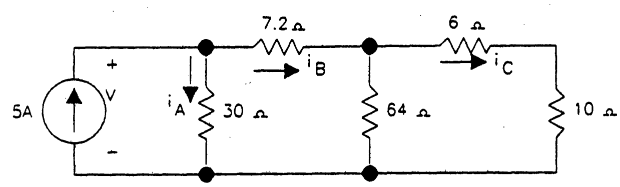
3

Simple Resistive Circuits

Drill Exercises

DE 3.1



$16//64 = 12.8\,\Omega,\qquad 12.8 + 7.2 = 20\,\Omega,\qquad 20//30 = 12\,\Omega$

[a] $v = 5(12) = 60\,\text{V}$

[b] $p_{5A}(\text{del}) = (5)(60) = 300\,\text{W}$

[c] $i_A = 60/30 = 2\,\text{A}$ $\qquad i_C = 3(64)/(80) = 2.4\,\text{A}$

$i_B = 5 - 2 = 3\,\text{A}$ $\qquad p_{10\Omega} = (2.4)^2 10 = 57.6\,\text{W}$

DE 3.2 [a] $v_o(\text{no load}) = 120(50)/80 = 75\,\text{V}$

[b] $50//450 = 45\,\text{k}\Omega,$ therefore $v_o = 120(45)/75 = 72\,\text{V}$

[c] $i = 120/30 = 4\,\text{mA},\qquad p_{30k} = (4 \times 10^{-3})^2(30{,}000) = 0.48\,\text{W}$

[d] Maximum dissipation at no load since v_o is maximum,

$$p = \frac{v_o^2}{50{,}000} = 0.1125\,\text{W}$$

DE 3.3 [a] $i = 0.5/80 = 6.25\,\text{mA}$

[b] $i_{\text{meas}} = 0.5/85 = 5.88\,\text{mA}$

DE 3.4 [a] $v = 40(75)/80 = 37.5\,\text{V}$

[b] $R_m = 150/10^{-3} = 150\,\text{k}\Omega, \qquad 150\,\text{k}\Omega//75\,\text{k}\Omega = 50\,\text{k}\Omega$

Therefore $v_{\text{meas}} = 50(40)/55 = 36.36\,\text{V}$

DE 3.5 [a] $1.5 = 50 \times 10^{-6}(23{,}000 + R), \qquad R = 7\,\text{k}\Omega$

[b] At midscale: $v_{R_x} = 1.5 - 25 \times 10^{-6}(30{,}000) = 0.75\,\text{V}$

Therefore $i_{11\Omega} = 0.75/11 = 68{,}181.818\,\mu\text{A}$,

Therefore $i_{R_x} = 68{,}206.818\,\mu\text{A}, \qquad R_x = 0.75/i_{R_x} \cong 11\,\Omega$

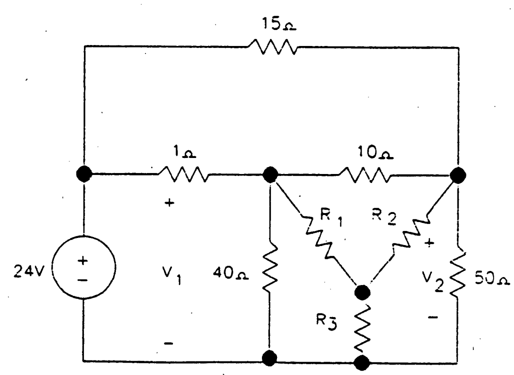
DE 3.6 [a] $100R_x = (1000)(150), \qquad R_x = 1.5\,\text{k}\Omega$

[b] The current supplied by the 5-V source is $i_s = \dfrac{5}{250} + \dfrac{5}{2500} = 0.022\,\text{A} = 22\,\text{mA}$

Therefore $p_{5\text{V}}(\text{delivered}) = 5(22) = 110\,\text{mW}$

Since total power delivered to bridge is less than 250 mW, bridge will not be damaged.

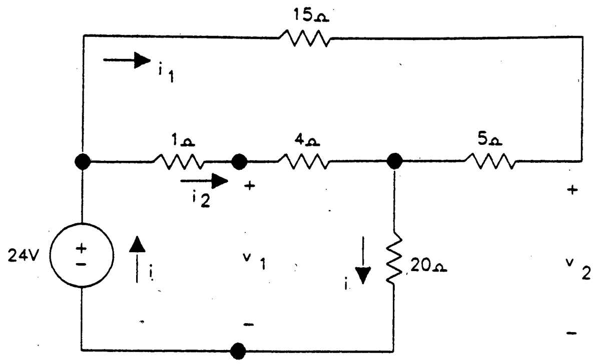
DE 3.7 [a] $R_1 = \dfrac{(40)(10)}{100} = 4\,\Omega; \qquad R_2 = \dfrac{(50)(10)}{100} = 5\,\Omega; \qquad R_3 = \dfrac{(40)(50)}{100} = 20\,\Omega$



The resistance seen by the 24-V source is

$R = (15 + 5)//(4 + 1) + 20 = 20//5 + 20 = 4 + 20 = 24\,\Omega$

$i = 24/24 = 1\,\text{A}$



[b] $i_1 = (5/25)i = 0.2\,\text{A}, \qquad i_2 = 1 - 0.2 = 0.8\,\text{A}$

$v_1 = 4i_2 + 20i + 3.2 + 20 = 23.2\,\text{V}$

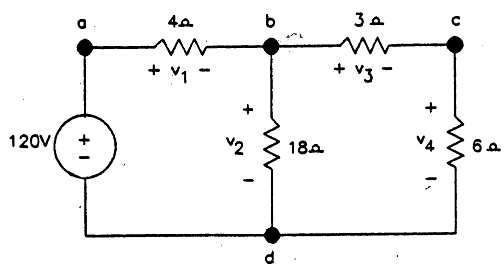
$v_2 = 5i_1 + 20i = 1.0 + 20 = 21.0\,\text{V}$

Problems

P 3.1 [a] From Ex. 3-1: $i_1 = 4\,\text{A}, \qquad i_2 = 8\,\text{A}, \qquad i_s = 12\,\text{A}$

at node x: $-12 + 4 + 8 = 0,$ at node y: $12 - 4 - 8 = 0$

[b]



$v_1 = 4i_s = 48\,\text{V} \qquad v_3 = 3i_2 = 24\,\text{V}$

$v_2 = 18i_1 = 72\,\text{V} \qquad v_4 = 6i_2 = 48\,\text{V}$

loop abda: $-120 + 48 + 72 = 0,$

loop bcdb: $-72 + 24 + 48 = 0,$

loop abcda: $-120 + 48 + 24 + 48 = 0$

P 3.2 [a] $p_{4\Omega} = i_s^2 4 = (12)^2 4 = 576\,\text{W} \qquad p_{18\Omega} = (4)^2 18 = 288\,\text{W},$

$p_{3\Omega} = (8)^2 3 = 192\,\text{W} \qquad p_{6\Omega} = (8)^2 6 = 384\,\text{W}$

[b] $p_{120\text{V}}(\text{delivered}) = 120i_s = 120(12) = 1440\,\text{W}$

[c] $p_{\text{diss}} = 576 + 288 + 192 + 384 = 1440\,\text{W}$

P 3.3 [a] $5\,\Omega // 20\,\Omega = 4\,\Omega \qquad \therefore\ R_{ab} = 10 + 4 + 6 = 20\,\Omega$

[b] $200\,\Omega // 50\,\Omega = 40\,\Omega$

$20\,\Omega + 40\,\Omega = 60\,\Omega$

$$\frac{1}{R_{ab}} = \frac{1}{30} + \frac{1}{60} + \frac{1}{60} = \frac{4}{60} = \frac{1}{15}\,\mho$$

$\therefore\ R_{ab} = 15\,\Omega$

P 3.4 [a] $R_{ab_a} = 4\,\Omega // 12\,\Omega = 3\,\Omega$

$R_{ab_b} = [2.5//7.5//5//15//5 + 7]//8 + 8 = 4 + 8 = 12\,\Omega$

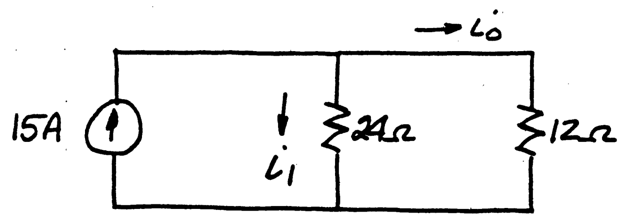
$R_{ab_c} = [6//3 + 3]//20 + 7 = 11\,\Omega$

[b] $P_{15V} = \dfrac{15^2}{3} = 75\ \text{W}$

$P_{48V} = \dfrac{48^2}{12} = 192\ \text{W}$

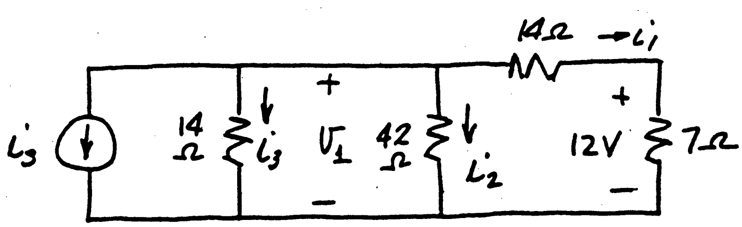
$P_{22V} = \dfrac{22^2}{11} = 44\ \text{W}$

P 3.5 $40\,\Omega // 60\,\Omega = 24\,\Omega$



$$i_o = \frac{(15)(24)}{(24 + 12)} = \frac{2}{3}(15) = 10\ \text{A}; \qquad p_{12\Omega} = (10)^2(12) = 1200\ \text{W}$$

P 3.6



$i_1 = \dfrac{12}{7}\ \text{A}; \qquad v_1 = (21)\left(\dfrac{12}{7}\right) = 36\ \text{V}$

$i_2 = \dfrac{36}{42} = \dfrac{6}{7}\ \text{A}; \qquad i_3 = \dfrac{36}{14} = \dfrac{18}{7}\ \text{A}$

$$i_s = -i_1 - i_2 - i_3 = -\frac{12}{7} - \frac{6}{7} - \frac{18}{7} = -\frac{36}{7}\ \text{A} = -5.14\ \text{A}$$

P 3.7 [a]

$5//20 = 100/25 = 4\,\Omega$

$9//18 = 162/27 = 6\,\Omega$

$R_{ab} = 5 + 12 + 3 = 20\,\Omega$

$5//20 + 9//18 + 10 = 20\,\Omega$

$20//30 = 600/50 = 12\,\Omega$

[b]

$5 + 15 = 20\,\Omega$

$20//60 = 1200/80 = 15\,\Omega$

$15 + 10 = 25\,\Omega$

$25//75 = 1875/100 = 18.75\,\Omega$

$18.75 + 11.25 = 30\,\Omega$

$30//20 = 600/50 = 12\,\Omega$

$3//6 = 18/9 = 2\,\Omega$

$3//6 + 30//20 = 2 + 12 = 14\,\Omega$

$26//14 = 364/40 = 9.1\,\Omega$

$R_{ab} = 2.5 + 9.1 + 3.4 = 15\,\Omega$

[c]

$3 + 5 = 8\,\Omega$

$8//12 = 96/20 = 4.8\,\Omega$

$4.8 + 5.2 = 10\,\Omega$

$45 + 15 = 60\,\Omega$

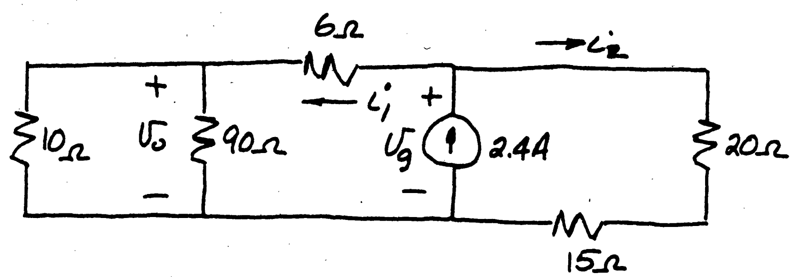
$60//40 = 2400/100 = 24\,\Omega$

$24 + 6 = 30\,\Omega$

$30//10 = 300/40 = 7.5\,\Omega$

$R_{ab} = 1.5 + 7.5 + 1.0 = 10\,\Omega$

P 3.8 [a]



