$R := 1000 \quad R_N := R \quad L := 0.4 \quad C := 0.5 \cdot 10^{-6}$

$L \cdot C = 2 \times 10^{-7} \qquad \frac{L}{R_N} = 4 \times 10^{-4}$

$$2 \cdot 10^{-7} \cdot \frac{d^2}{dt^2} i_L + 4 \cdot 10^{-4} \cdot \frac{d}{dt} i_L + i_L = 0$$



9-39 Continued (b)

$$\mathcal{L}\left(2\cdot 10^{-7}\cdot\frac{d^2}{dt^2}i_L + 4\cdot 10^{-4}\cdot\frac{d}{dt}i_L + i_L\right) = 2\cdot 10^{-7}\cdot\left(s^2\cdot I_L(s) - 5\cdot 10^{-3}\cdot s\right) + 4\cdot 10^{-4}\cdot\left(s\cdot I_L(s) - 5\cdot 10^{-3}\right) + I_L(s) = ($$

$$\left[I_L(s) = \frac{5\cdot 10^{-3}\cdot(s+2000)}{s^2 + 2000\cdot s + 5\cdot 10^6}\right]\frac{1}{200}\cdot\frac{s+2000}{(s+1000)^2 + 2000^2} = \frac{1}{200}\cdot\left[\frac{s+1000}{(s+1000)^2 + 2000^2} + \frac{(0.5)\cdot 2000}{(s+1000)^2 + 2000^2}\right.$$

$$i_L(t) = 5\cdot 10^{-3}\cdot\left(\exp(-1000\cdot t)\cdot\cos(2000\cdot t) + \frac{1}{2}\cdot\exp(-1000\cdot t)\cdot\sin(2000\cdot t)\right)\cdot u(t)$$

9-40 (a) For t < 0 the capacitor acts like an open ckt and the inductor like a short ckt. Hence:

$$v_C(0) = 0 \qquad i_L(0) = 0 \qquad \frac{d}{dt}i_L(0) = \frac{1}{L}\cdot v_C(0) = 0$$

For t > 0 the Norton ckt seen by the L & C in parallel is $i_N = V_A/2R = 10^{-3}$ and $R_N = 2R$.
The ckt diff eq. is

$$L\cdot C\cdot\frac{d^2}{dt^2}i_L + \frac{L}{R_N}\cdot\frac{d}{dt}i_L + i_L = i_N \qquad L := 10\cdot 10^{-3} \quad C := 16\cdot 10^{-12} \quad R_N := 2\cdot 5000$$

$$L\cdot C = 1.6\times 10^{-13} \qquad \frac{L}{R_N} = 1\times 10^{-6} \qquad 1.6\cdot 10^{-13}\cdot\frac{d^2}{dt^2}i_L + 10^{-6}\cdot\frac{d}{dt}i_L + i_L = 10^{-3}\cdot u(t)$$

(b) $$\mathcal{L}\left(1.6\cdot 10^{-13}\cdot\frac{d^2}{dt^2}i_L + 10^{-6}\cdot\frac{d}{dt}i_L + i_L\right) = 1.6\cdot 10^{-13}\cdot\left(s^2\cdot I_L(s)\right) + 10^{-6}\cdot\left(s\cdot I_L(s)\right) + I_L(s) = \frac{10^{-3}}{s}$$

$$I_L(s) = \frac{6.25\cdot 10^9}{\left[s\cdot\left(s^2 + 6.25\cdot 10^6\cdot s + 6.25\cdot 10^{12}\right)\right]} = \frac{6.25\cdot 10^9}{s\cdot\left[\left(s + 1.25\cdot 10^6\right)\cdot\left(s + 5\cdot 10^6\right)\right]} = \frac{K_0}{s} + \frac{K_1}{s + 1.25\cdot 10^6} + \frac{K_3}{s + 5\cdot 10^6}$$

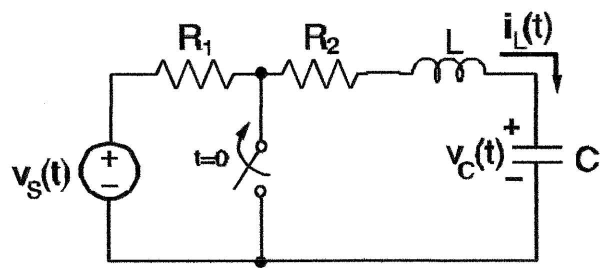
$$I_L(s) = \frac{10^{-3}}{s} - \frac{1.333\cdot 10^{-3}}{(s + 1250000)} + \frac{0.3333\cdot 10^{-3}}{(s + 5000000)}$$

$$i_L(t) = 10^{-3}\cdot(1 - 1.333\cdot\exp(-1250000\cdot t) + 0.3333\cdot\exp(-5000000\cdot t))\cdot u(t)$$

9-41

$$L\cdot\frac{d}{dt}i_L(t) + (R_1 + R_2)\cdot i_L(t) + \frac{1}{C}\cdot\int_0^t i_L(x)\,dx + v_C(0) - v_S(t) = 0$$

$$v_C(0) = 0 \qquad i_L(0) = 0$$



$$L := 0.5 \quad R_1 := 10^3 \quad R_2 := 2\cdot 10^3 \quad C := 250\cdot 10^{-9}$$

$$v_S(t) = (10\cdot\exp(-1000\cdot t))\cdot u(t)$$

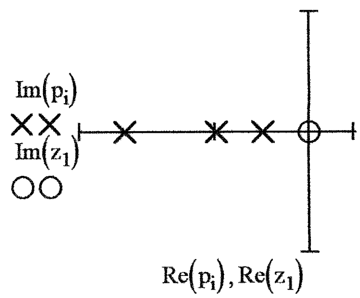
$$0.5\cdot\frac{d}{dt}i_L(t) + 3000\cdot i_L(t) + 4\cdot 10^6\cdot\int_0^t i_L(x)\,dx = 10\cdot\exp(-1000\cdot t)\cdot u(t)$$

9-41 Continued

(a) $$L\left(0.5\cdot\frac{d}{dt}i_L(t)+3000\cdot i_L(t)+4\cdot10^6\cdot\int_0^t i_L(x)\,dx\right)=L(10\cdot\exp(-1000\cdot t\quad\left(0.5\cdot s+3000+\frac{4\cdot10^6}{s}\right)\cdot I_L=\frac{10}{s+1000}$$

$$I_L(s)=\frac{20\cdot s}{(s+1000)\cdot\left(s^2+6000\cdot s+8\cdot10^6\right)}=\frac{20\cdot s}{(s+1000)\cdot(s+2000)\cdot(s+4000)}$$

(b) $i:=1,2..3\quad p_1:=-1000\quad p_2:=-2000\quad p_3:=-4000\quad z_1:=0$



forced response pole at s = -1000

natural response poles at s = -2000 and s = -4000

9-42

$$L\cdot C\cdot\frac{d^2}{dt^2}v_C(t)+(R_1+R_2)\cdot C\cdot\frac{d}{dt}v_C(t)+v_C(t)=v_S(t)$$

$v_C(0)=0\quad i_L(0)=0\quad L:=0.1$

$R_1:=100\quad R_2:=100\quad C:=2\cdot10^{-6}$

$v_S(t)=(10\cdot\sin(2000\cdot t))\cdot u(t)\quad L\cdot C=2\times10^{-7}\quad (R_1+R_2)\cdot C=4\times10^{-4}$

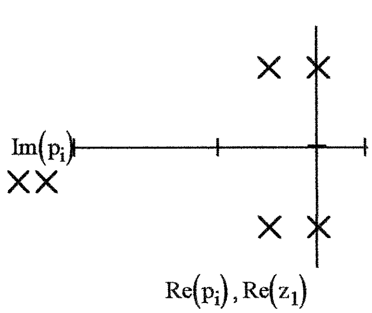
$$2\cdot10^{-7}\cdot\frac{d^2}{dt^2}v_C(t)+4\cdot10^{-4}\cdot\frac{d}{dt}v_C(t)+v_C(t)=10\cdot\sin(2000\cdot t)$$

(a) $$L\left[\left(2\cdot10^{-7}\cdot\frac{d^2}{dt^2}v_C(t)\right)+\left(4\cdot10^{-4}\cdot\frac{d}{dt}v_C(t)\right)+v_C(t)\right]=L(10\cdot\sin(2000\cdot t))$$

$$\left(2\cdot10^{-7}\cdot s^2+4\cdot10^{-4}\cdot s+1\right)\cdot V_C=\frac{2\cdot10^4}{s^2+2000^2}$$

$$V_C(s)=\frac{10^{11}}{\left[\left(s^2+2000^2\right)\cdot\left(s^2+2000\cdot s+5\cdot10^6\right)\right]}=\frac{10^{11}}{\left(s^2+2000^2\right)\cdot\left[(s+1000)^2+2000^2\right]}$$

(b) $i:=1,2..4\quad p_1:=-j\cdot2000\quad p_2:=j\cdot2000\quad p_3:=-1000+j\cdot2000\quad p_4:=-1000-j\cdot2000$

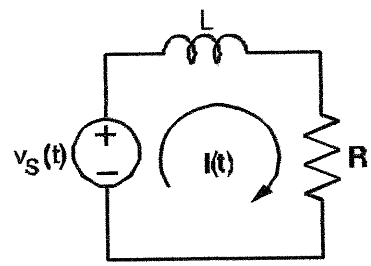


forced response poles at s = ±j2000

natural response poles at s = -1000 ± j2000

9-43 (a) $L \cdot \frac{d}{dt} i(t) + R \cdot i(t) = v_S(t)$ ---> $(L \cdot s + R) \cdot I(s) = V_S(s)$ ---> $I(s) = \frac{V_S(s)}{L \cdot s + R}$

(b) $R := 1000$ $L := 0.5$ $v_S(t) = 10 \cdot e^{-1000 \cdot t} \cdot u(t)$ ---> $\frac{V_S(s)}{L} = \frac{20}{s + 1000}$



$$I(s) = \frac{20}{(s + 1000) \cdot (s + 2000)}$$

Forced pole at s = - 1000 and natural poles at s = -2000

(c) If $I_L(s) = \left(\frac{V_S(s)}{L} \cdot \frac{1}{s + \frac{R}{L}} \right) = \frac{K}{(s + 1000) \cdot \left(s^2 + 1000^2\right)}$ $\frac{R}{L} = 1000$ let $L := 1$ then $R = 1000$

Let $V_S(s) = \frac{K}{s^2 + 1000^2} = \frac{K}{1000} \cdot \frac{1000}{s^2 + 1000^2}$ then $v_S(t) = \frac{K}{1000} \cdot \sin(1000 \cdot t) \cdot u(t)$

9-44 $C \cdot \frac{d}{dt} v_C(t) + \frac{v_C(t)}{R} + \frac{1}{L} \cdot \int_0^t v_C(x)\,dx + i_L(0) = 0$ $v_C(0) = 0$ $i_L(0) = I_0.$ $I_0 := 10 \cdot 10^{-3}$

(a) $\left(C \cdot s + \frac{1}{R} + \frac{1}{L \cdot s} \right) \cdot V_C + \frac{I_0}{s} = 0$ $R := 5000$ $C := 10 \cdot 10^{-9}$ $L := 0.1$

$$V_C(s) = \frac{\frac{-I_0}{C}}{s^2 + \frac{1}{R \cdot C} \cdot s + \frac{1}{L \cdot C}}$$ $\frac{I_0}{C} = 1 \times 10^6$ $\frac{1}{R \cdot C} = 2 \times 10^4$ $\frac{1}{L \cdot C} = 1 \times 10^9$

(b) $$V_C(s) = \frac{-10^6}{s^2 + 2 \cdot 10^4 \cdot s + 10^9} = \frac{-10^6 \cdot s}{\left(s + 10^4\right)^2 + \left(3 \cdot 10^4\right)^2} = \frac{-10^6}{3 \cdot 10^4} \cdot \left[\frac{3 \cdot 10^4}{\left(s + 10^4\right)^2 + \left(3 \cdot 10^4\right)^2} \right]$$

$$v_C(t) = \frac{-100}{3} \cdot (\exp(-10000 \cdot t) \cdot \sin(30000 \cdot t)) \cdot u(t) = -33.333 \cdot (\exp(-10000 \cdot t) \cdot \sin(30000 \cdot t)) \cdot u(t)$$

9-45 (a) $F_1(s) = \frac{16}{(s + 2) \cdot (s + 4) \cdot (s + 6)}$ $IV = \lim_{s \to \infty} s \cdot F_1(s) = 0$ $FV = \lim_{s \to 0} s \cdot F_1(s) = 0$

(b) $F_2(s) = \frac{s^2 + 4 \cdot s + 16}{\left(s^2 + 4^2\right) \cdot s}$ $IV = \lim_{s \to \infty} s \cdot F_2(s) = 1$ Final value thm does not apply since $F_2(s)$ has j-axis poles at s = $\pm$j4

9-46 (a) $F_1(s) = \frac{(s + 2) \cdot (s + 6)}{s \cdot (s + 4) \cdot (s + 8)}$ $IV = \lim_{s \to \infty} s \cdot F_1(s) = 1$ $FV = \lim_{s \to 0} s \cdot F_1(s) = \frac{3}{8} = 0.375$

(b) $F_2(s) = \frac{\left(s^2 + 45\right) \cdot \left(s^2 + 80\right)}{s \cdot \left(s^2 + 20\right) \cdot \left(s^2 + 60\right)}$ $IV = \lim_{s \to \infty} s \cdot F_1(s) = 1$ Final value thm does not applv since $F_2(s)$ has j-axis poles at s = $\pm j \cdot \sqrt{20}$ and s = $\pm j \cdot \sqrt{60}$

9-47 (a) $F_1(s) = \dfrac{2\cdot(s+2)\cdot(s+3)}{(s+2)\cdot(s+6)\cdot(s+12)}$ $IV = \lim_{s\to\infty} s\cdot F_1(s) = 2$ $FV = \lim_{s\to 0} s\cdot F_1(s) = 0$

(b) $F_2(s) = \dfrac{10\cdot\left(s^2+10\cdot s+40\right)}{s\cdot(s-10)\cdot(s+10)}$ $IV = \lim_{s\to\infty} s\cdot F_2(s) = 10$ **Final value thm does not apply since $F_2(s)$ has a RHP at s = +10**

9-48 (a) $F_1(s) = \dfrac{s\cdot(s+5)}{(s+3)^2}$ **Initial value thm does not apply since $F_1(s)$ is not a proper rational fucntion** $FV = \lim_{s\to 0} s\cdot F_1(s) = 0$

$F_2(s) = \dfrac{10\cdot\left(s^2+10\cdot s-20\right)}{s\cdot(s-j\cdot 10)\cdot(s+j\cdot 10)}$ $IV = \lim_{s\to\infty} s\cdot F_2(s) = 10$ **Final value thm does not apply since $F_2(s)$ has j-axis poles at s = $\pm$j10**

9-49(a) $F_1(s) = \dfrac{4\cdot(s+4)}{s\cdot\left(s^2+4\cdot s+8\right)}$ $IV = \lim_{s\to\infty} s\cdot F_1(s) = 0$ $FV = \lim_{s\to 0} s\cdot F_1(s) = 2$

(b) $F_2(s) = \dfrac{(s-1)^2}{(s+4)\cdot\left(s^2+2\cdot s+17\right)}$ $IV = \lim_{s\to\infty} s\cdot F_2(s) = 1$ $FV = \lim_{s\to 0} s\cdot F_2(s) = 0$

9-50(a) $F_1(s) := \dfrac{(s+50)^2}{(s+10)\cdot(s+100)}$ **Initial value thm does not apply since $F_1(s)$ is not a proper rational fucntion** $FV = \lim_{s\to 0} s\cdot F_1(s) = 0$

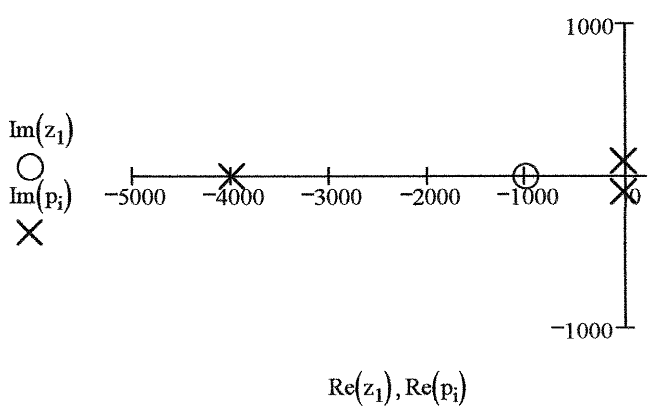
(b) $F_2(s) := \dfrac{s+1}{\left(s^2-2\cdot s+1\right)}$ $IV = \lim_{s\to\infty} s\cdot F_2(s) = 1$ **Final value thm does not apply since $F_2(s)$ has RHPs at s = 1**

9-51(a) $F(s) = \dfrac{10^6\cdot(s+1000)}{(s+4000)\cdot\left[(s+25)^2+100^2\right]} = \dfrac{-189.746}{(s+4000)} + \dfrac{(94.873-1229\cdot j)}{(s+25-100\cdot j)} + \dfrac{(94.873+1229\cdot j)}{(s+25+100j)}$

$A := 2\cdot|94.873-1229\cdot j|$ $A = 2.465\times 10^3$ $\phi := \arg(94.873-1229\cdot j)$ $\phi = -1.494$

$f(t) := -189.746\cdot e^{-4000\cdot t} + A\cdot e^{-25\cdot t}\cdot\cos(100\cdot t+\phi)$ $T_0 := \dfrac{2\cdot\pi}{100}$ $t := 0, \dfrac{T_0}{50} .. 2\cdot T_0$

(b) $z_1 := -1000$ $p_1 := -4000$ $p_2 := -25+100\cdot j$ $p_3 := \overline{p_2}$ $i := 1, 2 .. 3$

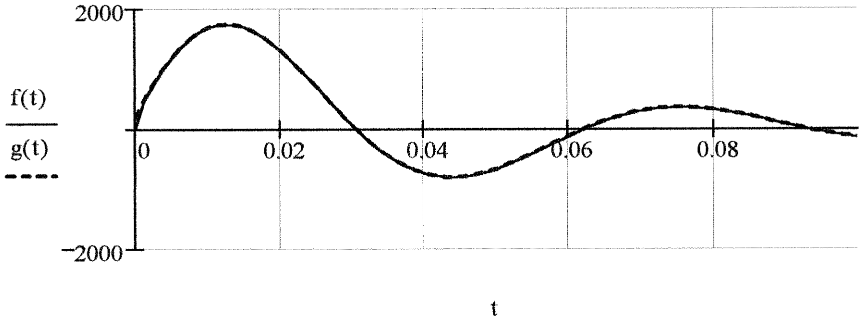


The complex poles are dominant

9-51 Continued

(c) $g(t) := A \cdot e^{-25 \cdot t} \cdot \cos(100 \cdot t + \phi) \cdot u(t)$ **<----Approx f(t)**

(d)



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> 0.0012

9-52 Let FV = f(∞) and IV = f(0) then:

t-domain to s-domain

$$f(t) = FV + (IV - FV) \cdot \exp\left(\frac{-t}{T_C}\right)$$

$$F(s) = \frac{FV}{s} + \frac{(IV - FV)}{s + \frac{1}{T_C}}$$

$$F(s) = \frac{FV + s \cdot T_C \cdot IV}{s \cdot (T_C \cdot s + 1)}$$

Comparing F(s) and G(s)

$$K = IV \quad \gamma = \frac{FV}{IV \cdot T_C} \quad \alpha = \frac{1}{T_C}$$

s-domain to t-domain

$$G(s) = \frac{K \cdot (s + \gamma)}{s \cdot (s + \alpha)}$$

$$G(s) = \frac{K}{\alpha} \cdot \frac{\gamma}{s} - \frac{K}{\alpha} \cdot \frac{(\gamma - \alpha)}{s + \alpha}$$

$$g(t) = \frac{K \cdot \gamma}{\alpha} - \frac{K \cdot \gamma}{\alpha} \cdot \exp(-\alpha \cdot t) + K \cdot \exp(-\alpha \cdot t)$$

Comparing g(t) and f(t)

$$FV = \frac{K \cdot \gamma}{\alpha} \quad IV = K \quad T_C = \frac{1}{\alpha}$$

9-53

$$L^{-1}\left(\frac{a + j \cdot b}{s + \alpha - j \cdot \beta} + \frac{a - j \cdot b}{s + \alpha + j \cdot \beta}\right) = L^{-1}\left[\frac{2 \cdot (a \cdot s + a \cdot \alpha - b \cdot \beta)}{(s + \alpha)^2 + \beta^2}\right] = L^{-1}\left[\frac{2 \cdot a \cdot (s + \alpha)}{(s + \alpha)^2 + \beta^2} - \frac{2 \cdot b \cdot \beta}{(s + \alpha)^2 + \beta^2}\right]$$

$$L^{-1}\left[\frac{2 \cdot a \cdot (s + \alpha)}{(s + \alpha)^2 + \beta^2}\right] = 2 \cdot a \cdot \exp(-\alpha \cdot t) \cdot \cos(\beta \cdot t) \qquad L^{-1}\left[\frac{2 \cdot b \cdot \beta}{(s + \alpha)^2 + \beta^2}\right] = 2 \cdot b \cdot \exp(-\alpha \cdot t) \cdot \sin(\beta \cdot t)$$

$$L^{-1}\left(\frac{a + j \cdot b}{s + \alpha - j \cdot \beta} + \frac{a - j \cdot b}{s + \alpha + j \cdot \beta}\right) = 2 \cdot \exp(-\alpha \cdot t) \cdot (a \cdot \cos(\beta \cdot t) - b \cdot \sin(\beta \cdot t))\text{QED}$$

9-54

$$\frac{d}{dt} v_C(t) = \frac{1}{C} \cdot i_L(t) \qquad v_C(0) = V_0 \qquad \frac{d}{dt} i_L(t) = \frac{-1}{L} \cdot v_C(t) - \frac{R}{L} \cdot i_L(t) \qquad i_L(0) = I_0$$

(a)

$$s \cdot V_C(s) - V_0 = \frac{1}{C} \cdot I_L(s) \qquad s \cdot I_L(s) - I_0 = \frac{-1}{L} \cdot V_C(s) - \frac{R}{L} \cdot I_L(s)$$

$$V_C(s) = \frac{V_0}{s} + \frac{I_L(s)}{C \cdot s} \quad \text{--->} \quad s \cdot I_L(s) - I_0 = \frac{-1}{L} \cdot \left(\frac{V_0}{s} + \frac{I_L(s)}{C \cdot s}\right) - \frac{R}{L} \cdot I_L(s)$$

Solving for $I_L(s)$ yields

$$I_L(s) = \frac{(I_0 \cdot L \cdot C \cdot s - V_0 \cdot C)}{(L \cdot C \cdot s^2 + R \cdot C \cdot s + 1)}$$

Hence $V_C(s)$ is

$$V_C(s) = \frac{V_0}{s} + \frac{1}{C \cdot s} \cdot \left[\frac{(I_0 \cdot L \cdot C \cdot s - V_0 \cdot C)}{(L \cdot C \cdot s^2 + R \cdot C \cdot s + 1)}\right]$$

9-54 Continued **The transforms of the state variables are**

$$V_C(s) = \frac{V_0 \cdot L \cdot C \cdot s + V_0 \cdot R \cdot C + I_0 \cdot L}{L \cdot C \cdot s^2 + R \cdot C \cdot s + 1} \qquad I_L(s) = \frac{L \cdot C \cdot s \cdot I_0 - V_0 \cdot C}{L \cdot C \cdot s^2 + R \cdot C \cdot s + 1}$$

(b) $R := 3000 \quad L := 0.1 \quad C := 50 \cdot 10^{-9} \quad I_0 := 5 \cdot 10^{-3} \quad V_0 := -5 \qquad L \cdot C = 5 \times 10^{-9} \quad R \cdot C = 1.5 \times 10^{-4}$

$V_0 \cdot R \cdot C + I_0 \cdot L = -2.5 \times 10^{-4} \quad V_0 \cdot L \cdot C = -2.5 \times 10^{-8} \quad L \cdot C \cdot I_0 = 2.5 \times 10^{-11} \quad -V_0 \cdot C = 2.5 \times 10^{-7}$

$$V_C(s) = \frac{-2.5 \cdot 10^{-8} \cdot s - 2.5 \cdot 10^{-4}}{5 \cdot 10^{-9} \cdot s^2 + 1.5 \cdot 10^{-4} \cdot s + 1} = \frac{-5 \cdot (s + 10000)}{(s + 10000) \cdot (s + 20000)} = \frac{-5}{s + 20000} \quad v_C(t) = -5 \cdot \exp(-20000 \cdot t) \cdot u(t)$$

$$I_L(s) = \frac{2.5 \cdot 10^{-7} + 2.5 \cdot 10^{-11} \cdot s}{5 \cdot 10^{-9} \cdot s^2 + 1.5 \cdot 10^{-4} \cdot s + 1} = \frac{5 \cdot 10^{-3} \cdot (s + 10000)}{(s + 10000) \cdot (s + 20000)} = \frac{5 \cdot 10^{-3}}{s + 20000} \, i_L(t) = 5 \cdot 10^{-3} \cdot \exp(-20000 \cdot t) \cdot u(t)$$

9-55

If $L(f(t)) = F(s)$ then $L(t \cdot f(t)) = -\frac{d}{ds} F(s)$

If $L(\exp(-\alpha \cdot t)) = \frac{1}{s + \alpha}$ then $L(t \cdot \exp(-\alpha \cdot t)) = -\frac{d}{ds}\left(\frac{1}{s + \alpha}\right) = \frac{1}{(s + \alpha)^2}$

If $L(\sin(\beta \cdot t)) = \frac{\beta}{s^2 + \beta^2}$ then $L(t \cdot \sin(\beta \cdot t)) = -\frac{d}{ds}\left(\frac{\beta}{s^2 + \beta^2}\right) = \frac{2 \cdot \beta \cdot s}{\left(s^2 + \beta^2\right)^2}$

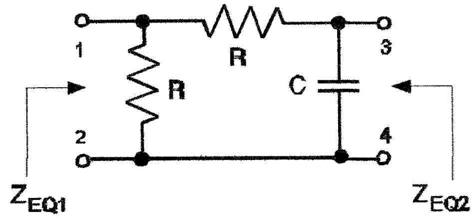
If $L(\cos(\beta \cdot t)) = \frac{s}{s^2 + \beta^2}$ then $L(t \cdot \cos(\beta \cdot t)) = -\frac{d}{ds}\left(\frac{s}{s^2 + \beta^2}\right) = \frac{\left(s^2 - \beta^2\right)}{\left(s^2 + \beta^2\right)^2}$

Chapter 10, Both Versions

10-1

$$Z_{EQ1} = \frac{1}{\frac{1}{R} + \frac{1}{R + \frac{1}{C\cdot s}}} = R\cdot\frac{(R\cdot C\cdot s + 1)}{2\cdot R\cdot C\cdot s + 1} = \frac{R}{2}\cdot\frac{s + \frac{1}{R\cdot C}}{s + \frac{1}{2\cdot R\cdot C}}$$

Pole at s = -1/2RC; zero at s = -1/RC



$$Z_{EQ2} = \frac{1}{C\cdot s + \frac{1}{2\cdot R}} = \frac{2\cdot R}{2\cdot R\cdot C\cdot s + 1} = \frac{1}{C}\cdot\frac{1}{s + \frac{1}{2\cdot R\cdot C}}$$

Pole at s = -1/2RC; zero at infinity

10-2

$$Z_{EQ} = \frac{1}{\frac{1}{2000 + \left(0.1\cdot 10^{-6}\cdot s\right)^{-1}} + \frac{1}{0.5\cdot s}} = \frac{2000\cdot[s\cdot(s + 5000)]}{s^2 + 4000\cdot s + 20000000} = \frac{2000\cdot s\cdot(s + 5000)}{(s + 2000)^2 + 4000^2}$$

Complex poles as s = -2000 ± j4000, zeros at s = 0 and s = -5000

10-3

$$Z_{EQ} = \frac{1}{\frac{1}{2000} + \frac{1}{0.1\cdot s + \frac{1}{10^{-7}\cdot s}}} = \frac{2000\cdot\left(s^2 + 10^8\right)}{\left(s^2 + 20000s + 10^8\right)} = \frac{2000\left(s + j\cdot 10^4\right)\cdot\left(s - j\cdot 10^4\right)}{\left(s + 10^4\right)^2}$$

Double real pole at s = -10,000 and imaginary zeros at s = ±j10,000

10-4

$$Z_{EQ}(s) = R + \frac{1}{\frac{1}{R} + \frac{1}{L\cdot s}} = R\cdot\left[\frac{(2\cdot L\cdot s + R)}{(L\cdot s + R)}\right] = 2\cdot R\cdot\left(\frac{s + \frac{R}{2\cdot L}}{s + \frac{R}{L}}\right) = 4000\cdot\frac{s + 10^5}{s + 2\cdot 10^5}$$

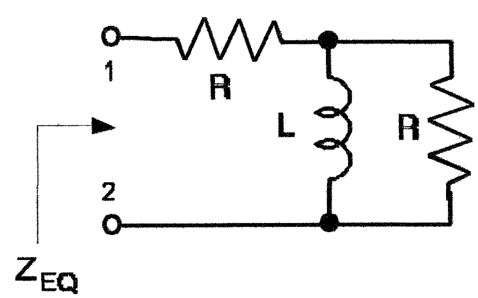
pole at s = -200000 and zero at s = -100000

$$Z_{EQ}(0) = R = 2000$$

at s = 0 the inductor acts like a short circuit

$$Z_{EQ}(\infty) = 2\cdot R = 4000$$

at s = ∞ the inductor acts like a open circuit

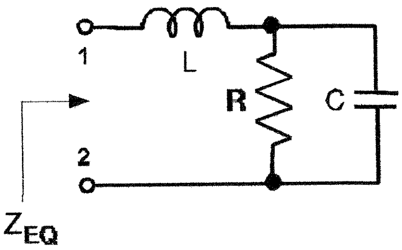


10-5

$$Z_{EQ} = \frac{1}{50\cdot 10^{-9}\cdot s} + \frac{1}{\frac{1}{1000} + \frac{1}{0.1\cdot s}} = 1000..\frac{\left(20000\cdot s + 200000000 + s^2\right)}{[s\cdot(s + 10000)]} = \frac{10^3\cdot\left[\left(s + 10^4\right)^2 + \left(10^4\right)^2\right]}{s\cdot(s + 10000)}$$

Real poles at s = 0 and s = -10000. Complex conjugate zeros at s = -10000 ± j110000

10-6 $Z_{EQ} = L\cdot s + \dfrac{1}{\dfrac{1}{R} + C\cdot s} = \dfrac{L\cdot C\cdot R\cdot s^2 + L\cdot s + R}{R\cdot C\cdot s + 1}$



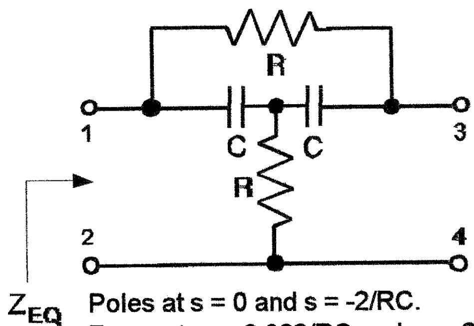
$Z_{EQ} = R$ at s = 0 and $Z_{EQ} = \infty$ at s = ∞.

At s = 0 the inductor acts like a short circuit & the capacitor like an open circuit hence $Z_{EQ} = R$ At s = ∞ the inductor acts like an open circuit, hence $Z_{EQ} = \infty$

Poles at s = -1/RC.

Zeros at s = $[1/(2LRC)][-L \pm (L^2 - 4LR^2C)^{1/2}]$

10-7 $Z_{EQ} = R + \dfrac{1}{C\cdot s + \dfrac{1}{R + \dfrac{1}{C\cdot s}}}$



$$Z_{EQ} = \frac{\left(R^2\cdot C^2\cdot s^2 + 3\cdot R\cdot C\cdot s + 1\right)}{C\cdot s\cdot(R\cdot C\cdot s + 2)}$$

$$Z_{EQ} = \frac{(R\cdot C\cdot s + 0.382)\cdot(R\cdot C\cdot s + 2.618)}{C\cdot s\cdot(R\cdot C\cdot s + 2)}$$

Poles at s = 0 and s = -2/RC.
Zeros at s = -0.382/RC and s = -2.618/RC

10-8 $$Z_{EQ} = \frac{1}{\dfrac{1}{R} + \dfrac{1}{\dfrac{1}{C\cdot s} + \dfrac{1}{C\cdot s + \dfrac{1}{R}}}} = \frac{1}{\dfrac{1}{R} + \dfrac{[C\cdot s\cdot(C\cdot s\cdot R + 1)]}{(2\cdot C\cdot s\cdot R + 1)}} = \frac{R\cdot(2\cdot R\cdot C\cdot s + 1)}{R^2\cdot C^2\cdot s^2 + 3\cdot R\cdot C\cdot s + 1}$$

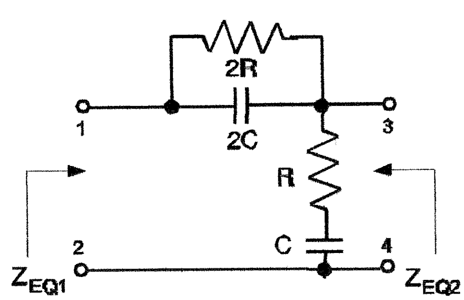
$$Z_{EQ}(s) = R\cdot\frac{(2\cdot R\cdot C\cdot s + 1)}{[(R\cdot C\cdot s + 0.382)\cdot(R\cdot C\cdot s + 2.618)]}$$

zero at s = -1/2RC and simple poles at s = -0.382/RC and s = -2.618/RC

10-9 (a)

$$Z_{EQ1} = R + \frac{1}{C\cdot s} + \frac{1}{2\cdot C\cdot s + \dfrac{1}{2\cdot R}} = \left[\frac{\left(4\cdot C^2\cdot s^2\cdot R^2 + 7\cdot C\cdot s\cdot R + 1\right)}{[C\cdot s\cdot(4\cdot C\cdot s\cdot R + 1)]}\right] = \frac{(R\cdot C\cdot s + 1.593)(R\cdot C\cdot s + 0.157)}{C\cdot s\cdot(R\cdot C\cdot s + 0.25)}$$

Zeros at s = -1.593/RC and s = -0.157/RC,
poles at s = 0 and s = -0.25/RC



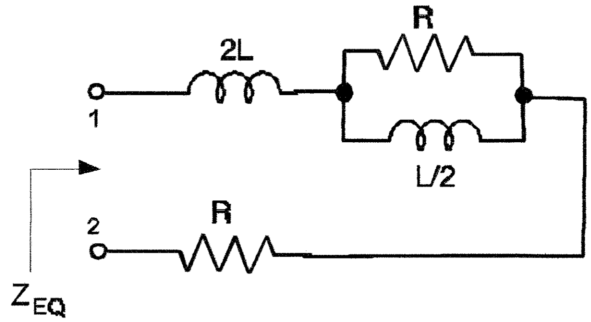
(b) $$Z_{EQ2} = \frac{1}{2\cdot C\cdot s + \dfrac{1}{2\cdot R} + \dfrac{1}{R + \dfrac{1}{C\cdot s}}} = \frac{2\cdot R\cdot(R\cdot C\cdot s + 1)}{4\cdot R^2\cdot C^2\cdot s^2 + 7\cdot R\cdot C\cdot s + 1} = \frac{0.5\cdot R\cdot(R\cdot C\cdot s + 1)}{(R\cdot C\cdot s + 1.593)(R\cdot C\cdot s + 0.157)}$$

Poles at s = -1.593/RC and s = -0.157/RC, zero at s = -1/RC

10-10 $Z_{EQ} = R + 2 \cdot L \cdot s + \dfrac{1}{\dfrac{1}{R} + \dfrac{2}{L \cdot s}}$

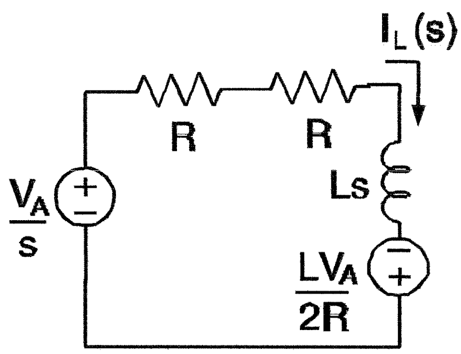
$$Z_{EQ} = 2 \cdot R \cdot \frac{\left(\frac{L}{R} \cdot s\right)^2 + 3 \cdot \left(\frac{L}{R} \cdot s\right) + 1}{\left(\frac{L}{R} \cdot s\right) + 2}$$

$$Z_{EQ} = 2 \cdot R \cdot \frac{\left(\frac{L}{R} \cdot s + 0.382\right) \cdot \left(\frac{L}{R} \cdot s + 2.618\right)}{\left(\frac{L}{R} \cdot s\right) + 2}$$



A pole at s = -5000 requires 2R/L = 5000.
Let L = 100 mH then R = 250 Ω. For these values there are zeros at s = $-0.3822500 = -955$ and s = $-2.6182500 = -6545$

10-11 With the switch open $i_L(0) = \dfrac{V_A}{2 \cdot R}$. With the switch closed the s-domain circuit is



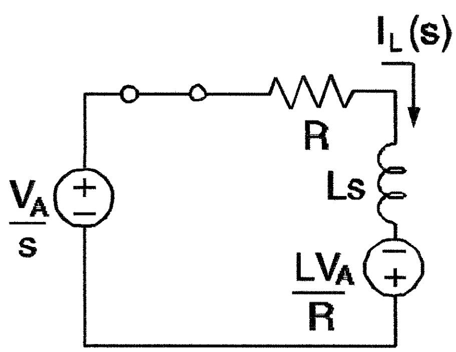
The inductor current is

$$I_L(s) = \frac{\frac{V_A}{s} + L \cdot \frac{V_A}{2 \cdot R}}{L \cdot s + R} = \frac{1}{2} \cdot V_A \cdot \frac{2 \cdot R + L \cdot s}{[s \cdot R \cdot (L \cdot s + R)]}$$

$$I_L(s) = \frac{V_A}{R \cdot s} - \frac{V_A}{2 \cdot R \cdot \left(s + \frac{R}{L}\right)} \quad \text{and}$$

$$i_L(t) = \left(\frac{V_A}{R}\right) \cdot \left(1 - 0.5 \cdot \exp\left(-\frac{R}{L} \cdot t\right)\right) \text{ A for } t > 0$$

10-12 With the switch close $i_L(0) = \dfrac{V_A}{R}$. With the switch open the s-domain circuit is



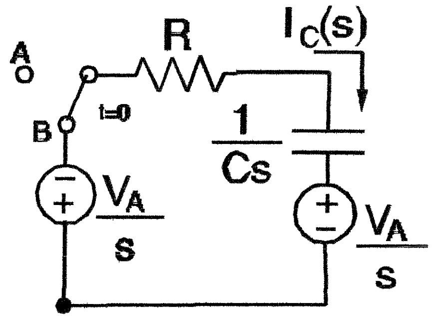
The inductor current is

$$I_L(s) = \frac{\frac{V_A}{s} + \frac{L \cdot V_A}{R}}{2 \cdot R + L \cdot s} = \frac{V_A}{R} \cdot \frac{L \cdot s + R}{s \cdot (L \cdot s + 2 \cdot R)}$$

$$I_L(s) = \frac{V_A}{2 \cdot R \cdot s} + \frac{V_A}{2 \cdot R \cdot \left(s + \frac{2 \cdot R}{L}\right)} \quad \text{and}$$

$$i_L(t) = \left(\frac{V_A}{2 \cdot R}\right) \cdot \left(1 + \exp\left(-\frac{2 \cdot R}{L} \cdot t\right)\right) \text{ A for } t > 0$$

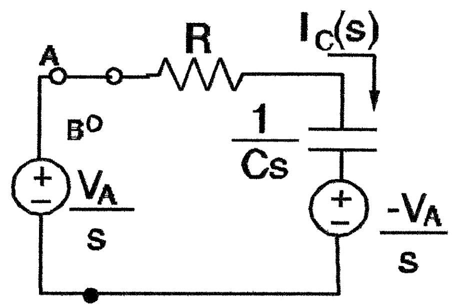
10-13 With the sw in position A $v_C(0) = V_A$
with the switch in position B the s-doman circuit is shown at right



$$I_C(s) = \frac{\frac{-V_A}{s} - \frac{V_A}{s}}{R + \frac{1}{C \cdot s}} = \frac{-2 \cdot V_A}{R} \cdot \frac{1}{s + \frac{1}{RC}}$$

$$i_C(t) = \frac{-2 \cdot V_A}{R} \cdot \exp\left(\frac{-t}{RC}\right) \cdot u(t)$$

10-14 With the sw in position B $v_C(0) = -V_A$ with the switch in position A the s-doman circuit is shown at right



$$V_{Czi}(s) = \frac{R}{R + \frac{1}{C \cdot s}} \cdot \left(\frac{-V_A}{s}\right) = \frac{-R \cdot C \cdot V_A}{R \cdot C \cdot s + 1}$$

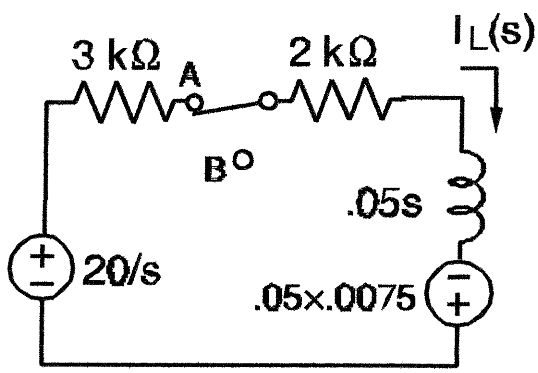
$$V_{Czs}(s) = \left(\frac{\frac{1}{C \cdot s}}{R + \frac{1}{C \cdot s}} \cdot \frac{V_A}{s}\right) = \frac{V_A}{s \cdot (R \cdot C \cdot s + 1)}$$

$$V_C(s) = \frac{V_A}{s \cdot (R \cdot C \cdot s + 1)} + \frac{-R \cdot C \cdot V_A}{R \cdot C \cdot s + 1} = \frac{V_A}{s} - \frac{2 \cdot R \cdot C \cdot V_A}{R \cdot C \cdot s + 1} \quad v_C(t) = V_A \cdot \left(1 - 2 \cdot \exp\left(\frac{-t}{R \cdot C}\right)\right) \cdot u(t) \text{ V}$$

10-15

With the switch closed $i_L(0) = \frac{15}{2000} = 0.0075$.

With the sw open the s-domain circuit is
The inductor current is



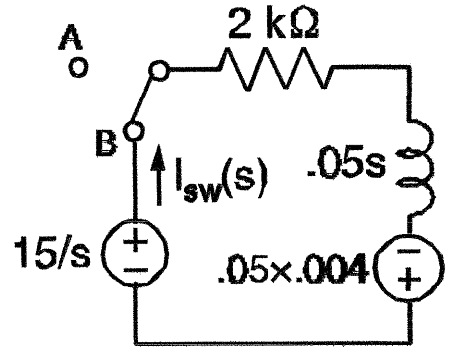
$$I_L(s) = \frac{\frac{20}{s} + 0.05 \cdot \frac{15}{2000}}{(0.05 \cdot s + 5000)} = 2.5 \cdot 10^{-3} \cdot \frac{(3 \cdot s + 16000)}{[s \cdot (s + 100000)]}$$

$$I_L(s) = \frac{\left(4 \cdot 10^{-3}\right)}{s} + \frac{\left(3.5 \cdot 10^{-3}\right)}{(s + 100000)}$$

$$i_L(t) = 10^{-3} \cdot (4 + 3.5 \cdot \exp(-100000 \cdot t)) \cdot u(t) \text{ A}$$

10-16 With the switch open $i_L(0) = \dfrac{20}{5000} = 0.004$.

With the sw closed the s-domain circuit is
The inductor current is



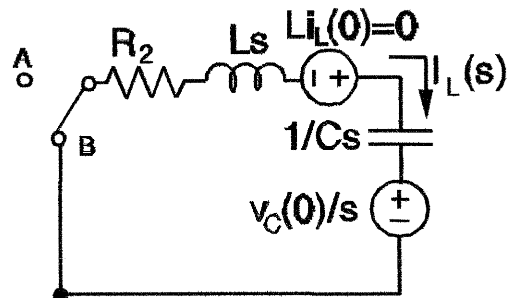
$$I_L(s) = \frac{\frac{15}{s} + 0.05\cdot\frac{20}{5000}}{0.05\cdot s + 2000} = \frac{0.004\cdot(s + 75000)}{s\cdot(s + 40000)}$$

$$I_L(s) = \frac{3}{400\cdot s} - \frac{7}{2000\cdot(s + 40000)} \quad \text{and}$$

$$i_L(t) = 7.5\cdot 10^{-3} - 3.5\cdot 10^{-3}\cdot \exp(-40000\cdot t) \text{ A}$$

10-17 With the sw in Pos. A the IC's are
$i_L(0) = 0$ and $v_C(0) = V_A$

With the sw in Pos B the s-domain ckt is
(a) The inductor current is



$$I_L(s) = \frac{\frac{V_A}{s}}{R_2 + L\cdot s + \frac{1}{C\cdot s}} = \frac{-V_A\cdot C}{L\cdot C\cdot s^2 + R_2\cdot C\cdot s + 1}$$

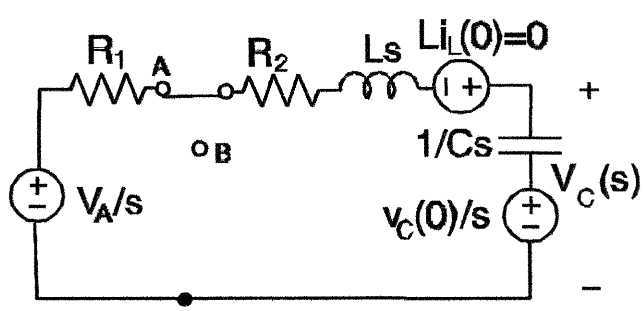
(b) Using the numerical values

$$I_L(s) = \frac{-60\cdot 10^{-6}}{10^{-6}\cdot s^2 + 2\cdot 10^{-3}\cdot s + 1} = \frac{-60}{s^2 + 2000\cdot s + 10^6} = \frac{-60}{(s + 1000)^2}$$

hence $i_L(t) = -60\cdot t\cdot \exp(-1000\cdot t)\cdot u(t)$ A

10-18 With the sw in Pos.B the IC's are
$i_L(0) = 0$ and $v_C(0) = 0$

With the sw in Pos B the s-domain ckt is
(a) The capacitor voltage is



$$V_C(s) = \left(\frac{\frac{1}{C\cdot s}}{R_1 + R_2 + L\cdot s + \frac{1}{C\cdot s}}\right)\cdot\left(\frac{V_A}{s}\right)$$

$$V_C(s) = \frac{V_A}{\left[L\cdot C\cdot s^2 + (R_1 + R_2)\cdot C\cdot s + 1\right]\cdot s}$$

(b) Using numerical values

$$V_C(s) = \frac{15}{\left(5\cdot 10^{-7}\cdot s^2 + 10^{-3}\cdot s + 1\right)\cdot s} = \left[\frac{15}{s} - \frac{15\cdot(s + 1000)}{(s + 1000)^2\cdot s + 1000^2} - \frac{15\cdot 1000}{(s + 1000)^2\cdot s + 1000^2}\right]$$

hence $v_C(t) = 15 - 15\cdot \exp(-1000\cdot t)\cdot \cos(1000\cdot t) - 15\cdot \exp(-1000\cdot t)\cdot \sin(1000\cdot t)$ V $t \geq 0$

10-19 With the sw in Pos. A the IC's are
$i_L(0) = 0$ and $v_C(0) = V_A$

With the sw in Pos B the s-domain ckt is

(a)

$$I_L(s) = \left(\frac{\frac{V_A}{s} - \frac{V_B}{s}}{R_2 + L\cdot s + \frac{1}{C\cdot s}}\right) = \frac{(V_A - V_B)\cdot C}{L\cdot C\cdot s^2 + R_2\cdot C\cdot s + 1}$$

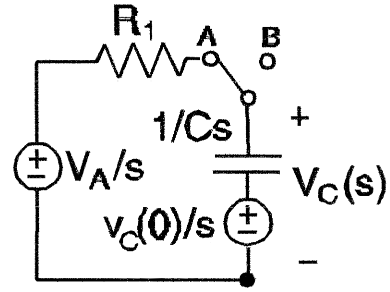
(b) $$I_L(s) = \frac{1.25\cdot 10^{-6}}{10^{-8}\cdot s^2 + 2.5\cdot 10^{-4}\cdot s + 1} = \frac{125}{(s+5000)\cdot(s+20000)}\ \frac{1}{120}\cdot\left(\frac{1}{s+5000} - \frac{1}{s+20000}\right)$$

hence for t > 0: $i_L(t) = 8.333\cdot(\exp(-5000\cdot t) - \exp(-20000\cdot t))$ mA

10-20 With the sw in Pos. B the IC's are $i_L(0) = 0$ and $v_C(0) = V_B$

(a) With the sw in Pos A the s-domain ckt is

$$V_C(s) = \left(\frac{\frac{1}{C\cdot s}}{R_1 + \frac{1}{C\cdot s}}\right)\cdot\frac{(V_A - V_B)}{s} + \frac{V_B}{s} \quad V_C(s) = \frac{V_A - V_B}{s\cdot(R_1\cdot C\cdot s + 1)} + \frac{V_B}{s}$$



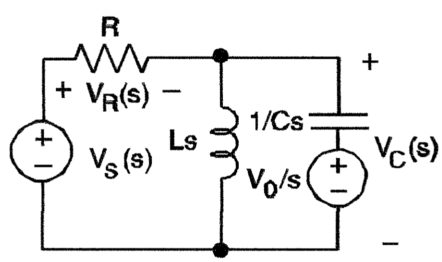
(b) Using the numerical values

$$V_C(s) = \frac{5}{s\cdot(5\cdot 10^{-5}\cdot s + 1)} + \frac{5}{s} = \frac{10}{s} - \frac{5}{(s+20000)}$$

hence for t > 0 $v_C(t) = (10 - 5\cdot\exp(-20000\cdot t))\cdot u(t)$ V

10-21

By voltage division



$$Z_{RL} = \frac{1}{\frac{1}{L\cdot s} + \frac{1}{R}}$$

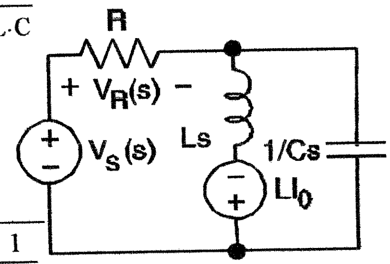
$$Z_C = \frac{1}{C\cdot s}$$

$$V_{Czs}(s) = \frac{\frac{1}{C\cdot s + \frac{1}{L\cdot s}}\cdot V_S(s)}{\frac{1}{C\cdot s + \frac{1}{L\cdot s}} + R} = \frac{L\cdot s\cdot V_S(s)}{R\cdot L\cdot C\cdot s^2 + L\cdot s + R} = \frac{\frac{s}{R\cdot C}\cdot V_S(s)}{s^2 + \frac{s}{R\cdot C} + \frac{1}{L\cdot C}}$$

$$V_{Czi}(s) = \frac{\left(\frac{1}{\frac{1}{L\cdot s} + \frac{1}{R}}\right)\cdot\frac{V_0}{s}}{\frac{1}{C\cdot s} + \left(\frac{1}{\frac{1}{L\cdot s} + \frac{1}{R}}\right)} = \frac{V_0\left(\frac{R\cdot Ls}{R + L\cdot s}\right)}{s\left(\frac{1}{C\cdot s} + \frac{R\cdot L\cdot s}{R + L\cdot s}\right)} = \frac{R\cdot L\cdot C\cdot s\cdot V_0}{R\cdot L\cdot C\cdot s^2 + L\cdot s + R}$$

10-22 By voltage division

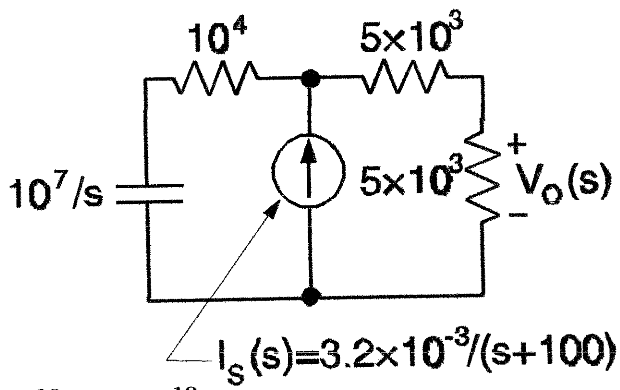
$$V_{Rzs}(s) = \frac{R\cdot V_S(s)}{\dfrac{1}{C\cdot s + \dfrac{1}{L\cdot s}} + R} = \frac{R\cdot V_S(s)\cdot\left(L\cdot C\cdot s^2 + 1\right)}{R\cdot L\cdot C\cdot s^2 + L\cdot s + R} = \frac{\left(s^2 + \dfrac{1}{L\cdot C}\right)\cdot V_S(s)}{s^2 + \dfrac{s}{R\cdot C} + \dfrac{1}{L\cdot C}}$$



$$V_{Rzi}(s) = \frac{\left(\dfrac{1}{C\cdot s + \dfrac{1}{R}}\right)\cdot L\cdot I_0}{L\cdot s + \left(\dfrac{1}{C\cdot s + \dfrac{1}{R}}\right)} = \frac{R\cdot L\cdot I_0}{R\cdot L\cdot C\cdot s^2 + L\cdot s + R} = \frac{\dfrac{I_0}{C}}{s^2 + \dfrac{s}{R\cdot C} + \dfrac{1}{L\cdot C}}$$

10-23 By current division the current through 5 kΩ resistor is

$$I_O(s) = \left(\frac{10000 + \dfrac{10^7}{s}}{10000 + \dfrac{10^7}{s} + 10000}\right)\cdot\frac{3.2\cdot 10^{-3}}{s + 100}$$



$$I_O(s) = \frac{1.6\cdot 10^{-3}\cdot(s + 1000)}{(s + 500)\cdot(s + 100)}$$

$$V_O(s) = 5000\cdot I_O(s) = \frac{8\cdot(s + 1000)}{(s + 500)\cdot(s + 100)} = \frac{-10}{(s + 500)} + \frac{18}{(s + 100)}$$

<-natural-><-forced->

Natural pole at s=-500 and forced pole at s=-100

Hence for t > 0 $v_O(t) = (-10\cdot\exp(-500\cdot t) + 18\cdot\exp(-100\cdot t))$ V

10-24 By current division the current through 5 kΩ resistor is

$$I_O(s) = \left(\frac{10000 + \dfrac{10^7}{s}}{10000 + \dfrac{10^7}{s} + 10000}\right)\cdot\frac{3.2\cdot 10^{-3}}{s + 1000}$$

$$I_O(s) = \frac{1.6\cdot 10^{-3}}{(s + 500)} \qquad V_O(s) = 5000\cdot I_O(s) = \frac{8}{(s + 500)}$$

forced pole canceled--no forced pole <-natural->

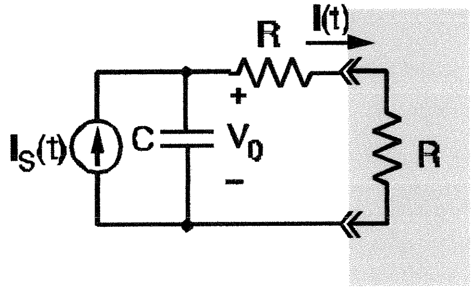
Natural pole at s=-500

Hence for t > 0 $v_O(t) = (8\cdot\exp(-500\cdot t))$ V

10-25 **(a)** The Thevenin equivalent is

$$Z_T = R + \frac{1}{C \cdot s} = \frac{R \cdot C \cdot s + 1}{C \cdot s} \text{ and}$$

$$V_T = \frac{1}{C \cdot s} \cdot I_S(s) + \frac{V_0}{s}$$



(b) For $i_S(t) = I_A \cdot u(t)$

$$I(s) = \frac{V_T}{R + Z_T} = \frac{\frac{1}{C \cdot s} \cdot \frac{I_A}{s} + \frac{V_0}{s}}{R + \frac{R \cdot C \cdot s + 1}{C \cdot s}}$$

$$I(s) = \frac{I_A}{(2 \cdot R \cdot C \cdot s + 1) \cdot s} + \frac{V_0 \cdot C}{2 \cdot R \cdot C \cdot s + 1}$$

<---zero state---> <-zero input->

10-26 **(a)** The Thevenin equivalent is

$$Z_T = R + \frac{1}{C \cdot s} = \frac{R \cdot C \cdot s + 1}{C \cdot s} \text{ and}$$

$$V_T = \frac{1}{C \cdot s} \cdot I_S(s) + \frac{V_0}{s}$$

(b) For $i_S(t) = I_A \cdot \cos(\beta \cdot t)$

$$I(s) = \frac{V_T}{R + Z_T} = \frac{\frac{1}{C \cdot s} \cdot \frac{I_A \cdot s}{s^2 + \beta^2} + \frac{V_0}{s}}{R + \frac{R \cdot C \cdot s + 1}{C \cdot s}}$$

$$I(s) = \frac{I_A \cdot s}{(2 \cdot R \cdot C \cdot s + 1) \cdot \left(s^2 + \beta^2\right)} + \frac{V_0 \cdot C}{2 \cdot R \cdot C \cdot s + 1}$$

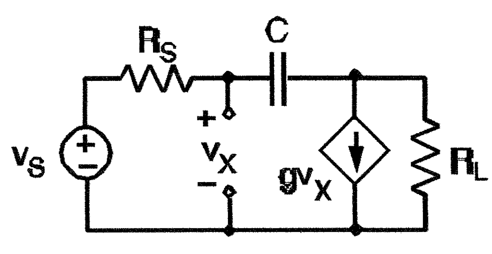
<------zero state---------> <-zero input->

10-27 Find the Thevenin resistance seen by the capacitor. With an open-circuit at the interface then KVL requires

$-V_X + V_{OC} - g \cdot V_X \cdot R_L = 0$ hence $V_{OC} = \left(1 + g \cdot R_L\right) \cdot V_X$

With a short circuit at the interface then KCL requires

$$I_{SC} - g \cdot V_X - \frac{V_X}{R_L} = 0 \text{ hence } I_{SC} = \frac{1 + g \cdot R_L}{R_L} \cdot V_X$$



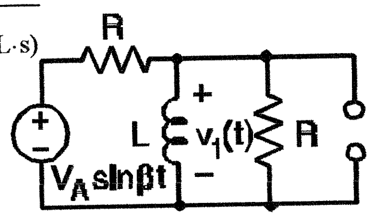
The Thevenin resistance is $R_T = \frac{V_{OC}}{I_{SC}} = R_L$

The circuit pole is to be located at $\frac{1}{R_T \cdot C} = 10 \cdot 10^6$

hence $C := \frac{1}{2000 \cdot 10 \cdot 10^6}$ and $C = 5 \times 10^{-11}$ F.

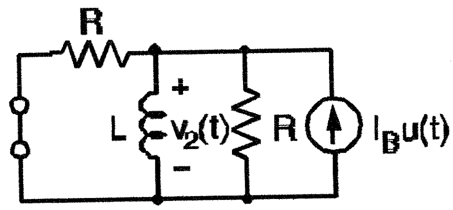
10-28 Using superposition. With the current source off

$$V_1(s) = \left[\frac{\left(\frac{1}{L\cdot s}+\frac{1}{R}\right)^{-1}}{R+\left(\frac{1}{L\cdot s}+\frac{1}{R}\right)^{-1}}\right]\cdot\frac{V_A\cdot\beta}{s^2+\beta^2} = \frac{\beta\cdot V_A\cdot L\cdot s}{\left(s^2+\beta^2\right)\cdot(R+2\cdot L\cdot s)}$$



with the voltage source off

$$V_2(s) = \frac{1}{\frac{1}{L\cdot s}+\frac{2}{R}}\cdot\frac{I_B}{s} = \frac{I_B\cdot L\cdot R}{R+2\cdot L\cdot s}$$



using superposition $V(s) = V_1(s) + V_2(s)$

$$V(s) = \frac{\beta\cdot V_A\cdot L\cdot s}{\left(s^2+\beta^2\right)\cdot(R+2\cdot L\cdot s)} + \frac{I_B\cdot L\cdot R}{R+2\cdot L\cdot s}$$

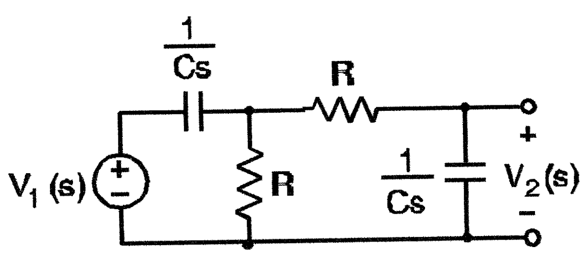
Natural pole at s = -R/2L and forced poles s = $\pm$ jβ.

10-29 $Z(s) = 1000\cdot\left(\frac{s+2000}{s}\right)$ $\quad v(t) = 10\cdot u(t)$ $\quad V(s) = \frac{10}{s}$

$$I(s) = \frac{V(s)}{Z(s)} = \frac{\frac{10}{s}}{\left[1000\cdot\left(\frac{s+2000}{s}\right)\right]} = \frac{10^{-2}}{s+2000} \quad i(t) = 10^{-2}\cdot\exp(-2000\cdot t)\cdot u(t)\text{ A}$$

10-30 The Thevenin equivalent to the left of the shunt R is

$$Z_T = \frac{1}{C\cdot s+\frac{1}{R}} = \frac{R}{R\cdot C\cdot s+1}$$



$$V_T = \frac{R}{R+\frac{1}{C\cdot s}}\cdot V_1(s) = \frac{R\cdot C\cdot s\cdot V_1(s)}{R\cdot C\cdot s+1}$$

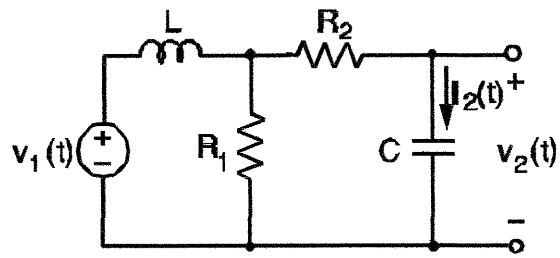
Using this Thevenin in a voltage divider yields

$$V_2(s) = \frac{\frac{1}{C\cdot s}\cdot V_T}{\frac{1}{C\cdot s}+R+Z_T} = \frac{\frac{1}{C\cdot s}\cdot\frac{R\cdot C\cdot s\cdot V_1(s)}{R\cdot C\cdot s+1}}{\frac{1}{C\cdot s}+R+\frac{R}{R\cdot C\cdot s+1}} = \left[\frac{\frac{R}{R\cdot C\cdot s+1}}{\frac{\left[(R\cdot C\cdot s)^2+3\cdot R\cdot C\cdot s+1\right]}{\left[C\cdot s\cdot(R\cdot C\cdot s+1)\right]}}\right]\cdot V_1(s)$$

$$V_2(s) = \left[\frac{R\cdot C\cdot s}{(R\cdot C\cdot s)^2+3\cdot R\cdot C\cdot s+1}\right]\cdot V_1(s)$$

10-31 (a) mesh-current eqs.

$$\begin{pmatrix} L\cdot s + R_1 & -R_1 \\ -R_1 & R_1 + R_2 + \dfrac{1}{C\cdot s} \end{pmatrix} \cdot \begin{pmatrix} I_A(s) \\ I_B(s) \end{pmatrix} = \begin{pmatrix} V_1(s) \\ 0 \end{pmatrix}$$



(b)

$$\Delta(s) = \frac{(R_1 + R_2)\cdot L\cdot C\cdot s^2 + (L + R_1\cdot R_2\cdot C)\cdot s + R_1}{C\cdot s}$$

$$\Delta_B(s) = R_1\cdot V_1(s)$$

$$I_2(s) = I_B(s) = \frac{R_1\cdot C\cdot s\cdot V_1(s)}{(R_1 + R_2)\cdot L\cdot C\cdot s^2 + (L + R_1\cdot R_2\cdot C)\cdot s + R_1}$$

(c) $R_1 := 2000$ $\quad R_2 := 2000$ $\quad L := 0.25$ $\quad C := 250\cdot 10^{-9}$ $\quad v_1(t) = 10\cdot u(t)$

$(R_1 + R_2)\cdot L\cdot C = 2.5\times 10^{-4}$ $\quad L + R_1\cdot R_2\cdot C = 1.25$ $\quad R_1\cdot C = 5\times 10^{-4}$ $\quad R_1\cdot C\cdot s\cdot V_S(s) = 5\cdot 10^{-3}$

$$I_2(s) = \frac{5\cdot 10^{-3}}{2.5\cdot 10^{-4}\cdot s^2 + 1.25\cdot s + 2000} = \frac{20}{s^2 + 5000\cdot s + 8\cdot 10^6} = \frac{20}{(s+2500)^2 + (1323)^2}$$

$$i_2(t) = \frac{20}{1323}\cdot \exp(-2500\cdot t)\cdot \sin(1323\cdot t) = 15.1\cdot 10^{-3}\cdot(\exp(-2500\cdot t)\cdot \sin(1323\cdot t))$$

10-32

(a) node-voltage eqs.

$$\begin{pmatrix} \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{L\cdot s} & \dfrac{-1}{R_2} \\ \dfrac{-1}{R_2} & \dfrac{1}{R_2} + C\cdot s \end{pmatrix} \cdot \begin{pmatrix} V_A(s) \\ V_B(s) \end{pmatrix} = \begin{pmatrix} \dfrac{V_1}{L\cdot s} \\ C\cdot V_0 \end{pmatrix}$$

(b)

$$\Delta(s) = \frac{(R_1 + R_2)\cdot L\cdot C\cdot s^2 + L\cdot s + R_1\cdot R_2\cdot C\cdot s + R_1}{R_1\cdot R_2\cdot (L\cdot s)}$$

$$\Delta_B(s) = \frac{(R_1 + R_2)\cdot L\cdot C\cdot V_0\cdot s + R_1\cdot R_2\cdot C\cdot V_0 + R_1\cdot V_1}{R_1\cdot R_2\cdot L\cdot s}$$

$$V_2 = V_B = \frac{\Delta_B}{\Delta} = \frac{(R_1 + R_2)\cdot L\cdot C\cdot V_0\cdot s + R_1\cdot R_2\cdot C\cdot V_0 + R_1\cdot V_1}{(R_1 + R_2)\cdot L\cdot C\cdot s^2 + L\cdot s + R_1\cdot R_2\cdot C\cdot s + R_1}$$

10-32 Continued

(c) $R_1 := 50 \quad R_2 := 50 \quad L := 1.25 \cdot 10^{-3} \qquad C := 2 \cdot 10^{-6} \qquad v_1(t) = 0 \quad V_0 := 10$

$(R_1 + R_2) \cdot L \cdot C = 2.5 \times 10^{-7} \quad L + R_1 \cdot R_2 \cdot C = 6.25 \times 10^{-3} \qquad R_1 \cdot C = 1 \times 10^{-4}$

$(R_1 + R_2) \cdot L \cdot C \cdot V_0 = 2.5 \times 10^{-6} \qquad R_1 \cdot R_2 \cdot C \cdot V_0 = 0.05$

$$V_2(s) = \frac{2.5 \cdot 10^{-6} \cdot s + 0.05}{2.5 \cdot 10^{-7} \cdot s^2 + 6.25 \cdot 10^{-3} \cdot s + 50} = \frac{10 \cdot (s + 20000)}{s^2 + 25000 \cdot s + 2 \cdot 10^8} = \frac{10(s + 20000)}{(s + 12500)^2 + 6614^2}$$

$$V_2(s) = \frac{10 \cdot (s + 20000)}{(s + 12500 - j \cdot 6614) \cdot (s + 12500 + j \cdot 6614)} = \frac{(5 - 5.67 \cdot i)}{(s + 12500 - 6614 \cdot i)} + \frac{(5 + 5.67 \cdot i)}{(s + 12500 + 6614 \cdot i)}$$

$2 \cdot |(5 - 5.67 \cdot i)| = 15.119 \qquad 57.296 \cdot \arg((5 - 5.67 \cdot i)) = -48.593$

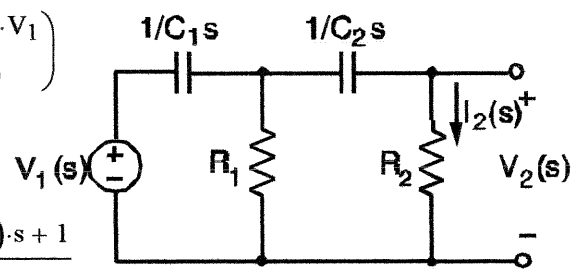
$v_2(t) = 15.119 \cdot \exp(-12500 \cdot t) \cdot \cos\left(6614 \cdot t - 48.593^{o}\right) \cdot u(t)$

$v_2(t) = (10 \cdot \exp(-12500 \cdot t) \cdot \cos(6614 \cdot t) + 11.34 \cdot \exp(-12500 \cdot t) \cdot \sin(6614 \cdot t)) \cdot u(\cdot V$

10-33

(a) node-voltage eqs.

$$\begin{pmatrix} \frac{1}{R_1} + C_1 \cdot s + C_2 \cdot s & -C_2 \cdot s \\ -C_2 \cdot s & \frac{1}{R_2} + C_2 \cdot s \end{pmatrix} \cdot \begin{pmatrix} V_A(s) \\ V_B(s) \end{pmatrix} = \begin{pmatrix} C_1 \cdot s \cdot V_1 \\ 0 \end{pmatrix}$$



(b)

$$\Delta(s) = \frac{R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 + (R_1 \cdot C_1 + R_1 \cdot C_2 + R_2 \cdot C_2) \cdot s + 1}{R_1 \cdot R_2}$$

$$\Delta_B(s) = C_1 \cdot C_2 \cdot s^2 \cdot V_1(s)$$

$$V_2(s) = V_B(s) = \frac{\Delta_B(s)}{\Delta(s)} = \frac{R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 \cdot V_1(s)}{R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 + (R_1 \cdot C_1 + R_1 \cdot C_2 + R_2 \cdot C_2) \cdot s + 1}$$

(c) $R_1 := 10^4 \qquad R_2 := 2 \cdot 10^4 \quad C_1 := 200 \cdot 10^{-9} \qquad C_2 := 100 \cdot 10^{-9} \quad V_1(s) = \frac{10}{s}$

$R_1 \cdot R_2 \cdot C_1 \cdot C_2 = 4 \times 10^{-6} \qquad R_1 \cdot C_1 + R_1 \cdot C_2 + R_2 \cdot C_2 = 5 \times 10^{-3}$

$$V_2(s) = \frac{4 \cdot 10^{-5} \cdot s}{\left(4 \cdot 10^{-6} \cdot s^2 + 5 \cdot 10^{-3} \cdot s + 1\right)} = \frac{10 \cdot s}{s^2 + 1250 \cdot s + 250000} = \frac{40}{[3 \cdot (s + 1000)]} - \frac{10}{[3 \cdot (s + 250)]}$$

$v_2(t) = \left(\frac{40}{3} \cdot \exp(-1000 \cdot t) - \frac{10}{3} \cdot \exp(-250 \cdot t)\right) \cdot u(t)$ V $\qquad \frac{40}{3} = 13.333 \qquad \frac{10}{3} = 3.333$

10-34

(a) mesh-current eqs.

$$\begin{pmatrix} R_1 + \frac{1}{C_1 \cdot s} & -R_1 \\ -R_1 & R_1 + R_2 + \frac{1}{C_2 \cdot s} \end{pmatrix} \cdot \begin{pmatrix} I_A(s) \\ I_B(s) \end{pmatrix} = \begin{pmatrix} V_1 \\ 0 \end{pmatrix}$$

(b)

$$\Delta(s) = \frac{R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 + (R_1 \cdot C_1 + R_1 \cdot C_2 + R_2 \cdot C_2) \cdot s + 1}{C_1 \cdot C_2 \cdot s^2}$$

$$\Delta_B(s) = R_1 \cdot V_1(s)$$

$$I_2(s) = I_B(s) = \frac{\Delta_B(s)}{\Delta(s)} = \frac{(R_1 \cdot C_1 \cdot C_2 \cdot s^2) \cdot V_1(s)}{R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 + (R_1 \cdot C_1 + R_1 \cdot C_2 + R_2 \cdot C_2) \cdot s + 1}$$

(c) $R_1 := 10^3$ $\quad R_2 := 1.25 \cdot 10^4$ $\quad C_1 := 6 \cdot 10^{-6}$ $\quad C_2 := \frac{2}{3} \cdot 10^{-6}$ $\quad V_1(s) = \frac{10}{s}$

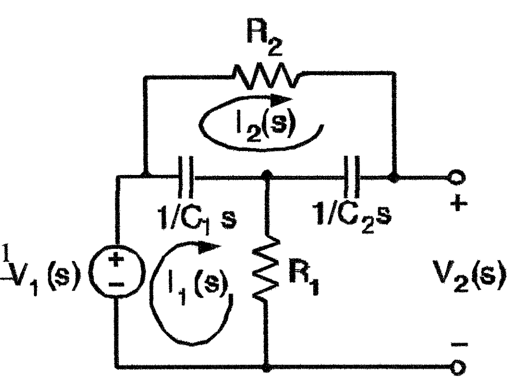
$R_1 \cdot C_1 \cdot C_2 = 4 \times 10^{-9}$ $\quad R_1 \cdot R_2 \cdot C_1 \cdot C_2 = 5 \times 10^{-5}$ $\quad R_1 \cdot C_1 + R_1 \cdot C_2 + R_2 \cdot C_2 = 0.015$

$$I_2(s) = \frac{4 \cdot 10^{-8} \cdot s}{5 \cdot 10^{-5} \cdot s^2 + 0.015 \cdot s + 1} = \frac{8 \cdot 10^{-4} \cdot s}{s^2 + 300 \cdot s + 20000} = \frac{1.6 \cdot 10^{-3}}{s + 200} - \frac{8 \cdot 10^{-4}}{s + 100}$$

$$i_2(t) = 10^{-3} \cdot (1.6 \cdot \exp(-200 \cdot t) - 0.8 \cdot \exp(-100 \cdot t)) \cdot u(t) \text{A}$$

10-35 (a) Mesh-current eqs.

$$\begin{pmatrix} R_1 + \frac{1}{C_1 \cdot s} & \frac{-1}{C_1 \cdot s} \\ \frac{-1}{C_1 \cdot s} & R_2 + \frac{1}{C_1 \cdot s} + \frac{1}{C_2 \cdot s} \end{pmatrix} \cdot \begin{pmatrix} I_A(s) \\ I_B(s) \end{pmatrix} = \begin{pmatrix} V_1(s) \\ 0 \end{pmatrix}$$



(b)

$$\Delta(s) = \frac{R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 + (R_1 \cdot C_1 + R_2 \cdot C_2 + R_1 \cdot C_2) \cdot s + 1}{C_1 \cdot C_2 \cdot s^2}$$

$$\Delta_A(s) = \frac{R_2 \cdot C_1 \cdot C_2 \cdot s + C_1 + C_2}{C_1 \cdot C_2 \cdot s} \cdot V_1(s)$$

$$I_A(s) = \frac{\Delta_A(s)}{\Delta(s)} \qquad Z(s) = \frac{V_1(s)}{I_A(s)} = \frac{R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 + (R_1 \cdot C_1 + R_2 \cdot C_2 + R_1 \cdot C_2) \cdot s + 1}{(R_2 \cdot C_1 \cdot C_2 \cdot s + C_1 + C_2) \cdot s}$$

10-35 Continued

(c)

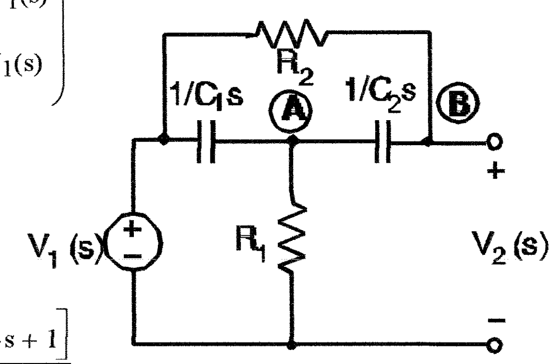
$$Z(s) = R_1 + \frac{1}{C_1 \cdot s + \frac{1}{R_2 + \frac{1}{C_2 \cdot s}}} = R_1 + \frac{1}{C_1 \cdot s + \frac{C_2 \cdot s}{C_2 \cdot s \cdot R_2 + 1}} = R_1 + \frac{R_2 \cdot C_2 \cdot s + 1}{\left[s \cdot \left(C_1 \cdot C_2 \cdot s \cdot R_2 + C_1 + C_2\right)\right]}$$

$$Z(s) = \frac{V_1(s)}{I_B(s)} = \frac{R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 + \left(R_1 \cdot C_1 + R_2 \cdot C_2 + R_1 \cdot C_2\right) \cdot s + 1}{R_2 \cdot C_1 \cdot C_2 \cdot s^2 + \left(C_1 + C_2\right) \cdot s} \qquad \text{QED}$$

10-36 (a)

Node-voltage eqs.

$$\begin{pmatrix} C_1 \cdot s + C_2 \cdot s + \frac{1}{R_1} & -C_2 \cdot s \\ -C_2 \cdot s & C_2 \cdot s + \frac{1}{R_2} \end{pmatrix} \cdot \begin{pmatrix} V_A(s) \\ V_B(s) \end{pmatrix} = \begin{pmatrix} C_1 \cdot s \cdot V_1(s) \\ \frac{1}{R_2} \cdot V_1(s) \end{pmatrix}$$



(b)

$$\Delta(s) = \frac{\left[R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 + \left(R_1 \cdot C_1 + R_2 \cdot C_2 + R_1 \cdot C_2\right) \cdot s + 1\right]}{\left(R_1 \cdot R_2\right)}$$

$$\Delta_B(s) = \frac{\left[R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 + \left(R_1 \cdot C_1 + R_1 \cdot C_2\right) \cdot s + 1\right]}{\left(R_1 \cdot R_2\right)} \cdot V_1(s)$$

$$V_2(s) = V_B(s) = \frac{\Delta_B(s)}{\Delta(s)} = \frac{R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 + \left(R_1 \cdot C_1 + R_1 \cdot C_2\right) \cdot s + 1}{R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 + \left(R_1 \cdot C_1 + R_1 \cdot C_2 + R_2 \cdot C_2\right) \cdot s + 1} \cdot V_1(s)$$

$R_1 := 3000 \qquad R_2 := 25 \cdot 10^3 \qquad C_1 := 80 \cdot 10^{-9} \qquad C_2 := 20 \cdot 10^{-9} \qquad V_1(s) = \frac{V_A}{s + 2000}$

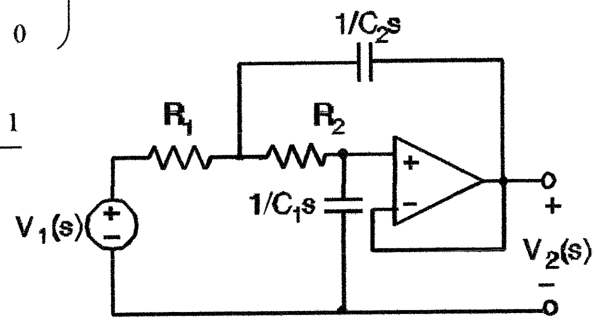
$R_1 \cdot R_2 \cdot C_1 \cdot C_2 = 1.2 \times 10^{-7} \qquad R_1 \cdot C_1 + R_1 \cdot C_2 + R_2 \cdot C_2 = 8 \times 10^{-4} \qquad R_1 \cdot C_1 + R_1 \cdot C_2 = 3 \times 10^{-4}$

$$V_2(s) = \frac{1.2 \cdot 10^{-7} \cdot s^2 + 3 \cdot 10^{-4} \cdot s + 1}{1.2 \cdot 10^{-7} \cdot s^2 + 8 \cdot 10^{-4} \cdot s + 1} \cdot \frac{V_A}{s + 2000} = \frac{(s + 1250)^2 + 2602^2}{(s + 1667) \cdot (s + 5000)} \cdot \left(\frac{V_A}{s + 2000}\right)$$

Complex zeros at s = -1250 ± j2602 and natural poles at s = -1667 and s = -5000
Forced pole at s = -2000

10-37 Node-voltage eqs.

$$\begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2}+C_2\cdot s & -C_2\cdot s-\frac{1}{R_2} \\ \frac{-1}{R_2} & \frac{1}{R_2}+C_1\cdot s \end{pmatrix}\cdot\begin{pmatrix} V_A(s) \\ V_B(s) \end{pmatrix} = \begin{pmatrix} \frac{V_1(s)}{R_1} \\ 0 \end{pmatrix}$$



$$\Delta(s) = \frac{R_1\cdot R_2\cdot C_1\cdot C_2\cdot s^2 + ((R_1\cdot C_1 + R_2\cdot C_1))\cdot s + 1}{R_1\cdot R_2}$$

$$\Delta_B(s) = \frac{V_1(s)}{R_1\cdot R_2}$$

$$V_B(s) = \frac{\Delta_B(s)}{\Delta(s)} = \frac{V_1(s)}{\left[R_1\cdot R_2\cdot C_1\cdot C_2\cdot s^2 + ((R_1\cdot C_1 + R_2\cdot C_1))\cdot s + 1\right]}$$

$$\omega_0 = \frac{1}{\sqrt{R_1\cdot R_2\cdot C_1\cdot C_2}} \qquad \frac{2\cdot\zeta}{\omega_0} = R_1\cdot C_1 + R_2\cdot C_1 \qquad 2\cdot\zeta = \frac{R_1\cdot C_1 + R_2\cdot C_1}{\sqrt{R_1\cdot R_2\cdot C_1\cdot C_2}}$$

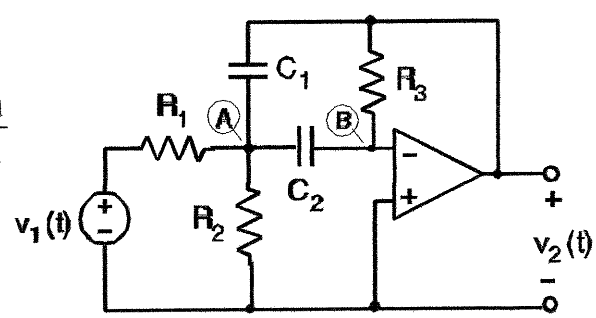
$$R_1 = R_2 = R \qquad \zeta = \frac{C_1 + C_1}{\sqrt{C_1\cdot C_2}} = \sqrt{\frac{C_1}{C_2}} \qquad \omega_0 = \frac{1}{R\cdot C_2\cdot\zeta} \qquad R := 10\cdot 10^3 \quad \omega_0 := 1000 \quad \zeta := 0.5$$

$$C_2 := \frac{1}{(R\cdot\zeta\cdot\omega_0)} \qquad C_2 = 2\times 10^{-7} \qquad C_1 := \zeta^2\cdot C_2 \qquad C_1 = 5\times 10^{-8}$$

10-38 The OP AMP requires $V_B(s) = 0$ since the noninverting input is grounded.
Writing node eqs. at A and B

$$\left(\frac{1}{R_1}+\frac{1}{R_2}+C_1\cdot s + C_2\cdot s\right)\cdot V_A - C_1\cdot s\cdot V_C = \frac{V_1}{R_1}$$

$$-C_2\cdot s\cdot V_A - \frac{V_C}{R_3} = 0$$



For $R_1 = R_2 = R_3 = R$, and $C_1 = C_2 = C$
the circuit determinant is

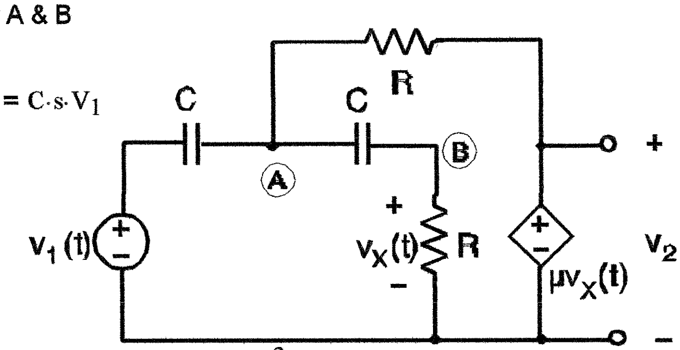
$$\Delta(s) = \left(\frac{2}{R}+2\cdot C\cdot s\right)\cdot\left(-\frac{1}{R}\right) - \left(C^2\cdot s^2\right) = -\frac{2 + 2\cdot R\cdot C\cdot s + (R\cdot C\cdot s)^2}{R^2} = -\frac{(R\cdot C\cdot s + 1)^2 + 1^2}{R^2}$$

The natural poles are at RCs = -1 ± j1. To place the poles at s = -1000 ± j1000, let RC = 1/1000 and select R = 10 kΩ then C = 100 nF.

10-39 **(a)** Writing node equations at A & B

$$\left(\frac{1}{R}+2\cdot C\cdot s\right)\cdot V_A - C\cdot s\cdot V_B - \frac{\mu\cdot V_B}{R} = C\cdot s\cdot V_1$$

$$-C\cdot s\cdot V_A + \left(\frac{1}{R}+C\cdot s\right)\cdot V_B = 0$$



Hence

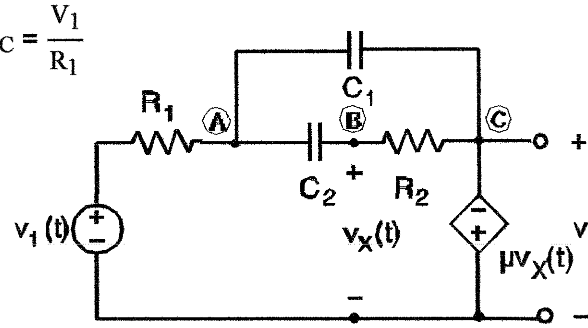
$$\Delta(s) = \left(\frac{1}{R}+2\cdot C\cdot s\right)\cdot\left(\frac{1}{R}+C\cdot s\right) - (C\cdot s)\cdot\left(C\cdot s+\frac{\mu}{R}\right) = \frac{(R\cdot C\cdot s)^2 + (3-\mu)\cdot(R\cdot C\cdot s) + 1}{R^2}$$

(b) Poles at s = $\pm j5000$ require $\mu = 3$ and RC = 1/5000. Let R = 10 kΩ, then C = 20 nF

10-40 (a) Writing node equations at A & B

$$\left(\frac{1}{R_1}+C_1\cdot s+C_2\cdot s\right)\cdot V_A - C_2\cdot s\cdot V_B - C_1\cdot s\cdot V_C = \frac{V_1}{R_1}$$

$$-C_2\cdot s\cdot V_A + \left(\frac{1}{R_2}+C_2\cdot s\right)\cdot V_B - \frac{V_C}{R_2} = 0$$



The dependent source requires $V_C = -\mu\cdot V_B$

Hence the determinant is

$$\Delta(s) = \left(\frac{1}{R_1}+C_1\cdot s+C_2\cdot s\right)\cdot\left(\frac{1}{R_2}+C_2\cdot s+\frac{\mu}{R_2}\right) - (-C_2\cdot s)\cdot(-C_2\cdot s + C_1\cdot s\cdot\mu)$$

$$\Delta(s) = \frac{(\mu+1)\cdot R_1\cdot R_2\cdot C_1\cdot C_2\cdot s^2 + \left[(\mu+1)\cdot(R_1\cdot C_1 + R_1\cdot C_2) + R_2\cdot C_2\right]\cdot s + (\mu+1)}{R_1\cdot R_2}$$

$$\Delta(s) = \frac{R_1\cdot R_2\cdot C_1\cdot C_2\cdot s^2 + \left(R_1\cdot C_1 + R_1\cdot C_2 + \frac{R_2\cdot C_2}{\mu+1}\right)\cdot s + 1}{R_1\cdot R_2}$$

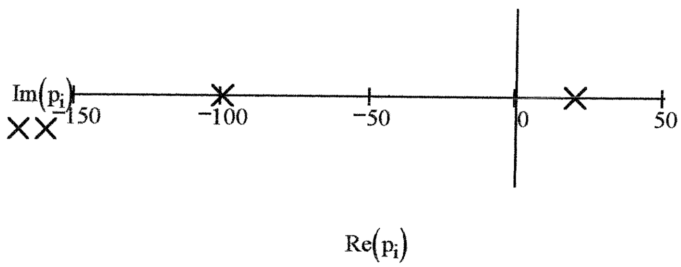
(b) For $R_1 := 10^4$, $R_2 := 2\cdot 10^4$, $C_1 := 10^{-8}$, $C_2 := C_1$, and $\mu := 5$

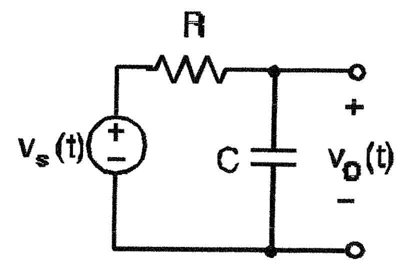
the poles are located at

$$\text{polyroots}\left(\begin{pmatrix} 1 \\ R_1\cdot C_1 + R_1\cdot C_2 + \frac{R_2\cdot C_2}{\mu+1} \\ R_1\cdot R_2\cdot C_1\cdot C_2 \end{pmatrix}\right) = \begin{pmatrix} -5.833\times 10^3 - 3.997j\times 10^3 \\ -5.833\times 10^3 + 3.997j\times 10^3 \end{pmatrix}$$

10-41 (a) $V_C(s) = \dfrac{500}{(s+100)\cdot(s-20)} = \dfrac{-25}{[6\cdot(s+100)]} + \dfrac{25}{[6\cdot(s-20)]}$ $v_C(t) = \dfrac{25}{6}\cdot\left(-e^{-100\cdot t} + e^{20\cdot t}\right)$

$i := 1, 2..2 \quad p_1 := -100 \quad p_2 := 20 \qquad z_1 := 0$

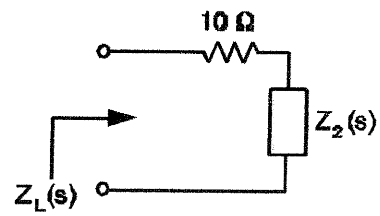




(b) $T_V(s) = \dfrac{1}{R\cdot C\cdot s+1} = \dfrac{100}{s+100} = \dfrac{V_C(s)}{V_S(s)}$ $V_S(s) = \dfrac{s+100}{100}\cdot V_C(s) = \dfrac{5}{s-20}$ $v_S(t) = 5\cdot e^{20\cdot t}$ V

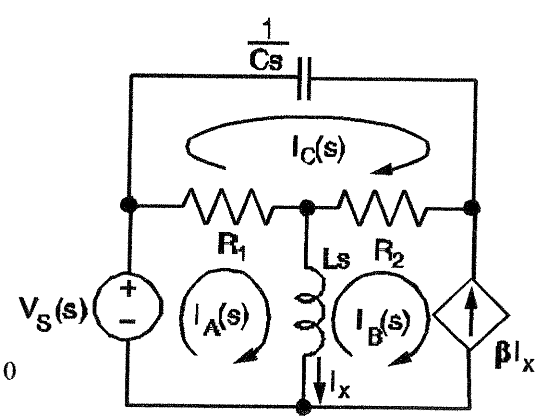
(c) The circuit is stable. Right half plane pole in V_C comes from the unbounded input.