$10//90 = 9\,\Omega, \quad 9 + 6 = 15\,\Omega, \quad 15 + 20 = 35\,\Omega$

$$i_1 = \frac{(2.4)(35)}{(35+15)} = \frac{35}{50}(2.4) = 1.68\,\text{A}$$

$v_o = (9)(1.68) = 15.12\,\text{V}$

[b] $i_2 = 2.4 - 1.68 = 0.72\,\text{A}$

$p_{20\Omega} = i_2^2(20) = (0.72)^2(20) = 10.368\,\text{W}$

[c] $v_g = 35i_2 = 35(0.72) = 25.20\,\text{V}$

$p_g(\text{dev}) = (25.2)(2.4) = 60.48\,\text{W}$

P 3.9 $24 + 16 = 40\,\Omega; \quad 40\,\Omega//10\,\Omega = 8\,\Omega; \quad 8\,\Omega + 12\,\Omega = 20\,\Omega$

$$\therefore\ i_{12\Omega} = \frac{80}{20} = 4\,\text{A}; \qquad i_o = i_{12\Omega}\left(\frac{40}{50}\right) = 0.8(4) = 3.2\,\text{A}$$

$$i_{4\Omega} = \frac{80}{8} = 10\,\text{A}; \qquad i_g = i_{4\Omega} + i_{12\Omega} = 10 + 4 = 14\,\text{A}$$

Check:

$P_{\text{dev}} = (80)i_g = 1120\,\text{W}$

$$\sum P_{\text{diss}} = (10)^2(8) + 16(12) + (3.2)^2(10) + (0.8)^2(40)$$

$$= 800 + 192 + 102.4 + 25.6 = 1120\,\text{W}$$

P 3.10 $20 + 60//30 = 40\,\Omega, \qquad 16 + 12 + 32 = 60\,\Omega$

$$\therefore\ i_{16\Omega} = (15)\frac{(40)}{(100)} = 6\text{ A}; \qquad v_o = (32)(6) = 192\text{ V}$$

The voltage across the three series connected resistors is

$$v_a = 6(16 + 12 + 32) = 360\text{ V}$$

$$\therefore\ v_g - 8(15) = 360; \qquad v_g = 360 + 120 = 480\text{ V}$$

Check:

$$P_{\text{dev}} = (480)(15) = 7200\text{ W}$$

$$P_{\text{diss}} = (15)^2(8) + (6)^2(60) + \frac{(360)^2}{40} = 7200\text{ W}$$

P 3.11 [a]

$20//80 = 16\,\Omega$

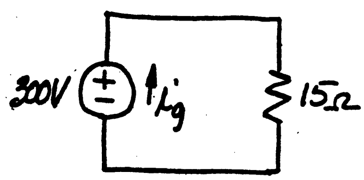
$16 + 4 = 20\,\Omega$

$20//30 = 12\,\Omega$

$12 + 8 = 20\,\Omega$

$20//60 = 15\,\Omega$

$i_g = 300/15 = 20\text{ A}$

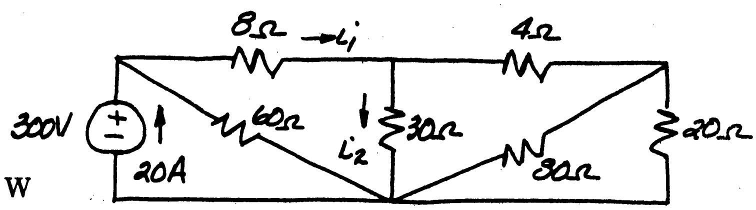


[b]

$$i_1 = 20\left(\frac{60}{80}\right) = 15\text{ A}$$

$$i_2 = 15\left(\frac{20}{50}\right) = 6\text{ A}$$

$$p_{30\Omega} = 6^2(30) = 1080\text{ W}$$



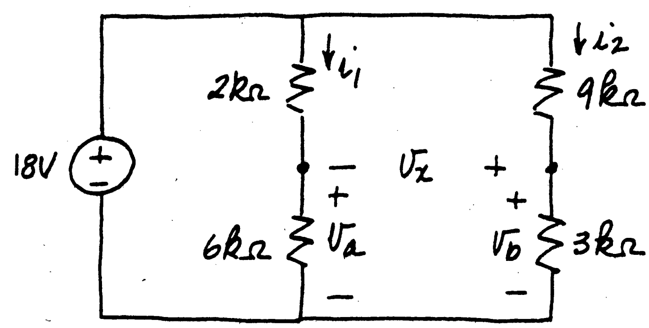
P 3.12 [a]

$$v_a = \frac{18}{8} \times 6 = 13.5\text{ V}$$

$$v_b = \frac{18}{12}(3) = 4.5\text{ V}$$

$$v_a + v_x = v_b$$

$$v_x = 4.5 - 13.5 = -9\text{ V}$$



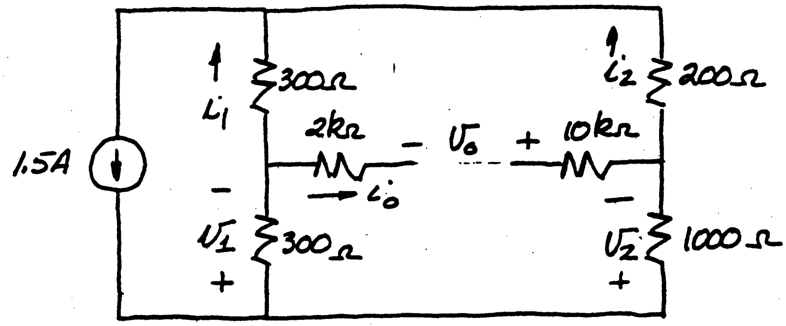
[b]

$$v_a = \frac{v_s}{8} \times 6 = 0.75v_s$$

$$v_b = \frac{v_s}{12}(3) = 0.25v_s$$

$$v_x = v_b - v_a = 0.25v_s - 0.75v_s = -0.5v_s$$

P 3.13



Since $i_o = 0$

$$i_1 = \frac{1.5(1200)}{(1800)} = 1.0\,\text{A}, \qquad \therefore\ v_1 = 300\,\text{V}$$

$$i_2 = \frac{1.5(600)}{(1800)} = 0.5\,\text{A}, \qquad \therefore\ v_2 = 500\,\text{V}$$

$$v_1 - v_o - v_2 = 0, \qquad \therefore\ v_o = 300 - 500 = -200\,\text{V}$$

P 3.14 [a] Voltage across 9-Ω resistor is $1(12 + 6) = 18$ V.

The current in the 9-Ω resistor is $18/9 = 2$ A. The current in the 2-Ω resistor is 2+1 or 3 A. Therefore the voltage across the 24-Ω resistor is $(2)(3)+18 = 24$ V.

The current in the 24-Ω resistor is 1 A. The current in the 3-Ω resistor is $1 + 2 + 1$ or 4 A. Therefore, the voltage across the 72-Ω resistor is $24 + 3(4) = 36$ V.

The current in the 72-Ω resistor is 36/72 or 0.5 A.

The 20 Ω//5 Ω resistors are equivalent to a 4-Ω resistor. The current in this equivalent resistor is $0.5 + 1 + 3$ or 4.5 A. Therefore the voltage across the 108-Ω resistor is $36 + 4.5(4) = 54$ V.

The current in the 108-Ω resistor is 54/108 or 0.5 A. The current in the 1.2-Ω resistor is $4.5 + 0.5$ or 5 A. Therefore $v_g = (1.2)(5) + 54 = 60$ V.

[b] The current in the 20-Ω resistor is $i_{20\Omega} = \frac{(4.5)(4)}{20} = \frac{18}{20} = 0.9$ A

The power dissipated in the 20-Ω resistor is $p_{20\Omega} = (0.9)^2(20) = 16.2$ W

P 3.15 [a] $R_{\text{cond}} = 845(0.0397) = 33.5465\,\Omega$

$R_{\text{total}} = 2(1/2)R_{\text{cond}} = 33.5465\,\Omega$

$P_{\text{loss}} = (2000)^2(33.5465) = 134.186$ MW

$P_{\text{calif}} = 800(2) - 134.186 = 1465.814$ MW

Efficiency $= (1465.814/1600) \times 100 = 91.61\%$

[b] $P_{\text{calif}} = 2000 - 134.86 = 1865.814\,\text{MW}$
Efficiency = 93.29%

[c] $P_{\text{loss}} = (3000)^2 \cdot 2 \cdot (1/3) \cdot 845 \cdot (0.0397) = 201.279\,\text{MW}$
$P_{\text{oregon}} = 3000\,\text{MW}, \quad P_{\text{calif}} = 3000 - 201.279 = 2798.7\,\text{MW}$
Efficiency = (2798.70/3000) × 100 = 93.29%

P 3.16 [a] $v_o = \dfrac{400(5.6)}{(39.2 + 5.6)} = 50\,\text{V}$

[b] $v_{R_1} = 350\,\text{V}$
$P_{R_1} = (350)^2/39{,}200 = 3.125\,\text{W}$
$v_{R_2} = 50\,\text{V}$
$P_{R_2} = (50)^2/5600 = 0.4464\,\text{W}$

[c] $\dfrac{R_2}{R_1 + R_2} = \dfrac{1}{8}, \quad \therefore\ 7R_2 = R_1, \quad R_2 = R_1/7$

$$P_{R_1} = \frac{(350)^2}{R_1} = 1\,\text{W}$$

$R_1 = 122.5\,\text{k}\Omega$

$$R_2 = \frac{122.5}{7} = 17.50\,\text{k}\Omega$$

$$P_{R_2} = \frac{2500}{17{,}500} \cong 0.1429\,\text{W} < 1\,\text{W}$$

Note if R_2 is selected to be a 1-W resistor,

$R_2 = 2500\,\Omega$
$R_1 = 7R_2 = 17{,}500\,\Omega$

$$P_{R_1} = \frac{(350)^2}{17{,}500} = 7\,\text{W} > 1\,\text{W}$$

Therefore design must be based on R_1 dissipating 1 W.

P 3.17 [a] $v_o = \dfrac{80R_2}{(R_1 + R_2)} = 20$

$$\therefore\ \frac{R_2}{R_1 + R_2} = \frac{20}{80} = \frac{1}{4}; \qquad \therefore\ 3R_2 = R_1$$

Let $R_e = R_2 /\!/ R_L = \dfrac{R_2 R_L}{R_2 + R_L}$

$$v_o = \frac{80R_e}{R_1 + R_e} = 18; \qquad \frac{R_e}{R_1 + R_e} = \frac{18}{80} = \frac{9}{40}$$

$$40R_e = 9R_1 + 9R_e; \qquad 31R_e = 9R_1 = 27R_2; \qquad R_e = \frac{(37.8)R_2}{R_2 + 37.8}$$

$$\frac{31(37.8)R_2}{R_2 + 37.8} = 27R_2$$

$$\therefore\ \frac{31(37.8)}{27} = R_2 + 37.8; \qquad R_2 = 5.6\,\text{k}\Omega; \qquad R_1 = 3(5.6) = 16.8\,\text{k}\Omega$$

[b] Power dissipated in R_1 will be maximum when the voltage across R_1 is maximum. This will occur under load conditions.

$$v_{R_1} = 80 - 18 = 62\,\text{V}$$

$$P_{R_1} = \frac{(62)^2}{16.8} \times 10^{-3} = 228.81\,\text{mW}$$

P 3.18 Refer to the solution of Problem 3.17. The divider will reach its dissipation limit when the power dissipated in R_1 equals 0.25 W.

Notes:

The dissipation in R_2 will be equal to or less than $P_{R_2} \leq \frac{400}{5.6} = 71.43\,\text{mW}$

The dissipation in R_1 can reach a maximum of $P_{R_1} = \frac{6400}{16.8} = 380.95\,\text{mW}$

When the dissipation in R_1 is 250 mW, the voltage across R_1 is

$$v_{R_1}^2 = 0.25(16.8 \times 10^3) = 4200, \qquad v_{R_1} = 64.81\,\text{V}$$

$$\therefore\ v_o = 80 - 64.81 = 15.19\,\text{V}$$

$$\therefore\ \frac{80R_e}{R_1 + R_e} = 15.19, \qquad \therefore\ R_e = 3.94\,\text{k}\Omega$$

$$\therefore\ \frac{5.6R_L}{5.6 + R_L} = 3.94, \qquad R_L = 13.27\,\text{k}\Omega \quad \text{(minimum value)}$$

P 3.19 $$\frac{(24)^2}{R_1 + R_2 + R_3} = 60, \qquad \therefore\ R_1 + R_2 + R_3 = 9.6\,\Omega$$

$$\frac{(R_1 + R_2)24}{(R_1 + R_2 + R_3)} = 12$$

$$\therefore\ 2(R_1 + R_2) = R_1 + R_2 + R_3$$

$$\therefore\ R_1 + R_2 = R_3; \qquad 2R_3 = 9.6; \qquad R_3 = 4.8\,\Omega$$

$$\frac{R_2(24)}{R_1 + R_2 + R_3} = 5$$

$$4.8R_2 = R_1 + R_2 + 4.8 = 9.6$$

$$R_2 = 2\,\Omega; \qquad R_1 = 9.6 - R_2 - R_3 = 2.8\,\Omega$$

P 3.20 [a] When the circuits are connected, the 25-kΩ resistor is paralleled by a 100-kΩ resistor, thus the output voltage of the first divider is

$$v'_{o_1} = \frac{380}{(75 + 20)}(20) = 80\,\text{V}$$

Therefore $v_o = \frac{80}{100}(60) = 48\,\text{V}$

[b] $i = \frac{380}{100} \times 10^{-3} = 3.8\,\text{mA}$

$(3.8)(10^{-3})(25 \times 10^3) = 95\,\text{V}$

$v_o = \frac{95}{(100)}(60) = 57\,\text{V}$

[c] It removes the loading effect on the voltage divider connected to the 380-V source.

This can be seen by noting the no-load value of v_o' is 95 V, which is identical to the voltage applied to the second divider circuit in Fig. P3.20(c).

P 3.21 [a] Let v_o be the voltage across the parallel branches, positive at the upper terminal, then

$$i_g = v_oG_1 + v_oG_2 + \cdots + v_oG_N = v_o(G_1 + G_2 + \cdots + G_N)$$

It follows that $v_o = \frac{i_g}{(G_1 + G_2 + \cdots + G_N)}$

The current in the k^{th} branch is $i_k = v_oG_k$; $\therefore\ i_k = \frac{i_gG_k}{[G_1 + G_2 + \cdot + G_N]}$

[b] $G_1 + G_2 + G_3 + G_4 + G_5 + G_6 = 2.5\,℧$

$G_{5\Omega} = 0.2\,℧$

$i_o = \frac{40(0.2)}{2.5} = 3.2\,\text{A}$

Check: $i_g = \frac{16}{0.5} + \frac{16}{5} + \frac{16}{8} + \frac{16}{10} + \frac{16}{20} + \frac{16}{40}$

$= 32 + 3.2 + 2 + 1.6 + 0.8 + 0.4 = 40\,\text{A}$

P 3.22 $i_4 = \frac{G_4}{\sum G}10^{-3}$ $\qquad i_2 = 2i_3 = \frac{4G_4}{\sum G} \times 10^{-3}$

$i_3 = 2i_4 = \frac{2G_4}{\sum G} \times 10^{-3}$ $\qquad i_1 = 2i_2 = \frac{8G_4}{\sum G} \times 10^{-3}$

$i_1 + i_2 + i_3 + i_4 = 10^{-3} = \frac{G_4}{\sum G} \times 10^{-3}(8 + 4 + 2 + 1)$

$\therefore\ \frac{15G_4}{\sum G} = 1$ or $15G_4 = \sum G$

$i_4R_4 = 1$

$\therefore\ \left(\frac{G_4}{\sum G} \times 10^{-3}\right)\left(\frac{1}{G_4}\right) = 1$

$\therefore\ \sum G = 10^{-3}$

$\therefore\ G_4 = \frac{10^{-3}}{15},\qquad R_4 = \frac{1}{G_4} = 15{,}000\,\Omega$

$$i_3 = \frac{G_3 \times 10^{-3}}{\sum G} = \frac{2G_4}{\sum G} \times 10^{-3}$$

$$\therefore\ G_3 = 2G_4 = \frac{2 \times 10^{-3}}{15}, \qquad R_3 = \frac{1}{G_3} = 7500\,\Omega$$

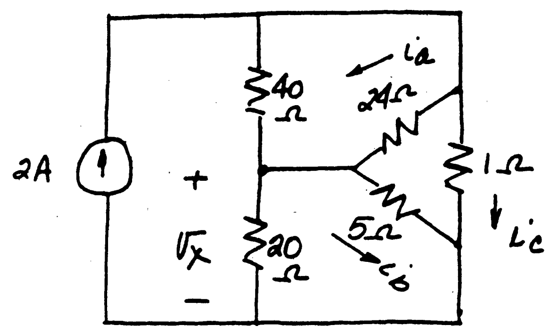
$$i_2 = \frac{G_2 10^{-3}}{\sum G} = \frac{4G_4 \times 10^{-3}}{\sum G}$$

$$\therefore\ G_2 = 4G_4, \qquad R_2 = \frac{15}{4} \times 10^3 = 3750\,\Omega$$

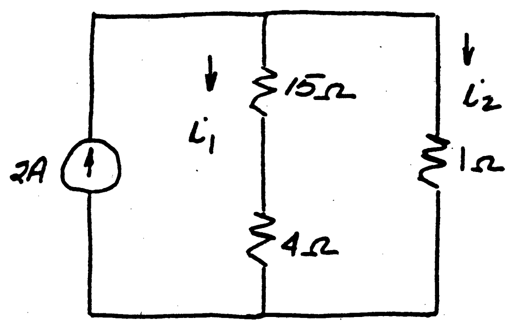
$$i_1 = \frac{G_1 \times 10^{-3}}{\sum G} = \frac{8G_4 \times 10^{-3}}{\sum G}$$

$$\therefore\ G_1 = 8G_4, \qquad R_1 = \frac{15 \times 10^3}{8} = 1875\,\Omega$$

P 3.23 [a]



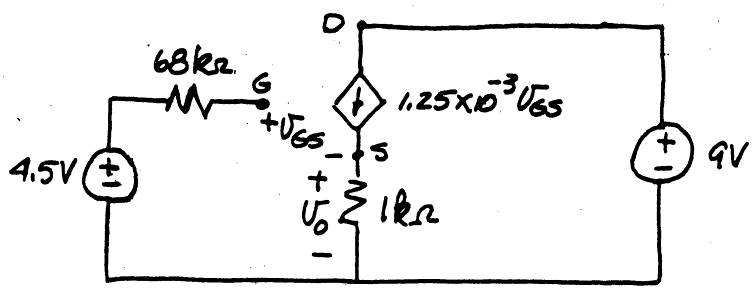
$40//24 = 15\,\Omega$
$20//5 = 4\,\Omega$



$$i_2 = \frac{2(19)}{20} = 1.9\,\text{A}$$
$$i_c = i_2 = 1.9\,\text{A}$$

$$i_1 = \frac{2(1)}{(19+1)} = 0.1\,\text{A}$$
$$i_a = \frac{i_1(40)}{64} = 62.5\,\text{mA}$$
$$i_b = \frac{0.1(20)}{25} = 0.08 = 80\,\text{mA}$$
$$v_x = 5i_b = 0.40\,\text{V}$$
$$P_{\text{device}} = 24i_a^2 + 5i_b^2 + 1i_c^2 = 3.74\,\text{W}$$

P 3.24



$$4.5 = v_{GS} + 1000(1.25\times10^{-3}v_{GS}) = 2.25v_{GS}, \qquad \therefore\ v_{GS} = \frac{4.5}{2.25} = 2\,\mathrm{V}$$

$$v_o = 1000(1.25\times10^{-3})(2) = 2.5\,\mathrm{V}$$

$$v_{DS} + v_o = 9\,\mathrm{V}, \qquad \therefore\ v_{DS} = 6.5\,\mathrm{V}$$

P 3.25 The current in the shunt resistor at full-scale deflection is $i_A = i_{\text{fullscale}} - 2\times10^{-3}$. The voltage across R_A at full-scale deflection is always 50mV, therefore

$$R_A = \frac{50\times10^{-3}}{i_{\text{fullscale}} - 2\times10^{-3}} = \frac{50}{1000i_{\text{fullscale}} - 2}$$

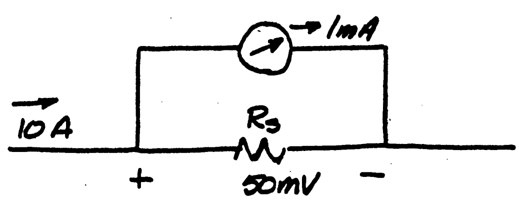
[a] $R_A = \dfrac{50}{10{,}000 - 2} = \dfrac{50}{9998} = 5.001\,\mathrm{m}\Omega$

[b] $R_A = \dfrac{50}{1000 - 2} = \dfrac{50}{998} = 50.10\,\mathrm{m}\Omega$

[c] $R_A = \dfrac{50}{48} = 1.042\,\Omega$

[d] $R_A = \infty$ (open)

P 3.26



Original meter: $R_e = \dfrac{50\times10^{-3}}{10} = 5\times10^{-3} = 0.005\,\Omega$

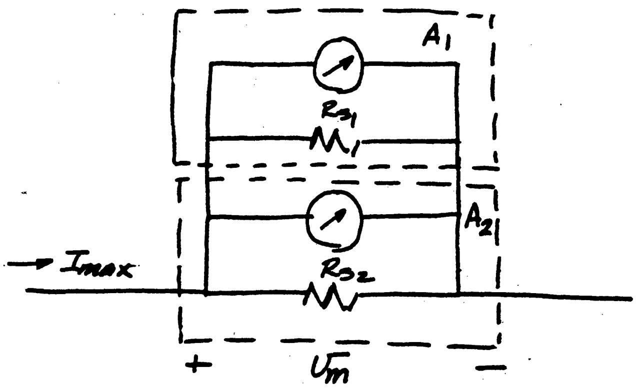
Modified meter: $R_e = \dfrac{(0.005)(0.01)}{0.015} = \dfrac{0.05}{15} = \dfrac{1}{300}\,\Omega$

$$\therefore\ (I_{\text{fullscale}})\left(\frac{1}{300}\right) = 50\times10^{-3}$$

$$\therefore\ I_{\text{fullscale}} = (300)(50)\times10^{-3} = 15\,\mathrm{A}$$

$$I_{\text{fullscale}} = 15\,\mathrm{A}$$

P 3.27



Note the maximum voltage across the parallel-connected ammeters is limited to 500 μV, otherwise ammeter 1 exceeds its full-scale reading.

When $v_m = 500\,\mu$V, A_1 reads 10 μA and ammeter 2 reads (1/2)(5) or 2.5 μA.