10-42 (a) The unknown impedance must be



$Z_2(s) = Z_L(s) - 10 = \dfrac{20\cdot(s+5)}{s+10} - 10 = \dfrac{10s}{s+10} = \dfrac{1}{\dfrac{1}{10}+\dfrac{1}{s}}$ <---**(b)** $Z_2(s)$ is a 10 Ω resistor in parallel with a 1 H inductor

10-43 (a)



$I_B = -\beta\cdot I_X = -\beta\cdot(I_A - I_B)$

hence $\quad I_B = \dfrac{\beta}{\beta-1}\cdot I_A$

(b)

Mesh A:

$(R_1 + L\cdot s)I_A - L\cdot s\cdot I_B - R_1\cdot I_C = V_S$

Mesh C: $\quad -R_1\cdot I_A - R_2\cdot I_B + \left(R_1 + R_2 + \dfrac{1}{C\cdot s}\right)\cdot I_C = 0$

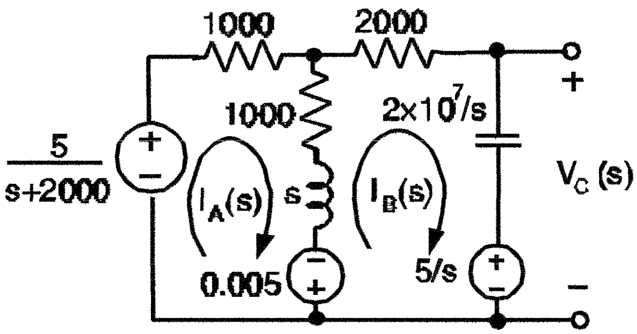
Using the result in (a) to eliminate I_B

Mesh A: $\quad \left(R_1 + \dfrac{1}{1-\beta}\cdot L\cdot s\right)I_A - R_1\cdot I_C = V_S$

Mesh C: $\quad -\left(R_1 + \dfrac{\beta}{\beta-1}\cdot R_2\right)\cdot I_A + \left(R_1 + R_2 + \dfrac{1}{C\cdot s}\right)\cdot I_C = 0$

10-44 For t < 0 the sw is in position A and a dc steady-state exists, the inductor acts like a short and the capacitor an open, hence $i_L(0) = \frac{10}{1000+1000} = 5\cdot 10^{-3}$ A $\quad v_C(0) = \frac{1000}{1000+1000}\cdot 10 = 5$ V

The s domain circuit is shown below



Writing two node-voltage equations in matrix form

$$\begin{pmatrix} \frac{1}{1000}+\frac{1}{2000}+\frac{1}{s+1000} & \frac{-1}{2000} \\ \frac{-1}{2000} & \frac{1}{2000}+50\cdot 10^{-9}\cdot s \end{pmatrix}\cdot\begin{pmatrix} V_A(s) \\ V_B(s) \end{pmatrix} = \begin{pmatrix} \frac{5\cdot 10^{-3}}{s+2000} - \frac{5\cdot 10^{-3}}{s+1000} \\ 25\cdot 10^{-8} \end{pmatrix}$$

Solving for $V_B(s)$

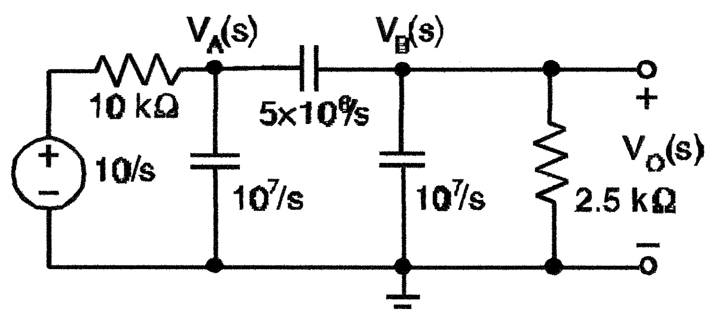
$$\Delta(s) = \frac{3\cdot s^2 + 25000\cdot s + 40000000}{4\cdot 10^{10}\cdot(s+1000)} \qquad \Delta_B(s) = \frac{1}{8\cdot 10^9}\cdot\frac{\left(3\cdot s^2 + 11000\cdot s - 10000000\right)}{[(s+1000)\cdot(s+2000)]}$$

$$V_C(s) = \frac{\Delta_B(s)}{\Delta(s)} = \frac{\frac{1}{8\cdot 10^9}\cdot\frac{\left(3\cdot s^2+11000\cdot s-10000000\right)}{[(s+1000)\cdot(s+2000)]}}{\frac{3\cdot s^2+25000\cdot s+40000000}{4\cdot 10^{10}\cdot(s+1000)}} = \frac{5\cdot\left(3\cdot s^2+11000\cdot s-10000000\right)}{\left[\left(3\cdot s^2+25000\cdot s+40000000\right)\cdot(s+2000)\right]}$$

$$V_C(s) = \frac{5}{3}\cdot\frac{\left(3\cdot s^2+11000\cdot s-10000000\right)}{(s+2000)\cdot(s+2159.734)\cdot(s+6173.6)} = \frac{-50}{s+2000} + \frac{51.37}{s+2159.7} + \frac{3.624}{s+6173.6}$$

$$v_C(t) = (-50\cdot\exp(-2000\cdot t) + 51.37\cdot\exp(-2159.7\cdot t) + 3.624\cdot\exp(-6173.6\cdot t))\cdot u\text{V}$$

10-45 The circuit is transformed to the s-domain below



10-45 Continue Writing two node equations at A and B:

A: $\left(10^{-4} + 3\cdot10^{-7}\cdot s\right)\cdot V_A - \left(2\cdot10^{-7}\cdot s\right)\cdot V_B = \frac{10^{-3}}{s}$

B: $-2\cdot10^{-7}\cdot s\cdot V_A + \left(\frac{1}{2500} + 3\cdot10^{-7}\cdot s\right)\cdot V_B = 0$

Solving for the voltage at Node B

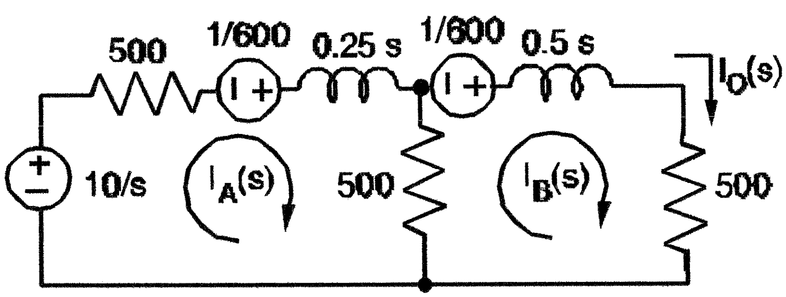
$$\Delta = \left(10^{-4} + 3\cdot10^{-7}\cdot s\right)\cdot\left(\frac{1}{2500} + 3\cdot10^{-7}\cdot s\right) - \left(2\cdot10^{-7}\cdot s\right)^2$$

$\Delta = \frac{s^2 + 3000\cdot s + 800000}{2\cdot10^{13}}$ and $\Delta_B(s) = 2\cdot10^{-10}$

$$V_O(s) = V_B = \frac{\Delta_B}{\Delta} = \frac{4000}{s^2 + 3000\cdot s + 800000} = \frac{4000}{(s + 295.8)\cdot(s + 2704)} = \frac{1.661}{s + 295.8} - \frac{1.661}{s + 2704}$$

and hence $v_O(t) = 1.661\cdot\left(e^{-295.8\cdot t} - e^{-2704\cdot t}\right)\cdot u(t)$ V

10-46 For t < 0 sw is open and $L_1\cdot i_{L1}(0) = \frac{1}{4}\cdot\frac{10}{1500} = \frac{1}{600}$ and $L_2\cdot i_{L2}(0) = 2\cdot L_1\cdot\frac{i_{L1}(0)}{2} = \frac{1}{600}$



Mesh A: $(1000 + 0.25\cdot s)\cdot I_A - 500\cdot I_B = \frac{10}{s} + \frac{1}{600}$ Mesh B: $-500\cdot I_A + (1000 + 0.5\cdot s)\cdot I_B = \frac{1}{600}$

solving for $I_O(s) = I_B(s) = \frac{\Delta_B(s)}{\Delta(s)}$

$$\Delta(s) = (1000 + 0.25\cdot s)\cdot(1000 + 0.5\cdot s) - 500^2 = 750000 + 750\cdot s + .125\cdot s^2 = \frac{(s + 1268)\cdot(s + 4732)}{8}$$

$$\Delta_B(s) = (1000 + 0.25\cdot s)\cdot\left(\frac{1}{600}\right) + 500\cdot\left(\frac{10}{s} + \frac{1}{600}\right) = \frac{s^2 + 6000\cdot s + 12000000}{2400\cdot s}$$

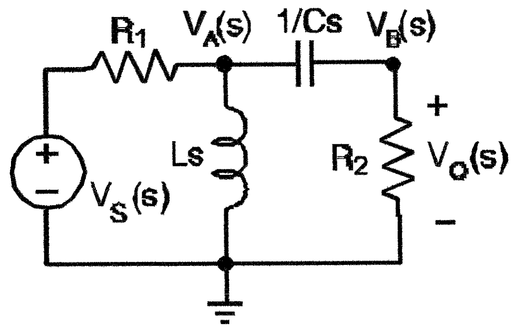
$$I_O(s) = I_B = \frac{\Delta_B}{\Delta} = \frac{1}{300}\cdot\frac{s^2 + 6000\cdot s + 12000000}{s\cdot(s + 1268)\cdot(s + 4732)} = \frac{6.666\cdot10^{-3}}{s} - \frac{4.553\cdot10^{-3}}{s + 1268} + \frac{1.22\cdot10^{-3}}{s + 4732}$$

Hence for t> 0 $i_O(t) = (6.666 - 4.553\cdot\exp(-1268\cdot t) + 1.22\cdot\exp(-4732\cdot t))$ mA

10-47 (a) Writing two node equations

Node A: $\left(\frac{1}{R_1}+\frac{1}{L\cdot s}+C\cdot s\right)\cdot V_A - C\cdot s\cdot V_B = \frac{V_S}{R_1}$

Node B: $-C\cdot s\cdot V_A + \left(\frac{1}{R_2}+C\cdot s\right)\cdot V_B = 0$



Solving for $V_O(s) = V_B(s) = \frac{\Delta_B(s)}{\Delta(s)}$

$$\Delta(s) = \left(\frac{1}{R_1}+\frac{1}{L\cdot s}+C\cdot s\right)\cdot\left(\frac{1}{R_2}+C\cdot s\right) - (C\cdot s)^2 = \frac{(R_1+R_2)\cdot L\cdot C\cdot s^2 + (L + R_1\cdot R_2\cdot C)\cdot s + R_1}{R_1\cdot R_2\cdot L\cdot s}$$

$$\Delta_B(s) = \frac{C\cdot s\cdot V_S(s)}{R_1}\ ;\ V_O(s) = \frac{\Delta_B}{\Delta} = \left[\frac{R_2\cdot L\cdot C\cdot s^2}{(R_1+R_2)\cdot L\cdot C\cdot s^2 + (L+R_1\cdot R_2\cdot C)\cdot s + R_1}\right]\cdot V_S(s)$$

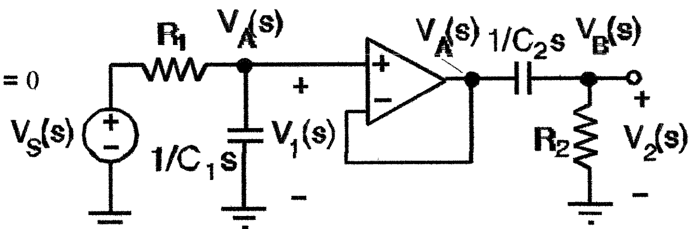
(b) $v_O(t) = \left[\frac{0.5\cdot s^2}{\left(s^2+10000\cdot s+5\cdot 10^7\right)}\right]\cdot\frac{10^4}{s^2} = \frac{5000}{(s+5000)^2+5000^2}$

hence for t > 0 $v_O(t) = \exp(-5000\cdot t)\cdot\sin(5000\cdot t)$ V

10-48 (a) Writing two node equations

Node A: $\left(\frac{1}{R_1}+C_1\cdot s\right)\cdot V_A = \frac{V_S}{R_1}$

Node B: $-C_2\cdot s\cdot V_A + \left(\frac{1}{R_2}+C_2\cdot s\right)\cdot V_B = 0$



solving the Node A equation

$V_1(s) = V_A = \left(\frac{1}{R_1\cdot C_1\cdot s+1}\right)\cdot V_S$

Using this in the Node B equation

$$V_2(s) = V_B(s) = \frac{R_2\cdot C_2\cdot s}{R_2\cdot C_2\cdot s+1}\cdot V_A = \left[\frac{R_2\cdot C_2\cdot s}{(R_1\cdot C_1\cdot s+1)\cdot(R_2\cdot C_2\cdot s+1)}\right]\cdot V_S(s)$$

poles are at $s = -\frac{1}{R_1\cdot C_1} = -2000$ and $s = -\frac{1}{R_2\cdot C_2} = -1000$

(b) $V_1(s) = V_A(s) = \left(\frac{1}{R_1\cdot C_1\cdot s+1}\right)\cdot\frac{10}{s}$ poles at s = 0 and $s = -\frac{1}{R_1\cdot C_1} = -2000$

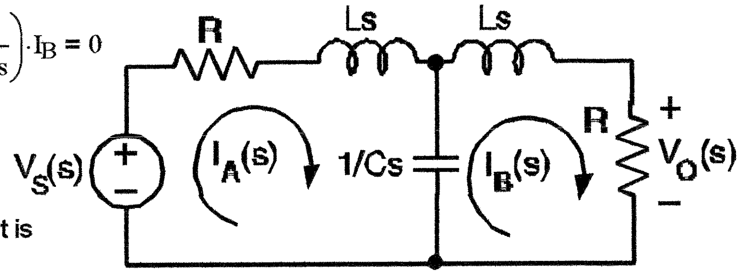
(c) $V_2(s) = \left[\frac{R_2\cdot C_2\cdot s}{(R_1\cdot C_1\cdot s+1)\cdot(R_2\cdot C_2\cdot s+1)}\right]\cdot\frac{10}{s} = \frac{10\cdot R_2\cdot C_2}{(R_1\cdot C_1\cdot s+1)\cdot(R_2\cdot C_2\cdot s+1)}$

Forced pole at s = 0 is cancelled by a natural zero;
Both natural poles at s = -1000 and s = -2000 are present in $V_2(s)$.

10-49 Writing two mesh equations

Mesh A: $\left(R + L\cdot s + \frac{1}{C\cdot s}\right)\cdot I_A - \left(\frac{1}{C\cdot s}\right)\cdot I_B = V_S$

Mesh B: $-\left(\frac{1}{C\cdot s}\right)\cdot I_A + \left(R + L\cdot s + \frac{1}{C\cdot s}\right)\cdot I_B = 0$



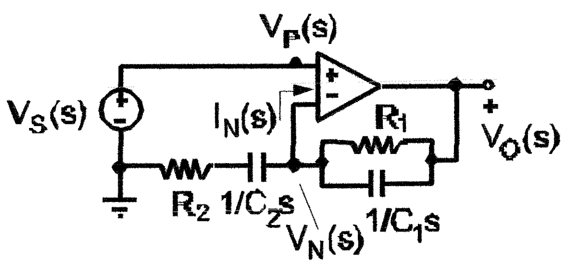
For L = R²C/8 the circuit determinant is

$$\Delta(s) = \left(R + L\cdot s + \frac{1}{C\cdot s}\right)^2 - \left(\frac{1}{C\cdot s}\right)^2 = \left(R + \frac{R^2\cdot C}{8}\cdot s + \frac{1}{C\cdot s}\right)^2 - \left(\frac{1}{C\cdot s}\right)^2$$

$$\Delta(s) = \frac{R}{64\cdot C\cdot s}\cdot\left[(R\cdot C\cdot s)^3 + 16\cdot(R\cdot C\cdot s)^2 + 80\cdot(R\cdot C\cdot s) + 128\right] = \frac{R}{4\cdot C\cdot s}\cdot\left[(R\cdot C\cdot s + 8)\cdot\left[(R\cdot C\cdot s + 4)^2\right]\right]$$

hence there is a simple pole at s = -8/RC and a double pole at s = -4/RC. QED

10-50



The KCL equation at the inverting input: $\frac{V_N}{R_2 + (C_2\cdot s)^{-1}} + \left(C_1\cdot s + \frac{1}{R_1}\right)\cdot(V_N - V_O) + I_N = 0$

The OP AMP I-V relations require $V_P = V_N = V_S$; $I_N = 0$ Hence the KCL equation becomes

$\left(\frac{C_2\cdot s}{R_2\cdot C_2\cdot s + 1} + \frac{R_1\cdot C_1\cdot s + 1}{R_1}\right)\cdot V_S - \left(\frac{R_1\cdot C_1\cdot s + 1}{R_1}\right)\cdot V_O = 0$ solving for V_O yields

$V_O(s) = \left[\frac{R_1\cdot R_2\cdot C_1\cdot C_2\cdot s^2 + (R_1\cdot C_1 + R_2\cdot C_2 + R_1\cdot C_2)\cdot s + 1}{(R_1\cdot C_1\cdot s + 1)\cdot(R_2\cdot C_2\cdot s + 1)}\right]\cdot V_S$, To produce the stated

poles requires $R_1\cdot C_1 = \frac{1}{200}$ and $R_2\cdot C_2 = \frac{1}{1000}$.

Let $R_1 = R_2 = 10^4$ then $C_1 = 0.5\cdot 10^{-6}$ $C_2 = 0.1\cdot 10^{-6}$

10-51 With an open-ckt:: $v_{oc}(t) = \left(9 \cdot e^{-10 \cdot t} - 6 \cdot e^{-40 \cdot t}\right) \cdot u(t)$; $V_{oc}(s) = \frac{3 \cdot (s+100)}{(s+10) \cdot (s+40)}$

With a 50-Ω load:

$v_L(t) = \left(5 \cdot e^{-10 \cdot t} - 5 \cdot e^{-40 \cdot t}\right) \cdot u(t)$ V

; $i_L(t) = \frac{v_L(t)}{50} = \frac{1}{10} \cdot \left(e^{-10 \cdot t} - e^{-40 \cdot t}\right) \cdot u(t)$ A ; $I_L(s) = \frac{3}{(s+10) \cdot (s+40)}$

$I_L(s) = \frac{V_{oc}(s)}{Z_T + 50}$ Hence $Z_T(s) = \frac{V_{oc}(s)}{I_L(s)} - 50 = \frac{\frac{3 \cdot (s+100)}{(s+10) \cdot (s+40)}}{\frac{3}{(s+10) \cdot (s+40)}} - 50 = s + 50$

10-52 for $\omega_0 < 10$ and $1 > \zeta > 0.5$ the allowable region in the s-plane falls within the heavy curves

$\alpha := -10, -9.9 .. 0$ $\beta(\alpha) := \begin{cases} \sqrt{100 - \alpha^2} & \text{if } -10 \le \alpha < -5 \\ -\sqrt{3} \cdot \alpha & \text{if } -5 \le \alpha \end{cases}$

$\Delta(s) = s^2 + (2 + 5 \cdot K_1)s + 9 \cdot K_2$

<-2ζω0-> <-ω0²->

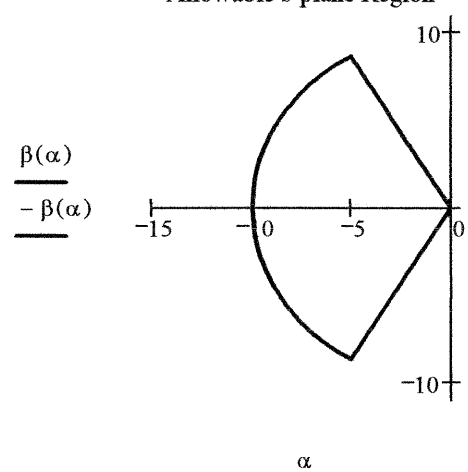
$\sqrt{9K_2} < 10$ $K_2 < 11.1$

$2 + 5 \cdot K_1 = 2 \cdot \zeta \cdot \omega_0$ $\zeta = \frac{2 + 5 \cdot K_1}{2 \cdot \omega_0}$

$1 > \frac{2 + 5 \cdot K_1}{2 \cdot \omega_0} > 0.5$

Hence $\frac{\omega_0 \cdot 2}{5} > K_1 > \frac{\omega_0 - 2}{5}$ Where ω_0 is <10

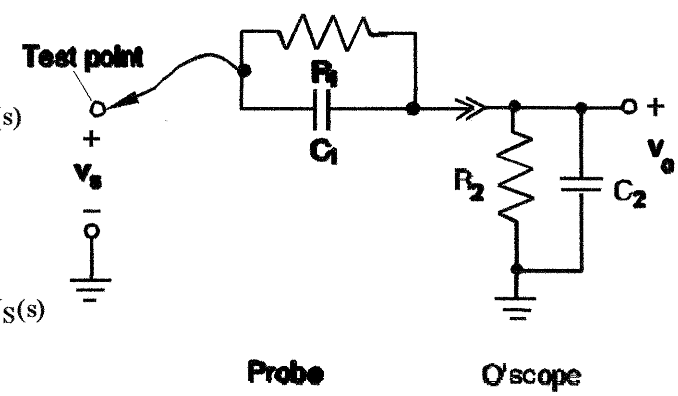
Allowable s-plane Region



10-53 **(a)** By voltage division

$$V_O(s) = \frac{\frac{1}{R_1} + C_1 \cdot s}{\frac{1}{R_1} + C_1 \cdot s + \left(\frac{1}{R_2} + C_2 \cdot s\right)} \cdot V_S(s)$$

$$V_O(s) = \frac{\frac{R_1 \cdot C_1 \cdot s + 1}{R_1}}{\frac{R_1 \cdot C_1 \cdot s + 1}{R_1} + \frac{R_2 \cdot C_2 \cdot s + 1}{R_2}} \cdot V_S(s)$$



(b) If $R_1 = R_2 = R$ and $C_1 = C_2 = C$ then $V_O(s) = 0.5 \cdot V_S(s)$

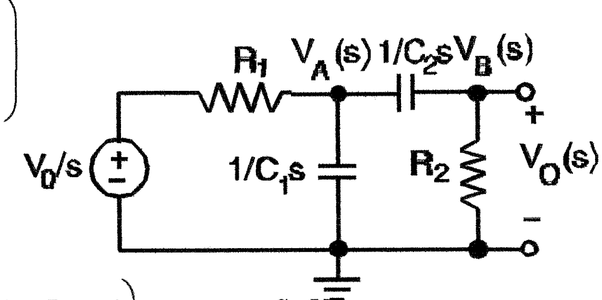
hence let $R_1 = R_2 = 10$ MΩ and $C_1 = C_2 = 2$ pF .

10-54 Transforming the pulse $v_O(t) := 500\cdot\left(e^{-2000\cdot t} - e^{-80000\cdot t}\right)$ into the s domain yields

$$V_O(s) = \frac{39000000}{(s+2000)\cdot(s+80000)} = \frac{39000000}{\left(s^2 + 82000\cdot s + 1.6\cdot 10^8\right)} = \frac{0.244}{6.25\cdot 10^{-9}\cdot s^2 + 5.125\cdot 10^{-4}\cdot s + 1}$$

Writing two node voltage equations

$$\begin{pmatrix} C_1\cdot s + C_2\cdot s + \frac{1}{R_1} & -C_2\cdot s \\ -C_2\cdot s & C_2\cdot s + \frac{1}{R_2} \end{pmatrix}\cdot\begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{pmatrix} \frac{V_0}{R_1\cdot s} \\ 0 \end{pmatrix}$$



Solving for $V_O(s) = V_B$

$$\Delta(s) = \frac{\left(C_1\cdot s^2\cdot R_1\cdot C_2\cdot R_2 + C_1\cdot s\cdot R_1 + C_2\cdot s\cdot R_1 + C_2\cdot s\cdot R_2 + 1\right)}{\left(R_1\cdot R_2\right)} \;;\; \Delta_B(s) = \frac{C_2\cdot V_0}{R_1}$$

$$V_O(s) = \frac{\Delta_B(s)}{\Delta(s)} = \frac{R_2\cdot C_2\cdot V_0}{R_1\cdot R_2\cdot C_1\cdot C_2\cdot s^2 + \left(R_1\cdot C_1 + R_2\cdot C_2 + R_1\cdot C_2\right)\cdot s + 1}$$

Comparing this result to the required transform

$$V_O(s) = \frac{0.244}{6.25\cdot 10^{-9}\cdot s^2 + 5.125\cdot 10^{-4}\cdot s + 1}$$ yields the following design constraints:

$R_2\cdot C_2\cdot V_0 = 0.244$; $R_1\cdot R_2\cdot C_1\cdot C_2 = 6.25\cdot 10^{-9}$; $R_1\cdot C_1 + R_2\cdot C_2 + R_1\cdot C_2 = 5.125\cdot 10^{-4}$

Defining $x_1 = R_1\cdot C_1$; $x_2 = R_2\cdot C_2$; $x_{12} = R_1\cdot C_2$ leads to the following constraints

$x_2\cdot V_0 = 0.244$ $\quad x_1\cdot x_2 = 6.25\cdot 10^{-9}$ $\quad x_1 + x_2 + x_{12} = 5.125\cdot 10^{-4}$ Let $\quad V_0 := 1000$

$x_2 := \frac{0.244}{V_0}$ $\quad x_1 := \frac{6.25\cdot 10^{-9}}{x_2}$ $\quad x_{12} := 5.125\cdot 10^{-4} - x_1 - x_2$

$x_1 = 2.561\times 10^{-5}$ $\quad x_2 = 2.44\times 10^{-4}$ $\quad x_{12} = 2.429\times 10^{-4}$

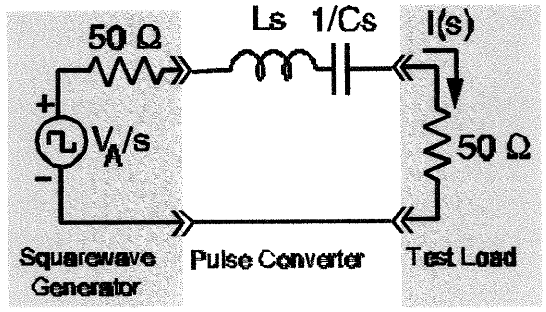
Let $\quad R_1 := 10^4$ $\quad C_1 := \frac{x_1}{R_1}$ $\quad C_2 := \frac{x_{12}}{R_1}$ $\quad R_2 := \frac{x_2}{C_2}$

$R_1 = 1\times 10^4$ $\quad C_1 = 2.561\times 10^{-9}$ $\quad C_2 = 2.429\times 10^{-8}$ $\quad R_2 = 1.005\times 10^4$ $\quad V_0 = 1\times 10^3$

Many other solutions are possible

10-55 For a 50 Ω load the impedance seen by the source is:

$$Z(s) = 10^{-2}\cdot s + \frac{1}{10^{-7}\cdot s} + 50 + 50 = \frac{s^2 + 10^4\cdot s + 10^9}{100\cdot s}$$



The current delivered to the load is

$$I(s) = \frac{V_A}{s}\cdot\frac{1}{Z(s)} = \frac{100\cdot V_A}{s^2 + 10^4\cdot s + 10^9}$$

For which $\omega_0 := \sqrt{10^9}$; $\omega_0 = 3.162 \times 10^4$

and $\zeta := \frac{10^4}{2\cdot\omega_0}$; $\zeta = 0.158$

The converter meet the specification requirements of $\omega_0 > 10^4$ and $\zeta < 0.5$ for a 50-Ω load.

For a 600-Ω load the impedance seen by the source is:

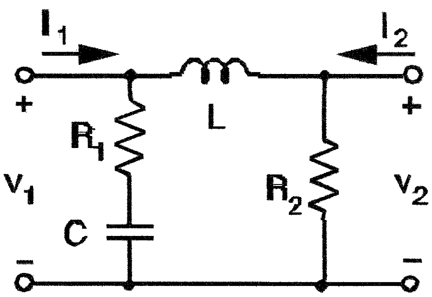
$$Z(s) = 10^{-2}\cdot s + \frac{1}{10^{-7}\cdot s} + 50 + 600 = \frac{s^2 + 65000\cdot s + 10^9}{100\cdot s}$$

For which $\omega_0 := \sqrt{10^9}$; $\zeta := \frac{65000}{2\cdot\omega_0}$ yielding $\omega_0 = 3.162 \times 10^4$ and $\zeta = 1.028$

The converter **does not** meet the requirements of $\omega_0 > 10^4$ and $\zeta < 0.5$ for a 600-Ω load.
It can be brought into compliance by changing the inductance from 100 mH to 50 mH.

CHAPTER 11, Both Versions

11-1 (a) $Z(s) = \dfrac{1}{\left(R_1 + \dfrac{1}{C\cdot s}\right)^{-1} + (L\cdot s + R_2)^{-1}}$



$$Z(s) = \frac{(L\cdot s + R_2)\cdot(R_1\cdot C\cdot s + 1)}{L\cdot C\cdot s^2 + (R_2 + R_1)\cdot C\cdot s + 1}$$

By voltage division: $T_V(s) = \dfrac{R_2}{L\cdot s + R_2}$

(b) $R_1 := 300 \quad R_2 := R_1$

$L := 0.05 \quad C := 4\cdot 10^{-6}$

$$\text{polyroots}\left[\begin{bmatrix} 1 \\ (R_1 + R_2)\cdot C \\ L\cdot C \end{bmatrix}\right] = \begin{pmatrix} -1.157\times 10^4 \\ -432.236 \end{pmatrix} \quad \frac{R_2}{L} = 6\times 10^3 \quad \frac{1}{R_1\cdot C} = 833.333$$

Z(s) has zeros at s = $-R_2/L$ = -6000 & s = $-1/R_1C$ = -833 and poles at about s = -432.2 and -11570
T_V(s) has a pole at s = $-R_2/L$ = -6000

11-2 (a) $Z(s) = \dfrac{1}{\left(R_1 + \dfrac{1}{C\cdot s}\right)^{-1} + (L\cdot s + R_2)^{-1}}$

$$Z(s) = \frac{(L\cdot s + R_2)\cdot(R_1\cdot C\cdot s + 1)}{L\cdot C\cdot s^2 + (R_2 + R_1)\cdot C\cdot s + 1}$$

By voltage division

$$V_2(s) = V_1(s)\cdot\frac{R_2}{L\cdot s + R_2} = (I_1(s)\cdot Z(s))\cdot\frac{R_2}{L\cdot s + R_2}$$

$$V_2(s) = \left[\frac{(L\cdot s + R_2)\cdot(R_1\cdot C\cdot s + 1)}{L\cdot C\cdot s^2 + (R_2 + R_1)\cdot C\cdot s + 1}\cdot\frac{R_2}{L\cdot s + R_2}\right]\cdot I_1(s)$$

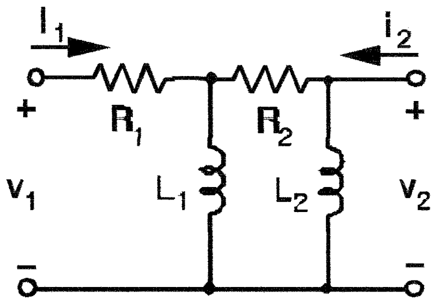
$$V_2(s) = \left[\frac{R_2\cdot(R_1\cdot C\cdot s + 1)}{L\cdot C\cdot s^2 + (R_2 + R_1)\cdot C\cdot s + 1}\right]\cdot I_1(s) \quad \text{hence} \quad T_Z(s) = \frac{R_2\cdot(R_1\cdot C\cdot s + 1)}{L\cdot C\cdot s^2 + (R_2 + R_1)\cdot C\cdot s + 1}$$

(b) $R_1 := 1000 \quad R_2 := R_1 \quad L := 0.1 \quad C := 50\cdot 10^{-9}$

$$\text{polyroots}\left[\begin{bmatrix} 1 \\ (R_1 + R_2)\cdot C \\ L\cdot C \end{bmatrix}\right] = \begin{pmatrix} -1\times 10^4 - i\times 10^4 \\ -1\times 10^4 + i\times 10^4 \end{pmatrix} \quad \frac{R_2}{L} = 1\times 10^4 \quad \frac{1}{R_1\cdot C} = 2\times 10^4$$

Z(s) has zeros at s = $-R_2/L$ = -10000 & s = $-1/R_1C$ = -20000 and poles at s = -10000±j10000
T_Z(s) has the same poles as Z(s) & has zeros at s = -20000 and infinity.

11-3 (a) $Z(s) = R_1 + \dfrac{1}{\dfrac{1}{L_1 \cdot s} + \dfrac{1}{R_2}}$



$$Z(s) = \frac{\left[(R_1 + R_2)\cdot L_1\cdot s + R_1\cdot R_2\right]}{(L_1\cdot s + R_2)}$$

By current division:

$$I_2(s) = \frac{-L_1\cdot s}{L_1\cdot s + R_2}\cdot I_1(s)$$

$$T_Y(s) = \frac{I_2(s)}{V_1(s)} = \left(\frac{I_2(s)}{I_1(s)}\cdot\frac{I_1(s)}{V_1(s)}\right) = \frac{I_2(s)}{I_1(s)}\cdot\frac{1}{Z(s)}$$

$$T_Y(s) = \frac{-L_1\cdot s}{L_1\cdot s + R_2}\cdot\frac{L_1\cdot s + R_2}{(R_1 + R_2)\cdot L_1\cdot s + R_1\cdot R_2} = \frac{-L_1\cdot s}{(R_1 + R_2)\cdot L_1\cdot s + R_1\cdot R_2}$$

(b) $R_1 := 100 \quad R_2 := 50 \quad L_1 := 0.75$

$$\text{polyroots}\left(\begin{pmatrix} R_2 \\ L_1 \end{pmatrix}\right) = -66.667$$

$$\text{polyroots}\left[\begin{bmatrix} R_1\cdot R_2 \\ L_1\cdot(R_1 + R_2) \end{bmatrix}\right] = -44.444$$

Z(s) has a zero at s = -44.4 and a pole at s = -66.7
T_Y(s) has a zero at s = 0 and a pole at s = -44.4

11-4 (a) $Z(s) = R_1 + \dfrac{1}{\dfrac{1}{L_1\cdot s} + \dfrac{1}{R_2}}$

$$Z(s) = \frac{\left[R_1\cdot R_2 + (R_1 + R_2)\cdot L_1\cdot s\right]}{(R_2 + L_1\cdot s)}$$

By current division:

$$I_2(s) = \frac{-L_1\cdot s}{L_1\cdot s + R_2}\cdot I_1(s)$$

Hence $T_I(s) = \dfrac{I_2(s)}{I_1(s)} = \dfrac{-L_1\cdot s}{L_1\cdot s + R_2}$

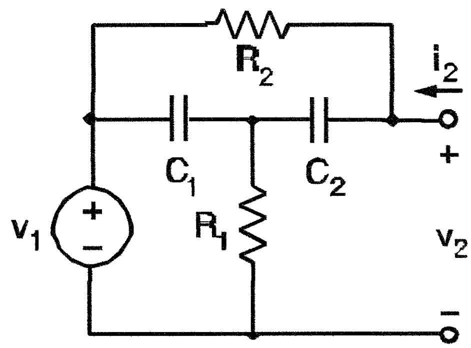
(b) $R_1 := 500 \quad R_2 := 2000 \; L_1 := 0.4$

$$\text{polyroots}\left(\begin{pmatrix} R_2 \\ L_1 \end{pmatrix}\right) = -5\times 10^3 \qquad \text{polyroots}\left[\begin{bmatrix} R_1\cdot R_2 \\ (R_1 + R_2)\cdot L_1 \end{bmatrix}\right] = -1\times 10^3$$

Z(s) has a zero at s = -1000 and a pole at s = -5000.
Tz(s) has a zero at s = 0 and a pole at s = -5000.

11-5 Writing node-voltage equations

$$\begin{pmatrix} C_1 \cdot s + C_2 \cdot s + \frac{1}{R_1} & -C_2 \cdot s \\ -C_2 \cdot s & C_2 \cdot s + \frac{1}{R_2} \end{pmatrix} \cdot \begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{pmatrix} C_1 \cdot s \cdot V_1 \\ \frac{1}{R_2} \cdot V_1 \end{pmatrix}$$



$$\Delta = \frac{\left[R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 + (R_1 \cdot C_1 + R_1 \cdot C_2 + R_2 \cdot C_2) \cdot s + 1\right]}{R_1 \cdot R_2}$$

$$\Delta_B = \det\begin{pmatrix} C_1 \cdot s + C_2 \cdot s + \frac{1}{R_1} & C_1 \cdot s \\ -C_2 \cdot s & \frac{1}{R_2} \end{pmatrix} \cdot V_1 = \frac{\left(C_1 \cdot s \cdot R_1 + C_2 \cdot s \cdot R_1 + 1 + R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2\right)}{(R_1 \cdot R_2)} \cdot V_1(s)$$

$$T_V(s) = \frac{V_B(s)}{V_1(s)} = \frac{\Delta_B}{\Delta} \cdot \frac{1}{V_1(s)} = \frac{R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 + (R_1 \cdot C_1 + R_1 \cdot C_2) \cdot s + 1}{R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 + (R_1 \cdot C_1 + R_1 \cdot C_2 + R_2 \cdot C_2) \cdot s + 1}$$

$R_1 := 1000 \quad R_2 := 4000 \quad C_1 := 1.5 \cdot 10^{-6} \quad C_2 := 0.5 \cdot 10^{-6}$

$$\text{polyroots}\left(\begin{pmatrix} 1 \\ R_1 \cdot C_1 + R_1 \cdot C_2 \\ R_1 \cdot R_2 \cdot C_1 \cdot C_2 \end{pmatrix}\right) = \begin{pmatrix} -333.333 - 471.405i \\ -333.333 + 471.405i \end{pmatrix}$$

$$\text{polyroots}\left(\begin{pmatrix} 1 \\ R_1 \cdot C_1 + R_1 \cdot C_2 + R_2 \cdot C_2 \\ R_1 \cdot R_2 \cdot C_1 \cdot C_2 \end{pmatrix}\right) = \begin{pmatrix} -1 \times 10^3 \\ -3.333 \times 10^2 \end{pmatrix}$$

Tv(s) has zeros at s = -333.3$\pm$j471,4 and poles at s = -333.3 and -1000

11-6 See circuit in 11-5 above

$$Z(s) = R_1 + \cfrac{1}{C_1 \cdot s + \cfrac{1}{R_2 + \cfrac{1}{C_2 \cdot s}}} = \frac{\left(R_1 \cdot s^2 \cdot C_1 \cdot R_2 \cdot C_2 + R_1 \cdot s \cdot C_1 + R_1 \cdot s \cdot C_2 + R_2 \cdot C_2 \cdot s + 1\right)}{\left[s \cdot (C_1 \cdot R_2 \cdot C_2 \cdot s + C_1 + C_2)\right]}$$

$R_1 := 10^3 \quad R_2 := 2 \cdot 10^3 \quad C_1 := 0.5 \cdot 10^{-6} \quad C_2 := 0.25 \cdot 10^{-6}$

Zeros

$$\text{polyroots}\left(\begin{pmatrix} 1 \\ R_1 \cdot C_1 + R_2 \cdot C_2 + R_1 \cdot C_2 \\ R_1 \cdot R_2 \cdot C_1 \cdot C_2 \end{pmatrix}\right) = \begin{pmatrix} -4 \times 10^3 \\ -1 \times 10^3 \end{pmatrix}$$

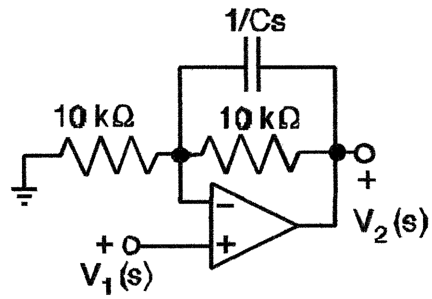
Poles

$$\text{polyroots}\left(\begin{pmatrix} 0 \\ C_1 + C_2 \\ R_2 \cdot C_1 \cdot C_2 \end{pmatrix}\right) = \begin{pmatrix} -3 \times 10^3 \\ 0 \end{pmatrix}$$

11-7

$$T_V(s) = \frac{R + \dfrac{1}{C\cdot s + \dfrac{1}{R}}}{R} = \left[\frac{(C\cdot s\cdot R + 2)}{(C\cdot s\cdot R + 1)}\right] = \frac{s + 400}{s + 200}$$

$$R := 10^4 \qquad C := \frac{1}{200\cdot R} \qquad C = 5 \times 10^{-7}$$



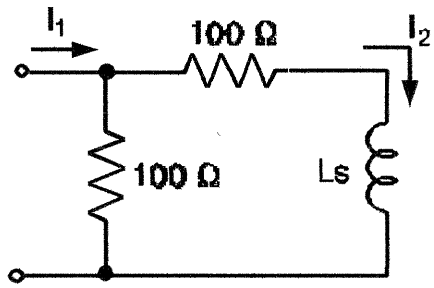
11-8

$$T_I(s) = \frac{R}{R + R + L\cdot s} = \frac{200}{400 + s}$$

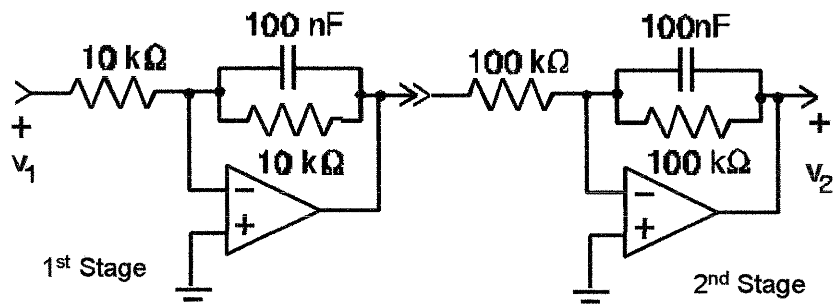
$$\frac{R}{L} = 200 \qquad R := 100 \qquad L := \frac{R}{200}$$

$$L = 0.5$$

$$Z(s) = \frac{1}{\dfrac{1}{100} + \dfrac{1}{100 + 0.5\cdot s}} = \frac{100\cdot(s + 200)}{s + 400}$$



11-9



$$Z_2(s) = \frac{1}{10^{-7}\cdot s + \dfrac{1}{10^4}} = \frac{10^7}{(s + 1000)}$$

$$Z_1(s) = 10^4 \qquad T_{V1}(s) = -\frac{Z_2}{Z_1} = \frac{-1000}{s + 1000}$$

$$Z_2(s) = \frac{1}{10^{-7}\cdot s + \dfrac{1}{10^5}} = \frac{10^7}{(s + 100)}$$

$$Z_1(s) = 10^5 \qquad T_{V2}(s) = -\frac{Z_2}{Z_1} = \frac{-100}{s + 100}$$

Using the chain rule

$$T_V(s) = T_{V1}(s)\cdot T_{V2}(s) = \left(\frac{-1000}{s + 1000}\right)\cdot\left(\frac{-100}{s + 100}\right)$$

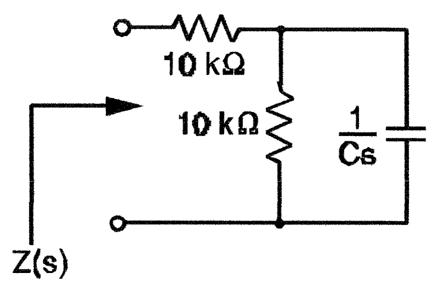
Tv(s) has poles at s = -1000, s = -100 and a double zero at infinity

11-10

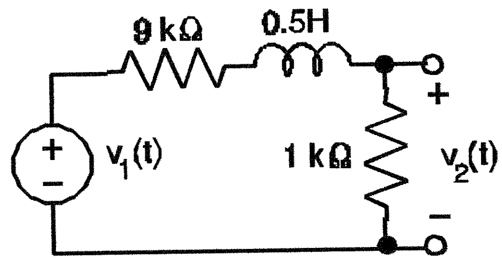
$$Z(s) = 10^4 + \frac{1}{C\cdot s + 10^{-4}} = 20000\cdot\frac{(5000\cdot C\cdot s + 1)}{(10000\cdot C\cdot s + 1)}$$

$$Z(s) = \frac{s + \dfrac{1}{5000\cdot C}}{s + \dfrac{1}{10000\cdot C}}\cdot 10^4 \qquad C = \frac{1}{5000\cdot 200} = \frac{1}{10000\cdot 100}$$

$$C = 10^{-6}\ \mathrm{F}$$



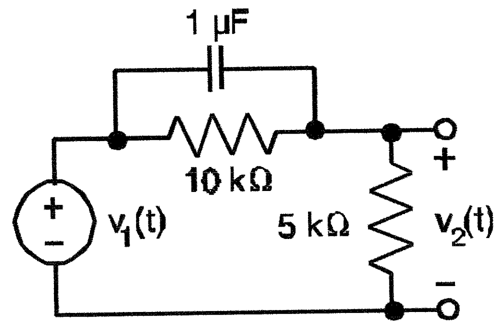
11-11 By voltage division:



$$T_V(s) = \frac{1000}{1000 + 0.5\cdot s + 9000} = \frac{2000.}{(s + 20000.)}$$

$$h(t) = L^{-1}\cdot\left[\frac{2000.}{(s + 20000.)}\right] = \left(2000\cdot e^{-20000\cdot t}\right)\cdot u(t)$$

11-12 By voltage division:

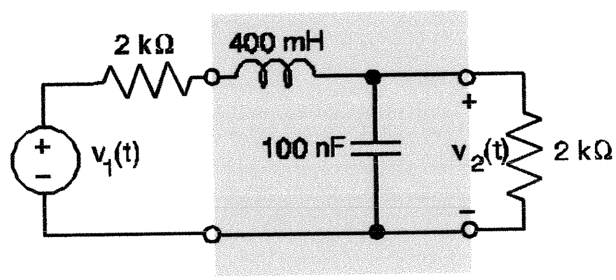


$$T_V(s) = \frac{5000}{5000 + \dfrac{1}{10^{-6}\cdot s + 10^{-4}}} = \frac{(s + 100)}{(s + 300)}$$

$$h(t) = L^{-1}\cdot\left[\frac{(s + 100)}{(s + 300)}\right] = L^{-1}\cdot\left(1 - \frac{200}{s + 300}\right)$$

$$h(t) = \delta(t) - 200\cdot e^{-300\cdot t}\cdot u(t)$$

11-13 Using voltage division



$$T_V(s) = \frac{\dfrac{1}{\dfrac{1}{2000} + 100\cdot 10^{-9}\cdot s}}{\dfrac{1}{\dfrac{1}{2000} + 100\cdot 10^{-9}\cdot s} + 2000 + 400\cdot 10^{-3}\cdot s}$$

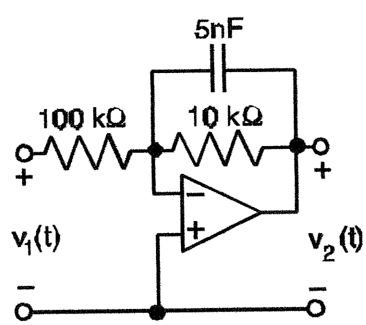
$$T_V(s) = \frac{25\cdot 10^{6}}{\left(50000000 + 10000\cdot s + s^{2}\right)} \quad \text{polyroots}\left(\begin{pmatrix} 50000000 \\ 10000 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} -5\times 10^{3} - 5i\times 10^{3} \\ -5\times 10^{3} + 5i\times 10^{3} \end{pmatrix}$$

$$T_V(s) = \frac{25\cdot 10^{6}}{(s + 5000)^{2} + 5000^{2}} \quad G(s) = \frac{T_V(s)}{s} = \frac{1}{s}\cdot\frac{25000000}{(s + 5000)^{2} + 5000^{2}} = \frac{1}{2\cdot s} - \frac{1}{2}\cdot\frac{(s + 5000) + 5000}{(s + 5000)^{2} + 5000^{2}}$$

$$g(t) = \frac{1}{2} - \frac{1}{2}\cdot\exp(-5000\cdot t)\cdot\cos(5000\cdot t) - \frac{1}{2}\cdot\exp(-5000\cdot t)\cdot\sin(5000\cdot t)$$

$$g(t) = \left(\frac{1}{2} + \frac{1}{\sqrt{2}}\cdot\exp(-5000\cdot t)\cdot\cos\left(5000\cdot t - \frac{5\cdot\pi}{4}\right)\right)\cdot u(t)$$

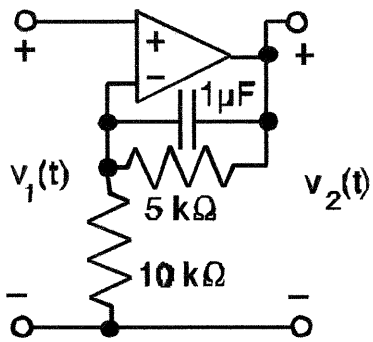
11-14



$$T_V(s) = -\frac{\dfrac{1}{\dfrac{1}{10\cdot 10^{3}} + 5\cdot 10^{-9}\cdot s}}{100\cdot 10^{3}} = \frac{-2000}{(20000 + s)}$$

$$G(s) = \frac{T_V(s)}{s} = \frac{-2000}{(20000 + s)\cdot s} = \frac{1}{[10\cdot(20000 + s)]} - \frac{1}{(10\cdot s)} \quad g(t) = \left(\frac{1}{10}\cdot\exp(-20000\cdot t) - \frac{1}{10}\right)\cdot u(t)$$

11-15



$$T_V(s) = \frac{10^4 + \dfrac{1}{\dfrac{1}{5\cdot 10^3} + 10^{-6}\cdot s}}{10^4} = \frac{(300 + s)}{(200 + s)}$$

$$H(s) = T_V(s) = \frac{(300 + s)}{(200 + s)} = 1 + \frac{100}{s + 200}$$

$h(t) = \delta(t) + [(100\cdot \exp(-200\cdot t))\cdot u(t)$

11-16 $h(t) = 15\cdot(\exp(-10\cdot t) - \exp(-30\cdot t))\cdot u(t)$ $\quad H(s) = \left[\dfrac{15}{(s + 10)} - \dfrac{15}{(s + 30)}\right] = \dfrac{300}{[(s + 10)\cdot(s + 30)]}$

$$G(s) = \frac{H(s)}{s} = \frac{300}{[(s + 10)\cdot(s + 30)]\cdot s} = \frac{-3}{[2\cdot(s + 10)]} + \frac{1}{[2\cdot(s + 30)]} + \frac{1}{s}$$

$$g(t) = \left(\frac{-3}{2}\cdot\exp(-10\cdot t) + \frac{1}{2}\cdot\exp(-30\cdot t) + 1\right)\cdot u(t)$$

Alternatively, operating in the time domain

$$g(t) = \int_0^t h(x)\,dx = \int_0^t 15\cdot(\exp(-10\cdot x) - \exp(-30\cdot x))\,dx = \left(\frac{-3}{2}\cdot\exp(-10\cdot t) + \frac{1}{2}\cdot\exp(-30\cdot t) + 1\right)$$

11-17 (a)

$g(t) = -\exp(-2000\cdot t)\cdot u(t)$ $\quad T(s) = s\cdot \mathsf{L}(g(t)) = \dfrac{-s}{(s + 2000)}$ $\quad H(s) = T(s) = \dfrac{-s}{(s + 2000)} = -1 + \dfrac{2000}{s + 2000}$

$$h(t) = \mathsf{L}^{-1}\cdot(H(s)) = \mathsf{L}^{-1}\cdot\left(-1 + \frac{2000}{s + 2000}\right) = -\delta(t) + 2000\cdot\exp(-2000\cdot t)\cdot u(t)$$

(b) $g(t) = (1 - \exp(-2000\cdot t))\cdot u(t)$ $\quad T(s) = s\cdot \mathsf{L}(g(t)) = 1 - \dfrac{s}{s + 2000} = \dfrac{2000}{(s + 2000)}$ $\quad H(s) = T(s)$

$$h(t) = \mathsf{L}^{-1}\cdot(H(s)) = \mathsf{L}^{-1}\cdot\left(\frac{2000}{s + 2000}\right) = 2000\cdot\exp(-2000\cdot t)\cdot u(t)$$

11-18 (a) $h(t) = -1000\cdot\exp(-2000\cdot t)\cdot u(t)$ $\quad T(s) = \mathsf{L}(h(t)) = \dfrac{-1000}{(s + 2000)}$ $\quad G(s) = \dfrac{T(s)}{s} = \dfrac{-1000}{s\cdot(s + 2000)}$

$G(s) = \dfrac{-0.5}{s} + \dfrac{0.5}{(s + 2000)}$ $\quad g(t) = 0.5\cdot(-1 + \exp(-2000\cdot t))\cdot u(t)$

(b) $h(t) = \delta(t) - 2000\cdot\exp(-2000\cdot t)$ $\quad T(s) = \mathsf{L}(h(t)) = 1 + \dfrac{-2000}{(s + 2000)} = \dfrac{s}{(s + 2000)}$

$G(s) = \dfrac{T(s)}{s} = \dfrac{1}{s + 2000}$ $\quad g(t) = \exp(-2000\cdot t)\cdot u(t)$

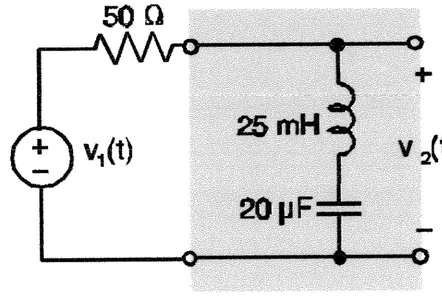
11-19 Given $g(t) = (\exp(-2000\cdot t)\cdot\sin(5000\cdot t))\cdot u(t)$ $\quad G(s) = \dfrac{5000}{\left[(s + 2000)^2 + 5000^2\right]}$

$T(s) = s\cdot G(s) = \dfrac{5000\cdot s}{\left[(s + 2000)^2 + 5000^2\right]}$ zeros at s = 0 and infinity, poles at s = -2000 $\pm$ j5000.

11-20 $I(s) = 1 \quad V(s) = L^{-1}\cdot\left(\delta(t) - 200\cdot\exp(-200\cdot t)\cdot\sin(200\cdot t)\right) = \dfrac{s^2 + 400\cdot s + 40000}{s^2 + 400\cdot s + 80000} = \dfrac{(s+200)^2}{(s+200)^2 + 200^2}$

$Z(s) = \dfrac{V(s)}{I(s)} = \dfrac{(s+200)^2}{(s+200)^2 + 200^2}$ **For v(t) = δ(t) we have I(s) = 1/Z(s)**

$I(s) = \dfrac{s^2 + 400\cdot s + 80000}{(s+200)^2} = 1 + \dfrac{40000}{(s+200)^2}$ $\quad i(t) = L^{-1}\cdot(I(s)) = \delta(t) + (40000\cdot t\cdot\exp(-200\cdot t))\cdot u(t)$



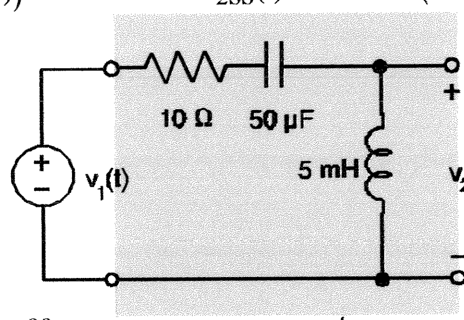
$$T_V(s) = \frac{25\cdot 10^{-3}\cdot s + \dfrac{1}{20\cdot 10^{-6}\cdot s}}{50 + \left(25\cdot 10^{-3}\cdot s + \dfrac{1}{20\cdot 10^{-6}\cdot s}\right)}$$

$$T_V(s) := \frac{\left(s^2 + 2000000\right)}{\left(2000\cdot s + s^2 + 2000000\right)}$$

$\omega := 500 \quad 5\cdot\left|T_V(j\cdot\omega)\right| = 4.341 \qquad \dfrac{180}{\pi}\cdot\arg\left(T_V(j\cdot\omega)\right) = -29.745 \qquad v_{2SS}(t) = 4.341\cdot\cos\left(500\cdot t - 29.745^o\right)$

$\omega := 1000 \quad 10\cdot\left|T_V(j\cdot\omega)\right| = 4.472 \qquad \dfrac{180}{\pi}\cdot\arg\left(T_V(j\cdot\omega)\right) = -63.435 \qquad v_{2SS}(t) = 4.472\cdot\cos\left(1000\cdot t - 63.435^o\right)$

11-22



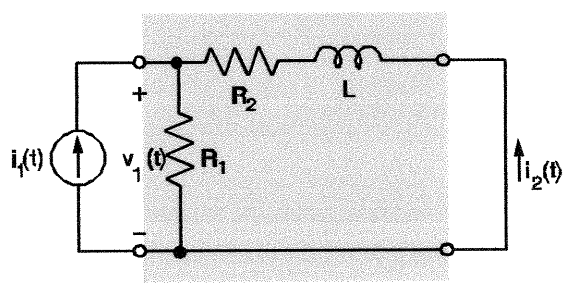
$$T_V(s) = \frac{5\cdot 10^{-3}\cdot s}{\left(10 + \dfrac{1}{50\cdot 10^{-6}\cdot s}\right) + 5\cdot 10^{-3}\cdot s}$$

$$T_V(s) := \frac{s^2}{\left(2000\cdot s + 4000000 + s^2\right)}$$

$\omega := 2000 \quad 10\cdot\left|T_V(j\cdot\omega)\right| = 10 \qquad \dfrac{180}{\pi}\cdot\arg\left(T_V(j\cdot\omega)\right) = 90 \qquad v_{2SS}(t) = 10\cdot\cos\left(2000\cdot t + 90^o\right)$

$\omega := 4000 \quad 3\cdot\left|T_V(j\cdot\omega)\right| = 3.328 \qquad \dfrac{180}{\pi}\cdot\arg\left(T_V(j\cdot\omega)\right) = 33.69 \qquad v_{2SS}(t) = 3.327\cdot\cos\left(4000\cdot t + 33.69^o\right)$

11-23



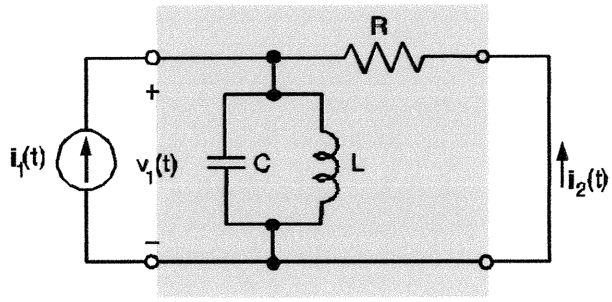
$$T_I(s) = \frac{-R_1}{R_1 + R_2 + L\cdot s}$$

$$T_I(s) := \frac{-100}{100 + 400 + 0.1\cdot s}$$

$\omega := 5000 \quad 5\cdot\left|T_I(j\cdot\omega)\right| = 0.707 \qquad \dfrac{180}{\pi}\cdot\arg\left(T_I(j\cdot\omega)\right) = 135 \qquad i_{2SS}(t) = 0.707\cdot\cos\left(5000\cdot t + 135^0\right)\text{mA}$

$\omega := 10^4 \quad 10\cdot\left|T_I(j\cdot\omega)\right| = 0.894 \qquad \dfrac{180}{\pi}\cdot\arg\left(T_I(j\cdot\omega)\right) = 116.6 \qquad i_{2SS}(t) = 0.894\cdot\cos\left(10^4\cdot t + 116.6^0\right)\text{mA}$

$$T_I(s) = \frac{-\frac{1}{R}}{\frac{1}{R} + C\cdot s + \frac{1}{L\cdot s}} = \frac{-L\cdot s}{R\cdot L\cdot C\cdot s^2 + L\cdot s + R}$$



$R := 1000 \quad C := 500\cdot 10^{-9} \quad L := 2 \quad R\cdot L\cdot C = 1\times 10^{-3}$

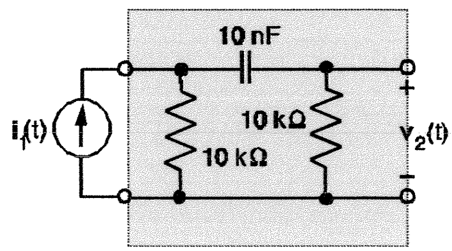
$$T_I(s) := \frac{-2\cdot s}{10^{-3}\cdot s^2 + 2\cdot s + 1000}$$

$\omega := 1000 \quad 5\cdot\left|T_I(j\cdot\omega)\right| = 5 \qquad \frac{180}{\pi}\cdot\arg\left(T_I(j\cdot\omega)\right) = 180 \qquad i_{2SS}(t) = 5\cdot\cos\left(1000\cdot t + 180^0\right)$ mA

$\omega := 1000 \quad 10\cdot\left|T_I(j\cdot\omega)\right| = 10 \qquad \frac{180}{\pi}\cdot\arg\left(T_I(j\cdot\omega)\right) - 90 = 90 \qquad i_{2SS}(t) = 10\cdot\cos\left(1000\cdot t + 90^0\right)$ mA

11-25

$$T_Z(s) = 10^4\cdot\left(\frac{10^4}{10^4 + 10^4 + \frac{1}{10\cdot 10^{-9}\cdot s}}\right)$$



$$T_Z(s) := \frac{5000\cdot s}{s + 5000}$$

$\omega := 5000 \quad 10\cdot 10^{-3}\cdot\left|T_Z(j\cdot\omega)\right| = 35.355 \qquad \frac{180}{\pi}\cdot\arg\left(T_Z(j\cdot\omega)\right) = 45 \qquad v_{2SS}(t) = 35.4\cdot\cos\left(5000\cdot t + 45^o\right)$

$\omega := 2500 \quad 5\cdot 10^{-3}\cdot\left|T_Z(j\cdot\omega)\right| = 11.18 \qquad \frac{180}{\pi}\cdot\arg\left(T_Z(j\cdot\omega)\right) = 63.435 \qquad v_{2SS}(t) = 11.2\cdot\cos\left(2500\cdot t + 63.4^o\right)$

11-26 $\quad T(s) := \frac{s + 200}{s + 50}$

$\omega := 200 \quad 5\cdot\left|T(j\cdot\omega)\right| = 6.86 \qquad \frac{180}{\pi}\cdot\arg\left(T(j\cdot\omega)\right) = -30.964 \qquad y_{SS}(t) = 6.86\cdot\cos\left(200\cdot t - 30.96^o\right)$

11-27 $g(t) = \exp(-1000\cdot t)\cdot u(t)$

$G(s) := \frac{1}{s + 1000} \qquad T(s) := s\cdot G(s) \qquad x(t) = 5\cdot\cos(2000\cdot t)$

$5\cdot\left|T(j\cdot 2000)\right| = 4.472 \qquad \frac{180}{\pi}\cdot\arg(T(j\cdot 2000)) = 26.565 \quad y_{SS}(t) = 4.472\cdot\cos\left(2000\cdot t + 26.565^o\right)$

11-28 $g(t) = 2\cdot\exp(-100\cdot t) - 1$ $G(s) = \frac{2}{s+100} - \frac{1}{s} = \frac{s-100}{s+100}\cdot\frac{1}{s}$ $\quad T(s) = s\cdot G(s) \quad T(s) := \frac{s-100}{s+100}$

$$5\cdot\left|T(j\cdot\omega)\right| = 5\cdot\frac{|j\cdot\omega - 100|}{|j\cdot\omega + 100|} = 5\cdot\frac{\sqrt{\omega^2 + 100^2}}{\sqrt{\omega^2 + 100^2}} = 5 \quad \text{QED}$$

11-29 $h(t) = -\delta(t) + 800\cdot\exp(-1000\cdot t)\cdot u(t)$ **then** $H(s) = \left(-1 + \frac{800}{s+1000}\right) = -\left(\frac{s+200}{s+1000}\right)$

$T(s) := -\frac{s+200}{s+1000} \qquad x(t) = 3\cdot\cos(400\cdot t - 90) \qquad AMP := 3\cdot\left|T(j\cdot 400)\right| \qquad AMP = 1.246$

$ANGLE := \frac{180}{\pi}\cdot\arg(T(j\cdot 400)) - 90 \qquad ANGLE = -228.366$ **or** $ANGLE = 131.634$

11-30 $h(t) = 400\cdot(\exp(-100\cdot t) - \exp(-5000\cdot t))$ **then**

$H(s) = 400\cdot\left(\frac{1}{s+100} - \frac{1}{s+5000}\right) = \frac{400\cdot 4900}{(s+100)\cdot(s+5000)}$

$T(s) := \frac{400\cdot 4900}{(s+100)\cdot(s+5000)} \qquad x(t) = 5\cdot\cos(700\cdot t) \qquad AMP := 5\cdot\left|T(j\cdot 700)\right| \qquad AMP = 2.745$

$ANGLE := \frac{180}{\pi}\cdot\arg(T(j\cdot 700)) \qquad ANGLE = -89.84$

$u(x) := \text{if}(x < 0, 0, 1)$ <---**Defines a step function**

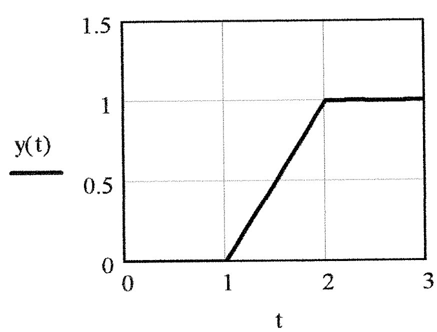
11-31 $h(t) := u(t) - u(t-1) \qquad x(t) := u(t-1)$

$y(t) := \int_0^t h(t-\tau)\cdot x(\tau)\,d\tau \qquad t := 0, .1 .. 3$

$y(t) = 0 \qquad t < 1$

$y(t) = t - 1 \qquad 1 \le t \le 2$

$y(t) = 1 \qquad 2 < t$



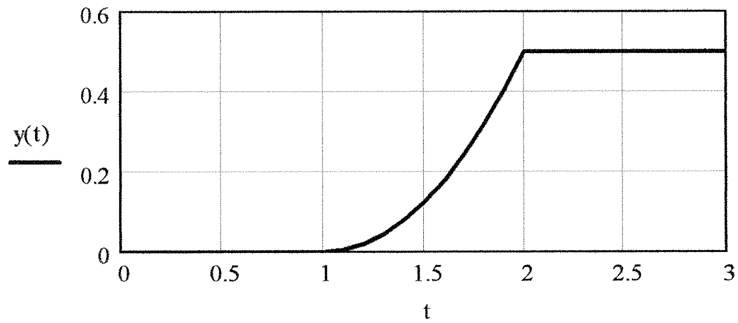
11-32 $h(t) := t\cdot(u(t) - u(t-1)) \qquad x(t) := u(t-1) \qquad y(t) := \int_0^t h(t-\tau)\cdot x(\tau)\,d\tau$

For t < 1 $\quad y(t) = 0$

For 1 ≤ t ≤ 2 $\quad y(t) = \int_1^t (t - \tau)\,d\tau = t\cdot\int_1^t 1\,d\tau - \int_1^t \tau\,d\tau = t\cdot(t-1) - \frac{t^2 - 1}{2} = \frac{t^2 - 2\cdot t + 1}{2}$

For 2 < t $\quad y(t) = \int_{t-1}^t (t - \tau)\,d\tau = \int_{t-1}^t 1\,d\tau - \int_{t-1}^t \tau\,d\tau = 0.5$

11-32 Continued



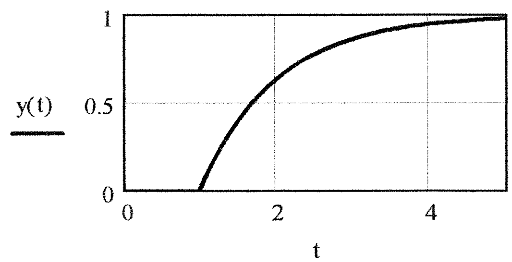
11-33 $h(t) := \exp(-t)$ $x(t) := u(t-1)$

$$y(t) := \int_0^t h(t-\tau)\cdot x(\tau)\, d\tau$$

$t := 0, .1 .. 5$

$y(t) = 0 \qquad t < 1$

$y(t) = 1 - \exp[-(t-1)] \qquad 1 \le t$

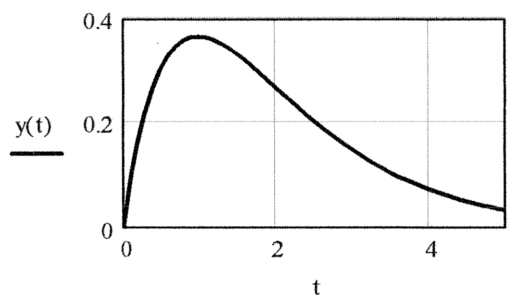


11-34 $h(t) := \exp(-t)$ $x(t) := \exp(-t)$

$$y(t) := \int_0^t h(t-\tau)\cdot x(\tau)\, d\tau \qquad t := 0, 0.1 .. 5$$

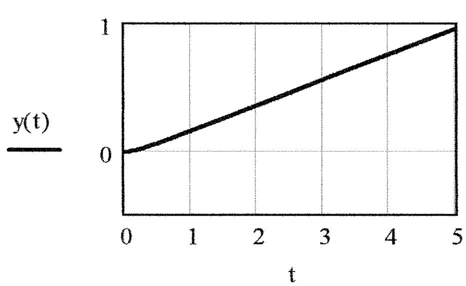
$y(t) = 0 \qquad t < 0$

$y(t) = t\cdot\exp(-t) \qquad 0 \le t$



11-35 $h(t) := \exp(-5\cdot t)$ $x(t) := t$

$$y(t) := \int_0^t h(t-\tau)\cdot x(\tau)\, d\tau \qquad t := 0, 0.1 .. 5 \qquad y(t) = 0 \qquad t < 0$$



$$\int_0^t \exp[-5\cdot(t-\tau)]\cdot\tau\, d\tau = \frac{-1}{25} + \frac{1}{5}\cdot t + \frac{1}{25}\cdot\exp(-5\cdot t) \quad 0 \le t$$

11-36 $H(s) = \dfrac{1}{s+5}$ $x(t) = t \cdot u(t)$

$$Y(s) = \frac{1}{s^{2} \cdot (s+5)}$$

$$y(t) = \left[\left(\frac{1}{25}\right) \cdot \exp(-5 \cdot t) - \frac{1}{25} + \frac{t}{5}\right] \cdot u(t)$$

11-37 $F(s) = \dfrac{s}{(s+1) \cdot (s+2)} = F_1(s) \cdot F_2(s)$ $F_1(s) = \dfrac{1}{s+1}$ $F_2(s) = \dfrac{s}{s+2}$

$f_1(t) = \exp(-t)$ $f_2(t) = \delta(t) - 2 \cdot \exp(-2 \cdot t)$

$f(t) = f_1(t)*f_2(t) = f_1(t)*\delta(t) - f_1(t)*2\exp(-2t) = f_1(t) - \exp(-t)*2\exp(-2t)$

$$f(t) = \exp(-t) - \int_0^t \exp[-(t-\tau)] \cdot 2 \cdot \exp(-2 \cdot \tau)\, d\tau$$

$$-\int_0^t \exp[-(t-\tau)] \cdot 2 \cdot \exp(-2 \cdot \tau)\, d\tau = 2 \cdot \exp(-2 \cdot t) - 2 \cdot \exp(-t) \qquad f(t) = -\exp(-t) + 2 \cdot \exp(-2 \cdot t)$$

Checking $F(s) = \dfrac{s}{(s+1) \cdot (s+2)} = \dfrac{-1}{s+1} + \dfrac{2}{s+2}$ $f(t) = -\exp(-t) + 2 \cdot \exp(-2 \cdot t)$

11-38 $h(t) = u(t)$ $y(t) = \displaystyle\int_0^t u(t-\tau) \cdot x(\tau)\, d\tau$

The integration on τ is on the range from 0 to t. On this range $u(t - \tau) = 1$, hence the integration reduces to

$$y(t) = \int_0^t x(\tau)\, d\tau \qquad \text{QED}$$

11-39 $x(t) = u(t)$ $y(t) = \displaystyle\int_0^t h(t-\tau) \cdot u(\tau)\, d\tau = \int_0^t u(t-\tau) \cdot h(\tau)\, d\tau$

The integration on τ is on the range from 0 to t. On this range $u(t - \tau) = 1$, hence the integration reduces to

$$y(t) = \int_0^t h(\tau)\, d\tau \qquad \text{QED}$$

11-40 $h_1(t) = \exp(-2 \cdot t)$ $h_2(t) = 4 \cdot \exp(-4 \cdot t)$ $h(t) = h_1(t) * h_2(t)$

$$h(t) = \int_0^t \exp[-2 \cdot (t - \tau)] \cdot 4 \cdot \exp(-4 \cdot \tau)\, d\tau = 2 \cdot \exp(-2 \cdot t) - 2 \cdot \exp(-4 \cdot t)$$

Checking $H(s) = \frac{1}{s+2} \cdot \frac{4}{s+4} = \frac{2}{s+2} - \frac{2}{s+4}$ $h(t) = 2 \cdot \exp(-2 \cdot t) - 2 \cdot \exp(-4 \cdot t)$

11-41 $h(t) = 1000 \cdot (\exp(-1000 \cdot t))$ **then** $H(s) = T(s) = 1000 \cdot \left(\frac{1}{s + 1000}\right) = \frac{1000}{(s + 1000)}$

$x(t) = 5 \cdot t \cdot u(t)$ $X(s) = \frac{5}{s^2}$ $Y(s) = T(s) \cdot X(s) = \frac{5000}{s^2 \cdot (s + 1000)} = \frac{5}{s^2} - \frac{1}{(200 \cdot s)} + \frac{1}{[200 \cdot (s + 1000)]}$

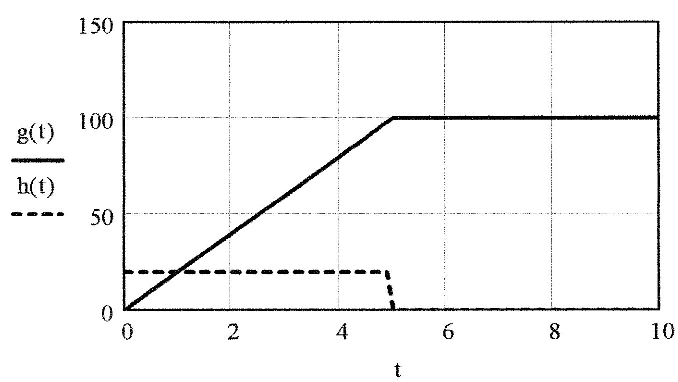
$$y(t) = \left[5t - \frac{1}{200} \cdot (1 - \exp(-1000 \cdot t)) \right] \cdot u(t)$$

11-42 $g(t) = 0.5 \cdot (1 - \exp(-100 \cdot t))$ **then** $T(s) = s \cdot G(s) = s \cdot 0.5 \cdot \left(\frac{1}{s} - \frac{1}{s + 100}\right) = \frac{50}{(s + 100)}$

$x(t) = \exp((-200 \cdot t)) \cdot u(t)$ $X(s) = \frac{1}{s + 200}$ $Y(s) = T(s) \cdot X(s) = \frac{50}{(s + 100) \cdot (s + 200)} = \frac{0.5}{(s + 100)} - \frac{0.5}{(s + 200)}$

$y(t) = 0.5 \cdot (\exp(-100 \cdot t) - \exp(-200 \cdot t)) \cdot u(t)$

11-43 $g(t) := 20 \cdot [\, t \cdot u(t) - (t - 5) \cdot u(t - 5)\,]$ $t := 0, .1 .. 10$ $h(t) = \frac{d}{dt} g(t)$ $h(t) := 20 \cdot (u(t) - u(t - 5))$



11-44 $T(s) = H(s) = \frac{s + 2000}{s + 1000}$ $x(t) = 5 \cdot (\exp(-1000 \cdot t)) \cdot u(t)$ $X(s) = \frac{5}{s + 1000}$ $Y(s) = \frac{5 \cdot (s + 2000)}{(s + 1000)^2}$

$Y(s) = \frac{5000}{(s + 1000)^2} + \frac{5}{(s + 1000)}$ $y(t) = [(5000 \cdot t + 5) \cdot \exp(-1000 \cdot t)] \cdot u(t)$

11-45 $h(t) = u(t) - u(t - 10)$ $T(s) = H(s) = \left(\frac{1}{s} - \frac{\exp(-10 \cdot s)}{s}\right) = \frac{1}{s} \cdot (1 - \exp(-10 \cdot s))$ $x(t) = u(t)$

$X(s) = \frac{1}{s}$ $Y(s) = \frac{1}{s^2} \cdot (1 - \exp(-10 \cdot s))$ $\mathcal{L}^{-1}\left(\frac{1}{s^2}\right) = t \cdot u(t)$ $y(t) = t \cdot u(t) - (t - 10) \cdot u(t - 10)$

11-46 $x(t) = t \cdot u(t)$ $y(t) = \exp(-100 \cdot t) \cdot u(t)$ $X(s) = \frac{1}{s^2}$ $Y(s) = \frac{1}{s + 100}$

$$T(s) = \frac{Y(s)}{X(s)} = \frac{s^2}{s + 100} \qquad G(s) = \frac{T(s)}{s} = \frac{s}{s + 100} = 1 - \frac{100}{s + 100} \quad g(t) = \delta(t) - 100 \cdot \exp(-100 \cdot t)$$

$$\frac{d}{dt}y(t) = \frac{d}{dt}\exp(-100 \cdot t) \cdot u(t) = -100 \cdot \exp(-100 \cdot t) \cdot u(t) + \exp(-100 \cdot t) \cdot \frac{d}{dt}u(t)$$

$\frac{d}{dt}y(t) = -100 \cdot \exp(-100 \cdot t) \cdot u(t) + \exp(-100 \cdot t) \cdot \delta(t)$ but $\exp(-100 \cdot t) \cdot \delta(t) = \delta(t)$ hence $\frac{d}{dt}y(t) = g(t)$

11-47 $H(s) = T(s) = \frac{s}{s + 200}$ $y(t) = \exp(-100 \cdot t)$ $Y(s) = \frac{1}{s + 100}$

$X(s) = \frac{Y(s)}{T(s)} = \left(\frac{1}{s + 100} \cdot \frac{s + 200}{s}\right) = \frac{s + 200}{s \cdot (s + 100)} = \frac{2}{s} - \frac{1}{(s + 100)}$ hence $x(t) = 2 - \exp(-100 \cdot t)$

11-48 $h(t) = 200 \cdot u(t) + \delta(t)$ $H(s) = T(s) = \frac{200}{s} + 1 = \frac{s + 200}{s}$ $x(t) = \exp(-200 \cdot t) \cdot u(t)$

$X(s) = \frac{1}{s + 200}$ $Y(s) = T(s) \cdot X(s) = \left(\frac{s + 200}{s} \cdot \frac{1}{s + 200}\right) = \frac{1}{s}$ $y(t) = u(t)$

11-49 $g(t) = 10 \cdot \exp(-1000 \cdot t) \cdot \sin(2000 \cdot t) \cdot u(t)$ $T(s) = s \cdot G(s) = \frac{20000 \cdot s}{(s + 1000)^2 + 4000000}$

If $x(t) = 10 \cdot \cos(2000 \cdot t)$ then $y_{SS}(t) = AMP \cdot \cos(2000 \cdot t + PHASE)$ where

$$AMP := 10 \cdot \left|\frac{20000 \cdot j \cdot 2000}{(j \cdot 2000 + 1000)^2 + 4000000}\right| \qquad AMP = 97.014$$

$$PHASE := \frac{180}{\pi} \cdot \arg\left[\frac{20000 \cdot j \cdot 2000}{(j \cdot 2000 + 1000)^2 + 4000000}\right] \qquad PHASE = 14.036$$

11-50 $h(t) = \delta(t - 0.002)$ $H(s) = T(s) = \exp(-0.002 \cdot s)$ $x(t) = 10 \cdot \exp(-1000 \cdot t) \cdot u(t)$

$X(s) = \frac{10}{s + 1000}$ $Y(s) = T(s) \cdot X(s) = \frac{10 \cdot \exp(-0.002 \cdot s)}{s + 1000}$

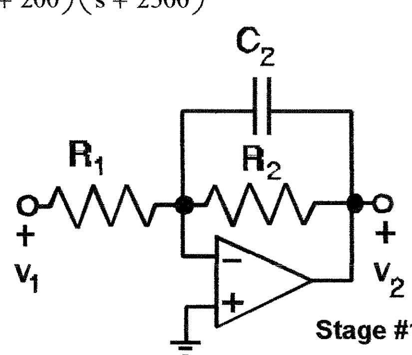
$y(t) = 10 \cdot [\exp[-1000 \cdot (t - 0.002)]] \cdot u(t - 0.002)$

11-51 $T_V(s) = \frac{100000}{(s + 200) \cdot (s + 2500)} = T_1(s) \cdot T_2(s) = \left(\frac{-100}{s + 200}\right)\left(\frac{-1000}{s + 2500}\right)$ Use a two-stage design

$$T_1(s) = \frac{-100}{s + 200} = \frac{-Y_1}{Y_2} = \frac{-\frac{1}{R_1}}{\frac{1}{R_2} + C_2 \cdot s}$$

$k_m := 10^7$ $R_1 := \frac{k_m}{100}$ $R_2 := \frac{k_m}{200}$ $C_2 := \frac{1}{k_m}$

$R_1 = 1 \times 10^5$ $R_2 = 5 \times 10^4$ $C_2 = 1 \times 10^{-7}$



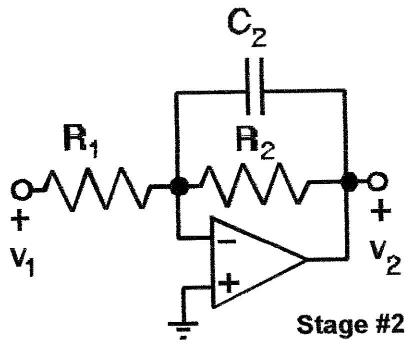
Stage #1

11-51 Continued

$$T_2(s) = \frac{-1000}{s+2500} = \frac{-Y_1}{Y_2} = \frac{-\frac{1}{R_1}}{\frac{1}{R_2} + C_2 \cdot s}$$

$$k_m := 10^8 \quad R_1 := \frac{k_m}{1000} \quad R_2 := \frac{k_m}{2500} \quad C_2 := \frac{1}{k_m}$$

$$R_1 = 1 \times 10^5 \qquad R_2 = 4 \times 10^4 \qquad C_2 = 1 \times 10^{-8}$$



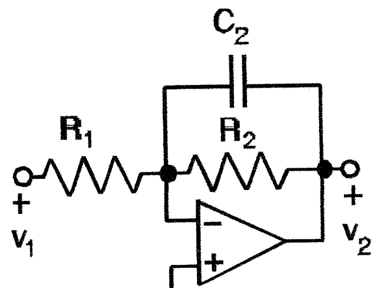
Stage #2

11-52 $T_V(s) = \frac{-100 \cdot (s+500)}{(s+200) \cdot (s+2500)} = T_1(s) \cdot T_2(s) = \left(\frac{-100}{s+200}\right)\left(\frac{s+500}{s+2500}\right)$ Use a two-stage design

$$T_1(s) = \frac{-100}{s+200} = \frac{-Y_1}{Y_2} = \frac{-\frac{1}{R_1}}{\frac{1}{R_2} + C_2 \cdot s}$$

$$k_m := 10^{10} \quad R_1 := \frac{k_m}{100} \quad R_2 := \frac{k_m}{200} \quad C_2 := \frac{1}{k_m}$$

$$R_1 = 1 \times 10^8 \qquad R_2 = 5 \times 10^7 \qquad C_2 = 1 \times 10^{-10}$$

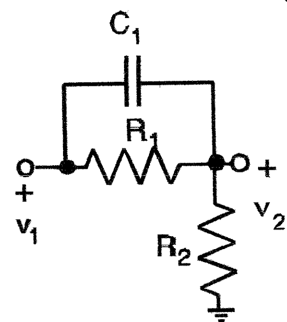


Stage #1

$$T_2(s) = \frac{s+500}{s+2500} = \frac{Y_1}{Y_1 + Y_2} = \frac{\frac{1}{R_1} + C_1 \cdot s}{\frac{1}{R_2} + \frac{1}{R_1} + C_1 \cdot s}$$

$$k_m := 10^{10} \quad R_1 := \frac{k_m}{500} \quad R_2 := \frac{k_m}{2500 - 500} \quad C_1 := \frac{1}{k_m}$$

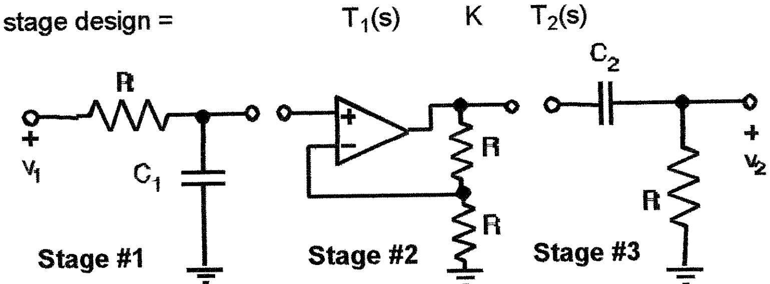
$$R_1 = 2 \times 10^7 \qquad R_2 = 5 \times 10^6 \qquad C_2 = 1 \times 10^{-10}$$



Stage #2

11-53 $T_V(s) = \frac{1000 \cdot s}{(s+500) \cdot (s+2000)} = \left(\frac{\frac{500}{s}}{1 + \frac{500}{s}}\right) \cdot (2) \cdot \left(\frac{1}{1 + \frac{2000}{s}}\right)$

Use a 3 stage design = $\quad T_1(s) \quad K \quad T_2(s)$

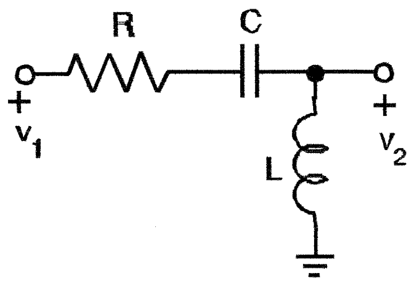


Stage #1 Stage #2 Stage #3

$$k_m := 10^4 \quad R := k_m \quad C_1 := \frac{1}{500 \cdot k_m} \quad C_2 := \frac{1}{2000 \cdot k_m} \quad R = 1 \times 10^4 \quad C_1 = 2 \times 10^{-7} \quad C_2 = 5 \times 10^{-8}$$

11-54 $T_V(s) = \dfrac{s^2}{(s+1000)\cdot(s+4000)} = \dfrac{s^2}{s^2 + 5000\cdot s + 4\cdot 10^6} = \dfrac{s}{s + 5000 + \dfrac{4\cdot 10^6}{s}}$ Use an RLC voltage divider

$$\frac{Z_2}{Z_2 + Z_1} = \frac{L\cdot s}{L\cdot s + R + \dfrac{1}{C\cdot s}} = \frac{s}{s + 5000 + \dfrac{1}{2.5\cdot 10^{-7}\cdot s}}$$

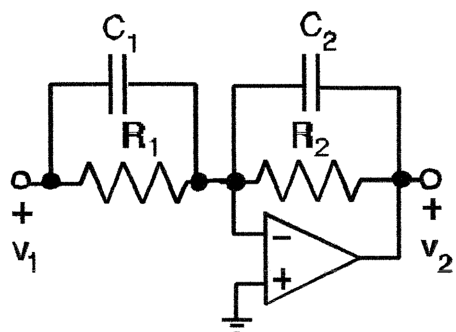


$k_m := 10^{-2}$ $L := k_m$ $R := 5000\cdot k_m$ $C := \dfrac{2.5\cdot 10^{-7}}{k_m}$

$L = 0.01$ $R := 50$ $C = 2.5 \times 10^{-5}$

11-55 $T_V(s) = \dfrac{-(s+100)\cdot(s+1000)}{(s+200)\cdot(s+500)} = T_1(s)\cdot T_2(s) = \left[\dfrac{-(s+1000)}{s+500}\right]\left(\dfrac{s+100}{s+200}\right)$ Use a two-stage design

$$T_1(s) = \frac{-(s+1000)}{s+500} = \frac{-Y_1}{Y_2} = \frac{-\left(\dfrac{1}{R_1} + C_1\cdot s\right)}{\dfrac{1}{R_2} + C_2\cdot s}$$

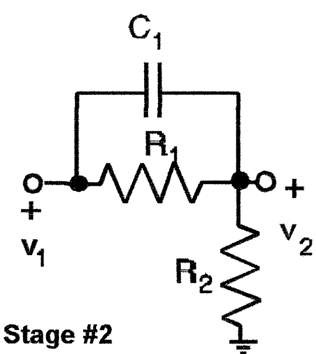


$k_m := 10^7$ $R_1 := \dfrac{k_m}{1000}$ $C_1 := \dfrac{1}{k_m}$

$R_2 := \dfrac{k_m}{500}$ $C_2 := \dfrac{1}{k_m}$

$R_1 = 1 \times 10^4$ $C_1 = 1 \times 10^{-7}$ $R_2 = 2 \times 10^4$ $C_2 = 1 \times 10^{-7}$

$$T_2(s) = \frac{s+100}{s+200} = \frac{Y_1}{Y_2 + Y_1} = \frac{\dfrac{1}{R_1} + C_1\cdot s}{\dfrac{1}{R_2} + \dfrac{1}{R_1} + C_1\cdot s}$$



$k_m := 10^6$ $R_1 := \dfrac{k_m}{100}$ $R_2 := \dfrac{k_m}{200 - 100}$ $C_1 := \dfrac{1}{k_m}$

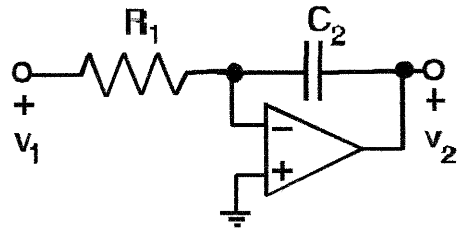
$R_1 = 1 \times 10^4$ $R_2 = 1 \times 10^4$ $C_1 = 1 \times 10^{-6}$

11-56 $T_V(s) = \dfrac{-400\cdot(s+100)}{s(s+200)} = T_1(s)\cdot T_2(s) = \left(\dfrac{-400}{s}\right)\left(\dfrac{s+100}{s+200}\right)$ Use a two-stage design

$$T_1(s) = \frac{-400}{s} = \frac{-Y_1}{Y_2} = \frac{-\left(\frac{1}{R_1}\right)}{C_2\cdot s}$$

$$k_m := 10^7 \qquad R_1 := \frac{k_m}{400} \qquad C_2 := \frac{1}{k_m}$$

$$R_1 = 2.5\times 10^4 \qquad C_2 = 1\times 10^{-7}$$



Stage #1

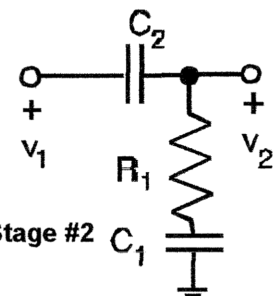
$$T_2(s) = \frac{s+100}{s+200} = \frac{1+\frac{100}{s}}{1+\frac{200}{s}} = \frac{Z_1}{Z_1+Z_2} = \frac{\left(R_1+\frac{1}{sC_1}\right)}{\left(R_1+\frac{1}{sC_1}\right)+\frac{1}{sC_2}}$$

$$k_m := 10^7 \qquad R_1 := \frac{k_m}{100}$$

$$C_1 := \frac{1}{k_m} \qquad C_2 := \frac{1}{k_m}$$

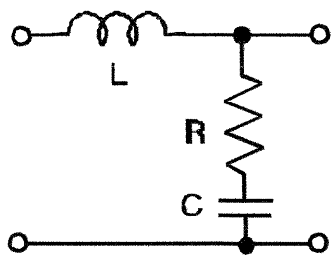
$$R_1 = 1\times 10^5 \qquad C_1 = 1\times 10^{-7} \qquad C_2 = 1\times 10^{-7}$$

Stage #2



11-57

$$T_V(s) = \frac{100\cdot s + 10^6}{s^2 + 100\cdot s + 10^6} = \frac{100+\frac{10^6}{s}}{s+100+\frac{10^6}{s}} = \frac{Z_2(s)}{Z_1(s)+Z_2(s)}$$

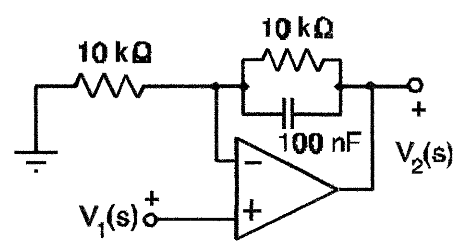


Use an RLC Voltage Divider

$$Z_2(s) = 100 + \frac{10^6}{s} = R + \frac{1}{C\cdot s} \quad Z_1(s) = s = L\cdot s \quad C = 10^{-6} \qquad R = 100 \qquad L = 1$$

11-58 (a) Noninverting OP AMP with K = 1

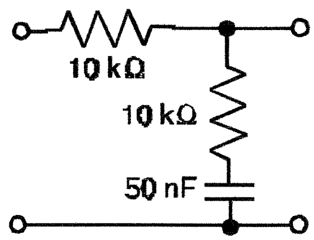
$$T_{V1}(s) = \frac{10\cdot 10^3 + \frac{1}{\left(100\cdot 10^{-9}\cdot s\right)+\left(10\cdot 10^3\right)^{-1}}}{10\cdot 10^3} = \frac{(s+2000)}{(s+1000)}$$



Design #1

Voltage divider with K = 0.5

$$T_{V2}(s) = \frac{10\cdot 10^3 + \frac{1}{50\cdot 10^{-9}\cdot s}}{10\cdot 10^3 + \left[10\cdot 10^3 + \left(50\cdot 10^{-9}\cdot s\right)^{-1}\right]} = \frac{1}{2}\cdot\frac{(s+2000)}{(s+1000)}$$



Design #2

11-58 Continued

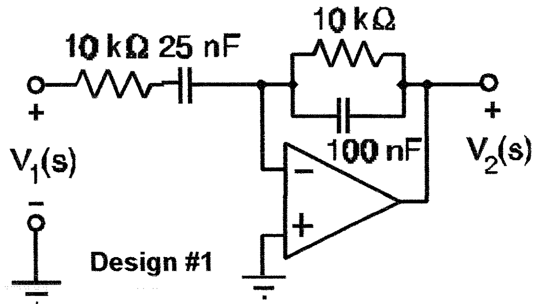
(b) Use the OP AMP circuit design since 1kΩ would load the passive voltage divider circuit.

(c) Either circuit would work since neither would load a 50 Ω source.

(d) Agree. The OP AMP circuit could drive the passive divider without loading and conversely the OP AMP circuit would not load the passive divider if it were first in the cascade.

11-59 **(a)** Inverting OP AMP circuit with K = -1

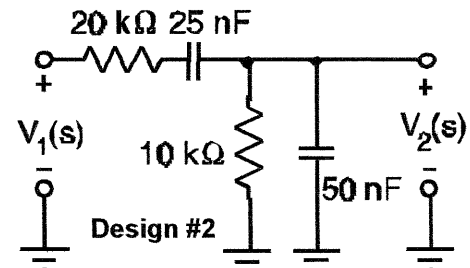
$$T_{V1}(s) = \frac{\dfrac{-1}{100\cdot 10^{-9}\cdot s + \left(10\cdot 10^{3}\right)^{-1}}}{10\cdot 10^{3} + \left(25\cdot 10^{-9}\cdot s\right)^{-1}} = \frac{-1000}{(s+1000)}\cdot\frac{s}{(s+4000)}$$



RC voltage divider with K = 1

$$T_{V2}(s) = \frac{\left(50\cdot 10^{-9}\cdot s + \dfrac{1}{10\cdot 10^{3}}\right)^{-1}}{20\cdot 10^{3} + \dfrac{1}{25\cdot 10^{-9}\cdot s} + \left(50\cdot 10^{-9}\cdot s + \dfrac{1}{10\cdot 10^{3}}\right)^{-1}}$$

$$T_{V2}(s) = \frac{1000\cdot s}{s^2 + 5000\cdot s + 4\cdot 10^{6}} = \frac{1000\cdot s}{(s+1000)\cdot(s+4000)}$$



(b) Use the OP AMP circuit design since 1kΩ would load the passive voltage divider circuit.

(c) Either circuit would work since neither would load a 50 Ω source.

(d) Disagree. The OP AMP circuit could drive the passive divider without loading and but the OP AMP circuit would load the passive divider if it were first in the cascade.

11-60 $g(t) = (1 - \exp(-50\cdot t) - 50\cdot t\cdot\exp(-50\cdot t))\cdot u(t)$ $\quad G(s) = \left[\dfrac{1}{s} - \dfrac{1}{(s+50)} - \dfrac{50}{(s+50)^2}\right] = \dfrac{2500}{\left[s\cdot(s+50)^2\right]}$

$$T(s) = s\cdot G(s) = \left(\frac{50}{s+50}\right)^2 = \left(\frac{-50}{s+50}\right)\cdot\left(\frac{-50}{s+50}\right)$$

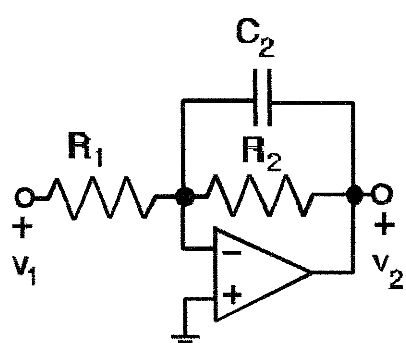
Use a cascade connection of two identical low-pass stages

$$T_1(s) = \frac{-50}{s+50} = \frac{-Y_1}{Y_2} = \frac{\dfrac{-1}{R_1}}{\dfrac{1}{R_2} + C_2\cdot s} \qquad k_m := 10^7$$

$$R_1 := \frac{k_m}{50} \qquad R_2 := \frac{k_m}{50} \qquad C_2 := \frac{1}{k_m}$$

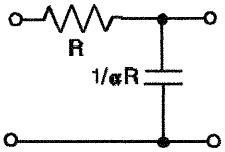
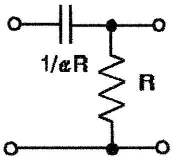
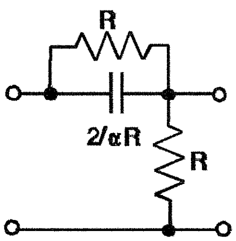
$$R_1 = 2\times 10^5 \quad R_2 = 2\times 10^5 \quad C_2 = 1\times 10^{-7}$$

2nd stage is identical to the 1st stage



Stages #1 and #2

11-61

Circuit	impulse response h(t)	step response g(t)
	$\alpha \cdot \exp(-\alpha \cdot t) \cdot u(t)$	$(1 - \exp(-\alpha \cdot t)) \cdot u(t)$
	$\delta(t) - \alpha \cdot \exp(-\alpha \cdot t) \cdot u(t)$	$\exp(-\alpha \cdot t) \cdot u(t)$
	$\delta(t) - 0.5 \cdot \alpha \cdot \exp(-\alpha \cdot t) \cdot u(t)$	$\dfrac{(1 + \exp(-\alpha \cdot t)}{2} \cdot u(t)$

11-62 (a) Time between successive crossings of the final value is about 1.4 μs, hence:

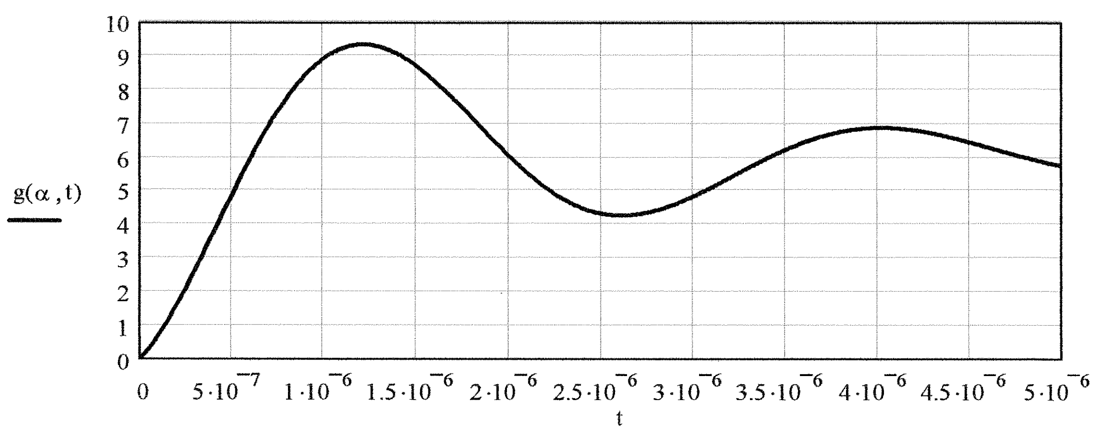
$T_0 := 2 \cdot 1.4 \cdot 10^{-6}$ $\quad \beta := \dfrac{2 \cdot \pi}{T_0}$ and $\beta = 2.244 \times 10^6$ The final value is g(∞)= 6 hence: $\dfrac{K}{\alpha^2 + \beta^2} = 6$

Assume $\alpha := \dfrac{\beta}{5}$ define $g(\alpha, t) := 6 \cdot \left[1 - \exp(-\alpha \cdot t) \cdot \left(\cos(\beta \cdot t) - \dfrac{\alpha}{\beta} \cdot \sin(\beta \cdot t)\right)\right]$

g(t) reaches its first maximum of 9.3 at t = 1.3 ms, hence

Given $g(\alpha, 1.3 \cdot 10^{-6}) = 9.3$ $\quad \alpha := \text{Find}(\alpha)$ $\quad K := 6 \cdot (\alpha^2 + \beta^2)$ $\quad \alpha = 4.768 \times 10^5$ $\quad K = 3.158 \times 10^{13}$

$t := 0, 10^{-8} .. 7 \cdot 10^{-6}$ $\quad \alpha = 4.768 \times 10^5$ $\quad \beta = 2.244 \times 10^6$



11-62 Continued

(b) $g(t) = \frac{K}{\alpha^2 + \beta^2} \cdot \left[1 - e^{-\alpha \cdot t} \cdot \cos(\beta \cdot t) - e^{-\alpha \cdot t} \cdot \left(\frac{\alpha}{\beta} \cdot \sin(\beta \cdot t) \right) \right] \cdot u(t)$

$$G(s) = \frac{K}{\alpha^2 + \beta^2} \cdot \left[\frac{1}{s} - \frac{s + \alpha}{(s + \alpha)^2 + \beta^2} - \frac{\alpha}{\beta} \cdot \frac{\beta}{(s + \alpha)^2 + \beta^2} \right] = \frac{K}{s \cdot (s^2 + 2 \cdot s \cdot \alpha + \alpha^2 + \beta^2)}$$

$$T(s) = s \cdot G(s) = \frac{K}{s^2 + 2 \cdot s \cdot \alpha + \alpha^2 + \beta^2}$$

(c) Partition T(s) into two stages: $T(s) = \left(\frac{K}{\alpha^2 + \beta^2} \right) \cdot \left(\frac{\alpha^2 + \beta^2}{s^2 + 2 \cdot s \cdot \alpha + \alpha^2 + \beta^2} \right)$

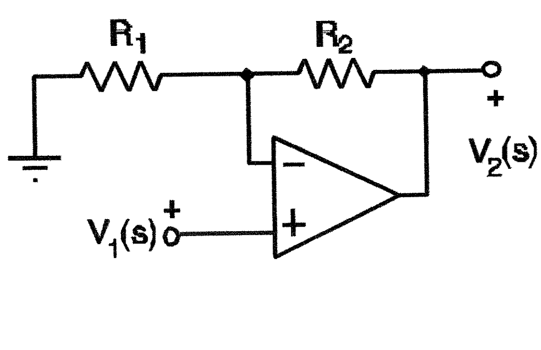
<-1st stage-><------2nd stage------>

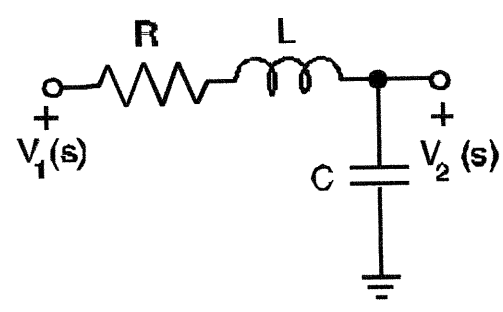
The 1st stage is a noninverting gain of $\frac{K}{\alpha^2 + \beta^2} = 6$ Let R_1 = 10 kΩ then R_2 = 50 kΩ.

The second stage can be an RLC voltage divider with: $\frac{Z_2}{Z_1 + Z_2} = \left(\frac{\frac{\alpha^2 + \beta^2}{s}}{s + 2 \cdot \alpha + \frac{\alpha^2 + \beta^2}{s}} \right)$

Z_2 is a capacitor $C = (\alpha^2 + \beta^2)^{-1} = 1.9 \cdot 10^{-13}$ F. Z_1 is an inductor L = 1 H in series with a resistor R = 2α = 953.5 kΩ. Using a scale factor of $k_m = 10^{-3}$ produces R = 953.5 Ω, L = 1 mH, and C = 0.19 nF.