Therefore $I_{max} = 12.5\,\mu A$

P 3.28 [a] $$i_m = \frac{i_{meas}(50/49)}{50 + (50/49)} = \frac{50 i_{meas}}{2500} = \frac{1}{50} i_{meas} = \text{one fiftieth of } i_{meas}$$

[b] $$R_s = \frac{50}{999}\,\Omega$$

$$i_m = \frac{i_{meas}(50/999)}{50 + (50/999)} = \frac{50 i_{meas}}{50(999) + 50} = \frac{i_{meas}}{1000}$$

$\therefore\ i_m$ = one-thousandth of i_{meas}

[c] Yes

P 3.29 Measured value:

$$i_g = \frac{400}{(16 + 64)} = 5\,\text{A}; \qquad i_{meas} = (5)\frac{80}{100} = 4\,\text{A}$$

True value:

$$i_g = \frac{400}{\left[\frac{(80)(19)}{99} + 64\right]} = \frac{2475}{491} \cong 5.04\,\text{A}; \qquad i_{true} = (5.04)\left(\frac{80}{99}\right) \cong 4.07\,\text{A}$$

$$\%\ \text{error} = \left[\frac{4}{4.07} - 1\right] \times 100 = -1.8\%$$

P 3.30 $$i_{true} = \frac{(300)(25)}{560} = 13.39\,\mu\text{A}$$

$$R_{\text{meter 1}} = \frac{500 \times 10^{-6}}{10 \times 10^{-6}} = 50\,\Omega$$

$$R_{\text{meter 2}} = \frac{1 \times 10^{-3}}{5 \times 10^{-6}} = 200\,\Omega$$

$\therefore$ The resistance of the paralleled meters is $R_m = \dfrac{(50)(200)}{250} = 40\,\Omega$

$$i_{\text{meas}} = \frac{(300)(25)}{600} = 12.5\,\mu\text{A}$$

$$\%\text{ error} = \left[\frac{12.5}{13.39} - 1\right] 100 = -6.67\%$$

P 3.31 At full scale the voltage across the shunt resistor will be 20 mV; therefore the power dissipated will be

$$P_A = \frac{(20 \times 10^{-3})^2}{R_A}.$$

Therefore R_A must be equal to or greater than $R_A \geq \dfrac{(20 \times 10^{-3})^2}{0.25} \geq 1600\,\mu\Omega$

Otherwise the power dissipated in R_A will exceed its power rating of 0.25 W

When $R_A = 1600\,\mu\Omega$, the shunt current will be $i_A = \dfrac{20 \times 10^{-3}}{1600 \times 10^{-6}} = 12.5\text{ A}$

The measured current will be $i_{\text{meas}} = 12.5 + 0.001 = 12.501\text{ A}$

$\therefore$ Full-scale reading is for practical purposes 12.5 A

P 3.32 For all full-scale readings the total resistance is $R_V + R_{\text{movement}} = \dfrac{\text{full-scale reading}}{10^{-3}}$

$\therefore\ R_V = 1000\ (\text{full-scale reading}) - 20$

[a] $R_V = 1000(100) - 20 = 99{,}980\,\Omega$

[b] $R_V = 1000(1) - 20 = 980\,\Omega$

[c] $R_V = 200 - 20 = 180\,\Omega$

[d] $R_V = 20 - 20 = 0\,\Omega$

P 3.33 [a] $v_{\text{meter}} = 320\text{ V}$

[b] $R_{\text{meter}} = (400)(600) = 240\text{ k}\Omega$

$48//240 = 40\text{ k}\Omega$

$$v_{\text{meter}} = \frac{320}{56} \times 40 = 228.57\text{ V}$$

[c] $16//240 = 15\text{ k}\Omega$

$$v_{\text{meter}} = \frac{320}{63}(15) = 76.19\text{ V}$$

[d] $v_{\text{meter a}} = 320\,\text{V}$

$v_{\text{meter b}} + v_{\text{meter c}} = 304.76,$ no

Because of the loading effect.

P 3.34 [a] $R_{\text{movement}} = 50\,\Omega$

$$R_1 + R_{\text{movement}} = \frac{30}{1 \times 10^{-3}} = 30\,\text{k}\Omega$$

$\therefore\ R_1 = 29{,}950\,\Omega$

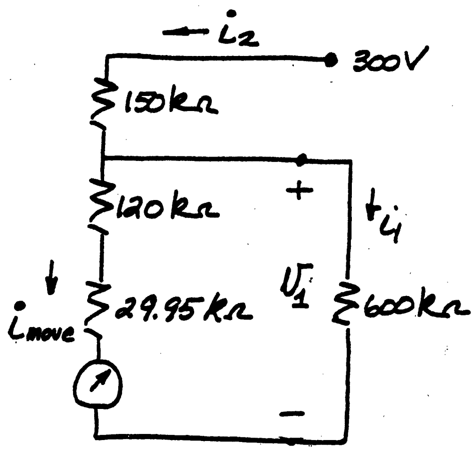
$$R_2 + R_1 + R_{\text{movement}} = \frac{150}{10^{-3}} = 150\,\text{k}\Omega$$

$\therefore\ R_2 = 120\,\text{k}\Omega$

$$R_3 + R_2 + R_1 + R_{\text{movement}} = \frac{300}{10^{-3}} = 300\,\text{k}\Omega$$

$\therefore\ R_3 = 150\,\text{k}\Omega$

[b]



$$i_{\text{move}} = \frac{240}{300}(1) = 0.8\,\text{mA}$$

$v_1 = (0.8)(150) = 120\,\text{V}$

$$i_1 = \frac{120}{600} = 0.2\,\text{mA}$$

$i_2 = i_{\text{move}} + i_1 = 0.8 + 0.2 = 1\,\text{mA}$

$v_{\text{meas}} = v_x = v_1 + 150 i_2 = 270\,\text{V}$

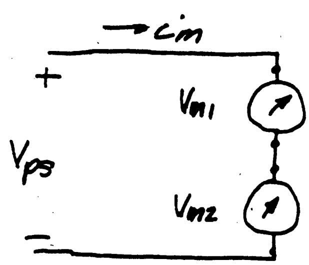
[c] $v_1 = 150\,\text{V}$

$$i_1 = \frac{150}{600} = 0.25\,\text{mA}$$

$i_2 = 1 + 0.25 = 1.25\,\text{mA}$

$v_{\text{meas}} = v_x = 150 + 150(1.25) = 337.50\,\text{V}$

P 3.35 [a] Since the unknown voltage is greater than either voltmeter's maximum reading, the only possible way to use the voltmeters would be to connect them in series.



$R_{m1} = (300)(900) = 270\,\text{k}\Omega$

$R_{m2} = (150)(1200) = 180\,\text{k}\Omega$

$\therefore\ R_{m1} + R_{m2} = 450\,\text{k}\Omega$

$$i_m = \frac{400}{450 \times 10^3} = \frac{8}{9}\,\text{mA}$$

$$V_{m1} = \frac{8}{9}(270) = 240\,\text{V}$$

$$V_{m2} = \frac{8}{9}(180) = 160\,\text{V}$$

Therefore the meters cannot be used since the reading of V_{m2} exceeds 150 V.

[b] The maximum current allowed in V_{m1} is $i_1(\text{max}) = \frac{300}{270} = \frac{10}{9}\,\text{mA}$.

The maximum current allowed in V_{m2} is $i_2(\text{max}) = \frac{150}{180} = \frac{5}{6}\,\text{mA}$.

∴ The 150-V meter limits the maximum voltage that can be measured. Thus

$$v_{\text{max}} = \frac{5}{6}(270) + \frac{5}{6}(180) = 225 + 150 = 375\,\text{V}$$

[c] $i_m = \frac{320}{450 \times 10^3} = 0.711\,\text{mA}$

$$V_{m1} = (0.711)(270) = 192\,\text{V}$$

$$v_{m2} = (0.711)(180) = 128\,\text{V}$$

P 3.36 The current in the series-connected voltmeters is

$$i_m = \frac{205.2}{270} = \frac{136.8}{180} = 0.76\,\text{mA}$$

$$v_{50\text{k}\Omega} = (0.76)(50) = 38\,\text{V}$$

$$V_{\text{power supply}} = 205.2 + 136.8 + 38 = 380\,\text{V}$$

P 3.37 [a] $R_{\text{meter}} = 400\,\text{k}\Omega + 200\,\text{k}\Omega // 200\,\text{k}\Omega = 500\,\text{k}\Omega$

$$500//125 = 100\,\text{k}\Omega$$

$$V_{\text{meter}} = \frac{750}{125}(100) = 600\,\text{V}$$

[b] What is the percent error in the measured voltage?

$$\text{True value} = \frac{750}{150}(125) = 625$$

$$\%\ \text{error} = \left(\frac{600}{625} - 1\right) 100 = -4\%$$

P 3.38 $R_{\text{meter}} = R_m + R_{\text{movement}} = 500\,\text{k}\Omega$

$$v_{\text{meas}} = 30 \times 10^{-3} R_e$$

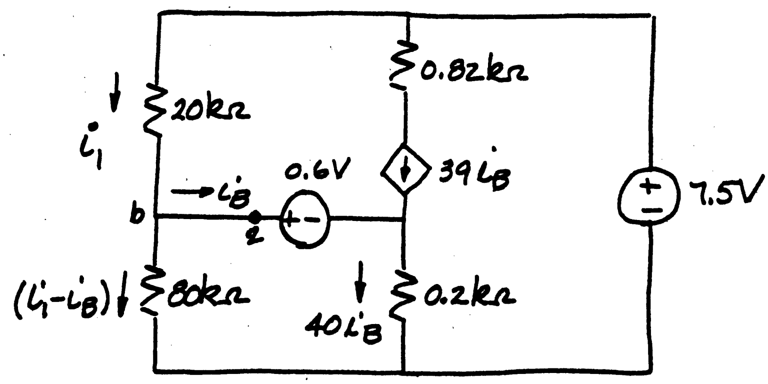
$$R_e = 25\,\text{k}\Omega // 125\,\text{k}\Omega // 500\,\text{k}\Omega = 20\,\text{k}\Omega$$

$$v_{\text{meas}} = 30(20) = 600\,\text{V}$$

$$v_{\text{true}} = (30 \times 10^{-3})\frac{(25)(125)}{150} \times 10^3 = 625\,\text{V}$$

$$\%\ \text{error} = \left(\frac{600}{625} - 1\right) 100 - 4\%$$

P 3.39 [a]



$$20 \times 10^3 i_1 + 80 \times 10^3(i_1 - i_B) = 7.5$$
$$80 \times 10^3(i_1 - i_B) = 0.6 + 40i_B(0.2) \times 10^3$$
$$\therefore\ 100i_1 - 80i_B = 7.5 \times 10^{-3}$$
$$80i_1 - 88i_B = 0.6 \times 10^{-3}$$
$$\therefore\ \Delta = \begin{vmatrix} 100 & -80 \\ 80 & -88 \end{vmatrix} = -2400$$
$$N_B = \begin{vmatrix} 100 & 7.5 \times 10^{-3} \\ 80 & 0.6 \times 10^{-3} \end{vmatrix} = -540 \times 10^{-3}$$
$$i_B = \frac{N_B}{\Delta} = \frac{-540 \times 10^{-3}}{-2400} = 0.225 \times 10^{-3} = 225 \times 10^{-6} = 225\,\mu\text{A}$$

[b] With the insertion of the ammeter the equations become

$$100i_1 - 80i_B = 7.5 \times 10^{-3} \quad \text{(no change)}$$
$$80 \times 10^3(i_1 - i_B) = 10^3 i_B + 0.6 + 40i_B(200)$$
$$80i_1 - 89i_B = 0.6 \times 10^{-3}$$
$$\Delta = \begin{vmatrix} 100 & -80 \\ 80 & -89 \end{vmatrix} = -8900 + 6400 = -2500$$
$$N_B = \begin{vmatrix} 100 & 7.5 \times 10^{-3} \\ 80 & 0.6 \times 10^{-3} \end{vmatrix} = -540 \times 10^{-3}$$
$$i_B = \frac{N_B}{\Delta} = \frac{-540 \times 10^{-3}}{-2500} = 0.216 \times 10^{-3} = 216\,\mu\text{A}$$

[c] $$\%\ \text{error} = \left(\frac{216}{225} - 1\right)100 = -4\%$$

P 3.40 [a] $$67.2 \times 10^{-3} = \frac{V_s(12 \times 10^6)}{R_s + 12 \times 10^6}$$
$$\therefore\ 67.2 \times 10^{-3}R_s + 806{,}400 = 12 \times 10^6 V_s$$
$$4\text{M}\Omega // 12\text{M}\Omega = 3\,\text{M}\,\Omega$$
$$\therefore\ 60 \times 10^{-3} = \frac{3 \times 10^6 V_s}{R_s + 3 \times 10^6}$$
$$\therefore\ 60 \times 10^{-3}R_s + 180{,}000 = 3 \times 10^6 V_s$$

Therefore the two simultaneous equations are

$$12\times10^6V_s - 67.2\times10^{-3}R_s = 806{,}400$$
$$3\times10^6V_s - 60\times10^{-3}R_s = 180{,}000$$

$$\therefore\ \Delta = \begin{vmatrix} 12\times10^6 & -67.2\times10^{-3} \\ 3\times10^6 & -60\times10^{-3} \end{vmatrix} = -720{,}000 + 201{,}600 = -518{,}400$$

$$N_V = \begin{vmatrix} 806{,}400 & -67.2\times10^{-3} \\ 180{,}000 & -60\times10^{-3} \end{vmatrix} = -48{,}384 + 12{,}096 = -36{,}288$$

$$V_s = \frac{N_V}{\Delta} = 0.07 = 70\times10^{-3} = 70\,\text{mV}$$

[b] $$N_R = \begin{vmatrix} 12\times10^6 & 806{,}400 \\ 3\times10^6 & 180{,}000 \end{vmatrix} = 216\times10^{10} - 24{,}192\times10^8 = -2592\times10^8$$

$$R_s = \frac{N_R}{\Delta} = \frac{-2592\times10^8}{-518{,}400} = 500{,}000\,\Omega$$

$$R_s = 500\,\text{k}\Omega$$

P 3.41 [a] $R_1 = (100/2)10^3 = 50\,\text{k}\Omega$

$R_2 = (10/2)10^3 = 5\,\text{k}\Omega$

$R_3 = (1/2)10^3 = 500\,\Omega$

[b] Let i_a = actual current in the movement

i_d = design current in the movement

Then % error $= \left(\frac{i_a}{i_d} - 1\right)100$

For the 100-V scale: $i_a = \frac{100}{50{,}000+25} = \frac{100}{50{,}025}$, $\quad i_d = \frac{100}{50{,}000}$

$\frac{i_a}{i_d} = \frac{50{,}000}{50{,}025} \cong 0.9995 \qquad$ % error $= (0.9995 - 1)100 = -0.05\%$

For the 10-V scale:

$\frac{i_a}{i_d} = \frac{5000}{5025} \cong 0.995 \qquad$ % error $= (0.995 - 1.0)100 = -0.4975\%$

For the 1-V scale:

$\frac{i_a}{i_d} = \frac{500}{525} \cong 0.9524 \qquad$ % error $= (0.9524 - 1.0)100 = -4.76\%$

P 3.42 [a] $\frac{9}{250}\times10^6 = 36\,\text{k}\Omega$

$\therefore\ 12 + R = 36, \qquad R = 24\,\text{k}\Omega$

[b] @ 80% full scale: $i = 0.8(250) = 200\,\mu\text{A}$

$R_t = \frac{9}{200}\times10^6 = 45\,\text{k}\Omega = 36 + R_x \qquad \therefore\ R_x = 9\,\text{k}\Omega$

50% full scale: $i = 0.5(250) = 125\,\mu A$

$R_t = \frac{9}{125} \times 10^6 = 72\,k\Omega = 36 + R_x \qquad \therefore\ R_x = 36\,k\Omega$

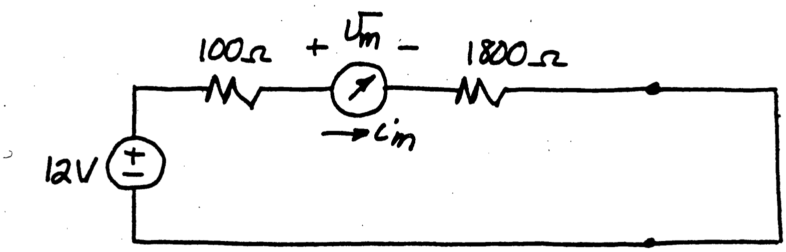
20% full scale: $i = 0.2(250) = 50\,\mu A$

$R_t = \frac{9}{50} \times 10^6 = 180\,k\Omega = 36 + R_x \qquad \therefore\ R_x = 144\,k\Omega$

0% full scale

$R_x = \infty$

P 3.43



When short circuited, $12 = 1900 i_m + v_m$

When the short circuit is replaced with a 2000-Ω resistor, the current is $0.5 i_m$; therefore $12 = 1900(0.5 i_m) + 0.5 v_m + 2000(0.5 i_m) = 1950 i_m + 0.5 v_m$

or $24 = 3900 i_m + v_m$

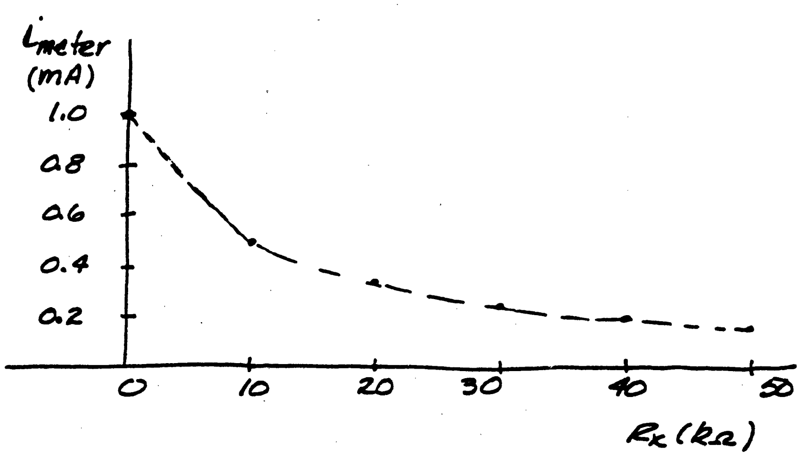
$\therefore\ 12 = 2000 i_m, \qquad i_m = 6\,mA$

$\therefore\ v_m = 0.6\,V = 600\,mV$

P 3.44 [a] $R_o = \frac{10}{10^{-3}} = 10\,k\Omega$

[b] $i_{meter} = \frac{10}{10 + R_x}\,mA$

R_x (kΩ)	0	10	20	30	40	50
i_{meter} (mA)	1.000	0.500	0.333	0.250	0.200	0.167

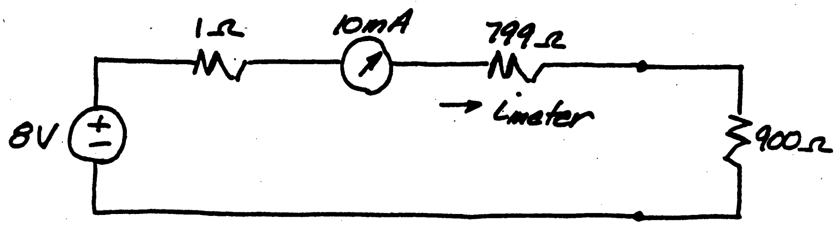


[c] Because the meter current varies nonlinearly with respect to R_x but the deflection of the meter is proportional to i_{meter}.

P 3.45 [a] $R_o + R_g = \frac{9}{10} \times 10^3 = 900$ $\quad\therefore\ R_o = 900 - 1 = 899\,\Omega$

[b] $R_o + R_g = \frac{8}{10} \times 1000 = 800\,\Omega$ $\quad\therefore\ R_o = 800 - 1 = 799\,\Omega$

[c]



$$i_{\text{meter}} = \frac{8}{800 + 900} = \frac{8}{1700}\text{ A}$$

The meter was calibrated using the relationship $i_{\text{meter}} = \frac{9}{900 + R_x}$.

$\therefore$ when $i_{\text{meter}} = \frac{8}{1700}$ A, the calibrated value of R_x is

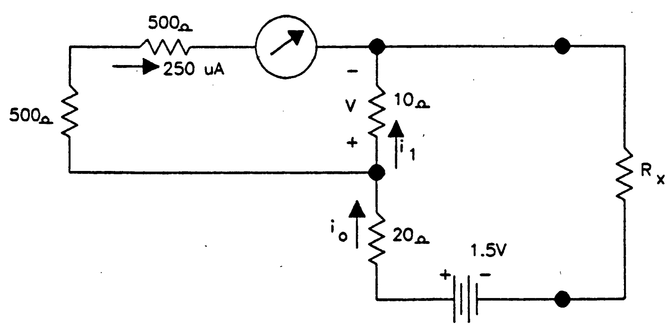
$$\frac{8}{1700} = \frac{9}{900 + R_x}$$

$$R_x = \frac{9(1700)}{8} - 900 = 1912.5 - 900 = 1012.5\,\Omega$$

Therefore the ohmmeter would read $1012.5\,\Omega$.

Note that R_o can be used to compensate for changes in R_g but not v_g.

P 3.46 [a] At midscale the microammeter carries $250\,\mu$A. With the switch in position 1, the circuit is



$v = 1000(250) \times 10^{-6} = 0.25\,\text{V}$, $i_1 = 0.25/10 = 25{,}000\,\mu\text{A}$,

$i_o = i_1 + 250 = 25{,}250\,\mu\text{A}$, $1.5/i_o = 59.4\,\Omega$,

$R_T = 20 + 1000//10 + R_x = 59.4\,\Omega$, $R_x = 29.5\,\Omega$

Similar analysis of the other positions yields

position #2:

$i_1 = 2272.73\,\mu\text{A}$ $R_T = 594.59\,\Omega$

$i_o = 2522.73\,\mu\text{A}$ $R_x = 295.50\,\Omega$

position #3:

$i_1 = 4629.63\,\mu\text{A}$ $R_T = 6148.01\,\Omega$

$i_o = 4879.63\,\mu\text{A}$ $R_x = 3146.77\,\Omega$

position #4:

$i_1 = 252.53\,\mu\text{A}$ $R_T = 59{,}698.49\,\Omega$

$i_o = 502.53\,\mu\text{A}$ $R_x = 29{,}701.01\,\Omega$

[b] Yes, the largest error is position #3, where the indicated value is 4.66% low.

P 3.47 $R_{\text{movement}} = \dfrac{100 \times 10^{-3}}{10 \times 10^{-6}} = 10\,\text{k}\Omega$

$R_{\text{movement}} + R_o = \dfrac{9}{10} \times 10^6 = 900\,\text{k}\Omega$

$R_o = 900 - 10 = 890\,\text{k}\Omega$

20% deflection $i_{\text{movement}} = 2\,\mu\text{A}$

$\therefore\ v_{500\Omega} = (2 \times 10^{-6})(900 \times 10^3) = 1.8\,\text{V}$

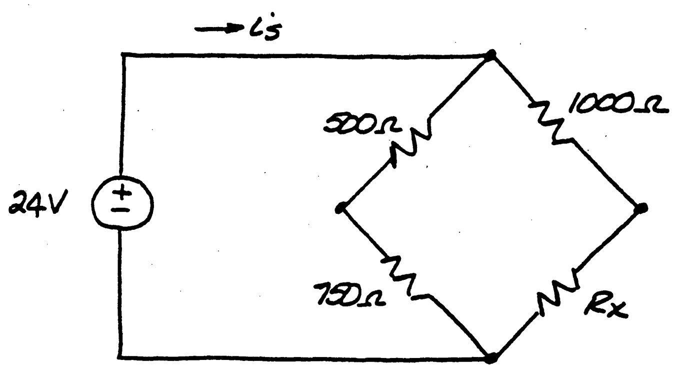
$i_{500\Omega} = \dfrac{1.8}{500} = \dfrac{3.6}{1000} = 3.6\,\text{mA} = 3600\,\mu\text{A}$

$i_x \cong i_{500} = 3600\,\mu\text{A}$

$v_x = 9 - 1.8 = 7.2\,\text{V}$

$R_x \cong \dfrac{7.2}{3600} \times 10^6 = 2000\,\Omega = 2\,\text{k}\Omega$

P 3.48 [a]



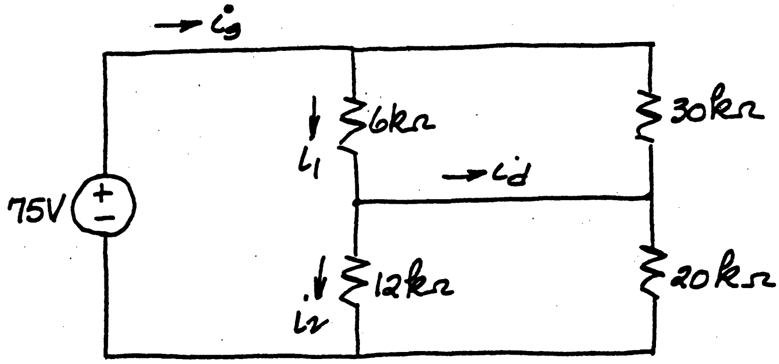
$R_x = \dfrac{(1000)(750)}{500} = 1500\,\Omega = 1.5\,\text{k}\Omega$

[b] $i_s = \frac{24}{1.25} + \frac{24}{2.5} = 19.2 + 9.6 = 28.80\,\text{mA}$

[c] 750-Ω resistor, $P = (19.2 \times 10^{-3})^2(750) = 276.48\,\text{mW}$

[d] 1000-Ω resistor, $P = (9.6 \times 10^{-3})^2(1000) = 92.16\,\text{mW}$

P 3.49 Redraw the circuit, replacing the detector branch with a short circuit.