1st Stage Noninverting Amplifier	2nd Stage Series RLC Voltage Divider
$R_1 = 10 \cdot 10^3\ \Omega$ $R_2 = 50 \cdot 10^3\ \Omega$	$R = 953.5\ \Omega$ $L = 10^{-3}$ H $C = 0.19 \cdot 10^{-9}$ F





11-63

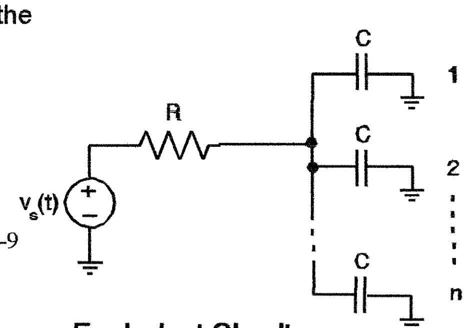
$g(t) = 2\exp(-\alpha\cdot t) - 1 \qquad G(s) = \dfrac{2}{s+\alpha} - \dfrac{1}{s} = \dfrac{s-\alpha}{s+\alpha}\cdot\dfrac{1}{s}$

$T(s) = s\cdot G(s) = \dfrac{s-\alpha}{s+\alpha}$ stable for $\alpha > 0$ and unstable for $\alpha < 0$ $\quad y(t) = \exp(-2\cdot\alpha\cdot t) \qquad Y(s) = \dfrac{1}{s+2\cdot\alpha}$

$$X(s) = \frac{Y(s)}{T(s)} = \frac{(s+\alpha)}{(s+2\cdot\alpha)\cdot(s-\alpha)} \qquad x(t) = \frac{1}{3}\cdot\exp(-2\cdot\alpha\cdot t) + \frac{2}{3}\cdot\exp(\alpha\cdot t)$$

X(s) has a pole in the right half plane at $s = \alpha$ if $\alpha > 0$ or $s = -2\alpha$ if $\alpha < 0$. Hence x(t) is unbounded. Does not change the conclusion about the stability of T(s) which depends only on natural poles.

11-64



$$v_{Load}(t) = 5\cdot\left(1 - \exp\left(\frac{-t}{R\cdot C}\right)\right) \qquad V_{Load}(10\cdot10^{-9}) > 3.7$$

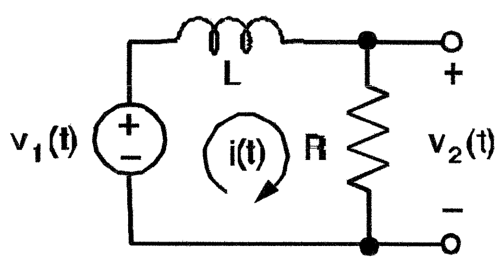
$$5\cdot\left(1 - \exp\left(\frac{-10\cdot10^{-9}}{RC}\right)\right) > 3.7 \text{ requires } \quad RC < 7.423499090116\cdot10^{-9}$$

$$C < \frac{7.4234990901164712440\cdot10^{-9}}{1000} = 7.423\cdot10^{-12} \qquad \frac{7.423\cdot10^{-12}}{3\cdot10^{-12}} = 2.474$$

Equivalent Circuit

fanout = 2

11-65 (a)



$$I(s) = \frac{V_1(s)}{Z(s)} \qquad Z(s) = R + L\cdot s$$

$$i_{SS}(t) = V_A\cdot|Y(j\cdot\omega)|\cdot\cos(\omega\cdot t + \arg(Y(j\cdot\omega)))$$

$$|Y(j\cdot\omega)| = \frac{1}{\sqrt{R^2 + (\omega\cdot L)^2}} \qquad \arg(Y(j\cdot\omega)) = -\operatorname{atan}\left(\frac{\omega\cdot L}{R}\right)$$

(b) Ckt diff eq. $\quad L\cdot\dfrac{d}{dt}i(t) + R\cdot i(t) = V_A\cdot\cos(\omega\cdot t)$

To find the forced response let: $\quad i_F(t) = I_A\cdot\cos(\omega\cdot t) + I_B\cdot\sin(\omega\cdot t)$

Subsituting into the diff eq yields:

$$-\omega\cdot L\cdot I_A\cdot\sin(\omega\cdot t) + \omega\cdot L\cdot I_B\cdot\cos(\omega\cdot t) + R\cdot I_A\cdot\cos(\omega\cdot t) + R\cdot I_B\cdot\sin(\omega\cdot t) = V_A\cdot\cos(\omega\cdot t)$$

Equating the sine and cosine terms yields two equations in I_A & I_B

$\omega\cdot L\cdot I_B + R\cdot I_A = V_A$ and $-\omega\cdot L\cdot I_A + R\cdot I_B = 0$ Whose solution is

$$I_A = \frac{R\cdot V_A}{\omega^2\cdot L^2 + R^2} \qquad I_B = \frac{-\omega\cdot L\cdot V_A}{(\omega^2\cdot L^2 + R^2)}$$

11-65 Continued

hence the forced response is

$$i_F(t) = \frac{R \cdot V_A}{\omega^2 \cdot L^2 + R^2} \cdot \cos(\omega \cdot t) + \frac{-\omega \cdot L \cdot V_A}{\omega^2 \cdot L^2 + R^2} \cdot \sin(\omega \cdot t) = \frac{\sqrt{\omega^2 \cdot L^2 + R^2}}{\omega^2 \cdot L^2 + R^2} \cdot V_A \cdot \cos(\omega \cdot t + \theta)$$

or $$i_F(t) = \frac{V_A}{\sqrt{\omega^2 \cdot L^2 + R^2}} \cdot \cos(\omega \cdot t + \theta)$$ where $$\theta = \operatorname{atan}\left(\frac{I_A}{I_B}\right) = \operatorname{atan}\left(\frac{-\omega \cdot L}{R}\right)$$

(c) The two methods produce the same result. The s-domain method is much simpler than the classical method based on the circuit differential equation.

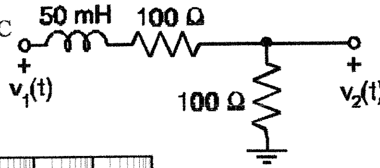
CHAPTER 12, Both Versions

The two functions below are used to generate the straight-line gain and phase plots for Prob 12-1 thru 10.

$$G_{SL}(\omega,\alpha) := \begin{cases} 1 & \text{if } 0 \le \omega < \alpha \\ \dfrac{\alpha}{\omega} & \text{if } \alpha \le \omega \end{cases}$$

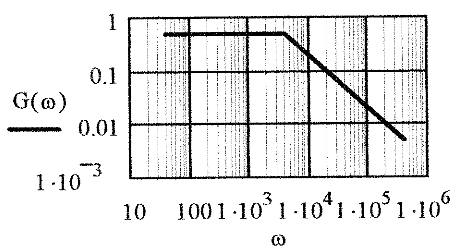
$$A_{SL}(\omega,\alpha) := \begin{cases} 0 & \text{if } 0 \le \omega < 0.1\cdot\alpha \\ -45\cdot\log\left(\dfrac{\omega}{0.1\cdot\alpha}\right) & \text{if } 0.1\cdot\alpha \le \omega < 10\cdot\alpha \\ -90 & \text{if } 10\cdot\alpha \le \omega \end{cases}$$

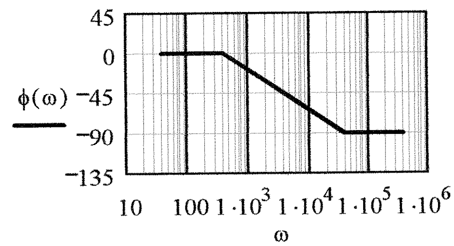
12-1 By voltage division $T_V(s) = \dfrac{Z_2}{Z_1 + Z_2} = \dfrac{100}{100 + 0.05\cdot s + 100}$ $\quad T_V(s) := \dfrac{2000}{(4000 + s)}$



(a) $T_V(0) = 0.5$; $\omega_C := 4000$ **low pass** $\omega := 0.01\cdot\omega_C, 0.02\cdot\omega_C .. 100\cdot\omega_C$

(b) $G(\omega) := 0.5\cdot G_{SL}(\omega,\omega_C)$ $\quad \phi(\omega) := A_{SL}(\omega,\omega_C)$

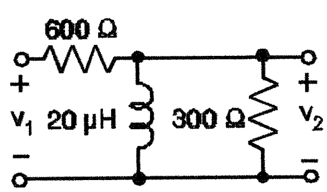




(c) $G(0.5\cdot\omega_C) = 0.5$ $\quad G(\omega_C) = 0.5$ $\quad G(2\cdot\omega_C) = 0.25$

12-2 By voltage division $T_V(s) = \dfrac{Y_1}{Y_1 + Y_2} = \dfrac{(600)^{-1}}{(600)^{-1} + \left(20\cdot10^{-6}\cdot s\right)^{-1} + (300)^{-1}}$ $\quad T_V(s) := \dfrac{1}{3}\cdot\dfrac{s}{s + 10^7}$

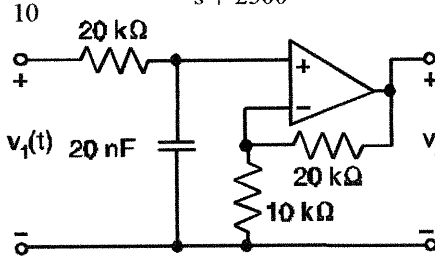
(a) $T_V(0) = 0$; $T_V(\infty) := 0.333$ $\quad \omega_C := 10^7$ **high pass**



(b) $G(\omega) := 0.333\cdot\dfrac{\omega}{\omega_C}\cdot G_{SL}(\omega,\omega_C)$

$G(0.5\cdot\omega_C) = 0.167$ $\quad G(\omega_C) = 0.333$ $\quad G(2\cdot\omega_C) = 0.333$

12-3 $T_V(s) = \left(\dfrac{Z_2}{Z_1 + Z_2}\cdot\dfrac{R_1 + R_2}{R_1}\right) = \dfrac{\left(20\cdot10^{-9}\cdot s\right)^{-1}}{20\cdot10^{3} + \left(20\cdot10^{-9}\cdot s\right)^{-1}}\cdot\dfrac{10 + 20}{10}$ $\quad T_V(s) := \dfrac{7500}{s + 2500}$

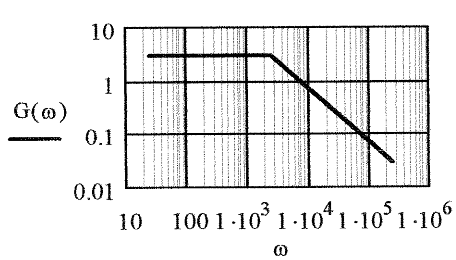


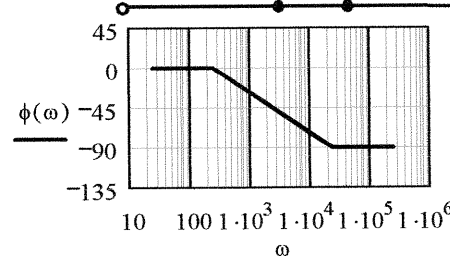
(a) $T_V(0) = 3$; $T_V(j\cdot\infty) = 0$; $\omega_C := 2500$ **low pass**

$\omega := 0.01\cdot\omega_C, 0.02\cdot\omega_C .. 100\cdot\omega_C$

$G(\omega) := 3\cdot G_{SL}(\omega,\omega_C)$ $\quad \phi(\omega) := A_{SL}(\omega,\omega_C)$

(b)





12-3 Continued

(c) $\phi(0.5\cdot\omega_C) = -31.454$ $\qquad \frac{180}{\pi}\cdot\arg(T_V(j\cdot 0.5\cdot\omega_C)) = -26.565$

$\phi(\omega_C) = -45$ $\qquad \frac{180}{\pi}\cdot\arg(T_V(j\cdot\omega_C)) = -45$

$\phi(2\cdot\omega_C) = -58.546$ $\qquad \frac{180}{\pi}\cdot\arg(T_V(j\cdot 2\cdot\omega_C)) = -63.435$

12-4 $T_V(s) = \frac{-Y_1}{Y_2} = \frac{-\left(10^4\right)^{-1}}{\left(100\cdot 10^3\right)^{-1} + 5\cdot 10^{-9}\cdot s}$ $\quad T_V(s) = \frac{-20000}{s + 2000}$

(a) $T_V(0) = -10$; $T_V(j\cdot\infty) = 0$; $\omega_C := 2000$ **low pass**

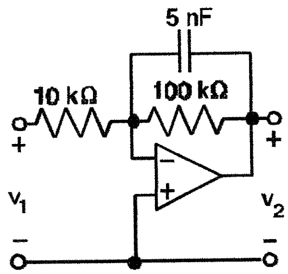
$\omega := 0.01\cdot\omega_C, 0.02\cdot\omega_C .. 100\cdot\omega_C$

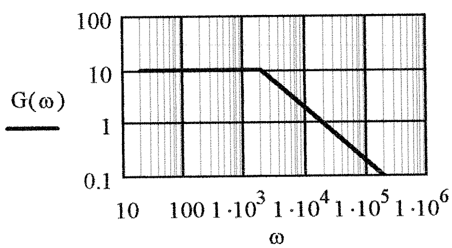
(b) $G(\omega) := 10\cdot G_{SL}(\omega, \omega_C)$

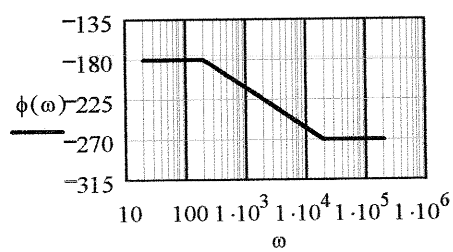
$\phi(\omega) := -180 + A_{SL}(\omega, \omega_C)$

(c) $V_O(\omega) := G(\omega)\cdot 5$ $\quad V_O(0.5\cdot\omega_C) = 50$

$V_O(\omega_C) = 50 \quad V_O(2\cdot\omega_C) = 25$





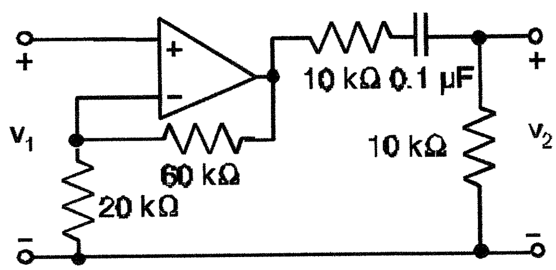


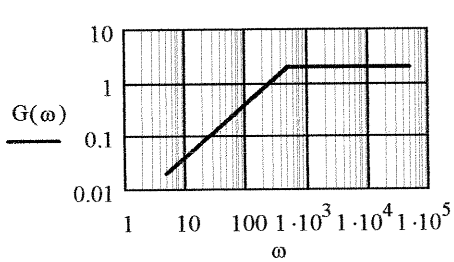
12-5 $T_V(s) = \left(\frac{60 + 20}{20}\right)\cdot\left[\frac{10^4}{10^4 + 10^4 + \left(10^{-7}\cdot s\right)^{-1}}\right]$ $\quad T_V(s) := \frac{2\cdot s}{s + 500}$

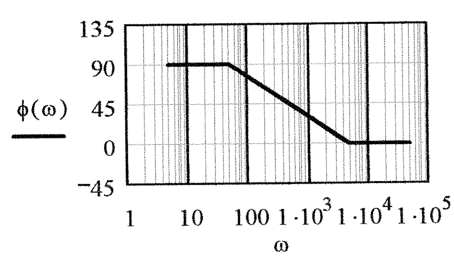
(a) $|T_V(0)| = 0 \quad |T_V(j\cdot\infty)| = 2 \quad \omega_C := 500$

(b) high pass $\omega := 0.01\cdot\omega_C, 0.02\cdot\omega_C .. 100\cdot\omega_C$

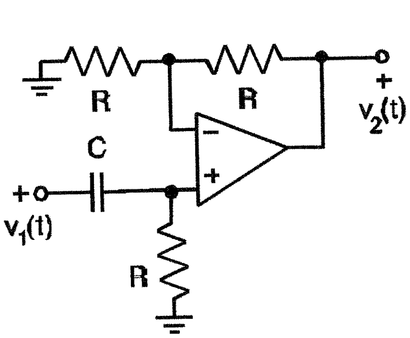
(c) $G(\omega) := \frac{\omega}{250}\cdot G_{SL}(\omega, \omega_C) \quad \phi(\omega) := 90 + A_{SL}(\omega, \omega_C)$







12-6 (a) $T_V(s) = T_1 \cdot T_2 = \left(\dfrac{R}{R + \dfrac{1}{C \cdot s}}\right) \cdot \left(\dfrac{R+R}{R}\right) = \left(\dfrac{2 \cdot R \cdot C \cdot s}{R \cdot C \cdot s + 1}\right)$



Ckt is highpass $\quad |T_V(\infty)| = 2$

$\omega_C = \dfrac{1}{R \cdot C} = 2 \cdot \pi \cdot 500$ Let $\quad R := 10^4 \quad C := \dfrac{1}{R \cdot 1000 \cdot \pi}$

$C = 3.183 \times 10^{-8} \quad \arg(T_V(j \cdot \omega_C)) = \arg\left(\dfrac{2 \cdot j}{1+j}\right) = \dfrac{\pi}{4}$

12-7 $\quad K = 10^{\frac{20}{20}} = 10 \quad \omega_C := 350 \quad T_V(s) := \dfrac{10}{\dfrac{s}{350} + 1} \quad G_{dB}(\omega) := 20 \cdot \log(|10 \cdot G_{SL}(\omega, \omega_C)|)$

$G_{dB}(200) = 20 \qquad G_{dB}(400) = 18.84 \qquad G_{dB}(800) = 12.82$

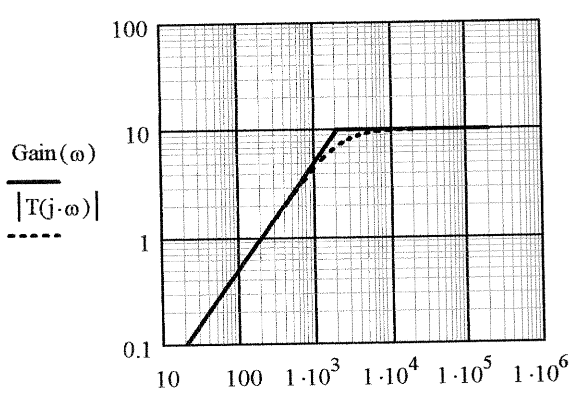
12-8 $\quad K := 10^{\frac{5}{20}} \qquad K = 1.778 \qquad \omega_C := 2000 \qquad T(s) := \dfrac{1.778 \cdot s}{s + 2000}$ The value of K does not effect the phase angle

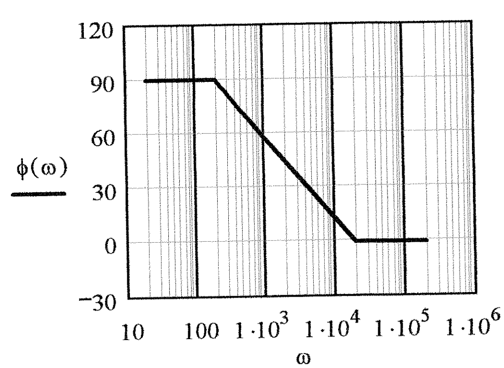
$\phi(\omega) := \dfrac{180}{\pi} \cdot \arg(T(j \cdot \omega)) \qquad \phi(0.5 \cdot \omega_C) = 63.435 \qquad \phi(\omega_C) = 45 \qquad \phi(2 \cdot \omega_C) = 26.565$

$\phi_{SL}(\omega) := 90 + A_{SL}(\omega, \omega_C) \qquad \phi_{SL}(0.5 \cdot \omega_C) = 58.546 \qquad \phi_{SL}(\omega_C) = 45 \qquad \phi_{SL}(2 \cdot \omega_C) = 31.454$

12-9 $\quad T(s) = \dfrac{0.1}{0.01 + \dfrac{20}{s}} \qquad T(s) := \dfrac{10 \cdot s}{s + 2000}$ **(a)** highpass $\qquad K := 10, \quad \omega_C := 2000$rad/s.

(b) $\quad \text{Gain}(\omega) := \dfrac{\omega}{200} \cdot G_{SL}(\omega, \omega_C) \quad \omega := 0.01 \cdot \omega_C, 0.02 \cdot \omega_C .. 100 \cdot \omega_C \qquad \phi(\omega) := 90 + A_{SL}(\omega, \omega_C)$

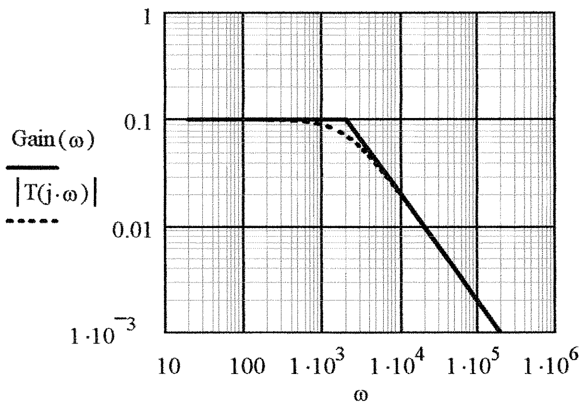


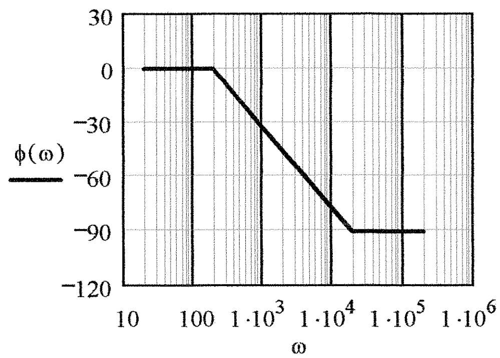


(c) $\quad \text{Gain}(1000) = 5 \qquad \text{Gain}(2000) = 10 \qquad \text{Gain}(4000) = 10$

12-10 $T(s) := \dfrac{10}{10^2 + \dfrac{s}{20}}$ $\quad T(s) := \dfrac{0.1}{\dfrac{s}{2000} + 1}$ $\quad$ **(a)** **lowpass** $K := 0.1,\ \omega_C := 2000$**rad/s.**

(b) $\mathrm{Gain}(\omega) := 0.1 \cdot G_{SL}(\omega, \omega_C)$ $\quad \omega := 0.01\cdot\omega_C, 0.02\cdot\omega_C .. 100\cdot\omega_C$ $\quad \phi(\omega) := 0 + A_{SL}(\omega, \omega_C)$

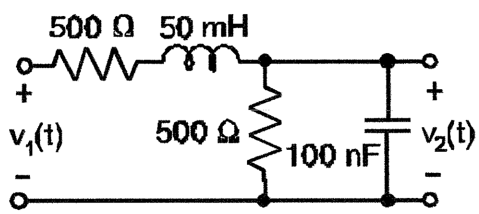




(c) $\mathrm{Gain}(1000) = 0.1$ $\quad \mathrm{Gain}(2000) = 0.1$ $\quad \mathrm{Gain}(4000) = 0.05$

12-11

$$T_V(s) = \frac{\dfrac{1}{\left(\dfrac{1}{500} + 10^{-7}\cdot s\right)}}{\dfrac{1}{\left(\dfrac{1}{500} + 10^{-7}\cdot s\right)} + 500 + 0.05\cdot s}$$



$$T_V(s) := \frac{2\cdot 10^8}{\left(4\cdot 10^8 + 30000\cdot s + s^2\right)}$$

(a) $\omega_0 := 2\cdot 10^4$ $\quad \zeta := \dfrac{30000}{2\cdot\omega_0}$ $\quad \zeta = 0.75$

(b) $T_{SL}(\omega) := \begin{cases} 0.5 & \text{if } 0 \le \omega < \omega_0 \\ \dfrac{2\cdot 10^8}{\omega^2} & \text{if } \omega_0 \le \omega \end{cases}$

$\omega := 0.01\cdot\omega_0, 0.02\cdot\omega_0 .. 100\cdot\omega_0$

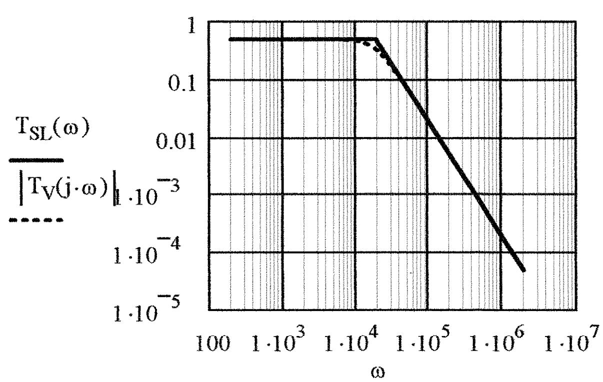
$|T_V(0) = 0.5|$ $\quad |T_V(\infty)| = 0$ $\quad$ **Ckt is low pass**

(c)

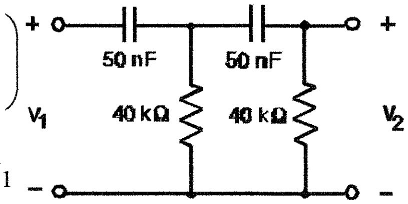
$T_{SL}(0.5\cdot\omega_0) = 0.5$ $\quad |T_V(j\cdot 0.5\cdot\omega_0)| = 0.471$

$T_{SL}(\omega_0) = 0.5$ $\quad |T_V(j\cdot\omega_0)| = 0.333$

$T_{SL}(2\cdot\omega_0) = 0.125$ $\quad |T_V(j\cdot 2\cdot\omega_0)| = 0.118$



12-12 Using node analysis



$$\begin{pmatrix} 2.5\cdot10^{-5} + 1\cdot10^{-7}\cdot s & -0.5\cdot10^{-7}\cdot s \\ -0.5\cdot10^{-7}\cdot s & 2.5\cdot10^{-5} + 0.5\cdot10^{-7}\cdot s \end{pmatrix}\begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{pmatrix} 0.5\cdot10^{-7}\cdot s\cdot V_1 \\ 0 \end{pmatrix}$$

$$\Delta(s) = 6.25\cdot10^{-10} + 3.75\cdot10^{-12}\cdot s + 2.5\cdot10^{-15}\cdot s^2 \quad \Delta_B(s) = 0.25\cdot10^{-14}\cdot s^2\cdot V_1$$

$$T_V(s) := \frac{0.25\cdot10^{-14}\cdot s^2}{6.25\cdot10^{-10} + 3.75\cdot10^{-12}\cdot s + 2.5\cdot10^{-15}\cdot s^2} \quad T_V(s) := \frac{s^2}{\left[(500)^2 + 1500\cdot s + s^2\right]}$$

(a) $\omega_0 := 500 \quad \zeta := \frac{1500}{2\cdot\omega_0} \quad \zeta = 1.5$ **(b)** $T_{SL}(\omega) := \text{if}\left[\omega > \omega_0, 1, \left(\frac{\omega}{\omega_0}\right)^2\right]$

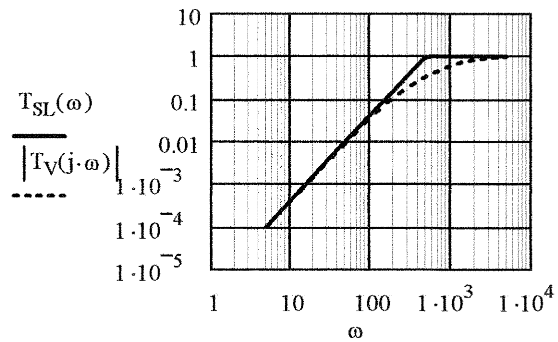
$\omega := 0.01\cdot\omega_0, 0.2\cdot\omega_0 .. 10\cdot\omega_0$

$|T_V(0)| = 0 \qquad |T_V(\infty)| = 1$ Ckt is high pass

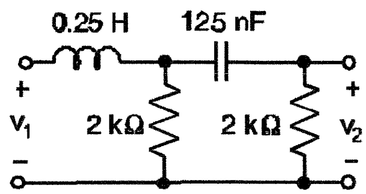
(c) $T_{SL}(0.5\cdot\omega_0) = 0.25$

$T_{SL}(\omega_0) = 1$

$T_{SL}(2\cdot\omega_0) = 1$



12-13 (a) Using node analysis



$$\begin{pmatrix} \frac{4}{s} + \frac{1}{2000} + 125\cdot10^{-9}\cdot s & -125\cdot10^{-9}\cdot s \\ -125\cdot10^{-9}\cdot s & \frac{1}{2000} + 125\cdot10^{-9}\cdot s \end{pmatrix}\cdot\begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{pmatrix} \frac{4}{s}\cdot V_1 \\ 0 \end{pmatrix}$$

$$\Delta(s) = \frac{s^2 + 6000\cdot s + 16\cdot10^6}{8\cdot10^9\cdot s} \quad \Delta_B = \frac{V_1}{2\cdot10^6} \quad T_V(s) := \frac{4000\cdot s}{s^2 + 6000\cdot s + 16\cdot10^6}$$

(b) $\omega_0 := 4\cdot10^3 \quad \zeta := \frac{6000}{2\cdot\omega_0} \quad \zeta = 0.75$ **(c)** $T_{SL}(\omega) := \begin{cases} \frac{4000\cdot\omega}{16\cdot10^6} & \text{if } 0 \le \omega < \omega_0 \\ \frac{4000}{\omega} & \text{if } \omega_0 \le \omega \end{cases}$

$\omega := 0.01\cdot\omega_0, 0.02\cdot\omega_0 .. 100\cdot\omega_0$ Ckt is bandpass

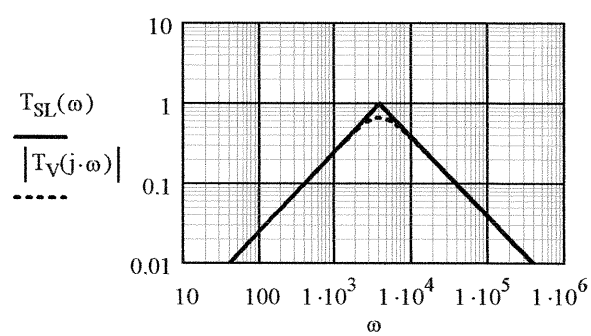
$|T_V(0)| = 0 \qquad |T_V(\infty)| = 0$

(d)

$T_{SL}(0.5\cdot\omega_0) = 0.5 \qquad |T_V(j\cdot0.5\cdot\omega_0)| = 0.471$

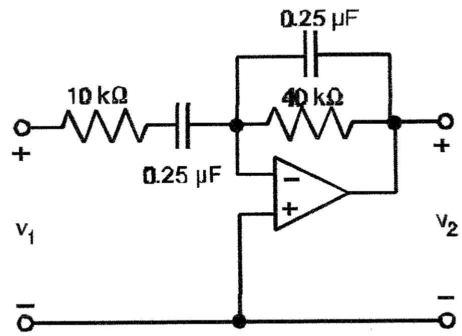
$T_{SL}(\omega_0) = 1 \qquad |T_V(j\cdot\omega_0)| = 0.667$

$T_{SL}(2\cdot\omega_0) = 0.5 \qquad |T_V(j\cdot2\cdot\omega_0)| = 0.471$



12-14 Inverting OP AMP circuit

$$T_V(s) = -\left(\frac{Y_1}{Y_2}\right) = -\frac{\left(\dfrac{1}{10000 + \dfrac{1}{0.25\cdot 10^{-6}\cdot s}}\right)}{\left(\dfrac{1}{40000} + 0.25\cdot 10^{-6}\cdot s\right)}$$



$$T_V(s) := \frac{-400\cdot s}{s^2 + 500\cdot s + 40000}$$

(a) $\omega_0 := 200$ $\quad \zeta := \dfrac{500}{2\cdot\omega_0}$ $\quad \zeta = 1.25$ $\quad |T_V(0)| = 0$ $\quad |T_V(\infty)| = 0$ $\quad$ Ckt is bandpass

(b) $T_{SL}(\omega) := \text{if}\left(\omega > \omega_0, \dfrac{400}{\omega}, \dfrac{\omega}{100}\right)$ $\quad T_{SL}(0.5\cdot\omega_0) = 1$ $\quad T_{SL}(\omega_0) = 2$ $\quad T_{SL}(2\cdot\omega_0) = 1$

12-15 (a) Using node analysis

Node A: $\left(40\cdot 10^{-9}\cdot s + 10^{-4}\right)\cdot V_A - 5\cdot 10^{-5}\cdot V_B - 5\cdot 10^{-5}\cdot V_2 = 40\cdot 10^{-9}\cdot s\cdot V_1$

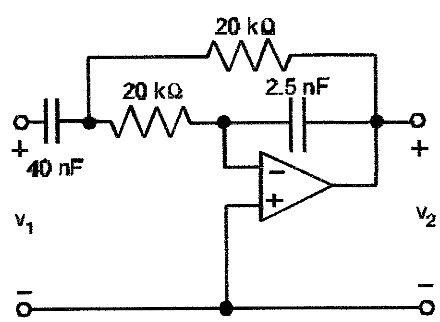
Node B: $-5\cdot 10^{-5}\cdot V_A + \left(5\cdot 10^{-5} + 2.5\cdot 10^{-9}\cdot s\right)\cdot V_B - 2.5\cdot 10^{-9}\cdot s\cdot V_2(s) = 0$

The ideal OP AMP i-v characteristics require $V_B(s) = 0$

since the noninverting input is grounded.

The node eqs. reduce to

$$\begin{pmatrix} 40\cdot 10^{-9}\cdot s + 10^{-4} & -5\cdot 10^{-5} \\ -5\cdot 10^{-5} & -2.5\cdot 10^{-9}\cdot s \end{pmatrix}\cdot\begin{pmatrix} V_A \\ V_2 \end{pmatrix} = \begin{pmatrix} 40\cdot 10^{-9}\cdot s\cdot V_1 \\ 0 \end{pmatrix}$$



$$\Delta(s) = -\left(1\cdot 10^{-16}\right)\cdot s^2 - 2.5\cdot 10^{-13}\cdot s - 2.5\cdot 10^{-9}$$

$$\Delta_2 = \left(2\cdot 10^{-12}\cdot s\right)\cdot V_1$$

$$T_V(s) = \frac{V_2(s)}{V_1(s)} = \frac{2\cdot 10^{-12}\cdot s}{-\left(1\cdot 10^{-16}\right)\cdot s^2 - 2.5\cdot 10^{-13}\cdot s - 2.5\cdot 10^{-9}}$$

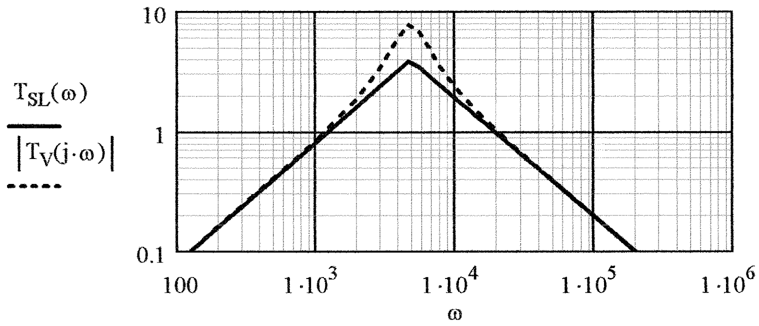
$$T_V(s) := \frac{-20000\cdot s}{s^2 + 2500\cdot s + 25000000}$$

(b) $\omega_0 := \dfrac{1}{\sqrt{4\cdot 10^{-8}}}$ $\quad \omega_0 = 5\times 10^3$ $\quad \zeta := \dfrac{10^{-4}\cdot\omega_0}{2}$ $\quad \zeta = 0.25$

$|T_V(0)| = 0$ $\quad |T_V(\infty)| = 0$ $\quad$ Ckt is bandpass

12-15 Continued

(c) $T_{SL}(\omega) := \left| \begin{array}{ll} 8\cdot 10^{-4}\cdot\omega & \text{if } 0 \le \omega < \omega_0 \\ \dfrac{2}{10^{-4}\cdot\omega} & \text{if } \omega_0 \le \omega \end{array} \right.$ $\omega := 0.01\cdot\omega_0, 0.2\cdot\omega_0 .. 100\cdot\omega_0$



(d) $T_{SL}(0.5\cdot\omega_0) = 2$ $\quad |T_V(j\cdot 0.5\cdot\omega_0)| = 2.53$

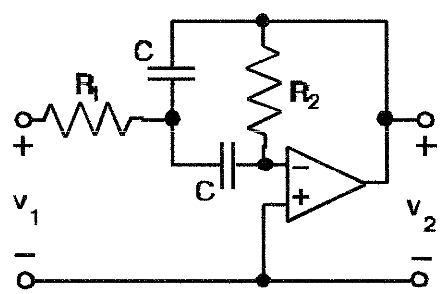
$T_{SL}(\omega_0) = 4$ $\quad |T_V(j\cdot\omega_0)| = 8$

$T_{SL}(2\cdot\omega_0) = 2$ $\quad |T_V(j\cdot 2\cdot\omega_0)| = 2.53$

12-16(a) Using node analysis

$$\left(2\cdot C\cdot s + \frac{1}{R_1}\right)\cdot V_A - C\cdot s\cdot V_B - C\cdot s\cdot V_2 = \frac{V_1}{R_1}$$

$$-C\cdot s\cdot V_A + \left(\frac{1}{R_2} + C\cdot s\right)\cdot V_B - \frac{V_2(s)}{R_2} = 0$$



But the ideal OP AMP i-v characteristics require that $V_B(s) = 0$ since the noninverting input is grounded. The node eqs. become

$$\begin{pmatrix} 2\cdot C\cdot s + \dfrac{1}{R_1} & -C\cdot s \\ -C\cdot s & -\dfrac{1}{R_2} \end{pmatrix}\cdot\begin{pmatrix} V_A \\ V_2 \end{pmatrix} = \begin{pmatrix} \dfrac{V_1}{R_1} \\ 0 \end{pmatrix} \quad \Delta(s) = -\frac{R_1\cdot R_2\cdot C^2\cdot s^2 + 2\cdot R_1\cdot C\cdot s + 1}{R_1\cdot R_2} \qquad \Delta_2 = \frac{C\cdot s\cdot V_1}{R_1}$$

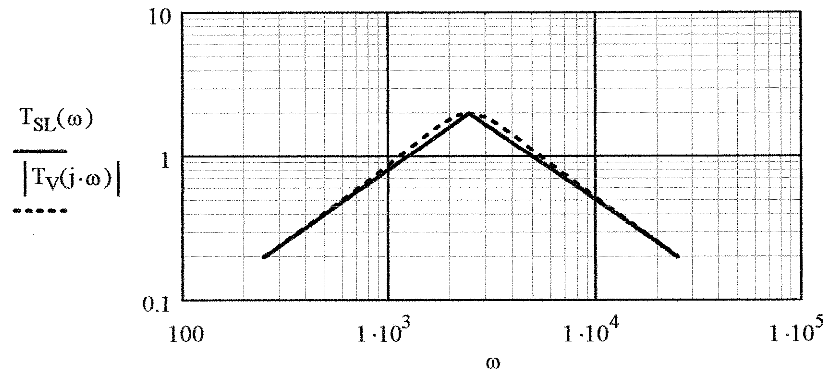
$$T_V(s) = \frac{V_2(s)}{V_1(s)} = \frac{-R_2\cdot C\cdot s}{R_1\cdot R_2\cdot C^2\cdot s^2 + 2\cdot R_1\cdot C\cdot s + 1} \qquad T_V(s) := \frac{-8\cdot 10^{-4}\cdot s}{\left(1.6\cdot 10^{-7}\cdot s^2\right) + 4\cdot 10^{-4}\cdot s + 1}$$

(b) $|T_V(0)| = 0$ $\quad |T_V(\infty)| = 0$ Ckt is bandpass $\quad R_1 := 10^4 \quad R_2 := 4\cdot 10^4 \quad C := 20\cdot 10^{-9}$

$\omega_0 := \dfrac{1}{C\cdot\sqrt{R_1\cdot R_2}}$ $\quad \omega_0 = 2.5\times 10^3$ $\quad \zeta := R_1\cdot C\cdot\omega_0$ $\quad \zeta = 0.5$

12-16 Continued

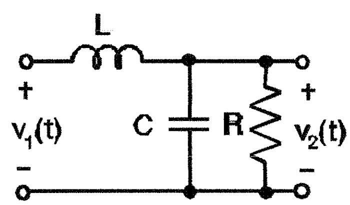
(c) $T_{SL}(\omega) := \begin{cases} 8\cdot 10^{-4}\cdot\omega & \text{if } 0 \le \omega < \omega_0 \\ \dfrac{5000}{\omega} & \text{if } \omega_0 \le \omega \end{cases}$ $\qquad \omega := 0.1\cdot\omega_0, 0.2\cdot\omega_0 .. 10\cdot\omega_0$



(d) $V_O(\omega) := T_{SL}(\omega)$ $\qquad V_O(0.5\cdot\omega_0) = 1$ $\qquad V_O(\omega_0) = 2$ $\qquad V_O(2\cdot\omega_0) = 1$

12-17 using voltage division

$$T_V(s) = \frac{Z_2}{Z_1 + Z_2} = \frac{\dfrac{1}{C\cdot s + \frac{1}{R}}}{L\cdot s + \dfrac{1}{C\cdot s + \frac{1}{R}}} = \frac{R}{R\cdot L\cdot C\cdot s^2 + L\cdot s + R}$$



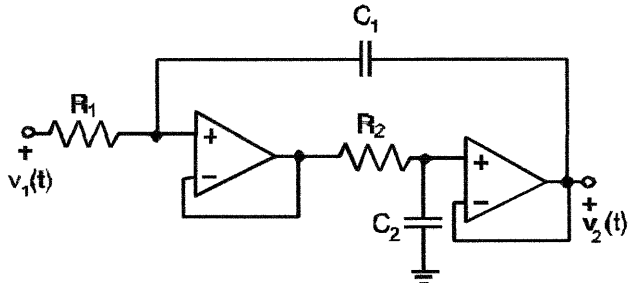
$$\omega_0 = \frac{1}{\sqrt{L\cdot C}} \qquad \zeta = \frac{1}{2}\cdot\frac{L}{R}\cdot\omega_0 = \frac{1}{2\cdot R}\cdot\sqrt{\frac{L}{C}}$$

$|T_V(0)| = 1$ $\qquad |T_V(\infty)| = 0$ $\qquad$ **Ckt is low pass**

For $\zeta = 2$ **&** $\omega_0 := 5000$ $\qquad L\cdot C = 4\cdot 10^{-8}$ $\qquad R = \frac{1}{4}\cdot\sqrt{\frac{L}{C}}$

Let $L := 10^{-1}$ $\quad C := \frac{4\cdot 10^{-8}}{L}$ $\quad C = 4 \times 10^{-7}$ $\quad R := \frac{1}{4}\cdot\sqrt{\frac{L}{C}}$ $\qquad R = 125\ \Omega$

12-18 Using node analysis



$$\begin{pmatrix} C_1\cdot s + \frac{1}{R_1} & -C_1\cdot s \\ -\frac{1}{R_2} & \frac{1}{R_2} + C_2\cdot s \end{pmatrix}\cdot\begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{pmatrix} \frac{V_1}{R_1} \\ 0 \end{pmatrix} \qquad \Delta(s) = \frac{R_1\cdot R_2\cdot C_1\cdot C_2\cdot s^2 + R_2\cdot C_2\cdot s + 1}{R_1\cdot R_2} \qquad \Delta_B(s) = \frac{V_1}{R_1\cdot R_2}$$

$$T_V(s) = \frac{V_2(s)}{V_1(s)} = \frac{V_B(s)}{V_1(s)} = \frac{1}{R_1\cdot R_2\cdot C_1\cdot C_2\cdot s^2 + R_2\cdot C_2\cdot s + 1} \qquad |T_V(0)| = 1 \qquad |T_V(\infty)| = 0$$

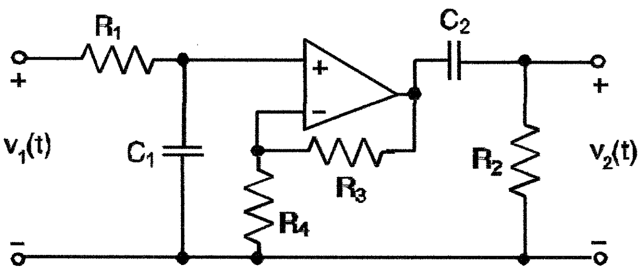
Circuit is low pass with a passband gain of unity for all values of the circuit parameters.

$$\omega_0 = \frac{1}{\sqrt{R_1\cdot R_2\cdot C_1\cdot C_2}} \qquad \zeta = \frac{1}{2}\cdot R_2\cdot C_2\cdot\omega_0 = \frac{1}{2}\cdot\sqrt{\frac{R_2\cdot C_2}{R_1\cdot C_1}}$$

Let $R_1 = R_2 = R$ then $C_1\cdot C_2 = (R\cdot\omega_0)^{-2}$ & $R\cdot C_2 = \frac{2\cdot\zeta}{\omega_0}$ For $\omega_0 := 2500$ & $\zeta := 0.7$

Let $R := 10^4$ $\quad C_2 := \frac{2\cdot\zeta}{\omega_0\cdot R}$ $\quad C_2 = 5.6\times 10^{-8}$ $\quad C_1 := \frac{(R\cdot\omega_0)^{-2}}{C_2}$ $\quad C_1 = 2.857\times 10^{-8}$

12-19



$$T_V(s) = T_1\cdot T_2\cdot T_3 = \left(\frac{\frac{1}{C_1\cdot s}}{R_1 + \frac{1}{C_1\cdot s}}\right)\cdot\left(\frac{R_3 + R_4}{R_4}\right)\cdot\left(\frac{R_2}{R_2 + \frac{1}{C_2\cdot s}}\right) = \left(\frac{R_3 + R_4}{R_4}\right)\cdot\left(\frac{1}{R_1\cdot C_1\cdot s + 1}\right)\cdot\left(\frac{R_2\cdot C_2\cdot s}{R_2\cdot C_2\cdot s + 1}\right)$$

$R_1 := 10^4$ $\quad R_2 := 10^4$ $\quad R_4 := 10^4$ $\quad \omega_{C1} := 500$ $\quad \omega_{C2} := 5\cdot 10^4$ $\quad K := 5$

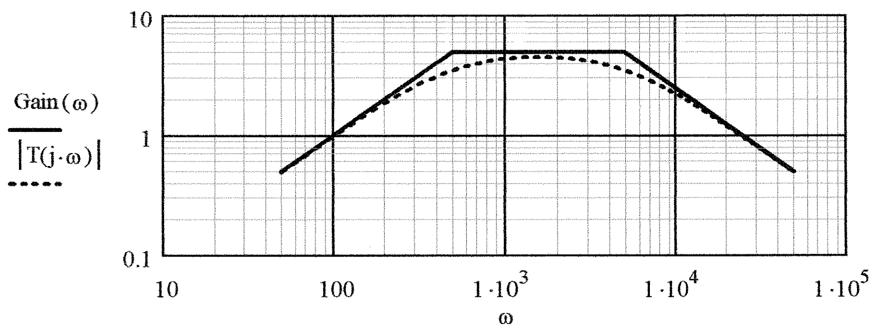
$K = \frac{R_3 + R_4}{R_4}$ $\quad \omega_{C1} = \frac{1}{R_2\cdot C_2}$ $\quad R_3 := (K - 1)\cdot R_4$ $\quad C_2 := \frac{1}{R_2\cdot\omega_{C1}}$ $\quad C_1 := \frac{1}{R_1\cdot\omega_{C2}}$

$C_1 = 2\times 10^{-9}$ $\quad C_2 = 2\times 10^{-7}$ $\quad R_3 = 4\times 10^4$

12-20 $T(s) := \dfrac{2.5\cdot 10^4\cdot s}{s^2 + 5.5\cdot 10^3\cdot s + 2.5\cdot 10^6}$ $\quad T(s) = \dfrac{2.5\cdot 10^4\cdot s}{(s+500)\cdot(s+5000)} = \left(\dfrac{\frac{s}{500}}{\frac{s}{500}+1}\right)\cdot(5)\cdot\left(\dfrac{1}{\frac{s}{5000}+1}\right)$

(a) $K := 5$, $\omega_{C1} := 500\text{rad/s}$, and $\omega_{C2} := 5000$ rad/s.

(b) $\text{Gain}(\omega) := \dfrac{\omega}{500}\cdot G_{SL}(\omega, \omega_{C1})\cdot 5\cdot G_{SL}(\omega, \omega_{C2})$ $\qquad \omega := 0.1\cdot\omega_{C1}, 0.2\cdot\omega_{C1} .. 10\cdot\omega_{C2}$



12-21 Given a series RLC circuit with: $\omega_0 := 40\cdot 10^6$ **rad/s** ; $R := 50\Omega$; $B := 8\cdot 10^6$ **rad/s**

Find: $L := \dfrac{R}{B}$; $L = 6.25\times 10^{-6}$ **H** ; $C := \dfrac{1}{\omega_0^{\,2}\cdot L}$; $C = 1\times 10^{-10}$ **F** ; $Q := \dfrac{\omega_0\cdot L}{R}$; $Q = 5$

$\omega_{C1} := -\dfrac{B}{2} + \sqrt{\left(\dfrac{B}{2}\right)^2 + \omega_0^{\,2}}$ $\quad \omega_{C1} = 3.62\times 10^7$ $\quad \omega_{C2} := \omega_{C1} + B$ $\quad \omega_{C2} = 4.42\times 10^7$

12-22 Given a parallel RLC circuit with: $\omega_0 := 10^8$ **rad/s** ; $C := 20\cdot 10^{-12}$ **F** ; $Q := 10$

Find: $B := \dfrac{\omega_0}{Q}$; $B = 1\times 10^7$; $L := \dfrac{1}{\omega_0^{\,2}\cdot C}$; $L = 5\times 10^{-6}$ **H** ; $R := \dfrac{1}{B\cdot C}$; $R = 5\times 10^3$

$\omega_{C1} := -\dfrac{B}{2} + \sqrt{\left(\dfrac{B}{2}\right)^2 + \omega_0^{\,2}}$ $\quad \omega_{C1} = 9.512\times 10^7$ $\quad \omega_{C2} := \omega_{C1} + B$ $\quad \omega_{C2} = 1.051\times 10^8$

12-23 Given a series RLC circuit with: $R := 100\Omega$; $L := 20\cdot 10^{-3}$ **H** ; $C := 200\cdot 10^{-12}$ **F** ; $V_A := 10V$.

(a) $\omega_0 := (L\cdot C)^{-0.5}$; $\omega_0 = 5\times 10^5$ **rad/s** ; $Q := \dfrac{\omega_0\cdot L}{R}$; $Q = 100$; $B := \dfrac{\omega_0}{Q}$; $B = 5000$ **rad/s;**

$\omega_{C1} := -\dfrac{B}{2} + \sqrt{\left(\dfrac{B}{2}\right)^2 + \omega_0^{\,2}}$; $\omega_{C1} = 4.975\times 10^5$ **rad/s** ; $\omega_{C2} := \omega_{C1} + B$; $\omega_{C2} = 5.025\times 10^5$ **rad/s.**

(b) $V_L := (j\cdot\omega_0\cdot L)\cdot\dfrac{V_A}{R}$; $|V_L| = 1\times 10^3$ **V** ; $V_C := \left(\dfrac{1}{j\cdot\omega_0\cdot C}\right)\cdot\dfrac{V_A}{R}$; $|V_C| = 1\times 10^3$ **V.**

12-24 Given a parallel RLC circuit with: $\omega_0 := 400\cdot 10^3$ **rad/s** ; $\omega_{C2} := 420\cdot 10^3$ **rad/s ; Find:**

$\omega_{C1} := \dfrac{\omega_0^{\,2}}{\omega_{C2}}$ $\quad \omega_{C1} = 3.81\times 10^5$ $\quad B := \omega_{C2} - \omega_{C1}$ $\quad B = 3.905\times 10^4$ $\quad Q := \dfrac{\omega_0}{B}$ $\quad Q = 10.244$

12-25 Given a parallel RLC circuit with: $R := 40\cdot10^3\ \Omega$; $\omega_0 := 2\cdot\pi\cdot10^8$ rad/s ; $B := 2\cdot\pi\cdot10^5$ rad/s.

$Q := \dfrac{\omega_0}{B}$; $Q := 1000$ $L := \dfrac{R}{Q\cdot\omega_0}$; $L = 6.366\times10^{-8}$ H ; $C := \dfrac{1}{\omega_0^2\cdot L}$; $C = 3.979\times10^{-11}$ F.

12-26 Given a series RLC circuit with: $R := 50\ \Omega$; $\omega_0 := 4\cdot\pi\cdot10^5$;

Since $B = \dfrac{\omega_0}{Q}$ we must maximize Q to minimize B. Since $Q = \dfrac{\omega_0\cdot L}{R}$ and ω_0 & R are fixed,

we must maximize L to maximize Q. Since $L := \dfrac{1}{\omega_0^2\cdot C}$ we must minimize C to minimize L.

Let $C := 120\cdot10^{-9}$ F ; then $L := \dfrac{1}{\omega_0^2\cdot C}$; $L = 5.277\times10^{-6}$ H .

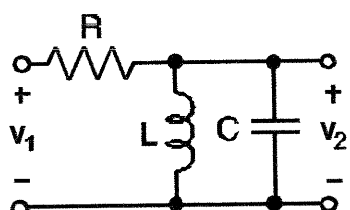
12-27 Given a series RLC circuit with: $R := 600\ \Omega$; $\omega_0 := 2\cdot\pi\cdot60$; $\omega_0 = 376.991$ rad/s ;
$\omega_{C2} < 2\cdot\pi\cdot200$; ω_{C2}<1257 rad/s.

Let $\omega_{C2} := 1200$ Given $\left(\dfrac{B}{2}\right) + \sqrt{\left(\dfrac{B}{2}\right)^2 + \omega_0^2} = 1200$; $B := \text{Find}(B)$; $B = 1.082\times10^3$ rad/s

hence $Q := \dfrac{\omega_0}{B}$; $Q = 0.349$; $L := Q\cdot\dfrac{R}{\omega_0}$; $L = 0.555$ H ; $C := \dfrac{1}{\omega_0^2\cdot L}$; $C = 1.268\times10^{-5}$ F.

12-28 By voltage division:

$$T_V(s) = \frac{\dfrac{1}{C\cdot s + \dfrac{1}{L\cdot s}}}{R + \dfrac{1}{C\cdot s + \dfrac{1}{L\cdot s}}} = \frac{\dfrac{1}{R\cdot C}\cdot s}{s^2 + \dfrac{1}{R\cdot C}\cdot s + \dfrac{1}{L\cdot C}}$$

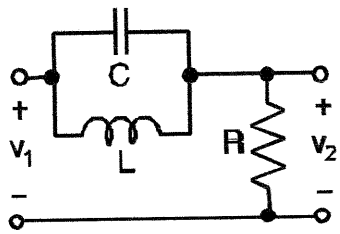


Comparing with the std form $\dfrac{K\cdot s}{s^2 + B\cdot s + \omega_0^2}$ yields $\omega_0 = \dfrac{1}{\sqrt{L\cdot C}}$ and $B = \dfrac{1}{R\cdot C}$

Hence $Q = \dfrac{\omega_0}{B} = R\cdot\sqrt{\dfrac{C}{L}}$; $\omega_{C1} = -\dfrac{B}{2} + \sqrt{\left(\dfrac{B}{2}\right)^2 + \omega_0^2} = \dfrac{-1}{2\cdot R\cdot C} + \sqrt{\left(\dfrac{1}{2\cdot R\cdot C}\right)^2 + \dfrac{1}{L\cdot C}}$

$$\omega_{C2} = \omega_{C1} + B = \frac{1}{2\cdot R\cdot C} + \sqrt{\left(\frac{1}{2\cdot R\cdot C}\right)^2 + \frac{1}{L\cdot C}}$$

12-29 (a) At low frequency the inductor acts like a short circuit and at high frequency the capacitor acts like a short circuit. In either case the input is directly connected to the output so the low- and high-frequency gains are unity.



(b) The impedance of the parallel LC branch is

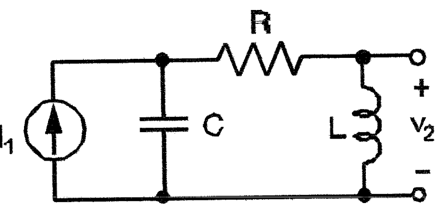
$$Z_{LC} = \frac{1}{C\cdot s + \frac{1}{L\cdot s}} = \frac{L\cdot s}{L\cdot C\cdot s^2 + 1}$$

This impedance has a pole at $s = \frac{j}{\sqrt{L\cdot C}}$ which means the impedance is infinite at $\omega_0 = \frac{1}{\sqrt{L\cdot C}}$ thereby disconnecting the input and output to produce the bandstop notch.

(c) The cutoff frequencies occur when $Z_{LC}(j\omega) = \pm jR$ or $\omega\cdot L = R\cdot\left(1 - \omega^2\cdot L\cdot C\right)$ and $\omega\cdot L = -R\cdot\left(1 - \omega^2\cdot L\cdot C\right)$

which yield $\omega_{C1} = \frac{-L + \sqrt{L^2 + 4\cdot R^2\cdot L\cdot C}}{2\cdot R\cdot L\cdot C}$ and $\omega_{C2} = \frac{L + \sqrt{L^2 + 4\cdot R^2\cdot L\cdot C}}{2\cdot R\cdot L\cdot C}$

12-30 By current division:



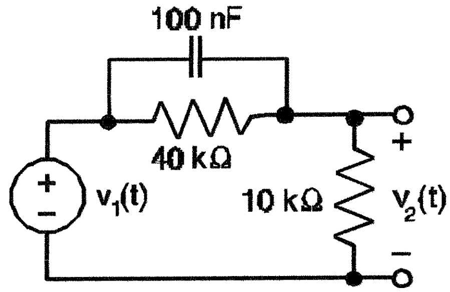
$$T_Z(s) = I_2\cdot L\cdot s = \left(\frac{\frac{1}{L\cdot s + R}}{\frac{1}{L\cdot s + R} + C\cdot s}\right)\cdot L\cdot s = \frac{\frac{s}{C}}{s^2 + \frac{R}{L}\cdot s + \frac{1}{L\cdot C}}$$

Comparing with the std form $\frac{K\cdot s}{s^2 + B\cdot s + \omega_0^2}$ yields $\omega_0 = \frac{1}{\sqrt{L\cdot C}}$ and $B = \frac{R}{L}$

Hence $Q = \frac{\omega_0}{B} = \frac{1}{R}\cdot\sqrt{\frac{L}{C}}$; $\omega_{C1} = -\frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_0^2} = \frac{-R}{2\cdot L} + \sqrt{\left(\frac{R}{2\cdot L}\right)^2 + \frac{1}{L\cdot C}}$

$$\omega_{C2} = \omega_{C1} + B = \frac{R}{2\cdot L} + \sqrt{\left(\frac{R}{2\cdot L}\right)^2 + \frac{1}{L\cdot C}}.$$

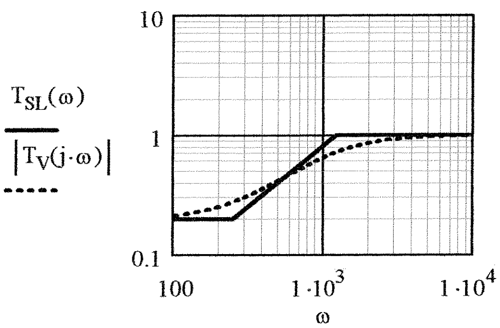
12-31 Using voltage division



$$T_V(s) = \frac{10^4}{10^4 + \dfrac{1}{10^{-7}\cdot s + \dfrac{1}{40\cdot 10^3}}}$$

$$T_V(s) := \frac{(s+250)}{(s+1250)} \qquad \omega := 100, 101 .. 10000$$

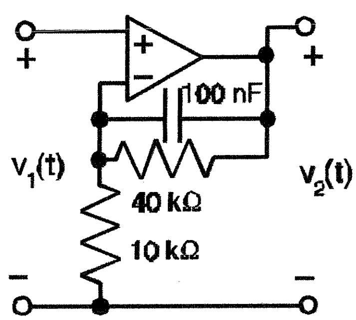
$$T_{SL}(\omega) := \begin{cases} \frac{1}{5} & \text{if } 0 \le \omega < 250 \\ \frac{\omega}{1250} & \text{if } 250 \le \omega < 1250 \\ 1 & \text{if } 1250 \le \omega \end{cases}$$



$A_{SL} := 10\cdot T_{SL}(500) \qquad A_{SL} = 4$

$A := 10\cdot |T_V(j\cdot 500)| \qquad A = 4.152$

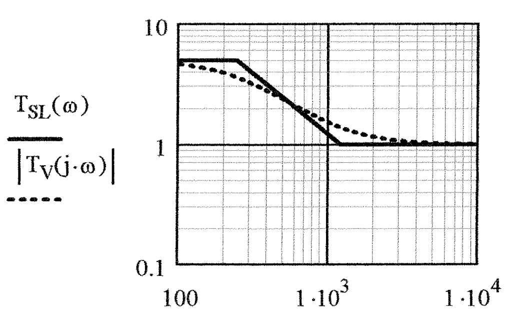
12-32 Noninverting amplifier



$$T_V(s) = \frac{10^4 + \dfrac{1}{10^{-7}\cdot s + \dfrac{1}{40\cdot 10^3}}}{10^4}$$

$$T_V(s) := \frac{(s+1250)}{(s+250)}$$

$$\omega := 100, 101 .. 10000 \qquad T_{SL}(\omega) := \begin{cases} 5 & \text{if } 0 \le \omega < 250 \\ \frac{1250}{\omega} & \text{if } 250 \le \omega < 1250 \\ 1 & \text{if } 1250 \le \omega \end{cases}$$



$A_{SL} := 10\cdot T_{SL}(500) \qquad A_{SL} = 25$

$A := 10\cdot |T_V(j\cdot 500)| \qquad A = 24.083$

****

The functions below define the straight-line approximations to the gain (G) and angle (A) of a first order factor. The are used to create the Bode plots for Prob. 12-33 thru 12-47

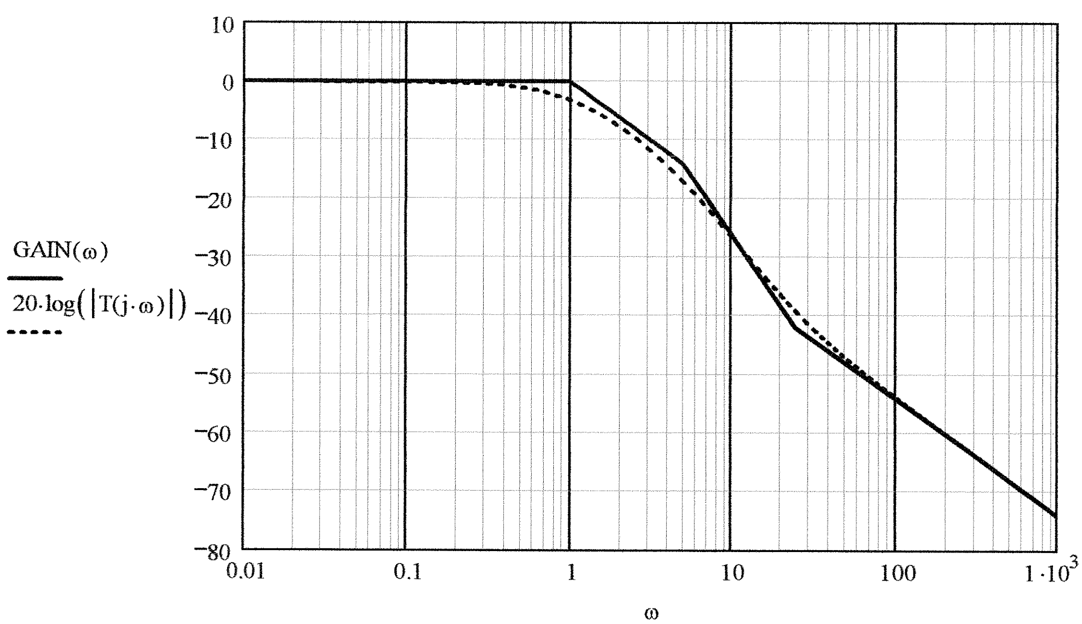
$$G_{SL}(\omega,\alpha) := \begin{cases} 0 & \text{if } 0 \le \omega < \alpha \\ 20\cdot\log\left(\dfrac{\omega}{\alpha}\right) & \text{if } \alpha \le \omega \end{cases}$$

$$A_{SL}(\omega,\alpha) := \begin{cases} 0 & \text{if } 0 \le \omega < 0.1\cdot\alpha \\ 45\cdot\log\left(\dfrac{\omega}{0.1\cdot\alpha}\right) & \text{if } 0.1\cdot\alpha \le \omega < 10\cdot\alpha \\ 90 & \text{if } 10\cdot\alpha \le \omega \end{cases}$$

****

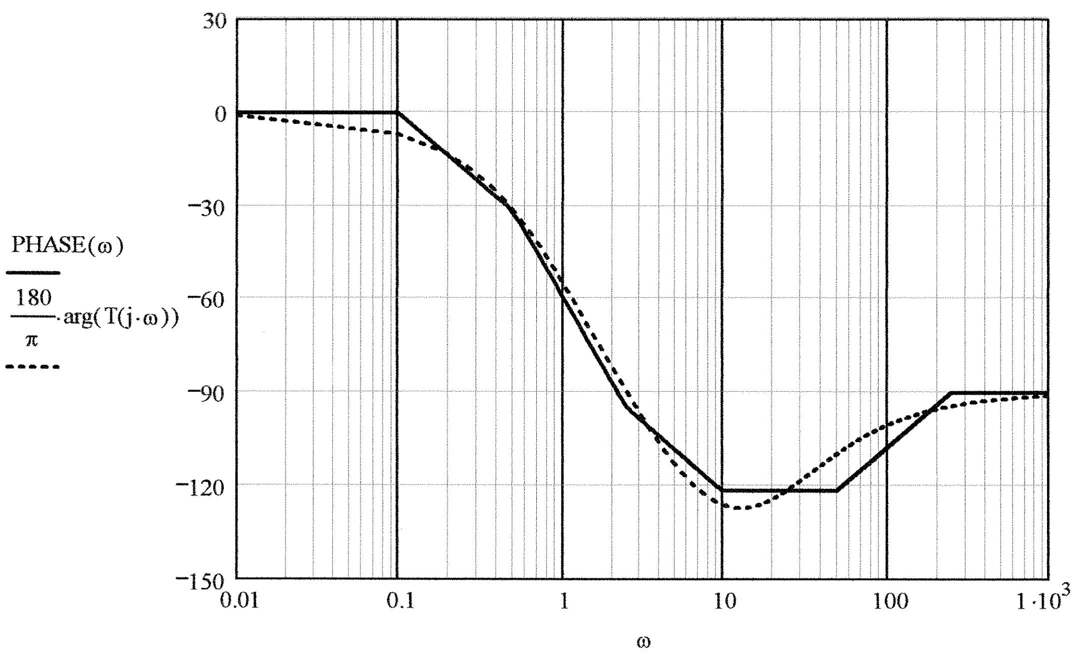
12-33 $T(s) := \dfrac{s + 25}{(5\cdot s + 5)\cdot(s + 5)}$ $\qquad \omega := 0.01, 0.1 .. 1000$

$GAIN(\omega) := G_{SL}(\omega,25) - G_{SL}(\omega,5) - G_{SL}(\omega,1)$



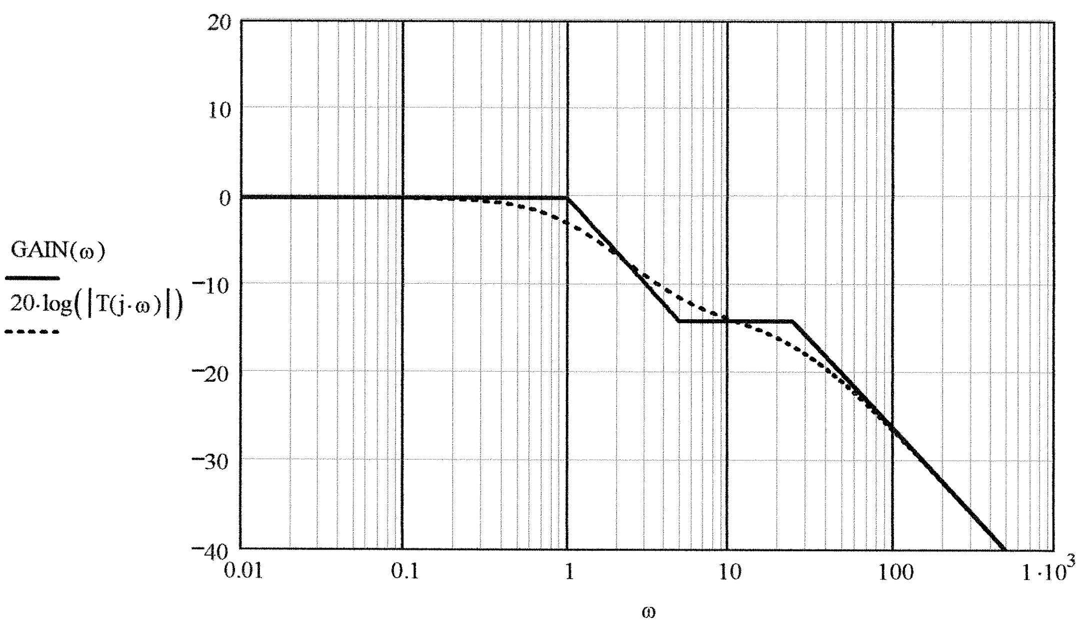
Circuit is Low pass with a passband gain of 0 dB and cutoff at about 1 rad/s

12-33 Continued $PHASE(\omega) := A_{SL}(\omega, 25) - A_{SL}(\omega, 5) - A_{SL}(\omega, 1)$



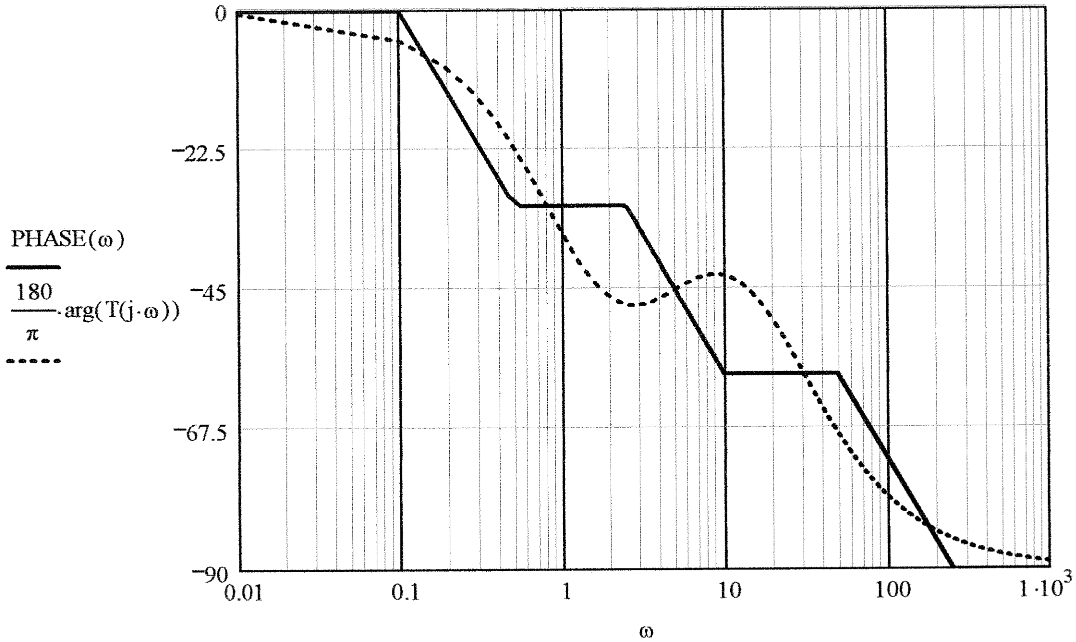
12-34 $T(s) := \dfrac{5\cdot(s+5)}{(s+1)\cdot(s+25)}$ $\omega := 0.01, 0.1 .. 1000$

$GAIN(\omega) := G_{SL}(\omega, 5) - G_{SL}(\omega, 25) - G_{SL}(\omega, 1)$



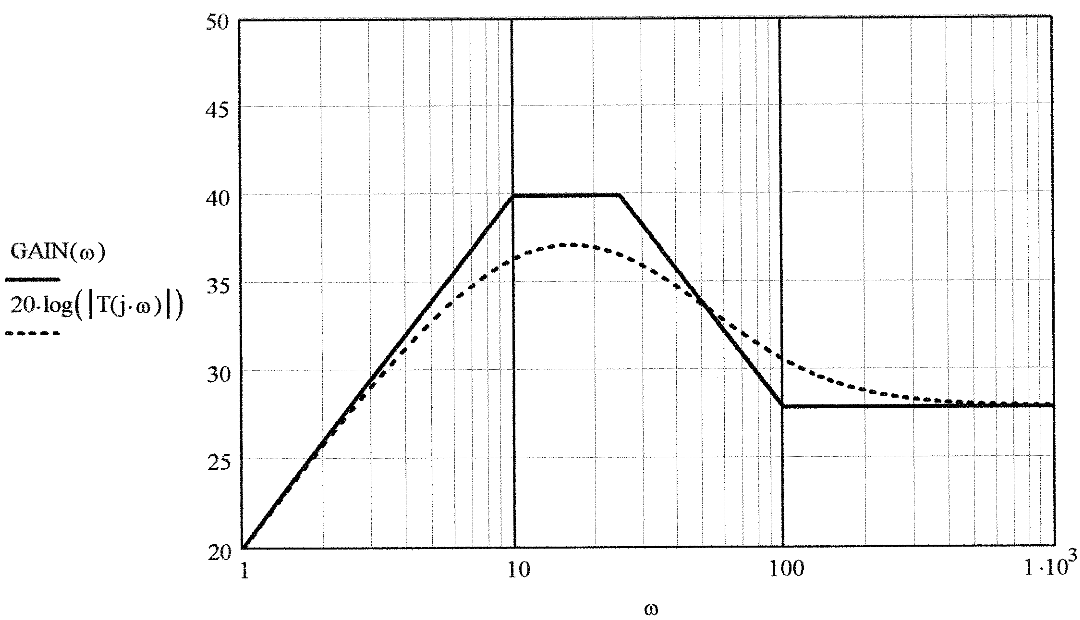
Low pass with a passband gain of 0 dB and cutoff at about 0.5 rad/s

12-34 Continued $\mathrm{PHASE}(\omega) := A_{SL}(\omega, 5) - A_{SL}(\omega, 25) - A_{SL}(\omega, 1)$



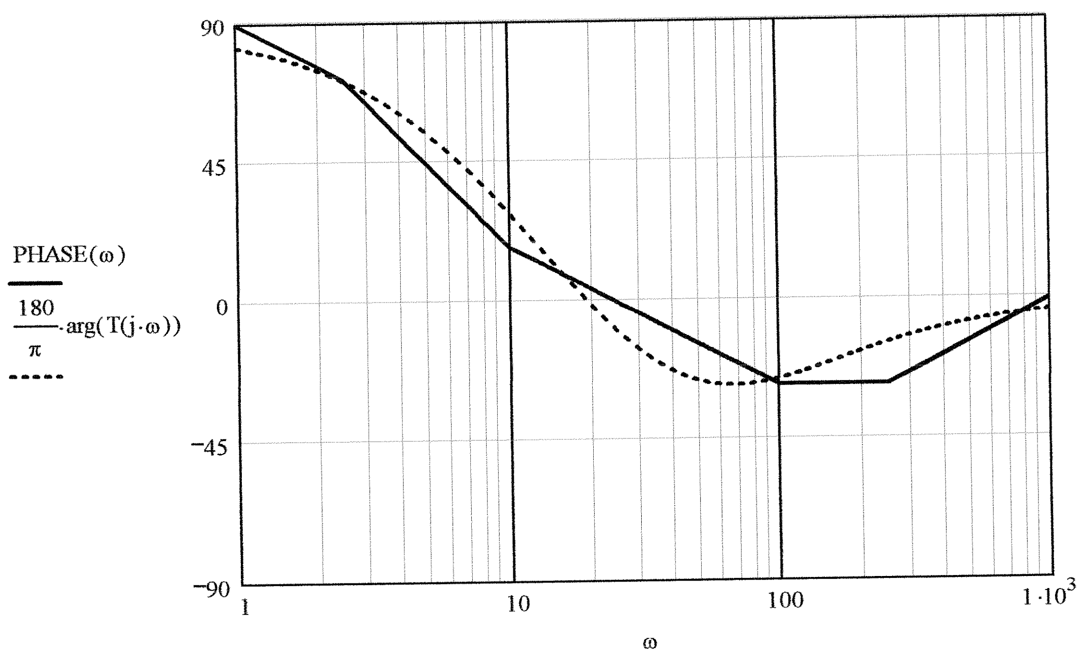
12-35 $T(s) := \dfrac{25\cdot s\cdot(s+100)}{(s+10)\cdot(s+25)}$ $\omega := 1, 1.5 .. 1000$

$\mathrm{GAIN}(\omega) := 20\cdot\log(10\cdot\omega) + G_{SL}(\omega, 100) - G_{SL}(\omega, 25) - G_{SL}(\omega, 10)$



Bandpass with a passband gain of 40 dB and cutoffs at about 10 & 25 rad/s

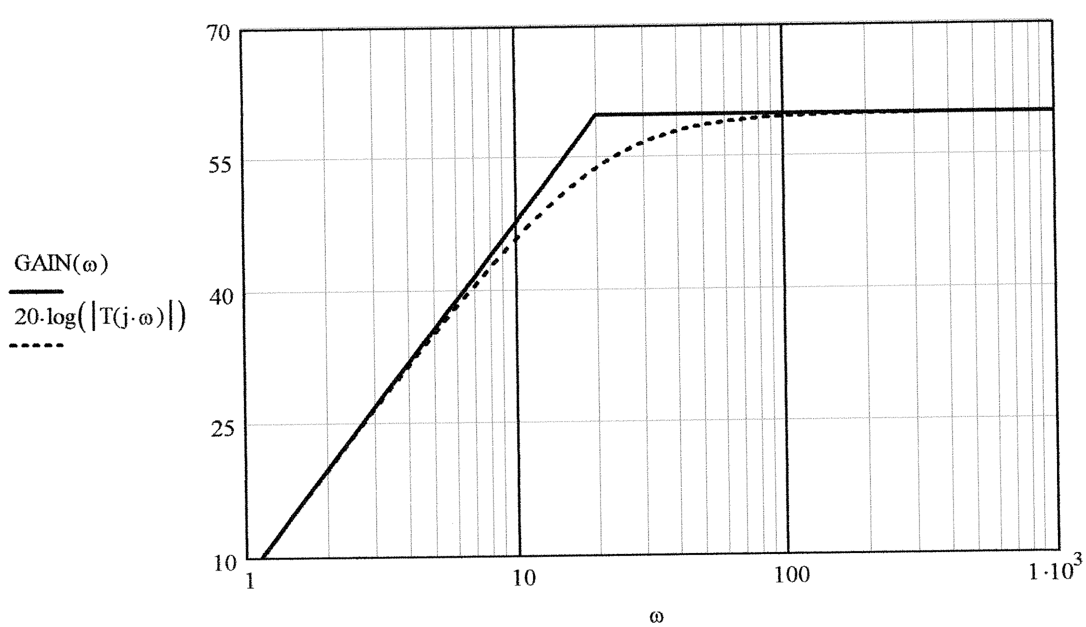
12-35 Continued $PHASE(\omega) := 90 + A_{SL}(\omega, 100) - A_{SL}(\omega, 25) - A_{SL}(\omega, 10)$



12-36

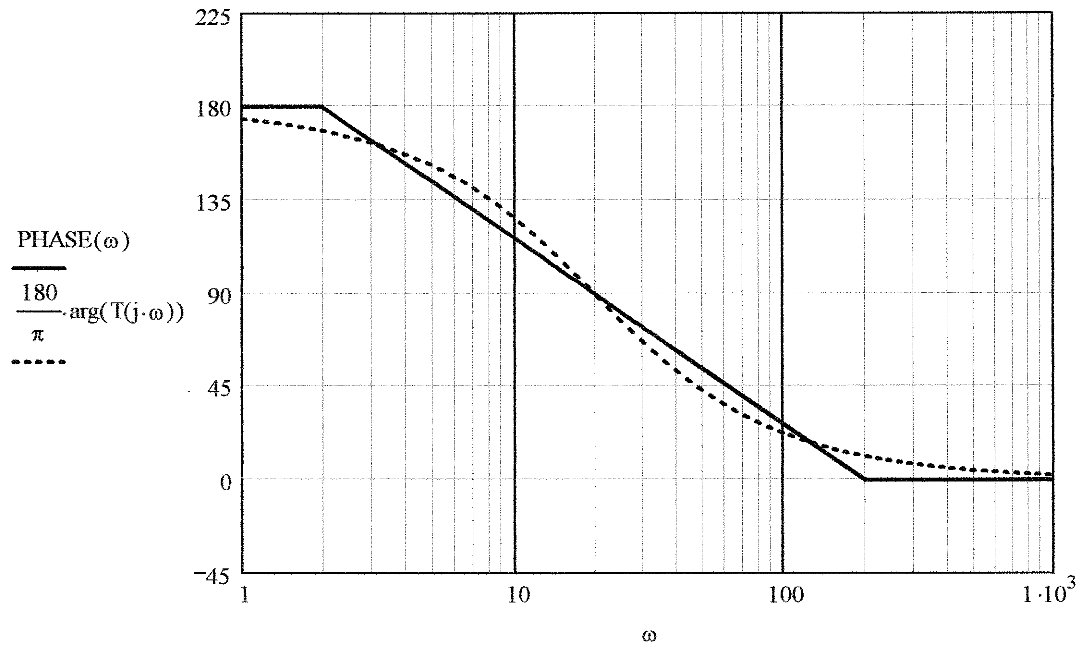
$$T(s) := \frac{2.5 \cdot s^2}{(0.05 \cdot s + 1)^2} \qquad \omega := 1, 1.5 .. 1000$$

$$GAIN(\omega) := 20 \log(2.5) + 40 \cdot \log(\omega) - 2 \cdot G_{SL}(\omega, 20)$$



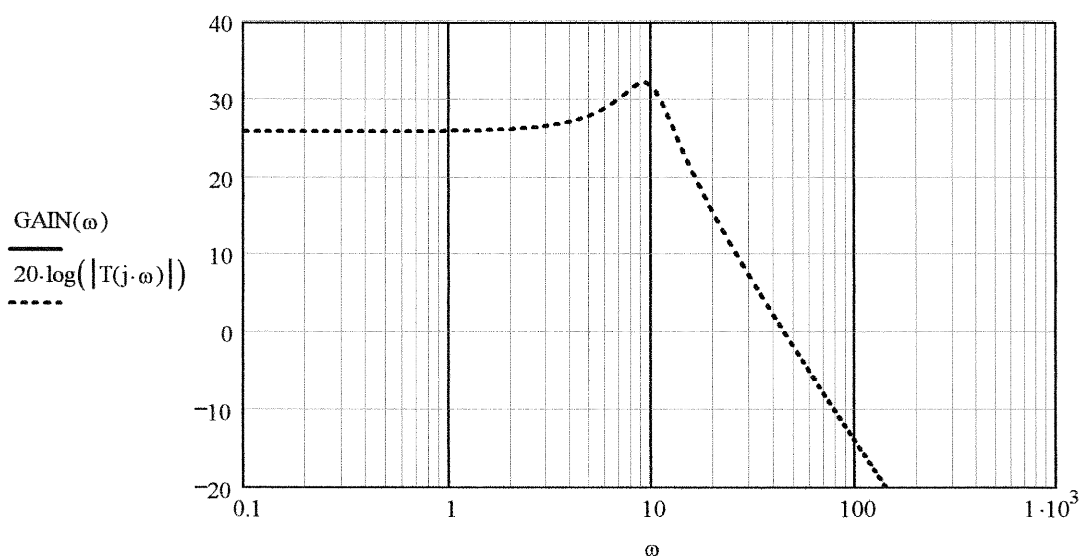
High pass a passband gain of 60 dB and cutoff at about 35 rad/s

12-36 Continued $PHASE(\omega) := 180 - 2 \cdot A_{SL}(\omega, 20)$



12-37 $T(s) := \dfrac{2000}{s^2 + 5 \cdot s + 100}$ $\omega := .1, .2 .. 1000$ $\omega_0 := 10$ $\zeta := \dfrac{5}{2 \cdot \omega_0}$ $\zeta = 0.25$

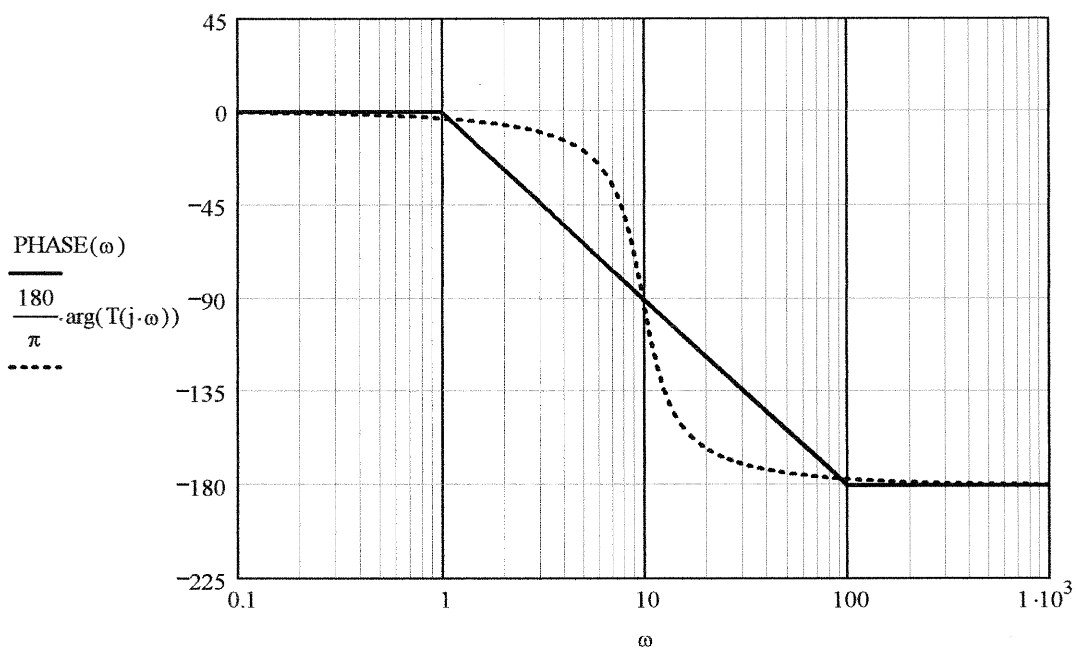
$GAIN(\omega) := 20 \cdot \log(20) - 2 \cdot G_{SL}(\omega, 10)$ $\omega_{max} := \omega_0 \cdot \sqrt{1 - 2 \cdot \zeta^2}$ $\omega_{max} = 9.354$



Low pass wih a passband gain of 26 dB. Max gain = $20 \cdot \log(|T(j \cdot \omega_{max})|) = 32.321$ **dB.**
Cutoff freq occurs at about 10 rad/s.

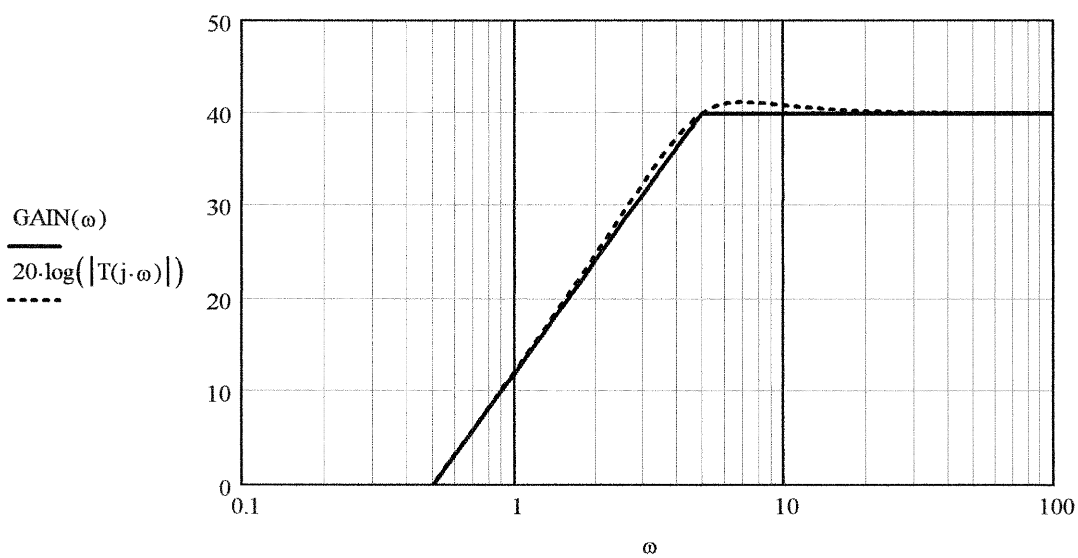
12-37 Continued

$PHASE(\omega) := -2 \cdot A_{SL}(\omega, 10)$



12-38 $T(s) := \frac{4 \cdot s^2}{0.04 \cdot s^2 + 0.2 \cdot s + 1}$ $\omega := 0.1, 0.2 .. 100$ $\omega_0 := 5$ $\zeta := \frac{5}{2 \cdot \omega_0}$ $\omega_{max} := \frac{\omega_0}{\sqrt{1 - 2 \cdot \zeta^2}}$

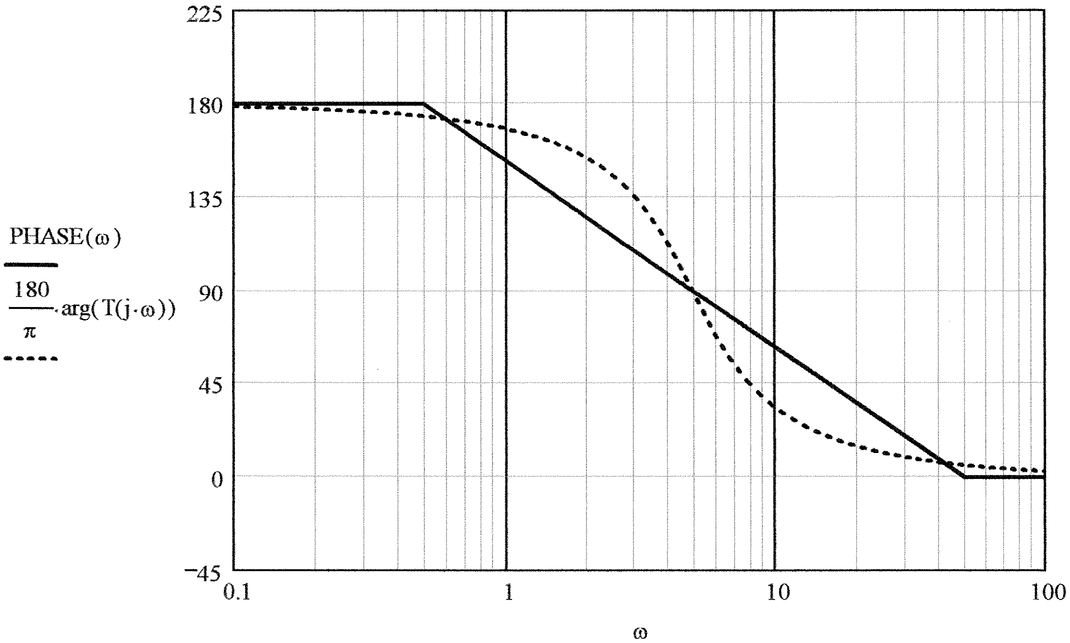
$GAIN(\omega) := 20 \cdot \log(4) + 40 \cdot \log(\omega) - 2 \cdot G_{SL}(\omega, 5)$ $\omega_{max} = 7.071$ $\zeta = 0.5$



Str-line High pass gain of 40 dB. Max gain of $20 \cdot \log(|T(j \cdot \omega_{max})|) = 41.249$ **dB**

The cutoff frequency occurs at about 5 rad/s.

12-38 Continued $PHASE(\omega) := 180 - 2 \cdot A_{SL}(\omega, 5)$

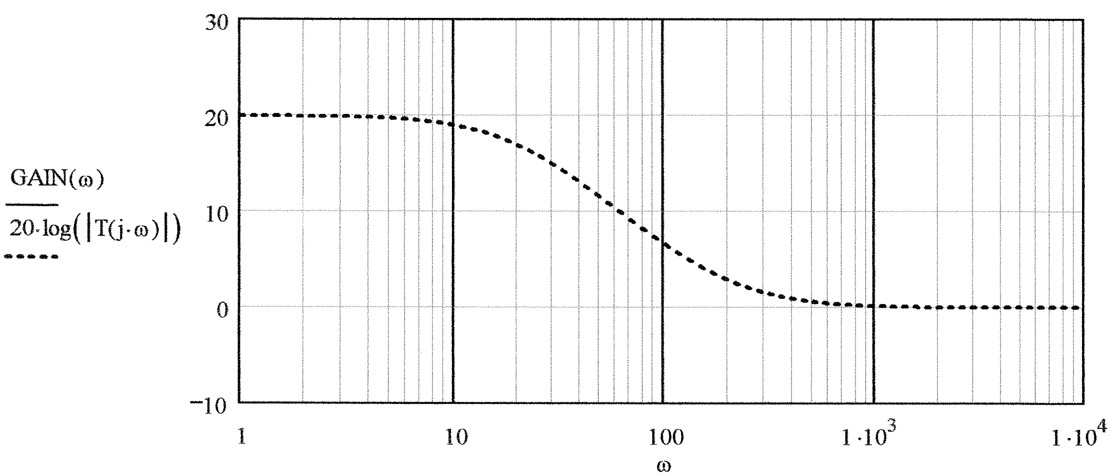


12-39 $GAIN(\omega) := 20 \cdot \log(10) - G_{SL}(\omega, 20) + G_{SL}(\omega, 200)$ $\quad \omega := 1, 3 .. 10000$

$$T(s) := \frac{10 \cdot \left(\frac{s}{200} + 1\right)}{\left(\frac{s}{20} + 1\right)}$$

$GAIN(20) = 20$ $\quad 20 \cdot \log(|T(j \cdot 20)|) = 40.262$

$GAIN(200) = 0$ $\quad 20 \cdot \log(|T(j \cdot 200)|) = 2.967$



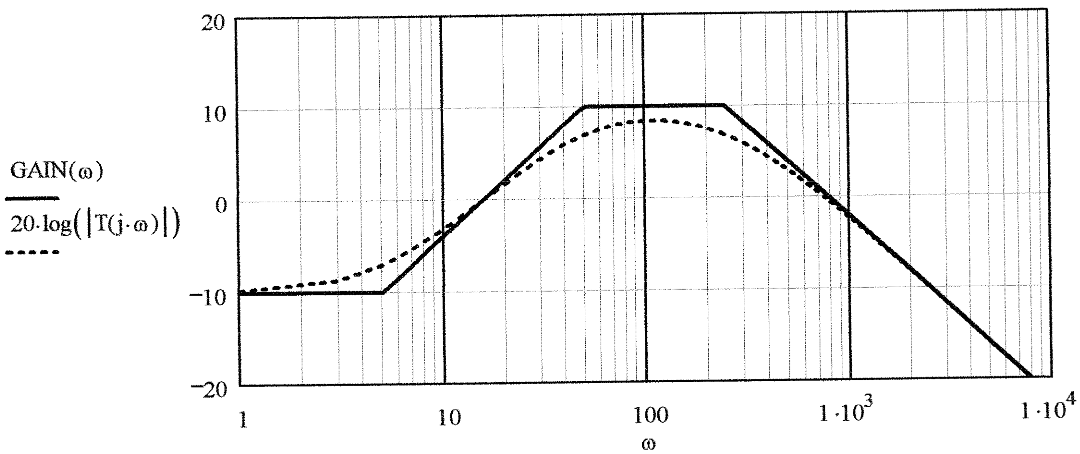
12-40 $GAIN(\omega) := G_{SL}(\omega,5) - G_{SL}(\omega,50) - G_{SL}(\omega,250) - 10$ $\qquad \omega := 1,3..10000$

$$T(s) := \frac{\frac{1}{\sqrt{10}}\cdot\left(\frac{s}{5}+1\right)}{\left(\frac{s}{50}+1\right)\cdot\left(\frac{s}{250}+1\right)}$$

$GAIN(5) = -10$ $\qquad 20\cdot\log(|T(j\cdot 5)|) = -7.035$

$GAIN(50) = 10$ $\qquad 20\cdot\log(|T(j\cdot 50)|) = 6.863$

$GAIN(250) = 10$ $\qquad 20\cdot\log(|T(j\cdot 250)|) = 6.821$



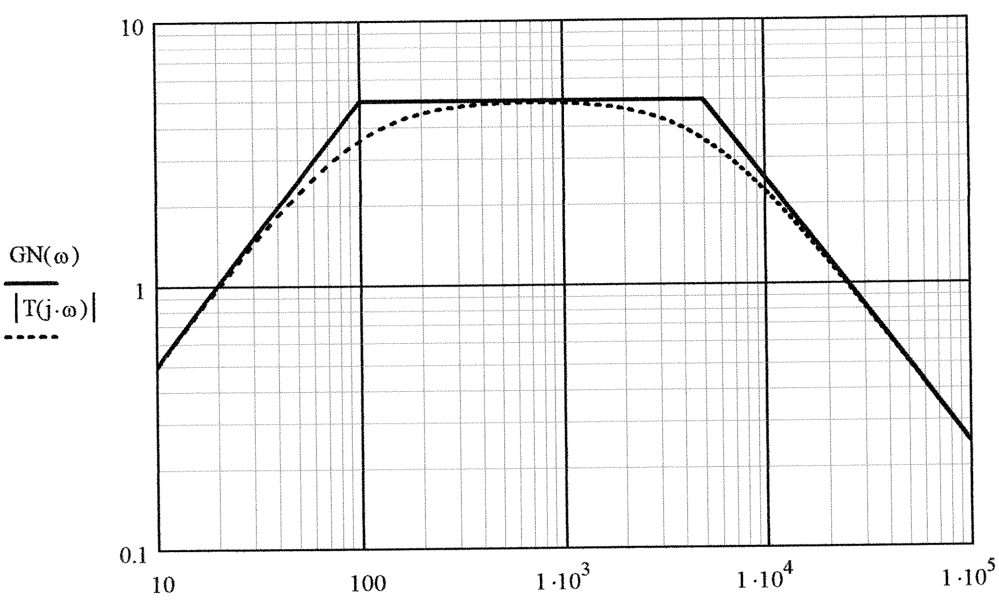
12-41 $GAIN(\omega) := 20\cdot\log\left(\frac{1}{20}\right) + 20\cdot\log(\omega) - G_{SL}(\omega,100) - G_{SL}(\omega,5000)$

$$GN(\omega) := 10^{\left(\frac{GAIN(\omega)}{20}\right)}$$

$$T(s) := \frac{1}{20}\cdot\frac{s}{\left(\frac{s}{100}+1\right)\cdot\left(\frac{s}{5000}+1\right)}$$

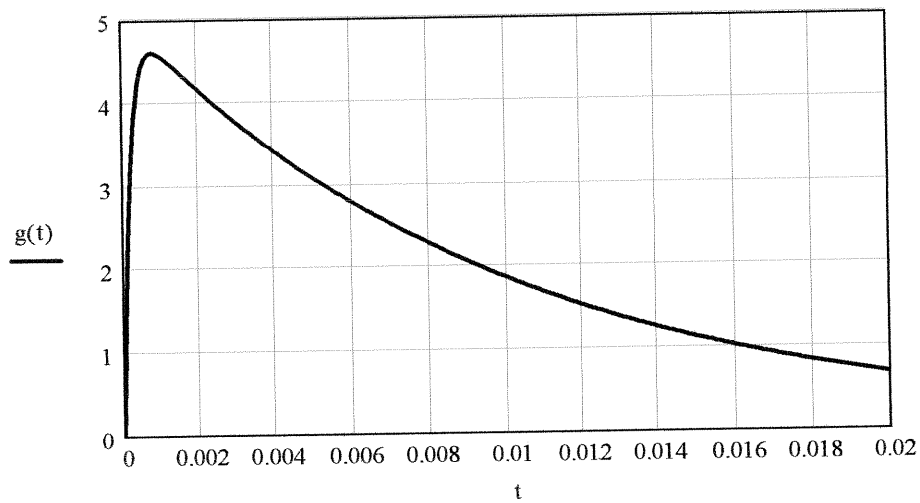
Note: The vertical axis in the plot is **NOT in dB**.