$6\,\text{k}\Omega // 30\,\text{k}\Omega = 5\,\text{k}\Omega$

$12\,\text{k}\Omega // 20\,\text{k}\Omega = 7.5\,\text{k}\Omega$

$i_g = \frac{75}{5 + 7.5} = 6\,\text{mA}$

$i_1 = 6\frac{(30)}{36} = 5\,\text{mA}$

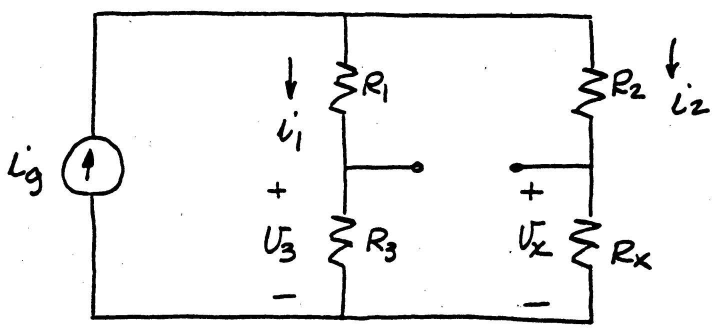
$i_2 = 6\frac{(20)}{32} = 3.75\,\text{mA}$

$i_1 = i_d + i_2$

$i_d = i_1 - i_2 = 5 - 3.75 = 1.25\,\text{mA}$

Check: $6i_1 + 12i_2 = 30 + 45 = 75\,\text{V}$

P 3.50 Since the bridge is balanced, we can remove the detector without disturbing the voltages and currents in the circuit.



It follows that

$$i_1 = \frac{i_g(R_2+R_x)}{R_1+R_2+R_3+R_x} = \frac{i_g(R_2+R_x)}{\sum R}$$

$$i_2 = \frac{i_g(R_1+R_3)}{R_1+R_2+R_3+R_x} = \frac{i_g(R_1+R_3)}{\sum R}$$

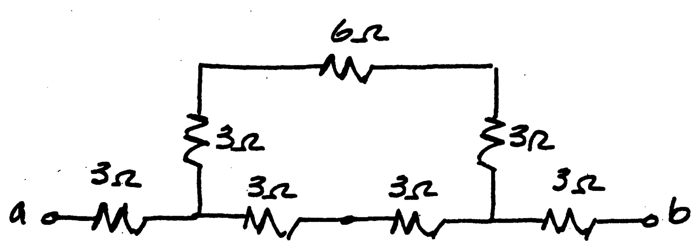
$$v_3 = R_3 i_3 = v_x = i_2 R_x$$

$$\therefore\ \frac{R_3 i_g(R_2+R_x)}{\sum R} = \frac{R_x i_g(R_1+R_3)}{\sum R}; \qquad \therefore\ R_3(R_2+R_x) = R_x(R_1+R_3)$$

From which $R_x = \dfrac{R_2R_3}{R_1}$

P 3.51 The top of the pyramid can be replaced by a resistor equal to $R_1 = \dfrac{(18)(9)}{27} = 6\,\Omega$

The lower left and right deltas can be replaced by wyes. Each resistance in the wye equals $3\,\Omega$. Thus our circuit can be reduced to

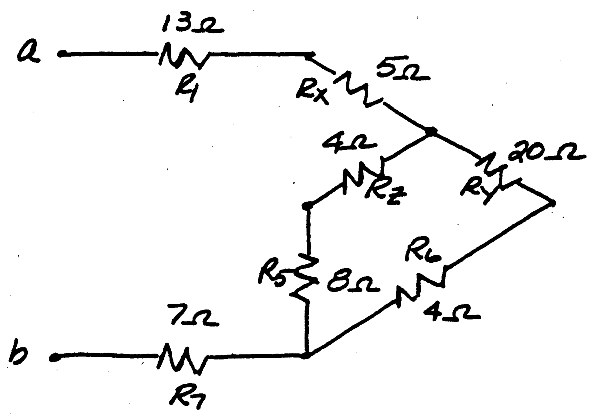


Now the $12\,\Omega$ in parallel with $6\,\Omega$ reduces to $R_2 = \dfrac{(12)(6)}{18} = 4\,\Omega$

$\therefore\ R_{ab} = 3+4+3 = 10\,\Omega$

P 3.52 [a] $R_X = \dfrac{(10)(50)}{100} = 5\,\Omega;\qquad R_Y = \dfrac{(40)(50)}{100} = 20\,\Omega;\qquad R_Z = \dfrac{(10)(40)}{100} = 4\,\Omega$

Replacing the R_2–R_3–R_4 delta with its equivalent Y gives



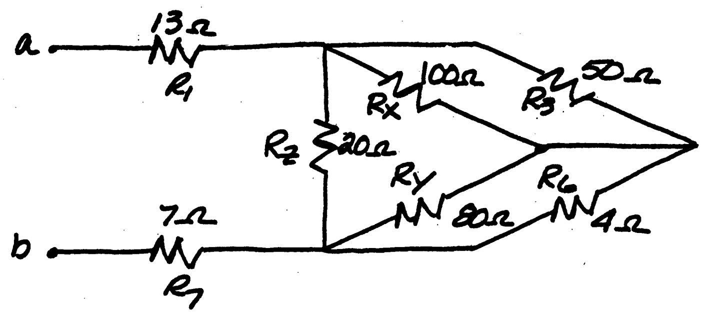
$R_{ab} = 13 + 5 + 12//24 + 7 = 18 + 8 + 7 = 33\,\Omega$

[b] $R_X = \dfrac{(10)(40) + (40)(8) + (8)(10)}{8} = \dfrac{800}{8} = 100\,\Omega$

$R_Y = \dfrac{800}{10} = 80\,\Omega$

$R_Z = \dfrac{800}{40} = 20\,\Omega$

Replacing the R_2, R_4, R_5 wye with its equivalent Δ gives



Now note that $100//50 = \dfrac{5000}{150} = \dfrac{100}{3}\,\Omega$; $\quad 80//4 = \dfrac{320}{84} = \dfrac{80}{21}\,\Omega$

$\therefore\ 100//50 + 80//4 = \dfrac{100}{3} + \dfrac{80}{21} = \dfrac{780}{21}\,\Omega$

$\therefore\ \dfrac{780}{21}//20 = \dfrac{(20)(780)}{21[20 + (780/21)]} = 13\,\Omega$

$\therefore\ R_{ab} = 13 + 13 + 7 = 33\,\Omega$

[c] Convert the delta connection R_4–R_5–R_6 to its equivalent wye.
Convert the wye connection R_3–R_4–R_6 to its equivalent delta.

P 3.53 In order that all four decades (1, 10, 100, 1000) that are used to set R_3 contribute to the balance of the bridge, the ratio R_2/R_1 should be set to 0.001.

P 3.54 Subtracting Eq. 3.44 from Eq. 3.45 gives

$$R_1 - R_2 = (R_cR_b - R_cR_a)/(R_a + R_b + R_c).$$

Adding this expression to Eq. 3.43 and solving for R_1 gives

$$R_1 = R_cR_b/(R_a + R_b + R_c).$$

To find R_2, subtract Eq. 3.45 from Eq. 3.43 and add this result to Eq. 3.44. To find R_3, subtract Eq. 3.43 from Eq. 3.44 and add this result to Eq. 3.45. Using the hint, Eq. 3.45 becomes

$$R_1 + R_3 = \frac{R_b[(R_2/R_3)R_b + (R_2/R_1)R_b]}{(R_2/R_1)R_b + R_b + (R_2/R_3)R_b} = \frac{R_b(R_1 + R_3)R_2}{(R_1R_2 + R_2R_3 + R_3R_1)}$$

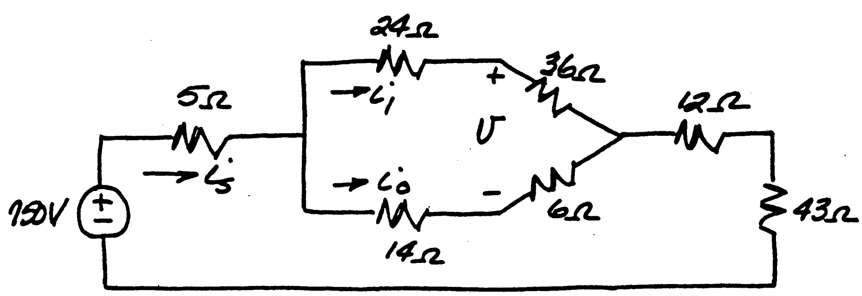
Solving for R_b gives $R_b = (R_1R_2+R_2R_3+R_3R_1)/R_2$. To find R_a: First use Eqs. 3.37–3.39 to obtain the ratios $(R_1/R_3) = (R_c/R_a)$ or $R_c = (R_1/R_3)R_a$ and $(R_1/R_2) = (R_b/R_a)$ or $R_b = (R_1/R_2)R_a$. Now use these relationships to eliminate R_b and R_c from Eq. 3.44. To find R_c, use Eqs. 3.37–3.39 to obtain the ratios $R_b = (R_3/R_2)R_c$ and $R_a = (R_3/R_1)R_c$. Now use the relationships to eliminate R_b and R_a from Eq. 3.43.

P 3.55

$$G_a = \frac{1}{R_a} = \frac{R_1}{R_1R_2 + R_2R_3 + R_3R_1}$$
$$= \frac{1/G_1}{(1/G_1)(1/G_2) + (1/G_2)(1/G_3) + (1/G_3)(1/G_1)}$$
$$= \frac{(1/G_1)(G_1G_2G_3)}{G_1 + G_2 + G_3} = \frac{G_2G_3}{G_1 + G_2 + G_3}$$

Similar manipulations generate the expressions for G_b and G_c.

P 3.56 [a] Replace the 60–120–20–Ω delta with a wye equivalent to get



$24 + 36 = 60\,\Omega$; $\quad 14 + 6 = 20\,\Omega$

$60\,\Omega // 20\,\Omega = 15\,\Omega$

$$\therefore\ i_s = \frac{750}{5 + 15 + 12 + 43} = \frac{750}{75} = 10\,\text{A}$$

$$\therefore\ i_1 = (10)\left(\frac{20}{80}\right) = 2.5\,\text{A}$$

[b] $i_o = 10\left(\frac{60}{80}\right) = 7.5\,\text{A}$

$24i_1 + v - 14i_o = 0$; $\quad \therefore\ v = 14(7.5) - 24(2.5) = 45\,\text{V}$

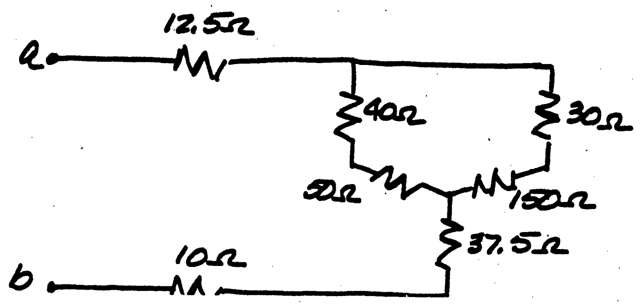
$\therefore\ i_{60\Omega} = \frac{45}{60} = 0.75\,\text{A}$; $\quad i_2 = i_o + i_{60\Omega} = 7.5 + 0.75 = 8.25\,\text{A}$

[c] $v = 45\,\text{V}$ [See calculation in part (b).]