$\omega := 10, 20..10^5$



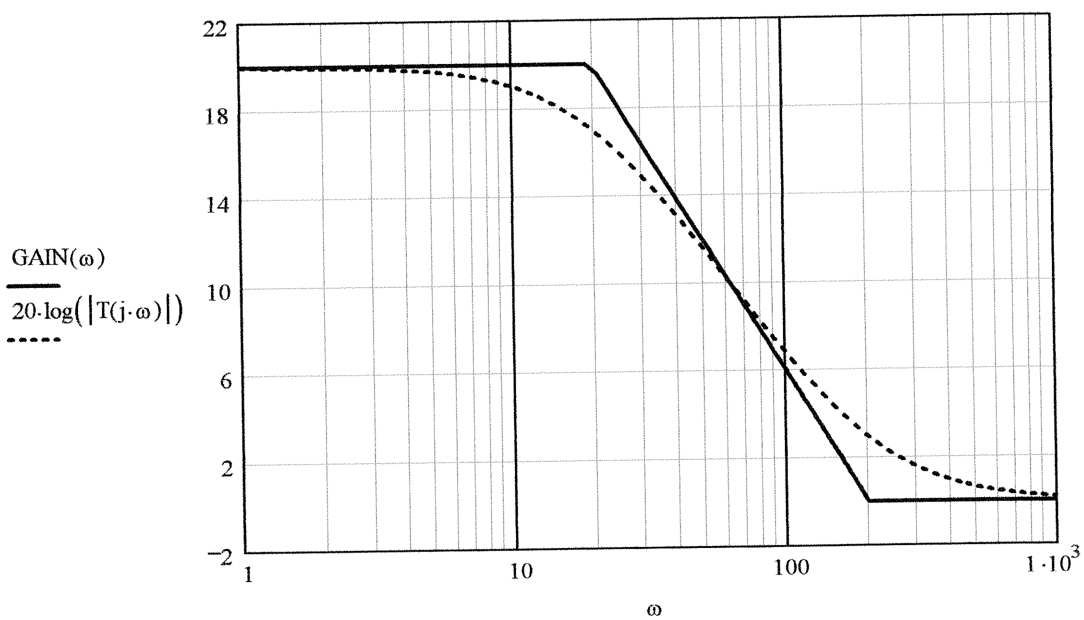
12-41Continued $G(s) = \frac{T(s)}{s} = \frac{1}{20}\cdot\frac{1}{\left(\frac{s}{100}+1\right)\cdot\left(\frac{s}{5000}+1\right)} = \frac{250}{49\cdot(s+100)} - \frac{250}{49\cdot(s+5000)}$ $u(t) := 1$

$$g(t) := \left(\frac{250}{49}\cdot\exp(-100\cdot t) - \frac{250}{49}\cdot\exp(-5000\cdot t)\right)\cdot u(t) \qquad t := 0, 0.0001 .. 0.02$$



12-42 $GAIN(\omega) := 20\cdot\log(10) - G_{SL}(\omega, 20) + G_{SL}(\omega, 200) \qquad \omega := 1, 3 .. 1000$

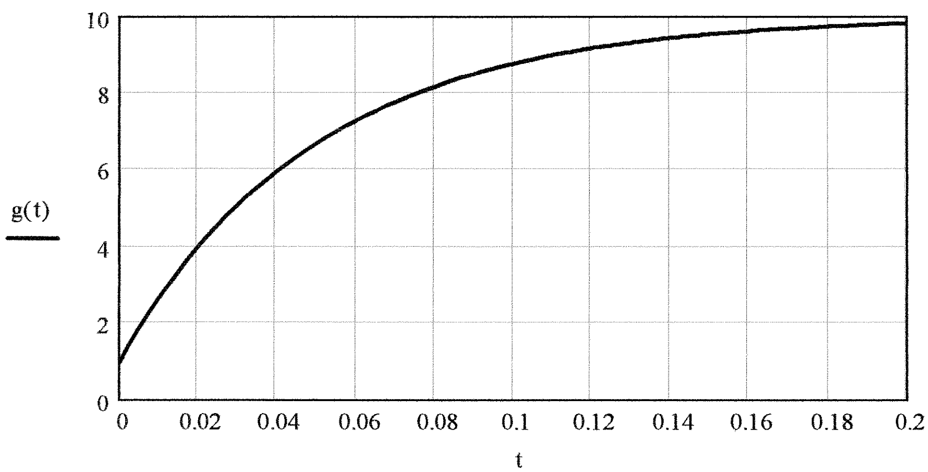
$$T(s) := \frac{10\cdot\left(\frac{s}{200}+1\right)}{\left(\frac{s}{20}+1\right)}$$



12-42 Continued

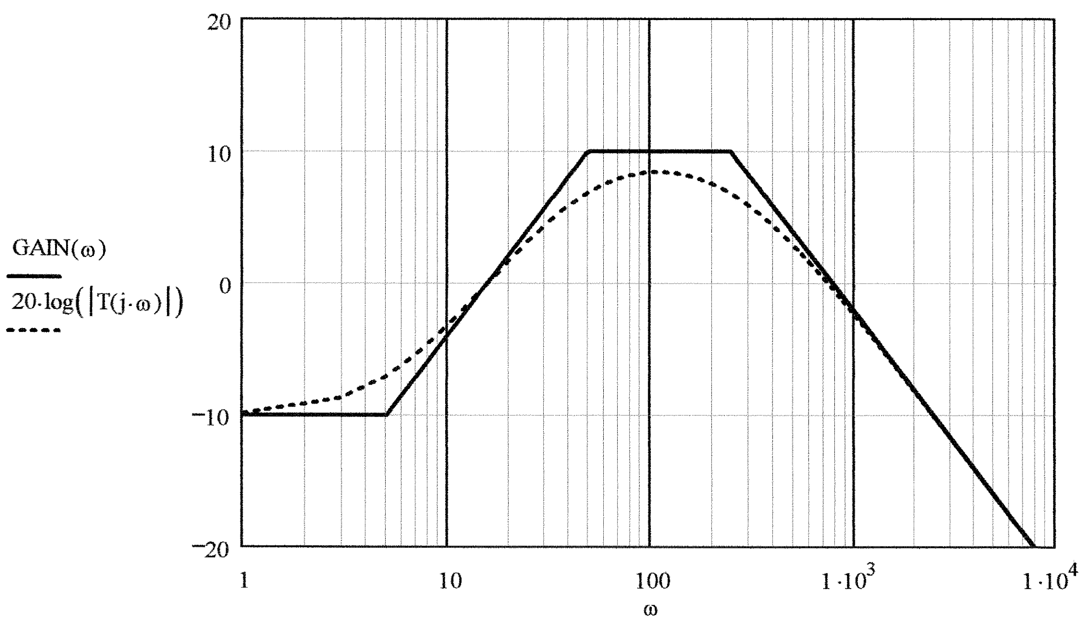
$$G(s) = \frac{T(s)}{s} = \frac{10\cdot\left(\frac{s}{200}+1\right)}{\left(\frac{s}{20}+1\right)\cdot s} = \frac{-9}{(s+20)} + \frac{10}{s}$$

$g(t) := (10 - 9\cdot\exp(-20\cdot t))\cdot u(t) \quad t := 0, 0.0025 .. 0.2$



12-43 $\mathrm{GAIN}(\omega) := G_{SL}(\omega, 5) - G_{SL}(\omega, 50) - G_{SL}(\omega, 250) - 10$

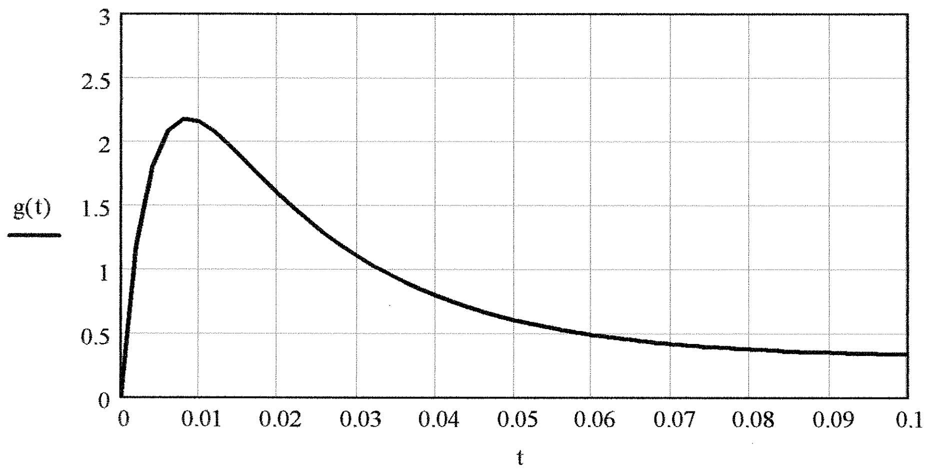
$$T(s) := \frac{\frac{1}{\sqrt{10}}\cdot\left(\frac{s}{5}+1\right)}{\left(\frac{s}{50}+1\right)\cdot\left(\frac{s}{250}+1\right)} \qquad \omega := 1, 3 .. 10000$$



12-43 Continued

$$G(s) = \frac{T(s)}{s} = \frac{\frac{1}{\sqrt{10}}\cdot\left(\frac{s}{5}+1\right)}{\left(\frac{s}{50}+1\right)\cdot\left(\frac{s}{250}+1\right)}\cdot\frac{1}{s} = \frac{1}{\sqrt{10}}\cdot\left[\frac{45}{[4\cdot(s+50)]} - \frac{49}{[4\cdot(s+250)]} + \frac{1}{s}\right]$$

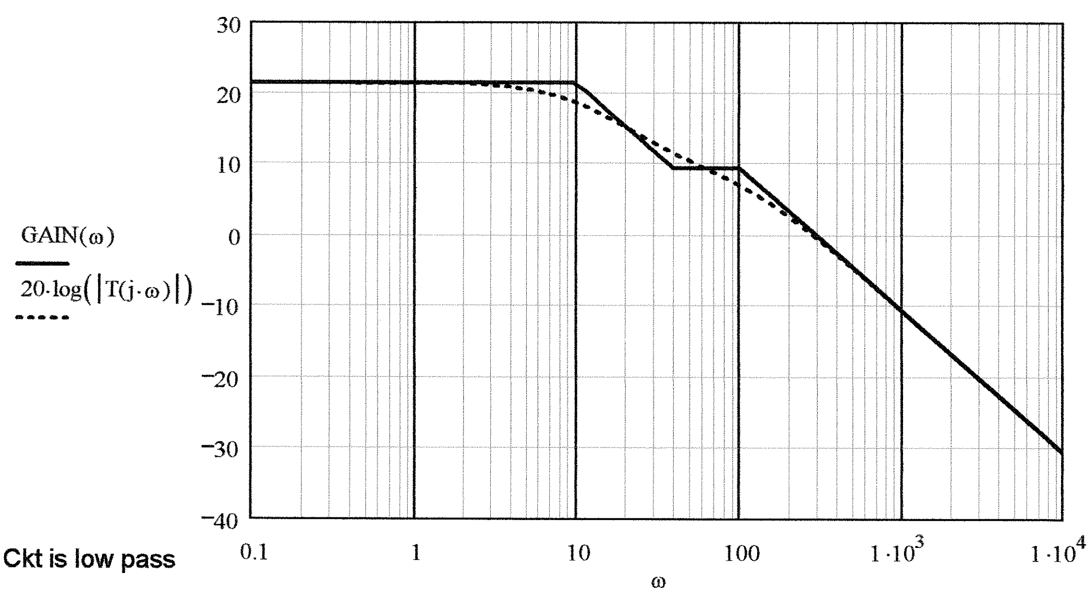
$$g(t) := \frac{1}{\sqrt{10}}\cdot\left(1 + \frac{45}{4}\cdot\exp(-50\cdot t) - \frac{49}{4}\cdot\exp(-250\cdot t)\right)\cdot u(t) \qquad t := 0, 0.002 .. 0.1$$



12-44 $g(t) = 12 - 10\cdot e^{-10\cdot t} - 2\cdot e^{-100\cdot t}$ $\quad G(s) = \frac{12}{s}\,\frac{-10}{s+10} - \frac{2}{s+100} = \frac{300\cdot(s+40)}{s\cdot(s+10)\cdot(s+100)}$

$T(s) = s\cdot G(s) \qquad T(s) := \frac{300\cdot(s+40)}{(s+10)\cdot(s+100)}$

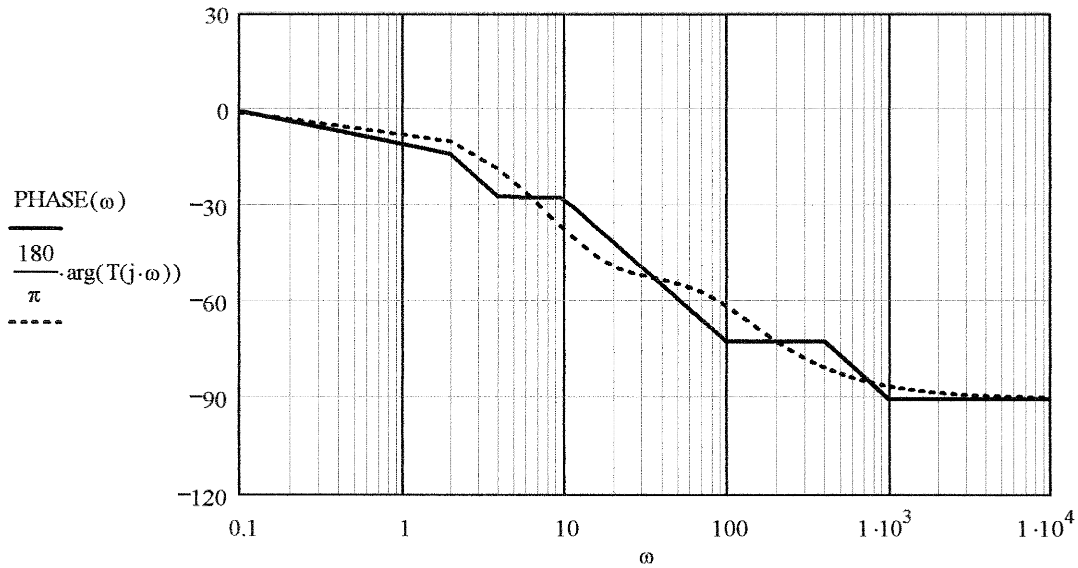
$GAIN(\omega) := (20\cdot\log(12)) - G_{SL}(\omega,10) + G_{SL}(\omega,40) - G_{SL}(\omega,100) \qquad \omega := .1, 2 .. 10000$



Ckt is low pass

12-44 Continued

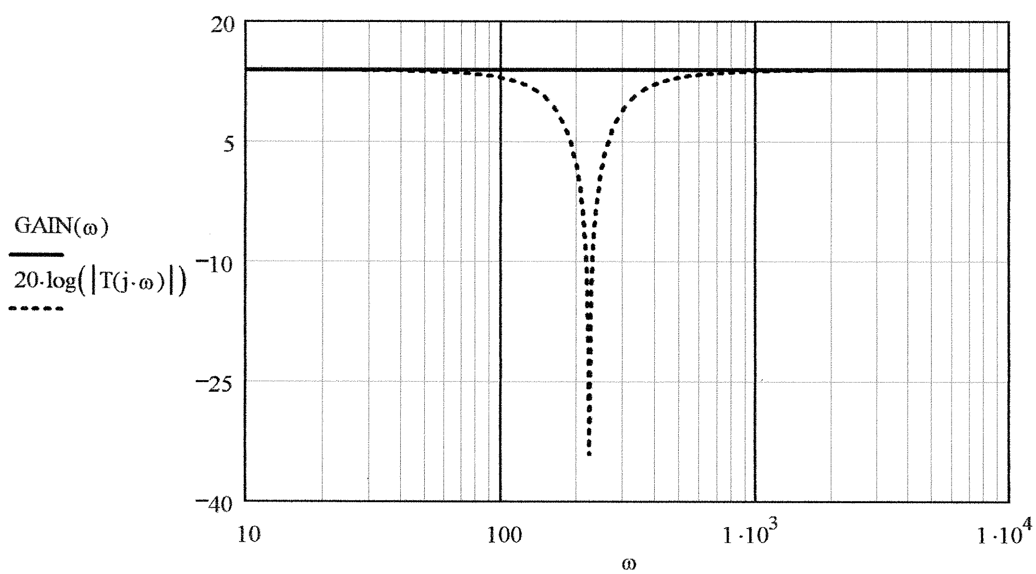
$PHASE(\omega) := -A_{SL}(\omega, 10) + A_{SL}(\omega, 40) - A_{SL}(\omega, 100)$



12-45 $g(t) = 5 - 5 \cdot e^{-100 \cdot t} \cdot \sin(200 \cdot t)$ $\qquad G(s) = \frac{5}{s} - \frac{1000}{(s+100)^2 + 200^2} = \frac{5 \cdot (s^2 + 50000)}{s \cdot (s^2 + 200 \cdot s + 50000)}$

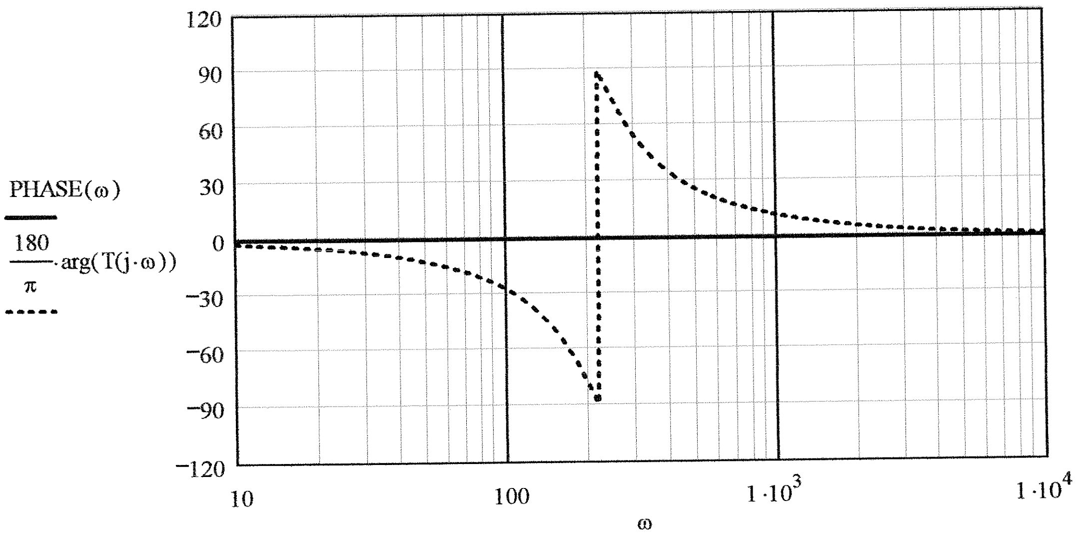
$$T(s) := \frac{5 \cdot \left(\frac{s^2}{50000} + 1\right)}{\left(\frac{s^2}{50000} + \frac{s}{250} + 1\right)} \qquad T(s) = s \cdot G(s)$$

$GAIN(\omega) := 20 \cdot \log(5) + 2 \cdot G_{SL}(\omega, \sqrt{50000}) - 2 \cdot G_{SL}(\omega, \sqrt{50000})$ $\qquad \omega := 10, 12 .. 10000$



Ckt is bandstop. Straight-line approximations do not work for transfer functions with j-axis zeros

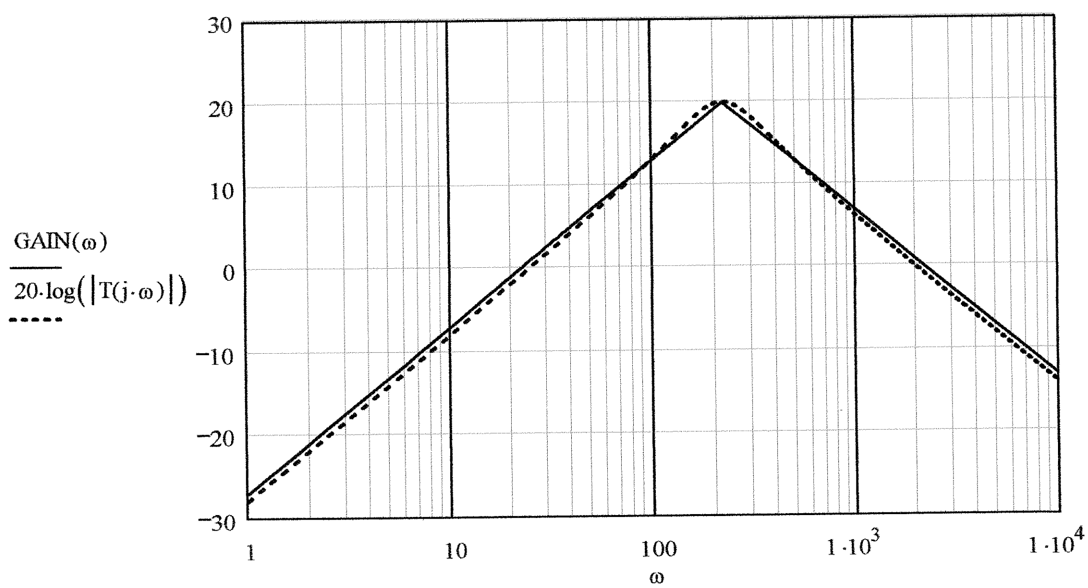
12-45 Continued $PHASE(\omega) := 2 \cdot A_{SL}(\omega, \sqrt{50000}) - 2 \cdot A_{SL}(\omega, \sqrt{50000})$



12-46 $g(t) = 10 \cdot e^{-100 \cdot t} \cdot \sin(200 \cdot t)$ $G(s) = \dfrac{2000}{(s+100)^2 + 200^2}$

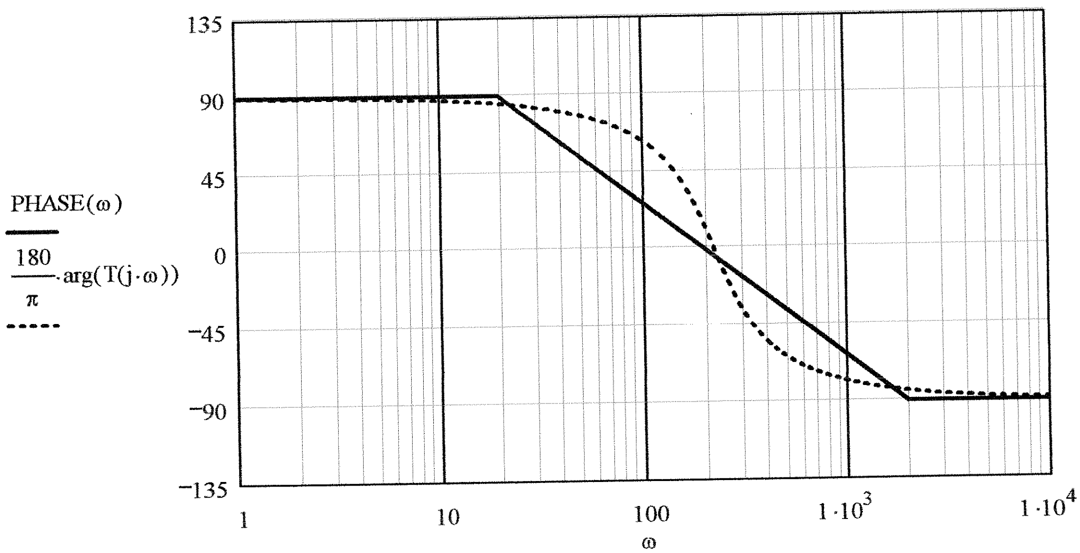
$T(s) = s \cdot G(s)$ $T(s) := \dfrac{2000 \cdot s}{(s+100)^2 + 200^2}$

$GAIN(\omega) := 10 \cdot \log\left(\dfrac{\omega^2}{5000}\right) - (2 \cdot G_{SL}(\omega, 223)) + 10$ $\omega := 1, 2 .. 10000$



Ckt is band pass

12-46 Continued $PHASE(\omega) := 90 - 2 \cdot A_{SL}(\omega, 200)$

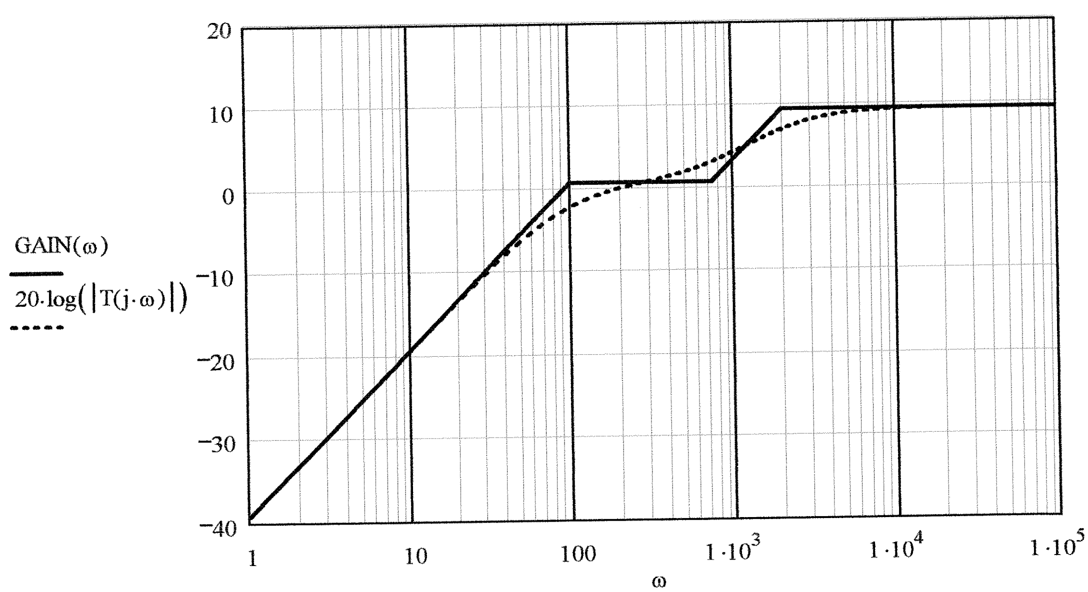


12-47

$g(t) = \exp(-100 \cdot t) + 2 \cdot \exp(-2000 \cdot t)$ $\qquad G(s) = \dfrac{3 \cdot s + 2200}{(s + 100) \cdot (s + 2000)}$

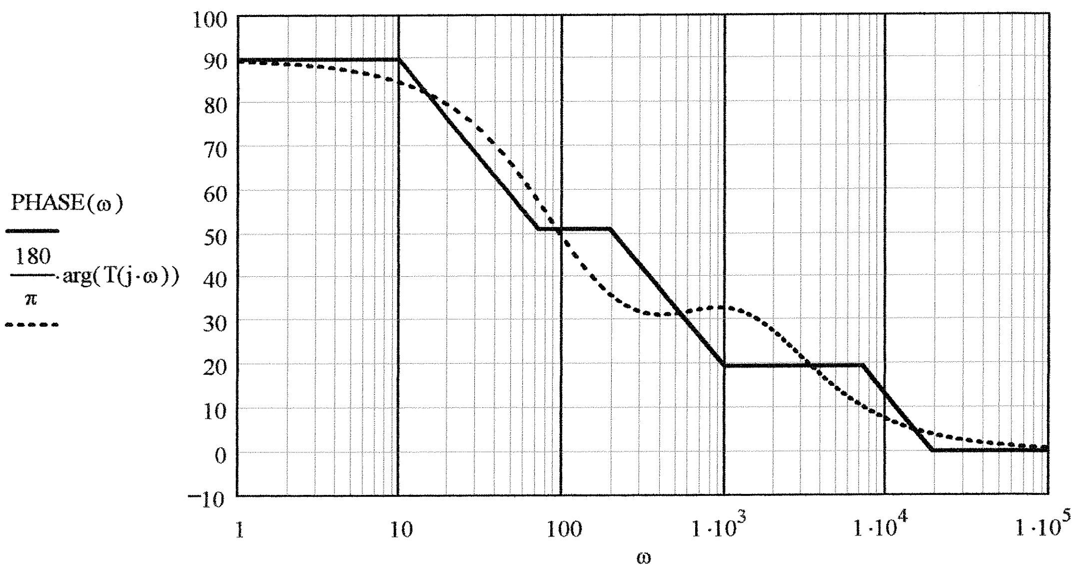
$T(s) = s \cdot G(s)$ $\qquad T(s) := \dfrac{s \cdot (3 \cdot s + 2200)}{(s + 100) \cdot (s + 2000)}$

$$GAIN(\omega) := 20 \cdot \log\left(\frac{22}{2000}\right) + 20 \cdot \log(\omega) - G_{SL}(\omega, 100) - G_{SL}(\omega, 2000) + G_{SL}\left(\omega, \frac{2200}{3}\right) \quad \omega := 1, 2 .. 100000$$



Ckt is high pass

12-47 Continued $PHASE(\omega) := 90 - A_{SL}(\omega, 100) - A_{SL}(\omega, 2000) + A_{SL}\left(\omega, \frac{2200}{3}\right)$



12-48 $T(s) = \dfrac{K}{\dfrac{s}{\omega_C} + 1}$ **T(0) = 10 implies K = 10, B < 250 rad/s implies $\omega_C < 250$,**

$G(s) = \dfrac{10}{s \cdot \left(\dfrac{s}{\omega_C} + 1\right)}$ $g(t) = 10 - 10 \cdot \exp(-\omega_C \cdot t)$

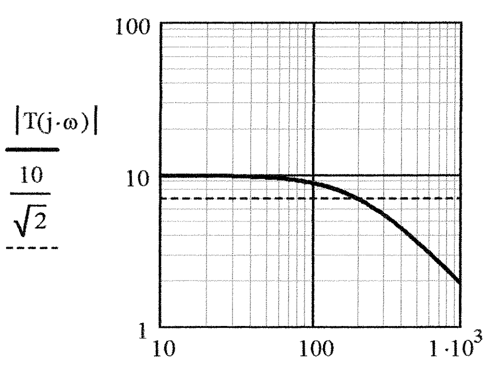
g(0.004) > 0.5x10 implies exp(ω_Cx0.004) > 2 or ω_C >ln(2)x250 = 173.28

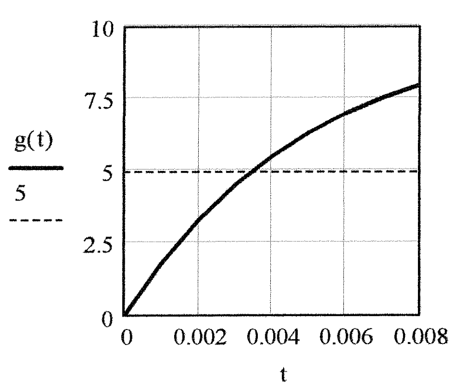
Hence 173.28 < ω_C < 250. Let ω_C = 200 rad/s then $T(s) = \dfrac{10}{\dfrac{s}{200} + 1}$ $T(s) := \dfrac{2000}{s + 200}$

Verifying that T(s) meets the design requirements

$g(t) := 10 \cdot \left(1 - e^{-200 \cdot t}\right)$

$\omega := 10, 20 .. 1000$ $t := 0, 0.001 .. 0.01$





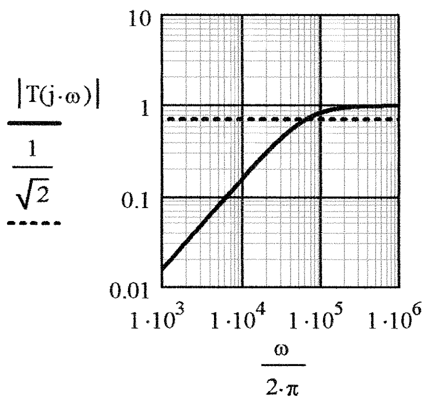
12-49 $T(s) = \frac{K \cdot s}{s + \omega_C}$ $\omega_C < 200000\pi = 628318$ $G(s) = \frac{K}{s + \omega_C}$ $g(t) = K \cdot \exp(-\omega_C \cdot t)$

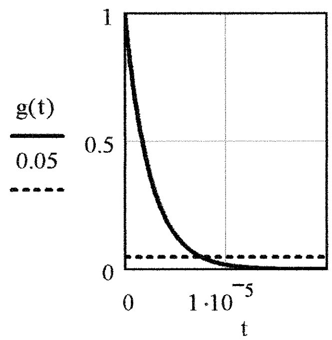
$g(10^{-5}) > 0.05xK$ implies $\exp(\omega_C x\, 10^{-5}) > 20$ or $\omega_C > \ln(20) x\, 10^5 = 299600$

Hence $299600 < \omega_C < 628318$ Let $\omega_C = 400$ krad/s & K = 1 $T(s) := \frac{s}{s + 400000}$

Verifying that T(s) meets the design requirements $g(t) := \exp(-400000 \cdot t)$

$\omega := 1000, 2000 .. 10000000$ $t := 0, 10^{-7} .. 9 \cdot 10^{-5}$





12-50 $T(s) = \frac{K \cdot s}{s^2 + 2 \cdot \zeta \cdot \omega_0 \cdot s + \omega_0^2}$; $\omega_0 := 2 \cdot \pi \cdot 100 \cdot 10^3$; $\omega_0 = 6.283 \times 10^5$; $T_{max} = \frac{K}{2 \cdot \zeta \cdot \omega_0} = 10$

$B = 2 \cdot \zeta \cdot \omega_0 < 2 \cdot \pi \cdot 50 \cdot 10^3$ $\zeta < \frac{\pi \cdot 5 \cdot 10^4}{\omega_0} = 0.25$ $G(s) = \frac{T(s)}{s} = \frac{K}{(s + \zeta \cdot \omega_0)^2 + \left(\omega_0 \cdot \sqrt{1 - \zeta^2}\right)^2}$

$g(t) = \frac{K}{\omega_0 \cdot \sqrt{1 - \zeta^2}} \cdot \exp(-\zeta \cdot \omega_0 \cdot t) \cdot \sin\left(\omega_0 \cdot \sqrt{1 - \zeta^2} \cdot t\right)$ $g_{max} < \frac{K}{\omega_0 \cdot \sqrt{1 - \zeta^2}}$

$g(t) = \left(\frac{K}{\omega_0 \cdot \sqrt{1 - \zeta^2}} \cdot \exp(-\zeta \cdot \omega_0 \cdot t) \cdot \sin(\omega_0) \cdot \sqrt{1 - \zeta^2} \cdot t\right) < \left(\frac{K}{\omega_0 \cdot \sqrt{1 - \zeta^2}} \cdot \exp(-\zeta \cdot \omega_0 \cdot t)\right)$

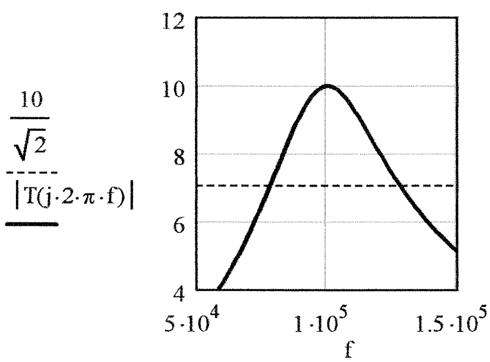
hence $g\left(5 \cdot 10^{-6}\right) < 0.2 \cdot g_{max}$ implies $\exp(\zeta\, \omega_0 x\, 5 \cdot 10^{-6}) > 5$ hence $\zeta > \ln(5)/\pi = 0.512$

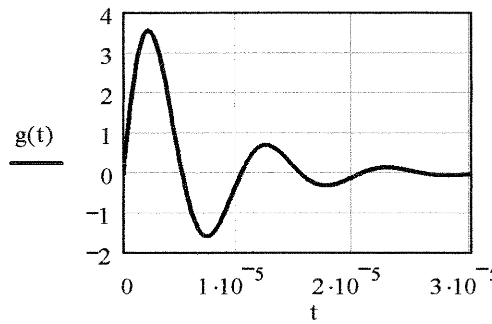
Bandwidth requirement is $\zeta < 0.25$, step response requirement is $\zeta > 0.512$.
There is no value of ζ, hence there is no T(s) that meets the design conditions.

To verify this conclusion let $\zeta := 0.25$ then $K := 10 \cdot 2 \cdot \zeta \cdot \omega_0$ $K = 3.142 \times 10^6$

$f := 50 \cdot 10^3, 5.1 \cdot 10^4 .. 150 \cdot 10^3$ $t := 0, 10^{-7} .. 3 \cdot 10^{-5}$

$T(s) := \frac{K \cdot s}{s^2 + 2 \cdot \zeta \cdot \omega_0 \cdot s + \omega_0^2}$ $g(t) := \frac{K}{\omega_0 \cdot \sqrt{1 - \zeta^2}} \cdot \exp(-\zeta \cdot \omega_0 \cdot t) \cdot \sin\left(\omega_0 \cdot \sqrt{1 - \zeta^2} \cdot t\right)$

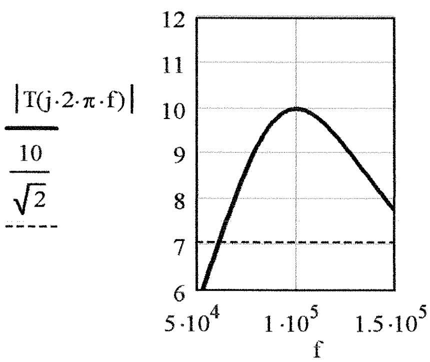


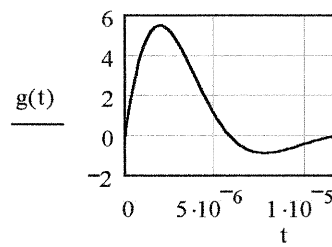


ζ = 0.25 meets frequency response bandwidth but does not meet step response settling time

Now let $\zeta := 0.512$ and $K := 10 \cdot 2 \cdot \zeta \cdot \omega_0$

$$T(s) := \frac{K \cdot s}{s^2 + 2 \cdot \zeta \cdot \omega_0 \cdot s + \omega_0^{\,2}} \qquad g(t) := \frac{K}{\omega_0 \cdot \sqrt{1 - \zeta^2}} \cdot \exp(-\zeta \cdot \omega_0 \cdot t) \cdot \sin\left(\omega_0 \cdot \sqrt{1 - \zeta^2} \cdot t\right)$$





ζ = 0.512 does not meet frequency response bandwidth but does meet step response settling time

12-51 $\quad v_1(t) = 2 \cdot \exp(-1000 \cdot t) \qquad V_1(s) = \frac{2}{s + 1000}$

$$v_2(t) = 5 \cdot \exp(-1000 \cdot t) \cdot (1 - \cos(2000 \cdot t)) \qquad V_2(s) = \frac{20000000}{\left[\left(s^2 + 2000 \cdot s + 5000000\right) \cdot (s + 1000)\right]}$$

$$T_V(s) = \frac{V_2(s)}{V_1(s)} = \frac{\dfrac{20000000}{\left[\left(s^2 + 2000 \cdot s + 5000000\right) \cdot (s + 1000)\right]}}{\dfrac{2}{s + 1000}} \qquad T_V(s) := \frac{10000000}{\left(s^2 + 2000 \cdot s + 5000000\right)}$$

(a) Input ac waveform: $v_1(t) = 4 \cdot \cos(2000 \cdot t)$

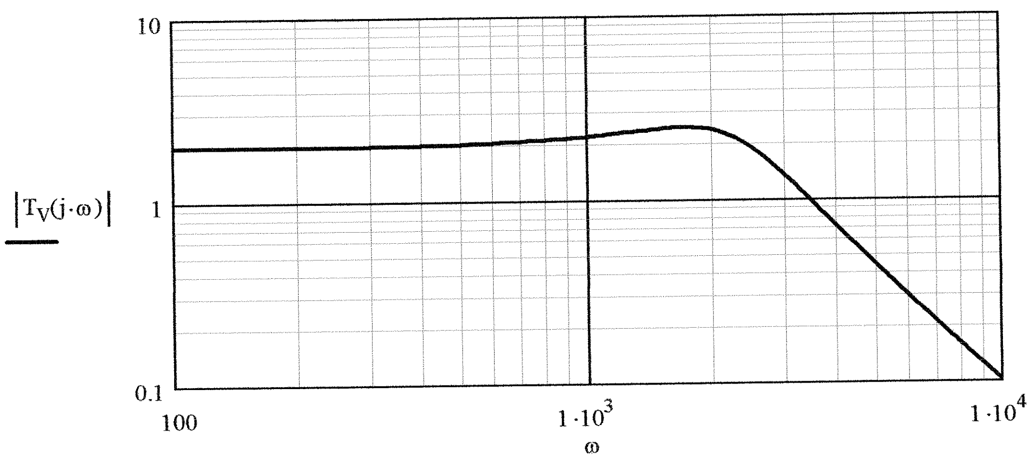
Steady-state output : $\quad 4 \cdot \left|T_V(j \cdot 2000)\right| = 9.701 \qquad \frac{180}{\pi} \cdot \arg\left(T_V(j \cdot 2000)\right) = -75.964$

$v_{2SS}(t) = 9.701 \cdot \cos\left(2000 \cdot t - 75.964^{o}\right)$ V

(b) Input dc waveform: $v_1(t) = 15$

DC output : $v_{2dc} := 15 \cdot \left|T_V(j \cdot 0)\right| \qquad v_{2dc} = 30$ V

12-51(c) Continued $\omega := 100, 200 .. 10000$



(d) $h(t) = \mathcal{L}^{-1}\cdot(T_V(s)) = \mathcal{L}^{-1}\cdot\left(\frac{10000000}{s^2 + 2000\cdot s + 5000000}\right) = \mathcal{L}^{-1}\cdot\left[\frac{5000\cdot 2000}{(s+1000)^2 + 2000^2}\right]$

$h(t) = 5000\cdot \exp(-1000\cdot t)\cdot \sin(2000\cdot t)$

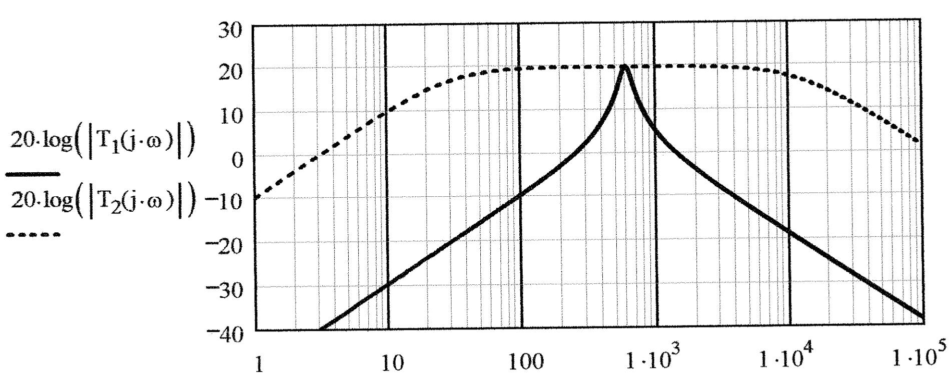
12-52 (a) $g_1(t) = 2\cdot \exp(-60\cdot t)\cdot \sin(600\cdot t)$ $G_1(s) = \frac{1200}{\left[(s+60)^2 + 600^2\right]}$ $T_1(s) = s\cdot G_1(s)$

$T_1(s) := \frac{1200\cdot s}{s^2 + 120\cdot s + 363600}$ $\omega_{01} := \sqrt{363600}$ $\omega_{01} = 602.993$ $BW_1 := 120$

$g_2(t) = 10\cdot(\exp(-30\cdot t) - \exp(-12000\cdot t))$ $G_2(s) = \frac{119700}{\left(s^2 + 12030\cdot s + 360000\right)}$ $T_2(s) = s\cdot G_2(s)$

$T_2(s) := \frac{119700\cdot s}{\left(s^2 + 12030\cdot s + 360000\right)}$ $\omega_{02} := \sqrt{360000}$ $\omega_{02} = 600$ $BW_2 := 12030$

$\omega := 1, 2 .. 100000$



12-52(b) Continued Input $v_1(t) = 8\cdot\cos(100\cdot t) + 6\cdot\cos(600\cdot t) + 10\cdot\cos(3000\cdot t)$

Frequency	Amplitude	Filter No. 1 Output	Filter No. 2 Output
$\omega := 200$	$A := 8$	$20\cdot\log(A\cdot\lvert T_1(j\omega)\rvert) = 15.442$	$20\cdot\log(A\cdot\lvert T_2(j\cdot\omega)\rvert) = 37.942$
$\omega := 600$	$A := 6$	$20\cdot\log(A\cdot\lvert T_1(j\omega)\rvert) = 35.552$	$20\cdot\log(A\cdot\lvert T_2(j\cdot\omega)\rvert) = 35.52$
$\omega := 3000$	$A := 10$	$20\cdot\log(A\cdot\lvert T_1(j\omega)\rvert) = 12.392$	$20\cdot\log(A\cdot\lvert T_2(j\cdot\omega)\rvert) = 39.715$

Filter No. 1 has a gain of about 20 dB at $\omega = 600$ and the amplitudes of the undesired outputs at ω = 200 & 3000 are at least 20 dB below the amplitude of the desired output.
Filter No. 2 amplifies both the desired and undesired signals by about 20 dB. Choose Filter No. 1.

(c) Input $v_1(t) = 6\cdot\cos(400\cdot t) + 6\cdot\cos(800\cdot t) + 6\cdot\cos(1600\cdot t)$

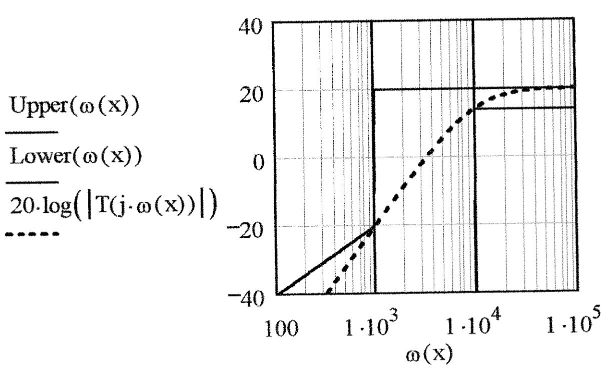
Frequency	Amplitude	Filter No. 1 Gain	Filter No. 2 Gain
$\omega := 400$	$A := 6$	$20\cdot\log(A\cdot\lvert T_1(j\omega)\rvert) = 22.777$	$20\cdot\log(A\cdot\lvert T_2(j\cdot\omega)\rvert) = 35.512$
$\omega := 800$	$A := 6$	$20\cdot\log(A\cdot\lvert T_1(j\omega)\rvert) = 25.883$	$20\cdot\log(A\cdot\lvert T_2(j\cdot\omega)\rvert) = 35.516$
$\omega := 1600$	$A := 6$	$20\cdot\log(A\cdot\lvert T_1(j\omega)\rvert) = 14.362$	$20\cdot\log(A\cdot\lvert T_2(j\cdot\omega)\rvert) = 35.463$

Filter No. 1 does not amplify all three desired frequencies equally.
Filter No. 2 amplifies all three desired frequencies by about 20 dB. Choose Filter No. 2.

12-53 $x := 2, 2.01 .. 5 \quad \omega(x) := 10^x$

$$\text{Upper}(\omega) := \begin{cases} \left(-40 + 20\cdot\log\left(\frac{\omega}{100}\right)\right) & \text{if } 10^2 \le \omega < 10^3 \\ 20 & \text{if } 10^3 \le \omega \le 10^5 \end{cases} \qquad \text{Lower}(\omega) := \begin{cases} -40 & \text{if } 100 \le \omega < 10^4 \\ 14 & \text{if } 10^4 \le \omega \le 10^5 \end{cases}$$

$$T(s) := \frac{10\cdot s^2}{\left(s + 10^4\right)^2}$$ <--- T(s) meets the gain requirement



$$20\cdot\log\left(\left|T\left(j\cdot 10^2\right)\right|\right) = -60.001$$

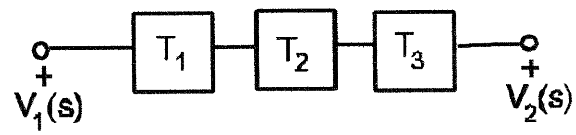
$$20\cdot\log\left(\left|T\left(j\cdot 10^3\right)\right|\right) = -20.086$$

$$20\cdot\log\left(\left|T\left(j\cdot 10^4\right)\right|\right) = 14$$

$$20\cdot\log\left(\left|T\left(j\cdot 10^5\right)\right|\right) = 19.914$$

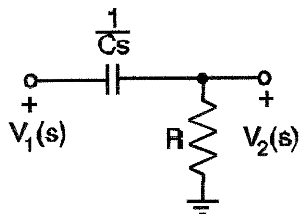
12-53 Continued Partition T(s) into three stages as:

$$T(s) = T_1(s) \cdot T_2(s) \cdot T_1(s) = \left(\frac{1}{1+\frac{10^4}{s}}\right) \cdot (10) \cdot \left(\frac{1}{1+\frac{10^4}{s}}\right)$$



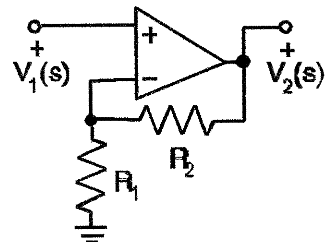
The first and last stages can be high-pass RC voltage dividers:

$$T_1(s) = \frac{Z_2}{Z_2 + Z_1} = \frac{1}{1+\frac{10^4}{s}}$$



Using $k_m := 50$ yields $R := k_m \cdot 1$

$$C := \frac{1}{k_m \cdot 10^4} \qquad R = 50 \qquad C = 2 \times 10^{-6}$$



The middle stage can be a noninverting amplifier

$$T_2 = \frac{R_1 + R_2}{R_1} = 10$$

Let $R_1 := 10^4$ then $R_2 := 9 \cdot R_1$ $\qquad R_2 = 9 \times 10^4$

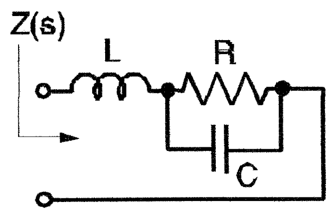
$\omega_C := 10^4$ Given $\left|T(j \cdot \omega_C)\right| = \frac{10}{\sqrt{2}}$ $\quad \omega_C := \text{Find}(\omega_C)$ $\qquad \omega_C = 1.554 \times 10^4$ <---Cutoff frequency

$Z_{IN}(\omega) := R + \frac{1}{j \cdot \omega \cdot C}$ $\qquad \left|Z_{IN}(\omega_C)\right| = 59.46$ $\qquad \left|Z_{IN}(2 \cdot \omega_C)\right| = 52.525$

$\left|Z_{IN}(4 \cdot \omega_C)\right| = 50.643$

Input impedance is around 50 Ω for frequencies above cutoff

12-54 (a) $Z(s) = L \cdot s + \frac{1}{C \cdot s + \frac{1}{R}} = \frac{R \cdot L \cdot C \cdot s^2 + L \cdot s + R}{R \cdot C \cdot s + 1}$



(b & c) $R := 10^4 \quad C := 4 \cdot 10^{-12} \quad L := 2 \cdot 10^{-6}$

$Z(s) := R \cdot \frac{L \cdot C \cdot s^2 + \frac{L}{R} \cdot s + 1}{R \cdot C \cdot s + 1}$ $\quad \frac{1}{R \cdot C} = 2.5 \times 10^7$ $\quad \omega_0 := \frac{1}{\sqrt{L \cdot C}}$ $\quad \omega_0 = 3.536 \times 10^8$ $\quad \zeta := \frac{L}{2 \cdot R} \cdot (\omega_0)$ $\quad \zeta = 0.0354$

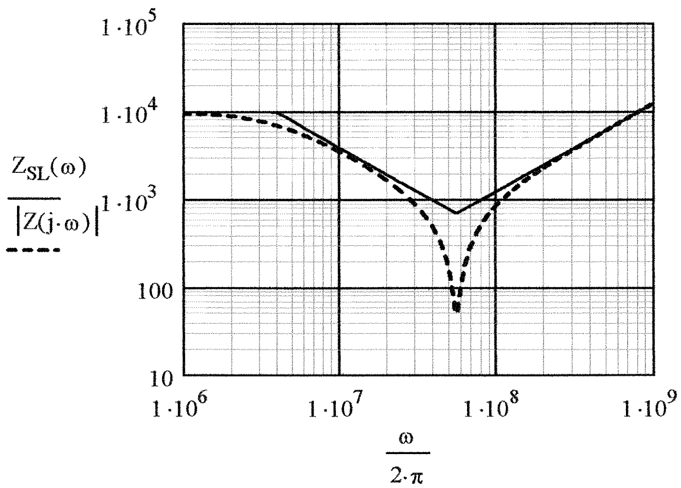
complex zeros

$$\left|Z(j \cdot \omega)\right| = R \cdot \sqrt{\frac{\left(1 - L \cdot C \cdot \omega^2\right)^2 + \left(\frac{L}{R} \cdot \omega\right)^2}{1 + (R \cdot C \cdot \omega)^2}}$$

$Z_{SL}(\omega) :=$

- R if $\omega < \frac{1}{R \cdot C}$ <--- slope is zero, acts like a 10 kΩ resistor
- $\frac{1}{\omega \cdot C}$ if $\frac{1}{R \cdot C} \le \omega < \frac{1}{\sqrt{L \cdot C}}$ <---slope is -1, acts like a 4 pF capacitor
- $\omega \cdot L$ if $\frac{1}{\sqrt{L \cdot C}} < \omega$ <---slope is +1, acts like a 2mH inductor

12-54 Continued $\omega := 2\cdot\pi\cdot 10^5, 10\cdot\pi\cdot 10^5 .. 2\cdot\pi\cdot 10^9$



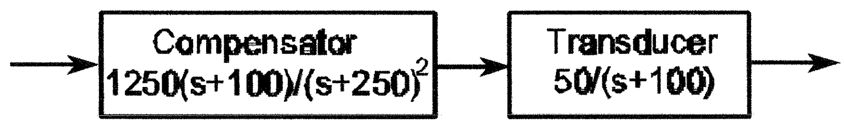
Resistor: f $3.979\text{x}10^6$

Capacitor: $3.979\text{x}10^6 < f < 5.627\text{x}10^7$

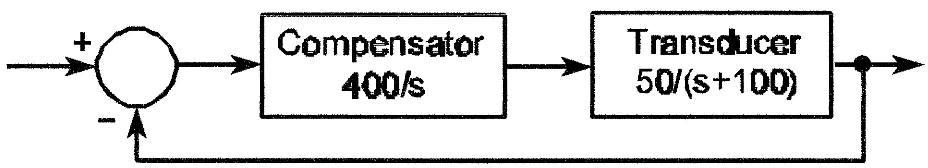
Inductor: $5.627\text{x}10^7 < f$

(d) Acts like a resistor for ω<1/RC or f < $\frac{1}{2\cdot\pi}\cdot\frac{1}{R\cdot C} = 3.979\times 10^6$

12-55



Cascade Design



Feedback Design

$$T_C(s) := \frac{1250\cdot(s+100)}{(s+250)^2}\cdot\frac{50}{s+100}$$

$$T_C(s) := \frac{62500}{(s+250)^2}$$

$$Y(s) = \frac{400}{s}\cdot\frac{50}{s+100}\cdot E(s)$$

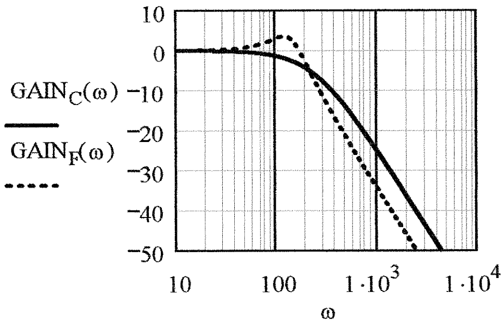
$$E(s) = X(s) - Y(s) \quad \text{hence}$$

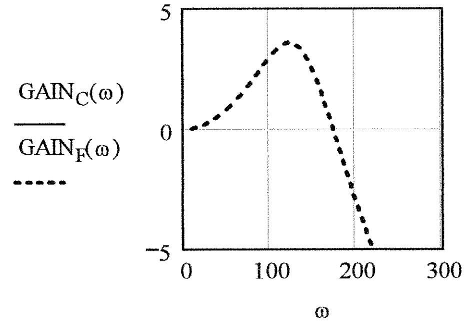
$$T_F(s) = \frac{Y(s)}{X(s)} = \frac{\frac{400}{s}\cdot\frac{50}{s+100}}{1+\frac{400}{s}\cdot\frac{50}{s+100}} \qquad T_F(s) := \frac{20000}{\left(s^2+100\cdot s+20000\right)}$$

(a) Verifying requirements $\omega := 10, 11 .. 10^4$

$$\text{GAIN}_C(\omega) := 20\cdot\log\left(\left|T_C(j\cdot\omega)\right|\right) \qquad \text{GAIN}_F(\omega) := 20\cdot\log\left(\left|T_F(j\cdot\omega)\right|\right)$$

12-55 C





Both circuits have 0 dB gain at dc and high-frequency slopes of -40 dB/decade

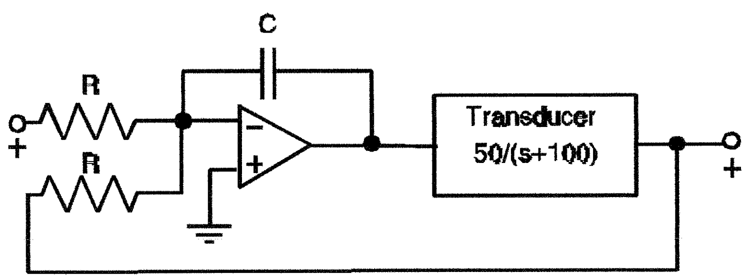
Both remain within ± 5 dB on the range from ω = 20 to 200 rad/s

(b) Use the feedback design because it can be realized using a single integrator/summer. The cascade design requires a second order circuit with two poles and one zero.

(c)

$R \cdot C = \frac{1}{400}$ Let $R := 10^4$ Then

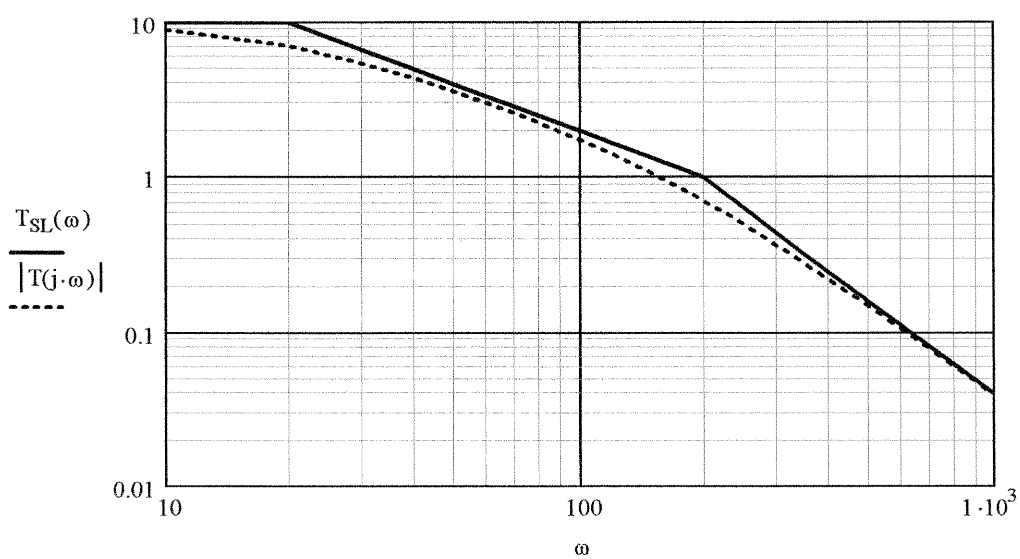
$C := \frac{1}{400 \cdot R}$ $C = 2.5 \times 10^{-7}$



12-56(a) $T_{SL}(\omega) := \begin{cases} 10 & \text{if } \omega < 20 \\ \frac{10 \cdot 20}{\omega} & \text{if } 20 \le \omega < 200 \\ \left(\frac{200}{\omega}\right)^2 & \text{if } 200 \le \omega \end{cases}$

$T(s) := \frac{10}{\left(\frac{s}{20} + 1\right) \cdot \left(\frac{s}{200} + 1\right)}$ $\omega := 10, 20 .. 1000$

$V_{max} := \frac{15}{T(0)}$ $V_{max} = 1.5$



12-55(b) Continued $V_{max} := \frac{15}{|T_{SL}(2 \cdot \pi \cdot 20)|}$ $V_{max} = 9.425$ <---Answer using straight-line gain

$V_{max} := \frac{15}{|T(j \cdot 2 \cdot \pi \cdot 20)|}$ $V_{max} = 11.271$ <---Exact answer

(c) The shortest time constant is $T_C := \frac{1}{200}$ The pulse duration should be about

$T_D := \frac{T_C}{5}$ $T_D = 1 \times 10^{-3}$

(d) The longest time constant is $T_C := \frac{1}{20}$ and the settling time is about $T_S := 5 \cdot T_C$ $T_S = 0.25$ sec.

Using the final value theorem $FV = \lim_{s \to inf} s \cdot T(s) = 0$

(e) The final value is $FV := 0.5 \cdot T(0)$ $FV = 5$ volts. The 50% rise time occurs when exp(-20t) = 0.5 or

$T_R := \frac{\ln(.45)}{-20}$ $T_R = 0.03993$ sec

(f) $RATIO := 20 \cdot \log\left(\frac{0.5 \cdot T_{SL}(100)}{0.25 \cdot T_{SL}(1000)}\right)$ $RATIO = 40$ <---Answer using straight-line gain

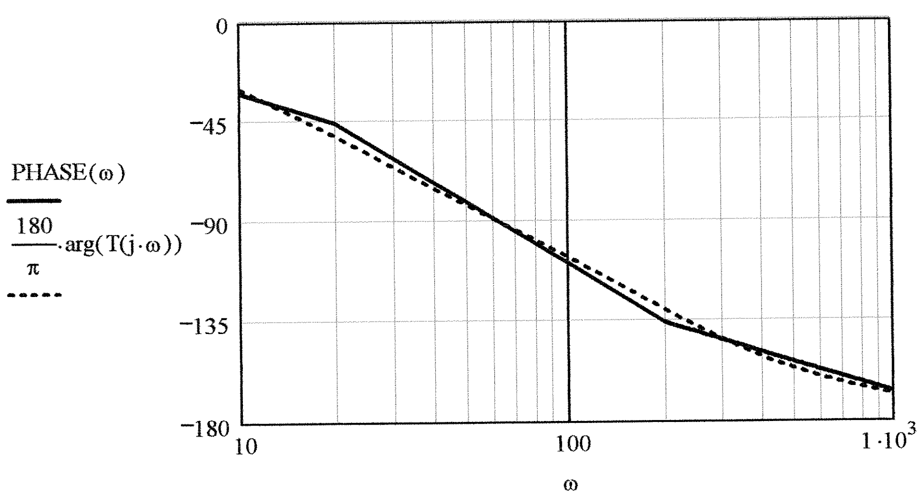
$RATIO := 20 \cdot \log\left(\frac{0.5\,|T(j \cdot 100)|}{0.25 \cdot |T(j \cdot 1000)|}\right)$ $RATIO = 39.033$ <---Exact answer

The longest time constant is $T_C := \frac{1}{20}$ and the steady state is reached after about $T_{SS} := 5 \cdot T_C$ or

$T_{SS} = 0.25$ sec.

$PHASE(\omega) := -A_{SL}(\omega, 20) - A_{SL}(\omega, 200)$

(g)



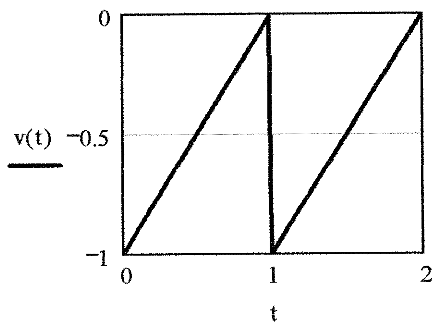
$\omega := 65$ Given $PHASE(\omega) = -90$ $Find(\omega) = 63.246$ <---Answer using straight-line phase

$\omega := 65$ Given $\frac{180}{\pi} \cdot \arg(T(j \cdot \omega)) = -90$ $Find(\omega) = 63.246$ <---Exact answer

CHAPTER 13, Both Versions

$V_A := 1 \quad T_0 := 1$

13-1 (a) $v_0(t) := \begin{cases} V_A\cdot\left(\dfrac{t}{T_0}-1\right) & \text{if } 0 \le t \le T_0 \\ 0 & \text{otherwise} \end{cases} \qquad t := 0, \dfrac{T_0}{100} .. 2\cdot T_0 \qquad v(t) := \sum_{n=0}^{1} v_0(t - n\cdot T_0)$



(b) $$a_0 = \frac{1}{T_0}\cdot\int_0^{T_0} V_A\cdot\left(\frac{t}{T_0}-1\right)dt = \frac{-V_A}{2}$$

$$a_n = \frac{2}{T_0}\cdot\int_0^{T_0} V_A\cdot\left(\frac{t}{T_0}-1\right)\cdot\cos\left(2\cdot\pi\cdot n\cdot\frac{t}{T_0}\right)dt = \frac{2}{T_0}\cdot\left[\frac{1}{4}\cdot\cos(2\cdot\pi\cdot n)\cdot T_0\cdot\frac{V_A}{\left(\pi^2\cdot n^2\right)} - \frac{1}{4}\cdot T_0\cdot\frac{V_A}{\left(\pi^2\cdot n^2\right)}\right]$$

$$a_n = \frac{1}{2}\cdot V_A\cdot\frac{(\cos(2\cdot\pi\cdot n)-1)}{\left(\pi^2\cdot n^2\right)} = 0 \qquad \textbf{for all n}$$

$$b_n = \frac{2}{T_0}\cdot\int_0^{T_0} V_A\cdot\left(\frac{t}{T_0}-1\right)\cdot\sin\left(2\cdot\pi\cdot n\cdot\frac{t}{T_0}\right)dt = \frac{2}{T_0}\cdot\left[\frac{1}{4}\cdot\sin(2\cdot\pi\cdot n)\cdot T_0\cdot\frac{V_A}{\left(\pi^2\cdot n^2\right)} - \frac{1}{[2\cdot(\pi\cdot n)]}\cdot T_0\cdot V_A\right]$$

$$b_n = \frac{-1}{2}\cdot V_A\cdot\frac{(-\sin(2\cdot\pi\cdot n)+2\cdot\pi\cdot n)}{\left(\pi^2\cdot n^2\right)} = \frac{-V_A}{n\cdot\pi} \qquad \textbf{for all n}$$

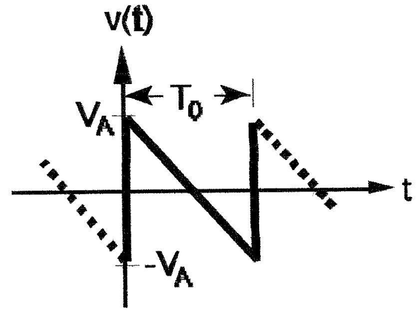
13-2 (a) $v(t) = \begin{cases} V_A & \text{if } 0 \le t < \dfrac{T_0}{2} \\ 0 & \text{if } \dfrac{T_0}{2} \le t < T_0 \end{cases}$ **(b)** $a_0 = \dfrac{1}{T_0}\cdot\displaystyle\int_0^{\frac{T_0}{2}} V_A\,dt = \dfrac{V_A}{2}$

$$a_n = \frac{2}{T_0}\cdot\int_0^{\frac{T_0}{2}} V_A\cdot\cos\left(2\cdot\pi\cdot n\cdot\frac{t}{T_0}\right)dt = \frac{2}{T_0}\cdot\left[\frac{V_A\cdot T_0}{2\cdot\pi n}\cdot\left(\sin\left(2\cdot\pi\cdot n\cdot\frac{T_0}{2\cdot T_0}\right) - \sin(2\cdot\pi\cdot n\cdot 0)\right)\right] = \frac{V_A\cdot\sin(\pi\cdot n)}{\pi\cdot n} = 0$$

$$b_n = \frac{2}{T_0}\cdot\int_0^{\frac{T_0}{2}} V_A\cdot\sin\left(2\cdot\pi\cdot n\cdot\frac{t}{T_0}\right)dt = \frac{2}{T_0}\cdot\frac{-V_A\cdot T_0}{2\cdot\pi\cdot n}\cdot\left[\left(\cos\left(2\cdot\pi\cdot n\cdot\frac{T_0}{2\cdot T_0}\right) - \cos(2\cdot\pi\cdot n\cdot 0)\right)\right] = V_A\cdot\left(\frac{1-\cos(n\cdot\pi)}{n\cdot\pi}\right)$$

$$a_0 = \frac{V_A}{2} \qquad a_n = 0 \quad \textbf{all n} \qquad b_n = \frac{2\cdot V_A}{n\cdot\pi} \quad \textbf{n odd}$$

13-3 $v(t) = 2\cdot V_A\cdot\left(\frac{1}{2} - \frac{t}{T_0}\right) \qquad 0 \le t \le T_0$



$$a_0 = \frac{1}{T_0}\cdot\int_0^{T_0} 2\cdot V_A\cdot\left(\frac{1}{2} - \frac{t}{T_0}\right) dt = 0$$

$$a_n = \frac{2}{T_0}\cdot\int_0^{T_0} 2\cdot V_A\cdot\left(\frac{1}{2} - \frac{t}{T_0}\right)\cdot\cos\left(2\cdot\pi\cdot n\cdot\frac{t}{T_0}\right) dt$$

$$a_n = \frac{2}{T_0}\cdot\left[\frac{-1}{2}\cdot T_0\cdot(\pi\cdot n\cdot\sin(2\cdot\pi\cdot n) + \cos(2\cdot\pi\cdot n))\cdot\frac{V_A}{\left(\pi^2\cdot n^2\right)} + \frac{1}{2}\cdot T_0\cdot\frac{V_A}{\left(\pi^2\cdot n^2\right)}\right]$$

$$a_n = -V_A\cdot\frac{(\pi\cdot n\cdot\sin(2\cdot\pi\cdot n) + \cos(2\cdot\pi\cdot n) - 1)}{\pi^2\cdot n^2} = 0 \qquad \text{for all n}$$

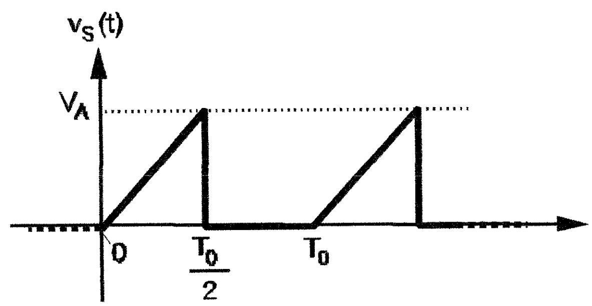
$$b_n = \frac{2}{T_0}\cdot\int_0^{T_0} 2\cdot V_A\cdot\left(\frac{1}{2} - \frac{t}{T_0}\right)\cdot\sin\left(2\cdot\pi\cdot n\cdot\frac{t}{T_0}\right) dt$$

$$b_n = \frac{2}{T_0}\cdot\left[\frac{1}{2}\cdot T_0\cdot(\pi\cdot n\cdot\cos(2\cdot\pi\cdot n) - \sin(2\cdot\pi\cdot n))\cdot\frac{V_A}{\left(\pi^2\cdot n^2\right)} + \frac{1}{[2\cdot(\pi\cdot n)]}\cdot T_0\cdot V_A\right]$$

$$b_n = V_A\cdot\frac{(\pi\cdot n\cdot\cos(2\cdot\pi\cdot n) - \sin(2\cdot\pi\cdot n) + \pi\cdot n)}{\left(\pi^2\cdot n^2\right)} = V_A\cdot\frac{\cos(2\cdot\pi\cdot n) + 1}{\pi\cdot n} = \frac{2\cdot V_A}{\pi\cdot n} \qquad \text{for all n}$$

13-4

$$v(t) = \begin{cases} V_A\cdot\frac{2\cdot t}{T_0} & \text{if } 0 \le t < \frac{T_0}{2} \\ 0 & \text{if } \frac{T_0}{2} \le t < T_0 \end{cases}$$



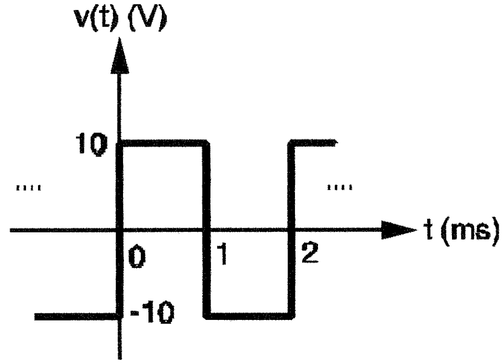
$$a_0 = \frac{1}{T_0}\cdot\int_0^{\frac{T_0}{2}} V_A\cdot\frac{2t}{T_0}\,dt = \frac{V_A}{4}$$

$$a_n = \frac{2}{T_0}\cdot\int_0^{\frac{T_0}{2}} V_A\cdot\frac{2\cdot t}{T_0}\cdot\cos\left(2\cdot\pi\cdot n\cdot\frac{t}{T_0}\right) dt = \frac{V_A}{n^2\cdot\pi^2}\cdot(\cos(n\cdot\pi) - 1) = \frac{-2\cdot V_A}{(n\cdot\pi)^2} \qquad \text{n odd}$$

$$b_n = \frac{2}{T_0}\cdot\int_0^{\frac{T_0}{2}} V_A\cdot\frac{2\cdot t}{T_0}\cdot\sin\left(2\cdot\pi\cdot n\cdot\frac{t}{T_0}\right) dt = \frac{-V_A\cdot(-\sin(\pi\cdot n) + \pi\cdot n\cdot\cos(\pi\cdot n))}{\left(\pi^2\cdot n^2\right)} = \frac{-V_A\cdot\cos(n\cdot\pi)}{n\cdot\pi} = \frac{(-1)^{n-1}\cdot V_A}{n\cdot\pi}$$

13-5 Square wave $A := 10 \quad T_0 := 2 \cdot 10^{-3} \quad f_0 := \frac{1}{T_0} \qquad n := 1, 3 .. 13$

$a_0 := 0 \quad a_n := 0 \quad b_n := \frac{4 \cdot A}{n \cdot \pi}$

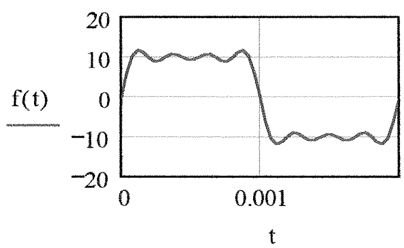


$n =$	$n \cdot f_0 =$	$a_n =$	$b_n =$
1	500	0	12.732
3	$1.5 \cdot 10^3$	0	4.244
5	$2.5 \cdot 10^3$	0	2.546
7	$3.5 \cdot 10^3$	0	1.819
9	$4.5 \cdot 10^3$	0	1.415
11	$5.5 \cdot 10^3$	0	1.157
13	$6.5 \cdot 10^3$	0	0.979

$t := 0, 0.02 \cdot T_0 .. T_0$

The first four nonzero terms are:

$f(t) := 12.732\sin(2 \cdot \pi \cdot 500t) + 4.244\sin(2 \cdot \pi \cdot 1500t) + 2.546\sin(2 \cdot \pi \cdot 2500t) + 1.819\sin(2 \cdot \pi \cdot 3500t)$



<---waveform agrees with f(t) in Fig. P13-5. The more terms that are added the better the agreement.

13-6 Shifted Triagular wave with $A := 20 \quad T_0 := 10^{-3} \quad f_0 := T_0^{-1} \quad f_0 = 1 \times 10^3$

Let g(t) be the triangular wave in Fig. 13-3, then the waveform in Fig. P13-6 is $f(t) = g(t - T_0/4)$.

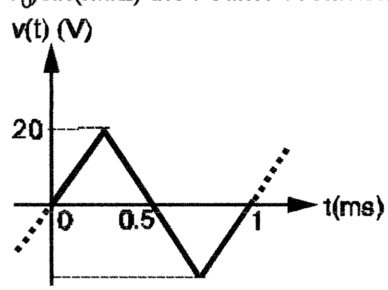
Since $\cos[2\pi n(t - 0.25T_0)/T_0] = \sin(2\pi nt/T_0)\sin(n\pi/2)$ the Fourier coefficients of f(t) are

$n := 1, 3 .. 15$

$a_0 := 0 \qquad a_n := 0$

$b_n := \sin\left(n \cdot \frac{\pi}{2}\right) \cdot \frac{8 \cdot A}{(n \cdot \pi)^2}$

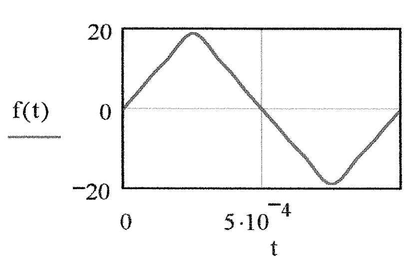
$t := 0, 0.02 \cdot T_0 .. T_0$



$b_1 = 16.211$	$b_2 = 0$	$b_3 = -1.801$	$b_4 = 0$	$b_5 = 0.648$	$b_6 = 0$
$b_7 = -0.331$	$b_8 = 0$	$b_9 = 0.2$	$b_{10} = 0$	$b_{11} = -0.134$	$b_{12} = 0$

The first four nonzero terms are

: $f(t) := \left[\left(16.2\sin\left(2 \cdot \pi \cdot 10^3 \cdot t\right) - 1.80\sin\left(2 \cdot \pi \cdot 3 \cdot 10^3 \cdot t\right)\right) + 0.648\sin\left(2 \cdot \pi \cdot 5 \cdot 10^3 \cdot t\right)\right] - 0.331\sin\left(2 \cdot \pi \cdot 7 \cdot 10^3 \cdot t\right)$

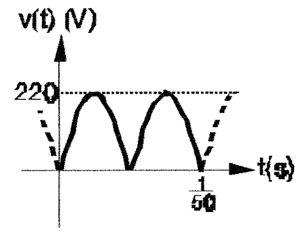


<---Waveform agrees with f(t) in Fig. P13-6

13-7 **Full-wave Rectified Sinewave with** $A := 220 \quad f_0 := 50 \quad T_0 := f_0^{-1} \quad T_0 = 2 \times 10^{-2}$

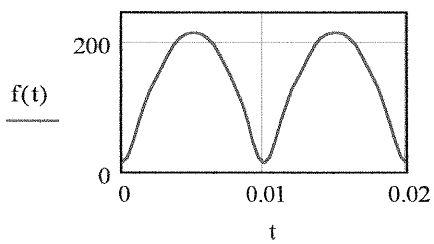
$n := 2, 4 .. 14 \quad a_0 := \frac{2 \cdot A}{\pi} \quad a_0 = 140.056 \quad b_n := 0 \quad a_n := \frac{4 \cdot A}{\pi \cdot \left(1 - n^2\right)}$ $\qquad t := 0 .. T_0$

$n =$	$n \cdot f_0 =$	$a_n =$	$b_n =$
2	100	-93.371	0
4	200	-18.674	0
6	300	-8.003	0
8	400	-4.446	0
10	500	-2.829	0
12	600	-1.959	0
14	700	-1.436	0



The first four nonzero harmonics besides the dc component are

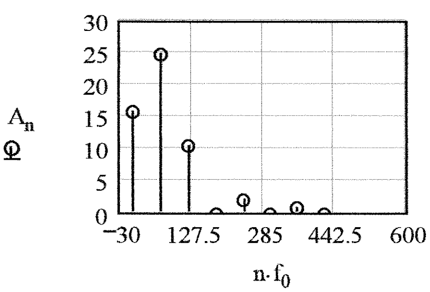
$f(t) := \left(140 - 93.4 \cos(4 \cdot \pi \cdot 50 \cdot t) - 18.7 \cos(8 \cdot \pi \cdot 50 \cdot t) - 8.0 \cos(12 \cdot \pi \cdot 50 \cdot t) - 4.44 \cos(16 \cdot \pi \cdot 50 \cdot t)\right)$



<---Waveform agrees with f(t) in Fig. P13-7

13-8 $V_A := 50 \quad f_0 := 60 \quad n := 2, 3 .. 8 \quad a_0 := \frac{V_A}{\pi} \quad a_n := \frac{2 \cdot V_A}{\pi \cdot \left(1 - n^2\right)} \cdot \left|\cos\left(n \cdot \frac{\pi}{2}\right)\right| \quad b_n := 0$

$a_0 = 15.915 \quad b_1 := \frac{V_A}{2} \qquad n := 0, 1 .. 7 \quad A_n := \sqrt{\left(a_n\right)^2 + \left(b_n\right)^2}$

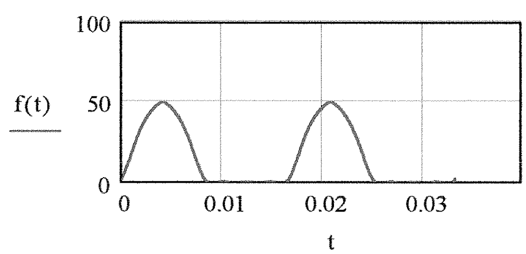


$T_0 := f_0^{-1}$

$t := 0, 0.005 \cdot T_0 .. 2 \cdot T_0$

$n \cdot f_0 =$	$a_n =$	$b_n =$
0	15.915	0
60	0	25
120	-10.61	0
180	0	0
240	-2.122	0
300	0	0
360	-0.909	0
420	0	0

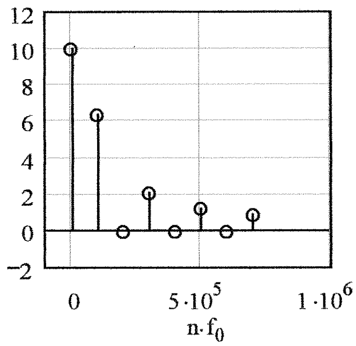
$f(t) := 15.915 + 25 \cdot \sin(2 \cdot \pi \cdot 60 \cdot t) - 10.61 \cdot \cos(2 \cdot \pi \cdot 120 \cdot t) - 2.122 \cdot \cos(2 \cdot \pi \cdot 240 \cdot t) - 0.909 \cdot \cos(2 \cdot \pi \cdot 360 \cdot t)$



<---halfwave sine with 50 Vp.

13-9 $V_A := 5 \quad f_0 := 100 \cdot 10^3 \quad n := 1, 2 .. 8 \quad a_0 := 10 \quad a_n := 0 \quad b_n := \frac{4 \cdot V_A}{n \cdot \pi} \cdot \left| \sin\left(n \cdot \frac{\pi}{2}\right) \right|$

$n := 0, 1 .. 7 \quad A_n := \sqrt{(a_n)^2 + (b_n)^2}$

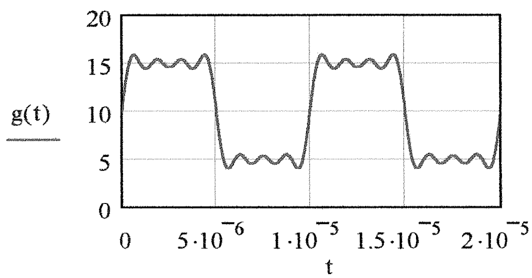


$n \cdot f_0 =$	$a_n =$	$b_n =$
0	10	0
$1 \cdot 10^5$	0	6.366
$2 \cdot 10^5$	0	0
$3 \cdot 10^5$	0	2.122
$4 \cdot 10^5$	0	0
$5 \cdot 10^5$	0	1.273
$6 \cdot 10^5$	0	0
$7 \cdot 10^5$	0	0.909

$T_0 := f_0^{-1}$

$t := 0, 0.005 \cdot T_0 .. 2 \cdot T_0$

$g(t) := 10 + 6.37 \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t) + 2.12 \cdot \sin(2 \cdot \pi \cdot 3 \cdot f_0 \cdot t) + 1.27 \cdot \sin(2 \cdot \pi \cdot 5 \cdot f_0 \cdot t) + 0.909 \cdot \sin(2 \cdot \pi \cdot 7 \cdot f_0 \cdot t)$



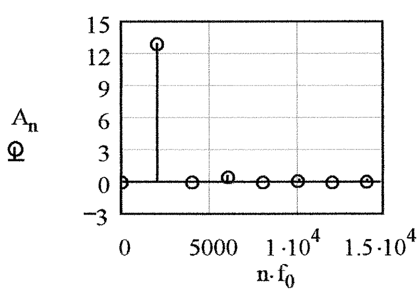
<---100 kHz square wave with 10 Vpp and 10 Vdc offset.

13-10 $V_A := 12.5 \quad f_0 := 2000 \quad n := 1, 2 .. 7 \quad a_0 := 0 \quad a_n := 0 \quad b_n := \frac{32 \cdot V_A}{(n \cdot \pi)^3} \cdot \left| \sin\left(n \cdot \frac{\pi}{2}\right) \right|$

$a_0 = 0 \quad n := 0, 1 .. 7 \quad A_n := \sqrt{(a_n)^2 + (b_n)^2}$

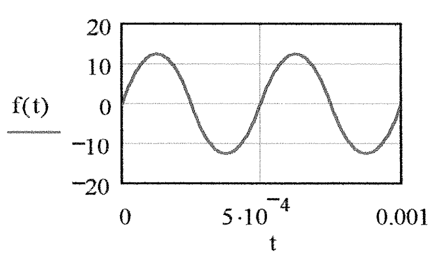
$T_0 := f_0^{-1}$

$n \cdot f_0 =$	$b_n =$
0	0
$2 \cdot 10^3$	12.901
$4 \cdot 10^3$	0
$6 \cdot 10^3$	0.478
$8 \cdot 10^3$	0
$1 \cdot 10^4$	0.103
$1.2 \cdot 10^4$	0
$1.4 \cdot 10^4$	0.038



$t := 0, 0.005 \cdot T_0 .. 2 \cdot T_0$

$f(t) := 12.9 \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t) + 0.478 \cdot \sin(2 \cdot \pi \cdot 3 \cdot f_0 \cdot t) + 0.103 \cdot \sin(2 \cdot \pi \cdot 5 \cdot f_0 \cdot t) + 0.038 \cdot \sin(2 \cdot \pi \cdot 7 \cdot f_0 \cdot t)$

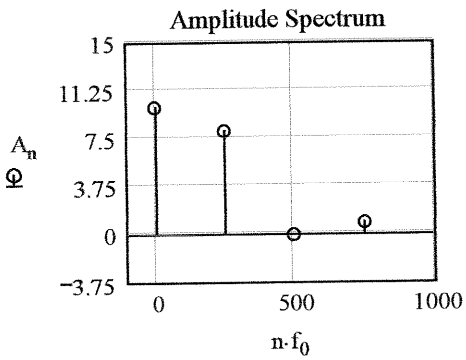


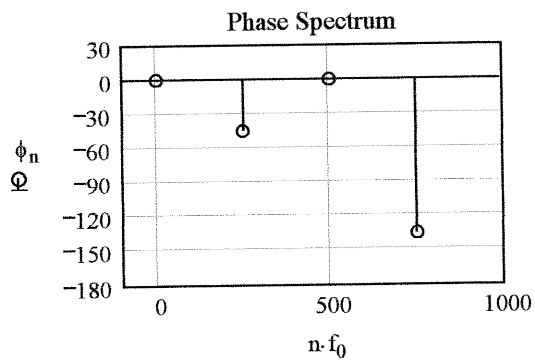
<---parabolic wave 25 Vpp and f_0 = 2kHz

13-11 (a) $T_0 := \frac{1}{250}$ $f_0 := \frac{1}{T_0}$ $\omega_0 := 2\cdot\pi\cdot f_0$ $T_0 = 4\times10^{-3}$ $f_0 = 250$ $\omega_0 = 1.571\times10^{3}$

(b) $A_0 := 10$ $A_1 := 8.11$ $A_2 := 0$ $A_3 := 0.901$ $\phi_0 := 0$ $\phi_1 := -45$ $\phi_2 := 0$ $\phi_3 := -135$

$n := 0, 1 .. 3$





(c) Converting phase angles to radians $45\cdot\frac{\pi}{180} = 0.785$ and $135\cdot\frac{\pi}{180} = 2.356$ yields

$f(t) := 10 + 8.11\cdot\cos(2\cdot\pi\cdot250t - 0.785) + 0.901\cdot\cos(2\cdot\pi\cdot750t - 2.356)$ which can be expanded as

$f(t) := 10 + 5.74\cdot\cos(500\pi\cdot t) + 5.74\sin(500\pi\cdot t) - .637\cdot\cos(1500\pi\cdot t) + .637\cdot\sin(1500\pi\cdot t)$

(d) The waveform is neither even or odd. It contains both sine and cosine terms plus a dc component.

13-12

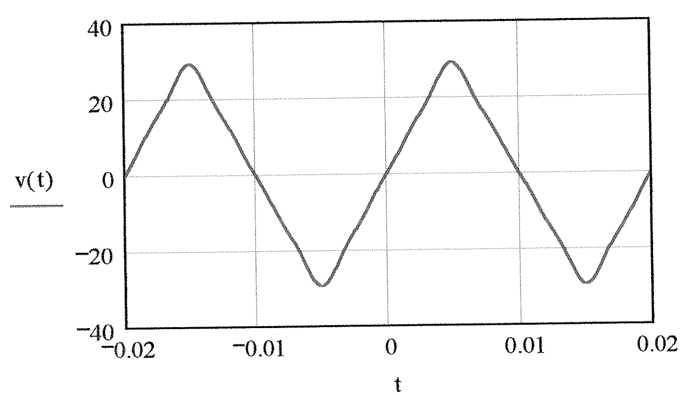
$$v(t) := 25\cdot\left(\sin(100\cdot\pi\cdot t) - \frac{1}{9}\cdot\sin(300\cdot\pi\cdot t) + \frac{1}{25}\cdot\sin(500\cdot\pi\cdot t) - \frac{1}{49}\cdot\sin(700\cdot\pi\cdot t)\right)$$

(a) $f_0 := 50\text{Hz}$; $\omega_0 := 2\cdot\pi\cdot f_0$; $\omega_0 = 314.159$ rad/s ; $T_0 := \frac{1}{f_0}$; $T_0 = 2\times10^{-2}$ s

(b) The waveform is odd since only sine terms are present.

(c) $t := -T_0, -T_0 + \frac{T_0}{200} .. T_0$

Signal appears to be a triangular wave passing through the origin at t = 0.



13-13

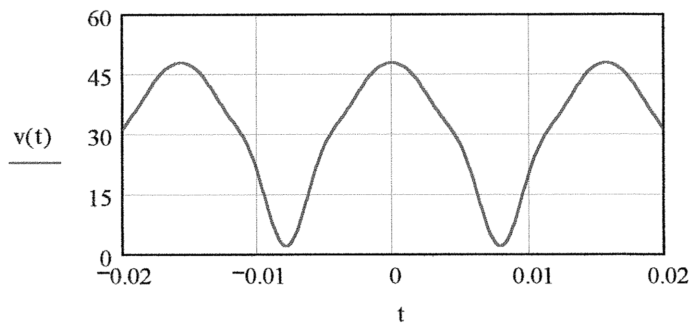
$$v(t) := 30 + 20\cdot\left(\cos(400\cdot t) - \frac{1}{5}\cdot\cos(800\cdot t) + \frac{1}{7}\cdot\cos(1200\cdot t) - \frac{1}{21}\cdot\cos(1600\cdot t)\right)$$

(a) $\omega_0 := 400\text{rad/s}$; $f_0 := \frac{\omega_0}{2\cdot\pi}$; $f_0 = 63.662$ Hz ; $T_0 := \frac{1}{f_0}$; $T_0 = 1.571\times 10^{-2}$ s.

(b) The waveform has even symmetry since only cosine terms are present.

(c) $t := -T_0, -(0.9995)\cdot T_0 .. T_0$

Signal appears to be a full-wave rectified cosine

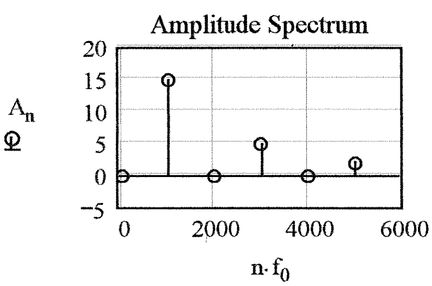


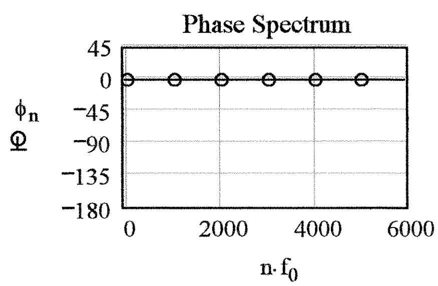
13-14 $f(t) = 15\cdot\cos(2000\cdot\pi\cdot t) + 5\cdot\cos(6000\cdot\pi\cdot t) + 2\cdot\cos(10^4\cdot\pi\cdot t)$

(a) $T_0 := \frac{1}{1000}$ $\quad f_0 := \frac{1}{T_0}$ $\quad \omega_0 := 2\cdot\pi\cdot f_0$ $\quad T_0 = 1\times 10^{-3}$ $\quad f_0 = 1\times 10^{3}$ $\quad \omega_0 = 6.283\times 10^{3}$

(b) $A_0 := 0$ $\quad A_1 := 15$ $\quad A_2 := 0$ $\quad A_3 := 5$ $\quad A_4 := 0$ $\quad A_5 := 2$

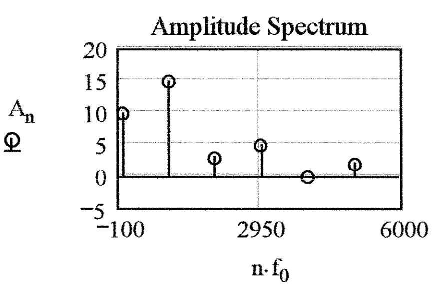
$\phi_0 := 0$ $\quad \phi_1 := 0$ $\quad \phi_2 := 0$ $\quad \phi_3 := 0$ $\quad \phi_4 := 0$ $\quad \phi_5 := 0$ $\quad n := 0, 1 .. 5$

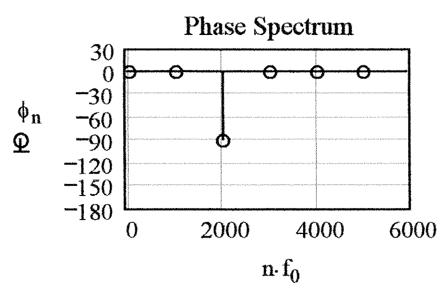




(c) The waveform has even symmetry - there are no sine terms.

(d) g(t) = f(t) +10 + 3 sin(4000πt) adds dc and a 2nd harmonic sine term. $A_0 := 10$, $A_2 := 3$ and $\phi_2 := -90$. g(t) has neither even or odd symmetry.