[d] $P_{\text{supplied}} = (750)(i_s) = 7500\,\text{W} = 7.5\,\text{kW}$

P 3.57 Replace the lower delta with an equivalent wye.

$$R_1 = \frac{(100)(400)}{800} = 50\,\Omega; \qquad R_2 = \frac{(400)(300)}{800} = 150\,\Omega; \qquad R_3 = \frac{(100)(300)}{800} = 37.5\,\Omega$$

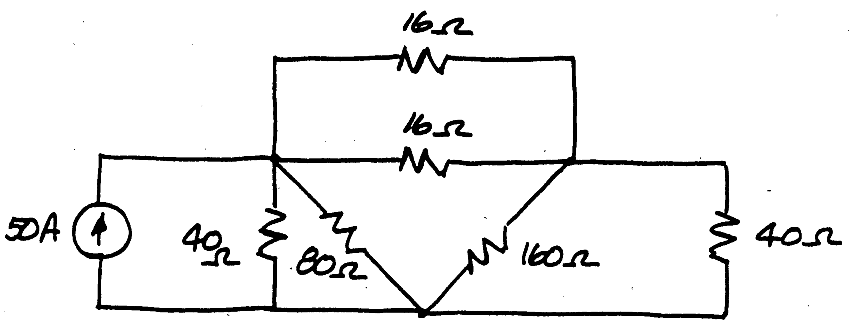


$50\,\Omega + 40\,\Omega = 90\,\Omega, \qquad 150\,\Omega + 30\,\Omega = 180\,\Omega$

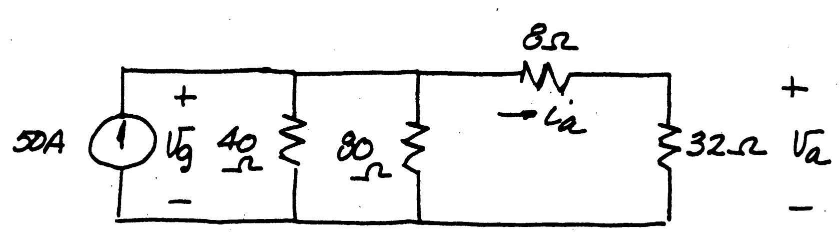
$90\,\Omega // 180\,\Omega = 60\,\Omega$

$R_{ab} = 12.5 + 60 + 37.5 + 10 = 120\,\Omega$

P 3.58 [a] After the $5\,\Omega$–$10\,\Omega$–$50\,\Omega$ wye is replaced by its equivalent delta, the circuit reduces to



Now the circuit can be reduced to



Now $40\,\Omega // 80\,\Omega // 40\,\Omega = 16\,\Omega$.

Therefore $v_g = (50)(16) = 800\text{ V}$

$i_a = \dfrac{800}{40} = 20\text{ A}$

$v_a = (32)(20) = 640\text{ V}$

It follows that $i_o = \dfrac{640}{40} = 16\text{ A}$

[b] The voltage across the 16-Ω resistor is $v_{16} = 8i_a = 160\text{ V}$

Therefore $i_1 = \dfrac{160}{16} = 10\text{ A}$

[c] It follows that the current in the 10-Ω resistor is

$$i_{10\Omega} = i_o - i_1 = 16 - 10 = 6\text{ A}$$

Therefore the voltage across the 50-Ω resistor is

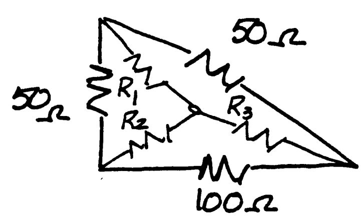
$$v_{50\Omega} = v_a + 10i_{10\Omega} = 640 + 60 = 700\text{ V}$$

$\therefore\ i_2 = \dfrac{v_{50\Omega}}{50} = 14\text{ A}$

[d] $P_{del} = (50)(800) = 40{,}000\text{ W} = 40\text{ kW}$

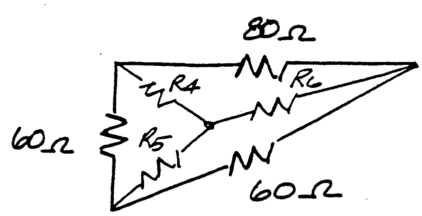
P 3.59 [a] Convert the upper delta to a wye.

$$R_1 = \frac{(50)(50)}{200} = 12.5\,\Omega$$
$$R_2 = \frac{(50)(100)}{200} = 25\,\Omega$$
$$R_3 = \frac{(50)(100)}{200} = 25\,\Omega$$

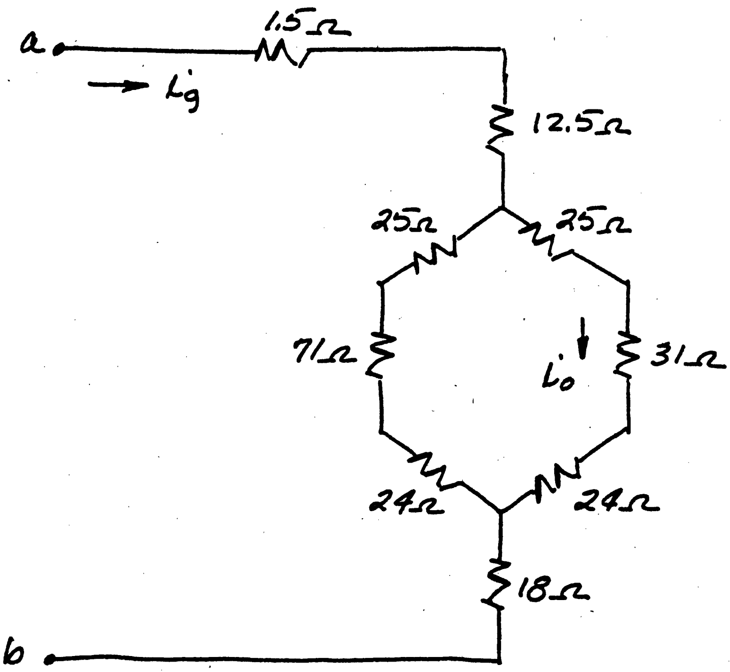


Convert the lower delta to a wye.

$$R_4 = \frac{(60)(80)}{200} = 24\,\Omega$$
$$R_5 = \frac{(60)(60)}{200} = 18\,\Omega$$
$$R_6 = \frac{(60)(80)}{200} = 24\,\Omega$$



Now redraw the circuit using the wye equivalents.



$$R_{ab} = 1.5 + 12.5 + \frac{(120)(80)}{200} + 18 = 14 + 48 + 18 = 80\,\Omega$$

[b] When $v_{ab} = 400$ V

$$i_g = \frac{400}{80} = 5\text{ A}$$

$$i_o = \frac{(5)(120)}{200} = 3\text{ A}$$

$$p_{31\Omega} = (9)(31) = 279\text{ W}$$

P 3.60 [a] At no load: $v_o = kv_s = \frac{R_2}{R_1 + R_2}v_s$.

At full load: $v_o = \alpha v_s = \frac{R_e}{R_1 + R_e}v_s$, where $R_e = \frac{R_o R_2}{R_o + R_2}$

Therefore $k = \frac{R_2}{R_1 + R_2}$ and $R_1 = \frac{(1-k)}{k}R_2$

$\alpha = \frac{R_e}{R_1 + R_e}$ and $R_1 = \frac{(1-\alpha)}{\alpha}R_e$

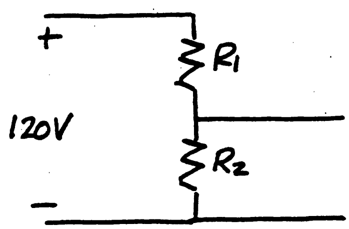
Therefore $\left(\frac{1-\alpha}{\alpha}\right)\left[\frac{R_2 R_o}{R_o + R_2}\right] = \frac{(1-k)}{k}R_2$

Solving for R_2 yields $R_2 = \frac{(k-\alpha)}{\alpha(1-k)}R_o$

[b] $R_1 = \left(\frac{0.15}{0.675}\right) R_o = \frac{2}{9}(36) = 8\,\text{k}\Omega$

$R_2 = \left(\frac{0.15}{0.075}\right) R_o = 2(36) = 72\,\text{k}\Omega$

[c]



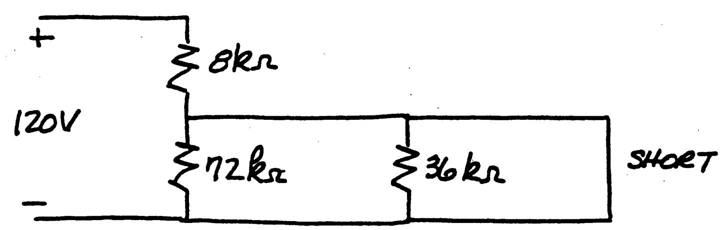
Maximum dissipation in R_2 occurs at no load, ∴

$$P_{R_2(\text{max})} = \frac{[(120)(0.9)]^2}{72{,}000} = 162\,\text{mW}$$

Maximum dissipation in R_1 occurs at full load.

$$P_{R_1(\text{max})} = \frac{[120 - 0.75(120)]^2}{8000} = 112.50\,\text{mW}$$

[d]



$$P_{R_1} = \frac{(120)^2}{8000} = 1.8\,\text{W} = 1800\,\text{mW}$$

$$P_{R_2} = \frac{(0)^2}{72{,}000} = 0\,\text{W}$$

P 3.61 [a] $R_{ab} = 2R_1 + \frac{R_2(2R_1 + R_L)}{2R_1 + R_2 + R_L} = R_L$

Therefore $2R_1 - R_L + \frac{R_2(2R_1 + R_L)}{2R_1 + R_2 + R_L} = 0$

Therefore $R_L^2 = 4R_1^2 + 4R_1R_2 = 4R_1(R_1 + R_2)$

When $R_{ab} = R_L$, the current into terminal a of the attenuator will be $\frac{v_i}{R_L}$

Using current division, the current in the R_L branch will be

$$\frac{v_i}{R_L} \cdot \frac{R_2}{2R_1 + R_2 + R_L}$$

Therefore $v_o = \frac{v_i}{R_L} \cdot \frac{R_2}{2R_1 + R_2 + R_L} R_L$ and $\frac{v_o}{v_i} = \frac{R_2}{2R_1 + R_2 + R_L}$

[b] $(600)^2 = 4(R_1 + R_2)R_1$

$9 \times 10^4 = R_1^2 + R_1R_2$

$\frac{v_o}{v_i} = 0.6 = \frac{R_2}{2R_1 + R_2 + 600}$

$\therefore\ 1.2R_1 + 0.6R_2 + 360 = R_2$

$0.4R_2 = 1.2R_1 + 360$

$R_2 = 3R_1 + 900$

$\therefore\ 9 \times 10^4 = R_1^2 + R_1(3R_1 + 900) = 4R_1^2 + 900R_1$

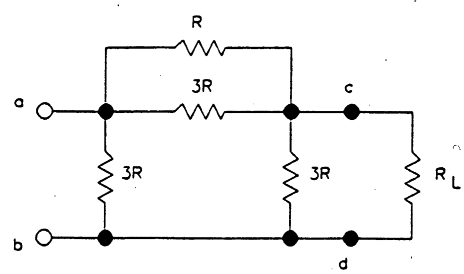
$\therefore\ R_1^2 + 225R_1 - 22{,}500 = 0$

$R_1 = -112.5 \pm \sqrt{(112.5)^2 + 22{,}500} = -112.5 \pm 187.5$

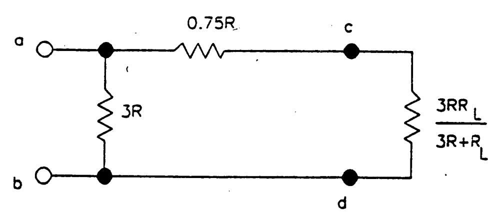
$\therefore\ R_1 = 75\,\Omega$

$\therefore\ R_2 = 3(75) + 900 = 1125\,\Omega$

P3.62 [a] After making the Y-to-Δ transformation, the circuit in Fig. P3.62 reduces to