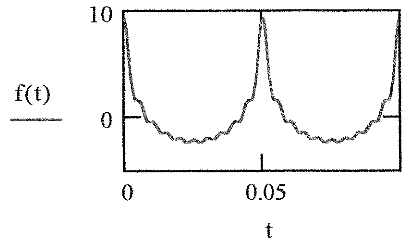


13-15 Given $n := 1, 2 .. 8$ $\quad a_0 := 0$ $\quad a_n := \dfrac{10}{n \cdot \pi}$ $\quad b_n := 0$ $\quad$ for all n > 0

$$f(t) := \sum_{n=1}^{10} \frac{10}{n \cdot \pi} \cdot \cos(40 \cdot n \cdot \pi \cdot t)$$

(a) $f_0 := 20$ $\quad T_0 := f_0^{-1}$ $\quad T_0 = 0.05$ $\quad t := 0, \dfrac{T_0}{100} .. 2 \cdot T_0$

$\omega_0 = 6.283 \times 10^3$ $\quad \omega_0 := 2 \cdot \pi \cdot f_0$

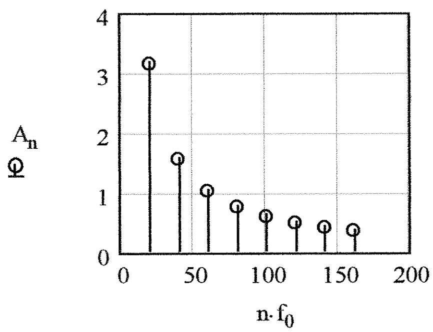


(b) The waveform has even symmetry

(c) $A_n := \sqrt{(a_n)^2 + (b_n)^2}$

$$f(t) := \sum_{n=1}^{8} \frac{10}{n \cdot \pi} \cdot \cos(40 \cdot n \cdot \pi \cdot t)$$

$n \cdot f_0 =$	$A_n =$
20	3.183
40	1.592
60	1.061
80	0.796
100	0.637
120	0.531
140	0.455
160	0.398



(d) Given $g(t) = 10 + 2 \cdot f(t - 0.005) = 10 + \displaystyle\sum_{n=1}^{\infty} \frac{20}{n \cdot \pi} \cdot \cos[40 \cdot n \cdot \pi \cdot (t - 0.005)]$ we expand as

$$\cos[40 \cdot n \cdot \pi \cdot (t - 0.005)] = \cos\left(40 \cdot \pi \cdot n \cdot t - n \cdot \frac{\pi}{4}\right) = \cos(40 \cdot \pi \cdot n \cdot t) \cdot \cos\left(n \cdot \frac{\pi}{4}\right) + \sin(40 \cdot \pi \cdot n \cdot t) \cdot \sin\left(n \cdot \frac{\pi}{4}\right)$$

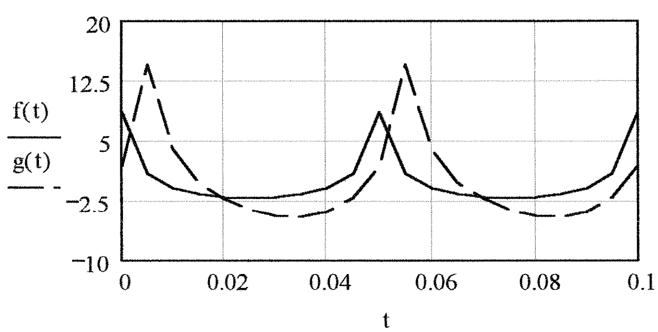
The Fourier coeff of g(t) are: $a_0 := 0$ $\quad a_n := \dfrac{20}{n \cdot \pi} \cdot \cos\left(n \cdot \dfrac{\pi}{4}\right)$ $\quad b_n := \dfrac{20}{n \cdot \pi} \cdot \sin\left(n \cdot \dfrac{\pi}{4}\right)$

g(t) has both cosine and sine terms has neither even or odd symmetry. Checking

$$g(t) := \sum_{n=1}^{8} \left(a_n \cdot \cos(2 \cdot \pi \cdot n \cdot f_0 \cdot t) + b_n \cdot \sin(2 \cdot \pi \cdot n \cdot f_0 \cdot t)\right)$$

$t := 0, 0.1 \cdot T_0 .. 2 \cdot T_0$

$a_n =$	$b_n =$
4.502	4.502
0	3.183
-1.501	1.501
-1.592	0
-0.9	-0.9
0	-1.061
0.643	-0.643
0.796	0



---- g(t) is shifted by 0.005.

13-16 Given are $V_A := 5$ $T_0 := 0.5 \cdot \pi \cdot 10^{-3}$

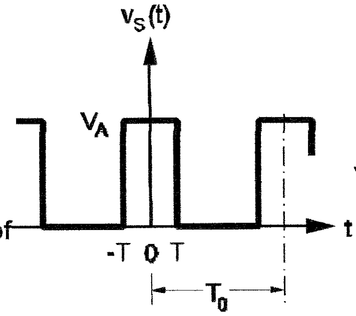
$T := 0.2 \cdot T_0$ $R := 200$ $L := 0.1$ From Fig 13-3

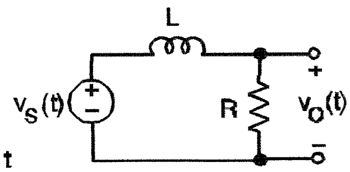
the Fourier coefficients ($n := 1, 2..10$) are

$a_0 := V_A \cdot \frac{T}{T_0}$ $a_n := \frac{2 \cdot V_A}{n \cdot \pi} \cdot \sin\left(n \cdot \pi \cdot \frac{T}{T_0}\right)$ $b_n := 0$

For $\omega_0 := 2 \cdot \pi \cdot T_0^{-1}$ For the transfer function of

RL circuit is $T_V(n) := \frac{R}{R + j \cdot n \cdot \omega_0 \cdot L}$ $n := 0, 1..5$





The output amplitudes $AMP_n := |a_n| \cdot |T_V(n)|$ phase angles $PH_n := \frac{180}{\pi} \cdot \arg(a_n \cdot T_V(n))$

$n =$	$n \cdot \omega_0 =$	$a_n =$	$\lvert T_V(n) \rvert =$	$AMP_n =$	$PH_n =$
0	0	1	1	1	0
1	$4 \cdot 10^3$	1.871	0.447	0.837	-63.435
2	$8 \cdot 10^3$	1.514	0.243	0.367	-75.964
3	$1.2 \cdot 10^4$	1.009	0.164	0.166	-80.538
4	$1.6 \cdot 10^4$	0.468	0.124	0.058	-82.875
5	$2 \cdot 10^4$	0	0.1	0	-84.289

$v_O(t) = 1 + 0.837 \cdot \cos\left(4 \cdot 10^3 \cdot t - 63.4^o\right) + 0.367 \cdot \cos\left(8 \cdot 10^3 \cdot t - 76.0^o\right) + 0.166 \cdot \cos\left(12 \cdot 10^3 \cdot t - 80.5^o\right)$

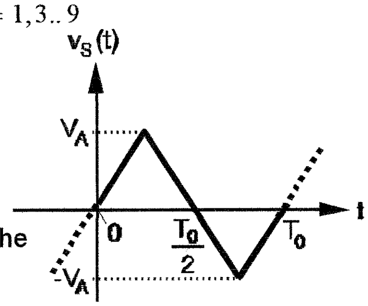
13-17 Given $V_A := 20$ $T_0 := 2 \cdot \pi \cdot 10^{-3}$ $n := 1, 3..9$

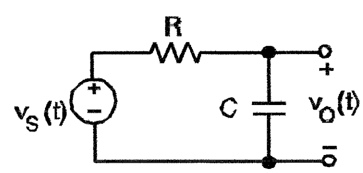
$R := 10^4$, $C := 50 \cdot 10^{-9}$ The Fourier coeffs.

are $a_0 := 0$ $a_n := 0$ $b_n := \frac{8 \cdot V_A}{(n \cdot \pi)^2} \cdot \sin\left(n \cdot \frac{\pi}{2}\right)$

For $\omega_0 := 2 \cdot \pi \cdot T_0^{-1}$ the transfer function the

RC circuit is $T_V(n) := \frac{1}{1 + j \cdot n \cdot \omega_0 \cdot R \cdot C}$





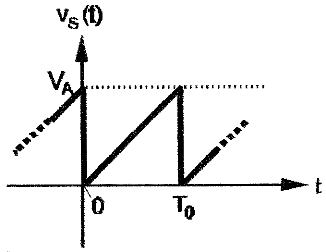
The output amplitude $AMP_n := |b_n| \cdot |T_V(n)|$ phase angle $PH_n := \frac{180}{\pi} \cdot \arg(b_n \cdot T_V(n))$

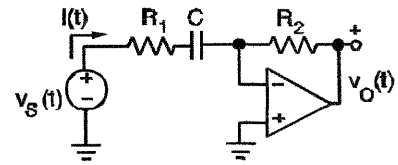
$n =$	$n \cdot \omega_0 =$	$b_n =$	$\lvert T_V(n) \rvert =$	$AMP_n =$	$PH_n =$	$PH_n - 90 =$
1	$1 \cdot 10^3$	16.211	0.894	14.5	-26.565	-116.565
3	$3 \cdot 10^3$	-1.801	0.555	0.999	123.69	33.69
5	$5 \cdot 10^3$	0.648	0.371	0.241	-68.199	-158.199
7	$7 \cdot 10^3$	-0.331	0.275	0.091	105.945	15.945
9	$9 \cdot 10^3$	0.2	0.217	0.043	-77.471	-167.471

$v_O(t) = 14.5 \cdot \sin\left(10^3 \cdot t - 26.6^o\right) + 0.999 \cdot \sin\left(3 \cdot 10^3 \cdot t + 124^o\right) + 0.241 \cdot \sin\left(5 \cdot 10^3 \cdot t - 68.2^o\right) + 0.091 \cdot \sin\left(7 \cdot 10^3 \cdot t + 10\right.$

$v_O(t) = 14.5 \cdot \cos\left(10^3 \cdot t - 117^o\right) + 0.999 \cdot \cos\left(3 \cdot 10^3 \cdot t + 34^o\right) + 0.241 \cdot \cos\left(5 \cdot 10^3 \cdot t - 168^o\right) + 0.091 \cdot \cos\left(7 \cdot 10^3 \cdot t + 16^o\right.$

13-18 Given $V_A := 5 \quad T_0 := 400\pi \cdot 10^{-6}$





$R_1 := 10^4 \quad R_2 := 5 \cdot 10^4 \quad C := 10 \cdot 10^{-9}$

The Fourier coefficients are $n := 1, 2..5$

$$a_0 := \frac{V_A}{2} \quad a_n := 0 \quad b_n := -\frac{V_A}{n \cdot \pi}$$

For $\omega_0 := 2 \cdot \pi \cdot T_0^{-1}$ the transfer function of the

OP AMP circuit is $T_V(n) := \dfrac{-j \cdot n \cdot \omega_0 \cdot R_2 \cdot C}{1 + j \cdot n \cdot \omega_0 \cdot R_1 \cdot C}$

The output amplitudes $AMP_n := |b_n| \cdot |T_V(n)|$ phase angles $PH_n := \dfrac{180}{\pi} \cdot \arg(b_n \cdot T_V(n))$

$n =$	$n \cdot \omega_0 =$	$b_n =$	$\lvert T_V(n) \rvert =$	$AMP_n =$	$PH_n =$	$PH_n - 90 =$
1	$5 \cdot 10^3$	-1.592	2.236	3.559	63.435	-26.565
2	$1 \cdot 10^4$	-0.796	3.536	2.813	45	-45
3	$1.5 \cdot 10^4$	-0.531	4.16	2.207	33.69	-56.31
4	$2 \cdot 10^4$	-0.398	4.472	1.779	26.565	-63.435
5	$2.5 \cdot 10^4$	-0.318	4.642	1.478	21.801	-68.199

$v_O(t) = 3.56 \cdot \sin(5 \cdot 10^3 \cdot t + 63.4^o) + 2.81 \cdot \sin(10^4 \cdot t + 45^o) + 2.21 \cdot \sin(15 \cdot 10^3 \cdot t + 33.7^o) + 1.78 \cdot \sin(2 \cdot 10^4 \cdot t + 26.6^o$

$v_O(t) = 3.56 \cdot \cos(5 \cdot 10^3 \cdot t - 26.6^o) + 2.81 \cdot \sin(10^4 \cdot t - 45^o) + 2.21 \cdot \sin(15 \cdot 10^3 \cdot t - 56.3^o) + 1.78 \cdot \sin(2 \cdot 10^4 \cdot t - 63.4^o$

13-19 Given $V_A := 4 \quad T_0 := 800\pi \cdot 10^{-6}$

$R_1 := 10^4, \quad R_2 := 5 \cdot 10^4, \quad C := 400 \cdot 10^{-9}$ See Figure 13-18 above.

The Fourier coefficients are $n := 1, 2..5 \quad a_0 := \dfrac{V_A}{2} \quad a_n := 0 \quad b_n := -\dfrac{V_A}{n \cdot \pi}$

For $\omega_0 := 2 \cdot \pi \cdot T_0^{-1}$ the ckt input admittance is $Y_{IN}(n) := \dfrac{j \cdot n \cdot \omega_0 \cdot C}{1 + j \cdot n \cdot \omega_0 \cdot R_1 \cdot C}$

Output amplitudes $a_0 \cdot Y_{IN}(0) = 0 \quad AMP_n := |b_n \cdot Y_{IN}(n)|$ phase angle $PH_n := \dfrac{180}{\pi} \cdot \arg(b_n \cdot Y_{IN}(n))$

$n =$	$n \cdot \omega_0 \cdot 10^{-3} =$	$b_n =$	$\lvert Y_{IN}(n) \rvert =$	$AMP_n \cdot 10^6 =$	$PH_n =$	$PH_n - 90 =$
1	2.5	-1.273	$9.95 \cdot 10^{-5}$	126.692	-174.289	-264.289
2	5	-0.637	$9.988 \cdot 10^{-5}$	63.583	-177.138	-267.138
3	7.5	-0.424	$9.994 \cdot 10^{-5}$	42.418	-178.091	-268.091
4	10	-0.318	$9.997 \cdot 10^{-5}$	31.821	-178.568	-268.568
5	12.5	-0.255	$9.998 \cdot 10^{-5}$	25.46	-178.854	-268.854

Input current in μA is:

$i(t) = 127 \cdot \sin(2.5 \cdot 10^3 - 174^o) + 63.6 \cdot \sin(5 \cdot 10^3 \cdot t - 177^o) + 42.4 \cdot \sin(7.5 \cdot 10^3 \cdot t - 178^o) + 31.8 \cdot \sin(10^4 \cdot t - 179^o)$

$i(t) = 127 \cdot \cos(2.5 \cdot 10^3 - 264^o) + 63.6 \cdot \cos(5 \cdot 10^3 \cdot t - 267^o) + 42.4 \cdot \cos(7.5 \cdot 10^3 \cdot t - 268^o) + 31.8 \cdot \cos(10^4 \cdot t - 269^o)$

13-20 Find i(t). Given $V_A := 5 \quad T_0 := 4 \cdot \pi \cdot 10^{-4}$

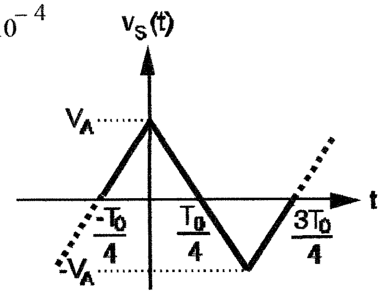
$R := 1 \quad C := 800 \cdot 10^{-9} \quad L := 2 \cdot 10^{-3}$

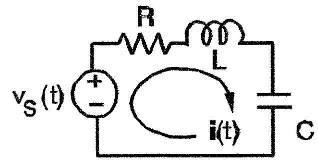
The input Fourier coeffs. are $n := 1, 3 .. 9$

$$a_0 := 0 \quad a_n := \frac{8 \cdot V_A}{(n \cdot \pi)^2} \quad b_n := 0$$

For $\omega_0 := 2 \cdot \pi \cdot T_0^{-1}$ the admittance of the RLC ckt is

$$Y(n) := \frac{1}{R + \frac{1}{j \cdot n \cdot \omega_0 \cdot C} + j \cdot n \cdot \omega_0 \cdot L}$$





The output amplitudes $AMP_n := \left|a_n \cdot Y(n)\right|$ phase angles $PH_n := \frac{180}{\pi} \cdot \arg\left(a_n \cdot Y(n)\right)$

$n =$	$n \cdot \omega_0 =$	$a_n =$	$\lvert Y(n) \rvert =$	$AMP_n \cdot 10^3 =$	$PH_n =$
1	$5 \cdot 10^3$	4.053	$4.167 \cdot 10^{-3}$	16.887	89.761
3	$1.5 \cdot 10^4$	0.45	0.019	8.442	88.926
5	$2.5 \cdot 10^4$	0.162	1	162.114	0
7	$3.5 \cdot 10^4$	0.083	0.029	2.411	-88.329
9	$4.5 \cdot 10^4$	0.05	0.016	0.804	-89.079

Response is dominated by the 5^{th} harmonic at 25 krad/s because the circuit resonant frequency is $\frac{1}{\sqrt{L \cdot C}} = 2.5 \times 10^4$ rad/s

13-21 The given parameters are $V_A := 15, \; T_0 := 40 \cdot \pi \cdot 10^{-3}$ and $T(s) := \frac{s}{s + 200}$.

Using Fig. 13-3 the Fourier coefficients of the input are

$$a_0 := 0 \qquad a_n := 0 \quad b_n := 0 \quad \text{n even} \qquad b_n := \frac{4 \cdot V_A}{n \cdot \pi}$$

For $\omega_0 := 2 \cdot \pi \cdot T_0^{-1}$ the steady-state output is

$$AMP_n := \left|b_n \cdot T(j \cdot n \cdot \omega_0)\right| \qquad n := 1, 3 .. 7 \quad a_0 \cdot T(0) = 0$$

		Input	Output
$n =$	$n \cdot \omega_0 =$	$b_n =$	$AMP_n =$
1	50	19.099	4.632
3	150	6.366	3.82
5	250	3.82	2.983
7	350	2.728	2.369

The ckt is a highpass filter with $\omega_C = 200$. The fundamental at $\omega_0 = 50$ is reduced more than the other harmonics which fall just below or slightly above the highpass cutoff frequency.

13-22 The given parameters are $V_A := 15,\ T_0 := 40\pi\cdot 10^{-3}$ and $T(s) := \dfrac{100}{s+100}$.

Using Fig. 13-3 the Fourier coefficients of the input are

$a_0 := 0 \qquad a_n := 0 \qquad b_n := 0$ n even $\qquad b_n := \dfrac{4\cdot V_A}{n\cdot\pi}$

For $\omega_0 := 2\cdot\pi\cdot T_0^{-1}$ the steady-state output is

$AMP_n := \left|b_n\cdot T(j\cdot n\cdot\omega_0)\right| \qquad n := 1,3..7 \quad a_0\cdot T(0) = 0$

		Input	Output
$n =$	$n\cdot\omega_0 =$	$b_n =$	$AMP_n =$
1	50	19.099	17.082
3	150	6.366	3.531
5	250	3.82	1.419
7	350	2.728	0.75

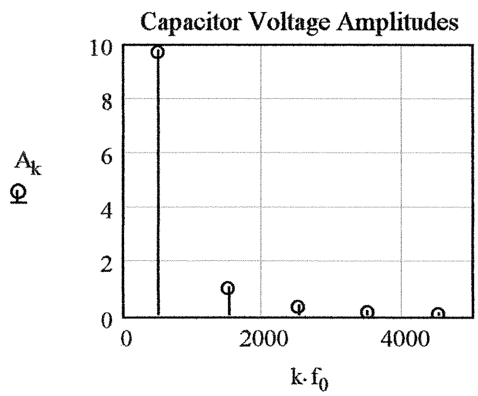
The ckt is a lowpass filter with $\omega_C = 100$. Since the fundamental frequency is $\omega_0 = 50$ the fundamental falls in the passband and is reduced less than the other harmonics which fall just above the filter cutoff frequency.

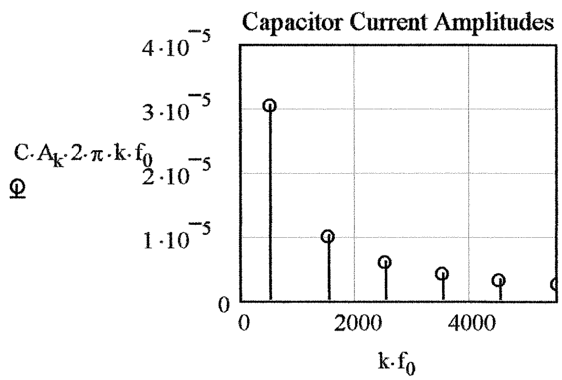
13-23 Triangular wave

$n := 0,1..20 \quad a_n := 0 \quad b_n := 0 \quad \phi_n := 0 \qquad V_A := 12 \quad T_0 := 2\cdot 10^{-3} \qquad f_0 := \dfrac{1}{T_0}$

$a_0 := 0 \quad n := 1,3..21 \quad b_n := 0\ a_n := \dfrac{8\cdot V_A}{(n\cdot\pi)^2} \qquad k := 1,3..11 \qquad A_k := \sqrt{(a_k)^2 + (b_k)^2} \qquad C := 10^{-9}$

$v_C(t) = \sum_k A_k\cdot\cos(2\cdot\pi\cdot k\cdot f_0\cdot t) \qquad i_C(t) = C\cdot\dfrac{d}{dt}v_C(t) = -\left(\sum_k C\cdot A_k\cdot 2\cdot\pi\cdot k\cdot f_0\cdot\sin(2\cdot\pi\cdot k\cdot f_0\cdot t)\right)$





Differentiation multiplies the each harmonic's voltage by its frequency. This increases the high frequency amplitudes of the current relative to the voltage. The voltage amplitudes decrease as $1/n^2$ while the current amplitudes fall off as $1/n$

13-24 $n := 0, 1..20 \quad a_n := 0 \quad b_n := 0 \quad \phi in_n := 0 \quad \phi out_n := 0 \quad f_0 := 500 \quad \omega_0 := 2\cdot\pi\cdot f_0 \quad T_D := 0.001$

The input is $v_{IN}(t) = 10 + 10\cdot\cos(2\cdot\pi\cdot 500\cdot t) + 3\cdot\cos(2\cdot\pi\cdot 1000\cdot t) + 2\cdot\cos(2\cdot\pi\cdot 4000\cdot t)$ and the input

spectrum is $a_0 := 10 \quad a_1 := 10 \quad a_2 := 3 \quad a_8 := 2 \quad n := 1, 2..8 \quad Ain_n := \sqrt{(a_n)^2 + (b_n)^2}$

With an ideal 1 ms delay the output signal is

$$v_{OUT}(t) = 10 + 10\cdot\cos\left[2\cdot\pi\cdot 500\cdot\left(t - 10^{-3}\right)\right] + 3\cdot\cos\left[2\cdot\pi\cdot 1000\cdot\left(t - 10^{-3}\right)\right] + 2\cdot\cos\left[2\cdot\pi\cdot 4000\cdot\left(t - 10^{-3}\right)\right]$$

$v_{OUT}(t) = 10 + 10\cdot\cos(2\cdot\pi\cdot 500\cdot t - \pi) + 3\cdot\cos(2\cdot\pi\cdot 1000\cdot t - 2\cdot\pi) + 2\cdot\cos(2\cdot\pi\cdot 4000\cdot t - 8\cdot\pi)$ hence

$Aout_n := Ain_n \qquad \phi out_1 := -\pi \quad \phi out_2 := -2\cdot\pi \quad \phi out_8 := -8\cdot\pi$

$n =$	$n\cdot f_0 =$	$Ain_n =$	$Aout_n =$	$\phi in_n =$	$\phi out_n =$	$-n\cdot\omega_0\cdot T_D =$
1	500	10	10	0	-3.142	-3.142
2	$1\cdot 10^3$	3	3	0	-6.283	-6.283
3	$1.5\cdot 10^3$	0	0	0	0	-9.425
4	$2\cdot 10^3$	0	0	0	0	-12.566
5	$2.5\cdot 10^3$	0	0	0	0	-15.708
6	$3\cdot 10^3$	0	0	0	0	-18.85
7	$3.5\cdot 10^3$	0	0	0	0	-21.991
8	$4\cdot 10^3$	2	2	0	-25.133	-25.133

Ideal delay introduces a linear phase shift of $-n\omega_0 T_D$, but does not change the amplitudes. .

13-25 The given parameters are $V_A := 5\cdot\pi$, $T_0 := 20\cdot\pi\cdot 10^{-3}$ and $T(s) := \dfrac{100 s}{(s + 50)^2 + 300^2}$.

The input Fourier coefficients for $n := 2, 4..12$ are $a_0 := 0 \quad a_n := 0 \quad b_n := 0$

The input Fourier coefficients for $n := 1, 3..11$ are $a_0 := 0 \quad a_n := 0 \quad b_n := \dfrac{4\cdot V_A}{n\cdot\pi}$

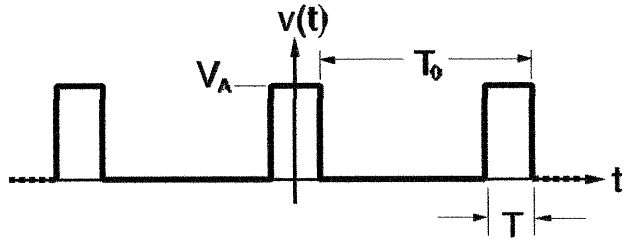
$f_0 := \dfrac{1}{T_0} \qquad \omega_0 := 2\cdot\pi\cdot f_0 \qquad \omega_0 = 100 \qquad n := 0, 1..7 \qquad AMP_n := \left|b_n\cdot T(j\cdot n\cdot\omega_0)\right|$

$n\cdot\omega_0 =$	$b_n =$	$\left\|T(j\cdot n\cdot\omega_0)\right\| =$	$AMP_n =$
0	0	0	0
100	20	0.12	2.407
200	0	0.356	0
300	6.667	0.997	6.644
400	0	0.51	0
500	4	0.303	1.21
600	0	0.219	0
700	2.857	0.173	0.496

The ckt is a bandpass filter centered at the 3rd harmonic of the input. The 3rd harmonic dominates the output eventhough its input amplitude is 1/3 of the amplitude of the input fundamental.

13-26 For a rectangular pulse train

$$V_{rms} = \sqrt{\frac{1}{T_0} \cdot \int_0^T V_A^2 \, dt} = V_A \cdot \sqrt{\frac{T}{T_0}}$$



Hence $P_{total} = \frac{V_{rms}^2}{R} = \frac{V_A^2}{R} \cdot \frac{T}{T_0}$

In general $P = \frac{(V_0)^2}{R} + \sum_{n=1}^{\infty} \frac{(V_n)^2}{2 \cdot R}$ For the rectangular pulse $V_0 = V_A \cdot \frac{T}{T_0}$ & $V_n = \frac{2 \cdot V_A}{n \cdot \pi} \cdot \sin\left(\frac{n \cdot \pi \cdot T}{T_0}\right)$

the dc component carries $\frac{P_0}{P_{total}} = \frac{V_0^2}{V_{rms}^2} = \frac{\left(\frac{T}{T_0}\right)^2}{\frac{T}{T_0}} = \left(\frac{T}{T_0}\right)$ and the ac components carry $1 - \frac{T}{T_0}$.

For 10% duty cycle the dc component carries 10% and the ac components 90%. At 50% duty cycle they each carry half of the average power. Decreasing the duty cycle increases the high-frequency content of the amplitude spectrum

13-27 The voltage $v(t) = 10 + 15 \cdot \cos(300 \cdot \pi \cdot t) - 20 \cdot \sin(900 \cdot \pi \cdot t)$ delivers $P = \frac{10^2}{50} + \frac{15^2}{2 \cdot 50} + \frac{20^2}{2 \cdot 50} = 8.25$ W

to a 50-Ω resistor. The rms value of the waveform is $V_{rms} = \sqrt{10^2 + \frac{15^2}{2} + \frac{20^2}{2}} = 20.31$ V.

13-28 For a square wave:

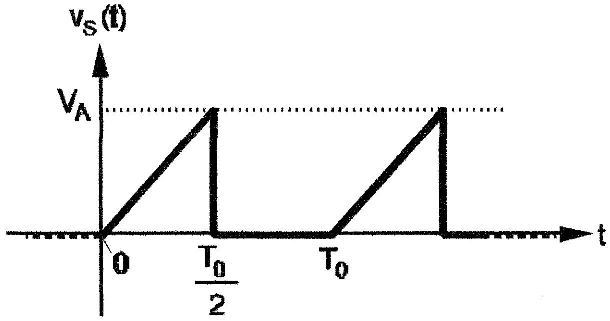
$$V_{rms} = \sqrt{\frac{1}{T_0} \cdot \left[\int_0^{\frac{T_0}{2}} V_A^2 \, dt + \int_{\frac{T_0}{2}}^{T_0} (-V_A)^2 \, dt\right]} = V_A$$

$V_0 := 0 \quad n := 1, 3 .. 5 \qquad V_n := \frac{4 \cdot V_A}{n \cdot \pi}$ Power fraction in the first three nonzero terms is

$$\frac{\frac{(V_1)^2}{2 \cdot R} + \frac{(V_3)^2}{2 \cdot R} + \frac{(V_5)^2}{2 \cdot R}}{\frac{V_{rms}^2}{R}} = \frac{\frac{1}{2} \cdot \left(\frac{4 \cdot V_A}{\pi}\right)^2 + \frac{1}{2} \cdot \left(\frac{4 \cdot V_A}{3 \cdot \pi}\right)^2 + \frac{1}{2} \cdot \left(\frac{4 \cdot V_A}{5 \cdot \pi}\right)^2}{V_A^2} = \frac{8}{\pi^2} \cdot \left(1 + \frac{1}{9} + \frac{1}{25}\right) = 0.933$$

The first three non zero harmonics carry 93.3% of the total average power.

13-29 For waveform in Fig. P13-29



$$V_{rms} = \sqrt{\frac{1}{T_0}\cdot\int_0^{\frac{T_0}{2}}\left[V_A\cdot\left(2\cdot\frac{t}{T_0}\right)\right]^2 dt} = V_A\cdot\sqrt{\frac{4}{3}\cdot\left(\frac{t}{T_0}\right)^3\Bigg|_0^{\frac{T_0}{2}}} = \frac{V_A}{\sqrt{6}} \quad \text{hence} \quad P = \frac{V_{rms}^2}{R} = \frac{1}{6}\cdot\frac{V_A^2}{R}$$

$$V_n = \sqrt{(a_n)^2 + (b_n)^2} = \frac{V_A}{n\cdot\pi}\cdot\sqrt{\left(\frac{\cos(n\cdot\pi) - 1}{n\cdot\pi}\right)^2 + (\cos(n\cdot\pi))^2} = \frac{V_A}{n\cdot\pi}\cdot\sqrt{\left(\frac{\cos(n\cdot\pi) - 1}{n\cdot\pi}\right)^2 + 1}$$

In general $P = \frac{(V_0)^2}{R} + \sum_{n=1}^{\infty}\frac{(V_n)^2}{2\cdot R}$ $V_0 = \frac{V_A}{4}$ The amplitudes of the first three nonzero ac terms are:

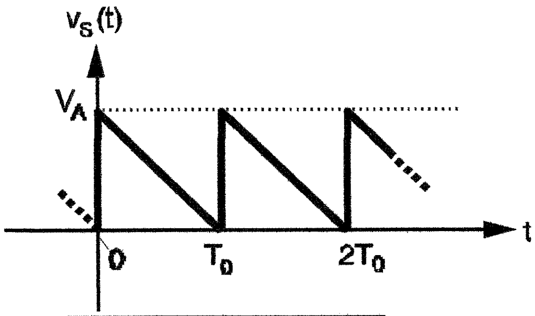
$V_1 = \frac{V_A}{\pi}\cdot\sqrt{\frac{4}{\pi^2} + 1}$ $V_2 = \frac{V_A}{2\cdot\pi}$ $V_3 = \frac{V_A}{3\cdot\pi}\cdot\sqrt{\frac{4}{9\cdot\pi^2} + 1}$ hence the required power fraction is

$$\frac{(V_0)^2 + \frac{(V_1)^2}{2} + \frac{(V_2)^2}{2} + \frac{(V_3)^2}{2}}{V_{rms}^2} = \frac{\left(\frac{1}{4}\right)^2 + \frac{1}{2}\cdot\left(\frac{1}{\pi}\cdot\sqrt{\frac{4}{\pi^2} + 1}\right)^2 + \frac{1}{2}\cdot\left(\frac{1}{2\cdot\pi}\right)^2 + \frac{1}{2}\cdot\left[\frac{1}{3\cdot\pi}\cdot\sqrt{\frac{4}{(3\cdot\pi)^2} + 1}\right]^2}{\left(\frac{1}{\sqrt{6}}\right)^2}$$

=0.913

91.3% carried by the dc plus first three nonzero ac components

13-30 For waveform in Fig. P13-30



$$V_{rms} = \sqrt{\frac{1}{T_0}\cdot\int_0^{T_0}\left[V_A\cdot\left(1-\frac{t}{T_0}\right)\right]^2 dt} = V_A\cdot\sqrt{\frac{-1}{3}\cdot\left(1-\frac{t}{T_0}\right)^3}\Bigg|_0^{T_0} = \frac{V_A}{\sqrt{3}} \text{ hence}$$

$$P = \frac{V_{rms}^2}{R} = \frac{V_A^2}{3\cdot R} \quad V_0 = \frac{1}{T_0}\cdot\int_0^{T_0} V_A\cdot\left(1-\frac{t}{T_0}\right) dt = \frac{V_A}{2} \quad P_0 = \frac{(V_0)^2}{R} = \frac{V_A^2}{4\cdot R} \quad \frac{P_0}{P} = \frac{3}{4}$$

$$\frac{P_{ac}}{P} = 1 - \frac{P_0}{P_{total}} = \frac{1}{4}$$ **75% carried by the dc component and 25% by the ac components**

13-31 $K := 10 \quad \omega_C := 500 \quad T(s) := \frac{K}{\frac{s}{\omega_C}+1} \quad x(t) = 6\cdot\sin(1000\cdot t) + 2\cos(3000\cdot t)$

$$y_{ss}(t) = Y_1\cdot\sin(1000\cdot t + \theta_1) + Y_2\cdot\cos(1300\cdot t + \theta_2)$$

$$Y_1 := 6\cdot|T(j\cdot 1000)| \quad Y_2 := 2\cdot|T(j\cdot 3000)| \quad Y_{rms} := \sqrt{\frac{Y_1^2}{2}+\frac{Y_2^2}{2}} \quad Y_{rms} = 19.116$$

13-32 $i(t) = \frac{2}{\pi}\cdot\left[\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\cdot\sin(n\cdot\omega_0\cdot t)\right] \quad P = \sum_{n=1}^{\infty}\frac{(i_n)^2}{2}\cdot R = \sum_{n=1}^{\infty}\left[\frac{2}{\pi}\cdot\frac{(-1)^n}{n}\right]^2\cdot\frac{R}{2}$

$k := 1,2..5 \quad p_k := \sum_{n=1}^{k}\left[\frac{2}{\pi}\cdot\frac{(-1)^n}{n}\right]^2\cdot\frac{10}{2}$

k =	p_k =
1	2.026
2	2.533
3	2.758
4	2.885
5	2.966

<---Estimate from the first three harmonics is 2.76 W

13-33 $v(t) = V_A \cdot \left[\frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cdot \cos(n \cdot \omega_0 \cdot t) \right]$ $\qquad$ $V_{rms} = V_A \cdot \sqrt{\left(\frac{\pi^2}{12}\right)^2 + \frac{1}{2} \cdot \sum_{n=1}^{\infty} \frac{1}{n^2}}$

$m := 1, 2 .. 3$ $\qquad$ $S(m) := \sqrt{\left(\frac{\pi^2}{12}\right)^2 + \frac{1}{2} \cdot \sum_{n=1}^{m} \frac{1}{n^2}}$ $\qquad$ $S(\infty) = \sqrt{\left(\frac{\pi^2}{12}\right)^2 + \frac{\pi^2}{12}} = 1.224$

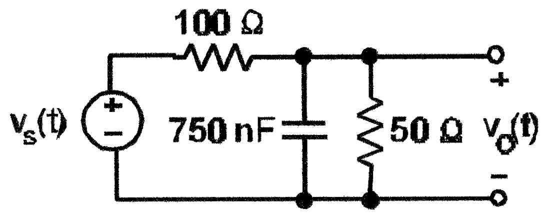
so the true rms value $V_{rms} = 1.224 \cdot V_A$

m =	S(m) =
1	1.085
2	1.141
3	1.165

<---m = 3 yields an estimate $V_{rms} = 1.165 \cdot V_A$ which is within 5% .

13-34 Using voltage division

$$T(s) = \frac{\dfrac{1}{750 \cdot 10^{-9} \cdot s + \frac{1}{50}}}{\dfrac{1}{750 \cdot 10^{-9} \cdot s + \frac{1}{50}} + 100}$$



$$T(s) := \frac{40000}{3 \cdot (40000 + s)}$$

For input $v_S(t) = 40 \cdot \cos(2 \cdot \pi \cdot 3000 \cdot t) + 15 \cdot \sin(2 \cdot \pi \cdot 9000 \cdot t)$ The amplitudes of the steady-state output are $V_1 := 40 \cdot |T(j \cdot 2 \cdot \pi \cdot 3000)|$ $\quad V_3 := 15 \cdot |T(j \cdot 2 \cdot \pi \cdot 9000)|$ hence the average power delivered to $R := 50$

is $P := \frac{(V_1)^2}{2 \cdot R} + \frac{(V_3)^2}{2 \cdot R}$ $\qquad P = 1.538$ W.

13-35 See Problem 13-34 above for circuit and analysis leading to $T(s) := \frac{\quad}{3 \cdot (40000 + s)}$

$T_0 := 50 \cdot \pi \cdot 10^{-3}$ $\qquad \omega_0 := 2 \cdot \frac{\pi}{T_0}$ $\qquad \omega_0 = 40$

For v(t) in Prob. 13-33 $v_S(t) = V_A \cdot \left[\frac{\pi^2}{3 \cdot 12} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cdot \cos(n \cdot \omega_0 \cdot t) \right]$ with $V_A := 4$ $\quad n := 1, 2 .. 5$

$V_0 := \frac{V_A \cdot \pi^2}{12} \cdot |T(0)|$ $\quad V_n := \frac{V_A}{n^2} \cdot (-1)^n \cdot |T(j \cdot n \cdot \omega_0)|$ the average power to the 50-Ω resistor is

$m := 1, 2 .. 4$

$$P(m) := \frac{1}{50} \cdot \left[(V_0)^2 + \sum_{n=1}^{m} \frac{(V_n)^2}{2} \right]$$

2	0.043
3	0.043
4	0.043

<---P is approximately 0.043 W.

13-36 (a) $n := 1,3..7 \quad A_n := \frac{14}{n} \quad n := 1,3..7 \quad \phi_n := 0 \quad \omega_0 := 10$

$n \cdot \omega_0 =$	$A_n =$	$\phi_n =$
10	14	0
30	4.667	0
50	2.8	0
70	2	0

$x(t) = 14 \cdot \cos(10 \cdot t) + 4.667 \cdot \cos(30 \cdot t) + 2.8 \cdot \cos(50 \cdot t) + 2 \cdot \cos(70 \cdot t)$

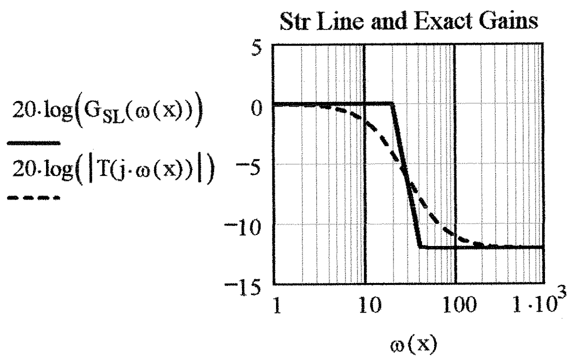
The straight line gain and phase of T(s) are

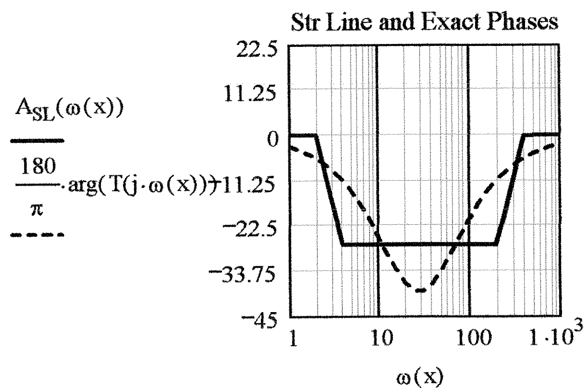
$$G_{SL}(\omega) := \begin{cases} 1 & \text{if } 0 < \omega < 20 \\ \dfrac{400}{\omega^2} & \text{if } 20 < \omega < 40 \\ 0.25 & \text{if } 40 < \omega \end{cases}$$

$$A_{SL}(\omega) := \begin{cases} 0 & \text{if } 0 < \omega < 2 \\ -90 \cdot \log\left(\dfrac{\omega}{2}\right) & \text{if } 2 \le \omega < 4 \\ -90 \cdot \log(2) & \text{if } 4 \le \omega < 200 \\ 90 \cdot \log\left(\dfrac{\omega}{400}\right) & \text{if } 200 \le \omega < 400 \\ 0 & \text{if } 400 < \omega \end{cases}$$

Double Pole at 20 and double zero at 40, hence $T(s) := \dfrac{0.25(s+40)^2}{(s+20)^2}$

comparing the straight line and exact gains $x := 0, 0.1..3 \quad \omega(x) := 10^x$





For the straight line plots the output amplitudes and phase angles are:

$B_n := A_n \cdot G_{SL}(n \cdot \omega_0) \quad \theta_n := A_{SL}(n \cdot \omega_0) \qquad y_{SL}(t) = \sum_{n=1}^{\infty} B_n \cdot \cos(n \cdot \omega_0 \cdot t + \theta_n) \qquad n := 1,3..7$

$n \cdot \omega_0 =$	$A_n =$	$G_{SL}(n \cdot \omega_0) =$	$B_n =$	$\phi_n =$	$\theta_n =$
10	14	1	14	0	-27.093
30	4.6667	0.4444	2.0741	0	-27.093
50	2.8	0.25	0.7	0	-27.093
70	2	0.25	0.5	0	-27.093

Straight-line output

$y_{SL}(t) = 14 \cdot \cos(10 \cdot t - 27.1) + 2.07 \cdot \cos(30 \cdot t - 27.1^o) + 0.7 \cdot \cos(50 \cdot t - 27.1^o) + 0.5 \cdot \cos(70 \cdot t - 27.1^o)$

13-36 Continued
(b)

$C_n := A_n \cdot \left|T(j \cdot n \cdot \omega_0)\right| \qquad \psi_n := \frac{180}{\pi} \cdot \arg(T(j \cdot n \cdot \omega_0)) \qquad y_{EX}(t) = \sum_{n=1}^{\infty} C_n \cdot \cos(n \cdot \omega_0 \cdot t + \psi_n)$

$n \cdot \omega_0 =$	$A_n =$	$B_n =$	$C_n =$	$\phi_n =$	$\theta_n =$	$\psi_n =$
10	14	14	11.9	0	-27.093	-25.058
30	4.667	2.074	2.244	0	-27.093	-38.88
50	2.8	0.7	0.99	0	-27.093	-33.717
70	2	0.5	0.613	0	-27.093	-27.599

Input
$x(t) = 14 \cdot \cos(10 \cdot t) + 4.667 \cdot \cos(30 \cdot t) + 2.8 \cdot \cos(50 \cdot t) + 2 \cdot \cos(70 \cdot t)$
Straight-line output
$y_{SL}(t) = 14 \cdot \cos\left(10 \cdot t - 27.1^{o}\right) + 2.07 \cdot \cos\left(300 \cdot t - 27.1^{o}\right) + 0.7 \cdot \cos\left(500 \cdot t - 27.1^{o}\right) + 0.5 \cdot \cos\left(700 \cdot t - 27.1^{o}\right)$
$y_{EX}(t) = 11.9 \cdot \cos\left(10 \cdot t - 25.1^{o}\right) + 2.24 \cdot \cos\left(300 \cdot t - 38.9^{o}\right) + 0.99 \cdot \cos\left(500 \cdot t - 33.7^{o}\right) + 0.613 \cdot \cos\left(700 \cdot t - 27.6^{o}\right)$

(c) The straight-line amplitudes generally agree within 20%. The straight-line phase is the same for all four harmonics which is not a very good approximation to the exact phase angles.

13-37 (a) For a periodic impulse train: $x(t) = T_0 \cdot \sum_{n=-\infty}^{\infty} \delta(t - n \cdot T_0) \qquad T_0 := 1$

$$a_0 = \frac{1}{T_0} \cdot \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t)\, dt = \frac{1}{T_0} \cdot \lim_{x \to 0} \int_{-x}^{x} T_0 \cdot \delta(t)\, dt = 1$$

$$a_n = \frac{2}{T_0} \cdot \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t) \cdot \cos\left(2 \cdot \pi \cdot n \cdot \frac{t}{T_0}\right) dt = \frac{1}{T_0} \cdot \lim_{x \to 0} \int_{-x}^{x} T_0 \cdot \delta(t) \cdot \cos\left(2 \cdot \pi \cdot n \cdot \frac{t}{T_0}\right) dt = 1$$

$$b_n = \frac{2}{T_0} \cdot \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t) \cdot \sin\left(2 \cdot \pi \cdot n \cdot \frac{t}{T_0}\right) dt = \frac{1}{T_0} \cdot \lim_{x \to 0} \int_{-x}^{x} T_0 \cdot \delta(t) \cdot \sin\left(2 \cdot \pi \cdot n \cdot \frac{t}{T_0}\right) dt = 0$$

$x(t) = 1 + \sum_{n=1}^{\infty} \cos\left(2 \cdot \pi \cdot n \cdot \frac{t}{T_0}\right)$ **hence** $n := 1, 2 .. 5 \quad A_n := 1 \quad \phi_n := 0$

13-37 Continued

(b) If $h(t) = \frac{1}{T_0} \cdot \exp\left(-\frac{t}{T_0}\right) \cdot u(t)$ then $T(s, T_0) := \frac{1}{T_0 \cdot s + 1}$

The amplitude and phase angles of the steady-state output for or a periodic impulse train input are:

$$B_n := A_n \cdot \left| T\left(\frac{j \cdot 2 \cdot \pi \cdot n}{T_0}, T_0\right) \right| \qquad \theta_n := \phi_n + \frac{180}{\pi} \cdot \arg\left(T\left(\frac{j \cdot 2 \cdot \pi \cdot n}{T_0}, T_0\right)\right)$$

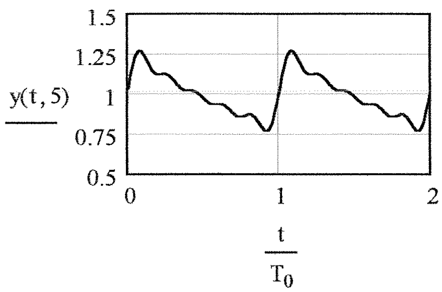
$n =$	$A_n =$	$B_n =$	$\phi_n =$	$\theta_n =$
1	1	0.157	0	-80.957
2	1	0.079	0	-85.45
3	1	0.053	0	-86.963
4	1	0.04	0	-87.721
5	1	0.032	0	-88.177

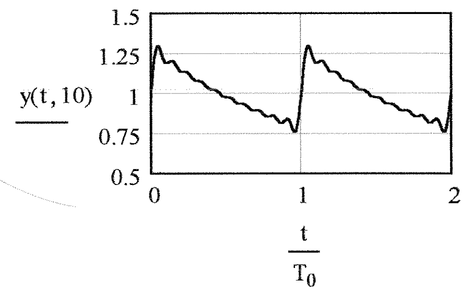
In general:

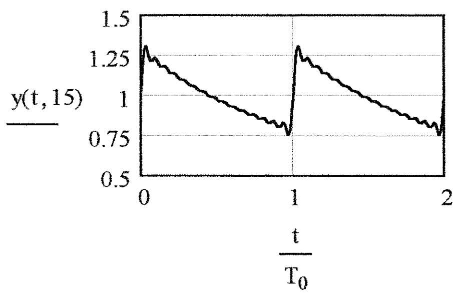
$$B_n := \left| \frac{1}{1 + j \cdot 2 \cdot \pi \cdot n} \right| \qquad \theta_n := \arg\left(\frac{1}{1 + j \cdot 2 \cdot \pi \cdot n}\right)$$

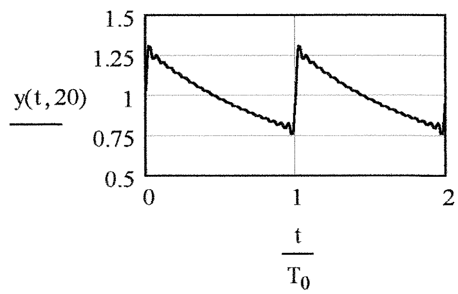
(c)
$$y(t) = \sum_{n=0}^{\infty} \left| \frac{1}{1 + j \cdot 2 \cdot \pi \cdot n} \right| \cdot \cos\left(2 \cdot \pi \cdot n \cdot \frac{t}{T_0} + \arg\left(\frac{1}{1 + j \cdot 2 \cdot \pi \cdot n}\right)\right)$$

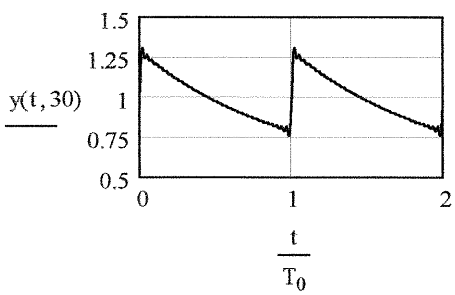
(d)
$$y(t, m) := \sum_{n=0}^{m} \left| \frac{1}{1 + j \cdot 2 \cdot \pi \cdot n} \right| \cdot \cos\left(2 \cdot \pi \cdot n \cdot \frac{t}{T_0} + \arg\left(\frac{1}{1 + j \cdot 2 \cdot \pi \cdot n}\right)\right) \qquad t := 0, \frac{T_0}{100} .. 2 \cdot T_0$$

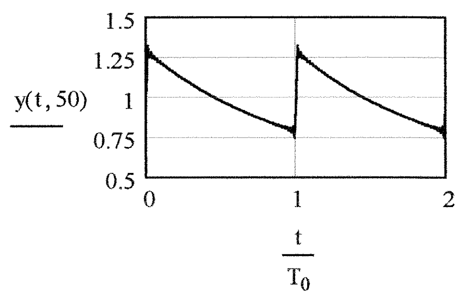












The steady-state output is a periodic exponential that does not decay to zero prior to the next impulse input.

13-38 Since the Butterworth gain decrease monotonically with frequency the overall requirements will be meet if the gain at the 3rd harmonic is at least -3 db and the gain at the 4th harmonic is less than -20 dB. The design constraints are:

$$20\cdot\log\left[\frac{1}{\sqrt{1+\left(\frac{3\cdot\omega_0}{\omega_C}\right)^{2\cdot m}}}\right] > -3 \qquad 20\cdot\log\left[\frac{1}{\sqrt{1+\left(\frac{4\cdot\omega_0}{\omega_C}\right)^{2\cdot m}}}\right] < -20$$

If we choose $\omega_C = 3\cdot\omega_0$ then $20\cdot\log\left[\frac{1}{\sqrt{1+\left(\frac{3\cdot\omega_0}{3\cdot\omega_0}\right)^{2\cdot m}}}\right] = -3.01 < -3$ regardless of m.

For $\omega_C = 3\cdot\omega_0$ the gain at the 3rd harmonic is slightly below - 3dB. So we try $\omega_C = 3.05\cdot\omega_0$ in which case the gain contraint at the 4th harmonic becomes

$$\frac{1}{\sqrt{1+\left(\frac{4}{3.05}\right)^{2\cdot m}}} < 10^{-1} \quad \text{or} \quad \left(\frac{4}{3.05}\right)^{2\cdot m} > 10^2 - 1 \quad \text{or} \quad m > \frac{1}{2}\cdot\frac{\log\left(10^2-1\right)}{\log\left(\frac{4}{3.05}\right)} \quad \text{or} \quad m > 8.473$$

Since m must be an integer choose m = 9. Checking both design requirements yields

$$20\cdot\log\left[\frac{1}{\sqrt{1+\left(\frac{3\cdot\omega_0}{3.05\cdot\omega_0}\right)^{18}}}\right] = -2.412 > -3 \qquad 20\cdot\log\left[\frac{1}{\sqrt{1+\left(\frac{4\cdot\omega_0}{3.05\cdot\omega_0}\right)^{18}}}\right] = -21.23 < -20$$

13-39 For $T_0 = 2\cdot\pi\cdot 10^{-6}$ $\quad \omega_0 := 10^6$ To be centered at the 7th harmonic $\omega_C := 7\cdot\omega_0$

and the transfer functiion is $T(s,Q) := \dfrac{\frac{7\cdot\omega_0}{Q}\cdot s}{s^2 + \frac{7\cdot\omega_0}{Q}\cdot s + (7\cdot\omega_0)^2}$ then since the waveform has even symmetry all harmonics are present and the two adjacent harmonics are the 6th and 8th. Try

$Q := 10, 20 .. 60$

$Q =$	$20\cdot\log\left(\left\lvert T(j\cdot 6\cdot\omega_0, Q)\right\rvert\right) =$	$20\cdot\log\left(\left\lvert T(j\cdot 8\cdot\omega_0, Q)\right\rvert\right) =$	
10	-10.245	-9.125	
20	-15.946	-14.727	
30	-19.406	-18.167	
40	-21.883	-20.637	<---Q = 40 is the minimum
50	-23.811	-22.562	
60	-25.389	-24.138	

13-40 For a 1 MHz sq. wave $f_0 := 10^6$ $V_n = \frac{4 \cdot V_A}{n \cdot \pi}$ where $K = \frac{(V_n)^2}{2 \cdot R} = \left(\frac{4 \cdot V_A}{\pi}\right)^2 \cdot \frac{1}{2 \cdot R}$ at

f = 1 MHz $10 \cdot \log(P_1) = 10 \cdot \log(K) = 12.1$ hence $K := 10^{1.21}$ $K = 16.218$ $n := 1, 3 .. 11$ $P_n := \frac{K}{n^2}$

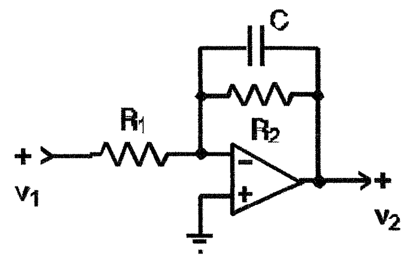
$n \cdot f_0 \cdot 10^{-6} =$	$P_n =$	$10 \cdot \log(P_n) =$
1	16.218	12.1
3	1.802	2.558
5	0.649	-1.879
7	0.331	-4.802
9	0.2	-6.985
11	0.134	-8.728

<---These results agree with the measurements so the analyzer is properly calibrated.

CHAPTER 14, Both Versions

14-1 $T(s) = \left[\frac{K}{\left(\frac{s}{\omega_C}\right)+1}\right]$ $\omega_C := 2\cdot\pi\cdot 1500$ $\omega_C = 9.425\times 10^3$ $K := 10^{\frac{26}{20}}$ $K = 19.953$

For the ckt at right $T(s) = \frac{-R_2}{R_1}\cdot\frac{1}{R_2\cdot C\cdot s+1}$

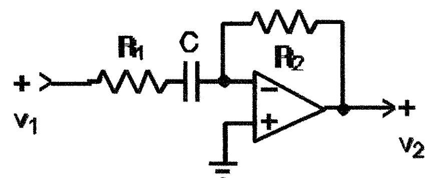


Let $R_1 := 10^4$ then $R_2 := K\cdot R_1$ $C := \frac{1}{\omega_C\cdot R_2}$

$R_2 = 1.995\times 10^5$ $C = 5.318\times 10^{-10}$

14-2 $T(s) = \frac{K\cdot s}{s+\omega_C}$ $\omega_C := 2\cdot\pi\cdot 500$ $K := 10^{(0.5)}$ $K = 3.162$ $\omega_C = 3.142\times 10^3$

For the ckt at right $T(s) = \frac{-R_2\cdot C\cdot s}{R_1\cdot C\cdot s+1}$



Let $R_1 := 10^4$ then $R_2 := K\cdot R_1$ $C := \frac{1}{\omega_C\cdot R_1}$

$R_2 = 3.162\times 10^4$ $C = 3.183\times 10^{-8}$ Checking

$T(s) := \frac{-R_2\cdot C\cdot s}{R_1\cdot C\cdot s+1}$ $20\cdot\log(|T(j\cdot 100\cdot\omega_C)|) = 10$ $20\cdot\log(|T(j\cdot\omega_C)|) = 6.99$

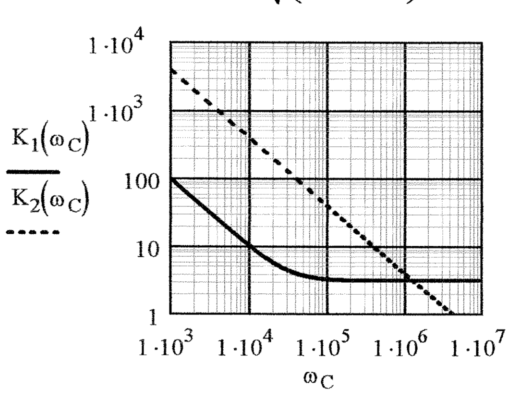
14-3 $T(s) = \frac{K}{\frac{s}{\omega_C}+1}$ ω_C = unk $\omega_1 := 2\cdot\pi\cdot 5000$ $\omega_2 := 2\cdot\pi\cdot 2\cdot 10^6$

K = unk **Design constraints are:**

$20\cdot\log(|T(j\cdot\omega_1)|) \geq 10$ and $20\cdot\log(|T(j\cdot\omega_2)|) \leq -10$

Then $K \geq \left[10^{\frac{10}{20}}\cdot\sqrt{\left(\frac{\omega_1}{\omega_C}\right)^2+1}\right] = 3.166$ $K \leq \left[10^{\frac{-10}{20}}\cdot\sqrt{\left(\frac{\omega_2}{\omega_C}\right)^2+1}\right] = 6.332$ Thus $3.166 \leq K \leq 6.322$

Define $K_1(\omega_C) := \sqrt{10}\cdot\sqrt{\left(2\cdot\pi\cdot\frac{5000}{\omega_C}\right)^2+1}$ $K_2(\omega_C) := \frac{1}{\sqrt{10}}\cdot\sqrt{\left(2\cdot\pi\cdot\frac{2\cdot 10^6}{\omega_C}\right)^2+1}$ $\omega_C := 10^3, 2\cdot 10^3 .. 10^7$

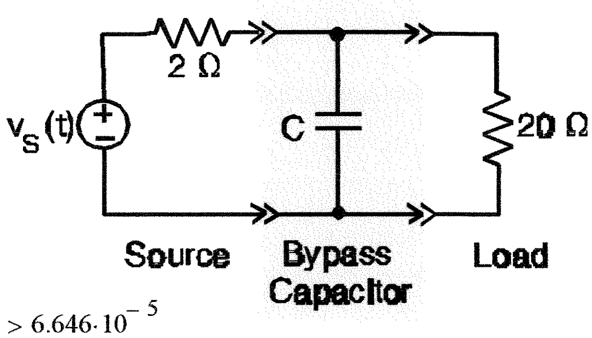


$\omega_C = 10^6$

K must fall below the dotted line but above the solid curve. Hence, for example, we could choose K=5 & Other solutions are possible.

14-4

$$T_V(s) = \frac{\frac{1}{C\cdot s + (20^{-1})}}{\frac{1}{C\cdot s + (20^{-1})} + 2} = \frac{10}{11 + 20\cdot C\cdot s}$$



A_{out} = $\left|T_V(j\cdot 4000\cdot\pi)\right|\cdot 0.02 < 0.01$ hence

$\left|T_V(j\cdot 4000\cdot\pi)\right| < 0.5$ Requires

$\frac{10}{\sqrt{11^2 + (20\cdot C\cdot\pi\cdot 4000)^2}} < 0.5$. solving for C yields $C > 6.646\cdot 10^{-5}$

14-5 The required T(s) is $T(s,\omega_C) = \frac{5}{\frac{s}{\omega_C} + 1}$ The design requirement is

$20\cdot\log\left(\left|T\left(j\cdot\pi\cdot 4\cdot 10^4,\omega_C\right)\right|\right) < -20$ which means $\left|\frac{5}{\sqrt{\left(\frac{\pi\cdot 4\cdot 10^4}{\omega_C}\right)^2 + 1}}\right| < 10^{-1}$

which yields

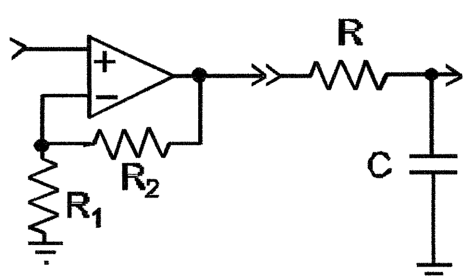
$\omega_C < 2.514\cdot 10^3$ let $\omega_C = 2000$ then $T(s) = \frac{5}{\frac{s}{2000} + 1}$

For the circuit at right $T(s) = \frac{\frac{R_1+R_2}{R_1}}{R\cdot C\cdot s + 1}$

$\frac{R_1 + R_2}{R_1} = 5$ and $R\cdot C = (2000)^{-1}$

Let $R_1 := 10^4$ $R := 10^4$ then

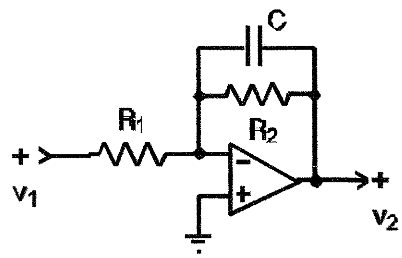
$R_2 := 40\cdot 10^3$ $C := 50\cdot 10^{-9}$



Checking $20\cdot\log\left(\left|\frac{\frac{R_1+R_2}{R_1}}{R\cdot C\cdot j\cdot\pi\cdot 4\cdot 10^4 + 1}\right|\right) = -21.985$ <--- gain < -20 dB

14-6 $T(s) = \dfrac{8 \cdot 10^6}{(s+500)\cdot(s+2000)} = \left(\dfrac{-4}{\dfrac{s}{500}+1}\right)\cdot\left(\dfrac{-2}{\dfrac{s}{2000}+1}\right) = T_1(s)\cdot T_2(s)$

For the circuit at right $T(s) = \dfrac{-R_2}{R_1}\cdot\dfrac{1}{R_2\cdot C\cdot s+1}$



This circuit can be used for both $T_1(s)$ and $T_2(s)$

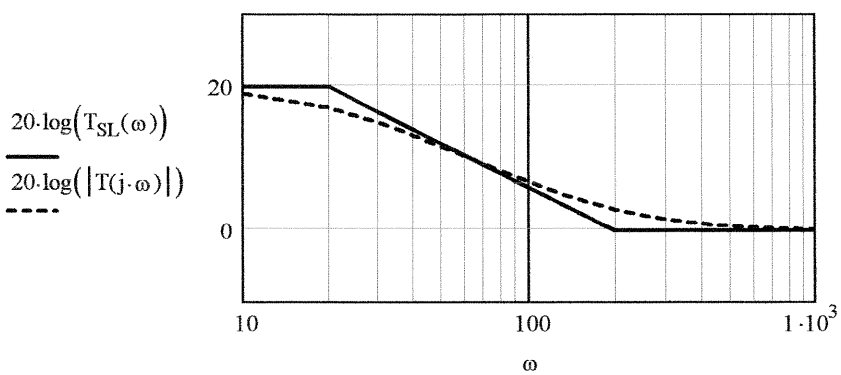
For $T_1(s)$ $R_2 = 4\cdot R_1$ $R_2\cdot C = (500)^{-1}$

Let R_1 = 10 kΩ then R_2 = 40 kΩ and C = 50 nF.

For $T_2(s)$ $R_2 = 2\cdot R_1$ $R_2\cdot C = (2000)^{-1}$

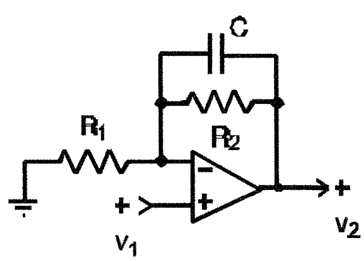
Let R_1 = 10 kΩ then R_2 = 20 kΩ and C = 25 nF

14-7(a) $T_{SL}(\omega) := \begin{cases} 10 & \text{if } \omega < 20 \\ \dfrac{200}{\omega} & \text{if } 20 \le \omega < 200 \\ 1 & \text{if } 200 \le \omega \end{cases}$ $T(s) := \dfrac{10\cdot\left(\dfrac{s}{200}+1\right)}{\left(\dfrac{s}{20}+1\right)}$ $\omega := 10, 20 .. 1000$



(b) Use a non-inverting amplifier

$T(s) = \dfrac{\dfrac{s}{20}+10}{\dfrac{s}{20}+1} = \dfrac{Z_1+Z_2}{Z_1} = \dfrac{Y_1+Y_2}{Y_2}$



$Y_2 = 1 + \dfrac{s}{20} = \dfrac{1}{R_2} + C\cdot s$ $Y_1 = 9 = \dfrac{1}{R_1}$

$R_1 = \dfrac{1}{9}$ $R_2 = 1$ $C = \dfrac{1}{20}$

Let $k_m := 9\cdot 10^4$ then $R_1 := \dfrac{k_m}{9}$ $R_2 := k_m$ $C := \dfrac{1}{20\cdot k_m}$

$R_1 = 1\times 10^4$ $R_2 = 9\times 10^4$

$C = 5.556\times 10^{-7}$

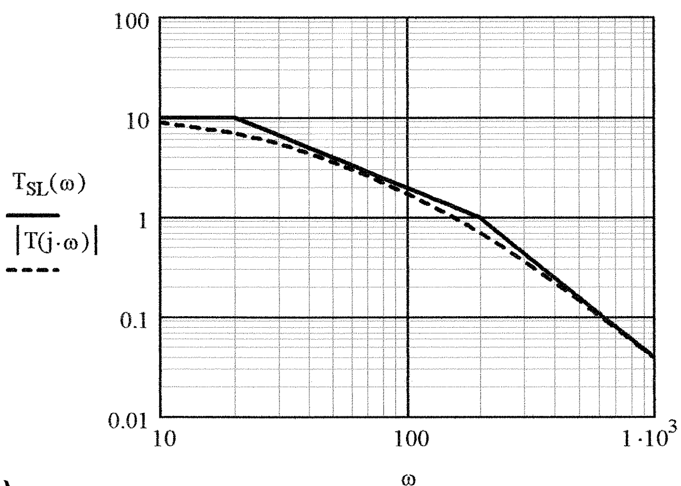
14-7 Continue Checking:

$$T(s) = \frac{R_1 + R_2}{R_1} \cdot \frac{\left(\frac{R_1 \cdot R_2 \cdot C \cdot s}{R_1 + R_2} + 1\right)}{R_2 \cdot C \cdot s + 1} \qquad \frac{R_1 + R_2}{R_1} = 10 \qquad \frac{R_1 + R_2}{R_1 \cdot R_2 \cdot C} = 200 \qquad \frac{1}{R_2 \cdot C} = 20$$

14-8(a)

$$T_{SL}(\omega) := \begin{cases} 10 & \text{if } \omega < 20 \\ \frac{10 \cdot 20}{\omega} & \text{if } 20 \le \omega < 200 \\ \left(\frac{200}{\omega}\right)^2 & \text{if } 200 \le \omega \end{cases} \qquad T(s) := \frac{10}{\left(\frac{s}{20} + 1\right) \cdot \left(\frac{s}{200} + 1\right)}$$

$$\omega := 10, 20 .. 1000$$



(b)

$$T(s) = \left(\frac{-2}{\frac{s}{20} + 1}\right) \cdot \left(\frac{-5}{\frac{s}{200} + 1}\right) = T_1 \cdot T_2$$

$$T_1 = \frac{-Z_2}{Z_1} = \frac{-Y_1}{Y_2} = \frac{-2}{\frac{s}{20} + 1} \quad Y_1 = 2 \quad Y_2 = \frac{s}{20} + 1$$

$$T_2 = \frac{-Z_4}{Z_3} = \frac{-Y_3}{Y_4} = \frac{-5}{\frac{s}{200} + 1} \quad Y_3 = 5 \quad Y_4 = \frac{s}{200} + 1$$

Let $k_m := 10^4$ then

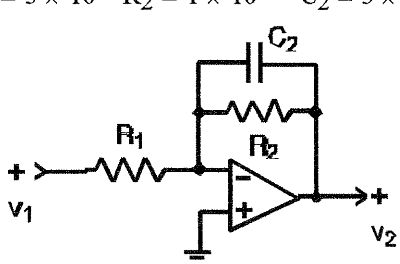
$$R_1 := \frac{k_m}{2} \qquad R_2 := k_m \qquad C_2 := \frac{1}{20 \cdot k_m}$$

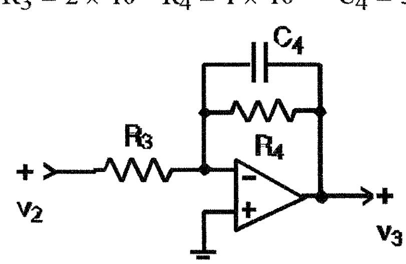
$$R_1 = 5 \times 10^3 \quad R_2 = 1 \times 10^4 \quad C_2 = 5 \times 10^{-6}$$

Let $k_m := 10^4$ then

$$R_3 := \frac{k_m}{5} \qquad R_4 := k_m \qquad C_4 := \frac{1}{200 \cdot k_m}$$

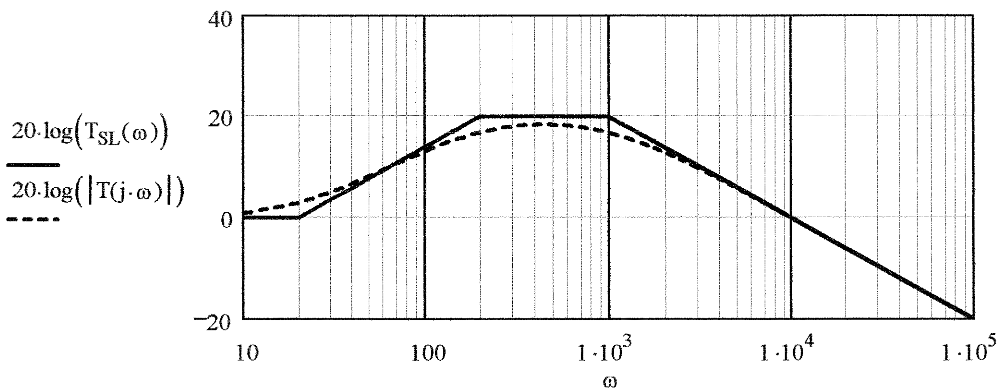
$$R_3 = 2 \times 10^3 \quad R_4 = 1 \times 10^4 \quad C_4 = 5 \times 10^{-7}$$





14-9(a) $T_{SL}(\omega) := \begin{cases} 1 \text{ if } \omega < 20 \\ \frac{\omega}{20} \text{ if } 20 \le \omega < 200 \\ 10 \text{ if } 200 \le \omega < 10^3 \\ \frac{10^4}{\omega} \text{ if } \left(10^3 \le \omega\right) \end{cases}$ $\quad T(s) := \dfrac{\left(\frac{s}{20}+1\right)}{\left(\frac{s}{200}+1\right)\cdot\left(\frac{s}{10^3}+1\right)} = \left(\dfrac{\frac{s}{20}+1}{\frac{s}{200}+1}\right)\cdot\left(\dfrac{1}{\frac{s}{10^3}+1}\right)$

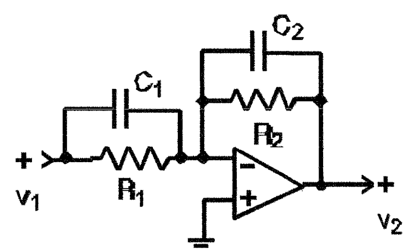
$\omega := 10, 20 .. 10^5$



Use a two-stage design $\quad T(s) = T_1(s)\cdot T_2(s)$

(b) First stage use an inverting amplifier

$$T(s) = -\frac{\frac{s}{20}+1}{\frac{s}{200}+1} = -\frac{Y_1}{Y_2} \quad Y_1 = 1 + \frac{s}{20} = \frac{1}{R_1} + C_1\cdot s$$



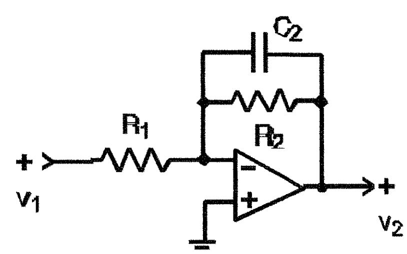
$Y_2 = 1 + \dfrac{s}{200} = \dfrac{1}{R_2} + C_2\cdot s$ Let $k_m := 10^4$ then

$$R_1 := k_m \quad R_2 := k_m \quad C_1 := \frac{1}{20\cdot k_m} \quad C_2 := \frac{1}{200\cdot k_m}$$

$$R_1 = 1\times10^4 \quad R_2 = 1\times10^4 \quad C_1 = 5\times10^{-6} \quad C_2 = 5\times10^{-7}$$

Second stage use an inverting amplifier

$$T(s)\frac{1}{\frac{s}{10^3}+1} = -\frac{Y_1}{Y_2} \quad Y_1 = 1 = \frac{1}{R_1}$$

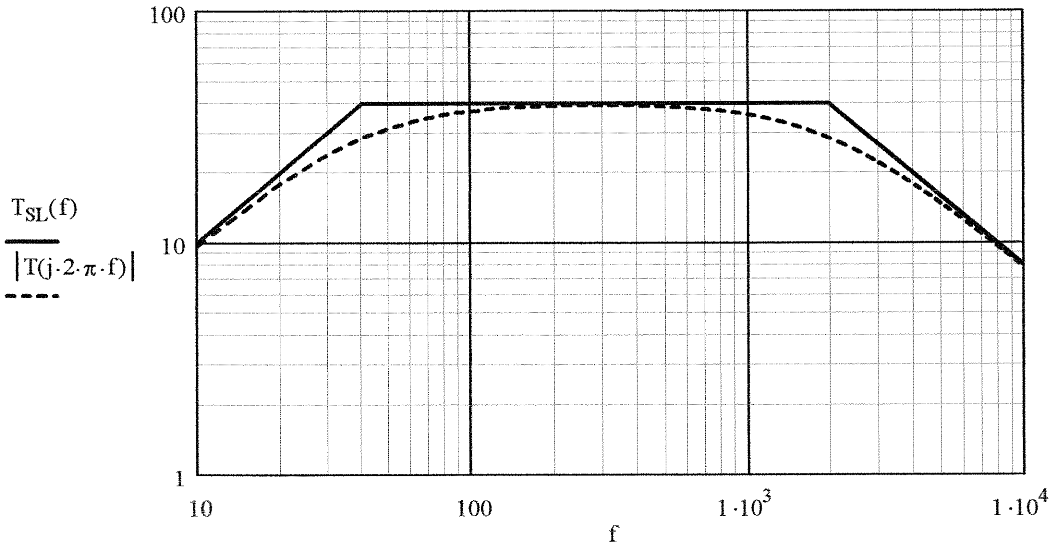


$Y_2 = 1 + \dfrac{s}{10^3} = \dfrac{1}{R_2} + C_2\cdot s$ Let $k_m := 10^4$ then

$$R_1 := k_m \quad R_2 := k_m \quad C_2 := \frac{1}{10^3\cdot k_m}$$

$$R_1 = 1\times10^4 \quad R_2 = 1\times10^4 \quad C_2 = 1\times10^{-7}$$

14-10(a) $T_{SL}(f) := \begin{cases} f & \text{if } f < 40 \\ 40 & \text{if } 40 \le f < 2000 \\ \dfrac{2000 \cdot 40}{f} & \text{if } 2000 \le f \end{cases}$ $\qquad T(s) := \dfrac{\dfrac{s}{2\cdot\pi}}{\left(\dfrac{s}{2\cdot\pi\cdot 40}+1\right)\cdot\left(\dfrac{s}{2\cdot\pi\cdot 2000}+1\right)}$ $\qquad f := 10, 20 .. 10000$

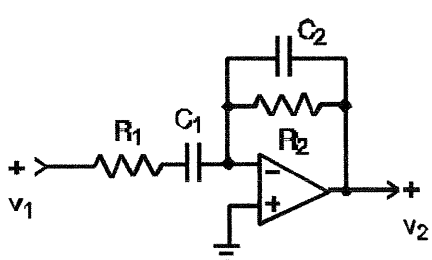


(b) $$T(s) = \frac{\dfrac{s}{2\cdot\pi}}{\left(\dfrac{s}{2\cdot\pi\cdot 40}+1\right)\cdot\left(\dfrac{s}{2\cdot\pi\cdot 2000}+1\right)} = \frac{-1}{\left(\dfrac{1}{40}+\dfrac{2\cdot\pi}{s}\right)\cdot\left(\dfrac{s}{2\cdot\pi\cdot 2000}+1\right)} = \frac{-Z_2}{Z_1} = \frac{-1}{Y_2\cdot Z_1}$$

$$Z_1 = \frac{1}{40}+\frac{2\cdot\pi}{s} = R_1 + \frac{1}{C_1\cdot s} \qquad Y_2 = \frac{s}{2\cdot\pi\cdot 2000}+1 = C_2\cdot s + \frac{1}{R_2}$$

Let $k_m := 10^5$ then $R_1 := \dfrac{k_m}{40}$ $\quad C_1 := \dfrac{1}{2\cdot\pi\cdot k_m}$ $\quad R_2 := k_m$ $\quad C_2 := \dfrac{1}{2\cdot\pi\cdot 2000\cdot k_m}$

$R_1 = 2.5\times10^3$ $\quad C_1 = 1.592\times10^{-6}$ $\quad R_2 = 1\times10^5$ $\quad C_2 = 7.958\times10^{-10}$

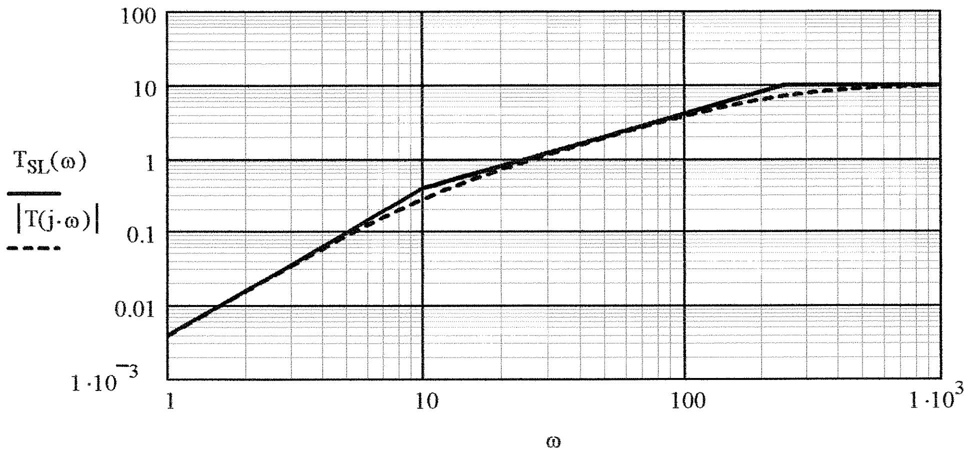


14-11(a)

$\omega := 1, 2 .. 10^3$

$$T_{SL}(\omega) := \begin{cases} \frac{\omega^2}{250} & \text{if } \omega < 10 \\ \frac{\omega}{25} & \text{if } 10 \le \omega < 25(\\ 10 & \text{if } 250 \le \omega \end{cases}$$

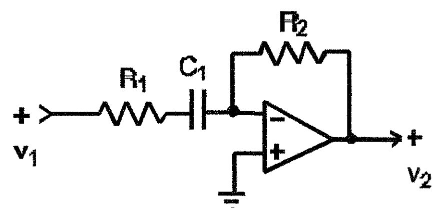
$$T(s) := \frac{\frac{s^2}{250}}{\left(\frac{s}{10}+1\right)\cdot\left(\frac{s}{250}+1\right)} = \left(\frac{\frac{s}{10}}{\frac{s}{10}+1}\right)\cdot\left(\frac{\frac{s}{25}}{\frac{s}{250}+1}\right)$$



Use a two-stage design $T(s) = T_1(s)\cdot T_2(s)$

(b) First stage use an inverting amplifier

$$T(s) = -\frac{1}{1+\frac{10}{s}} = -\frac{Z_2}{Z_1} \quad Z_1 = 1 + \frac{10}{s} = R_1 + \frac{1}{C_1\cdot s}$$



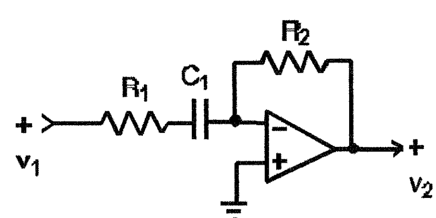
$Z_2 = 1 = R_2$ Let $k_m := 10^5$ then

$R_1 := k_m \quad R_2 := k_m \quad C_1 := \frac{1}{10\cdot k_m}$

$R_1 = 1 \times 10^5 \qquad R_2 = 1 \times 10^5 \qquad C_1 = 1 \times 10^{-6}$

Second stage use an inverting amplifier

$$T(s) = \frac{1}{\frac{1}{10}+\frac{25}{s}} = -\frac{Z_2}{Z_1} \quad Z_1 = \frac{1}{10} + \frac{25}{s} = R_1 + \frac{1}{C_1\cdot s}$$



$Z_2 = 1 = R_2$ Let $k_m := 10^5$ then

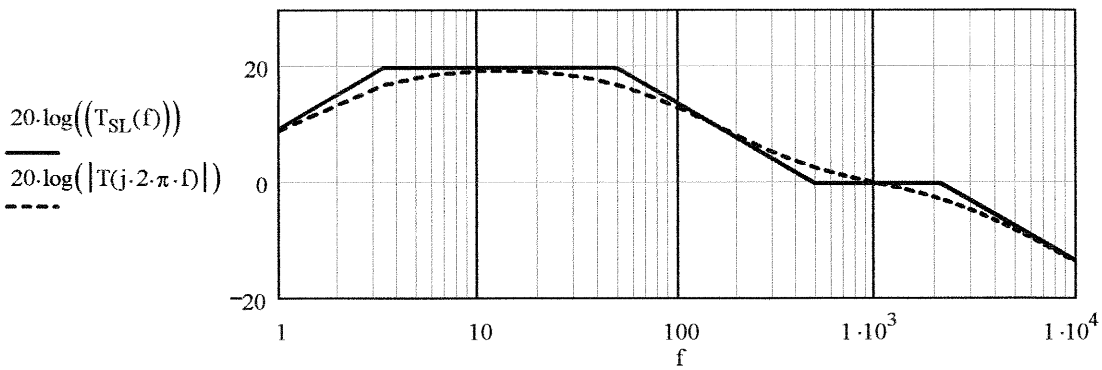
$R_1 := \frac{k_m}{10} \quad R_2 := k_m \quad C_1 := \frac{1}{25\cdot k_m}$

$R_1 = 1 \times 10^4 \qquad R_2 = 1 \times 10^5 \qquad C_1 = 4 \times 10^{-7}$

14-12 $T_{SL}(f) := \begin{cases} \dfrac{10\cdot f}{3.4} & \text{if } f < 3.4 \\ 10 & \text{if } 3.4 \le f < 50 \\ \dfrac{500}{f} & \text{if } 50 \le f < 500 \\ 1 & \text{if } 500 \le f < 2120 \\ \dfrac{2120}{f} & \text{if } 2120 \le f \end{cases}$

$$T(s) := \frac{\dfrac{s}{2\cdot\pi\cdot 0.34}\cdot\left(\dfrac{s}{2\cdot\pi\cdot 500}+1\right)}{\left(\dfrac{s}{2\cdot\pi\cdot 3.4}+1\right)\cdot\left(\dfrac{s}{2\cdot\pi\cdot 50}+1\right)\cdot\left(\dfrac{s}{2\cdot\pi\cdot 2120}+1\right)}$$

$f := 1, 3.4 .. 10000$



Single-stage design required.

Use an inverting amplifier

$$T(s) = \frac{(-1)\cdot\dfrac{s}{2\cdot\pi\cdot 0.34}\cdot\left(\dfrac{s}{2\cdot\pi\cdot 500}+1\right)}{\left(\dfrac{s}{2\cdot\pi\cdot 3.4}+1\right)\cdot\left(\dfrac{s}{2\cdot\pi\cdot 50}+1\right)\cdot\left(\dfrac{s}{2\cdot\pi\cdot 2120}+1\right)} = -\frac{\dfrac{\left(\dfrac{s}{2\cdot\pi\cdot 500}+1\right)}{\left(\dfrac{s}{2\cdot\pi\cdot 50}+1\right)\cdot\left(\dfrac{s}{2\cdot\pi\cdot 2120}+1\right)}}{\left(\dfrac{\dfrac{s}{2\cdot\pi\cdot 3.4}+1}{\dfrac{s}{2\cdot\pi\cdot 0.34}}\right)} = -\frac{Z_2}{Z_1}$$

$$Z_1 = \frac{\dfrac{s}{2\cdot\pi\cdot 3.4}+1}{\dfrac{s}{2\cdot\pi\cdot 0.34}} = 0.1 + \frac{2\cdot\pi\cdot 0.34}{s} = R_1 + \frac{1}{C_1\cdot s}$$

Let $k_m := 10^5$ $\quad R_1 := \dfrac{k_m}{10}$ $\quad C_1 := \dfrac{1}{k_m\cdot(2\cdot\pi\cdot 0.34)}$

$$Z_2 = \frac{\left(\dfrac{s}{2\cdot\pi\cdot 500}+1\right)}{\left(\dfrac{s}{2\cdot\pi\cdot 50}+1\right)\cdot\left(\dfrac{s}{2\cdot\pi\cdot 2120}+1\right)} = \frac{2120}{23}\cdot\frac{\pi}{(s+100\cdot\pi)} + \frac{7632}{23}\cdot\frac{\pi}{(s+4240\cdot\pi)}$$

$$Z_2 = \frac{1}{\left(\dfrac{23\cdot s}{2120\cdot\pi}+\dfrac{2300}{2120}\right)} + \frac{1}{\left(\dfrac{23\cdot s}{7632\cdot\pi}+\dfrac{23\cdot 4240}{7632}\right)} = \frac{1}{C_2\cdot s+\dfrac{1}{R_2}} + \frac{1}{C_3\cdot s+\dfrac{1}{R_3}}$$

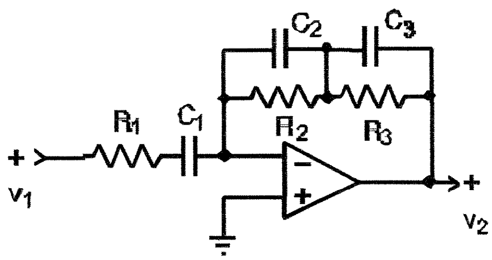
let $k_m := 10^5$ **then**

$C_2 := \dfrac{23}{2120\cdot\pi\cdot k_m}$ $\quad R_2 := \dfrac{2120}{2300}\cdot k_m$ $\quad R_3 := \dfrac{7632}{23\cdot 4240}\cdot k_m$ $\quad C_3 := \dfrac{23}{7632\cdot\pi\cdot k_m}$

14-12 Continued

$R_1 = 1\times 10^4 \quad C_1 = 4.681\times 10^{-6} \quad R_2 = 9.217\times 10^4 \quad C_2 = 3.453\times 10^{-8}$

$R_3 = 7.826\times 10^3 \quad C_3 = 9.593\times 10^{-9}$



14-13 Design requirement $T(s) = T_{pre}\cdot T_{pwr} = \dfrac{10}{\left(\dfrac{s}{2\cdot\pi\cdot 5000}+1\right)}$ given $T_{pwr} = \dfrac{2.239}{\left(\dfrac{s}{2\cdot\pi\cdot 2000}+1\right)}$

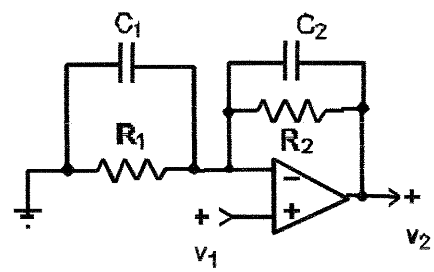
Use an non-inverting amplifier

$$T_{pre} = \frac{T(s)}{T_{pwr}} = \frac{10\cdot\left(\dfrac{s}{2\cdot\pi\cdot 2000}+1\right)}{\dfrac{2.239\cdot s}{2\cdot\pi\cdot 5000}+2.239} = \frac{Y_1+Y_2}{Y_2}$$

$$Y_2 = \frac{2.239\cdot s}{2\cdot\pi\cdot 5000}+2.239 = C_2\cdot s+\frac{1}{R_2}$$

$$Y_1 = \left(10\frac{s}{2\cdot\pi\cdot 2000}+10\right)-Y_2$$

$$Y_1 = 7.245\cdot 10^{-4}\cdot s+7.761 = C_1\cdot s+\frac{1}{R_1}$$



Let $k_m := 10^4 \quad R_1 := 7.761^{-1}\cdot k_m \quad R_2 := 2.239^{-1}\cdot k_m \quad C_1 := \dfrac{7.245\,10^{-4}}{k_m} \quad C_2 := \dfrac{2.239}{2\cdot\pi\cdot 5000 k_m}$

$R_1 = 1.288\times 10^3 \quad R_2 = 4.466\times 10^3 \quad C_1 = 7.245\times 10^{-8} \quad C_2 = 7.127\times 10^{-9}$

14-14 Design requirement $T(s) = T_{pre}\cdot T_{pwr} = \dfrac{\dfrac{s}{2\cdot\pi}}{\left(\dfrac{s}{2\cdot\pi\cdot 10}+1\right)\cdot\left(\dfrac{s}{2\cdot\pi\cdot 10000}+1\right)}$

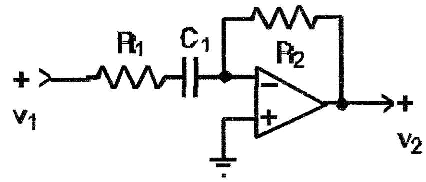
given $T_{pwr} = \dfrac{-\sqrt{10}}{\left(\dfrac{s}{2\cdot\pi\cdot 10000}+1\right)}$

Use an inverting amplifier

$$T_{pre} = \frac{T(s)}{T_{pwr}} = \frac{\dfrac{1}{2\cdot\pi}}{\left(\dfrac{\sqrt{10}}{2\cdot\pi\cdot 10}+\dfrac{\sqrt{10}}{s}\right)} = \frac{-Z_2}{Z_1}$$

14-14 Continued

$$Z_1 = \frac{\sqrt{10}}{2\cdot\pi\cdot 10} + \frac{\sqrt{10}}{s} = R_1 + \frac{1}{C_1\cdot s} \quad Z_2 = \frac{1}{2\cdot\pi} = R_2$$



Let $k_m := 2\cdot\pi\cdot 5\cdot 10^5$

$$R_1 := \frac{\sqrt{10}}{2\cdot\pi\cdot 10}\cdot k_m \quad R_2 := \frac{1}{2\cdot\pi}\cdot k_m$$

$$C_1 := \frac{1}{\sqrt{10}\cdot k_m} \quad R_1 = 1.581\times 10^5 \quad C_1 = 1.007\times 10^{-7} \quad R_2 = 5\times 10^5$$

14-15 Use the passive RC voltage divider

$$Z_1 = R_1 + \frac{1}{C_1\cdot s} = \frac{R_1\cdot C_1\cdot s + 1}{C_1\cdot s} \quad Z_2 = \frac{1}{C_2\cdot s + \frac{1}{R_2}} = \frac{R_2}{R_2\cdot C_2\cdot s + 1}$$

$$Z_1 + Z_2 = \frac{R_1\cdot C_1\cdot s + 1}{C_1\cdot s} + \frac{R_2}{R_2\cdot C_2\cdot s + 1} = \frac{R_1\cdot R_2\cdot C_1\cdot C_2\cdot s^2 + (R_1\cdot C_1 + R_2\cdot C_2 + R_2\cdot C_1)\cdot s + 1}{C_1\cdot s\cdot(R_2\cdot C_2\cdot s + 1)}$$

$$T_V(s) = \frac{Z_2}{Z_1 + Z_2} = \frac{R_2\cdot C_1\cdot s}{R_1\cdot R_2\cdot C_1\cdot C_2\cdot s^2 + (R_1\cdot C_1 + R_2\cdot C_2 + R_2\cdot C_1)\cdot s + 1}$$

The required transfer function is: $T(s) = \dfrac{K\cdot s}{(s + 10^2)\cdot(s + 10^5)} = \dfrac{K\cdot 10^{-7}\cdot s}{10^{-7}\cdot s^2 + 1001\cdot 10^{-5}\cdot s + 1}$

The design requirements are: $R_1\cdot R_2\cdot C_1\cdot C_2 = 10^{-7}$ $\quad R_1\cdot C_1 + R_2\cdot C_2 + R_2\cdot C_1 = 1001\cdot 10^{-5}$

Let $R_1 := 10^4$ $\quad C_1 := 5.6\cdot 10^{-9}$ $\quad C_2 := 5.6\cdot 10^{-9}$ $\quad R_2 := \dfrac{10^{-8}}{R_1\cdot C_1\cdot C_2}$ These are initial guesses

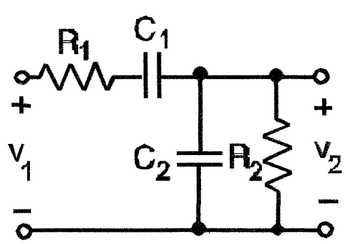
Given $\quad R_1\cdot R_2\cdot C_1\cdot C_2 = 10^{-7}$ $\quad R_1\cdot C_1 + R_2\cdot C_2 + R_2\cdot C_1 = 1001\cdot 10^{-5}$ $\quad C_1 = 5.6\cdot 10^{-9}$ $\quad C_2 = 5.6\cdot 10^{-9}$

$$\text{Find}(R_1, R_2, C_1, C_2) = \begin{pmatrix} 3.575\times 10^3 \\ 8.92\times 10^5 \\ 5.6\times 10^{-9} \\ 5.6\times 10^{-9} \end{pmatrix}$$

<---Use R_1 = 3.6 kΩ

<---Use R_2 = 8.92 kΩ

<---Capacitors are $C_1 = C_2$ = 5.6 nF



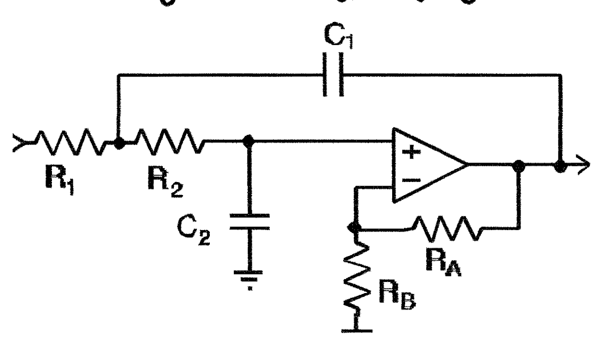
14-16 Low Pass $\omega_0 := 2000$ $\quad \zeta := 0.5$ $\quad R := 10\cdot 10^3$

Equal element design $R_1 := R$ $\quad R_2 := R$ $\quad R_B := R$

$$C := \frac{1}{R\cdot\omega_0} \quad C_1 := C \quad C_2 := C \quad R_A := 2\cdot(1 - \zeta)\cdot R_B$$

$$R_1 = 1\times 10^4 \quad R_2 = 1\times 10^4 \quad R_A = 1\times 10^4$$

$$R_B = 1\times 10^4 \quad C_1 = 5\times 10^{-8} \quad C_2 = 5\times 10^{-8}$$



14-17 Low Pass $\omega_0 := 2000$ $\zeta := 0.25$ $\mu := 10$

Use a gain stage with $R_B := 10^4$

$R_A := (\mu - 1) \cdot R_B$ $R_A = 9 \cdot 10^4$

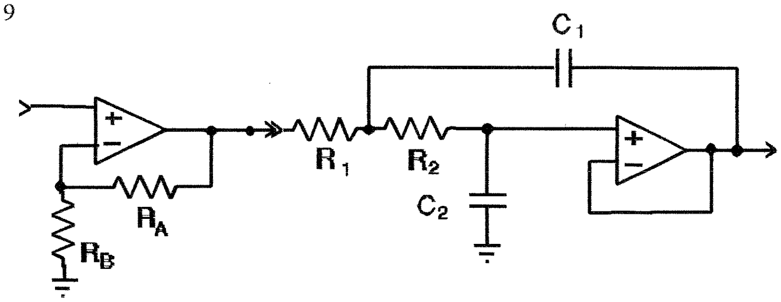
and a unity gain stage with $C_1 := 10^{-9}$

$C_2 := C_1 \cdot \zeta^2$ $R := \dfrac{1}{\sqrt{C_1 \cdot C_2} \cdot \omega_0}$

$R_1 := R$ $R_2 := R$

$R_1 = 2 \times 10^6$ $R_2 = 2 \times 10^6$

$C_1 = 1 \times 10^{-9}$ $C_2 = 6.25 \times 10^{-11}$



14-18 Low Pass $\omega_0 := 2000$ $\zeta := \left(\sqrt{2}\right)^{-1}$

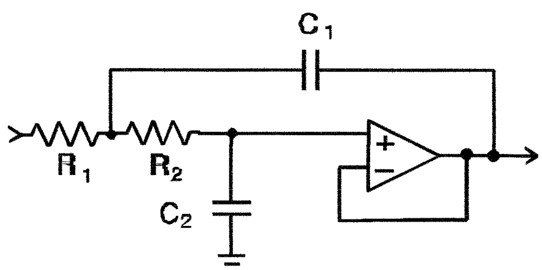
Unity gain design $\mu := 1$ $C_1 := 10^{-9}$

$C_2 := C_1 \cdot \zeta^2$ $R := \dfrac{1}{\sqrt{C_1 \cdot C_2} \cdot \omega_0}$

$R_1 := R$ $R_2 := R$

$R_1 = 7.071 \times 10^5$ $R_2 = 7.071 \times 10^5$

$C_1 = 1 \times 10^{-9}$ $C_2 = 5 \times 10^{-10}$



14-19 High Pass $\omega_0 := 2000$ ζ = unk

Using the equal element design Eq. (14-7) indicates that

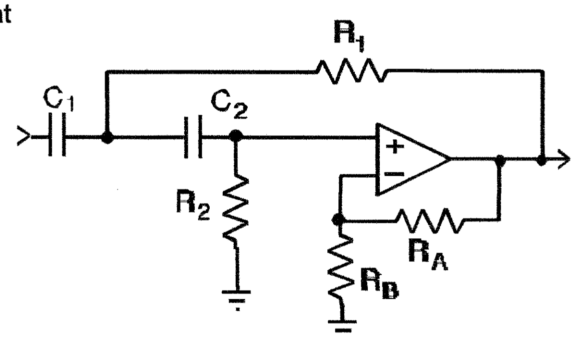
$\left|T(j \cdot \omega_0)\right| = \left|T\left(j \cdot \dfrac{1}{R \cdot C}\right)\right| = \dfrac{\mu}{3 - \mu} = 10$ hence $\mu := \dfrac{30}{11}$

Let $C := 20 \cdot 10^{-9}$ $C_1 := C$ $C_2 := C$ $R_B := 10^4$

$R := \dfrac{1}{C \cdot \omega_0}$ $R_1 := R$ $R_2 := R$ $R_A := (\mu - 1) \cdot R_B$

$R_1 = 2.5 \times 10^4$ $R_2 = 2.5 \times 10^4$ $R_A = 1.727 \times 10^4$

$R_B = 1 \times 10^4$ $C_1 = 2 \times 10^{-8}$ $C_2 = 2 \times 10^{-8}$



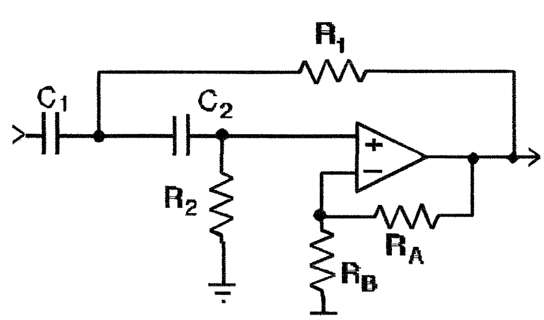
14-20 High Pass $\omega_0 := 200$ $\zeta := 0.5$ $C := 200 \cdot 10^{-9}$

Equal element design $C_1 := C$ $C_2 := C$ $R_B := 10^4$

$R := \dfrac{1}{C \cdot \omega_0}$ $R_1 := R$ $R_2 := R$ $R_A := 2 \cdot (1 - \zeta) \cdot R_B$

$R_1 = 2.5 \times 10^4$ $R_2 = 2.5 \times 10^4$ $R_A = 1 \times 10^4$

$R_B = 1 \times 10^4$ $C_1 = 2 \times 10^{-7}$ $C_2 = 2 \times 10^{-7}$

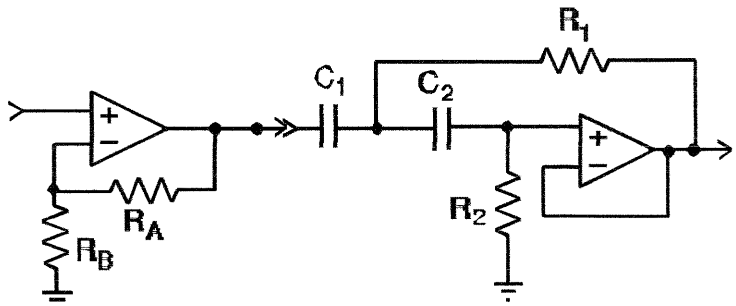


14-21 **High Pass** $\omega_0 := 1000 \quad \zeta := 0.75 \quad \mu := 100$

Use a gain stage with $R_B := 10^4$

$R_A := (\mu - 1) \cdot R_B \quad R_A = 9.9 \cdot 10^5$

and a unity gain stage with $R_2 := 10^4$



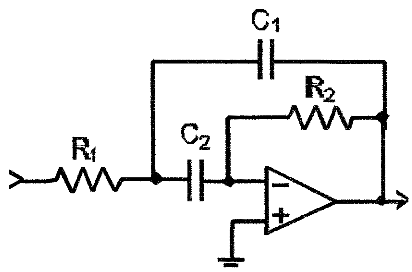
$R_1 := R_2 \cdot \zeta^2 \quad C := \dfrac{1}{\sqrt{R_1 \cdot R_2} \cdot \omega_0}$

$C_1 := C \quad C_2 := C$

$R_1 = 5.625 \times 10^3 \quad R_2 = 1 \times 10^4$

$C_1 = 1.333 \times 10^{-7} \quad C_2 = 1.333 \times 10^{-7}$

14-22 **Bandpass** $\omega_0 := 5000 \quad Q := 5 \quad \zeta := \dfrac{1}{2 \cdot Q} \quad \zeta = 0.1$

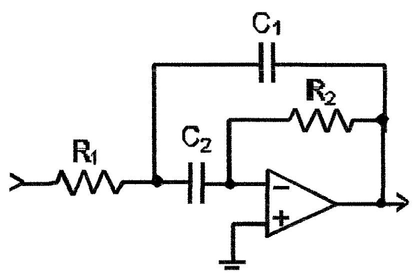


Equal capacitor design $R_2 := 10^5 \quad R_1 := \zeta^2 \cdot R_2 \quad \omega_0 = 5 \times 10^3$

$C := \dfrac{1}{\omega_0 \cdot \sqrt{R_1 \cdot R_2}} \quad C_1 := C \quad C_2 := C \quad C_1 = 2 \times 10^{-8}$

$C_2 = 2 \times 10^{-8} \quad R_1 = 1 \times 10^3 \quad R_2 = 1 \times 10^5$

14-23 **Bandpass** $\omega_0 := 5000 \quad B := 400 \quad \zeta := \dfrac{B}{2 \cdot \omega_0} \quad \zeta = 0.04$



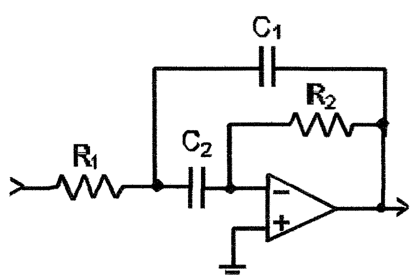
Equal capacitor design $R_2 := 10^5 \quad R_1 := \zeta^2 \cdot R_2$

$C := \dfrac{1}{\omega_0 \cdot \sqrt{R_1 \cdot R_2}} \quad C_1 := C \quad C_2 := C \quad C_1 = 5 \times 10^{-8}$

$C_2 = 5 \times 10^{-8} \quad R_1 = 160 \quad R_2 = 1 \times 10^5$

14-24 **Bandpass** $\omega_0 := 2 \cdot \pi \cdot \sqrt{0.95 \cdot 1.05} \cdot 10^3 \quad \omega_0 = 6.275 \times 10^3$

$B := 2 \cdot \pi \cdot 10^3 \cdot (1.05 - 0.95) \quad \zeta := \dfrac{B}{2 \cdot \omega_0} \quad \zeta = 5.006 \times 10^{-2}$



Equal capacitor design $R_2 := 10^5 \quad R_1 := \zeta^2 \cdot R_2$

$C := \dfrac{1}{\omega_0 \cdot \sqrt{R_1 \cdot R_2}} \quad C_1 := C \quad C_2 := C \quad C_1 = 3.183 \times 10^{-8}$

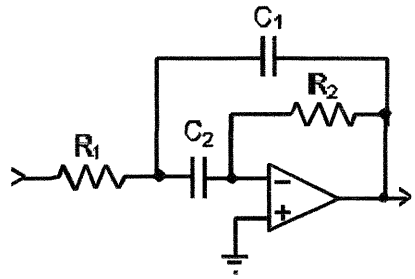
$C_2 = 3.183 \times 10^{-8} \quad R_1 = 250.627 \quad R_2 = 1 \times 10^5$

14-25 Bandpass $\omega_0 := 2\cdot\pi\cdot 5\cdot 10^3 \quad \omega_0 = 3.142\times 10^4$

$\omega_{C2} = 2\omega_{C1} \quad B = \omega_{C2} - \omega_{C1} = \omega_{C1}$

$\omega_0 = \sqrt{\omega_{C1}\cdot\omega_{C2}} = \sqrt{2}\cdot\omega_{C1} \quad \omega_{C1} := \dfrac{\omega_0}{\sqrt{2}} \quad \omega_{C1} = 2.221\cdot 10^4$

$B := \omega_{C1} \quad \zeta := \dfrac{B}{2\cdot\omega_0} \quad \zeta := \dfrac{1}{2\cdot\sqrt{2}} \quad \zeta = 0.354$



Equal capacitor design $R_2 := 10^5 \quad R_1 := \zeta^2\cdot R_2$

$C := \dfrac{1}{\omega_0\cdot\sqrt{R_1\cdot R_2}} \quad C_1 := C \quad C_2 := C \quad C_1 = 9.003\times 10^{-10}$

$C_2 = 9.003\times 10^{-10} \quad R_1 = 1.25\times 10^4 \quad R_2 = 1\times 10^5$

14-26 Low Pass $\omega_0 := 2\cdot\pi\cdot 1000$

Cutoff at 1 kHz requires $\zeta := \left(\sqrt{2}\right)^{-1}$

2nd order has -40 dB/dec rolloff

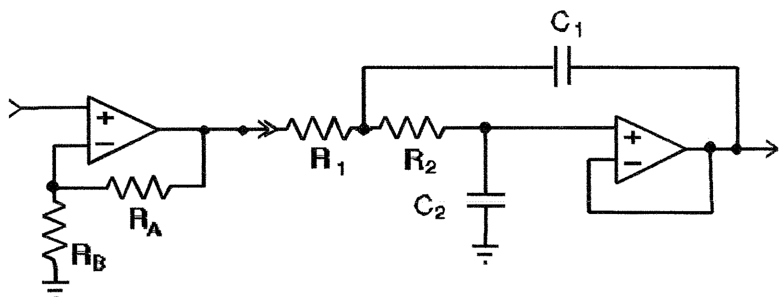
Unity gain stage with $C_1 := 10^{-9}$

$C_2 := C_1\cdot\zeta^2 \quad R := \dfrac{1}{\sqrt{C_1\cdot C_2}\cdot\omega_0}$

$R_1 := R \quad R_2 := R$

$R_1 = 2.251\times 10^5 \quad R_2 = 2.251\times 10^5$

$C_1 = 1\times 10^{-9} \quad C_2 = 5\times 10^{-10}$



Gain stage with $\mu := 10$ and $R_B := 10^4$

$R_A := (\mu - 1)\cdot R_B \quad R_A = 9\times 10^4$

14-27 High Pass $\omega_0 := 2\cdot\pi\cdot 1000$

Cutoff at 1 kHz requires $\zeta := \left(\sqrt{2}\right)^{-1}$

2nd order has +40 dB/dec rolloff

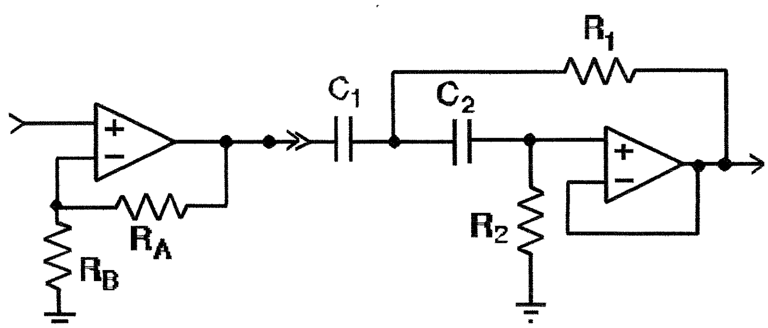
Unity gain stage with $R_2 := 10^4$

$R_1 := R_2\cdot\zeta^2 \quad C := \dfrac{1}{\sqrt{R_1\cdot R_2}\cdot\omega_0}$

$C_1 := C \quad C_2 := C$

$R_1 = 5\times 10^3 \quad R_2 = 1\times 10^4$

$C_1 = 2.251\times 10^{-8} \quad C_2 = 2.251\times 10^{-8}$

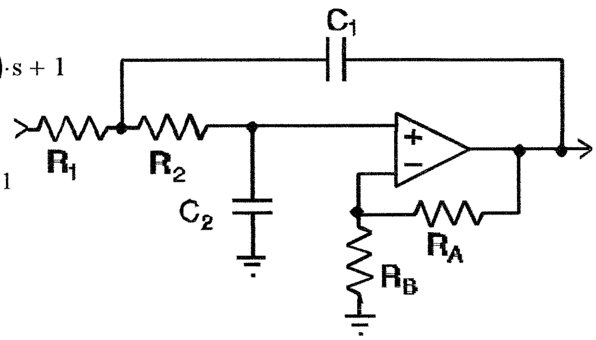


Gain stage with $\mu := 10$ and $R_B := 10^4$

$R_A := (\mu - 1)\cdot R_B \quad R_A = 9\times 10^4$

14-28 By Eq. (14-2) the denominator of T(s) is

$$R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 + (R_1 \cdot C_1 + R_2 \cdot C_2 + R_1 \cdot C_2 - \mu \cdot R_1 \cdot C_1) \cdot s + 1$$



for $\mu = 2$ and $R_1 \cdot C_1 = R_2 \cdot C_2$ this reduces to

$R_1 \cdot C_1 \cdot s^2 + R_1 \cdot C_2 \cdot s + 1$ which means $\omega_0 = (R_1 \cdot C_1)^{-1}$

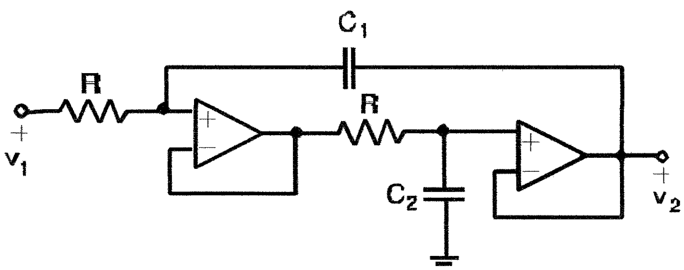
and $2 \cdot \zeta = R_1 \cdot C_2 \cdot \omega_0 = \dfrac{C_2}{C_1}$

Given ω_0 and ζ a design method is select C_1

then $C_2 = 2 \cdot \zeta \cdot C_1$ $R_1 = (\omega_0 \cdot C_1)^{-1}$ and $R_2 = (\omega_0 \cdot C_2)^{-1}$

$\mu = 2$ is obtained by setting $R_A = R_B$ in the OP AMP realization shown in the figure.

14-29 (a) The node-voltage equations for the circuit in martix form are:



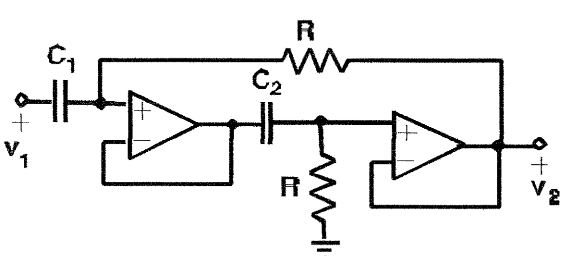
$$\begin{pmatrix} \frac{1}{R} + C_1 \cdot s & -C_1 \cdot s \\ -\frac{1}{R} & \frac{1}{R} + C_2 \cdot s \end{pmatrix} \cdot \begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{pmatrix} \frac{V_1}{R} \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} V_A(s) \\ V_B(s) \end{pmatrix} = \begin{pmatrix} \frac{1}{R} + C_1 \cdot s & -C_1 \cdot s \\ -\frac{1}{R} & \frac{1}{R} + C_2 \cdot s \end{pmatrix}^{-1} \cdot \begin{pmatrix} \frac{V_1(s)}{R} \\ 0 \end{pmatrix} = \begin{bmatrix} \frac{(1 + C_2 \cdot s \cdot R)}{(1 + C_2 \cdot s \cdot R + C_1 \cdot s^2 \cdot R^2 \cdot C_2)} \cdot V_1(s) \\ \frac{1}{(1 + C_2 \cdot s \cdot R + C_1 \cdot s^2 \cdot R^2 \cdot C_2)} \cdot V_1(s) \end{bmatrix}$$

Since $V_2(s) = V_B(s)$ we have $T(s) = \dfrac{V_B(s)}{V_1(s)} = \dfrac{1}{(1 + C_2 \cdot s \cdot R + C_1 \cdot s^2 \cdot R^2 \cdot C_2)}$ QED

(b) Since $R^2 \cdot C_1 \cdot C_2 = \dfrac{1}{\omega_o^{\,2}}$ and $\dfrac{2 \cdot \zeta}{\omega_0} = R \cdot C_2$ Select R, then $C_2 = \dfrac{2 \cdot \zeta}{R \cdot \omega_0}$ & $C_1 = \dfrac{1}{R \cdot 2 \cdot \zeta \cdot \omega_0}$

14-30 (a) The node-voltage equations for the circuit in martix form are:



$$\begin{pmatrix} \frac{1}{R} + C_1 \cdot s & -\frac{1}{R} \\ -C_2 \cdot s & \frac{1}{R} + C_2 \cdot s \end{pmatrix} \cdot \begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{pmatrix} C_1 \cdot s \cdot V_1 \\ 0 \end{pmatrix}$$

14-30 Continued

$$\begin{pmatrix} V_A(s) \\ V_B(s) \end{pmatrix} = \left[\begin{pmatrix} \frac{1}{R} + C_1 \cdot s & -\frac{1}{R} \\ -C_2 \cdot s & \frac{1}{R} + C_2 \cdot s \end{pmatrix}^{-1} \cdot \begin{pmatrix} C_1 \cdot s \cdot V_1(s) \\ 0 \end{pmatrix} \right] = \left[\begin{array}{c} \frac{(1 + C_2 \cdot s \cdot R) \cdot R \cdot C_1 \cdot s}{\left(1 + C_1 \cdot s \cdot R + C_1 \cdot s^2 \cdot R^2 \cdot C_2\right)} \cdot V_1(s) \\ \frac{R^2 \cdot C_1 \cdot C_2 \cdot s^2}{1 + C_1 \cdot s \cdot R + C_1 \cdot s^2 \cdot R^2 \cdot C_2} \cdot V_1(s) \end{array} \right]$$

Since $V_2(s) = V_B(s)$ we have $T(s) = \frac{V_B(s)}{V_1(s)} = \frac{R^2 \cdot C_1 \cdot C_2 \cdot s^2}{1 + R \cdot C_1 \cdot s + R^2 \cdot C_1 \cdot C_2 \cdot s^2}$ QED

(b) Since $R^2 \cdot C_1 \cdot C_2 = \frac{1}{\omega_o^{\;2}}$ & $\frac{2 \cdot \zeta}{\omega_0} = R \cdot C_1$ Select R, then $C_1 = \frac{2 \cdot \zeta}{R \cdot \omega_0}$ & $C_2 = \frac{1}{R \cdot 2 \cdot \zeta \cdot \omega_0}$

14-31 For a 1st order cascade with n = 4 and 17 dB gain at $\omega_C := 1000$

$\alpha := \omega_C \cdot \left(2^{.25} - 1\right)^{-0.5}$ $\alpha = 2298.959$ $K_{dB} = 17 + 3 = 20$ dB $K := 10$ $\omega := 100, 200 .. 10000$

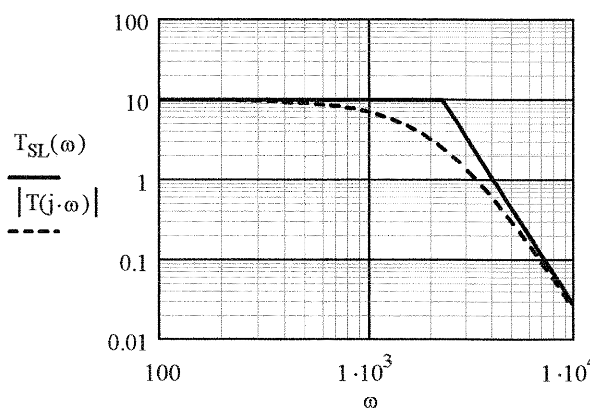
$$T(s) := \frac{K}{\left(\frac{s}{\alpha} + 1\right)^4} \qquad T_{SL}(\omega) := \begin{cases} K & \text{if } 0 < \omega \le \alpha \\ K \cdot \left(\frac{\alpha}{\omega}\right)^4 & \text{if } \alpha < \omega \end{cases}$$

$20 \cdot \log(|T(j \cdot \omega_C)|) = 16.99$

$20 \cdot \log(|T(j \cdot 2 \cdot \omega_C)|) = 10.211$

$20 \cdot \log(|T(j \cdot 5 \cdot \omega_C)|) = -10.327$

$20 \cdot \log(|T(j \cdot 10 \cdot \omega_C)|) = -31.972$



14-32 For a Butterworth response with n = 3, 10 dB pass band gain, $\omega_C := 500$ $\omega_{corner} := \omega_C$

$$T(s) := \frac{K}{\left[\left(\frac{s}{\omega_C}\right) + 1\right] \cdot \left[\left(\frac{s}{\omega_C}\right)^2 + \left(\frac{s}{\omega_C}\right) + 1\right]} \qquad K := 10 \qquad \omega := 100, 200 .. 1000$$

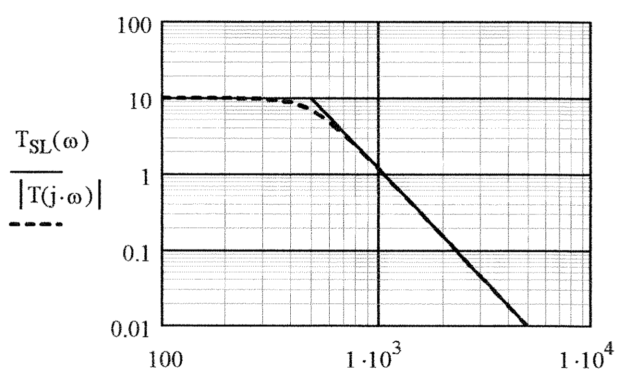
$$T_{SL}(\omega) := \begin{cases} K & \text{if } 0 < \omega \le \omega_{corner} \\ K \cdot \left(\frac{\omega_C}{\omega}\right)^3 & \text{if } \omega_{corner} < \omega \end{cases}$$

$20 \cdot \log(|T(j \omega_C)|) = 16.99$

$20 \cdot \log(|T(j \cdot 2 \cdot \omega_C)|) = 1.871$

$20 \cdot \log(|T(j \cdot 5 \cdot \omega_C)|) = -21.938$

$20 \cdot \log(|T(j \cdot 10 \cdot \omega_C)|) = -40$



14-33 For a Chebychev response with n = 4, 0 dB pass band gain ($K := 1$), $\omega_C := 500$

$$T(s) := \frac{K\cdot\left(\sqrt{2}\right)^{-1}}{\left[\left(\frac{s}{0.9502\cdot\omega_C}\right)^2 + 0.1789\cdot\left(\frac{s}{0.9502\cdot\omega_C}\right) + 1\right]\cdot\left[\left(\frac{s}{0.4425\cdot\omega_C}\right)^2 + 0.9276\cdot\left(\frac{s}{0.4425\cdot\omega_C}\right) + 1\right]}$$

$\omega_{corner} := \sqrt{.9502\cdot 0.4425}\cdot\omega_C$ $\qquad \omega_{corner} = 324.216$

$$T_{SL}(\omega) := \begin{cases} \dfrac{K}{\sqrt{2}} & \text{if } 0 < \omega \le \omega_{corner} \\ \dfrac{K}{\sqrt{2}}\cdot\left(\dfrac{0.9502\cdot\omega_C}{\omega}\right)^2\cdot\left(\dfrac{0.4425\cdot\omega_C}{\omega}\right)^2 & \text{if } \omega_{corner} < \omega \end{cases}$$

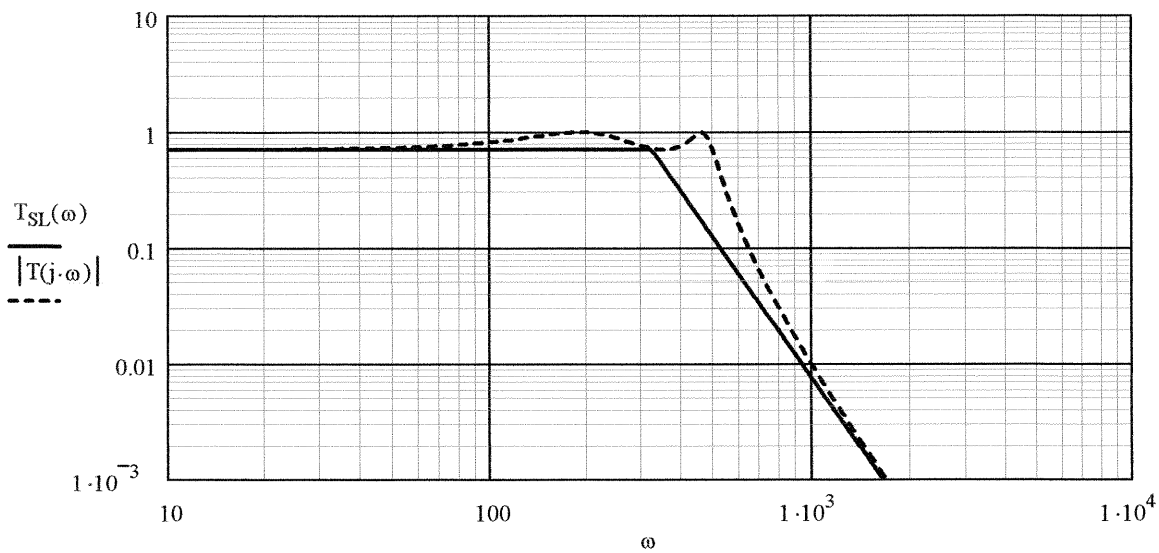
$20\cdot\log\left(\left|T\left(j\cdot 1\cdot\omega_C\right)\right|\right) = -3.009$

$20\cdot\log\left(\left|T\left(j\cdot 2\cdot\omega_C\right)\right|\right) = -39.735$

$20\cdot\log\left(\left|T\left(j\cdot 5\cdot\omega_C\right)\right|\right) = -73.626$

$20\cdot\log\left(\left|T\left(j\cdot 10\cdot\omega_C\right)\right|\right) = -97.974$

$\omega := 10, 20 .. 5000$



14-34 $\omega_C := 2000$; $T_{MAX} := 1$; $T_{MIN} := 10^{-1.5}$; $\omega_{MIN} := 10^4$

(a) First-order cascade:

$$T(s,n) := \frac{T_{MAX}}{\left(\dfrac{s\cdot\sqrt{2^{\frac{1}{n}} - 1}}{\omega_C} + 1\right)^n}$$

$20\cdot\log\left(\left|T\left(j\cdot\omega_{MIN}, 3\right)\right|\right) = -26.248$

$20\cdot\log\left(\left|T\left(j\cdot\omega_{MIN}, 4\right)\right|\right) = -30.327$ <--- n = 4 is the smallest integer

(b) Butterworth:

$$\frac{1}{2}\cdot\frac{\ln\left[\left(\frac{T_{MAX}}{T_{MIN}}\right)^2 - 1\right]}{\ln\left(\frac{\omega_{MIN}}{\omega_C}\right)} = 2.146 \quad \text{<--- n = 3}$$

Chebychev:

$$\frac{\operatorname{acosh}\left[\sqrt{\left(\frac{T_{MAX}}{T_{MIN}}\right)^2 - 1}\right]}{\operatorname{acosh}\left(\frac{\omega_{MIN}}{\omega_C}\right)} = 1.809 \quad \text{<--- n = 2}$$

14-35 First order cascade with $\omega_C := 1500$ $n := 4$ and $K := \sqrt{10}$

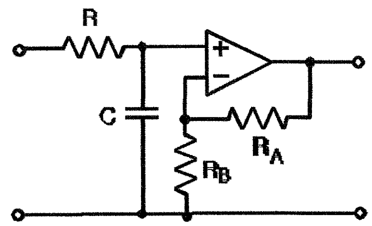
$K_n := K^{\frac{1}{n}}$ the transfer function is $T(s) = \left(\dfrac{K_n}{\dfrac{s}{\alpha}+1}\right)^n$ where

$\alpha := \left[\omega_C\cdot\left(2^{n^{-1}}-1\right)^{-0.5}\right]$ $\quad \alpha = 3.448\times 10^3$ $\quad K_n = \dfrac{R_A+R_B}{R_B}$ $\quad R\cdot C = \dfrac{1}{\alpha}$

Let $\quad R_B := 10^4 \quad R_A := R_B\cdot(K_n-1) \qquad R_A = 3.335\times 10^3 \quad R := 10^4 \quad C := (R\cdot\alpha)^{-1} \quad C = 2.9\times 10^{-8}$

$K_n = 1.334$

Checking $T(s) := \dfrac{\left(\dfrac{R_A+R_B}{R_B}\right)^n}{(R\cdot C\cdot s+1)^n}$



$20\cdot\log(|T(j\cdot 0)|) = 10$ <----Passband gain requirement

$20\cdot\log(|T(j\cdot 1500)|) = 6.99$ <----Cutoff frequency requirement

Need four identical stages

14-36 Butterworth with $\omega_C := 300 \quad n := 3 \quad K := \sqrt{10}$

the transfer function is

$$T(s) = \left[\frac{1}{\left(\dfrac{s}{\omega_C}\right)^2+\left(\dfrac{s}{\omega_C}\right)+1}\right]\cdot\left[\frac{K}{\left(\dfrac{s}{\omega_C}\right)+1}\right] = T_1(s)\cdot T_2(s)$$

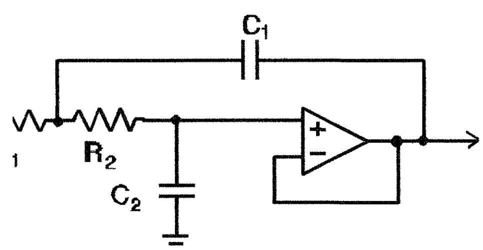
1st stage: use a unity-gain design with $\omega_0 := \omega_C$

and $\zeta := 0.5$ Let $C_1 := 10^{-7}$ then $C_2 := \zeta^2\cdot C_1$

$R := \left[\left(\sqrt{C_1\cdot C_2}\right)\cdot\omega_0\right]^{-1} \quad R_1 := R \quad R_2 := R$

$C_1 = 1\times 10^{-7} \quad C_2 = 2.5\times 10^{-8}$

$R_1 = 6.667\times 10^4 \quad R_2 = 6.667\times 10^4$



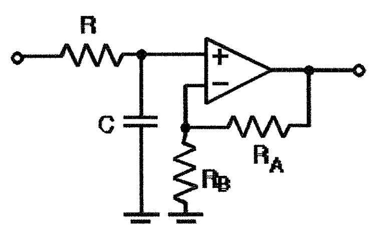
2nd stage: use a first-order design with $RC = \dfrac{1}{\omega_C}$

and $K = \dfrac{R_A+R_B}{R_B}$ Let $R := 10^4$ $C := \dfrac{1}{R\cdot\omega_C}$

then $C = 3.333\times 10^{-7}$

let $R_B := 10^4$ then $R_A := R_B\cdot(K-1)$

$R_A = 2.162\times 10^4$ Checking



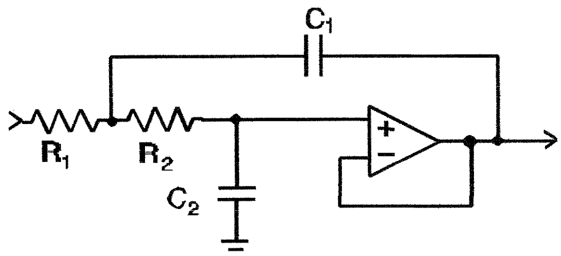
$$T(s) := \frac{1}{R_1\cdot R_2\cdot C_1\cdot C_2\cdot s^2+(R_2\cdot C_2+R_1\cdot C_2)\cdot s+1}\cdot\frac{1}{R\cdot C\cdot s+1}\cdot\frac{R_A+R_B}{R_B}$$

$20\cdot\log(|T(j\cdot 0)|) = 10$ <--Passband gain requirement $20\cdot\log(|T(j\cdot 300)|) = 6.99$ <--Cutoff freq. requiremen

14-37 Chebychev with $\omega_C := 8000$ $n := 3$ $K := 10$ the transfer function is

$$T(s) = \left[\frac{1}{\left(\frac{s}{0.9159\cdot\omega_C}\right)^2 + 0.3254\cdot\left(\frac{s}{0.9159\cdot\omega_C}\right) + 1}\right]\cdot\left[\frac{K}{\left(\frac{s}{0.2980\cdot\omega_C}\right) + 1}\right] = T_1(s)\cdot T_2(s)$$

1st stage: use a unity-gain design with $\omega_0 := 0.9159\cdot\omega_C$



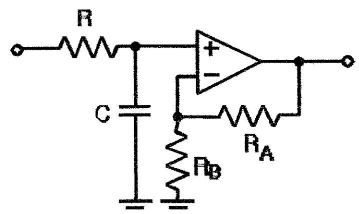
and $\zeta := \frac{0.3254}{2}$ Let $C_1 := 10^{-7}$ then $C_2 := \zeta^2\cdot C_1$

$R := \left[\left(\sqrt{C_1\cdot C_2}\right)\cdot\omega_0\right]^{-1}$ $R_1 := R$ $R_2 := R$

$C_1 = 1\times10^{-7}$ $C_2 = 2.647\times10^{-9}$

$R_1 = 8.388\times10^3$ $R_2 = 8.388\times10^3$

2nd stage: use a first-order design with $RC = \frac{1}{.2980\cdot\omega_C}$



and $K = \frac{R_A + R_B}{R_B}$ Let $R := 10^4$ $C := \frac{1}{R\cdot0.2980\cdot\omega_C}$

then $C = 4.195\times10^{-8}$

let $R_B := 10^4$ then $R_A := R_B\cdot(K-1)$ $R_A = 9\times10^4$

Checking design requirements

$$T(s) := \frac{1}{R_1\cdot R_2\cdot C_1\cdot C_2\cdot s^2 + (R_2\cdot C_2 + R_1\cdot C_2)\cdot s + 1}\cdot\frac{\frac{R_A+R_B}{R_B}}{R\cdot C\cdot s + 1}$$

$20\cdot\log(|T(j\cdot0)|) = 20$ <--Passband gain requirement

$20\cdot\log(|T(j\cdot8000)|) = 16.99$ <---Cutoff frequency requirement

14-38 $\omega_C := 2\cdot\pi\cdot1000$ $T_{MAX} := 1$

3rd Harmonic $T_{MIN} := 10^{\frac{-25}{20}}$ $\omega_{MIN} := 3\cdot\omega_C$ $\frac{1}{2}\cdot\frac{\ln\left[\left(\frac{T_{MAX}}{T_{MIN}}\right)^2 - 1\right]}{\ln\left(\frac{\omega_{MIN}}{\omega_C}\right)} = 2.618$ <---n = 3

5th Harmonic $T_{MIN} := 10^{\frac{-40}{20}}$ $\omega_{MIN} := 5\cdot\omega_C$ $\frac{1}{2}\cdot\frac{\ln\left[\left(\frac{T_{MAX}}{T_{MIN}}\right)^2 - 1\right]}{\ln\left(\frac{\omega_{MIN}}{\omega_C}\right)} = 2.861$ <---n = 3

Both requirements are met by a 3rd order Butterworth of the form:

$$T(s) = T_1\cdot T_2 = \left[\frac{K_1}{\left(\frac{s}{\omega_C}\right)^2 + \left(\frac{s}{\omega_C}\right) + 1}\right]\cdot\left(\frac{K_2}{\frac{s}{\omega_C} + 1}\right)$$

14-38 Continued

$$T_1 = \frac{K_1}{\left(\frac{s}{\omega_0}\right)^2 + 2\cdot\zeta\cdot\left(\frac{s}{\omega_0}\right) + 1} \qquad \omega_0 := \omega_C \qquad \zeta := 0.5 \qquad T_2 = \frac{K_2}{\left(\frac{s}{\alpha}\right) + 1} \qquad K_1\cdot K_2 = 1 \qquad \alpha := \omega_0$$

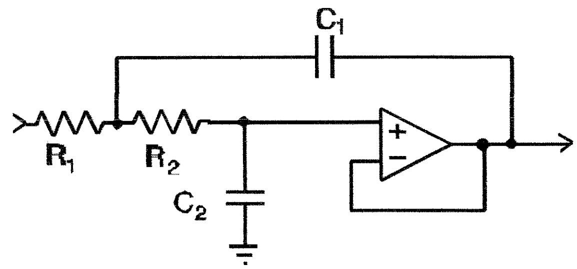
1st stage: use unity gain design

$C_1 := 10^{-7}$ $C_2 := \zeta^2\cdot C_1$

$K_1 := 1$ $R_1 := \left(\omega_0\cdot\sqrt{C_1\cdot C_2}\right)^{-1}$ $R_2 := R_1$

$R_1 = 3.183\times 10^3$ $R_2 = 3.183\times 10^3$

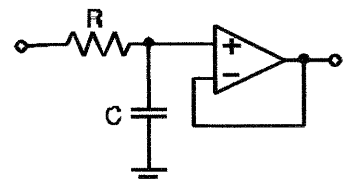
$C_1 = 1\times 10^{-7}$ $C_2 = 2.5\times 10^{-8}$



2nd stage: use a first-order design with $RC = (\alpha)^{-1}$

and $K_2 := 1$ Let $C := 10^{-7}$

$R := (C\cdot\alpha)^{-1}$ $R = 1.592\times 10^3$ Checking



$$T(s) := \frac{K_1}{R_1\cdot R_2\cdot C_1\cdot C_2\cdot s^2 + (R_1\cdot C_1 + R_2\cdot C_2 + R_1\cdot C_2 - K_1\cdot R_1\cdot C_1)\cdot s + 1}\cdot\frac{K_2}{R\cdot C\cdot s + 1}$$

$20\cdot\log(|T(j\cdot 0)|) = 0$ <--Meets passband gain requirement

$20\cdot\log(|T(j\cdot\omega_C)|) = -3.01$ <---Meets cutoff frequency requirement at 1 kHz

$20\cdot\log(|T(j\cdot 2\cdot\pi\cdot 3000)|) = -28.633$ <---Meets stopband gain requirement at 3 kHz

$20\cdot\log(|T(j\cdot 2\cdot\pi\cdot 5000)|) = -41.938$ <---Meets stopband gain requirement at 5 kHz

14-39 $f_C := 3200$ $T_{MAX} := 2$ $T_{MIN1} := 10^{-1}$ $f_{MIN1} := 6400$ $T_{MIN2} := 10^{-2}$ $f_{MIN2} := 12800$

Butterworth

$$\frac{1}{2}\cdot\frac{\ln\left[\left(\frac{T_{MAX}}{T_{MIN1}}\right)^2 - 1\right]}{\ln\left(\frac{f_{MIN1}}{f_C}\right)} = 4.32 \quad \text{<--- n = 5}$$

$$\frac{1}{2}\cdot\frac{\ln\left[\left(\frac{T_{MAX}}{T_{MIN2}}\right)^2 - 1\right]}{\ln\left(\frac{f_{MIN2}}{f_C}\right)} = 3.822 \quad \text{<--- n = 4}$$

Chebychev

$$\frac{\operatorname{acosh}\left[\sqrt{\left(\frac{T_{MAX}}{T_{MIN1}}\right)^2 - 1}\right]}{\operatorname{acosh}\left(\frac{f_{MIN1}}{f_C}\right)} = 2.8 \quad \text{<--- n = 3}$$

$$\frac{\operatorname{acosh}\left[\sqrt{\left(\frac{T_{MAX}}{T_{MIN2}}\right)^2 - 1}\right]}{\operatorname{acosh}\left(\frac{f_{MIN2}}{f_C}\right)} = 2.904 \quad \text{<--- n = 3}$$

Butterworth requires n = 5 Chebychev requires n = 3, use Chebychev $\omega_C := 2\cdot\pi\cdot f_C$ $\omega_C = 20106$

$$T(s) = T_1\cdot T_2 = \left[\frac{1}{\left(\frac{s}{0.9159\cdot\omega_C}\right)^2 + 0.3254\cdot\left(\frac{s}{0.9159\cdot\omega_C}\right) + 1}\right]\cdot\left(\frac{K}{\frac{s}{0.2980\cdot\omega_C} + 1}\right) \quad K := T_{MAX}$$

14-39 Continued

$$T_1 = \frac{1}{\left(\frac{s}{0.9159\omega_C}\right)^2 + 0.3254\cdot\left(\frac{s}{0.9159\cdot\omega_C}\right) + 1} = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + 2\cdot\zeta\cdot\left(\frac{s}{\omega_0}\right) + 1} \quad \omega_0 := 0.9159\cdot\omega_C \quad \zeta := \frac{0.3254}{2}$$

$$T_2 = \frac{K}{\left(\frac{s}{\alpha}\right) + 1} \qquad K = 2 \qquad \alpha := 0.2980\cdot\omega_C$$

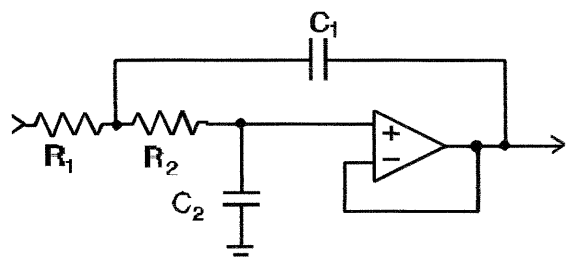
1st stage: use a unity-gain design with $\omega_0 = 1.842\times 10^4$

and $\zeta = 0.163$ Let $C_1 := 10^{-8}$ then $C_2 := \zeta^2\cdot C_1$

$R := \left[\left(\sqrt{C_1\cdot C_2}\right)\cdot\omega_0\right]^{-1}$ $R_1 := R$ $R_2 := R$

$C_1 = 1\times 10^{-8}$ $C_2 = 2.647\times 10^{-10}$

$R_1 = 3.338\times 10^4$ $R_2 = 3.338\times 10^4$

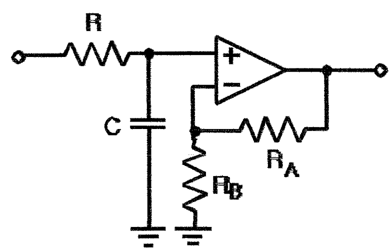


2nd stage: use a first-order design with $RC = \frac{1}{\alpha}$

and $K = \frac{R_A + R_B}{R_B}$ Let $R := 10^4$ $C := \frac{1}{R\cdot\alpha}$

then $C = 1.669\times 10^{-8}$ let $R_B := 10^4$

then $R_A := R_B\cdot(K - 1)$ $R_A = 1\times 10^4$



Checking design requirements

$$T(s) := \frac{1}{R_1\cdot R_2\cdot C_1\cdot C_2\cdot s^2 + (R_1\cdot C_1 + R_2\cdot C_2 + R_1\cdot C_2 - R_2\cdot C_1)\cdot s + 1}\cdot\frac{1}{R\cdot C\cdot s + 1}\cdot\frac{R_A + R_B}{R_B}$$

$20\cdot\log(|T(j\cdot 2\cdot\pi\cdot 0)|) = 6.021$ <-- 6 dB as required K = 2

$20\cdot\log(|T(j\cdot 2\cdot\pi\cdot 3200)|) = 3.01$ <-- (6 - 3) dB as required

$20\cdot\log(|T(j\cdot 2\cdot\pi\cdot 6400)|) = -22.286$ <--- less then -20 dB as required

$20\cdot\log(|T(j\cdot 2\cdot\pi\cdot 12800)|) = -41.728$ <--- less than -40 dB as required

14-40 $f_C := 25000$ $T_{MAX} := 1$ $T_{MIN} := 10^{-1.5}$ $f_{MIN} := 200\cdot 10^3$

Butterworth

$$\frac{1}{2}\cdot\frac{\ln\left[\left(\frac{T_{MAX}}{T_{MIN}}\right)^2 - 1\right]}{\ln\left(\frac{f_{MIN}}{f_C}\right)} = 1.661$$ <--- n = 2

Chebychev

$$\frac{\operatorname{acosh}\left[\sqrt{\left(\frac{T_{MAX}}{T_{MIN}}\right)^2 - 1}\right]}{\operatorname{acosh}\left(\frac{f_{MIN}}{f_C}\right)} = 1.498$$ <--- n = 2

Butterworth & Chebychev require n = 2, use Chebychev. $\omega_C := 2\cdot\pi\cdot f_C$

14-40 Continued

$$T(s) = \left[\frac{1}{\left(\frac{s}{0.8409\cdot\omega_C}\right)^2 + 0.7654\cdot\left(\frac{s}{0.8409\cdot\omega_C}\right) + 1}\right] = \frac{1}{\left(\frac{s}{\omega_o}\right)^2 + 2\cdot\zeta\cdot\left(\frac{s}{\omega_0}\right) + 1}$$

$$\omega_0 := 0.8409\cdot\omega_C \qquad \zeta := \frac{0.7654}{2}$$

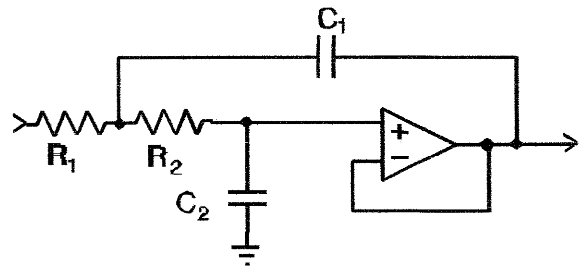
use a unity gain design with $\omega_0 = 1.321\times 10^5$

and $\zeta = 0.383$ Let $C_1 := 10^{-8}$ then $C_2 := \zeta^2\cdot C_1$

$$R := \left[\left(\sqrt{C_1\cdot C_2}\right)\cdot\omega_0\right]^{-1} \quad R_1 := R \quad R_2 := R$$

$$C_1 = 1\times 10^{-8} \qquad C_2 = 1.465\times 10^{-9}$$

$$R_1 = 1.978\times 10^3 \qquad R_2 = 1.978\times 10^3$$ Checking



$$T(s) := \frac{1}{R_1\cdot R_2\cdot C_1\cdot C_2\cdot s^2 + \left(R_1\cdot C_1 + R_2\cdot C_2 + R_1\cdot C_2 - R_2\cdot C_1\right)\cdot s + 1}$$

$$20\cdot\log\left(\left|T\left(j\cdot 2\cdot\pi\cdot 25\cdot 10^3\right)\right|\right) = -2.375\times 10^{-4}$$ <--- gain essentially unchanged

$$20\cdot\log\left(\left|T\left(j\cdot 2\cdot\pi\cdot 200\cdot 10^3\right)\right|\right) = -39.066$$ <--- less than -30 dB as required

14-41 The transfer function is question is

$$T(s) = \frac{7.751\cdot 10^9}{\left[(s + 468.2)^2 + 2839^2\right]\cdot(s + 936.2)} = \frac{7.751\cdot 10^9}{\left(s^2 + 936.4\cdot s + 8.279\cdot 10^6\right)\cdot(s + 936.2)}$$

A third-order Chebychev has the form

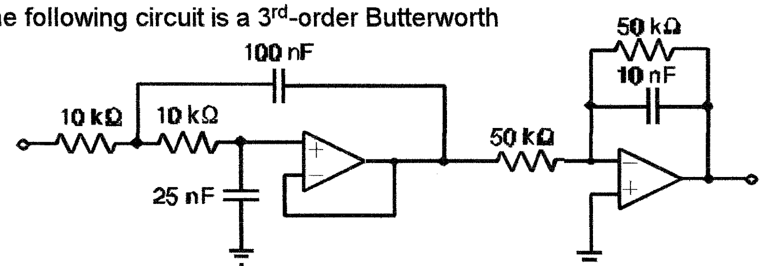
$$T(s) = \frac{1}{\left[\left(\frac{s}{0.9159\cdot\omega_C}\right)^2 + 0.3254\cdot\left(\frac{s}{0.9159\cdot\omega_C}\right) + 1\right]\cdot\left[\left(\frac{s}{.2980\cdot\omega_C}\right) + 1\right]}$$

For $\omega_C := 2\cdot\pi\cdot 500$ $\qquad \omega_C = 3.142\times 10^3$ This becomes

$$T(s) = \frac{\left(0.9159\cdot\omega_C\right)^2\cdot\left(0.2980\cdot\omega_C\right)}{\left[s^2 + 0.3254\cdot\left(0.9159\cdot\omega_C\right)\cdot s + \left(0.9159\cdot\omega_C\right)^2\right]\cdot\left(s + 0.2980\cdot\omega_C\right)} = \frac{7.751\cdot 10^9}{\left(s^2 + 936.3\cdot s + 8.279\cdot 10^6\right)\cdot(s + 936.2)}$$

This checks reansonable well. The proposed T(s) is a 3rd order Chebychev

14-42 Verify that the following circuit is a 3rd-order Butterworth



14-42 Continued The circuit is a 2nd order Sallen-Key low pass in cascade with an inverting 1st order low pass.

For $R_1 := 10^4$ $R_2 := 10^4$ $C_1 := 10^{-7}$ $C_2 := 2.5\cdot 10^{-8}$ $R_3 := 5\cdot 10^4$ $R_4 := 5\cdot 10^4$ $C_4 := 10^{-8}$

The transfer function of the circuit is

$$T(s) = \left[\frac{1}{R_1\cdot R_2\cdot C_1\cdot C_2\cdot s^2 + (R_1\cdot C_1 + R_2\cdot C_2 + R_1\cdot C_2 - R_1\cdot C_1)\cdot s + 1}\right]\cdot\left(\frac{1}{R_4\cdot C_4\cdot s + 1}\right)\cdot\left(\frac{-R_4}{R_3}\right)$$

for the given element values the coefficients are

$R_1\cdot R_2\cdot C_1\cdot C_2 = 2.5\times 10^{-7}$ $R_1\cdot C_1 + R_2\cdot C_2 + R_1\cdot C_2 - R_1\cdot C_1 = 5\times 10^{-4}$ $R_4\cdot C_4 = 5\times 10^{-4}$ $\frac{-R_4}{R_3} = -1$

$$T(s) = \left(\frac{1}{2.5\cdot 10^{-7}\cdot s^2 + 5\cdot 10^{-4}\cdot s + 1}\right)\cdot\left(\frac{1}{5\cdot 10^{-4}\cdot s + 1}\right)\cdot(-1)$$

A unity gain third-order Butterworth low pass has the form $\omega_C := 2000$ $\omega_0 := \omega_C$ $\zeta := 0.5$

$$T(s) = \left[\frac{1}{\left(\frac{s}{\omega_0}\right)^2 + 2\cdot\zeta\cdot\left(\frac{s}{\omega_0}\right) + 1}\right]\cdot\left(\frac{1}{\frac{s}{\omega_0} + 1}\right)$$ $\frac{1}{\omega_0^{\,2}} = 2.5\times 10^{-7}$ $\frac{2\cdot\zeta}{\omega_0} = 5\times 10^{-4}$ $\frac{1}{\omega_0} = 5\times 10^{-4}$

A third-order Butterworth with a cutoff frrequency of 2 krad/s is

$$T(s) = \left(\frac{1}{2.5\cdot 10^{-7}\cdot s^2 + 5\cdot 10^{-4}\cdot s + 1}\right)\cdot\left(\frac{1}{5\cdot 10^{-4}\cdot s + 1}\right)$$

Which checks with the transfer function realized by the circuit, except for the -1 which does not change the gain response, only the phase.

14-43 Highpass requirements $\omega_C := 20000$ $T_{MAX} := 10$ $T_{MIN} := 0.1$ $\omega_{MIN} := 5000$

(a) First-order cascade: Low pass prototype $\omega_{MIN} := \frac{\omega_C^{\,2}}{\omega_{MIN}}$ $\omega_{MIN} = 5\times 10^3$

$$T(s,n) := \frac{T_{MAX}}{\left(\frac{s\cdot\sqrt{2^{\frac{1}{n}} - 1}}{\omega_C} + 1\right)^n}$$

$20\cdot\log(|T(j\cdot\omega_{MIN}, 23)|) = -19.801$

$20\cdot\log(|T(j\cdot\omega_{MIN}, 24)|) = -20.074$ <--- n = 24 is the smallest integer

(b) Butterworth:

$$\frac{1}{2}\cdot\frac{\ln\left[\left(\frac{T_{MAX}}{T_{MIN}}\right)^2 - 1\right]}{\ln\left(\frac{\omega_{MIN}}{\omega_C}\right)} = 3.322$$ <--- n = 4

Chebychev:

$$\frac{\operatorname{acosh}\left[\sqrt{\left(\frac{T_{MAX}}{T_{MIN}}\right)^2 - 1}\right]}{\operatorname{acosh}\left(\frac{\omega_{MIN}}{\omega_C}\right)} = 2.568$$ <--- n = 3

(c) First-order cascade with n = 24 is not practical. Either the Butterworth or the Chebychev are fine:

Chebychev:	Butterworth:
(1) Requires one 2nd order & one 1st order stage.	(1) Requires two 2nd order stages & a gain stage.
(2) Does not require a gain stage.	(2) Requires a gain stage.
(3) Five R's three C's two OP AMPs	(3) Six R's four C's three OP AMPs
(4) Greater stop band attenuation.	(4) Better step response settling time.

14-44 High pass requirements $\omega_C := 25000 \quad T_{MAX} := 1 \quad T_{MIN1} := 0.1 \quad \omega_{MIN1} := 10000$

$\omega_{MIN2} := 5000 \quad T_{MIN2} := 10^{-\frac{40}{20}}$ **Lowpass prototype:** $\omega_{MIN1} := \dfrac{\omega_C^2}{\omega_{MIN1}} \quad \omega_{MIN2} := \dfrac{\omega_C^2}{\omega_{MIN2}}$

$\omega_{MIN1} = 6.25 \times 10^4 \quad \omega_{MIN2} = 1.25 \times 10^5$

(a) $\dfrac{\operatorname{acosh}\left[\sqrt{\left(\dfrac{T_{MAX}}{T_{MIN1}}\right)^2 - 1}\right]}{\operatorname{acosh}\left(\dfrac{\omega_{MIN1}}{\omega_C}\right)} = 1.907$ <--- n = 2 $\quad \dfrac{\operatorname{acosh}\left[\sqrt{\left(\dfrac{T_{MAX}}{T_{MIN2}}\right)^2 - 1}\right]}{\operatorname{acosh}\left(\dfrac{\omega_{MIN2}}{\omega_C}\right)} = 2.311$ <--- n = 3

The -20 dB requirement at 10 krad/s requires n = 2

The -40 dB requirement at 4 krad/s requires n = 3

3rd order Chebychev high pass respones is required.

$$T_{LP}(s) \frac{1}{\left[\left(\dfrac{s}{0.9159\cdot\omega_C}\right)^2 + 0.3254\cdot\left(\dfrac{s}{0.9159\cdot\omega_C}\right) + 1\right]\cdot\left[\left(\dfrac{s}{0.2980\cdot\omega_C}\right) + 1\right]}$$

<---3rd order Chebychev low pass

$$T_{HP}(s) = T_{LP}\left(\frac{\omega_C^2}{s}\right) = \frac{1}{\left[\left(\dfrac{\omega_C}{0.9159\cdot s}\right)^2 + 0.3254\cdot\left(\dfrac{\omega_C}{0.9159\cdot s}\right) + 1\right]\cdot\left[\left(\dfrac{\omega_C}{0.2980\cdot s}\right) + 1\right]}$$

<---3rd order Chebychev high pass

$$T_{HP}(s) := \frac{s^3}{\left(7.45\cdot 10^8 + 8.882\cdot 10^3\cdot s + s^2\right)\cdot\left(8.389\cdot 10^4 + s\right)}$$

Insert ω_C = 25,000 and evaluate coefficients.

(b) Stopband requirement at ω = 10 krad/s requires n = 2. The requirement at ω = 5 krad/s requires n = 3. Filter order determined by the stopband requirement at ω = 5 krad/s.

$20\cdot\log(|T_{HP}(j\cdot 5000)|) = -53.715$ <---less than -40 db as required

$20\cdot\log(|T_{HP}(j\cdot 10^4)|) = -34.808$ <---less than -20 dB as required

14-45 (a) Low pass $\omega_C := 3000 \quad \omega_{MIN} := 40000 \quad T_{MAX} := 1 \quad T_{MIN} := 10^{-2}$

For Butterworth poles

$$\frac{1}{2}\cdot\frac{\ln\left[\left(\dfrac{T_{MAX}}{T_{MIN}}\right)^2 - 1\right]}{\ln\left(\dfrac{\omega_{MIN}}{\omega_C}\right)} = 1.778 \qquad n = 2$$

$$T_{LP}(s) = \frac{1}{\left(\dfrac{s}{3000}\right)^2 + \sqrt{2}\cdot\left(\dfrac{s}{3000}\right) + 1} = \frac{9\cdot 10^6}{s^2 + \sqrt{2}\cdot 3000\cdot s + 9\cdot 10^6}$$

14-45 Continued

High pass $\omega_C := 100 \quad \omega_{MIN} := 10 \quad T_{MAX} := 1 \quad T_{MIN} := 10^{-2}$

For Buttrworth poles

$$\frac{1}{2}\cdot\frac{\ln\left[\left(\frac{T_{MAX}}{T_{MIN}}\right)^2 - 1\right]}{\ln\left(\frac{\omega_C}{\omega_{MIN}}\right)} = 2 \qquad n = 2 \quad T_{HP}(s) = \frac{1}{\left(\frac{100}{s}\right)^2 + \sqrt{2}\cdot\left(\frac{100}{s}\right) + 1} = \frac{s^2}{10^4 + \sqrt{2}\cdot 100\cdot s + s^2}$$

The Butterworth bandpass is $T_{BU}(s) := \frac{9\cdot 10^6}{s^2 + \sqrt{2}\cdot 3000\cdot s + 9\cdot 10^6}\cdot\frac{s^2}{10^4 + \sqrt{2}\cdot 100\cdot s + s^2}$

(b) Checking

$20\cdot\log(|T_{BU}(j\cdot 100)|) = -3.01$ <---lower cutoff at ω = 100 rad/s

$20\cdot\log(|T_{BU}(j\cdot 3000)|) = -3.01$ <---upper cutoff at ω = 3 krad/s

$20\cdot\log(|T_{BU}(j\cdot 10)|) = -40$ <---stopband gain -40 dB at ω = 10 rad/s

$20\cdot\log(|T_{BU}(j\cdot 30000)|) = -40$ <---stopband gains less than -40 dB at ω = 40 krad/s

$T_{BU}(s)$ meets requirements

14-46 Bandpass requirements:

$T_{MAX} := 1 \quad \omega_{C1} := 100 \quad \omega_{C2} := 2000 \quad T_{MIN} := 0.1 \quad \omega_{MIN1} := 25 \quad \omega_{MIN2} := 8000$

(a) Butterworth low-pass requirements

$$\frac{1}{2}\cdot\frac{\ln\left[\left(\frac{T_{MAX}}{T_{MIN}}\right)^2 - 1\right]}{\ln\left(\frac{\omega_{MIN2}}{\omega_{C2}}\right)} = 1.657 \quad \text{<---n = 2}$$

Butterworth high-pass requirements

$$\frac{1}{2}\cdot\frac{\ln\left[\left(\frac{T_{MAX}}{T_{MIN}}\right)^2 - 1\right]}{\ln\left(\frac{\omega_{C1}}{\omega_{MIN1}}\right)} = 1.657 \quad \text{<--- n = 2}$$

$$T_B(s) := \left[\frac{1}{\left(\frac{s}{\omega_{C2}}\right)^2 + 1.414\left(\frac{s}{\omega_{C2}}\right) + 1}\right]\cdot\left(\frac{s^2}{s^2 + 1.414\omega_{C1}\cdot s + \omega_{C1}^2}\right) \quad \text{<---Butterworth T(s)}$$

(b) Chebychev low-pass requirements

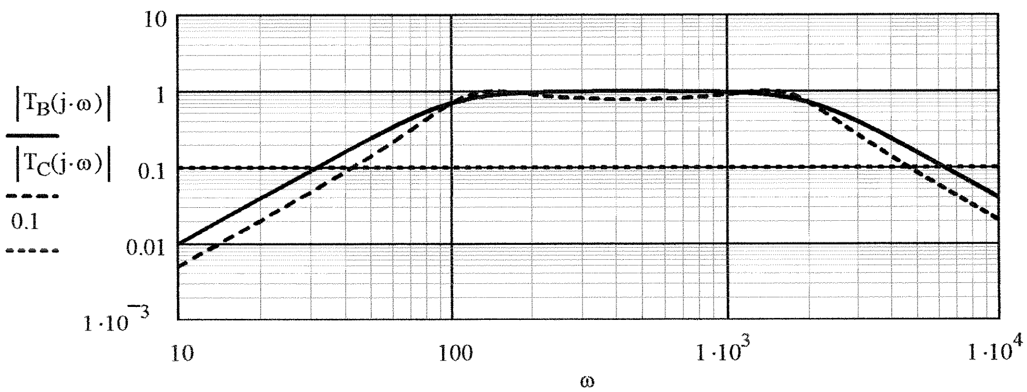
$$\frac{\operatorname{acosh}\left[\sqrt{\left(\frac{T_{MAX}}{T_{MIN}}\right)^2 - 1}\right]}{\operatorname{acosh}\left(\frac{\omega_{MIN2}}{\omega_{C2}}\right)} = 1.448 \quad \text{<--- n = 2}$$

Chebychev high-pass requirements

$$\frac{\operatorname{acosh}\left[\sqrt{\left(\frac{T_{MAX}}{T_{MIN}}\right)^2 - 1}\right]}{\operatorname{acosh}\left(\frac{\omega_{C1}}{\omega_{MIN1}}\right)} = 1.448 \quad \text{<--- n = 2}$$

$$T_C(s) := \left[\frac{(\sqrt{2})^{-1}}{\left(\frac{s}{0.8409\omega_{C2}}\right)^2 + 0.7654\left(\frac{s}{0.8409\omega_{C2}}\right) + 1}\right]\cdot\left[\frac{s^2}{s^2 + 0.7654\left(\frac{\omega_{C1}}{0.8409}\right)\cdot s + \left(\frac{\omega_{C1}}{0.8409}\right)^2}\right]$$

14-46 Continued $\omega := 10, 20 .. 10000$



(c) Butterworth gains

$20 \cdot \log(|T_B(j \cdot 25)|) = -24.099$

$20 \cdot \log(|T_B(j \cdot 100)|) = -3.009$

$20 \cdot \log(|T_B(j \cdot 2000)|) = -3.009$

$20 \cdot \log(|T_B(j \cdot 8000)|) = -24.099$

Chebychev gains

$20 \cdot \log(|T_C(j \cdot 25)|) = -29.83$ <--gains less than -20 dB

$20 \cdot \log(|T_C(j \cdot 100)|) = -2.989$ <---cutoff at 100 rad/s

$20 \cdot \log(|T_C(j \cdot 2000)|) = -2.989$ <---cutoff at 2 krad/s

$20 \cdot \log(|T_C(j \cdot 8000)|) = -29.83$ <--gains less than -20dB

(d) Either is acceptable. Both require two 2nd order filters. The Chebychev has more stopband attenuation. The Butterworth has a better step response.

14-47 Butterworth with $\omega_C := 2000$ $n := 3$ $K := 10^{0.85}$ the low-pass prototype is

$$T_{LP}(s) = \left[\frac{1}{\left(\frac{s}{\omega_C}\right)^2 + \left(\frac{s}{\omega_C}\right) + 1}\right] \cdot \left[\frac{K}{\left(\frac{s}{\omega_C}\right) + 1}\right]$$

The resulting high-pass transfer function is

$$T_{HP}(s) = T_{LP}\left(\frac{\omega_C^2}{s}\right) = \left(\frac{s^2}{s^2 + \omega_C \cdot s + \omega_C^2}\right) \cdot \left(\frac{K \cdot s}{s + \omega_C}\right) = (T_1(s)) \cdot (T_2(s))$$ <---two-stage design

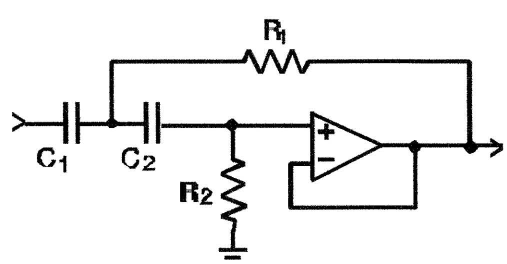
1st stage: use a unity-gain design with $\omega_0 := \omega_C$

and $\zeta := 0.5$ Let $R_2 := 10^4$ then $R_1 := \zeta^2 \cdot R_2$

$C := \left[\left(\sqrt{R_1 \cdot R_2}\right) \cdot \omega_0\right]^{-1}$ $C_1 := C$ $C_2 := C$

$C_1 = 1 \times 10^{-7}$ $C_2 = 1 \times 10^{-7}$

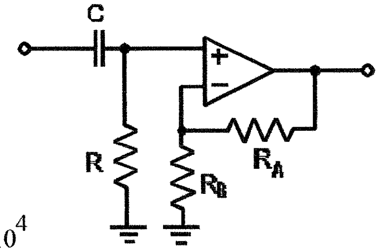
$R_1 = 2.5 \times 10^3$ $R_2 = 1 \times 10^4$



2nd stage: use a first-order design with $RC = \frac{1}{\omega_C}$

and $K = \frac{R_A + R_B}{R_B}$ Let $R := 10^4$ $C := \frac{1}{R \cdot \omega_C}$

then $C = 5 \times 10^{-8}$ let $R_B := 10^4$ then $R_A := R_B \cdot (K - 1)$ $R_A = 6.079 \times 10^4$



14-47 Continued Checking the design

$$T(s) := \frac{R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2}{R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 + (R_1 \cdot C_2 + R_1 \cdot C_1) \cdot s + 1} \cdot \frac{R \cdot C \cdot s}{R \cdot C \cdot s + 1} \cdot \frac{R_A + R_B}{R_B}$$

$20 \cdot \log(|T(j \cdot 10^9)|) = 17$ <---Passband gain is 17 dB

$20 \cdot \log(|T(j \cdot 2000)|) = 13.99$ <---Gain = 17 -14 = 3 dB down at the cutoff frequency

14-48 Butterworth high pass with $f_C := 2000 \quad f_{MIN} := 400 \quad T_{MAX} := 1 \quad T_{MIN} := 10^{-1.25}$

$$\frac{1}{2} \cdot \frac{\ln\left[\left(\frac{T_{MAX}}{T_{MIN}}\right)^2 - 1\right]}{\ln\left(\frac{f_C}{f_{MIN}}\right)} = 1.787$$ Second-order Butterworth required with $\omega_C := 2 \cdot \pi \cdot f_C$

$$T_{HP}(s) = \frac{1}{\left(\frac{\omega_C}{s}\right)^2 + \sqrt{2} \cdot \left(\frac{\omega_C}{s}\right) + 1} = \frac{s^2}{(\omega_C)^2 + \sqrt{2} \cdot (\omega_C) \cdot s + s^2}$$

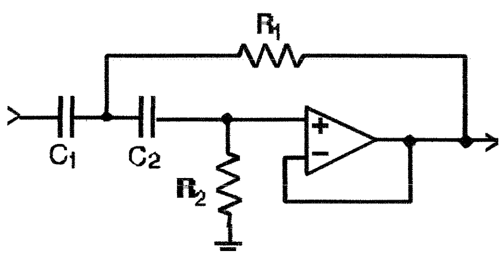
Use unity-gain design with $\omega_0 := \omega_C$

and $\zeta := \frac{\sqrt{2}}{2}$ Let $R_2 := 10^4$ then $R_1 := \zeta^2 \cdot R_2$

$C := \left[\left(\sqrt{R_1 \cdot R_2}\right) \cdot \omega_0\right]^{-1} \quad C_1 := C \quad C_2 := C$

$C_1 = 1.125 \times 10^{-8} \quad C_2 = 1.125 \times 10^{-8}$

$R_1 = 5 \times 10^3 \quad R_2 = 1 \times 10^4$ Checking design



$$T(s) := \frac{R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2}{R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 + [(R_1 \cdot C_1 + R_1 \cdot C_2) \cdot s + 1]}$$

$20 \cdot \log(|T(j \cdot 1000 \cdot \omega_C)|) = -4.344 \times 10^{-12}$ <---0 dB passband gain

$20 \cdot \log(|T(j \cdot 2 \cdot \pi \cdot 2000)|) = -3.01$ <--- Cutoff frequency at 2 kHz

$20 \cdot \log(|T(j \cdot 2 \cdot \pi \cdot 400)|) = -27.966$ <---Gain less than -25dB at 400 Hz

14-49 4th order Butterworth bandpass requirements:
passband gain of 0 dB $\quad \omega_{C1} := 400 \quad \omega_{C2} := 2500$

Butterworth low-pass Butterworth high-pass

$$T(s) := \left[\frac{1}{\left(\frac{s}{\omega_{C2}}\right)^2 + 1.414\left(\frac{s}{\omega_{C2}}\right) + 1}\right] \cdot \left(\frac{s^2}{s^2 + 1.414\omega_{C1} \cdot s + \omega_{C1}^2}\right)$$ <-----4th order Butterworth bandpass response

14-49 Continued

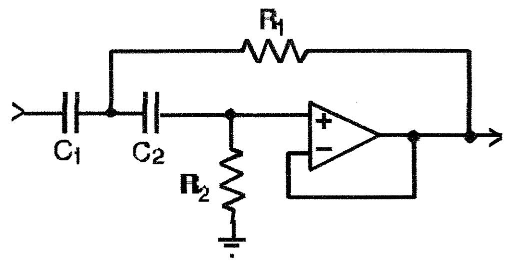
HP-stage use unity-gain design with $\omega_0 := \omega_{C1}$

and $\zeta := \frac{\sqrt{2}}{2}$ Let $R_2 := 10^4$ then $R_1 := \zeta^2 \cdot R_2$

$C := \left[\left(\sqrt{R_1 \cdot R_2}\right) \cdot \omega_0\right]^{-1}$ $C_1 := C$ $C_2 := C$

$C_1 = 3.536 \times 10^{-7}$ $C_2 = 3.536 \times 10^{-7}$

$R_1 = 5 \times 10^3$ $R_2 = 1 \times 10^4$



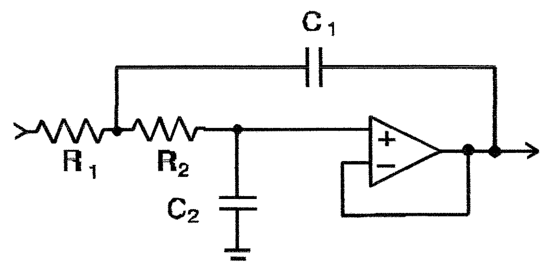
LP-stage use unity-gain design with $\omega_0 := \omega_{C2}$

and $\zeta := \frac{\sqrt{2}}{2}$ Let $C_3 := 10^{-7}$ then $C_4 := \zeta^2 \cdot C_3$

$R := \left[\left(\sqrt{C_3 \cdot C_4}\right) \cdot \omega_0\right]^{-1}$ $R_3 := R$ $R_4 := R$

$C_3 = 1 \times 10^{-7}$ $C_4 = 5 \times 10^{-8}$

$R_3 = 5.657 \times 10^3$ $R_4 = 5.657 \times 10^3$



Checking design

$$T(s) := \frac{R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2}{R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 + \left[\left(R_1 \cdot C_1 + R_1 \cdot C_2\right) \cdot s + 1\right]} \cdot \frac{1}{R_3 \cdot R_4 \cdot C_3 \cdot C_4 \cdot s^2 + \left[\left(R_4 \cdot C_4 + R_3 \cdot C_4\right) \cdot s + 1\right]}$$

$20 \cdot \log\left(\left|T\left(j \cdot \sqrt{400 \cdot 2500}\right)\right|\right) = -0.22$ <---0 dB passband gain

$20 \cdot \log\left(\left|T(j \cdot 2500)\right|\right) = -3.013$ <--- Upper cutoff frequency at 2500 rad/s

$20 \cdot \log\left(\left|T(j \cdot 400)\right|\right) = -3.013$ <---Lower cutoff frequency at 400 rad/s

14-50 (a) Low pass $\omega_C = 3000$ $\omega_{MIN} = 10000$ $T_{MAX} = 1$ $T_{MIN} = 10^{-0.75}$

For Butterworth poles

$$\frac{1}{2} \cdot \frac{\ln\left[\left(10^{.75}\right)^2 - 1\right]}{\ln\left(\frac{10}{3}\right)} = 1.421 \quad n = 2 \quad T_{LP}(s) = \frac{1}{\left(\frac{s}{3000}\right)^2 + \sqrt{2} \cdot \left(\frac{s}{3000}\right) + 1} = \frac{9 \cdot 10^6}{s^2 + \sqrt{2} \cdot 3000 \cdot s + 9 \cdot 10^6}$$

High pass $\omega_C = 40000$ $\omega_{MIN} = 10000$ $T_{MAX} = 1$ $T_{MIN} = 10^{-.75}$

For Butterworth poles

$$\frac{1}{2} \cdot \frac{\ln\left[\left(10^{.75}\right)^2 - 1\right]}{\ln(4)} = 1.234 \quad n = 2$$

$$T_{HP}(s) = \frac{1}{\left(\frac{40000}{s}\right)^2 + \sqrt{2} \cdot \left(\frac{40000}{s}\right) + 1} = \frac{s^2}{1.6 \cdot 10^9 + \sqrt{2} \cdot 40000 \cdot s + s^2}$$

The Butterworth bandstop is $T_{BU}(s) := \left(\frac{9 \cdot 10^6}{s^2 + \sqrt{2} \cdot 3000 \cdot s + 9 \cdot 10^6}\right) + \left(\frac{s^2}{1.6 \cdot 10^9 + \sqrt{2} \cdot 40000 \cdot s + s^2}\right)$

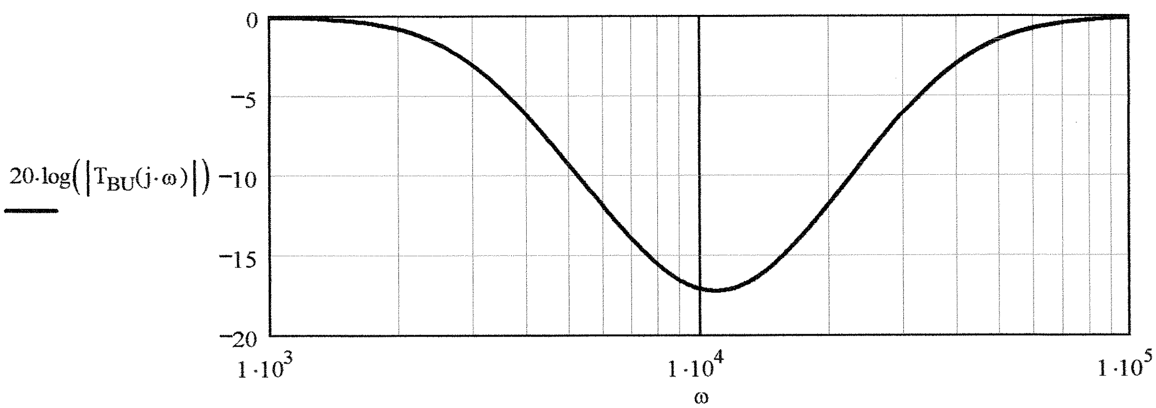
14-50(b) Checking

$20\cdot\log(|T_{BU}(j\cdot 3000)|) = -3.017$ <---lower cutoff at ω = 3 krad/s

$20\cdot\log(|T_{BU}(j\cdot 10000)|) = -17.046$ <---stopband gain less than -15 dB at ω = 10 krad/s

$20\cdot\log(|T_{BU}(j\cdot 40000)|) = -3.017$ <---upper cutoff at ω = 40 krad/s. $T_{BU}(s)$ meets requirements

$\omega := 1000, 1100 .. 10^5$



14-51 The filter requirements are defined by passband gain (0 dB), the minimum stopband gain (-10dB), and the cutoff frequency at 4 Hz. The stopband must begin at 5 Hz which is the lower edge of the adjacent theta band (5-7 Hz). The filter requirements are

Low pass with $\omega_C := 2\cdot\pi\cdot 4$ $\quad T_{MAX} := 1$ $\quad T_{MIN} := \frac{1}{\sqrt{10}}$ $\quad \omega_{MIN} := 2\cdot\pi\cdot 5$

Butterworth order

$$\frac{1}{2}\cdot\frac{\ln\left[\left(\frac{T_{MAX}}{T_{MIN}}\right)^2 - 1\right]}{\ln\left(\frac{\omega_{MIN}}{\omega_C}\right)} = 4.923$$

<---n = 5

Chebychev order

$$\frac{\operatorname{acosh}\left[\sqrt{\left(\frac{T_{MAX}}{T_{MIN}}\right)^2 - 1}\right]}{\operatorname{acosh}\left(\frac{\omega_{MIN}}{\omega_C}\right)} = 2.543$$

<---n = 3

Use 3rd order Chebychev

$$T(s) = \left[\frac{1}{\left(\frac{s}{.9159\cdot\omega_C}\right)^2 + 0.3254\cdot\left(\frac{s}{.9159\cdot\omega_C}\right)^2 + 1}\right]\cdot\left(\frac{1}{\frac{s}{0.2980\cdot\omega_C} + 1}\right) = T_1\cdot T_2$$

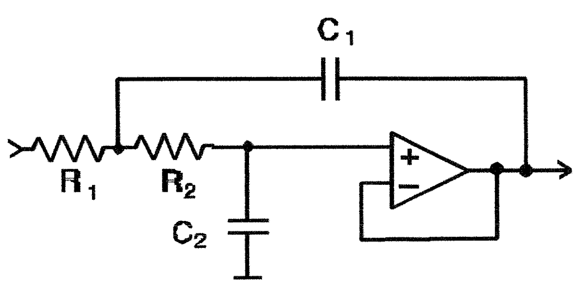
1st stage use unity-gain design with $\omega_0 := \omega_C\cdot 0.9159$

and $\zeta := \frac{0.3254}{2}$ Let $C_1 := 10^{-6}$ then $C_2 := \zeta^2\cdot C_1$

$R := \left[\left(\sqrt{C_1\cdot C_2}\right)\cdot\omega_0\right]^{-1}$ $\quad R_1 := R \quad R_2 := R$

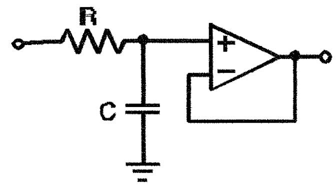
$C_1 = 1\times 10^{-6}$ $\quad C_2 = 2.647\times 10^{-8}$

$R_1 = 2.67\times 10^5$ $\quad R_2 = 2.67\times 10^5$



14-51 Continued. 2nd stage use 1st order unity-gain $\alpha := 0.2980\omega_C$

Let $R := 10^5$ then $C := \frac{1}{\alpha \cdot R}$



$R = 1 \times 10^5$ $\quad C = 1.335 \times 10^{-6}$

Checking the design

$$T(s) := \left[\frac{1}{R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 + (R_2 \cdot C_2 + R_1 \cdot C_2) \cdot s + 1}\right] \cdot \left(\frac{1}{R \cdot C \cdot s + 1}\right)$$

$20 \cdot \log(|T(j \cdot 2 \cdot \pi \cdot 4)|) = -3.01$ <---Cutoff at 4 Hz

$20 \cdot \log(|T(j \cdot 2 \cdot \pi \cdot 5)|) = -12.431$ <---gain less than -10 dB at 5 Hz

$20 \cdot \log(|T(j \cdot 2 \cdot \pi \cdot \sqrt{5 \cdot 7})|) = -18.653$ <---gain in mid Theta band

$20 \cdot \log(|T(j \cdot 2 \cdot \pi \cdot \sqrt{8 \cdot 13})|) = -35.365$ <---gain in mid Alpha band

$20 \cdot \log(|T(j \cdot 2 \cdot \pi \cdot \sqrt{14 \cdot 30})|) = -54.364$ <---gain in mid Beta band

14-52 Existing filter: $T_1(s) := \frac{1}{\left(\frac{s}{2 \cdot \pi \cdot 10^4}\right)^2 + \sqrt{2} \cdot \left(\frac{s}{2 \cdot \pi \cdot 10^4}\right) + 1}$

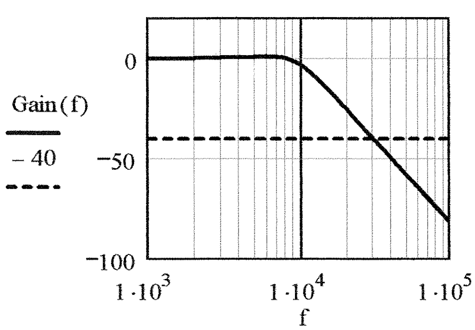
Added filter: $T_2(s, \zeta, \omega_0) := \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + 2 \cdot \zeta \cdot \left(\frac{s}{\omega_0}\right) + 1}$

Design requirement: $\left|T_2(j \cdot 2 \cdot \pi \cdot 10^4, \zeta, \omega_0)\right| = 1$ and $\left|T_2(j \cdot 2 \cdot \pi \cdot 3 \cdot 10^4, \zeta, \omega_0)\right| = 0.1$

$\omega_0 := 2 \cdot \pi \cdot 10^4$ $\quad \zeta := 0.5$ $\quad$ Given $\left|T_2(j \cdot 2 \cdot \pi \cdot 10^4, \zeta, \omega_0)\right| = 1$ $\quad \left|T_2(j \cdot 2 \cdot \pi \cdot 3 \cdot 10^4, \zeta, \omega_0)\right| = 0.1$

$\begin{pmatrix} \zeta \\ \omega_0 \end{pmatrix} := \mathrm{Find}(\zeta, \omega_0)$ $\quad \begin{pmatrix} \zeta \\ \omega_0 \end{pmatrix} = \begin{pmatrix} 0.455 \\ 5.802 \times 10^4 \end{pmatrix}$ $\quad \zeta = 0.455$ $\quad \omega_0 = 5.802 \times 10^4$ $\quad \frac{\omega_0}{2 \cdot \pi} = 9.235 \times 10^3$

$\mathrm{Gain}(f) := 20 \cdot \log(|T_1(j \cdot 2 \cdot \pi \cdot f)| \cdot |T_2(j \cdot 2 \cdot \pi \cdot f, \zeta, \omega_0)|)$ $\quad f := 1000, 2000 .. 100000$

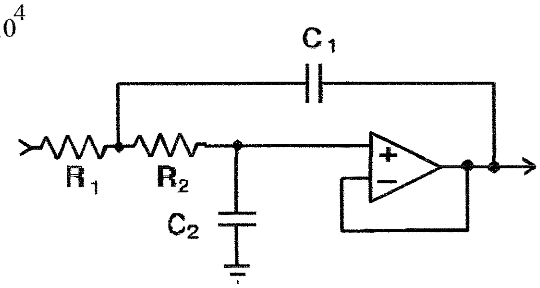


$\mathrm{Gain}(10 \cdot 10^3) = -3.01$ <---Cutoff frequency is unchanged

$20 \cdot \log(|T_1(j \cdot 2 \cdot \pi \cdot 30 \cdot 10^3)|) = -19.138$ <-Original gain at 30 kHz

$\mathrm{Gain}(30 \cdot 10^3) = -39.138$ <---Gain at 30 kHz reduced by 20 dB

14-52 Continued. Use a unity-gain design with $\omega_0 := 5.802\,10^4$ and $\zeta := 0.455$ Let $C_1 := 10^{-8}$ then $C_2 := \zeta^2 \cdot C_1$



$R := \left[\left(\sqrt{C_1 \cdot C_2}\right) \cdot \omega_0\right]^{-1}$ $R_1 := R$ $R_2 := R$

$C_1 = 1 \times 10^{-8}$ $C_2 = 2.07 \times 10^{-9}$

$R_1 = 3.788 \times 10^3$ $R_2 = 3.788 \times 10^3$

checking the circuit design

$$T_{ckt}(s) := \frac{1}{R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 + \left(R_1 \cdot C_1 + R_2 \cdot C_2 + R_1 \cdot C_2 - R_1 \cdot C_1\right) \cdot s + 1}$$

$$\text{Gain}(f) := 20 \log\left(\left|T_1(j \cdot 2 \cdot \pi \cdot f)\right| \cdot \left|T_{ckt}(j \cdot 2 \cdot \pi \cdot f)\right|\right)$$

$\text{Gain}\left(10 \cdot 10^3\right) = -3.015$ <---cutoff frequency unchanged

$20 \log\left(\left|T_1\left(j \cdot 2 \cdot \pi \cdot 30 \cdot 10^3\right)\right|\right) = -19.138$ <---original gain at 30 kHz

$\text{Gain}\left(30 \cdot 10^3\right) = -39.14$ <---gain at 30 kHz reduced by 20 dB

14-53 (a) At cutoff:

$$\left[\frac{|K|}{\sqrt{1 + \left(\frac{\omega_C}{\omega_0}\right)^4}}\right]^n = \frac{(|K|)^n}{\sqrt{2}} \quad \text{or} \quad \left[1 + \left(\frac{\omega_C}{\omega_0}\right)^4\right]^n = 2 \quad \text{or} \quad \frac{\omega_C}{\omega_0} = \left(2^{\frac{1}{n}} - 1\right)^{\frac{1}{4}}$$

(b) In general:

$$\left|T_n(j \cdot \omega)\right| = \frac{(|K|)^n}{\left[1 + \left(\frac{\omega}{\omega_0}\right)^4\right]^{\frac{n}{2}}}$$

At zero freq. $T_n(0) = (|K|)^n$

At high freq. $\left|T_n(j \cdot \omega)\right| = \dfrac{(|K|)^n}{\left(\frac{\omega}{\omega_0}\right)^{2 \cdot n}}$

The low and high freq. asymptotes intersect at the corner freq. i.e. when $(|K|)^n = \dfrac{(|K|)^n}{\left(\frac{\omega}{\omega_0}\right)^{2 \cdot n}}$

hence $\omega_{corner} = \omega_0$

(c) For $\omega_C = 1$ $\quad \omega_0 := \left(2^{\frac{1}{2}} - 1\right)^{-\frac{1}{4}}$

$$\text{Gain}_{BUC}(\omega) := \left[\frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^4}}\right]^2$$

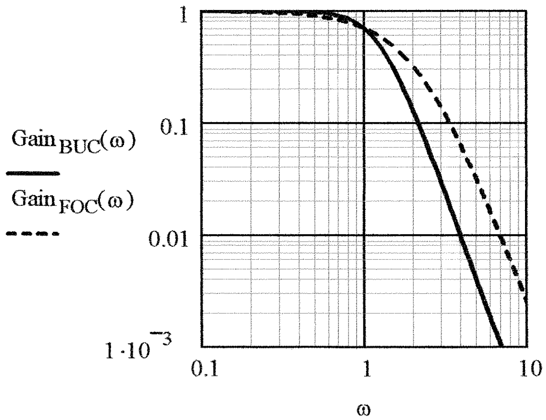
<---2nd order Butterworth cascade gain

$\omega := 0.1, 0.2 .. 10$ $\quad \alpha := \dfrac{1}{\sqrt{2^{\frac{1}{4}} - 1}}$

$$\text{Gain}_{FOC}(\omega) := \left[\frac{1}{\sqrt{1 + \left(\frac{\omega}{\alpha}\right)^2}}\right]^4$$

<---1st order cascade gain

14-53 Continued



The 2nd order Butterworth cascade (the solid line) produces more stopband attenuation than the 1st order cascade (the dashed line).

(d) The 2nd order Butterworth cascade:

(1) Produces a smooth (no ripple) passband response.

(2) Has more stopband attenuation than a 1st order cascade with the same number of poles.

(3) Involves a cascade connection of identical stages

(4) Has a faster step response than either a regular Butterworth or a Chebychev response with the same number of poles.

14-54 (a) The required equalizer is found from the conditions

$$G(s)\cdot T(s) = G(s)\cdot\frac{K_T\cdot s}{s + 2\cdot\pi\cdot 0.1} = \frac{K_T\cdot s}{s + 2\cdot\pi\cdot 0.01} \quad \text{which yields } G(s) = \frac{s + 2\cdot\pi\cdot 0.1}{s + 2\cdot\pi\cdot 0.01} = \frac{s + 0.62832}{s + 0.062832}$$

(b) Use a noninverting amplifier to realize G(s)

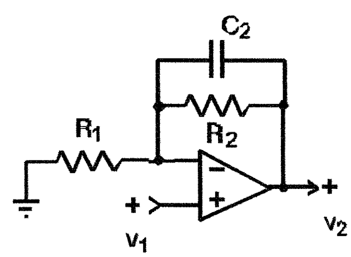
$$\frac{Z_1 + Z_2}{Z_1} = \frac{Y_1 + Y_2}{Y_2} = \frac{s + 0.62832}{s + 0.062832} \quad \text{hence} \quad Y_2 = s + 0.062832 \text{ and } Y_1 = 0.62832 - 0.062832 = 0.565$$

In the prototype Y_2 is a capacitor $C_2 = 1$ in parallel with a resistor $R_2 = \frac{1}{0.062832} = 15.915$ and

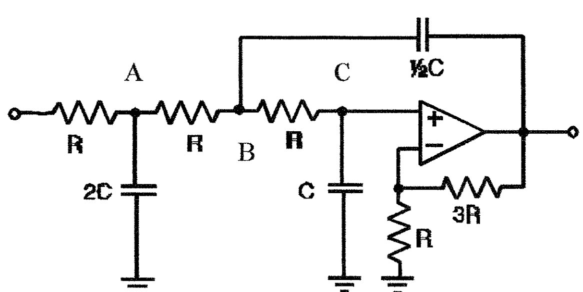
Y_1 is a resistor $R_1 = \frac{1}{0.565} = 1.77$ To get practical elements we use a scale factor of $k_m := 10^5$

hence in the final design $C_2 = 10\ \mu F$, $R_2 = 1.59\ M\Omega$, and $R_1 = 177\ k\Omega$.

This design is shown on the right --->.



14-55 (a) Writing node voltage equations: Note that $V_O = 4\cdot V_C$



$$\begin{pmatrix} \frac{2}{R} + 2\cdot C\cdot s & -\frac{1}{R} & 0 \\ -\frac{1}{R} & \frac{2}{R} + \frac{C\cdot s}{2} & -\frac{1}{R} - 2\cdot C\cdot s \\ 0 & -\frac{1}{R} & \frac{1}{R} + C\cdot s \end{pmatrix}\cdot\begin{pmatrix} V_A \\ V_B \\ V_C \end{pmatrix} = \begin{pmatrix} \frac{V_S}{R} \\ 0 \\ 0 \end{pmatrix}$$

14-55 Continued

$$\Delta(s) = \frac{(R\cdot C\cdot s)^3 + 2\cdot(R\cdot C\cdot s)^2 + 2\cdot(R\cdot C\cdot s) + 1}{R^3} \quad \Delta_C(s) = \frac{V_S(s)}{R^3}$$

$$T_V = \frac{V_O}{V_S} = \frac{4\cdot V_C}{V_S} = \frac{4}{V_S}\cdot\frac{\Delta_C}{\Delta} = \frac{4}{(RCs)^3 + 2\cdot(RCs)^2 + 2\cdot(RCs) + 1} = \frac{4}{\left[(RCs)^2 + (RCs) + 1\right]\cdot(RCs + 1)}$$

Which is of the form of a 3rd order Butterworth low pass response (see Table 14-1) with $\omega_C = 1/RC$ and K = 4. QED

(b) The 3rd order Butterworth requires a 2nd order stage in cascade with a 1st order stage.

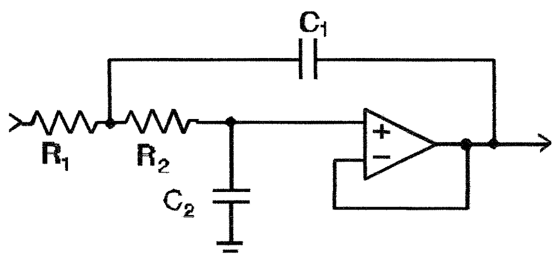
$$T_B(s) = T_1(s)\cdot T_2(s) = \left[\frac{1}{\left(\frac{s}{\omega_C}\right)^2 + \left(\frac{s}{\omega_C}\right) + 1}\right]\cdot\left(\frac{4}{\frac{s}{\omega_C} + 1}\right)$$

1st Stage use unity-gain design

$$T_1(s) = \frac{1}{\left(\frac{s}{\omega_C}\right)^2 + \left(\frac{s}{\omega_C}\right) + 1}$$

$$\zeta = 0.5 \quad C_1 = C \quad C_2 = \zeta^2\cdot C_1 = \frac{C}{4}$$

$$R = \frac{1}{C\cdot\omega_C} \quad R_1 = R \quad R_2 = R$$

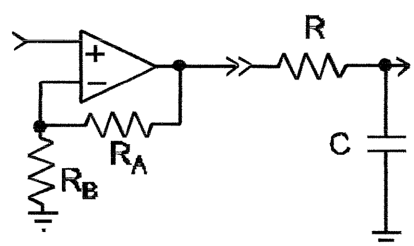


2nd Stage first-order circuit

$$T_2(s) = \frac{4}{\frac{s}{\omega_C} + 1}$$

Let $R\cdot C = \frac{1}{\omega_C}$ and

$R_B = R$ then $R_A = (K - 1)\cdot R_B = 3\cdot R_B$



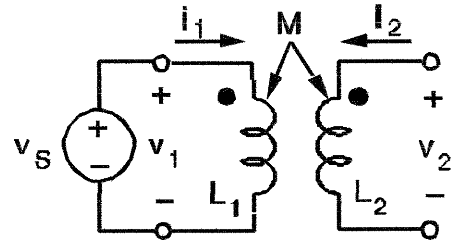
(c) Comparing the two designs:

Design	No. R's	Diff Values	No. C's	Diff Values	No. Op AMPs	OP AMP gains
3rd order ckt	5	R, 3R	3	C/2, C, 2C	1	4
2nd/1st order ckt	5	R, 3R	3	C/4, C	2	1 & 4

(1) The 3rd order design requires one less OP AMP.
(2) Both require gain 4 from OP AMPs. In the 2nd/1st order ckt the Gain-BW could be less for each stage.
(3) The cascade design requires fewer different element values.
(4) The cascade is easier to tune by adjusting its RC products of each stage.

CHAPTER 15, Standard Version
CHAPTER 16, Laplace-Early Version

15/16-1



(a) $L_1 := 10 \cdot 10^{-3}$ $L_2 := 5 \cdot 10^{-3}$ $M := 7 \cdot 10^{-3}$

$i_2(t) := 0$ $v_S(t) := 10 \cdot \sin(2000 \cdot t)$

$v_1(t) = L_1 \cdot \frac{d}{dt} i_1(t) + M \cdot \frac{d}{dt} i_2(t)$ $v_2(t) = M \cdot \frac{d}{dt} i_1(t) + L_2 \cdot \frac{d}{dt} i_2(t)$

(b) $v_1(t) = v_S(t) = 10 \cdot \sin(2000 \cdot t)$ with $i_2(t) = 0$

$v_1(t) = L_1 \cdot \frac{d}{dt} i_1(t)$ $\frac{d}{dt} i_1(t) = \frac{1}{L_1} \cdot v_1(t)$ $v_2(t) = M \cdot \frac{d}{dt} i_1(t) = \frac{M}{L_1} \cdot v_1(t) = 7 \cdot \sin(2000 \cdot t)$

15/16-2 See figure in Prob.15-1 $L_1 := 10 \cdot 10^{-3}$ $L_2 := 5 \cdot 10^{-3}$ $M := 7 \cdot 10^{-3}$ $v_S(t) = 5 \cdot \sin(500 \cdot t)$

(a) $v_1(t) = L_1 \cdot \frac{d}{dt} i_1(t) + M \cdot \frac{d}{dt} i_2(t)$ $v_2(t) = M \cdot \frac{d}{dt} i_1(t) + L_2 \cdot \frac{d}{dt} i_2(t)$

(b) $v_1(t) = v_S(t) = 5 \cdot \sin(500 \cdot t)$ with $v_2(t) = 0$ $\frac{d}{dt} i_1(t) = \frac{-L_2}{M} \cdot \frac{d}{dt} i_2(t)$

$$v_1(t) = L_1 \cdot \left(-\frac{L_2}{M} \cdot \frac{d}{dt} i_2(t) \right) + M \cdot \frac{d}{dt} i_2(t) = \left(\frac{M^2 - L_1 \cdot L_2}{M} \right) \cdot \frac{d}{dt} i_2(t) = 5 \cdot \sin(500 \cdot t)$$

$$i_2(t) = \frac{-5 \cdot M}{\left(M^2 - L_1 \cdot L_2\right) \cdot 500} \cdot \cos(500 \cdot t) = 70 \cdot \cos(500 \cdot t) \quad i_1(t) = \frac{-L_2}{M} \cdot i_2(t) = -50 \cdot \cos(1000 \cdot t)$$

15/16-3 See figure in Prob.15-1 $L_1 := 10 \cdot 10^{-3}$ $L_2 := 5 \cdot 10^{-3}$ $M := 7 \cdot 10^{-3}$

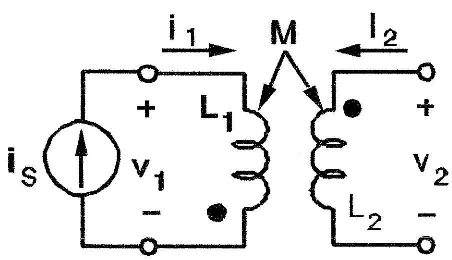
(a) $v_1(t) = L_1 \cdot \frac{d}{dt} i_1(t) + M \cdot \frac{d}{dt} i_2(t)$ $v_2(t) = M \cdot \frac{d}{dt} i_1(t) + L_2 \cdot \frac{d}{dt} i_2(t)$

(b) $i_2(t) = 35 \cdot \sin(500 \cdot t)$ with $v_2(t) = 0$ $\frac{d}{dt} i_1(t) = \frac{-L_2}{M} \cdot \frac{d}{dt} i_2(t)$

$$v_1(t) = L_1 \cdot \left(-\frac{L_2}{M} \cdot \frac{d}{dt} i_2(t) \right) + M \cdot \frac{d}{dt} i_2(t) = \left(\frac{M^2 - L_1 \cdot L_2}{M} \right) \cdot \left(\frac{d}{dt} i_2(t) \right) = \left(\frac{M^2 - L_1 \cdot L_2}{M} \right) \cdot 1.75 \cdot 10^4 \cdot \cos(500 \cdot t)$$

$v_1(t) = -2.5 \cdot \cos(500 \cdot t)$ $v_S(t) = v_1(t) = -2.5 \cdot \cos(500 \cdot t)$

15/16-4 $L_1 := 3 \cdot 10^{-3}$ $L_2 := 3 \cdot 10^{-3}$ $M := 2 \cdot 10^{-3}$ $i_S(t) = 10 \cdot \sin(500 \cdot t)$



(a) $v_1(t) = L_1 \cdot \frac{d}{dt} i_1(t) - M \cdot \frac{d}{dt} i_2(t)$

$v_2(t) = -M \cdot \frac{d}{dt} i_1(t) + L_2 \cdot \frac{d}{dt} i_2(t)$

(b) $i_1(t) = i_S(t) = 10 \cdot \sin(500 \cdot t)$ with $i_2(t) = 0$

$v_1(t) = L_1 \cdot \frac{d}{dt} i_1(t)$ hence

$v_1(t) = L_1 \cdot 10 \cdot 500 \cdot \cos(500 \cdot t) = 15 \cdot \cos(500 \cdot t)$ $v_2(t) = -M \cdot \frac{d}{dt} i_1(t) = -M \cdot 10 \cdot 500 \cdot \cos(500 \cdot t) = -10 \cdot \cos(500 \cdot t)$

15/16-5 See Prob. 15-4 for figure $L_1 := 3\cdot 10^{-3}$ $L_2 := 3\cdot 10^{-3}$ $M := 2\cdot 10^{-3}$ $i_2(t) = 0.5\cdot\sin(1000\cdot t)$

with $v_2(t) = 0$ $\quad -M\cdot\frac{d}{dt}i_1(t) + L_2\cdot\frac{d}{dt}i_2(t) = 0$ $\quad i_1(t) = \frac{L_2}{M}\cdot i_2(t) = 0.75\cdot\sin(1000\cdot t)$

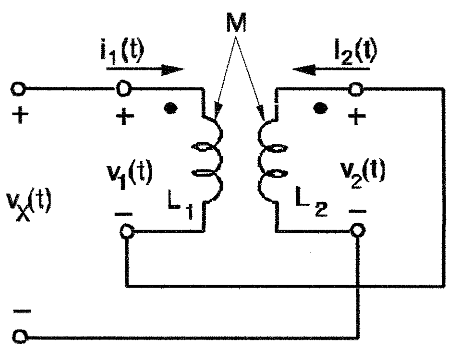
$$v_1(t) = L_1\cdot\frac{d}{dt}i_1(t) - M\cdot\frac{d}{dt}i_2(t) = 3\cdot 10^{-3}\cdot\frac{d}{dt}0.75\cdot\sin(1000\cdot t) - 2\cdot 10^{-3}\cdot\frac{d}{dt}0.5\sin(1000\cdot t) = 1.25\cdot\cos(1000\cdot t)$$

15/16-6 $L_1 := 1.2$ $\quad L_2 := 2.4$ $\quad v_1(t) = 3\cdot\sin(1000\cdot t)$ $\quad v_2(t) = 4\cdot\sin(1000\cdot t)$

with $i_2(t) = 0$ $\quad v_2(t) = \frac{M}{L_1}\cdot\left[L_1\cdot\left(\frac{d}{dt}i_1(t)\right)\right] = \frac{M}{L_1}\cdot v_1(t)$ hence $4 = \frac{M}{L_1}\cdot 3$ $\quad M = \frac{4\cdot L_1}{3} = 1.6$

$k = M\cdot(L_1\cdot L_2)^{-0.5} = 0.943$ The coupling is additive because v_1 and v_2 have the same polarity.

15/16-7 $L_1 := 10\cdot 10^{-3}$ $\quad L_2 := 10\cdot 10^{-3}$ $\quad M := 9\cdot 10^{-3}$ $\quad i_1(t) = 2\cdot\cos(1000\cdot t)$



By KCL $i_2(t) = i_1(t)$ hence

$$v_1(t) = L_1\cdot\frac{d}{dt}i_1(t) + M\cdot\frac{d}{dt}i_1(t) = 19\cdot 10^{-3}\cdot\frac{d}{dt}2\cdot\cos(1000\cdot t)$$

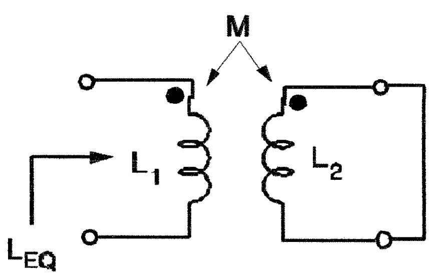
$v_1(t) = -38\cdot\sin(1000\cdot t)$ likewise

$$v_2(t) = M\cdot\frac{d}{dt}i_1(t) + L_2\cdot\frac{d}{dt}i_1(t) = 19\cdot 10^{-3}\cdot\frac{d}{dt}2\cdot\cos(1000\cdot t)$$

$v_2(t) = -38\cdot\sin(1000\cdot t)$

by KVL $\quad v_x(t) = v_1(t) + v_2(t) = -76\cdot\sin(1000\cdot t)$

15/16-8 with $v_2 = 0$ the i-v relationships are $\quad v_1(t) = L_1\cdot\frac{d}{dt}i_1(t) + M\cdot\frac{d}{dt}i_2(t)$ $\quad M\cdot\frac{d}{dt}i_1(t) + L_2\cdot\frac{d}{dt}i_2(t) = 0$



or $\frac{d}{dt}i_2(t) = \frac{-M}{L_2}\cdot\frac{d}{dt}i_1(t)$

and $v_1(t) = \left(L_1 - \frac{M^2}{L_2}\right)\cdot\frac{d}{dt}i_1(t)$ $\quad L_{EQ} = \frac{L_1\cdot L_2 - M^2}{L_2}$

$L_1 := 75\cdot 10^{-3}$ $\quad L_2 := 0.4$ $\quad k := 0.97$ $\quad M := k\cdot\sqrt{L_1\cdot L_2}$

$M = 0.168$ $\quad L_{EQ} := \frac{L_1\cdot L_2 - M^2}{L_2}$ $\quad L_{EQ} = 4.432\times 10^{-3}$

15/16-9 $L_1 := 40\cdot 10^{-3}$ $\quad L_2 := 60\cdot 10^{-3}$ for $k := 1$ $\quad M := k\cdot\sqrt{L_1\cdot L_2}$ $\quad M = 4.899\times 10^{-2}$

15/16-10 $L_1 := 0.11$ $\quad L_2 := 0.266$ $\quad L_{EQ} := 0.237$ since $L_{EQ} = 0.237 < L_1 + L_2 = 0.376$ then

by Example (15-2) $L_{EQ} = L_1 + L_2 - 2\cdot M$ $\quad M := \frac{L_1 + L_2 - L_{EQ}}{2}$ $\quad M = 6.95\times 10^{-2}$

$k := \frac{M}{\sqrt{L_1\cdot L_2}}$ $\quad k = 0.406$

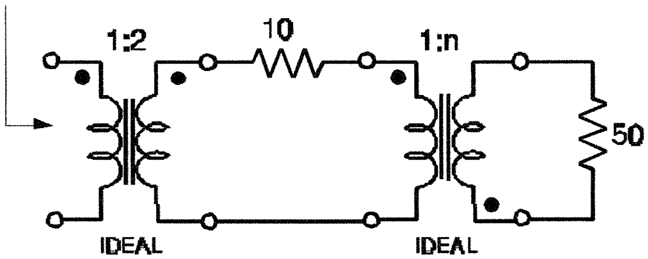
15/16-11 (a) $\frac{N_2}{N_1} = \frac{480}{120} = 4$ **(b)** $v_1(t) = 120\cdot\cos(2\cdot\pi\cdot 60\cdot t)$ $R_{IN} := \left(\frac{1}{4}\right)^2\cdot 800$ $R_{IN} = 50$

$i_1(t) = \frac{v_1(t)}{R_{IN}} = 2.4\cdot\cos(2\cdot\pi\cdot 60\cdot t)$ A

15/16-12 $n = \frac{500}{50} = 10$ $v_S(t) = 120\cdot\cos(2\cdot\pi\cdot 60\cdot t)$ $R_S := 4$ $R_L := 600$ $R_{IN} := \left(\frac{1}{10}\right)^2\cdot R_L$

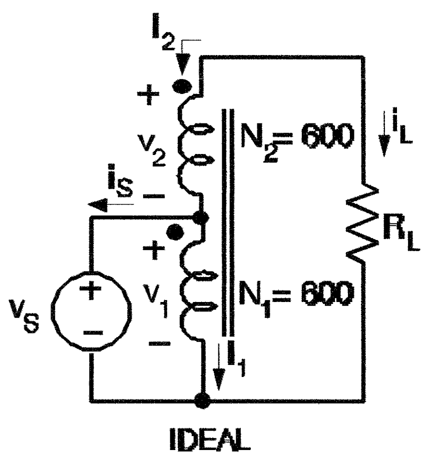
$R_{IN} = 6$ $i_1(t) = \frac{v_S(t)}{R_S + R_{IN}} = 12\cdot\cos(2\cdot\pi\cdot 60\cdot t)$ $i_2(t) = \frac{-1}{n}\cdot i_1(t) = -1.2\cdot\cos(2\cdot\pi\cdot 60\cdot t)$ A

15/16-13 for $n := \frac{1}{3}$ and $R_{EQ1} := \frac{50}{n^2}$ hence $R_{EQ} := \frac{10 + R_{EQ1}}{2^2}$ $R_{EQ} = 115$



15/16-14 See Prob 15-13 above for figure. For equal power $R_{EQ1} = \frac{50}{n^2} = 10$ hence $n = \sqrt{\frac{50}{10}} = 2.236$

15/16-15 ideal transformer constraints:



$n = \frac{N_2}{N_1} = 1$ $v_1(t) = v_2(t)$ $i_1(t) = -i_2(t)$

KVL: $v_1(t) = v_S(t)$ $v_L(t) = v_1(t) + v_2(t) = 2\cdot v_S(t)$

KCL: $i_L(t) = -i_2(t)$ $i_S(t) = i_2(t) - i_1(t) = -2\cdot i_L(t)$

Load: $i_S(t) = -2\cdot i_L(t) = -2\cdot\frac{v_L(t)}{R_L} = \frac{-4\cdot v_S(t)}{R_L}$

$v_S(t) = 200\cdot\sin(400\cdot t)$ V $R_L := 50$

$i_S(t) = \frac{-4\cdot 200\cdot\sin(400\cdot t)}{50} = -16\cdot\sin(400\cdot t)$ A

$i_L(t) = \left(\frac{-1}{2}\right)\cdot(-16\cdot\sin(400\cdot t)) = 8\cdot\sin(400\cdot t)$ A

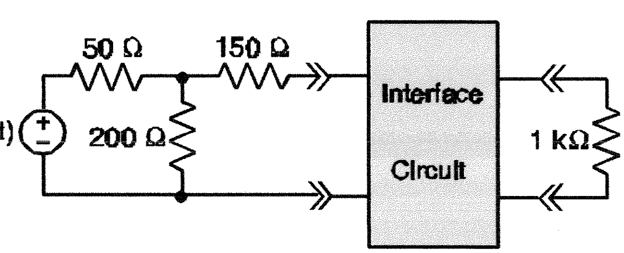
15/16-16 The look-back resistance in the secondary with $n := 5$ and $R_S := 100$ is $R_{EQ} := n^2\cdot R_S$

$R_{EQ} = 2.5 \times 10^3$ Max power requires $R_L = R_{EQ} = 2500\ \Omega$

15/16-17 $R_T := 150 + \frac{50\cdot 200}{(50 + 200)}$ $R_T = 190$

$R_L := 1000$

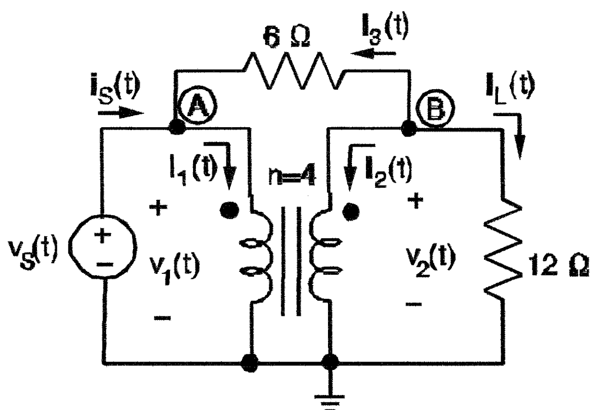
$n := \sqrt{\frac{R_L}{R_T}}$ $n = 2.294$



15/16-18 $v_T(t) = 5\cdot\sin(1000\cdot t)$ $\quad R_T := 50 \quad n := 5 \quad R_L := 300 \quad R_{EQ} := \dfrac{R_L}{n^2} \quad R_{EQ} = 12$

$$i_1(t) = \frac{v_T(t)}{R_T + R_{EQ}} = 0.0806\cdot\sin(1000\cdot t) \qquad p_L(t) = i_1(t)^2\cdot R_{EQ} = 0.0780\cdot\sin(1000\cdot t)^2 \text{ W}$$

15/16-19



$n := 4 \quad v_S := 12$

for ideal transformer $v_2 := n\cdot v_S \quad v_2 = 48$

by ohm's law $i_L := \dfrac{v_2}{12} \quad i_L = 4$

by KVL and Ohm's law $i_3 := \dfrac{v_2 - v_S}{6} \quad i_3 = 6$

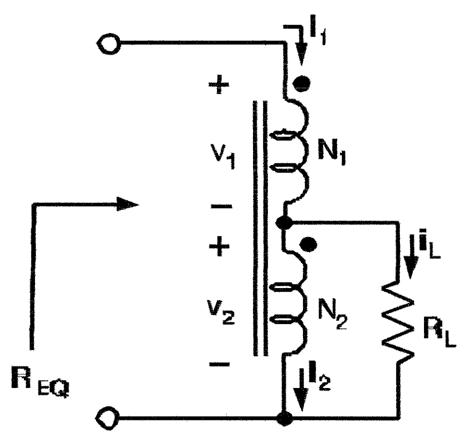
by KCL at node B $i_2 := -(i_L + i_3) \quad i_2 = -10$

for ideal transformer $i_1 := -n\cdot i_2 \quad i_1 = 40$

by KCL at node A $i_S := i_1 - i_3$ So finally

$i_S = 34$

15/16-20



for ideal transformer $N_1\cdot v_2 = N_2\cdot v_1$ and

$N_2\cdot i_2 = -N_1\cdot i_1$ by Ohm's law $i_L = \dfrac{v_2}{R_L}$

by KCL $i_1 = i_2 + i_L = \dfrac{-N_1}{N_2}\cdot i_1 + \dfrac{v_2}{R_L}$ hence

$i_1 = \dfrac{N_2}{N_1 + N_2}\cdot\dfrac{v_2}{R_L}$ by definition

$$R_{EQ} = \frac{v_1 + v_2}{i_1} = \left(\frac{N_1}{N_2} + 1\right)\cdot v_2\cdot\left(\frac{N_1 + N_2}{N_2}\right)\cdot\frac{R_L}{v_2}$$

so finally $R_{EQ} = \left(\dfrac{N_1 + N_2}{N_2}\right)^2\cdot R_L$

15/16-21 $L_1 := 0.03 \quad L_2 := 0.12 \quad M := 0.05 \quad V_1 := 20 + j\cdot 0 \qquad \omega := 1000$

$X_1 := \omega\cdot L_1 \quad X_2 := \omega\cdot L_2 \quad X_M := \omega\cdot M \qquad X_1 = 30 \quad X_2 = 120 \quad X_M = 50$

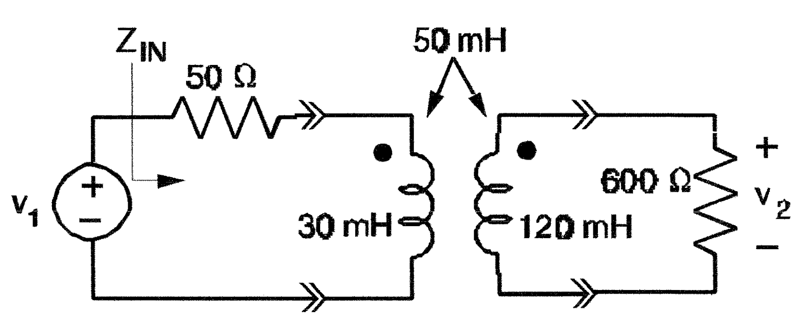
$I_A := 1 + j\cdot 1 \quad I_B := 1 + j\cdot 1$ <----Initial guesses

Given

$(50 + j\cdot X_1)\cdot I_A - j\cdot X_M\cdot I_B = 20$

$-j\cdot X_M\cdot I_A + (600 + j\cdot X_2)\cdot I_B = 0$

$\begin{pmatrix} I_A \\ I_B \end{pmatrix} := \text{Find}(I_A, I_B)$



15/16-21 Continued

$$\begin{pmatrix} I_A \\ I_B \end{pmatrix} = \begin{pmatrix} 2.866 \times 10^{-1} - 1.549i \times 10^{-1} \\ 1.701 \times 10^{-2} + 2.048i \times 10^{-2} \end{pmatrix} \qquad Z_{IN} := \frac{V_1}{I_A} \qquad Z_{IN} = 54.006 + 29.199i$$

$$V_2 := I_B \cdot 600 \qquad |V_2| = 15.972 \qquad \frac{180}{\pi} \cdot \arg(V_2) = 50.292 \qquad v_2(t) = 15.97 \cdot \cos(1000 \cdot t + 50.29^{o})\ V$$

15/16-22 $L_1 := 0.2 \quad L_2 := 0.45 \quad k := 0.98 \qquad M := k \cdot \sqrt{L_1 \cdot L_2} \quad M = 0.294 \quad R_L := 75 \qquad \omega := 377$

$$X_1 := \omega \cdot L_1 \qquad X_2 := \omega \cdot L_2 \qquad X_M := \omega \cdot M \quad Z_L := R_L \qquad V_S := 100 + j \cdot 0$$

Initial guesses for solve block

$$I_1 := 1 + j \qquad I_2 := 1 + j$$

$$V_1 := 100 \qquad V_2 := 100 + j \cdot 100$$

Given

Coupled coil constraints

$$V_1 = j \cdot X_1 \cdot I_1 + j \cdot X_M \cdot I_2$$

$$V_2 = j \cdot X_M \cdot I_1 + j \cdot X_2 \cdot I_2$$

Connection constraints

$$V_2 = -Z_L \cdot I_2 \qquad V_1 = V_S$$

Solution

(a) $$\begin{pmatrix} I_1 \\ I_2 \\ V_1 \\ V_2 \end{pmatrix} := \text{Find}(I_1, I_2, V_1, V_2) \qquad \begin{pmatrix} I_1 \\ I_2 \\ V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 2.858 - 1.582i \\ -1.944 + 0.174i \\ 100 \\ 145.83 - 13.063i \end{pmatrix}$$

(b) $$Z_{IN} := \frac{V_1}{I_1} \qquad Z_{IN} = 26.78 + 14.825i \quad \Omega$$

(c) $$P_L := \frac{(|I_2|)^2}{2} \cdot R_L \quad P_L = 142.913$$

$$P_S := P_L \qquad P_S = 142.913 \quad W$$

15/16-23 $L_1 := 0.01$ $L_2 := 0.06$ $M := 0.02$ $V_S := 20 + j\cdot 0$ $\omega := 2\cdot 10^4$

$X_1 := \omega\cdot L_1$ $X_2 := \omega\cdot L_2$ $X_M := \omega\cdot M$ $X_1 = 200$ $X_2 = 1.2\times 10^3$ $X_M = 400$

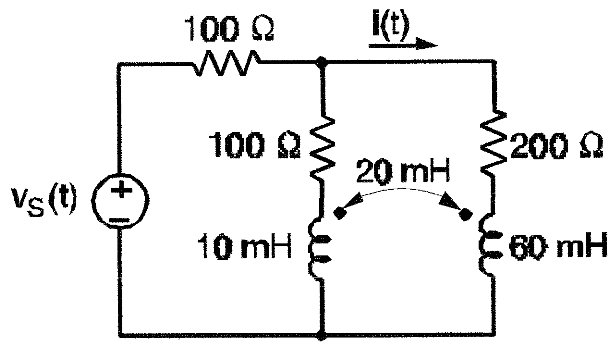
$I_1 := 1 + j$ $I_2 := 1 + j$ $V_1 := 10 + j\cdot 10$ $V_2 := 10 + j\cdot 10$ <---Initial guesses

Given

KVL constraints

$0 = -20 + 100\cdot(I_1 + I_2) + 100\cdot I_1 + V_1$

$0 = -20 + 100\cdot(I_1 + I_2) + 200\cdot I_2 + V_2$



Coupled coil constraints

$V_1 = j\cdot X_1\cdot I_1 + j\cdot X_M\cdot I_2$

$V_2 = j\cdot X_M\cdot I_1 + j\cdot X_2\cdot I_2$

Solution

$$\begin{pmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{pmatrix} := \text{Find}(V_1, V_2, I_1, I_2) \qquad \begin{pmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 8.114 + 6.166i \\ 18.824 + 4.706i \\ 0.069 - 0.028i \\ -0.019 - 6.491i\times 10^{-3} \end{pmatrix}$$

$I := I_2$ $|I| = 2.014\times 10^{-2}$ $30\cdot\pi^{-1}\cdot \arg(I) = -161.2$ $i(t) := 2.014\cdot 10^{-2}\cdot\cos(2\cdot 10^4\cdot t - 161.2)$ A

15/16-24 $L_1 := 0.01$ $L_2 := 0.06$ $M := 0.02$ $V_S := 20 + j\cdot 0$ $\omega := 10^4$

$X_1 := \omega\cdot L_1$ $X_2 := \omega\cdot L_2$ $X_M := \omega\cdot M$ $X_1 = 100$ $X_2 = 600$ $X_M = 200$

$I_1 := 1 + j$ $I_2 := 1 + j$ $V_1 := 10 + j\cdot 10$ $V_2 := 10 + j\cdot 10$ <---Initial guesses

Given

KVL constraints

$0 = -20 + 100\cdot(I_1 + I_2) + 100\cdot I_1 + V_1$

$0 = -20 + 100\cdot(I_1 + I_2) + 200\cdot I_2 + V_2$

Coupled coil constraints

$V_1 = j\cdot X_1\cdot I_1 + j\cdot X_M\cdot I_2$

$V_2 = j\cdot X_M\cdot I_1 + j\cdot X_2\cdot I_2$

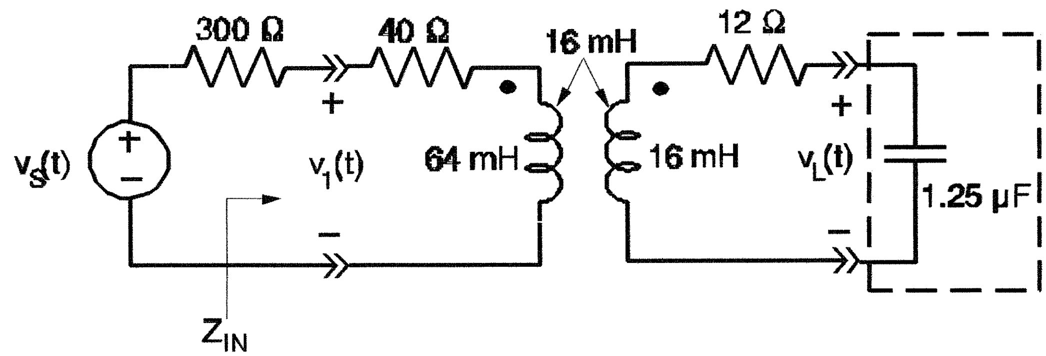
Solution

$$\begin{pmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{pmatrix} := \text{Find}(V_1, V_2, I_1, I_2) \qquad \begin{pmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 5.846 + 5.231i \\ 1.6\times 10^1 + 8i \\ 7.692\times 10^{-2} - 1.538i\times 10^{-2} \\ -1.231\times 10^{-2} - 2.154i\times 10^{-2} \end{pmatrix}$$

$I := I_2$ $|I| = 2.481\times 10^{-2}$ $180\cdot\pi^{-1}\cdot \arg(I) = -119.745$ $i(t) = 2.481\cdot 10^{-2}\cdot\cos(10^4\cdot t - 119.7)$ A

15/16-25 $L_1 := 0.064 \quad L_2 := 0.016 \quad M := 0.016 \quad \omega := 2000 \quad V_S := 0 - j \cdot 150 \quad Z_L := \dfrac{1}{\left(j \cdot \omega \cdot 1.25 \cdot 10^{-6}\right)}$

$X_1 := \omega \cdot L_1 \quad X_2 := \omega \cdot L_2 \quad X_M := \omega \cdot M$ **Inital guesses for the solve block:** $I_A := 1 + j \cdot 1 \quad I_B := 1 + j \cdot 1$



Given

$V_S = (300 + 40 + j \cdot X_1) \cdot I_A - j \cdot (X_M \cdot I_B) \qquad 0 = -j \cdot X_M \cdot I_A + (j \cdot X_2 + 12 + Z_L) \cdot I_B$

$\begin{pmatrix} I_A \\ I_B \end{pmatrix} := \text{Find}(I_A, I_B) \qquad \begin{pmatrix} I_A \\ I_B \end{pmatrix} = \begin{pmatrix} -0.148 - 0.384i \\ 0.014 + 0.033i \end{pmatrix}$

$Z_{IN} := \dfrac{V_S}{I_A} - 300 \qquad Z_{IN} = 40.091 + 130.78i \qquad Z_L = -400i$

$V_1 := V_S - I_A \cdot 300 \qquad V_1 = 44.327 - 34.728i \qquad V_L := I_B \cdot Z_L \qquad V_L = 13.183 - 5.569i \text{ V}$

15/16-26 Use the equivalent T-ckt $L_1 := 0.064 \quad L_2 := 0.016 \quad M := 0.016$

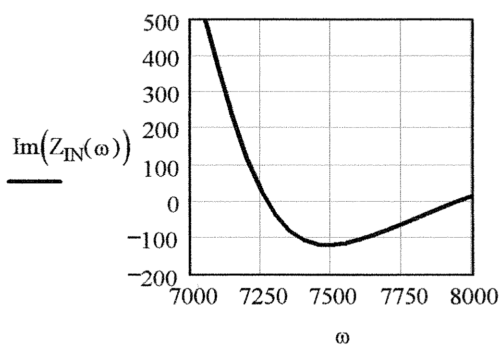
$X_1(\omega) := \omega \cdot (L_1 + M) \quad X_2(\omega) := \omega \cdot (L_2 + M)$

$X_M(\omega) := -\omega \cdot M$

$Z_L(\omega) := 12 + \left(j \cdot \omega \cdot 1.25 \cdot 10^{-6}\right)^{-1}$

$Z_{IN}(\omega) := 40 + j \cdot X_1(\omega) + \dfrac{1}{\dfrac{1}{j \cdot X_M(\omega)} + \dfrac{1}{j \cdot X_2(\omega) + Z_L(\omega)}}$

$\omega := 7000, 7050 .. 8000$



There are two solutions

$\omega := 7400$ Given $\quad \text{Im}(Z_{IN}(\omega)) = 0$

$\text{Find}(\omega) = 7271.6$ <---First solution

$Z_{IN}(7271.6) = 922.56 - 0.061i$ <---Checking

$\omega := 8000$ Given $\quad \text{Im}(Z_{IN}(\omega)) = 0$

$\text{Find}(\omega) = 7939.8$ <---Second solution

$Z_{IN}(7939.8) = 272.049 - 0.014i$ <---Checking

15/16-27 $L_1 := 0.5 \quad L_2 := 0.5 \quad M := 0.25 \; \omega := 4000$

$V_S := 10 + j\cdot 0 \qquad Z_L := \frac{1}{\left(2000^{-1} + j\cdot\omega\cdot 125\cdot 10^{-9}\right)}$

$X_1 := \omega\cdot L_1 \quad X_2 := \omega\cdot L_2 \quad X_M := \omega\cdot M$

Inital guesses for the solve block

$I_1 := 1 + j \quad I_2 := 1 + j \quad V_1 := 1 + j \quad V_2 := 1 + j$

Given

Coupled coil constraints

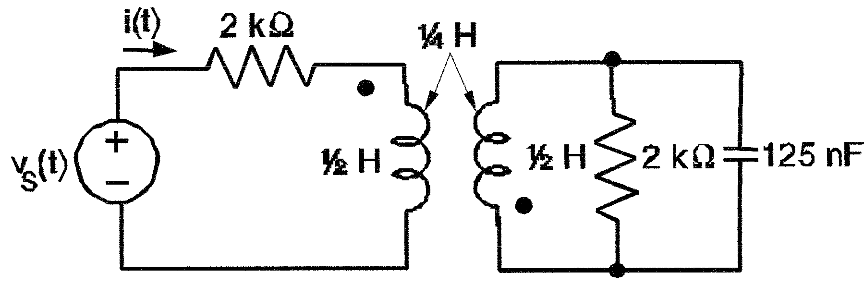
$V_1 = j\cdot X_1\cdot I_1 - j\cdot X_M\cdot I_2$

$V_2 = -j\cdot X_M\cdot I_1 + j\cdot X_2\cdot I_2$

Connection Constraints

$V_2 = -Z_L\cdot I_2$

$V_S - 2000\cdot I_1 = V_1$



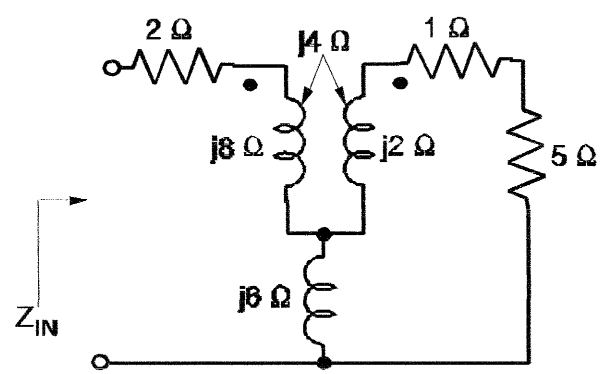
Solution

$$\begin{pmatrix} I_1 \\ I_2 \\ V_1 \\ V_2 \end{pmatrix} := \text{Find}(I_1, I_2, V_1, V_2) \qquad \begin{pmatrix} I_1 \\ I_2 \\ V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 2.941\times 10^{-3} - 1.765i\times 10^{-3} \\ 2.353\times 10^{-3} + 5.882i\times 10^{-4} \\ 4.118 + 3.529i \\ -2.941 + 1.765i \end{pmatrix}$$

$Z_{IN} := \frac{V_S}{I_1} \qquad Z_{IN} = 2.5\times 10^3 + 1.5i\times 10^3 \qquad |I_1| = 3.43\times 10^{-3} \quad \frac{180}{\pi}\cdot\arg(I_1) = -30.964$

$i_1(t) = 3.43\cdot 10^{-3}\cdot\cos\left(4000\cdot t - 30.964^o\right)\text{ A} \quad P_{IN} := \frac{\left(|I_1|\right)^2}{2}\cdot\text{Re}(Z_{IN}) \qquad P_{IN} = 1.471\times 10^{-2} \text{ W}$

15/16-28 Write Mesh Eqs.



$V_S = (2 + j\cdot 8 + j\cdot 6)\cdot I_A - (j\cdot 4 + j\cdot 6)\cdot I_B$

$0 = -(j\cdot 4 + j\cdot 6)\cdot I_A + (1 + 5 + j\cdot 2 + j\cdot 6)\cdot I_B$

$\Delta := (2 + j\cdot 14)\cdot(6 + j\cdot 8) - (j\cdot 10)^2$

$\Delta = 100i$

$\Delta_A := (6 + j\cdot 2 + j\cdot 6)\cdot V_S$

$I_A = \frac{\Delta_A}{\Delta} = \frac{(6 + j\cdot 8)\cdot V_S}{j\cdot 100} \qquad Z_{EQ} = \frac{V_S}{I_A} \qquad Z_{EQ} := \frac{j\cdot 100}{6 + j\cdot 8} \qquad Z_{EQ} = 8 + 6i \quad \Omega$

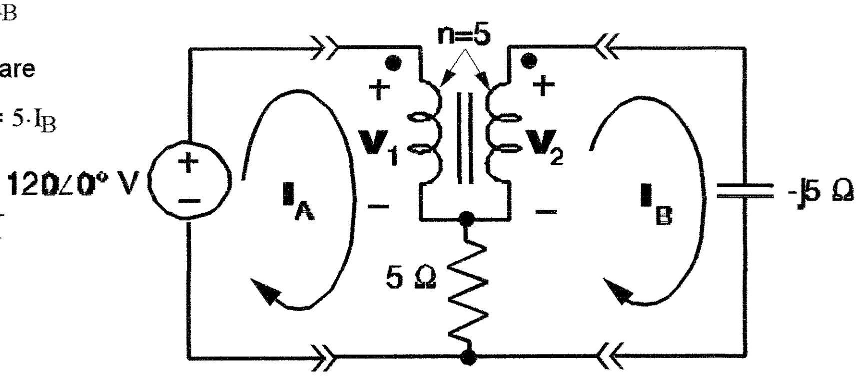
15/16-29 Write KVL Eqs.

$120 = V_1 + 5\cdot(I_A - I_B)$

$0 = 5\cdot(I_B - I_A) - V_2 + (-j\cdot 5)\cdot I_B$

The ideal xformer constraints are

$V_2 = n\cdot V_1 = 5\cdot V_1 \quad I_A = n\cdot I_B = 5\cdot I_B$



The KVL eqs. become

$120 = V_1 + 5\cdot(4\cdot I_B) \quad j := \sqrt{-1}$

$0 = -20\cdot I_B - 5\cdot V_1 - j\cdot 5\cdot I_B$

which yield

$V_1 := -29.416 - 9.339\cdot j\cdot 1 V_2 := 5\cdot V_1 \quad V_2 = -147.08 - 46.695i$

$I_B := 7.471 + 0.467\cdot j\cdot 1 \quad I_A := 5 I_B \quad I_A = 37.355 + 2.335i$

Checking solution in KVL eqs.

$V_1 + 5\cdot(I_A - I_B) = 120$ <---checks mesh A voltage sum

$5\cdot(I_B - I_A) - V_2 + (-j\cdot 5)\cdot I_B = -1.998\cdot 10^{-14}$ <---checks mesh B voltage sum

15/16-30 $L_1 := 5\cdot 10^{-3} \quad L_2 := 20\cdot 10^{-3} \quad M := 10^{-2} \quad \omega := 10^4 \quad X_1 := \omega\cdot L_1 \quad X_2 := \omega\cdot L_2 \quad X_M := \omega\cdot M$

$Z_L := 200 + j\cdot 100 \quad V_S := 1 + j\cdot 0$ <---Assume 1 V input

$I_1 := 1 + j \quad I_2 := I_1 \quad V_1 := 1 + j \quad V_2 := V_1$ <---Initial guesses for solve block

Given $V_1 = j\cdot X_1\cdot I_1 + j\cdot X_M\cdot I_2 \quad V_2 = j\cdot X_M\cdot I_1 + j\cdot X_2\cdot I_2$ <---Coupled coil constraints

$V_2 = -I_2\cdot Z_L \quad V_1 = V_S$ <---Connection constraints

Solution

$$\begin{pmatrix} I_1 \\ I_2 \\ V_1 \\ V_2 \end{pmatrix} := \text{Find}(I_1, I_2, V_1, V_2) \qquad \begin{pmatrix} I_1 \\ I_2 \\ V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 1.6\times 10^{-2} - 2.8i\times 10^{-2} \\ -8\times 10^{-3} + 4i\times 10^{-3} \\ 1 \\ 2 \end{pmatrix}$$

$Z_{IN} := \dfrac{V_1}{I_1} \qquad Z_{IN} = 15.385 + 26.923i\ \Omega$

$T_V := \dfrac{V_2}{V_1} \qquad T_V = 2$

15/16-31 With secondary open $|V_1| = 100$ $|V_2| = 220$ $|I_1| = 0.12$ $\omega := 2\cdot\pi\cdot 400$ and

$V_1 = j\cdot X_1\cdot I_1$ $V_2 = j\cdot X_M\cdot I_1$ hence $X_1 = \frac{|V_1|}{|I_1|} = 833.3$ $X_M = \frac{|V_2|}{|I_1|} = 1833$

With secondary shorted $|I_1| = 10$ $|I_2| = 2.2$ $V_2 = 0$ and

$0 = j\cdot X_M\cdot I_1 + j\cdot X_2\cdot I_2$ hence $|X_2| = X_M\cdot\frac{|I_1|}{|I_2|} = 8333.3$

so finally $L_1 = \frac{X_1}{\omega} = 0.332\,\text{H}$ $L_2 = \frac{X_2}{\omega} = 3.316\,\text{H}$ $M = \frac{X_M}{\omega} = 0.729\,\text{H}$

15/16-32 $n := 0.1$ $V_S := 2500 + j\cdot 0$ $Z_L := 25 + j\cdot 10$ $Z_S := 0 + j\cdot 2$

$I_1 := \frac{V_S}{Z_S + \frac{Z_L}{(n^2)}}$ $I_1 = 0.862 - 0.345i$ $I_2 := \frac{-I_1}{n}$ $I_2 = -8.616 + 3.453i$ $P_L := \frac{(|I_2|)^2}{2}\cdot\mathrm{Re}(Z_L)$

$P_L = 1.077\times 10^3$ W

15/16-33 $P_L := 25\cdot 10^3$ $V_2 := 480$ $V_1 := 2400$ $n := \frac{V_2}{V_1}$ $n = 0.2$ $I_2 := \frac{P_L}{V_2}$ $I_2 = 52.083$

$I_1 := -n\cdot I_2$ $|I_1| = 10.417$ A Use a 15 A circuit breaker.

15/16-34 $P_L := 50\cdot 10^3$ $V_2 := 480$ $V_1 := 7200$ $n := \frac{V_2}{V_1}$ $n = 0.066667$

$I_2 := \frac{P_L}{V_2}$ $|I_2| = 104.167$ A

$I_1 := -n\cdot I_2$ $|I_1| = 6.944$ A

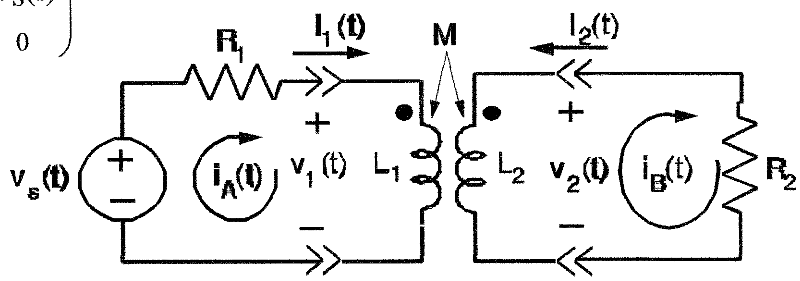
15/16-35 $N_1 := 480$ $N_2 := 120$ $n := \frac{N_2}{N_1}$ $n = 0.25$ $V_1 := 480$ $V_2 := n\cdot V_1$ $V_2 = 120$

$P_L := 5000$ $I_2 := \frac{2\cdot P_L}{V_2}$ $I_2 = 83.333$ (peak) $I_1 := -n\cdot I_2$ $|I_1| = 20.833$ (peak)

$R_L := \frac{2\cdot P_L}{(|I_2|)^2}$ $R_L = 1.44$ $R_{IN} := \frac{R_L}{n^2}$ $R_{IN} = 23.04\ \Omega$

15/16-36 (a) The s-domain mesh equations are

$$\begin{pmatrix} L_1\cdot s + R_1 & -M\cdot s \\ -M\cdot s & L_2\cdot s + R_2 \end{pmatrix}\cdot\begin{pmatrix} I_A(s) \\ I_B(s) \end{pmatrix} = \begin{pmatrix} V_S(s) \\ 0 \end{pmatrix}$$



for which

$$\Delta_2 = M\cdot s\cdot V_S(s)$$

$$\Delta = \left(L_1\cdot L_2 - M^2\right)\cdot s^2 + \left(R_1\cdot L_2 + R_2\cdot L_1\right)\cdot s + R_1\cdot R_2$$ since $V_2(s) = R_2\cdot I_B(s) = R_2\cdot\frac{\Delta_2}{\Delta}$ we have

$$V_2(s) = \frac{R_2\cdot M\cdot s\cdot V_S(s)}{\left(L_1\cdot L_2 - M^2\right)\cdot s^2 + \left(R_1\cdot L_2 + R_2\cdot L_1\right)\cdot s + R_1\cdot R_2} = \frac{\frac{R_2\cdot M\cdot s}{\left(L_1\cdot L_2 - M^2\right)}\cdot V_S(s)}{s^2 + \frac{\left(R_1\cdot L_2 + R_2\cdot L_1\right)}{\left(L_1\cdot L_2 - M^2\right)}\cdot s + \frac{R_1\cdot R_2}{\left(L_1\cdot L_2 - M^2\right)}}$$

(b) For $L_1 := 0.1$ $L_2 := 0.5$ $M := 0.1$ $R_1 := 10$ $R_2 := 40$ $V_S(s) = \frac{1}{s}$ this becomes

$$V_2(s) = \frac{100\cdot s}{s^2 + 225\cdot s + 10^4} = \frac{100}{(s + 164)\cdot(s + 60.96)} = \frac{0.97}{s + 60.96} - \frac{0.97}{s + 164}$$

$$v_2(t) = 0.97\cdot(\exp(-60.96\cdot t) - \exp(-164\cdot t))\ V$$

15/16-37 (a) s-domain mesh equation written in 15-36 above
(b) $V_2(s)$ was found in terms of $V_S(s)$ in 15-36 above. This solution yields the transfer function $V_2(s)/V_S(s)$, which has the form of a 2nd order bandpass filter

$$T(s) = \frac{V_2(s)}{V_S(s)} = \frac{\frac{R_2\cdot M\cdot s}{L_1\cdot L_2 - M^2}}{s^2 + \frac{\left(R_1\cdot L_2 + R_2\cdot L_1\right)}{\left(L_1\cdot L_2 - M^2\right)}\cdot s + \frac{R_1\cdot R_2}{\left(L_1\cdot L_2 - M^2\right)}} = \frac{K\cdot s}{s^2 + 2\cdot\zeta\cdot\omega_0\cdot s + \omega_0^{\ 2}}$$

(c) for $L_1 := 0.25$ $L_2 := 1$ $M := 0.499$ $R_1 := 50$ $R_2 := 200$

$$\omega_0 := \sqrt{\frac{R_1\cdot R_2}{\left(L_1\cdot L_2 - M^2\right)}} \quad \omega_0 = 3.164\times 10^3 \quad B := \frac{R_1\cdot L_2 + R_2\cdot L_1}{L_1\cdot L_2 - M^2} \quad B = 1.001\times 10^5 \quad \zeta := \frac{B}{2\cdot\omega_0}$$

$$\zeta = 15.819 \quad \omega_{C1} := \omega_0\cdot\left(-\zeta + \sqrt{1 + \zeta^2}\right) \quad \omega_{C1} = 99.9 \quad \omega_{C2} := \omega_{C1} + B \quad \omega_{C2} = 1.002\times 10^5$$

This is a broad band characteristic since $B >> \omega_0$.

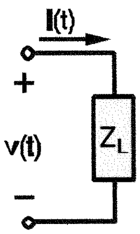
(d) $g(t) := \left(0.998\cdot e^{-t} - 0.998\cdot e^{-100099\cdot t}\right)$

CHAPTER 16, Standard Version
CHAPTER 17, Laplace-Early Version

16/17-1 (a) $v(t) = 1200\cdot\cos\left(\omega\cdot t - 145^{o}\right)$ $i(t) = 2\cdot\cos\left(\omega\cdot t + 50^{o}\right)$ $\theta := (-145) - (50)$ $\theta = -195$

$$P := \frac{1200\cdot 2}{2}\cdot\cos\left(\frac{\pi}{180}\cdot\theta\right) \quad Q := \frac{1200\cdot 2}{2}\cdot\sin\left(\frac{\pi}{180}\cdot\theta\right) \quad pf := \cos\left(\frac{\pi}{180}\cdot\theta\right)$$

$P = -1.159\times 10^{3}$ $Q = 310.583$ $pf = -0.966$ **P < 0 delivering**



(b) $v(t) = 280\cdot\cos\left(\omega\cdot t + 60^{o}\right)$ $i(t) = 15\cdot\cos\left(\omega\cdot t - 20^{o}\right)$ $\theta := (60) - (-20)$ $\theta = 80$

$$P := \frac{280\cdot 15}{2}\cdot\cos\left(\frac{\pi}{180}\cdot\theta\right) \quad Q := \frac{280\cdot 15}{2}\cdot\sin\left(\frac{\pi}{180}\cdot\theta\right) \quad pf := \cos\left(\frac{\pi}{180}\cdot\theta\right)$$

$P = 364.661$ $Q = 2.068\times 10^{3}$ $pf = 0.174$ **P > 0 absorbing**

16/17-2 (a) $v(t) = 135\cdot\cos\left(\omega\cdot t - 90^{o}\right)$ $i(t) = 1.5\cdot\cos\left(\omega\cdot t + 30^{o}\right)$ $\theta := (-90) - (30)$ $\theta = -120$

$$P := \frac{135\cdot 1.5}{2}\cdot\cos\left(\frac{\pi}{180}\cdot\theta\right) \quad Q := \frac{135\cdot 1.5}{2}\cdot\sin\left(\frac{\pi}{180}\cdot\theta\right) \quad pf := \cos\left(\frac{\pi}{180}\cdot\theta\right)$$

$P = -50.625$ $Q = -87.685$ $pf = -0.5$ **P < 0 delivering**

(b) $v(t) = 370\cdot\cos(\omega\cdot t)$ $i(t) = 11.5\cdot\cos\left(\omega\cdot t + 130^{o}\right)$ $\theta := (0 - 130)$ $\theta = -130$

$$P := \frac{370\cdot 11.5}{2}\cdot\cos\left(\frac{\pi}{180}\cdot\theta\right) \quad Q := \frac{370\cdot 11.5}{2}\cdot\sin\left(\frac{\pi}{180}\cdot\theta\right) \quad pf := \cos\left(\frac{\pi}{180}\cdot\theta\right)$$

$P = -1.368\times 10^{3}$ $Q = -1.63\times 10^{3}$ $pf = -0.643$ **P < 0 delivering**

16/17-3 (a) $V_{rms} := 250$ $\phi_V := 0$ $I_{rms} := 0.25$ $V := V_{rms}\cdot\left(\cos(\phi_V) + j\cdot\sin(\phi_V)\right)$ $\phi_I := -15\cdot\frac{\pi}{180}$

$I := I_{rms}\cdot\left(\cos(\phi_I) + j\cdot\sin(\phi_I)\right)$ $S := V\cdot\bar{I}$ $P := \mathrm{Re}(S)$ $Q := \mathrm{Im}(S)$ $pf := \frac{P}{|S|}$

$P = 60.37$ $Q = 16.176$ $pf = 0.966$ **P > 0 absorbing**

(b) $V_{rms} := 120$ $\phi_V := 135\cdot\frac{\pi}{180}$ $I_{rms} := 12.5$ $V := V_{rms}\cdot\left(\cos(\phi_V) + j\cdot\sin(\phi_V)\right)$ $\phi_I := 165\cdot\frac{\pi}{180}$

$I := I_{rms}\cdot\left(\cos(\phi_I) + j\cdot\sin(\phi_I)\right)$ $S := V\cdot\bar{I}$ $P := \mathrm{Re}(S)$ $Q := \mathrm{Im}(S)$ $pf := \frac{P}{|S|}$

$P = 1.299\times 10^{3}$ $Q = -750$ $pf = 0.866$ **P > 0 absorbing**

16/17-4 (a) $V_{rms} := 2400$ $\phi_V := 45\cdot\frac{\pi}{180}$ $\theta := -10.5\cdot\frac{\pi}{180}$ $Z_L := 250\cdot\left(\cos(\theta) + j\cdot\sin(\theta)\right)$

$V := V_{rms}\cdot\left(\cos(\phi_V) + j\cdot\sin(\phi_V)\right)$ $I := \frac{V}{Z_L}$ $I = 5.437 + 7.912j$

$S := V\cdot\bar{I}$ $P := \mathrm{Re}(S)$ $Q := \mathrm{Im}(S)$ $pf := \frac{P}{|S|}$

$P = 2.265\times 10^{4}$ $Q = -4.2\times 10^{3}$ $pf = 0.983$ **P > 0 absorbing**

(b) $I_{rms} := 0.120$ $\phi_I := 25\cdot\frac{\pi}{180}$ $I := I_{rms}\cdot\left(\cos(\phi_I) + j\cdot\sin(\phi_I)\right)$ $Z_L := 300 - j\cdot 400$

$V := I\cdot Z_L$ $V = 52.913 - 28.289j$ $|V| = 60$ $\frac{180}{\pi}\cdot\arg(V) = -28.13$

$S := V\cdot\bar{I}$ $P := \mathrm{Re}(S)$ $Q := \mathrm{Im}(S)$ $pf := \frac{P}{|S|}$ $pf = 0.6$

$P = 4.32$ $Q = -5.76$ **P > 0 absorbing**

16/17-5 (a) $S := 800\cdot 10^3 + j\cdot 250\cdot 10^3$ $P := 800\cdot 10^3$ $Q := 250\cdot 10^3$ $pf := \frac{P}{|S|}$ $pf = 0.954$ $Q > 0$ lagging

(b) $Q := -9000$ $P := \sqrt{12000^2 - Q^2}$ $S := P + j\cdot Q$ $pf := \frac{P}{|S|}$ $pf = 0.661$ $Q < 0$ leading

16/17-6 (a) $P := 10^4$ $S := 12\cdot 10^3$ $V_{rms} := 240$ $Q := \sqrt{S^2 - P^2}$ $Q = 6.633 \times 10^3$ $S := P + j\cdot Q$

$I_{rms} := \frac{|S|}{V_{rms}}$ $I_{rms} = 50$ $pf := \frac{P}{|S|}$ $pf = 0.833$

(b) $Z_L := \frac{S}{I_{rms}^2}$ $Z_L = 4 + 2.653j$ $|Z_L| = 4.8$

16/17-7(a) $V_{rms} := 2400$ $P := 20000\cdot 0.75$ $Q := \sqrt{20000^2 - P^2}$ $Q = 1.323 \times 10^4$ $S := P + j\cdot Q$

$S = 1.5 \times 10^4 + 1.323j \times 10^4$ $I_{rms} := \frac{|S|}{V_{rms}}$ $I_{rms} = 8.333$

(b) $Z_L := \frac{S}{I_{rms}^2}$ $Z_L = 216 + 190.494j$ $|Z_L| = 288$

16/17-8 $Z_L := 4 - j\cdot 1.5$ $S := 10\cdot 10^3$ $pf := \frac{Re(Z_L)}{|Z_L|}$ $P := S\cdot pf$ $Q := -\sqrt{S^2 - P^2}$

$P = 9.363 \times 10^3$ $Q = -3.511 \times 10^3$ $I_{rms} := \sqrt{\frac{P}{Re(Z_L)}}$ $I_{rms} = 48.382$ $V_{rms} := \frac{S}{I_{rms}}$ $V_{rms} = 206.688$

16/17-9 $I_{rms} := 12$ $P := 4800$ $Q := 2400$ $S := P + j\cdot Q$ $V_{rms} := \frac{|S|}{I_{rms}}$

$pf := \frac{P}{|S|}$ $Z_L := \frac{S}{I_{rms}^2}$ $pf = 0.894$ $Z_L = 33.333 + 16.667j$ $|Z_L| = 37.268$

16/17-10 $V_{rms} := 440$ $I_{rms} := 10$ $P := 4\cdot 10^3$ $Q := \sqrt{(V_{rms}\cdot I_{rms})^2 - P^2}$ $Q = 1.833 \times 10^3$

$S := P + j\cdot Q$ $Z_L := \frac{S}{I_{rms}^2}$ $Z_L = 40 + 18.33j$ $|Z_L| = 44$

16/17-11 $R := 100$ $V_{rms} := 240$ $\omega := 2\cdot\pi\cdot 60$ $L := 0.15$ $Y := R^{-1} + (j\cdot\omega\cdot L)^{-1}$

$Y = 1 \times 10^{-2} - 1.768j \times 10^{-2}$ $S := (V_{rms})^2\cdot\overline{Y}$ $S = 576 + 1.019j \times 10^3$

16/17-12 $R := 50$ $V_{rms} := 110$ $\omega := 2\cdot\pi\cdot 133$ $L := 10^{-1}$ $Z := R + j\cdot\omega\cdot L$ $Z = 50 + 83.566j$

$Y := Z^{-1}$ $S := (V_{rms})^2\cdot\overline{Y}$ $S = 63.796 + 106.624j$

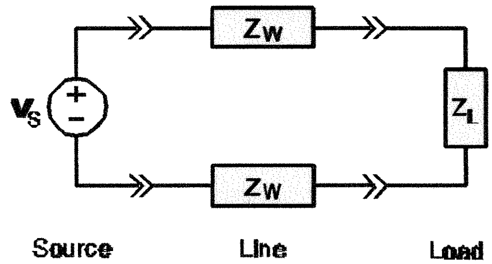
16/17-13 $S := 10 - j\cdot 12$ $\omega := 2\cdot\pi\cdot 60$ $V_{rms} := 440$

$I := \overline{\frac{S}{V_{rms}}}$ $I = 2.27 \times 10^{-2} + 2.73j \times 10^{-2}$ $Y := \frac{I}{V_{rms}}$

$Y = 5.165 \times 10^{-5} + 6.198j \times 10^{-5}$ $R := \frac{1}{Re(Y)}$ $C := \frac{1}{\omega}\cdot Im(Y)$ $R = 1.936 \times 10^4$ $C = 1.644 \times 10^{-7}$

16/17-14 $I_{rms} := 12 \quad Y_L := 200^{-1} + (j \cdot 50)^{-1}$

$Z_L := (Y_L)^{-1} \qquad Z_W := 1 + j \cdot 10$



(a) $V_S := (Z_L + 2 \cdot Z_W) \cdot I_{rms}$

$V_S = 165.176 + 804.706j \quad |V_S| = 821.483$

(b) $S_L := (|I_{rms}|)^2 \cdot Z_L \quad S_L = 1.694 \times 10^3 + 6.776j \times 10^3$

$S_W := (|I_{rms}|)^2 \cdot 2 \cdot Z_W \quad S_W = 288 + 2.88j \times 10^3$

(c) $S_S := S_L + S_W \qquad \eta := \frac{Re(S_L)}{Re(S_S)} \cdot 100 \quad \eta = 85.47$

16/17-15 See figure with Prob. 16-14 above $V_{rms} := 220 \qquad Z_L := 60 - j \cdot 40 \qquad Z_W := 2 + j \cdot 5$

(a) $I := \frac{V_{rms}}{Z_L + 2 \cdot Z_W} \qquad |I| = 3.113$

(b) $S_L := (|I|)^2 \cdot Z_L \qquad S_L = 581.265 - 387.51j \qquad S_W := (|I|)^2 \cdot 2 \cdot Z_W \qquad S_W = 38.751 + 96.878j$

(c) $S_S := S_L + S_W \qquad \eta := \frac{Re(S_L)}{Re(S_S)} \cdot 100 \qquad \eta = 93.75$

16/17-16 $V_{rms} := 2400 \quad S_{mag} := 50000 \qquad I_{rms} := \frac{S_{mag}}{V_{rms}}$

$R_W := \frac{0.03 \cdot V_{rms}}{I_{rms}} \qquad X_W := \frac{0.12 \cdot V_{rms}}{I_{rms}}$

$Z_W := R_W + j \cdot X_W \qquad Z_W = 3.456 + 13.824j$

When $R_L := 200 \qquad I := \frac{V_{rms}}{R_L + 2 \cdot Z_W}$

$P_L := (|I|)^2 \cdot R_L \qquad P_S := Re\left(V_{rms} \cdot \overline{I}\right) \qquad \eta := \frac{P_L}{P_S} \cdot 100 \qquad \eta = 96.659$

16/17-17 See figure with Prob 16-16 above $P_L := 25000 \qquad \eta := 90$

$P_W := P_L \cdot \left(\frac{100}{\eta} - 1\right) \qquad P_W = 2.778 \times 10^3 \quad W$

When the second line is added the wire resistance is reduced by 1/2. The line power is reduced by 1/2 since the same line current exists because the load power is still 25 kW. Hence:

$P_W := 0.5 \cdot P_W \qquad P_W = 1.389 \times 10^3 \quad W \qquad \eta := \frac{P_L}{P_L + P_W} \cdot 100 \qquad \eta = 94.737$

16/17-18 $V_{rms} := 110$

$Z_1 := 25 + j\cdot 6 \quad Z_2 := 16 + j\cdot 8 \quad Z_3 := 10 + j\cdot 2$

(a) $I_1 := \dfrac{V_{rms}}{Z_1} \quad I_2 := \dfrac{V_{rms}}{Z_2} \quad I_3 := \dfrac{2\cdot V_{rms}}{Z_3}$

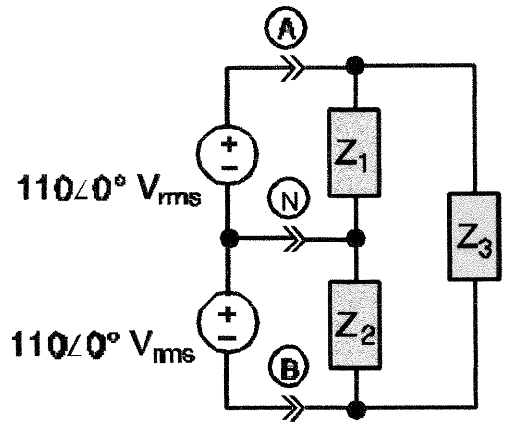
$I_A := I_1 + I_3 \quad I_N := I_2 - I_1 \quad I_B := -I_2 - I_3$

$I_A = 25.314 - 5.229j \quad I_N = 1.34 - 1.752j$

$I_B = -26.654 + 6.981j$

(b) $S_{upper} := V_{rms}\cdot\overline{I_A} \quad S_{upper} = 2.785\times 10^3 + 575.218j$

$S_{lower} := V_{rms}\cdot\left(\overline{-I_B}\right) \quad S_{lower} = 2.932\times 10^3 + 767.885j$



16/17-19 See figure with Problem 16-18 above $V_S := 110$

$S_1 := 1250 + j\cdot 500 \quad S_2 := 800 + j\cdot 0 \quad S_3 := 2000 + j\cdot 400$

(a) $I_1 := \dfrac{\overline{S_1}}{V_S} \quad I_2 := \dfrac{\overline{S_2}}{V_S} \quad I_3 := \dfrac{\overline{S_3}}{2\cdot V_S} \quad I_A := I_1 + I_3 \quad I_N := I_2 - I_1 \quad I_B := -I_2 - I_3$

$I_A = 20.455 - 6.364j \quad I_N = -4.091 + 4.545j \quad I_B = -16.364 + 1.818j$

(b) $S_{upper} := V_S\cdot\overline{I_A} \quad S_{upper} = 2.25\times 10^3 + 700j \quad S_{lower} := V_S\cdot\overline{-I_B} \quad S_{lower} = 1.8\times 10^3 + 200j$

16/17-20 The load ratings are:

$V_L := 440 \quad I_L := 30 \quad P_L := 10000$

$S_L := V_L\cdot I_L \quad Q_L := \sqrt{{S_L}^2 - {P_L}^2} \quad Q_L = 8.616\times 10^3$

$Z_L := \dfrac{P_L + j\cdot Q_L}{\left(|I_L|\right)^2} \quad Z_L = 11.111 + 9.574j$

When connected to the power source via the line

$V_S := 440 \quad Z_W := 0.3 + j\cdot 2 \quad I := \dfrac{V_S}{Z_L + 2\cdot Z_W} \quad V_L := I\cdot Z_L \quad V_L = 356.049 - 52.982j$

$|V_L| = 359.969 \quad S_L := V_L\cdot\bar{I} \quad S_L = 6.693\times 10^3 + 5.767j\times 10^3 \quad |S_L| = 8.835\times 10^3$

16/17-21 See figure with Problem 16-14

$V_L := 2400 \quad P_L := 25000 \quad pf := 0.85 \quad Z_W := 2 + j\cdot 8$

$Q_L := P_L\cdot\sqrt{\dfrac{1}{pf^2} - 1} \quad S_L := P_L + j\cdot Q_L \quad I_L := \dfrac{\overline{S_L}}{V_L} \quad V_S := V_L + I_L\cdot 2\cdot Z_W \quad S_S := V_S\cdot\overline{I_L}$

$S_S = 2.56\times 10^4 + 1.79j\times 10^4 \quad \eta := \dfrac{P_L}{\mathrm{Re}(S_S)}\cdot 100 \quad \eta = 97.653$

16/17-22 See figure with Prob. 16-14 $V_L := 2400$ $pf := 0.75$ $P_L := 30000 \cdot pf$

$Z_W := 2 + j \cdot 10$ $Q_L := P_L \cdot \sqrt{\frac{1}{pf^2} - 1}$

$S_L := P_L + j \cdot Q_L$ $I_L := \overline{\left(\frac{S_L}{V_L}\right)}$

$V_S := V_L + I_L \cdot 2 \cdot Z_W$

$V_S = 2.603 \times 10^3 + 154.428j$ $|V_S| = 2.607 \times 10^3$ $S_S := V_S \cdot \overline{I_L}$ $S_S = 2.313 \times 10^4 + 2.297j \times 10^4$

$pf_S := \frac{Re(S_S)}{|S_S|}$ $pf_S = 0.71$

16/17-23 See figure with Prob. 16-14 $P_S := 37000$ $S_L := 35000 + j \cdot 20000$ $Z_W := 2.1 + j \cdot 12$

$V_L := 2400$ $V_S := V_L + j \cdot 100$ $I_L := \frac{V_S - V_L}{Z_W}$ <---Initial guesses

Given $V_L \cdot \overline{I_L} = S_L$ $Re(V_S \cdot \overline{I_L}) = P_S$ $V_S = V_L + 2 \cdot Z_W \cdot I_L$ <---Constraints

$$\begin{pmatrix} V_L \\ I_L \\ V_S \end{pmatrix} := Find(V_L, I_L, V_S) \qquad \begin{pmatrix} V_L \\ I_L \\ V_S \end{pmatrix} = \begin{pmatrix} 1.83 \times 10^3 + 254.145j \\ 20.256 - 8.117j \\ 2.11 \times 10^3 + 706.196j \end{pmatrix}$$

<---Solution (many others are possible)
All have the same voltage magnitudes

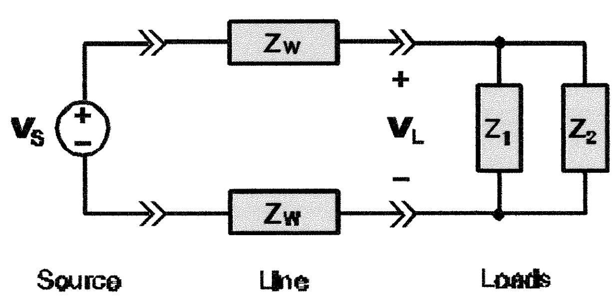
$|V_S| = 2.225 \times 10^3$ $|V_L| = 1.847 \times 10^3$

$V_L \cdot \overline{I_L} = 3.5 \times 10^4 + 2j \times 10^4$ $Re(V_S \cdot \overline{I_L}) = 3.7 \times 10^4$ <---Checking solution

16/17-24 $S_1 := 12000 + j \cdot 6000$ $pf_2 := 0.7$ $S_2 := 25000 \cdot \left(pf_2 + j \cdot \sqrt{1 - pf_2^2}\right)$

$S_2 = 1.75 \times 10^4 + 1.785j \times 10^4$

$V_L := 2400$ $Z_W := 2 + j \cdot 8$



(a) $S_L := S_1 + S_2$ $I_L := \overline{S_L \cdot V_L^{-1}}$

$V_S := V_L + 2 \cdot Z_W \cdot I_L$ $I_L = 12.292 - 9.939j$

$V_S = 2.608 \times 10^3 + 156.911j$

$|V_S| = 2.613 \times 10^3$

(b) $S_S := V_S \cdot \overline{I_L}$ $S_S = 3.05 \times 10^4 + 2.785j \times 10^4$ **(c)** $\eta := \frac{Re(S_L)}{Re(S_S)} \cdot 100$ $\eta = 96.723$

16/17-25 $pf_l := 0.75 \quad pf_S := 0.8$

$$S_l := 16000\left(pf_l + j\cdot\sqrt{1 - pf_l^2}\right)$$

$$S_2 := 25000 + j\cdot 0 \quad Z_W := 0.1 + j\cdot 0.5$$

$$S_S := 50000\left(pf_S + j\cdot\sqrt{1 - pf_S^2}\right)$$

$$S_L := S_l + S_2 \qquad S_W := S_S - S_L$$

$$I_L := \sqrt{\left|\frac{S_W}{2\cdot Z_W}\right|} \qquad I_L = 138.801 \qquad V_L := \frac{S_L}{I_L} \qquad V_L = 266.568 + 76.246j \quad |V_L| = 277.258$$

$$V_S := V_L + 2\cdot I_L\cdot Z_W \quad V_S = 294.328 + 215.047j \qquad |V_S| = 364.519 \text{ V}$$

16/17-26 $I_L := 15 \quad pf := 0.75 \quad P_L := 5000 \quad Q_L := P_L\cdot\sqrt{\frac{1}{pf^2} - 1} \quad S_L := P_L + j\cdot Q_L \quad V_L := \frac{S_L}{I_L}$

$$V_L = 333 + 294j \quad |V_L| = 444 \qquad Q_C := -Q_L \qquad C := \frac{-Q_C}{2\cdot\pi\cdot 60\cdot\left(|V_L|\right)^2}$$

$C = 5.921\times 10^{-5}$ **Checking** $Y_L := \frac{\left(|I_L|\right)^2}{S_L} \qquad Y_C := j\cdot 2\cdot\pi\cdot 60\cdot C \qquad Y_L + Y_C = 0.025$

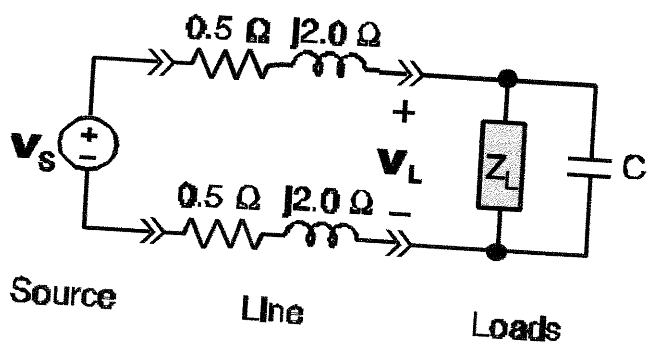
16/17-27 $V_L := 440 \quad pf := 0.75 \quad P_L := 33000pf \quad Q_L := 33000\sqrt{1 - pf^2} \quad S_L := P_L + j\cdot Q_L$

(a) For pf > 0.95 $\quad Q_L + Q_C < \frac{P_L\cdot\sqrt{1 - 0.95^2}}{0.95}$

since $\quad \frac{P_L\cdot\sqrt{1 - 0.95^2}}{0.95} - Q_L = -1.369\times 10^4$

$Q_C < -1.369\cdot 10^4$ but $\quad Q_C := -2\cdot\pi\cdot 60\cdot C\cdot V_L^2$

$C > \frac{1.369\cdot 10^4}{2\cdot\pi\cdot 60\cdot V_L^2}$ = 187.6 mF



(b) Without the capacitor: $\quad I_L := \overline{\frac{S_L}{V_L}} \qquad I_L = 56.25 - 49.608j \qquad R_W := 0.5 \qquad P_W := 2\cdot R_W\cdot\left(|I_L|\right)^2$

$$\eta := \frac{P_L}{P_L + P_W}\cdot 100 \qquad \eta = 81.5$$

With the capacitor $\qquad C := 187.6\cdot 10^{-6} \quad Q_C := -2\cdot\pi\cdot 60\cdot C\cdot V_L^2 \quad Q_L := Q_L + Q_C \quad S_L := P_L + j\cdot Q_L$

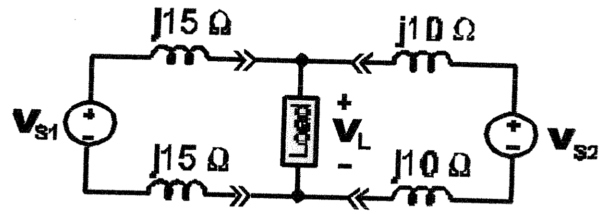
$$I_L := \overline{\frac{S_L}{V_L}} \qquad I_L = 56.25 - 18.489j \qquad P_W := 2\cdot R_W\cdot\left(|I_L|\right)^2 \qquad \eta := \frac{P_L}{P_L + P_W}\cdot 100 \quad \eta = 87.6$$

Power factor correction raises the transmission eff. from 81.5% to 87.6%.

16/17-28 $V_L := 440$ $pf := 0.6$ $S_L := 25000$ $P_L := S_L \cdot pf$ $Q_L := \sqrt{S_L^2 - P_L^2}$ For pf = 0.9

$$Q_C := \frac{P_L \cdot \sqrt{1 - 0.9^2}}{0.9} - Q_L \quad Q_C = -1.274 \times 10^4 \qquad C := \frac{|Q_C|}{2 \cdot \pi \cdot 60 \cdot V_L^2} \quad C = 1.745 \times 10^{-4}$$

16/17-29 $V_L := 12000$ $pf := 0.8$



$$S_L := 1.2 \cdot 10^6 \cdot \left(pf + j \cdot \sqrt{1 - pf^2}\right)$$

$V_{S1} := 12700 + j \cdot 960$ $\quad |V_{S1}| = 1.274 \times 10^4$

$$I_L := \frac{\overline{S_L}}{V_L} \qquad I_1 := \frac{V_{S1} - V_L}{2 \cdot j \cdot 15}$$

$I_1 = 32 - 23.3j$ $\quad S_1 := V_{S1} \cdot \overline{I_1}$ $\quad S_1 = 3.84 \times 10^5 + 3.271j \times 10^5$ $\quad I_2 := I_L - I_1$ $\quad I_2 = 48 - 36.7j$

$V_{S2} := V_L + 2 \cdot j \cdot 10 \cdot I_2$ $\quad V_{S2} = 1.273 \times 10^4 + 960j$ $\quad |V_{S2}| = 1.277 \times 10^4$

$S_2 := V_{S2} \cdot \overline{I_2}$ $\quad S_2 = 5.76 \times 10^5 + 5.13j \times 10^5$ $\quad$ Checking $\quad S_1 + S_2 = 9.6 \times 10^5 + 8.4j \times 10^5$

$S_W := 2 \cdot j \cdot 15 \cdot (|I_1|)^2 + 2 \cdot j \cdot 10 \cdot (|I_2|)^2$ $\quad S_L + S_W = 9.6 \times 10^5 + 8.4j \times 10^5$

16/17-30 $P_1 := 20 \cdot 10^3$ $pf_1 := 0.8$ $P_2 := 12 \cdot 10^3$ $pf_2 := 0.9$ $I_L := 8$ $Z_W := 1.2 + j \cdot 9$

$$Q_1 := P_1 \cdot \sqrt{\frac{1}{pf_1^2} - 1} \quad S_1 := P_1 + j \cdot Q_1 \quad Q_2 := P_2 \cdot \sqrt{\frac{1}{pf_2^2} - 1} \quad S_2 := P_2 + j \cdot Q_2 \quad S_L := S_1 + S_2$$

$$Z_L := \frac{S_L}{(|I_L|)^2} \quad Z_L = 500 + 325.185j \quad S_S := (Z_L + 2 \cdot Z_W) \cdot I_L^2 \quad S_S = 3.215 \times 10^4 + 2.196j \times 10^4$$

$$pf_S := \frac{Re(S_S)}{|S_S|} \quad pf_S = 0.826$$

16/17-31 $V_L := 480$ $\quad V_P := \frac{V_L}{\sqrt{3}}$ $\quad V_{AN} := V_P$ $\quad V_{BN} := V_P \cdot \left[\exp\left[j \cdot \left(-2 \cdot \frac{\pi}{3}\right)\right]\right]$ $\quad V_{CN} := V_P \cdot \left[\exp\left[j \cdot \left(-4 \cdot \frac{\pi}{3}\right)\right]\right]$

$V_{AB} := V_{AN} - V_{BN}$ $\quad V_{BC} := V_{BN} - V_{CN}$ $\quad V_{CA} := V_{CN} - V_{AN}$

$V_{AN} = 277 \cdot e^{j \cdot 0}$

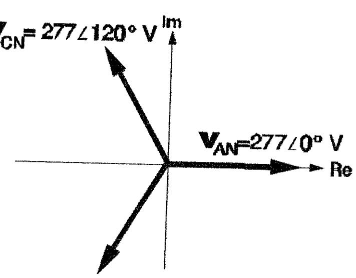
$V_{BN} = -138 - 240 \cdot j = 277e^{-j \cdot 120}$

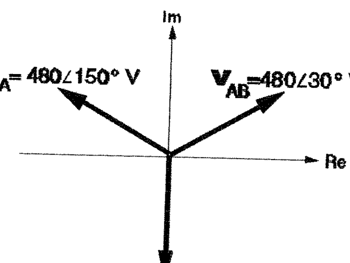
$V_{CN} = -138 + 240 \cdot j = 277 \cdot e^{j \cdot 120}$

$V_{AB} = 416 + 240 \cdot j = 480 \cdot e^{j \cdot 30}$

$V_{BC} = -480 \cdot j = 480 \cdot e^{-j \cdot 90}$

$V_{CA} = -416 + 240 \cdot j = 480 \cdot e^{j \cdot 150}$





16/17-32 $V_L := 2400$ $V_{AB} := V_L$ $V_{BC} := V_L \cdot \left[\exp\left[j \cdot \left(-2 \cdot \frac{\pi}{3}\right)\right]\right]$ $V_{CA} := V_L \cdot \left[\exp\left[j \cdot \left(-4 \cdot \frac{\pi}{3}\right)\right]\right]$

$V_{AN} := \frac{V_{AB}}{\sqrt{3}} \cdot \exp\left(-j \cdot \frac{\pi}{6}\right)$ $V_{BN} := V_{AN} \cdot \exp\left(-j \cdot \frac{2 \cdot \pi}{3}\right)$ $V_{CN} := V_{BN} \cdot \exp\left(-j \cdot \frac{2 \cdot \pi}{3}\right)$

$|V_{AB}| = 2.4 \times 10^3$ $\frac{180}{\pi} \cdot \arg(V_{AB}) = 0$

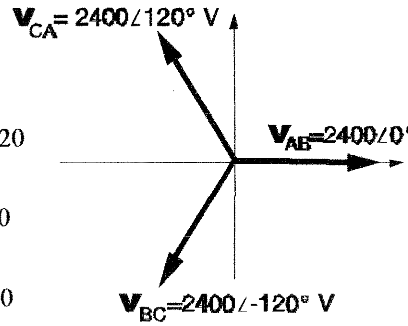
$|V_{BC}| = 2.4 \times 10^3$ $\frac{180}{\pi} \cdot \arg(V_{BC}) = -120$

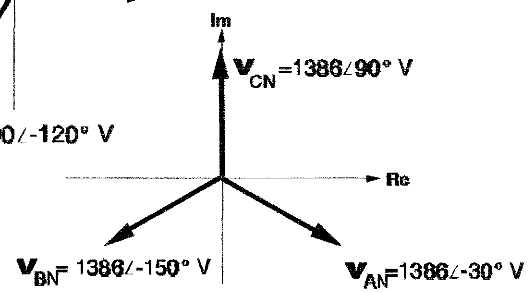
$|V_{CA}| = 2.4 \times 10^3$ $\frac{180}{\pi} \cdot \arg(V_{CA}) = 120$

$|V_{AN}| = 1.386 \times 10^3$ $\frac{180}{\pi} \cdot \arg(V_{AN}) = -30$

$|V_{BN}| = 1.386 \times 10^3$ $\frac{180}{\pi} \cdot \arg(V_{BN}) = -150$

$|V_{CN}| = 1.386 \times 10^3$ $\frac{180}{\pi} \cdot \arg(V_{CN}) = 90$





16/17-33 $I_P := 3.5$ $I_L := I_P \cdot \sqrt{3}$ $I_A := I_L$ $I_B := I_L \cdot \left[\exp\left[j \cdot \left(-4 \cdot \frac{\pi}{3}\right)\right]\right]$ $I_C := I_L \cdot \left[\exp\left[j \cdot \left(-2 \cdot \frac{\pi}{3}\right)\right]\right]$

$I_1 := I_P \cdot \exp\left(-j \cdot \frac{\pi}{6}\right)$ $I_2 := I_P \cdot \exp\left[j \cdot \left(-\frac{\pi}{6} - \frac{4 \cdot \pi}{3}\right)\right]$ $I_3 := I_P \cdot \exp\left[j \cdot \left(-\frac{\pi}{6} - \frac{2 \cdot \pi}{3}\right)\right]$

$|I_A| = 6.062$ $\frac{180}{\pi} \cdot \arg(I_A) = 0$

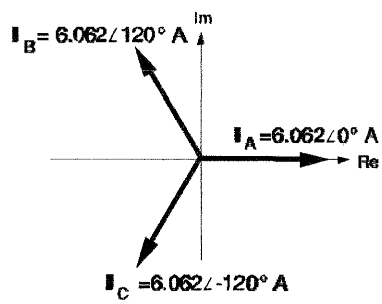
$|I_B| = 6.062$ $\frac{180}{\pi} \cdot \arg(I_B) = 120$

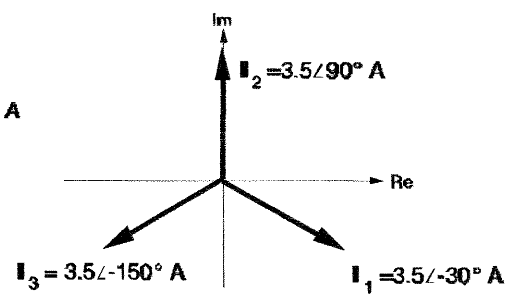
$|I_C| = 6.062$ $\frac{180}{\pi} \cdot \arg(I_C) = -120$

$|I_1| = 3.5$ $\frac{180}{\pi} \cdot \arg(I_1) = -30$

$|I_2| = 3.5$ $\frac{180}{\pi} \cdot \arg(I_2) = 90$

$|I_3| = 3.5$ $\frac{180}{\pi} \cdot \arg(I_3) = -150$





16/17-34 $V_L := 480$ $V_P := \frac{V_L}{\sqrt{3}}$ $V_{AN} := V_P$ $V_{BN} := V_P \cdot \left[\exp\left[j \cdot \left(-2 \cdot \frac{\pi}{3}\right)\right]\right]$ $V_{CN} := V_P \cdot \left[\exp\left[j \cdot \left(-4 \cdot \frac{\pi}{3}\right)\right]\right]$

$Z := 20 + j \cdot 15$

(a) $I_A := \frac{V_{AN}}{Z}$ $I_B := \frac{V_{BN}}{Z}$ $I_C := \frac{V_{CN}}{Z}$

$I_A = 8.868 - 6.651j$ $I_B = -10.194 - 4.354j$ $I_C = 1.326 + 11.006j$

(b) $S_L := \sqrt{3} \cdot V_L \cdot |I_A| \cdot \exp(j \cdot \arg(Z))$ $S_L = 7.373 \times 10^3 + 5.53j \times 10^3$

16/17-34 Continued

(c) $|I_A| = 11.1 \quad \frac{180}{\pi}\cdot\arg(I_A) = -36.87$

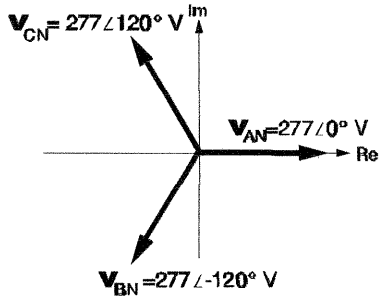
$|I_B| = 11.1 \quad \frac{180}{\pi}\cdot\arg(I_B) = -156.87$

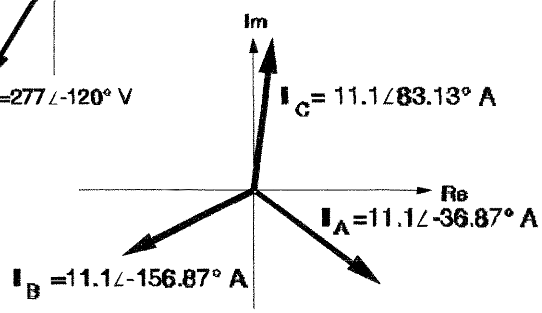
$|I_C| = 11.1 \quad \frac{180}{\pi}\cdot\arg(I_C) = 83.13$

$|V_{AN}| = 277 \quad \frac{180}{\pi}\cdot\arg(V_{AN}) = 0$

$|V_{BN}| = 277 \quad \frac{180}{\pi}\cdot\arg(V_{BN}) = -120$

$|V_{CN}| = 277 \quad \frac{180}{\pi}\cdot\arg(V_{CN}) = 120$





16/17-35 $V_L := 480 \quad V_{AB} := V_L \quad V_{BC} := V_L\cdot\left[\exp\left[j\cdot\left(-4\cdot\frac{\pi}{3}\right)\right]\right] \quad V_{CA} := V_L\cdot\left[\exp\left[j\cdot\left(-2\cdot\frac{\pi}{3}\right)\right]\right]$

$Z := 15 + j\cdot 12$ negative phase sequence

(a) $I_1 := \frac{V_{AB}}{Z} \quad I_2 := \frac{V_{BC}}{Z} \quad I_3 := \frac{V_{CA}}{Z} \quad I_A := I_1 - I_3 \quad I_B := I_2 - I_1 \quad I_C := I_3 - I_2$

$I_A = 42.8 - 6.5j \quad I_B = -15.7 + 40.3j \quad I_C = -27 - 33.8j$

(b) $S_L := \sqrt{3}\cdot V_L\cdot|I_A|\cdot\exp(j\cdot\arg(Z)) \quad S_L = 2.81\times 10^4 + 2.248j\times 10^4$

(c) $|I_A| = 43.3 \quad \frac{180}{\pi}\cdot\arg(I_A) = -8.66$

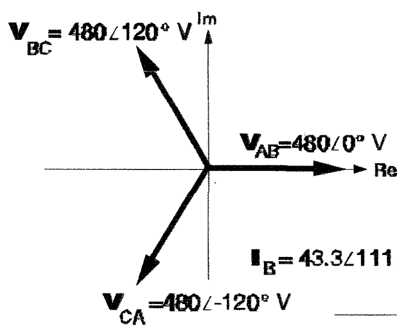
$|I_B| = 43.3 \quad \frac{180}{\pi}\cdot\arg(I_B) = 111.34$

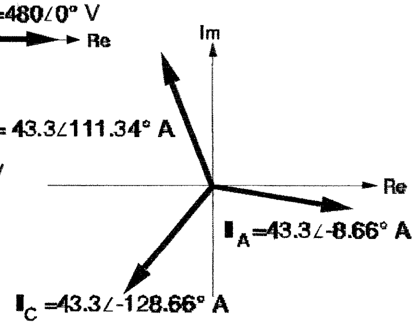
$|I_C| = 43.3 \quad \frac{180}{\pi}\cdot\arg(I_C) = -128.66$

$|V_{AB}| = 480 \quad \frac{180}{\pi}\cdot\arg(V_{AB}) = 0$

$|V_{BC}| = 480 \quad \frac{180}{\pi}\cdot\arg(V_{BC}) = 120$

$|V_{CA}| = 480 \quad \frac{180}{\pi}\cdot\arg(V_{CA}) = -120$





16/17-36 $V_L := 480 \quad Z := 50 + j\cdot 15 \quad I_P := V_L\cdot Z^{-1} \quad I_P = 8.807 - 2.642j$

(a) $I_L := \sqrt{3}\cdot|I_P| \quad I_A := I_L \quad I_B := I_L\cdot\exp\left(j\cdot\frac{-2\cdot\pi}{3}\right) \quad I_C := I_L\cdot\exp\left(j\cdot\frac{-4\cdot\pi}{3}\right)$

$I_A = 15.926 \quad I_B = -7.963 - 13.793j \quad I_C = -7.963 + 13.793j$

(b) $S_L := \sqrt{3}\cdot V_L\cdot|I_A|\cdot\exp(j\cdot\arg(Z)) \quad S_L = 1.268\times 10^4 + 3.805j\times 10^3$

16/17-37 $Z_\Delta := 8 - j\cdot 6$ $Z_Y := 12\cdot\exp\left(j\cdot 40\cdot\frac{\pi}{180}\right)$ $Z_{EQ} := \dfrac{1}{\dfrac{3}{Z_\Delta} + \dfrac{1}{Z_Y}}$ **<---Equivalent Y-load**

$Z_{EQ} = 2.805 - 1.167j$

$V_L := 480$ $V_P := \dfrac{V_L}{\sqrt{3}}$ $V_P = 277.128$

$I_L := \dfrac{V_P}{|Z_{EQ}|}$ $I_L = 91.2$ $pf := \dfrac{Re(Z_{EQ})}{|Z_{EQ}|}$ $pf = 0.923$

16/17-38 $I_A := 25\cdot\exp\left[j\cdot\left(-40\cdot\frac{\pi}{180}\right)\right]$ $V_{AB} := 480\cdot\exp\left[j\cdot\left(30\cdot\frac{\pi}{180}\right)\right]$ $I_1 := \dfrac{I_A}{\sqrt{3}}\cdot\exp\left(\dfrac{j\cdot 30\cdot\pi}{180}\right)$

$I_A = 19.151 - 16.07j$ $I_1 = 14.214 - 2.506j$ $Z := \dfrac{V_{AB}}{I_1}$ $Z = 25.475 + 21.376j$

16/17-39 $pf := 0.75$ $S_L := 30000\cdot\left(pf + j\cdot\sqrt{1 - pf^2}\right)$ $S_L = 2.25\times 10^4 + 1.984j\times 10^4$ $V_L := 480$

(a) $I_L := \dfrac{|S_L|}{\sqrt{3}\cdot V_L}$ $I_L = 36.084$

(b) $V_P := \dfrac{V_L}{\sqrt{3}}$ $V_P = 277.128$ $Z := \dfrac{V_P}{I_L}\cdot\exp(j\cdot\arg(S_L))$ $Z = 5.76 + 5.08j$

16/17-40 $P_L := 20\cdot 10^3$ $V_L := 480$ $I_L := 32$ $pf := \dfrac{P_L}{\sqrt{3}\cdot V_L\cdot I_L}$ $pf = 0.752$

$\theta := acos(pf)$ $\theta\cdot\dfrac{180}{\pi} = 41.257$ $V_P := \dfrac{V_L}{\sqrt{3}}$ $Z := \dfrac{V_P}{I_L}\cdot\exp(j\cdot\theta)$ $Z = 6.51 + 5.711j$

16/17-41 $pf := 0.75$ $S_L := 30000\cdot\left(pf + j\cdot\sqrt{1 - pf^2}\right)$ $S_L = 2.25\times 10^4 + 1.984j\times 10^4$ $V_L := 2400$

(a) $I_L := \dfrac{|S_L|}{\sqrt{3}\cdot V_L}$ $I_L = 7.217$

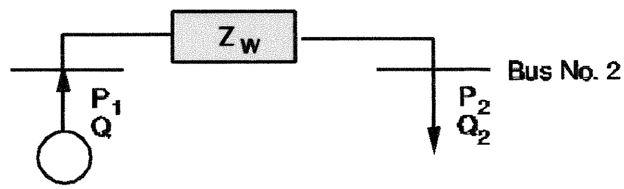
(b) $I_P := \dfrac{I_L}{\sqrt{3}}$ $I_P = 4.167$ $Z := \dfrac{S_L}{3\cdot(|I_P|)^2}$ $Z = 432 + 381j$

16/17-42 $pf := 0.85$ $S_\Delta := 30000\cdot\left(pf + j\cdot\sqrt{1 - pf^2}\right)$ $V_L := 480$ $V_P := \dfrac{V_L}{\sqrt{3}}$ $Z_Y := 20 + j\cdot 15$

$I_{LY} := \dfrac{V_P}{|Z_Y|}$ $S_Y := \sqrt{3}\cdot V_L\cdot I_{LY}\cdot\exp(j\cdot\arg(Z_Y))$ $S_Y = 7.373\times 10^3 + 5.53j\times 10^3$

$S_L := S_Y + S_\Delta$ $S_L = 3.287\times 10^4 + 2.133j\times 10^4$ $I_L := \dfrac{|S_L|}{\sqrt{3}\cdot V_L}$ $I_L = 47.136$

16/17-43 When $V_{L2} := 480$ $S_2 := 16000 + j \cdot 12000$ $Z_{Y2} := \frac{V_{L2}^2}{\overline{S_2}}$ $Z_W := 0.6 + j \cdot 4$



$V_{L1} := 440$ $V_{P1} := \frac{V_{L1}}{\sqrt{3}}$ $V_{P1} = 254.034$

$I_L := \frac{V_{P1}}{Z_W + Z_{Y2}}$ $V_{P2} := V_{P1} - I_L \cdot Z_W$

$V_{L2} := \sqrt{3} \cdot |V_{P2}|$ $V_{L2} = 345.348$

$S_1 := 3 \cdot V_{P1} \cdot \overline{I_L}$ $S_1 = 8.822 \times 10^3 + 9.806j \times 10^3$

Checking $S_2 := 3 \cdot V_{P2} \cdot \overline{I_L}$ $S_W := 3 \cdot (|I_L|)^2 \cdot Z_W$ $S_2 + S_W = 8.822 \times 10^3 + 9.806j \times 10^3$

16/17-44 See figure with Prob. 16-43 above $V_{P1} := 4160$ $Z_{Y1} := j \cdot 2.2$ $Z_W := 1.2 + j \cdot 5$ $Z_{Y2} := 80 + j \cdot 66$

(a) $I_L := \frac{V_{P1}}{Z_{Y1} + Z_W + Z_{Y2}}$ $|I_L| = 38.052$

(b) $V_{P2} := I_L \cdot Z_{Y2}$ $|V_{P2}| = 3.946 \times 10^3$ $S_2 := 3 \cdot V_{P2} \cdot \overline{I_L}$ $S_2 = 3.475 \times 10^5 + 2.867j \times 10^5$

(c) $S_1 := 3 \cdot V_{P1} \cdot \overline{I_L}$ $S_1 = 3.527 \times 10^5 + 3.18j \times 10^5$ **(d)** $\eta := \frac{\mathrm{Re}(S_2)}{\mathrm{Re}(S_1)} \cdot 100$ $\eta = 98.522$

16/17-45 See figure with Prob. 16-43 above $V_{L2} := 7200$ $pf_2 := 0.8$ $Z_W := 0 + j \cdot 8$

$S_2 := 8 \cdot 10^5 \cdot \left(pf_2 + j \cdot \sqrt{1 - pf_2^2}\right)$ $I_L := \frac{|S_2|}{\sqrt{3} \cdot V_{L2}}$ $S_W := 3 \cdot I_L^2 \cdot Z_W$ $S_1 := S_2 + S_W$ $V_{L1} := \frac{|S_1|}{\sqrt{3} \cdot I_L}$

$I_L = 64.15$ $V_{L1} = 7.766 \times 10^3$ $S_1 = 6.4 \times 10^5 + 5.79j \times 10^5$ $pf_1 := \frac{\mathrm{Re}(S_1)}{|S_1|}$

$|S_1| = 8.629 \times 10^5$ $pf_1 = 0.742$

16/17-46 See figure with Prob. 16-43 above

$V_{L1} := 7.2 \cdot 10^3$ $P_2 := 3 \cdot 10^5$ $pf_2 := 0.8$ $S_2 := P_2 + j \cdot P_2 \cdot \sqrt{\frac{1}{pf_2^2} - 1}$ $Z_W := 3 + j \cdot 6$

constraints on complex power at Bus No. 1 are: $|S_1| = \left|S_2 + 3 \cdot (|I_L|)^2 \cdot Z_W\right|$ and $|S_1| = \sqrt{3} \cdot V_{L1} \cdot |I_L|$

Since S_2, Z_W, and V_{L1} are known these equations yield one equation in the unknown line current. Using this eq in a solve block:

$I_L := 10$ Given $\left|S_2 + 3 \cdot (|I_L|)^2 \cdot Z_W\right| = \sqrt{3} \cdot V_{L1} \cdot |I_L|$ $I_L := \mathrm{Find}(I_L)$ $V_{L2} := \frac{|S_2|}{\sqrt{3} \cdot I_L}$

$I_L = 31.512$ $V_{L2} = 6.871 \times 10^3$ <---Solution (Many others are possible)

$|S_2| = 3.75 \times 10^5$ $\sqrt{3} \cdot I_L \cdot V_{L2} = 3.75 \times 10^5$ <---checking

16/17-47 $P_2 := 4.5 \cdot 10^3$ $\quad pf := \left(\frac{1}{\sqrt{2}}\right)$

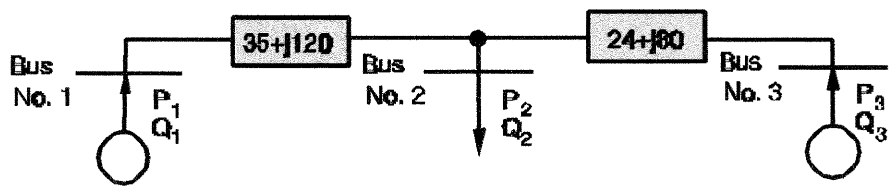
$S_2 := P_2 + j \cdot \frac{P_2}{pf} \cdot \sqrt{1 - pf^2}$ $\quad V_{L2} := 440$

$I_L := \frac{\overline{S_2}}{\sqrt{3} \cdot V_{L2}}$ $\quad Z_W := 0.5 + j \cdot 2$

$I_L = 5.905 - 5.905j$ $\quad V_{L1} := I_L \cdot Z_W \cdot \sqrt{3} + V_{L2}$ $\quad V_{L1} = 465.568 + 15.341j$ $\quad |V_{L1}| = 465.821$

16/17-48 $P_2 := 100 \cdot 10^6$ $\quad pf := 0.8$ $\quad S_2 := P_2 + j \cdot P_2 \cdot \sqrt{\frac{1}{pf^2} - 1}$ $\quad V_{P2} := 90000 + j \cdot 0$

$Z_{W1} := 35 + j \cdot 120$ $\quad Z_{W2} := 24 + j \cdot 80$ $\quad V_{P3} := 90000 \cdot \exp\left(j \cdot 12 \cdot \frac{\pi}{180}\right)$



$I_{L2} := \frac{V_{P3} - V_{P2}}{Z_{W2}}$ $\quad I_L := \frac{\overline{S_2}}{3 \cdot V_{P2}}$ $\quad I_{L1} := I_L - I_{L2}$ $\quad V_{P1} := V_{P2} + I_{L1} \cdot Z_{W1}$

$I_L = 370.37 - 277.778j$ $\quad I_{L1} = 162.549 - 364.708j$ $\quad I_{L2} = 207.822 + 86.93j$

$V_{P1} = 1.395 \times 10^5 + 6.741j \times 10^3$ $\quad V_{P2} = 9 \times 10^4$ $\quad V_{P3} = 8.803 \times 10^4 + 1.871j \times 10^4$

$S_1 := 3 \cdot V_{P1} \cdot \overline{I_{L1}}$ $\quad S_3 := 3 \cdot V_{P3} \cdot \overline{I_{L2}}$

$S_1 = 6.063 \times 10^7 + 1.559j \times 10^8$ $\quad S_3 = 5.977 \times 10^7 - 1.129j \times 10^7$ $\quad S_1 + S_3 = 1.204 \times 10^8 + 1.446j \times 10^8$

Checking complex power balance

$S_{W1} := 3 \cdot (|I_{L1}|)^2 \cdot Z_{W1}$ $\quad S_{W2} := 3 \cdot (|I_{L2}|)^2 \cdot Z_{W2}$ $\quad S_{W1} + S_{W2} + S_2 = 1.204 \times 10^8 + 1.446j \times 10^8$

16/17-49 $Z_{Y1} := 200 + j \cdot 100$ $\quad Z_{\Delta 2} := 2700 - j \cdot 1200$ $\quad S_3 := 110 \cdot 10^3 + j \cdot 95 \cdot 10^3$ $\quad Z_W := 1 + j \cdot 10$

$V_P := 7200$ $\quad Z_{Y3} := \frac{3 \cdot (|V_P|)^2}{\overline{S_3}}$ $\quad Z_{Y3} = 809.808 + 699.38j$ $\quad Z_Y := \frac{1}{\frac{1}{Z_{Y1}} + \frac{3}{Z_{\Delta 2}} + \frac{1}{Z_{Y3}}}$

$Z_Y = 154.016 + 60.087j$ $\quad I_L := \frac{V_P}{Z_Y}$ $\quad I_L = 40.573 - 15.829j$ $\quad S_L := 3 \cdot V_P \cdot \overline{I_L}$ $\quad S_L = 8.764 \times 10^5 + 3.419j \times 10^5$

$S_S := S_L + 3 \cdot (|I_L|)^2 \cdot Z_W$ $\quad S_S = 8.821 \times 10^5 + 3.988j \times 10^5$

16/17-50 $V_L := 480 \quad V_P := \frac{V_L}{\sqrt{3}} \quad Z_\Delta := 60 + j\cdot 25 \quad Z_Y := \frac{Z_\Delta}{3} \quad V_P = 277.128 \quad Z_Y = 20 + 8.333j$

(a) $I_L := \frac{V_P}{Z_Y} \quad S_L := 3\cdot V_P\cdot \overline{I_L} \quad S_L = 9.816\times 10^3 + 4.09j\times 10^3 \quad Z_W := 5 + j\cdot 15$

$S_S := S_L + 3\cdot\left(\left|I_L\right|\right)^2\cdot Z_W \quad S_S = 1.227\times 10^4 + 1.145j\times 10^4 \quad \eta := \frac{Re(S_L)}{Re(S_S)}\cdot 100 \quad \eta = 80$

(b) $Z_W := \frac{1}{\frac{1}{5 + j\cdot 15} + \frac{1}{2 + j\cdot 15}} \quad S_S := S_L + 3\cdot\left(\left|I_L\right|\right)^2\cdot Z_W \quad \eta := \frac{Re(S_L)}{Re(S_S)}\cdot 100 \quad \eta = 92.024$

16/17-51 $Z_A := 100 \quad Z_B := 100 \quad Z_C := 50 + j\cdot 100 \quad V_P := 208$

(a) $V_{AN} := V_P \quad V_{BN} := V_P\cdot\exp\left(-j\cdot 120\cdot\frac{\pi}{180}\right) \quad V_{CN} := V_P\cdot\exp\left(-j\cdot 240\cdot\frac{\pi}{180}\right)$

Writing a node-voltage equation at N' with point N grounded:

$$\frac{V_N - V_{AN}}{Z_A} + \frac{V_N - V_{BN}}{Z_B} + \frac{V_N - V_{CN}}{Z_C} = 0 \text{ hence } V_N := \frac{V_{AN}\cdot Z_A^{-1} + V_{BN}\cdot Z_B^{-1} + V_{CN}\cdot Z_C^{-1}}{Z_A^{-1} + Z_B^{-1} + Z_C^{-1}}$$

$I_A := \frac{V_{AN} - V_N}{Z_A} \quad I_B := \frac{V_{BN} - V_N}{Z_B} \quad I_C := \frac{V_{CN} - V_N}{Z_C}$

$I_A = 1.275 - 0.165j \quad I_B = -1.845 - 1.966j \quad I_C = 0.571 + 2.131j$

$S_L := V_{AN}\cdot\overline{I_A} + V_{BN}\cdot\overline{I_B} + V_{CN}\cdot\overline{I_C} \quad S_L = 1.136\times 10^3 + 486.72j \quad Re(S_L) = 1.136\times 10^3$

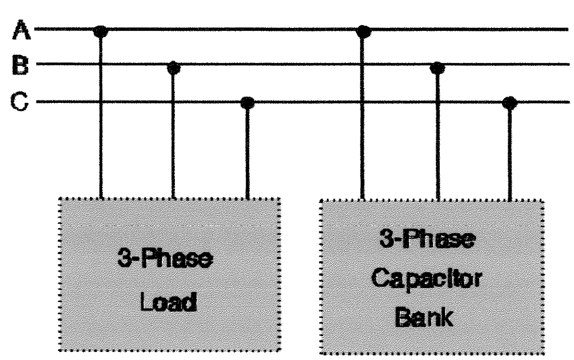
(b) With point N grounded $V_N = 0$ and hence:

$I_A := \frac{V_{AN}}{Z_A} \quad I_B := \frac{V_{BN}}{Z_B} \quad I_C := \frac{V_{CN}}{Z_C} \quad I_N := I_A + I_B + I_C$

$I_A = 2.08 \quad I_B = -1.04 - 1.801j \quad I_C = 1.025 + 1.553j \quad I_N = 2.065 - 0.249j$

$S_L := V_{AN}\cdot\overline{I_A} + V_{BN}\cdot\overline{I_B} + V_{CN}\cdot\overline{I_C} \quad S_L = 1.038\times 10^3 + 346.112j \quad Re(S_L) = 1.038\times 10^3$

16/17-52



16/17-52 Continued

$V_L := 480 \quad pf := 0.8 \quad S_L := 105 \cdot 10^3 \cdot \left(pf + j\cdot\sqrt{1 - pf^2}\right) \qquad P_L := Re(S_L) \qquad Q_L := Im(S_L)$

To make pf > 0.95 requires $\dfrac{P_L}{|S_L - j\cdot Q_C|} = \dfrac{P_L}{\sqrt{P_L^{\,2} + (Q_L - Q_C)^2}} > 0.95$ or $\left(\dfrac{P_L}{0.95}\right)^2 - P_L^{\,2} > (Q_L - Q_C)^2$

and finally $Q_C > Q_L - 0.329 \cdot P_L = 35.4 \cdot 10^3$ Checking let $Q_C := 35.4 \cdot 10^3$ then $\dfrac{P_L}{|S_L - j\cdot Q_C|} = 0.95$

Increasing the power factor above 0.95 requires a minimum of 35.4 kVAR of capacitive reactance.

The least expensive option is the 40 KVAR unit (PN=1N0240A17) unit whose cost is $900.

Installing this unit reduces the power cost by: $\dfrac{P_L}{1000} \cdot 8 \cdot (0.09 - 0.07) = 13.44$ $/day. Hence it takes about 100 days to recover the capital cost.

16/17-53 Three-Phase Power Transformer

	Primary		Secondary	
	$V_1 := 2540 \cdot \exp\left(-j\cdot\dfrac{\pi}{6}\right)$	$I_1 := 43.3 \cdot \exp\left(-j\cdot\dfrac{66\cdot\pi}{180}\right)$	$V_2 := 240$	$I_2 := 417 \cdot \exp\left(-j\cdot\dfrac{\pi}{6}\right)$

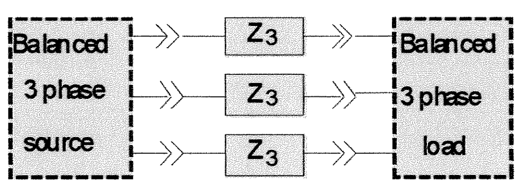
(a) $S_1 := \sqrt{3}\, V_1 \cdot \overline{I_1} \qquad S_1 = 1.541 \times 10^5 + 1.12j \times 10^5 \quad pf_1 := \dfrac{Re(S_1)}{|S_1|} \qquad pf_1 = 0.809$

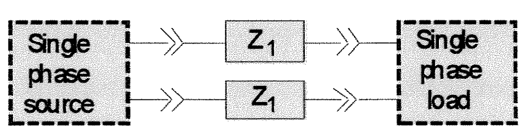
(b) $S_2 := \sqrt{3} \cdot V_2 \cdot \overline{I_2} \qquad S_2 = 1.501 \times 10^5 + 8.667j \times 10^4 \quad pf_2 := \dfrac{Re(S_2)}{|S_2|} \qquad pf_2 = 0.866$

(c) Transformer is not ideal $S_1 - S_2 = 3.993 \times 10^3 + 2.53j \times 10^4$ there is power loss

(d) $\eta := 100 \cdot \dfrac{Re(S_2)}{Re(S_1)} \qquad \eta = 97.409$

16/17-54





16/17-54 Continued

	Three phase system	single phase system
Line voltage	V_{L3}	V_{L1}

Same line voltage means $V_{L3} = V_{L1} = V_L$

Hencecurrent	I_3	I_1
Load power	P_{L3}	P_{L1}
	$P_{L3} = \sqrt{3}\cdot V_L\cdot I_3\cdot \cos(\theta)$	$P_{L1} = V_L\cdot I_1\cdot \cos(\theta)$

The same load power means: $P_{L3} = \sqrt{3}\cdot V_L\cdot I_3\cdot \cos(\theta) = P_{L1} = V_L\cdot I_1\cdot \cos(\theta)$ hence $I_1 = \sqrt{3}\cdot I_3$

$$P_{W3} = 3\cdot I_3^{\;2}\cdot R_3 \qquad\qquad P_{W1} = 2\cdot I_1^{\;2}\cdot R_1$$

Same efficiency means: $P_{W3} = 3\cdot I_3^{\;2}\cdot R_3 = P_{W1} = 2\cdot I_1^{\;2}\cdot R_1 = 2\cdot\left(\sqrt{3}\cdot I_3\right)^2\cdot R_1$ hence $R_3 = 2\cdot R_1$

Same length (L) means: $\dfrac{R_3}{R_1} = 2 = \dfrac{\frac{\rho\cdot L}{A_3}}{\frac{\rho\cdot L}{A_1}}$ Hence $\dfrac{A_3}{A_1} = \dfrac{1}{2}$

So finally: Copper in 1ϕ line = 2(L)(A_1)

Copper in 3ϕ line = 3(L)(A_3) = 3(L)(A_1/2) = 1.5(L)(A_1) = 0.75(copper in 1ϕ line)

Hence the 3ϕ line take 25% less copper than the 1ϕ line. QED

CHAPTER W1, Fourier Transforms

W1-1 $f(t) = A\cdot(u(t) - u(t-1))$

$$F(\omega) = \int_0^1 A\cdot e^{-j\cdot\omega\cdot t}\,dt = \frac{-1}{j\cdot\omega}\cdot\exp(-j\cdot\omega)\cdot A + \frac{1}{j\cdot\omega}\cdot A = \frac{A}{j\cdot\omega}\cdot(1 - \exp(-j\cdot\omega))$$

W1-2 $f(t) = A\cdot t\cdot(u(t) - u(t-1))$

$$F(\omega) = \int_0^1 A\cdot t\cdot\exp(-j\cdot\omega\cdot t)\,dt = A\cdot\frac{(j\cdot\omega + 1)}{\omega^2}\cdot\exp(-j\cdot\omega) - \frac{1}{\omega^2}\cdot A = \frac{A}{\omega^2}\cdot\left[-1 + (1 + j\cdot\omega)\cdot\exp(-j\cdot\omega)\right]$$

W1-3 $f(t) = A\cdot\cos\left(\frac{\pi}{2}\cdot t\right)\cdot(u(t+1) - u(t-1))$

$$F(\omega) = A\cdot\int_{-1}^{1} \cos\left(\frac{\pi}{2}\cdot t\right)\cdot e^{(-j\cdot\omega\cdot t)}\,dt = A\cdot\left[\frac{e^{-j\cdot\omega\cdot t}\cdot\left(-j\cdot\omega\cdot\cos\left(\frac{\pi}{2}\cdot t\right) + \frac{\pi}{2}\cdot\sin\left(\frac{\pi}{2}\cdot t\right)\right)}{(-j\cdot\omega)^2 + \left(\frac{\pi}{2}\right)^2}\right]\Bigg|_{-1}^{1}$$

$$F(\omega) = A\cdot\left[-2\cdot\pi\cdot\frac{\exp(-i\cdot\omega)}{\left(4\cdot\omega^2 - \pi^2\right)} - 2\cdot\pi\cdot\frac{\exp(i\cdot\omega)}{\left(4\cdot\omega^2 - \pi^2\right)}\right] = A\cdot\left[-4\cdot\pi\cdot\frac{\cos(\omega)}{\left(4\cdot\omega^2 - \pi^2\right)}\right]$$

W1-4 $F(\omega) = 10\cdot\pi\cdot(u(\omega + 1) - u(\omega - 1))$

$$f(t) = \frac{1}{2\cdot\pi}\cdot\int_{-1}^{1} 10\cdot\pi\cdot\exp(j\cdot\omega\cdot t)\,d\omega = \frac{1}{2\cdot\pi}\cdot\left(-j\cdot 10\cdot\pi\cdot\frac{\exp(j\cdot t)}{t} + j\cdot 10\cdot\pi\cdot\frac{\exp(-j\cdot t)}{t}\right)$$

$$f(t) = \frac{10\cdot(\exp(j\cdot t) - \exp(-j\cdot t))}{2\cdot j\cdot t} = \frac{10\cdot\sin(t)}{t}$$

W1-5 $F(\omega) = j\,10\cdot\pi\cdot(-u(\omega + 1) + 2\cdot u(\omega) - u(\omega - 1))$

$$f(t) = \frac{j}{2\cdot\pi}\cdot\int_{-1}^{0} -10\cdot\pi\cdot\exp(j\cdot\omega\cdot t)\,d\omega + \frac{j}{2\cdot\pi}\cdot\int_0^1 10\cdot\pi\cdot\exp(j\cdot\omega\cdot t)\,d\omega$$

$$f(t) = \frac{j}{(2\cdot\pi)}\cdot\left(10\cdot\frac{j}{t}\cdot\pi - 10\cdot\frac{j}{t}\cdot\exp(-j\cdot t)\cdot\pi\right) + \frac{j}{(2\cdot\pi)}\cdot\left(-10\cdot\frac{j}{t}\cdot\exp(j\cdot t)\cdot\pi + 10\cdot\frac{j}{t}\cdot\pi\right) = 10\cdot\frac{(\cos(t) - 1)}{t}$$

W1-6 $F(\omega) = \cos\left(\frac{\pi}{2}\cdot\omega\right)\cdot(u(\omega + 1) - u(\omega - 1))$

$$f(t) = \frac{1}{2\cdot\pi}\cdot\int_{-1}^{1} \cos\left(\frac{\pi}{2}\cdot\omega\right)\cdot\exp(j\cdot\omega\cdot t)\,d\omega = \frac{1}{(2\cdot\pi)}\cdot\left[-2\cdot\pi\cdot\frac{\exp(j\cdot t)}{\left(4\cdot t^2 - \pi^2\right)} - 2\cdot\pi\cdot\frac{\exp(j\,t)}{\left(4\cdot t^2 - \pi^2\right)}\right] = \frac{2\cdot\cos(t)}{\pi^2 - 4\cdot t^2}$$

W1-7(a) $F_1(\omega) = \dfrac{400}{(j\cdot\omega + 20)\cdot(j\cdot\omega + 40)} = \dfrac{20}{j\cdot\omega + 20} - \dfrac{20}{j\cdot\omega + 40}$ **hence**

$f_1(t) = 20\cdot\left(e^{-20\cdot t} - e^{-40\cdot t}\right)\cdot u(t)$

(b) $F_2(\omega) = \dfrac{j\cdot\omega}{(j\cdot\omega + 20)\cdot(j\cdot\omega + 40)} = \dfrac{-1}{j\cdot\omega + 20} + \dfrac{2}{j\cdot\omega + 40}$ **hence** $f_2(t) = \left(-e^{-20\cdot t} + 2\cdot e^{-40\cdot t}\right)\cdot u(t)$

W1-8(a) $F_1(\omega) = \dfrac{400}{j\cdot\omega\cdot(j\cdot\omega + 20)\cdot(j\cdot\omega + 40)} = \dfrac{0.5}{j\cdot\omega} - \dfrac{1}{j\cdot\omega + 20} + \dfrac{0.5}{j\cdot\omega + 40}$

$f_1(t) = \dfrac{1}{4}\cdot\text{sgn}(t) + \left(-e^{-20\cdot t} + 0.5\cdot e^{-40\cdot t}\right)\cdot u(t)$

(b) $F_2(\omega) = \dfrac{-\omega^2}{(j\cdot\omega + 20)\cdot(j\cdot\omega + 40)} = 1 + \dfrac{20}{j\cdot\omega + 20} - \dfrac{80}{j\cdot\omega + 40}$

$f_2(t) = \delta(t) + \left(20\cdot e^{-20\cdot t} - 80\cdot e^{-40\cdot t}\right)\cdot u(t)$

W1-9(a) $F_1(\omega) = \dfrac{5000}{j\cdot\omega\cdot(-j\cdot\omega + 50)\cdot(j\cdot\omega + 50)} = \dfrac{-1}{j\cdot\omega + 50} + \dfrac{1}{j\cdot(-\omega) + 50} + \dfrac{2}{j\cdot\omega}$

$f_1(t) = -\exp(-50\cdot t)\cdot u(t) + \exp(50\cdot t)\cdot u(-t) + \text{sgn}(t)$

(b) $F_2(\omega) = \dfrac{500\cdot j\cdot\omega}{(-j\cdot\omega + 50)\cdot(j\cdot\omega + 50)} = \dfrac{250}{[j\cdot(-\omega) + 50]} - \dfrac{250}{j\cdot\omega + 50}$

$f_2(t) = 250\cdot e^{50\cdot t}\cdot u(-t) - 250\cdot e^{-50\cdot t}\cdot u(t)$

W1-10 (a) $f_1(t) = 2\cdot u(t) - 2$ $\quad F_1(\omega) = 2\left(\dfrac{1}{j\cdot\omega} + \pi\cdot\delta(\omega)\right) - 4\cdot\pi\cdot\delta(\omega) = \dfrac{2}{j\cdot\omega} - 2\cdot\pi\cdot\delta(\omega)$

(b) $f_2(t) = 2\cdot\text{sgn}(t) - 2\cdot u(t)$ $\quad F_2(\omega) = \dfrac{4}{j\cdot\omega} - 2\cdot\left(\dfrac{1}{j\cdot\omega} + \pi\cdot\delta(\omega)\right) = \dfrac{2}{j\cdot\omega} - 2\cdot\pi\cdot\delta(\omega)$

(c) $f_3(t) = \text{sgn}(t) - 1$ $\quad F_3(\omega) = \dfrac{2}{j\cdot\omega} - 2\cdot\pi\cdot\delta(\omega)$

W1-11 (a) $f_1(t) = 2\cdot e^{-2\cdot t}\cdot u(t) + 2\cdot\text{sgn}(t)$; $F_1(\omega) = \dfrac{2}{j\cdot\omega + 2} + \dfrac{4}{j\cdot\omega} = \dfrac{6\cdot j\cdot\omega + 8}{j\cdot\omega\cdot(j\cdot\omega + 2)}$

(b) $f_2(t) = 2\cdot e^{-2\cdot t}\cdot u(t) + 2\cdot u(t)$; $F_4(\omega) = \dfrac{2}{j\cdot\omega + 2} + 2\cdot\left(\dfrac{1}{j\cdot\omega} + \pi\cdot\delta(\omega)\right) = \dfrac{4\cdot(j\cdot\omega + 1)}{j\cdot\omega\cdot(j\cdot\omega + 2)} + 2\cdot\pi\cdot\delta(\omega)$

W1-12 (a) $f_1(t) = 2\cdot\sin(t) + \cos(t)$; $F_1(\omega) = -j\cdot 2\cdot\pi[\delta(\omega - 1) - \delta(\omega + 1) + \pi\cdot(\delta(\omega - 1) + \phi(\omega + 1))]$

$F_1(\omega) = \pi[(1 - j\cdot 2)\cdot\delta(\omega - 1) + (1 + j\cdot 2)\cdot\delta(\omega + 1)]$

(b) $f_2(t) = 2\cdot\dfrac{\sin(t)}{t} + \cos(t)$; $F_2(\omega) = 2\cdot\pi(u(\omega + 1) - u(\omega - 1)) + \pi\cdot(\delta(\omega + 1) + \delta(\omega - 1))$

W1-13 (a) $f_1(t) = 2\cdot e^{-2\cdot t}\cdot\cos(4\cdot t)\cdot u(t)$; $F_1(\omega) = \dfrac{2\cdot(j\cdot\omega + 2)}{(j\cdot\omega + 2)^2 + 4^2} = \dfrac{j\cdot 2\omega + 4}{\left(-\omega^2 + 4\cdot j\cdot\omega + 20\right)}$

(b) $f_2(t) = \left(2\cdot e^{-2\cdot t} - e^{-4\cdot t}\right)\cdot u(t)$; $F_2(\omega) = \dfrac{2}{(j\cdot\omega + 2)} - \dfrac{1}{j\cdot\omega + 4} = \dfrac{j\cdot\omega + 6}{(j\cdot\omega + 2)\cdot(j\cdot\omega + 4)}$

W1-14 (a) $f_1(t) = 3\cdot\cos(2\cdot\pi\cdot t)$; $F_1(\omega) = 3\cdot\pi\cdot(\delta(\omega - \pi) + \delta(\omega + 2\cdot\pi))$

(b) $f_2(t) = 3\cdot e^{j\cdot 2\cdot\pi\cdot t}$; $F_2(\omega) = 3\cdot(2\cdot\pi\cdot\delta(\omega - 2\cdot\pi))$

W1-15(a) $F_1(\omega) = 4\cdot\pi\cdot\delta(\omega) + 4\cdot\pi\cdot\delta(\omega - 2) + 4\cdot\pi\cdot\delta(\omega + 2)$; $f_1(t) = 2 + 2\cdot\cos(2\cdot t)$

(b) $F_2(\omega) = 4\cdot\pi\cdot\delta(\omega) + \dfrac{2}{j\cdot\omega} = 2\cdot(2\cdot\pi\cdot\delta(\omega)) + \dfrac{2}{j\cdot\omega}$; $f_2(t) = 2 + \mathrm{sgn}(t)$

(c) $F_3(\omega) = 2\cdot\pi\cdot\delta(\omega) + \dfrac{2}{j\cdot\omega} = 2\cdot\left(\pi\cdot\delta(\omega) + \dfrac{1}{j\cdot\omega}\right)$; $f_3(t) = 2\cdot u(t)$

W1-16(a) $F_1(\omega) = 4\cdot\cos(2\cdot\omega)$; $f_1(t) = 2\cdot\delta(t + 2) + 2\cdot\delta(t - 2)$

(b) $F_2(\omega) = 4\cdot u(\omega) - 2$; $f_2(t) = \dfrac{2\cdot j}{\pi\cdot t}$

(c) $F_3(\omega) = 4\cdot e^{-|2\cdot\omega|}$; $f_3(t) = \dfrac{2}{\pi\cdot\left(4 + t^2\right)}$

W1-17(a) $F_1(\omega) = \left(4\cdot\pi\cdot\delta(\omega) + \dfrac{2}{j\cdot\omega}\right)\cdot e^{-j\cdot 2\cdot\omega}$; $f_1(t) = 2 + \mathrm{sgn}(t - 2)$

(b) $F_2(\omega) = \dfrac{2}{j\cdot\omega + 2}\cdot e^{-j\cdot 2\cdot\omega}$; $f_2(t) = 2\cdot\exp[-2\cdot(t - 2)]\cdot u(t - 2)$

(c) $F_3(\omega) = \dfrac{4\cdot\cos(2\cdot\omega)}{j\cdot\omega} = \dfrac{4}{j\cdot\omega}\cdot\left(\dfrac{e^{j\cdot 2\cdot\omega} + e^{-j\cdot 2\cdot\omega}}{2}\right) = \dfrac{2}{j\cdot\omega}\cdot e^{j\cdot 2\cdot\omega} + \dfrac{2}{j\cdot\omega}\cdot e^{-j\cdot 2\cdot\omega}$;

$f_3(t) = \mathrm{sgn}(t + 2) + \mathrm{sgn}(t - 2) = -2\cdot u(-t - 2) + 2\,u(t - 2)$

W1-18 (a) $f_1(t) = 2\cdot u(-t)$; $F_1(\omega) = 2\left(\dfrac{1}{-j\cdot\omega} + \pi\cdot\delta(-\omega)\right) = \dfrac{-2}{j\cdot\omega} + 2\cdot\pi\cdot\delta(\omega)$

$f_2(t) = 1 - \mathrm{sgn}(t)$; $F_2(\omega) = \dfrac{-2}{j\cdot\omega} + 2\cdot\pi\cdot\delta(\omega)$ thus $F_1(\omega) = F_2(\omega)$ QED

(b) No, since $f_2(t) = u(-t) + u(t) - (u(t) - u(-t)) = 2\cdot u(-t) = f_1(t)$

W1-19 $G(\omega) = \dfrac{F(\omega)}{j\cdot\omega} + \pi\cdot F(0)\cdot\delta(\omega) = \dfrac{2}{j\cdot\omega\cdot(j\cdot\omega + 2)\cdot(j\cdot\omega + 4)} + \dfrac{\pi}{4}\cdot\delta(\omega$ Expanding by paritial fractions

$G(\omega) = \dfrac{1}{4}\cdot\left(\dfrac{1}{j\cdot\omega}\right) + \dfrac{-1}{2}\cdot\left(\dfrac{1}{j\cdot\omega + 2}\right) + \dfrac{1}{4}\cdot\left(\dfrac{1}{j\cdot\omega + 4}\right) + \dfrac{\pi}{4}\cdot\delta(\omega$ Performing the inverse transform term by term

$$g(t) = \frac{1}{8}\cdot\mathrm{sgn}(t) - \frac{1}{2}\cdot e^{-2\cdot t}\cdot u(t) + \frac{1}{4}\cdot e^{-4\cdot t}\cdot u(t) + \frac{1}{8}$$

but $\dfrac{1}{8}\cdot\mathrm{sgn}(t) + \dfrac{1}{8} = \dfrac{1}{4}\cdot u(t)$ hence $g(t) = \left(\dfrac{1}{4} - \dfrac{1}{2}\cdot e^{-2\cdot t} + \dfrac{1}{4}\cdot e^{-4\cdot t}\right)\cdot u(t)$

W1-20 Given that $$\mathcal{F}(A\cdot\exp(-\alpha\cdot t)\cdot u(t)) = \frac{A}{j\cdot\omega+\alpha}$$

Using reversal $$\mathcal{F}[A\cdot\exp[-\alpha\cdot(-t)]\cdot u(-t)] = \frac{A}{-j\cdot\omega+\alpha}$$

Then since $$A\cdot\text{sgn}(t)\cdot\exp(-\alpha\cdot|t|) = -A\cdot u(-t)\cdot\exp[-\alpha\cdot(-t)] + A\cdot u(t)\cdot\exp(-\alpha\cdot t)$$

Using linearity $$\mathcal{F}(A\cdot\text{sgn}(t)\cdot\exp(-\alpha\cdot|t|)) = \frac{-A}{-j\cdot\omega+\alpha} + \frac{A}{j\cdot\omega+\alpha} = \frac{-2\cdot A\cdot j\cdot\omega}{\omega^2+\alpha^2} \quad \text{QED}$$

W1-21 Using Euler's eq. $$f(t)\cdot\sin(\beta\cdot t) = f(t)\cdot\left(\frac{e^{j\cdot\beta\cdot t}-e^{-j\cdot\beta\cdot t}}{2\cdot j}\right) = \frac{f(t)\cdot e^{j\cdot\beta\cdot t}}{2\cdot j} - \frac{f(t)\cdot e^{-j\cdot\beta\cdot t}}{2\cdot j}$$

Using linearity $$\mathcal{F}(f(t)\cdot\sin(\beta t)) = \mathcal{F}\left(\frac{f(t)\cdot e^{j\cdot\beta\cdot t}}{2\cdot j}\right) - \mathcal{F}\left(\frac{f(t)\cdot e^{-j\cdot\beta\cdot t}}{2\cdot j}\right)$$

Using frequency shifting $$\mathcal{F}(f(t)\cdot\sin(\beta t)) = \frac{F(\omega-\beta)}{2\cdot j} - \frac{F(\omega+\beta)}{2\cdot j} \quad \text{QED}$$

W1-22 Using Euler's eq. $$\cos(\beta\cdot t)\cdot u(t) = u(t)\cdot\left(\frac{e^{j\cdot\beta\cdot t}+e^{-j\cdot\beta\cdot t}}{2}\right) = \frac{u(t)\cdot e^{j\cdot\beta\cdot t}}{2} + \frac{u(t)\cdot e^{-j\cdot\beta\cdot t}}{2}$$

Using linearity $$\mathcal{F}(\cos(\beta t)\cdot u(t)) = \mathcal{F}\left(\frac{u(t)\cdot e^{j\cdot\beta\cdot t}}{2}\right) + \mathcal{F}\left(\frac{u(t)\cdot e^{-j\cdot\beta\cdot t}}{2}\right)$$

Using freq. shifting $$\mathcal{F}(\cos(\beta t)\cdot u(t)) = \frac{1}{2}\cdot\left[\frac{1}{j\cdot(\omega-\beta)} + \pi\cdot\delta(\omega-\beta) + \frac{1}{j\cdot(\omega+\beta)} + \pi\cdot\delta(\omega+\beta)\right]$$

Simplifying $$\mathcal{F}(\cos(\beta t)\cdot u(t)) = \frac{j\cdot\omega}{\beta^2-\omega^2} + \frac{\pi}{2}\cdot(\delta(\omega-\beta)+\delta(\omega+\beta)) \quad \text{QED}$$

The function cos(βt)u(t) is not absolutely integrable hence its Fourier transform can not be found by replacing s by jω in its Laplace transform.

W1-23

By definition $$F(\omega) = \int_{-\infty}^{\infty} f(t)\cdot e^{-j\cdot\omega\cdot t}\,dt$$ hence

$$\frac{d}{d\omega}F(\omega) = \frac{d}{d\omega}\left(\int_{-\infty}^{\infty} f(t)\cdot e^{-j\cdot\omega\cdot t}\,dt\right) = \int_{-\infty}^{\infty}\frac{d}{d\omega}\left(f(t)\cdot e^{-j\cdot\omega\cdot t}\right)dt = \int_{-\infty}^{\infty}(-j\cdot t\cdot f(t))\cdot e^{-j\cdot\omega\cdot t}\,dt$$

But, by definition

$$\int_{-\infty}^{\infty}(-j\cdot t\cdot f(t))\cdot e^{-j\cdot\omega\cdot t}\,dt = \mathcal{F}(-j\cdot t\cdot f(t)) \quad \text{QED}$$

W1-24

By definition $$F(\omega) = \int_{-\infty}^{\infty} f(t)\cdot e^{-j\cdot\omega\cdot t}\,dt$$ hence if ω = 0

$$F(0) = \int_{-\infty}^{\infty} f(t)\cdot e^{0}\,dt = \int_{-\infty}^{\infty} f(t)\,dt \quad \text{QED}$$

W1-25 By definition $F(\omega) = \int_{-\infty}^{\infty} f(t)\cdot\exp(-j\cdot\omega\cdot t)\,dt$

Define $G(\omega) = \int_{-\infty}^{\infty} f(t-T)\cdot\exp(-j\cdot\omega\cdot t)\,dt$ Change variables to $\tau = t - T$ then

$$G(\omega) = \int_{-\infty}^{\infty} f(\tau)\cdot\exp[-j\cdot\omega\cdot(\tau+T)]\,d\tau = \exp(-j\cdot\omega\cdot T)\cdot\left(\int_{-\infty}^{\infty} f(\tau)\cdot\exp(-j\cdot\omega\cdot\tau)\,d\tau\right)$$

By definition the integral inside the large parenthesis produces $F(\omega)$.

Hence $F(f(t-T)) = G(\omega) = \exp(-j\cdot\omega\cdot T)\cdot F(\omega)$ QED

W1-26 $v_1(t) = 10\cdot\mathrm{sgn}(t)$ $V_1(\omega) = \dfrac{20}{j\cdot\omega}$

$$T_V(s) = \frac{R}{R + L\cdot s + \dfrac{1}{C\cdot s}} = \frac{\dfrac{R}{L}\cdot s}{s^2 + \dfrac{R}{L}\cdot s + \dfrac{1}{L\cdot C}}$$

$$T_V(s) = \frac{1000\cdot s}{s^2 + 1000\cdot s + 1000^2}$$

$$V_2(\omega) = T_V(j\cdot\omega)\cdot V_1(\omega)$$

$$V_2(\omega) = \left[\frac{1000\cdot j\cdot\omega}{(j\cdot\omega)^2 + 1000\cdot j\cdot\omega + 1000^2}\right]\cdot\left(\frac{20}{j\cdot\omega}\right) = \frac{20000}{866}\cdot\left[\frac{866}{(j\cdot\omega+500)^2 + 866^2}\right]$$

$$v_2(t) = \frac{20000}{866}\cdot\left(e^{-500\cdot t}\cdot\sin(866\cdot t)\right)\cdot u(t) = 23.1\cdot\left(e^{-500\cdot t}\cdot\sin(866\cdot t)\right)\cdot u(t)$$

W1-27 $v_1(t) = 20\cdot e^{5\cdot t}\cdot u(-t)$ $V_1(\omega) = \dfrac{20}{j\cdot(-\omega)+5}$

$$T_V(s) = \frac{\dfrac{1}{C\cdot s+\dfrac{1}{R}}}{\dfrac{1}{C\cdot s+\dfrac{1}{R}} + R} = \frac{1}{R\cdot C\cdot s + 2} = \frac{10}{s+20}$$

$$V_2(\omega) = T_V(j\cdot\omega)\cdot V_1(\omega)$$

$$V_2(j\cdot\omega) = \left(\frac{10}{j\cdot\omega+20}\right)\cdot\left[\frac{20}{j\cdot(-\omega)+5}\right] = \frac{8}{j\cdot(-\omega)+5} + \frac{8}{j\cdot\omega+20}$$

$$v_2(t) = 8\cdot\exp(5\cdot t)\cdot u(-t) + 8\cdot\exp(-20\cdot t)\cdot u(t)$$

W1-28 $v_1(t) = 20\cdot\text{sgn}(t)$ $V_1(\omega) = \dfrac{40}{j\cdot\omega}$ Circuit same as Prob W1-27 above $T_V(s) = \dfrac{1}{R\cdot C\cdot s + 2} = \dfrac{10}{s + 20}$

$$V_2(\omega) = T_V(j\cdot\omega)\cdot V_1(\omega) = \left(\frac{10}{j\cdot\omega + 20}\right)\cdot\left(\frac{40}{j\cdot\omega}\right) = \frac{20}{j\cdot\omega} - \frac{20}{(j\cdot\omega + 20)} \qquad v_2(t) = 10\cdot\text{sgn}(t) - 20\cdot e^{-20\cdot t}\cdot u(t)$$

W1-29 $v_1(t) = 10\cdot u(-t)$

$$V_1(\omega) = \frac{10}{j\cdot(-\omega)} + 10\cdot\pi\cdot\delta(-\omega)$$

$$T_V(s) = \frac{-R}{R + \dfrac{1}{C\cdot s}} = \frac{-s}{s + \dfrac{1}{R\cdot C}} = \frac{-s}{s + 100}$$

$$V_2(\omega) = T_V(j\cdot\omega)\cdot V_1(\omega)$$

$$V_2(\omega) = \left(\frac{-j\cdot\omega}{j\cdot\omega + 100}\right)\cdot\left[\frac{10}{j\cdot(-\omega)} + 10\cdot\pi\cdot\delta(-\omega)\right] = \frac{10}{j\cdot\omega + 100} - \frac{j\cdot 10\pi\cdot\omega\cdot\delta(\omega)}{j\cdot\omega + 100} = \frac{10}{j\cdot\omega + 100}$$

$$v_2(t) = 10\cdot\exp((-100\cdot t))\cdot u(t)$$

W1-30

Circuit same as W1-29 $v_1(t) = 20\cdot e^{-10\cdot|t|}$ $V_1(\omega) = \dfrac{400}{\omega^2 + 100}$ $T_V(s) = \dfrac{-R}{R + \dfrac{1}{C\cdot s}} = \dfrac{-s}{s + \dfrac{1}{R\cdot C}} = \dfrac{-s}{s + 100}$

$$V_2(\omega) = T_V(j\cdot\omega)\cdot V_1(\omega) = \left(\frac{-j\cdot\omega}{j\cdot\omega + 100}\right)\cdot\left(\frac{400}{\omega^2 + 100}\right) = \frac{-20}{11}\cdot\frac{1}{[j\cdot(-\omega) + 10]} + \frac{20}{9}\cdot\frac{1}{(j\cdot\omega + 10)} - \frac{400}{99}\cdot\frac{1}{(j\cdot\omega + 1}$$

$$v_2(t) = \frac{-20}{11}\cdot\exp(10\cdot t)\cdot u(-t) + \frac{20}{9}\cdot\exp(-10\cdot t)\cdot u(t) - \frac{400}{99}\cdot\exp(-100\cdot t)\cdot u(t)$$

W1-31 $h(t) = \exp(-2\cdot t)\,u(t)$ $H(\omega) = \dfrac{1}{j\cdot\omega + 2}$ $x(t) = u(-t)$ $X(\omega) = \dfrac{1}{j\cdot(-\omega)} + \pi\cdot\delta(-\omega)$

$$Y(\omega) = H(\omega)\cdot X(\omega) = \frac{1}{j\cdot\omega + 2}\cdot\left[\frac{1}{j\cdot(-\omega)} + \pi\cdot\delta(-\omega)\right] = \frac{1}{j\cdot(-\omega)\cdot(j\cdot\omega + 2)} + \frac{\pi}{j\cdot\omega + 2}\cdot\delta(-\omega)$$

$$Y(\omega) = \frac{1}{2\cdot j\cdot(-\omega)} + \frac{1}{2\cdot(j\cdot\omega + 2)} + \frac{\pi}{2}\cdot\delta(-\omega) = \frac{1}{2}\cdot\left[\left[\frac{1}{j\cdot(-\omega)} + \pi\cdot\delta(-\omega)\right] + \frac{1}{j\cdot\omega + 2}\right]$$

$$y(t) = \frac{1}{2}\cdot u(-t) + \frac{1}{2}\cdot\exp(-2\cdot r)\cdot u(t)$$

W1-32 $h(t) = \exp(-2\cdot|t|)$ $\quad H(\omega) = \dfrac{4}{(j\cdot\omega + 2)\cdot[j\cdot(-\omega) + 2]}$ $\quad x(t) = u(t)$ $\quad X(\omega) = \dfrac{1}{j\cdot(\omega)} + \pi\cdot\delta(\omega)$

$$Y(\omega) = H(\omega)\cdot X(\omega) = \frac{4}{(j\cdot\omega + 2)\cdot[j\cdot(-\omega) + 2]}\cdot\left[\frac{1}{j\cdot(\omega)} + \pi\cdot\delta(\omega)\right]$$

$$Y(\omega) = \frac{-1}{[2\cdot(j\cdot\omega + 2)]} + \frac{1}{[2\cdot[j\cdot(-\omega) + 2]]} + \frac{1}{j\cdot\omega} + \frac{4\cdot\pi}{\omega^2 + 2^2}\cdot\delta(\omega) = \frac{1}{2}\cdot\left[\frac{-1}{j\cdot\omega + 2} + \frac{1}{j\cdot(-\omega) + 2}\right] + \frac{1}{j\cdot\omega} + \pi\cdot\delta$$

$$y(t) = \frac{-1}{2}\cdot\exp(-2\cdot t)\cdot u(t) + \frac{1}{2}\cdot\exp(2\cdot t)\cdot u(-t) + u(t)$$

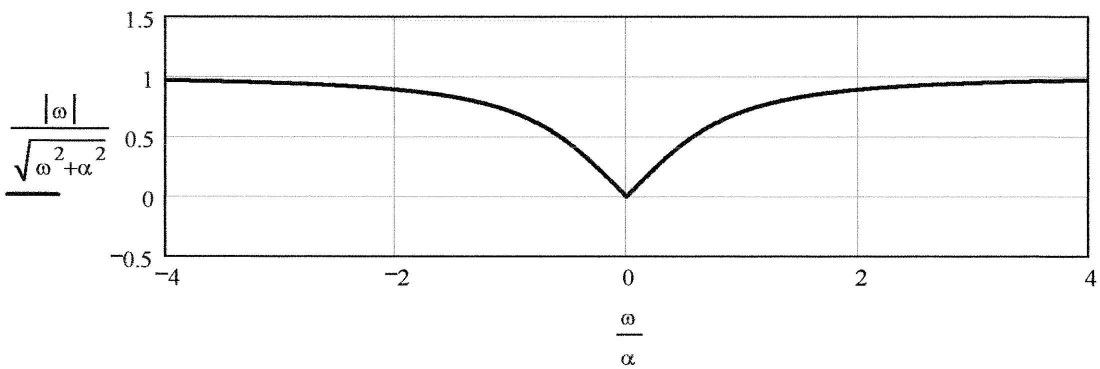
W1-33 $h(t) = \delta(\omega) - (2\cdot\exp(-t))\cdot u(t)$ $\quad H(\omega) = 1 - \dfrac{2}{j\cdot\omega + 1} = \dfrac{j\cdot\omega - 1}{j\cdot\omega + 1}$ $x(t) = \mathrm{sgn}(t)$ $\quad X(\omega) = \dfrac{2}{j\cdot\omega}$

$$Y(\omega) = H(\omega)\cdot X(\omega) = \frac{j\cdot\omega - 1}{j\cdot\omega + 1}\cdot\left(\frac{2}{j\cdot\omega}\right) = \frac{4}{j\cdot\omega + 1} - \frac{2}{j\cdot\omega} \quad y(t) = 4\cdot\exp(-t)\cdot u(t) - \mathrm{sgn}(t)$$

W1-34 $h(t) = A\cdot(\delta(t) - \alpha\cdot u(t)\cdot\exp(-\alpha\cdot t))$ **from Fourier transform Table**

$$A := 1$$

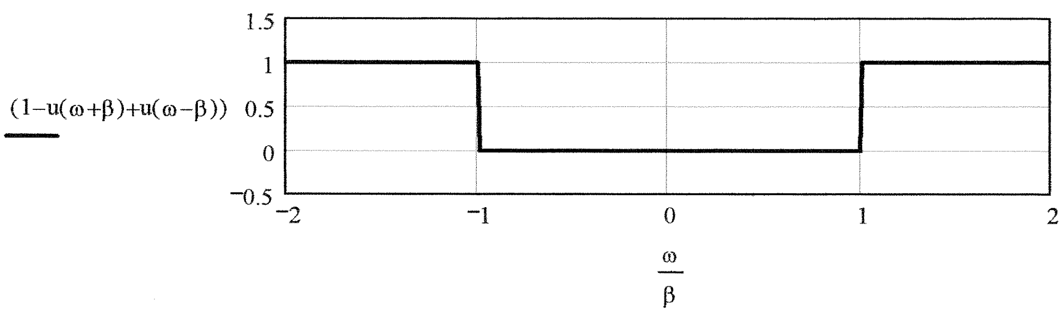
$$H(\omega) = A\left(1 - \frac{\alpha}{j\cdot\omega + \alpha}\right) = A\cdot\frac{j\cdot\omega}{j\cdot\omega + \alpha} \quad |H(\omega)| = |A|\cdot\left(\frac{|\omega|}{\sqrt{\omega^2 + \alpha^2}}\right) \qquad \alpha := 1 \quad \omega := -4, -3.99 .. 4$$



System is a 1st order high-pass filter with passband gain of |A| and cutoff freq. $\omega_C = \pm\alpha$.

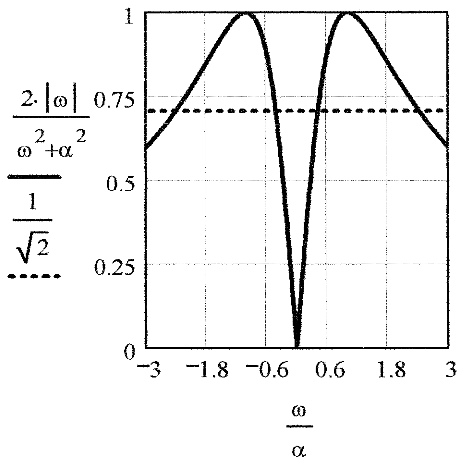
W1-35 $h(t) = A\cdot\left(\delta(t) - \dfrac{\sin(\beta\cdot t)}{\pi\cdot t}\right)$ **from Fourier transform Table** $\quad u(x) := \begin{cases} 0 & \text{if } 0 > x \\ 1 & \text{if } 0 \le x \end{cases}$ $\quad \beta := 1$

$H(\omega) = A(1 - u(\omega + \beta) + u(\omega - \beta))$ System is an ideal high-pass filter with cutoff freq. $\omega_C = \pm\beta$.



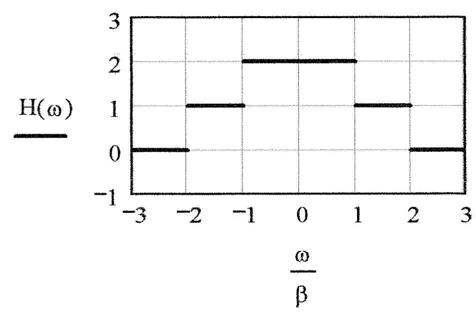
W1-36 $h(t) = A\cdot(\exp(\alpha\cdot t)\cdot u(-t) - \exp(-\alpha\cdot t)\cdot u(t))$ from Fourier transform Table and reversal prop.

$$H(\omega) = \frac{A}{j\cdot(-\omega)+\alpha} - \frac{A}{j\cdot\omega+\alpha} = \frac{2\cdot j\cdot\omega\cdot A}{\omega^2+\alpha^2} \qquad \omega := -4,-3.99..4$$



system is a bandpass filter with a pass-band gain of |A| and cutoff frequencies at
positive: $\omega_C = \alpha\cdot(\sqrt{2}+1)$ & $\omega_C = \alpha\cdot(\sqrt{2}-1)$
negative: $\omega_C = \alpha\cdot(-\sqrt{2}+1)$ & $\omega_C = \alpha\cdot(-\sqrt{2}-1)$

W1-37 $\beta := 1$ $H(\omega) := \begin{cases} 2 \text{ if } -\beta < \omega < \beta \\ 1 \text{ if } \beta < |\omega| < 2\cdot\beta \\ 0 \text{ if } 2\cdot\beta < |\omega| \end{cases}$



$$h(t) = \frac{1}{2\cdot\pi}\cdot\int_{-\beta}^{\beta} 2\cdot\exp(j\cdot\omega\cdot t)\,d\omega + \frac{1}{2\cdot\pi}\cdot\int_{-2\cdot\beta}^{-\beta} \exp(j\cdot\omega\cdot t)\,d\omega + \frac{1}{2\cdot\pi}\cdot\int_{\beta}^{2\cdot\beta} \exp(j\cdot\omega\cdot t)\,d\omega$$

$$h(t) = \frac{1}{2\cdot\pi}\left[\left(\frac{2\cdot\exp(j\cdot\beta\cdot t)}{j\cdot t} - \frac{2\cdot\exp(-j\cdot\beta\cdot t)}{j\cdot t}\right) + \left[\frac{\exp(-j\cdot\beta\cdot t)}{(j\cdot t)} - \frac{\exp(-2\cdot j\cdot\beta\cdot t)}{(j\cdot t)}\right] + \left[\frac{\exp(2\cdot j\cdot\beta\cdot t)}{(j\cdot t)} - \frac{\exp(j\cdot\beta\cdot t)}{(j\cdot t)}\right]\right]$$

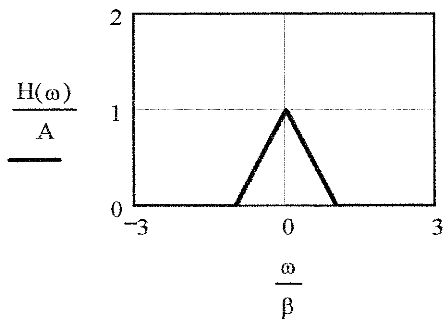
$$h(t) = \frac{1}{\pi\cdot t}\cdot\left(\frac{\exp(j\cdot\beta\cdot t) - \exp(-j\cdot\beta\cdot t)}{2\cdot j} + \frac{\exp(j\cdot 2\cdot\beta\cdot t) - \exp(-j\cdot 2\cdot\beta\cdot t)}{2\cdot j}\right) = \frac{\sin(\beta\cdot t) + \sin(2\cdot\beta\cdot t)}{\pi\cdot t}$$

Alternatively, use linearity and the table of Fourier transforms

$F(\omega) = F_1(\omega) + F_2(\omega)$ $\qquad F_1(\omega) = u(\omega+\beta) - u(\omega-\beta)$ $\quad F_2(\omega) = u(\omega+2\cdot\beta) - u(\omega-2\cdot\beta)$

$f_1(t) = \frac{\sin(\beta\cdot t)}{\pi\cdot t}$ $\qquad f_2(t) = \frac{\sin(2\cdot\beta\cdot t)}{\pi\cdot t}$ $\qquad f(t) = f_1(t) + f_2(t) = \frac{\sin(\beta\cdot t) + \sin(2\cdot\beta\cdot t)}{\pi\cdot t}$

W1-38 $H(\omega) := \begin{cases} \frac{A}{\beta}\cdot(\beta+\omega) & \text{if } -\beta < \omega < 0 \\ \frac{A}{\beta}\cdot(\beta-\omega) & \text{if } 0 < \omega < \beta \\ 0 & \text{if } \beta < |\omega| \end{cases}$



$$h(t) = \frac{1}{2\cdot\pi}\cdot\int_{-\beta}^{0}\frac{A}{\beta}\cdot(\beta+\omega)\cdot\exp(j\cdot\omega\cdot t)\,d\omega + \frac{1}{2\cdot\pi}\cdot\int_{0}^{\beta}\frac{A}{\beta}\cdot(\beta-\omega)\cdot\exp(j\cdot\omega\cdot t)\,d\omega$$

$$h(t) = \frac{1}{(2\cdot\pi)}\cdot\left[-j\cdot(\beta\cdot t+j)\cdot\frac{A}{(t^2\cdot\beta)} - \exp(-j\cdot\beta\cdot t)\cdot\frac{A}{(t^2\cdot\beta)}\right] + \frac{1}{(2\cdot\pi)}\cdot\left[-\exp(j\cdot\beta\cdot t)\cdot\frac{A}{(t^2\cdot\beta)} + j\cdot(\beta\cdot t-j)\cdot\frac{A}{(t^2\cdot\beta)}\right]$$

$$h(t) = \frac{A}{2\cdot\pi}\cdot\left[\frac{-j\cdot(\beta\cdot t+j)}{\beta\cdot t^2} - \frac{\exp(-j\cdot\beta\cdot t)}{\beta\cdot t^2}\right] + \frac{A}{2\cdot\pi}\cdot\left[\frac{-\exp(j\cdot\beta\cdot t)}{\beta\cdot t^2} + \frac{j\cdot(t\cdot\beta-j)}{\beta\cdot t^2}\right]$$

$$h(t) = \frac{A}{\pi\cdot\beta\cdot t^2}\cdot\left(\frac{1-j\cdot\beta\cdot t}{2} - \frac{\exp(-j\cdot\beta\cdot t)}{2} - \frac{\exp(j\cdot\beta\cdot t)}{2} + \frac{1+j\cdot\beta\cdot t}{2}\right) = A\cdot\frac{1-\cos(\beta\cdot t)}{\pi\cdot\beta\cdot t^2}$$

W1-39 Using the Fourier transform table for $h(t) = 2\sin(2\beta t)/\pi t$ and $x(t) = (A\pi/\beta)\sin(\beta t)/\pi t$ yields

$H(\omega) = 2\cdot(u(\omega+2\cdot\beta) - u(\omega-2\cdot\beta))$ and $X(\omega) = \frac{\pi\cdot A}{\beta}\cdot(u(\omega+\beta) - u(\omega-\beta))$

The input transform is zero everywhere except in the range $-\beta < \omega < \beta$. Throughout this range the transfer function is $H(\omega) = 2$, hence $Y(\omega) = H(\omega)X(\omega) = 2X(\omega)$ and $y(t) = 2\cdot A\cdot\frac{\sin(\beta\cdot t)}{\beta\cdot t}$. In other words, $X(\omega)$ is a bandlimited signal whose spectrum lies entirely within the passband of $H(\omega)$.

W1-40 $F(\omega) = \frac{2}{\omega^2+1}$ $\quad W_{1\Omega} = \frac{1}{\pi}\cdot\int_{0}^{\infty}\frac{4}{(\omega^2+1)^2}\,d\omega = \frac{4}{\pi}\cdot\left[\frac{\omega}{2\cdot(\omega^2+1)} + \frac{1}{2}\cdot\tan^{-1}(\omega)\right]\Big|_0^\infty$

$$W_{1\Omega} = \frac{4}{\pi}\cdot\left[0 + \frac{1}{2}\cdot\left[\tan^{-1}(\infty)\right] - 0 - \tan^{-1}(0)\right] = \frac{4}{\pi}\cdot\frac{1}{2}\cdot\frac{\pi}{2} = 1$$

W-41 $f(t) = A\cdot e^{\alpha\cdot t}\cdot u(-t)$ $\quad F(\omega) = \frac{A}{[j\cdot(-\omega)+\alpha]}$

$$W_{1\Omega} = \frac{1}{2\cdot\pi}\cdot\int_{-\infty}^{\infty}(|F(\omega)|)^2\,d\omega = \frac{A^2}{2\cdot\pi}\cdot\int_{-\infty}^{\infty}\frac{1}{(\omega^2+\alpha^2)}\,d\omega = \frac{A^2}{2\cdot\pi\cdot\alpha}\cdot\left[\tan^{-1}\left(\frac{\omega}{\alpha}\right)\right]\Big|_{-\infty}^{\infty}$$

$$W_{1\Omega} = \frac{A^2}{2\cdot\pi\cdot\alpha}\cdot\left[\tan^{-1}(\infty) - \tan^{-1}(-\infty)\right] = \frac{A^2}{2\cdot\pi\cdot\alpha}\cdot\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = \frac{A^2}{2\cdot\alpha}$$

W-42 $f(t) = A\cdot\frac{\sin(\beta\cdot t)}{\pi\cdot\beta\cdot t}$ $F(\omega) = \frac{A}{\beta}\cdot(u(\omega+\beta) - u(\omega-\beta))$

$$W_{1\Omega} = \frac{1}{2\cdot\pi}\cdot\int_{-\infty}^{\infty}(|F(\omega)|)^2\,d\omega = \frac{1}{2\cdot\pi}\cdot\int_{-\beta}^{\beta}\left(\frac{A}{\beta}\right)^2 d\omega = \frac{1}{2\cdot\pi}\cdot\left(\frac{A}{\beta}\right)^2\cdot[\beta-(-\beta)] = \frac{A^2}{\pi\cdot\beta}$$

f(t) is a bandlimited waveform whose amplitude spectrum all falls between $|\omega| < \beta$

W1-43

$$W_{1\Omega} = \frac{A^2}{2\cdot\pi}\cdot\int_{-\infty}^{\infty}\frac{\omega^2}{\left(\omega^2+\alpha^2\right)^2}\,d\omega = \frac{A^2}{2\cdot\pi}\cdot\left[\frac{-\omega}{2\cdot\left(\alpha^2+\omega^2\right)} + \frac{1}{2\cdot\alpha}\cdot\left[\tan^{-1}\cdot\left(\frac{\omega}{\alpha}\right)\right]\right]\Bigg|_{-\infty}^{\infty}$$

$$W_{1\Omega} = \frac{A^2}{2\cdot\pi}\cdot\frac{1}{2\cdot\alpha}\cdot\left[\tan^{-1}\cdot(\infty) - \tan^{-1}\cdot(-\infty)\right] = \frac{A^2}{4\cdot\alpha}\cdot\frac{1}{\pi}\cdot\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = \frac{A^2}{4\cdot\alpha}$$

$$W_{\alpha} = \frac{A^2}{2\cdot\pi}\cdot\int_{-\alpha}^{\alpha}\frac{\omega^2}{\left(\omega^2+\alpha^2\right)^2}\,d\omega = \frac{A^2}{2\cdot\pi}\cdot\left[\frac{-\omega}{2\cdot\left(\alpha^2+\omega^2\right)} + \frac{1}{2\cdot\alpha}\cdot\operatorname{atan}\left(\frac{\omega}{\alpha}\right)\right]\Bigg|_{-\alpha}^{\alpha}$$

$$W_{\alpha} = \frac{A^2}{2\cdot\pi}\cdot\left[\frac{-1}{4\cdot\alpha} + \frac{1}{2\cdot\alpha}\cdot\frac{\pi}{4} - \left(\frac{1}{4\cdot\alpha}\right) - \frac{1}{2\cdot\alpha}\cdot\left(-\frac{\pi}{4}\right)\right] = \frac{A^2}{2\cdot\pi}\cdot\left[\frac{-1}{(2\cdot\alpha)} + \frac{1}{(4\cdot\alpha)}\cdot\pi\right] = \frac{A^2}{4\cdot\alpha}\cdot\left(\frac{\pi-2}{2\cdot\pi}\right)$$

$$\frac{W_{\alpha}}{W_{1\Omega}} = \frac{\pi-2}{2\cdot\pi} = 0.182$$ **18.2 %**

W1-44 $x(t) = 10\cdot\exp(-2000\cdot t)\cdot u(t)$ $X(\omega) = \frac{10}{j\cdot\omega+2000}$ $H(\omega) = u(-\omega-2000) + u(\omega-2000)$

The input is a casual exponential whose 1Ω energy is

$$W_{IN} = \frac{1}{\pi}\cdot\int_{0}^{\infty}\frac{100}{2000^2+\omega^2}\,d\omega = \frac{10^2}{2\cdot 2000} = \frac{1}{40} = 0.025$$

$$W_{out} = \frac{1}{2\cdot\pi}\cdot\int_{-\infty}^{-2000}(X(\omega))^2\,d\omega + \frac{1}{2\cdot\pi}\cdot\int_{2000}^{\infty}(X(\omega))^2\,d\omega$$

$$W_{out} = \frac{100}{2\cdot\pi}\cdot\left(\int_{-\infty}^{-2000}\frac{1}{\omega^2+2000^2}\,d\omega + \int_{2000}^{\infty}\frac{1}{\omega^2+2000^2}\,d\omega\right) = \frac{100}{2\cdot\pi}\cdot\left(\frac{\pi}{8000}+\frac{\pi}{8000}\right) = \frac{1}{80} = 0.0125$$

$$\frac{W_{out}}{W_{in}} = \frac{0.0125}{0.025} = 0.5$$ **50% of the input energy falls in the filter passband**

W1-45 $x(t) = 20\cdot e^{-5\cdot t}\cdot u(t)$ $\quad X(\omega) = \frac{20}{j\cdot\omega + 5}$ $\quad h(t) = 10\cdot e^{-20\cdot t}\cdot u(t)$ $\quad H(\omega) = \frac{10}{j\cdot\omega + 20}$

$$Y(\omega) = H(\omega)\cdot X(\omega) = \left(\frac{10}{j\cdot\omega + 20}\right)\cdot\left(\frac{20}{j\cdot\omega + 5}\right)$$

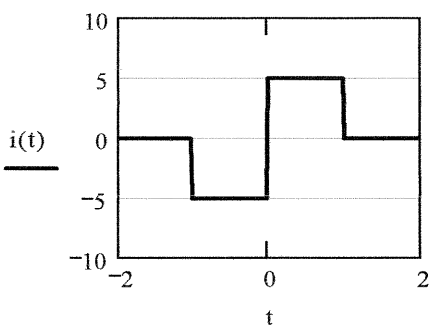
$$W_{out} = \frac{1}{\pi}\cdot\int_0^{\infty} \left(\left|Y(\omega)\right|\right)^2 d\omega = \frac{1}{\pi}\cdot\int_0^{\infty} \frac{100}{\omega^2 + 20^2}\cdot\frac{400}{\omega^2 + 5^2} d\omega = \frac{1}{\pi}\cdot\int_0^{\infty}\left[\frac{-320}{3\cdot\left(\omega^2 + 20^2\right)} + \frac{320}{3\cdot\left(\omega^2 + 5^2\right)}\right] d\omega$$

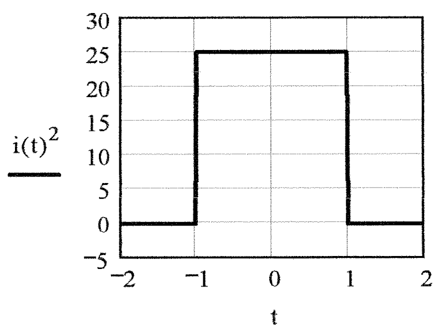
$$\frac{-320}{3\cdot\pi}\cdot\int_0^{\infty} \frac{1}{\omega^2 + 20^2} d\omega = \frac{-320}{3\cdot\pi}\cdot\left[\frac{1}{20}\cdot\tan^{-1}(\infty) - \frac{1}{20}\cdot\tan^{-1}(0)\right] = \frac{-320}{3\cdot\pi}\cdot\frac{1}{20}\cdot\frac{\pi}{2} = \frac{-8}{3}$$

$$\frac{320}{3\cdot\pi}\cdot\int_0^{\infty} \frac{1}{\omega^2 + 5^2} d\omega = \frac{320}{3\cdot\pi}\cdot\left[\frac{1}{5}\cdot\tan^{-1}(\infty) - \frac{1}{5}\cdot\tan^{-1}(0)\right] = \frac{320}{3\cdot\pi}\cdot\frac{1}{5}\cdot\frac{\pi}{2} = \frac{32}{3}$$

$$W_{out} = \frac{32}{3} - \frac{8}{3} = 8$$

W1-46 $i(t) := -5\cdot u(t + 1) + 10\cdot u(t) - 5\cdot u(t - 1)$ $\quad t := -2, -1.99 .. 2$





$$W = \int_{-\infty}^{\infty} R\cdot i(t)^2 dt = 500\cdot\int_{-1}^{1} 25\, dt = 500\cdot 25\cdot[1 - (-1)] = 25\cdot 10^3$$

W1-47 $H(\omega) := \begin{cases} 0 & \text{if } 0 < \omega \le 1000 \\ 1 & \text{if } 1000 < \omega \le 1100 \\ 0 & \text{if } 1100 \le \omega \end{cases}$ $\quad x(t) = 10\cdot e^{-500\cdot t}\cdot u(t)$ $\quad X(\omega) := \frac{10}{j\cdot\omega + 500}$

$$W_{out} = \frac{1}{\pi}\cdot\int_{1000}^{1100} \frac{100}{\omega^2 + 500^2} d\omega = \frac{1}{\pi}\cdot\left(\frac{1}{50}\cdot\operatorname{atan}\left(\frac{11}{5}\right) - \frac{1}{50}\cdot\operatorname{atan}(2)\right) = 2.357\cdot 10^{-4}$$

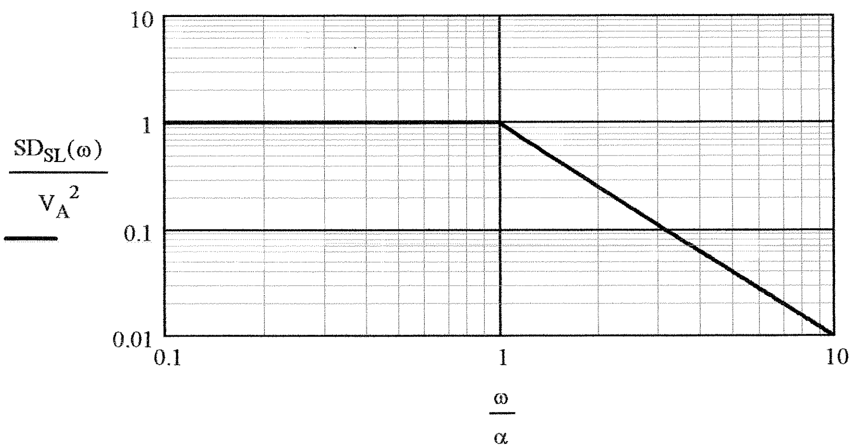
$W_{in} = \frac{A^2}{2\cdot\alpha} = \frac{10^2}{2\cdot 500} = 0.1$ $\quad \frac{W_{out}}{W_{in}} = 2.357\cdot 10^{-3}$ **<--0.236% of the input energy is in the output**
The filter is narrow band

W1-48 $v(t) = V_A \cdot \exp(-\alpha \cdot t) \cdot u(t)$ $\quad V(\omega) = \dfrac{V_A}{j \cdot \omega + \alpha}$ $\quad (|V(\omega)|)^2 = \dfrac{(V_A)^2}{(\omega^2 + \alpha^2)}$ $\quad W_{1\Omega} = \dfrac{(V_A)^2}{2 \cdot \alpha}$

(a) At high frequency ($\omega >> \alpha$) $(|V(\omega)|)^2 = \dfrac{(V_A)^2}{\omega^2}$ At low frequency ($\omega << \alpha$) $(|V(\omega)|)^2 = \dfrac{(V_A)^2}{\alpha^2}$.

These asymptotes intersect when $\dfrac{V_A}{\omega} = \dfrac{V_A}{\alpha}$ which occurs at $\omega = \alpha$

$$SD_{SL}(\omega) := \begin{cases} \left(\dfrac{V_A}{\alpha}\right)^2 & \text{if } |\omega| \le \alpha \\ \left(\dfrac{V_A}{\omega}\right)^2 & \text{if } \alpha < |\omega| \end{cases} \qquad \alpha := 1 \qquad V_A := 1 \qquad \omega := 10^{-6} \cdot \alpha, 0.1 \cdot \alpha .. 10 \cdot \alpha$$



(b)

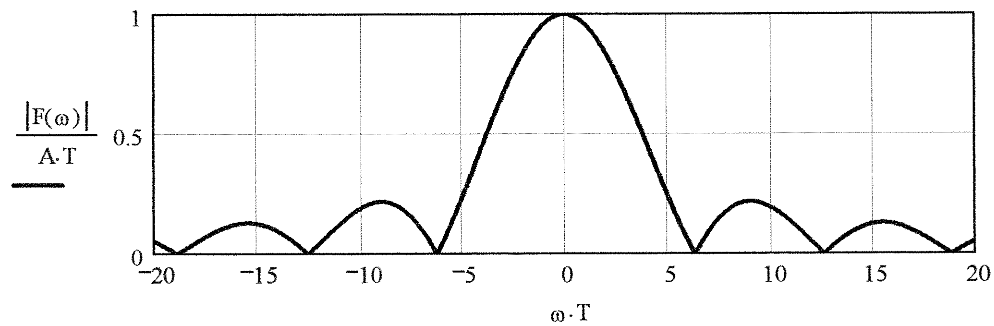
$$W_{SL} = \frac{1}{\pi} \cdot \int_0^{\infty} SD_{SL}(\omega)\, d\omega = \frac{1}{\pi} \cdot \int_0^{\alpha} \left(\frac{V_A}{\alpha}\right)^2 d\omega + \frac{1}{\pi} \cdot \int_{\alpha}^{\infty} \left(\frac{V_A}{\omega}\right)^2 d\omega$$

$$\frac{1}{\pi} \cdot \int_0^{\alpha} \left(\frac{V_A}{\alpha}\right)^2 d\omega = \frac{V_A^2}{\pi \cdot \alpha} \qquad \frac{1}{\pi} \cdot \int_{\alpha}^{\infty} \left(\frac{V_A}{\omega}\right)^2 d\omega = \frac{V_A^2}{\pi \cdot \alpha} \qquad W_{SL} = \frac{2 \cdot V_A^2}{\pi \cdot \alpha}$$

(c) $\dfrac{W_{SL}}{W_{1\Omega}} = \dfrac{2 \cdot V_A^2}{\pi \cdot \alpha} \cdot \dfrac{2 \cdot \alpha}{V_A^2} = \dfrac{4}{\pi} = 1.273$ <---W_{SL} overstates energy by 27.3%

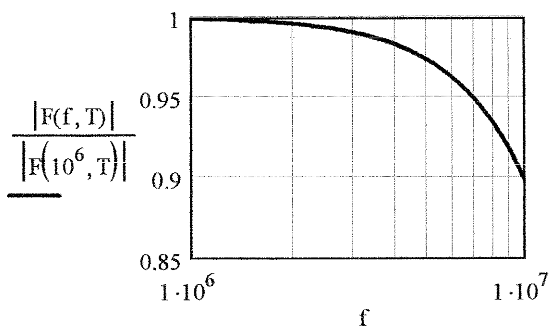
W1-49 $f(t) = A \cdot \left(u\left(t + \frac{T}{2}\right) - u\left(t - \frac{T}{2}\right) \right)$ $A := 1$ $T := 1$ $\omega := -20, -19.93 .. 20$

$$F(\omega) := \frac{A \cdot T \cdot \sin\left(\omega \cdot \frac{T}{2}\right)}{\omega \cdot \frac{T}{2}}$$ <-----From Example W1-1



$$F(f, T) := \frac{|A \cdot T \cdot \sin(\pi \cdot f \cdot T)|}{\pi \cdot f \cdot T}$$ $f := 10^6, 1.1 \cdot 10^6 .. 10^7$

$T := 2 \cdot 10^{-8}$ Given $F\left(10^7, T\right) = 0.9 \cdot F\left(10^6, T\right)$ $T := \text{Find}(T)$ $T = 2.516 \times 10^{-8}$



<---Verification of a 10% change on the 1 Mhz to 10 Mhz range for T = 25.16 ns.

Any pulse duration T < 25.16 ns with meet the requirement

CHAPTER W2, Two-Port Networks

W2-1

$$z_{11} := 100 + \frac{600\cdot(400+200)}{600+400+200}$$

$$z_{21} := \frac{600}{600+400+200}\cdot 200 \quad z_{12} := z_{21}$$

$$z_{22} := \frac{200\cdot(400+600)}{200+400+600} \qquad \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} = \begin{pmatrix} 400 & 100 \\ 100 & 166.667 \end{pmatrix}$$

W2-2

$$y_{11} := \frac{1}{100 + \dfrac{400\cdot 600}{400+600}} \qquad y_{21} := y_{11}\cdot\frac{-600}{600+400} \qquad y_{12} := y_{21}$$

$$y_{22} := \frac{1}{200} + \frac{1}{400 + \dfrac{600\cdot 100}{600+100}} \qquad \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} = \begin{pmatrix} 2.941\times 10^{-3} & -1.765\times 10^{-3} \\ -1.765\times 10^{-3} & 7.059\times 10^{-3} \end{pmatrix}$$

Checking

$$\begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix}^{-1} = \begin{pmatrix} 2.941\times 10^{-3} & -1.765\times 10^{-3} \\ -1.765\times 10^{-3} & 7.059\times 10^{-3} \end{pmatrix} \qquad \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix}^{-1} = \begin{pmatrix} 400 & 100 \\ 100 & 166.667 \end{pmatrix}$$

W2-3

$$z_{11} := \frac{1}{\dfrac{1}{j\cdot 50} + \dfrac{1}{100 - j\cdot 100}}$$

$$z_{21} := \frac{j\cdot 50}{j\cdot 50 + 100 - j\cdot 100}\cdot 100 \qquad z_{12} := z_{21}$$

$$z_{22} := \frac{1}{\dfrac{1}{100} + \dfrac{1}{-j\cdot 100 + j\cdot 50}} \qquad \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} = \begin{pmatrix} 20+60i & -20+40i \\ -20+40i & 20-40i \end{pmatrix}$$

W2-4

$$y_{11} := \frac{1}{j\cdot 50} + \frac{1}{-j\cdot 100} \qquad y_{21} := \frac{-1}{-j\cdot 100} \qquad y_{12} := y_{21} \qquad y_{22} := \frac{1}{100} + \frac{1}{-j\cdot 100}$$

$$\begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} = \begin{pmatrix} -i\times 10^{-2} & -i\times 10^{-2} \\ -i\times 10^{-2} & 1\times 10^{-2} + i\times 10^{-2} \end{pmatrix}$$

Checking

$$\begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix}^{-1} = \begin{pmatrix} -i\times 10^{-2} & -i\times 10^{-2} \\ -i\times 10^{-2} & 1\times 10^{-2} + i\times 10^{-2} \end{pmatrix}$$

$$\begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix}^{-1} = \begin{pmatrix} 20+60i & -20+40i \\ -20+40i & 20-40i \end{pmatrix}$$

W2-5 with $I_2 = 0$

$$V_1 = (R_1 + R_2)\cdot I_1 + \beta\cdot R_2\cdot I_1$$

$$z_{11} = R_1 + (\beta + 1)\cdot R_2$$

$$V_2 = -\beta\cdot I_2\cdot R_3 \quad z_{21} = -\beta\cdot R_3$$

with $I_1 = 0$ the current source is an open circuit

$$z_{22} = R_3 \quad z_{12} = 0 \quad \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} = \begin{bmatrix} R_1 + (\beta + 1)\cdot R_2 & 0 \\ -\beta\cdot R_3 & R_3 \end{bmatrix}$$

W2-6 with $V_2 = 0$ $\quad V_1 = (R_1 + R_2)\cdot I_1 + \beta\cdot R_2\cdot I_1 \quad y_{11} = \dfrac{1}{z_{11}} = \dfrac{1}{[R_1 + (\beta + 1)\cdot R_2]}$

$$I_2 = \beta\cdot I_1 = \beta\cdot\frac{V_1}{[R_1 + (\beta + 1)\cdot R_2]} \quad y_{21} = \frac{I_2}{V_1} = \frac{\beta}{[R_1 + (\beta + 1)\cdot R_2]}$$

with $V_1 = 0$ there is no path from output to input, henc $I_1 = 0$ $\quad y_{12} = 0 \quad y_{22} = \dfrac{1}{R_3}$

$$\begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} = \begin{bmatrix} \dfrac{1}{[R_1 + (\beta + 1)\cdot R_2]} & 0 \\ \dfrac{\beta}{[R_1 + (\beta + 1)\cdot R_2]} & \dfrac{1}{R_3} \end{bmatrix}$$

Checking

$$\begin{bmatrix} R_1 + (\beta + 1)\cdot R_2 & 0 \\ -\beta\cdot R_3 & R_3 \end{bmatrix}\cdot\begin{bmatrix} \dfrac{1}{[R_1 + (\beta + 1)\cdot R_2]} & 0 \\ \dfrac{\beta}{[R_1 + (\beta + 1)\cdot R_2]} & \dfrac{1}{R_3} \end{bmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

This checks that z-matrix from Prob. W2-5 is the inverse of the y-matrix from Prob W2-6, and vise versa.

W2-7 $\begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} := \begin{pmatrix} 1000 & 500 \\ 500 & 500 \end{pmatrix} \quad V_1 = 12 \qquad V_2 = -250\cdot I_2 \quad I_1 := 1 \qquad I_2 := 1$

Given $\quad 12 = 1000\cdot I_1 + 500\cdot I_2 \quad -250\cdot I_2 = 500\cdot I_1 + 500\cdot I_2 \quad \text{Find}(I_1, I_2) = \begin{pmatrix} 1.8\times 10^{-2} \\ -1.2\times 10^{-2} \end{pmatrix}$

W2-8 With $I_2 = 0$ and $\quad V_1 = z_{11}\cdot I_1 \qquad V_2 = z_{21}\cdot I_1 \quad T_V = \dfrac{V_2}{V_1} = \dfrac{z_{21}}{z_{11}}$

with $\begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} := \begin{pmatrix} 6\cdot 10^4 & 0 \\ -2.5\cdot 10^5 & 5\cdot 10^3 \end{pmatrix} \qquad \dfrac{z_{21}}{z_{11}} = -4.167$

W2-9 Given $I_1 = 4\cdot10^{-3}\cdot V_1 - 2\cdot10^{-3}\cdot V_2$; $I_2 = -2\cdot10^{-3}\cdot V_1 + 2\cdot10^{-3}\cdot V_2$ $V_2 = -1500\cdot I_2$ $V_1 := 12$

$$I_2 = \frac{-V_2}{1500} \text{ ----> } \frac{-V_2}{1500} = -2\cdot10^{-3}\cdot V_1 + 2\cdot10^{-3}\cdot V_2 \text{ ----> } V_2 := \frac{3}{4}\cdot V_1 \text{ ----> } V_2 = 9$$

$$I_1 := 4\cdot10^{-3}\cdot V_1 - 2\cdot10^{-3}\cdot\left(\frac{3}{4}\cdot V_1\right) \text{ ----> } I_1 = 3\times10^{-2} \text{ ----> } I_2 := \frac{-V_2}{1500} \text{ ----> } I_2 = -6\times10^{-3}$$

W2-10 Given $I_1 = (5 + j\cdot20)\cdot10^{-3}\cdot V_1 - j\cdot20\cdot10^{-3}\cdot V_2$ $I_2 = -\left(j\cdot20\cdot10^{-3}\cdot V_1\right)$ $V_2 = -50\cdot I_2$

$$I_2 = \frac{-V_2}{50} \text{ ----> } \frac{-V_2}{50} = -\left(j\cdot20\cdot10^{-3}\cdot V_1\right) \text{ ----> } V_2 = j\cdot V_1 \text{ ----> }$$

$$I_1 = (5 + j\cdot20)\cdot10^{-3}\cdot V_1 - j\cdot20\cdot10^{-3}\cdot(j\cdot V_1) \text{ ----> } I_1 = \left(25\cdot10^{-3} + j\cdot20\cdot10^{-3}\right)\cdot V_1 \text{ ----> }$$

$$Y_{IN} = \frac{I_1}{v_1} = (25 + j\cdot20)\cdot10^{-3}$$

W2-11 Given $V_1 = z_{11}\cdot I_1 + z_{12}\cdot I_2$

$$V_2 = z_{21}\cdot I_1 + z_{22}\cdot I_2 \qquad V_2 = -Z_L\cdot I_2$$

$$I_2 = \frac{-V_2}{Z_L} \text{ ----> } V_2 = z_{21}\cdot I_1 + z_{22}\cdot\left(\frac{-V_2}{Z_L}\right) \text{ ----> } I_1 = \frac{z_{22} + Z_L}{z_{21}\cdot Z_L}\cdot V_2 \text{ ----> }$$

$$V_1 = z_{11}\cdot I_1 + z_{12}\cdot I_2 = z_{11}\cdot\left(\frac{z_{22} + Z_L}{z_{21}\cdot Z_L}\cdot V_2\right) + z_{12}\cdot\left(\frac{-V_2}{Z_L}\right) = \frac{\left(z_{11}\cdot Z_L + z_{11}\cdot z_{22} - z_{12}\cdot z_{21}\right)}{\left(z_{21}\cdot Z_L\right)}\cdot V_2$$

$$T_V = \frac{V_2}{V_1} = \frac{z_{21}\cdot Z_L}{z_{11}\cdot Z_L + \Delta_z}$$

W2-12 Given $I_1 = y_{11}\cdot V_1 + y_{12}\cdot V_2$ $I_2 = y_{21}\cdot V_1 + z_{22}\cdot V_2$ $V_2 = -Z_L\cdot I_2$ ---->

$$I_2 = y_{21}\cdot V_1 + y_{22}\cdot\left(-I_2\cdot Z_L\right) \text{ ----> } V_1 = \frac{\left(1 + y_{22}\cdot Z_L\right)}{y_{21}}\cdot I_2 \text{ ----> }$$

$$I_1 = y_{11}\cdot V_1 + y_{12}\cdot V_2 = y_{11}\cdot\left[\frac{\left(1 + y_{22}\cdot Z_L\right)}{y_{21}}\cdot I_2\right] + y_{12}\cdot\left(-Z_L\cdot I_2\right) = \frac{\left[y_{11} + \left(y_{11}\cdot y_{22} - y_{12}\cdot y_{21}\right)\cdot Z_L\right]}{y_{21}}\cdot I_2$$

$$T_I = \frac{I_2}{I_1} = \frac{y_{21}}{y_{11} + \Delta_y\cdot Z_L}$$

W2-13 With $V_2 = 0$, $V_1 = 0 = h_{11} \cdot I_1$ ----> $h_{11} = 0$

$I_2 = -I_1 = h_{21} \cdot V_2$ ----> $h_{21} = -1$

With $I_1 = 0$, $V_1 = V_2 = h_{12} \cdot V_2$ ----> $h_{12} = 1$

$I_2 = Y \cdot V_2 = h_{22} \cdot V_2$ ----> $h_{22} = Y$

$$\begin{pmatrix} h_{11} & h_{21} \\ h_{21} & h_{22} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & Y \end{pmatrix}$$

With $I_2 = 0$, $V_1 = V_2 = A \cdot V_2$----> $A = 1$ $I_1 = Y \cdot V_2 = C \cdot V_2$ ----> $C = Y$

With $V_2 = 0$, $V_1 = 0 = B \cdot (-I_2)$----> $B = 0$ $I_1 = D \cdot (-I_2) = (-I_2)$ ----> $D = 1$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ Y & 1 \end{pmatrix} \qquad h_{11} = \frac{B}{D} = 0$$

W2-14 With $V_2 = 0$, $V_1 = Z \cdot I_1 = h_{11} \cdot I_1$ ----> $h_{11} = Z$

$I_2 = -I_1 = h_{21} \cdot V_2$ ----> $h_{21} = -1$

With $I_1 = 0$, $V_1 = V_2 = h_{12} \cdot V_2$ ----> $h_{12} = 1$

$I_2 = 0 \cdot V_2 = h_{22} \cdot V_2$ ----> $h_{22} = 0$

$$\begin{pmatrix} h_{11} & h_{21} \\ h_{21} & h_{22} \end{pmatrix} = \begin{pmatrix} Z & 1 \\ -1 & 0 \end{pmatrix}$$

With $I_2 = 0$, $V_1 = V_2 = A \cdot V_2$ ----> $A = 1$ $I_1 = 0 \cdot V_2 = C \cdot V_2$ ----> $C = 0$

With $V_2 = 0$, $V_1 = Z \cdot (-I_2) = B \cdot (-I_2)$----> $B = Z$ $I_1 = D \cdot (-I_2) = (-I_2)$ ----> $D = 1$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & Z \\ 0 & 1 \end{pmatrix}$$

W2-15 With $V_2 = 0$,

$V_1 = (R_1 + R_2) \cdot I_1 = h_{11} \cdot I_1$ ----> $h_{11} = R_1 + R_2$

$I_2 = \beta \cdot I_1 = h_{21} \cdot I_1$ ----> $h_{21} = \beta$

With $I_1 = 0$, $V_1 = 0 = h_{12} \cdot V_2$ ----> $h_{12} = 0$

$I_2 = G_3 \cdot V_2 = h_{22} \cdot V_2$ ----> $h_{22} = G_3$

$$\begin{pmatrix} h_{11} & h_{21} \\ h_{21} & h_{22} \end{pmatrix} = \begin{pmatrix} R_1 + R_2 & 0 \\ \beta & G_3 \end{pmatrix}$$

W2-16 With $I_2 = 0$,

$$V_2 = -\beta \cdot R_3 \cdot I_1 = -\beta \cdot R_3 \cdot \frac{V_1}{R_1 + R_2} = \frac{1}{A} \cdot V_1$$

$$A = -\frac{R_1 + R_2}{\beta \cdot R_3}$$

$$I_1 = \frac{V_1}{R_1 + R_2} = \frac{1}{R_1 + R_2} \cdot \frac{R_1 + R_2}{-\beta \cdot R_3} \cdot V_2 = C \cdot V_2$$

$$\text{---->} \quad C = \frac{1}{-\beta \cdot R_3}$$

With $V_2 = 0$, $\quad I_2 = \beta \cdot I_1 = \frac{-1}{D} \cdot I_1$ ----> $\quad D = \frac{-1}{\beta}$

$$V_1 = (R_1 + R_2) \cdot I_1 = (R_1 + R_2) \cdot \left(\beta^{-1} \cdot I_2\right) = -B \cdot I_2 \text{---->} \qquad B = -\beta^{-1} \cdot (R_1 + R_2)$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{bmatrix} -\dfrac{R_1 + R_2}{\beta \cdot R_3} & \dfrac{-(R_1 + R_2)}{\beta} \\ \dfrac{1}{-\beta \cdot R_3} & \dfrac{-1}{\beta} \end{bmatrix}$$

Checking with answer to Prob.W2-15

$$\begin{pmatrix} h_{11} & h_{21} \\ h_{21} & h_{22} \end{pmatrix} = \begin{pmatrix} \dfrac{B}{D} & \dfrac{\Delta_t}{D} \\ \dfrac{-1}{D} & \dfrac{C}{D} \end{pmatrix} = \begin{pmatrix} R_1 + R_2 & 0 \\ \beta & G_3 \end{pmatrix} \quad \text{Checks}$$

Note that $\quad \Delta_t = 0$

W2-17 With $V_2 = 0$,

$V_1 = (R_1) \cdot I_1 = h_{11} \cdot I_1$ ----> $h_{11} = R_1$

$I_2 = \frac{-V_1}{R_2} = \frac{-R_1 \cdot I_1}{R_2} = h_{21} \cdot I_1$ ----> $h_{21} = \frac{-R_1}{R_2}$

With $I_1 = 0$, $\quad V_1 = 0 = h_{12} \cdot V_2$ ----> $h_{12} = 0$

$V_2 = R_2 \cdot I_2 = (h_{22})^{-1} \cdot I_2$ ----> $h_{22} = G_2$

$$\begin{pmatrix} h_{11} & h_{21} \\ h_{21} & h_{22} \end{pmatrix} = \begin{pmatrix} R_1 & 0 \\ \dfrac{-R_1}{R_2} & G_2 \end{pmatrix} \qquad \Delta_h = \frac{R_1}{R_2}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} -\Delta_h & -h_{11} \\ -h_{22} & -1 \end{pmatrix} \cdot \frac{1}{h_{21}} = \begin{pmatrix} 1 & R_2 \\ \dfrac{1}{R_1} & \dfrac{R_2}{R_1} \end{pmatrix}$$

W2-18 With $I_2 = 0$,

$$V_2 = V_1 = \frac{1}{A}\cdot V_1 \qquad \text{---->} \qquad A = 1$$

$$I_1 = \frac{V_1}{R_1} = \frac{V_2}{R_1} = C\cdot V_2 \qquad \text{---->} \qquad C = \frac{1}{R_1}$$

With $V_2 = 0$, $\quad -I_2 = \frac{V_1}{R_2} = \frac{R_1}{R_2}\cdot I_1 = \frac{1}{D}I_1$ ----> $D = \frac{R_2}{R_1}$

$$V_1 = R_1\cdot I_1 = (R_1)\cdot\frac{R_2}{R_1}\cdot(-I_2) = B\cdot(-I_2) \qquad \text{---->} \qquad B = R_2$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & R_2 \\ \frac{1}{R_1} & \frac{R_2}{R_1} \end{pmatrix}$$

Checking with answer to Prob.W2-17

$$\begin{pmatrix} h_{11} & h_{21} \\ h_{21} & h_{22} \end{pmatrix} = \begin{pmatrix} \frac{B}{D} & \frac{\Delta_t}{D} \\ \frac{-1}{D} & \frac{C}{D} \end{pmatrix} = \begin{pmatrix} R_1 & 0 \\ \frac{-R_1}{R_2} & \frac{1}{R_2} \end{pmatrix}$$ Checks

Note that $\Delta_t = 0$

W2-19 Given $V_1 = 12 \quad V_1 = 500\cdot I_1 + V_2 \quad I_2 = -I_1 + 2\cdot 10^{-3}\cdot V_2$ Substitute 1st eq into 2nd

$$12 = 500\cdot I_1 + V_2 \;\text{--->}\; I_1 = \frac{3}{125} - \frac{1}{500}\cdot V_2 \text{--->}\; I_2 = -\left(\frac{3}{125} - \frac{1}{500}\cdot V_2\right) + 2\cdot 10^{-3}\cdot V_2 = \frac{-3}{125} + \frac{1}{25[illegible]}$$

---> $V_2 = 6 + 250\cdot I_2 = V_T + R_T\cdot I_2$ hence $V_T = 6 \quad R_T = 250$

W2-20 Given $V_1 = 6\cdot 10^4\cdot I_1 \; I_2 = 50\cdot I_1 + 2\cdot 10^{-4}\cdot V_2 \; V_2 = -20\cdot 10^3 I_2$ Substitute 3rd eq. into 2nd.

$$I_2 = 50\cdot I_1 + 2\cdot 10^{-4}\cdot\left(-20\cdot 10^3 I_2\right) \quad \text{--->} \quad I_2 = 10\cdot I_1 \quad T_I = \frac{I_2}{I_1} = 10$$

W2-21 Given $A := 2 \quad B := 400 \quad C := 2.5\cdot 10^{-3} \; D := 1 \qquad V_2 = R_L\cdot(-I_2)$

$$R_{IN} = \frac{A\cdot V_2 + B\cdot(-I_2)}{C\cdot V_2 + D\cdot(-I_2)} = \frac{A\cdot[R_L\cdot(-I_2)] + B\cdot(-I_2)}{C\cdot[R_L\cdot(-I_2)] + D\cdot(-I_2)} = \frac{A\cdot R_L + B}{C\cdot R_L + D} \text{ hence } \quad R_{IN}(R_L) := \frac{A\cdot R_L + B}{C\cdot R_L + D}$$

For an open circuit $\quad R_L := \infty \quad R_{IN}(R_L) = 800$

For an short circuit $\quad R_L := 0 \quad R_{IN}(R_L) = 400$

For $\quad R_L := 400 \quad R_{IN}(R_L) = 600$

W2-22 Given $A := 0 \quad B := -j\cdot 50 \quad C := -j\cdot 20\cdot 10^{-3} \quad D := 1 - j\cdot 0.25 \quad V_2 = 50\cdot(-I_2)$

$$Y_{IN} = \frac{I_1}{V_2} = \frac{C\cdot V_2 + D\cdot(-I_2)}{A\cdot V_2 + B(-I_2)} = \frac{C\cdot[50\cdot(-I_2)] + D\cdot(-I_2)}{A\cdot[50\cdot(-I_2)] + B(-I_2)} \text{ ---->} \quad Y_{IN} = \frac{50\cdot C + D}{50\cdot A + B} = (25 + j\cdot 20)\cdot 10^{-3}$$

W2-23 given $V_1 = h_{11}\cdot I_1 + h_{12}\cdot V_2 \quad I_2 = h_{21}\cdot I_1 + h_{22}\cdot V_2 \qquad V_2 = -Z_L\cdot I_2$

using the 2nd h-parameter eq. $I_2 = h_{21}\cdot I_1 + h_{22}\cdot(-Z_L\cdot I_2)$ ----> $I_2 = \dfrac{h_{21}\cdot I_1}{(1 + h_{22}\cdot Z_L)}$

$$T_I = \frac{I_2}{I_1} = \frac{h_{21}}{(1 + h_{22}\cdot Z_L)} \qquad \text{QED}$$

W2-24 given $V_1 = h_{11}\cdot I_1 + h_{12}\cdot V_2 \quad I_2 = h_{21}\cdot I_1 + h_{22}\cdot V_2 \qquad V_2 = -Z_L\cdot I_2$ ---->

$V_1 = h_{11}\cdot I_1 + h_{12}\cdot(-Z_L\cdot I_2)$ and $I_2 = h_{21}\cdot I_1 + h_{22}\cdot(-Z_L\cdot I_2)$ solving the 2nd eq for I_2

$I_2 = \dfrac{h_{21}\cdot I_1}{(1 + h_{22}\cdot Z_L)}$ substituting into the 1st eq $V_1 = h_{11}\cdot I_1 + h_{12}\cdot\left[-Z_L\cdot\left[\dfrac{h_{21}\cdot I_1}{(1 + h_{22}\cdot Z_L)}\right]\right]$

$$\text{---->} \quad V_1 = \left(h_{11} - \frac{h_{12}\cdot h_{21}\cdot Z_L}{1 + h_{22}\cdot Z_L}\right)\cdot I_1 \quad \text{---->} \quad Z_{IN} = \frac{V_1}{I_1} = h_{11} - \frac{h_{12}\cdot h_{21}\cdot Z_L}{1 + h_{22}\cdot Z_L} \qquad \text{QED}$$

W2-25 given $V_1 = h_{11}\cdot I_1 + h_{12}\cdot V_2 \quad I_2 = h_{21}\cdot I_1 + h_{22}\cdot V_2$ solve the 2nd eq. for V_2

$$V_2 = \frac{-h_{21}}{h_{22}}\cdot I_1 + \frac{1}{h_{22}}\cdot I_2 = z_{21}\cdot I_1 + z_{22}\cdot I_2 \quad \text{---->} \quad z_{21} = \frac{-h_{21}}{h_{22}} \qquad z_{22} = \frac{1}{h_{22}} \qquad \text{QED}$$

Using this result to eliminate V2 from the first eq. $V_1 = h_{11}\cdot I_1 + h_{12}\cdot\left(\dfrac{-h_{21}}{h_{22}}\cdot I_1 + \dfrac{1}{h_{22}}\cdot I_2\right)$ ---->

$$V_1 = \frac{h_{11}\cdot h_{22} - h_{12}\cdot h_{21}}{h_{22}}\cdot I_1 + \frac{h_{12}}{h_{22}}\cdot I_2 = z_{11}\cdot I_1 + z_{12}\cdot I_2 \quad \text{---->} \quad z_{11} = \frac{\Delta_h}{h_{22}} \quad z_{12} = \frac{h_{12}}{h_{22}} \quad \text{QED}$$

W2-26 given $V_1 = A\cdot V_2 + B\cdot(-I_2) \quad I_1 = C\cdot V_2 + D\cdot(-I_2)$ solve the 2nd eq. for V_2

$$V_2 = \frac{1}{C}\cdot I_1 + \frac{D}{C}\cdot I_2 = z_{21}\cdot I_1 + z_{22}\cdot I_2 \text{ ---->} \quad z_{21} = \frac{1}{C} \qquad z_{22} = \frac{D}{C} \qquad \text{QED}$$

Using this result to eliminate V_2 from the first eq. $V_1 = A\cdot\left(\dfrac{1}{C}\cdot I_1 + \dfrac{D}{C}\cdot I_2\right) + B\cdot(-I_2)$ ---->

$$V_1 = \frac{A}{C}\cdot I_1 + \frac{A\cdot D - B\cdot C}{C}\cdot I_2 = z_{11}\cdot I_1 + z_{12}\cdot I_2 \text{ ---->} \quad z_{11} = \frac{A}{C} \qquad z_{12} = \frac{\Delta_t}{C} \qquad \text{QED}$$

W2-27 given $V_1 = 2000 \cdot I_1 - 20 \cdot V_2$ $I_2 = 50 \cdot I_1 + 10^{-2} \cdot V_2$ solve the 1st eq. for I_1

$$I_1 = \frac{1}{2000} \cdot V_1 + \frac{1}{100} \cdot V_2 = y_{11} \cdot V_1 + y_{12} \cdot V_2 \text{---->} \quad y_{11} := \frac{1}{2000} \quad y_{12} := \frac{1}{100}$$

Using this result to eliminate I_1 from the 2nd eq. $I_2 = 50 \cdot \left(\frac{1}{2000} \cdot V_1 + \frac{1}{100} \cdot V_2 \right) + 10^{-2} \cdot V_2$---->

$$I_2 = \frac{1}{40} \cdot V_1 + \frac{51}{100} \cdot V_2 = y_{21} \cdot V_1 + y_{22} \cdot V_2 \text{ ---->} \quad y_{21} := \frac{1}{40} \quad y_{22} := \frac{51}{100}$$

$$\begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} = \begin{pmatrix} 5 \times 10^{-4} & 1 \times 10^{-2} \\ 2.5 \times 10^{-2} & 5.1 \times 10^{-1} \end{pmatrix}$$

Circuit is not reciprocal y_{12} is not equal to y_{21}

W2-28 given $V_1 = 5000 \cdot I_1 + 20 I_2$ $V_2 = 500 \cdot I_1 + 3000 \cdot I_2$ solve the 2nd eq. for I_1

$$I_1 = \frac{1}{500} \cdot V_2 - 6 \cdot I_2 = C \cdot V_2 - D \cdot I_2 \text{ ---->} \quad C := \frac{1}{500} \quad D := 6$$

Using this result to eliminate I_1 from the 1st eq. $V_1 = 5000 \cdot \left(\frac{1}{500} \cdot V_2 - 6 \cdot I_2 \right) + 20 I_2$---->

$$V_1 = 10 \cdot V_2 - 29980 \cdot I_2 = A \cdot V_2 - B \cdot I_2 \text{---->} \quad A := 10 \quad B := 29980$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 \times 10^{1} & 2.998 \times 10^{4} \\ 2 \times 10^{-3} & 6 \end{pmatrix} \quad \Delta_t := A \cdot D - B \cdot C \quad \Delta_t = 4 \times 10^{-2}$$

Circuit is not reciprocal Δ_t is not equal to 1

W2-29 given $h_{11} := 10^4$ $h_{12} := 0$ $h_{21} := -10$ $h_{22} := 10^{-3}$ $\Delta_h := h_{11} \cdot h_{22} - h_{12} \cdot h_{21}$

Convert h-parameters to t-parameters The t-parameters of the cascade connection are

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} := \begin{pmatrix} \frac{-\Delta_h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{pmatrix} \quad \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 10 & 2.998 \times 10^{4} \\ 2 \times 10^{-3} & 6 \end{pmatrix}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1.1 & 1.1 \times 10^{3} \\ 1.1 \times 10^{-4} & 0.11 \end{pmatrix}$$

Convert the t-parameters of the cascade connection into H-parameters

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} := \begin{pmatrix} 1.1 & 1.1 \cdot 10^{3} \\ 1.1 \cdot 10^{-4} & 0.11 \end{pmatrix} \quad H_{11} := \frac{b}{d} \quad H_{12} := \frac{a \cdot d - b \cdot c}{d} \quad H_{21} := \frac{-1}{d} \quad H_{22} := \frac{c}{d}$$

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} = \begin{pmatrix} 1 \times 10^{4} & 0 \\ -9.091 & 1 \times 10^{-3} \end{pmatrix}$$

<---H-parameters of the cascade connection

W2-30- given $h_{11} := 10^4$ $h_{12} := 0$ $h_{21} := -10$ $h_{22} := 10^{-3}$ $\Delta_h := h_{11} \cdot h_{22} - h_{12} \cdot h_{21}$

Convert h-parameters to y-parameters

$$\begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} := \begin{pmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_h}{h_{11}} \end{pmatrix} \qquad \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} = \begin{pmatrix} 1 \times 10^{-4} & 0 \\ -1 \times 10^{-3} & 1 \times 10^{-3} \end{pmatrix}$$

The y-parameters of the parallel connection is

$$\begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} := \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} + \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \qquad \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} = \begin{pmatrix} 2 \times 10^{-4} & 0 \\ -2 \times 10^{-3} & 2 \times 10^{-3} \end{pmatrix}$$

Convert the Y-parameters of the parallel connection into h-parameters

$$H_{11} := \frac{1}{Y_{11}} \qquad H_{12} := \frac{-Y_{12}}{Y_{11}} \qquad H_{21} := \frac{Y_{21}}{Y_{11}} \qquad H_{22} := \frac{Y_{11} \cdot Y_{22} - Y_{12} \cdot Y_{21}}{Y_{11}}$$

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} = \begin{pmatrix} 5 \times 10^{3} & 0 \\ -10 & 2 \times 10^{-3} \end{pmatrix}$$ <---H-parameters of the parallel connection

W2-31- From Table W2-2 $z_{12} = \frac{\Delta_t}{C} = z_{21} \cdot \Delta_t$ $y_{12} = \frac{-\Delta_t}{B} = \Delta_t \cdot y_{21}$ $h_{12} = \frac{\Delta_t}{D} = -\Delta_t \cdot h_{21}$

If excitation at the input produces a nonzero output then z_{21}, y_{21}, and h_{21} are not zero. When $\Delta_t = 0$ the reverse transfer functions z_{12}, y_{12}, and h_{12} are all zero, hence excitation applied at the output port produces zero response at the input port.

W2-32- with $V_2 = -Z_L \cdot I_2$ $\qquad \frac{V_1}{I_1} = \frac{A \cdot V_2 + B \cdot (-I_2)}{C \cdot V_2 + D \cdot (-I_2)} = \frac{A \cdot Z_L + B}{C \cdot Z_L + D}$

with $A = D$ and $B = C \cdot Z_L^{\,2}$ $\qquad \frac{V_1}{I_1} = \frac{A \cdot Z_L + C \cdot Z_L^{\,2}}{C \cdot Z_L + A} = Z_L \cdot \frac{C \cdot Z_L + A}{C \cdot Z_L + A} = Z_L$ QED