Combining the parallel resistors reduces the circuit to



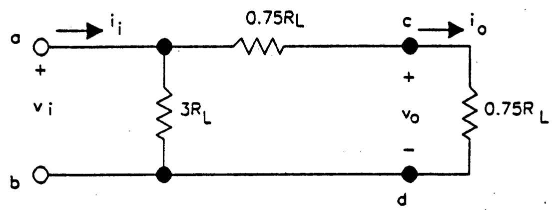
Now note: $0.75R + \frac{3RR_L}{3R + R_L} = \frac{2.25R_L^2 + 3.75RR_L}{3R + R_L}$

Therefore $R_{ab} = \frac{3R\left(\frac{2.25R_L^2 + 3.75RR_L}{3R + R_L}\right)}{3R + \left(\frac{2.25R_L^2 + 3.75RR_L}{3R + R_L}\right)} = \frac{3R(3R + 5R_L)}{15R + 9R_L}$

When $R_{ab} = R_L$, we have $15RR_L + 9R_L^2 = 9R^2 + 15RR_L$

Therefore $R_L^2 = R^2$ or $R_L = R$

[b] When $R = R_L$, the circuit reduces to



$$i_o = \frac{i(3R_L)}{4.5R_L} = \frac{1}{1.5}i = \frac{1}{1.5}\frac{v_i}{R_L}, \qquad v_o = 0.75R_L i_o = \frac{1}{2}v_i,$$

Therefore $\frac{v_o}{v_i} = 0.5$

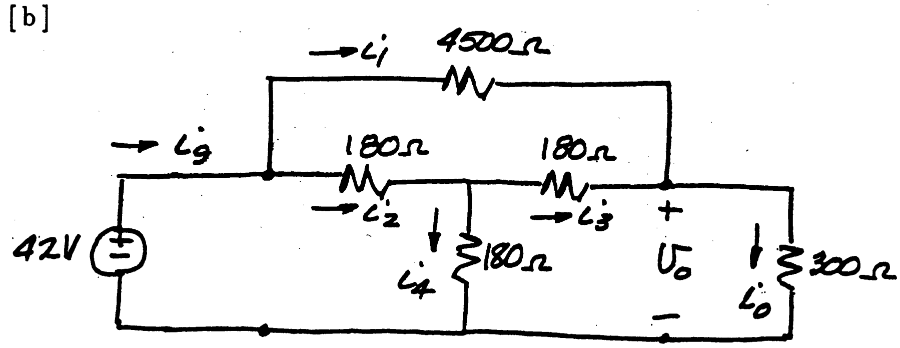
P 3.63 [a] $3.5(3R - R_L) = 3R + R_L$

$10.5R - 1050 = 3R + 300$

$7.5R = 1350, \qquad R = 180\,\Omega$

$$R_2 = \frac{2(180)(300)^2}{3(180)^2 - (300)^2} = 4500\,\Omega$$

[b]



$$v_o = \frac{v_i}{3.5} = \frac{42}{3.5} = 12\,\text{V}$$

$$i_o = \frac{12}{300} = 40\,\text{mA}$$

$$i_1 = \frac{42 - 12}{4500} = \frac{30}{4500} = 6.67\,\text{mA}$$

$$i_g = \frac{42}{300} = 140\,\text{mA}$$

$i_2 = 140 - 6.67 = 133.33\,\text{mA}$

$i_3 = 40 - 6.67 = 33.33\,\text{mA}$

$i_4 = 133.33 - 33.33 = 100\,\text{mA}$

$p_{4500\Omega} = (6.67 \times 10^{-3})^2(4500) = 0.20\,\text{W}$

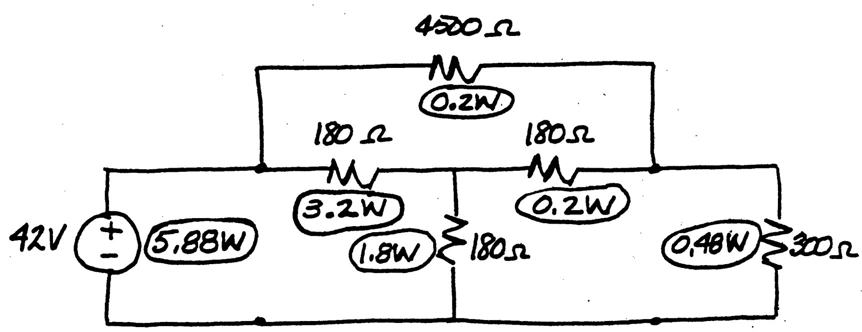
$p_{180\Omega\ \text{left}} = (133.33 \times 10^{-3})^2(180) = 3.20\,\text{W}$

$p_{180\Omega\ \text{right}} = (33.33 \times 10^{-3})^2(180) = 0.20\,\text{W}$
$p_{180\Omega\ \text{vertical}} = (100 \times 10^{-3})^2(180) = 1.8\,\text{W}$
$p_{300\Omega} = (40 \times 10^{-3})^2(300) = 0.48\,\text{W}$
The 180-Ω resistor carrying i_2

[c] $p_{180\Omega\ \text{left}} = 3.20\,\text{W}$

[d] Two resistors dissipate minimum power, the 4500-Ω and the 180-Ω carrying i_3.

[e] Both resistors dissipate 0.20 W or 200 mW



Check:

$$\sum P_{\text{diss}} = 0.2 + 3.2 + 1.8 + 0.2 + 0.48 = 5.88\,\text{W}$$
$$\sum P_{\text{dev}} = 42(0.14) = 5.88\,\text{W}$$

4

Techniques of Circuit Analysis

Drill Exercises

DE 4.1 [a] 11,8 resistors, 2 independent sources, 1 dependent source

[b] 9

[c] 9, $R_4 - R_5$ forms an essential branch as does $R_8 - 10\,\text{V}$. The remaining seven branches contain a single element.

[d] 7 [e] 6 [f] 4 [g] 6

DE 4.2 Solution given in text.

DE 4.3 Solution given in text.

DE 4.4 Solution given in text.

DE 4.5 [a] The two-node voltage equations are

$$\left.\begin{aligned} 5 + \frac{v_1}{16} + \frac{v_1 - v_2}{2} &= 0 \\ \frac{v_2 - v_1}{2} + \frac{v_2}{20} + \frac{v_2}{80} - 12 &= 0 \end{aligned}\right\} \quad \begin{aligned} v_1 &= 48\,\text{V} \\ v_2 &= 64\,\text{V} \end{aligned}$$

$$i_1 = \frac{v_1 - v_2}{2} = -8\,\text{A}$$

[b] $p_{12_A}(\text{del}) = (12)(64) = 768\,\text{W}$

[c] $p_{5A} = 5(48) = 240\,\text{W}$, therefore 5-A source delivers $-240\,\text{W}$ to the circuit.

DE 4.6 Use the lower node as the reference node. Let v_1 = node voltage across 1-Ω resistor and v_2 = node voltage across 12-Ω resistor. Then

$$\left.\begin{aligned} &\frac{v_1}{1} + \frac{v_1 - v_2}{8} = 4.5 \\ &\frac{v_2}{12} + \frac{v_2 - v_1}{8} + \frac{v_2 - 30}{4} = 0 \end{aligned}\right\} \quad \begin{aligned} &v_1 = 6\,\text{V} \\ &v_2 = 18\,\text{V} \\ &i = (v_1 - v_2)/8 = -1.5\,\text{A} \\ &v = v_2 + 2i = 15\,\text{V} \end{aligned}$$

DE 4.7 Use the lower node as the reference node. Let v_1 = node voltage across the 4-Ω resistor, let v_2 = node voltage across the 20-Ω resistor. Then

$$\left.\begin{aligned} &\frac{v_1}{4} + \frac{v_1 - v_2}{2} + 6i_2 = 1.5 \\ &\frac{v_2}{20} + \frac{v_2 - v_1}{2} - 6i_2 + \frac{v_2 - 80}{5} = 0 \\ &i_2 = \frac{80 - v_2}{5} \end{aligned}\right\} \quad \begin{aligned} &v_1 = 10\,\text{V} \\ &v_2 = 60\,\text{V} \\ &i_2 = 4\,\text{A} \end{aligned}$$

$p_{15A} = -1.5v_1 = -15\,\text{W}$ (delivering)
$p_{80V} = -4(80) = -320\,\text{W}$ (delivering)
$p_{6i_2} = -6i_2(v_2 - v_1) = -1200\,\text{W}$ (delivering)

DE 4.8 Use the lower node as the reference node. Let v_1 = node voltage across the 7.5-Ω resistor and v_2 = node voltage across the 2.5-Ω resistor. Place the dependent voltage source inside a supernode between the node voltages v and v_2. The node voltage equations are

$$\text{node 1:}\quad \frac{v_1}{7.5} + \frac{v_1 - v}{2.5} = 4.8$$

$$\text{supernode:}\quad \frac{v - v_1}{2.5} + \frac{v}{10} + \frac{v_2}{2.5} + \frac{v_2 - 12}{1} = 0$$

We also have: $v + i_x = v_2$ and $i_x = v_1/7.5$. Solving this set of equations for v gives $v = 8\,\text{V}$.

DE 4.9 $$\frac{v_1 - 150}{25} + \frac{v_1}{80} + \frac{v_1 - (150 - 15i_\phi)}{40} = 0, \qquad i_\phi = \frac{150 - v_1}{25}$$

Therefore $v_1 = 120\,\text{V}$

DE 4.10 $$\frac{v_o}{40} + \frac{v_o - 10}{10} + \frac{v_o + 20i_\Delta}{20} = 0, \qquad i_\Delta = \frac{10 - v_o}{10} + \frac{10 + 20i_\Delta}{30}$$

Therefore $v_o = 24\,\text{V}$

DE 4.11 Define three clockwise mesh currents i_1, i_2, and i_3 in the lower left, upper, and lower right windows. The three mesh-current equations are

$$100 = 45i_1 - 5i_2 - 40i_3$$
$$0 = -5i_1 + 35i_2 - 10i_3$$
$$0 = -40i_1 - 10i_2 + 65i_3$$

[a] Solve for i_1 and get $i_1 = 6\,\text{A}$, therefore 100-V source is delivering 600 W to the circuit.

[b] Solve for i_3 and get $i_3 = 4\,\text{A}$, therefore $p_{15\Omega} = (16)(15) = 240\,\text{W}$.

DE 4.12 [a] $b = 8, \quad n = 6, \quad b - n + 1 = 3$

[b] $b_e = 6, \quad n_e = 4, \quad b_e - n_e + 1 = 3$

[c] Define three clockwise mesh currents i_1, i_2, and i_3 in the upper, lower left, and lower right windows. The three mesh-current equations are

$$-(-3v_\phi) + 19i_1 - 2i_2 - 3i_3 = 0$$
$$25 - 10 = -2i_1 + 7i_2 - 5i_3$$
$$10 = -3i_1 - 5i_2 + 9i_3$$

We also have $v_\phi = 3(i_3 - i_1)$

Solving for i_1 and i_3 gives $i_1 = -1\,\text{A}, \quad i_3 = 3\,\text{A}$

Therefore $v_\phi = 12\,\text{V}$ and $p_{3v_\phi} = -(-3v_\phi)i_1 = -36\,\text{W}$

DE 4.13 Let i_a = lower left mesh current cw, let i_b = upper mesh cw current, and i_c = lower right cw mesh current. Then

$$15 = 10(i_a - i_b) + 40(i_a - i_c)$$
$$0 = 50i_b + 20(i_b - i_c) + 10(i_b - i_a)$$
$$0 = 40(i_c - i_a) + 20(i_c - i_b) - 40i_\phi$$
$$i_\phi = i_a, \quad i_a = -1\,\text{A}, \quad i_c = -1.5\,\text{A}$$
$$v_o = 40(i_a - i_c) = 20\,\text{V}$$

DE 4.14 Define two mesh currents i_1 and i_2 (clockwise) in the left and center windows of the circuit. Then $30 = 11i_1 - 2i_2$ and $0 = -2i_1 + 19i_2 - 5(-16)$. Solving for i_1 and i_2 gives $i_1 = 2\,\text{A}$ and $i_2 = -4\,\text{A}$. The current in the 2-Ω resistor is $i_1 - i_2 = 6\,\text{A}$, therefore $p_{2\Omega} = (6)^2(2) = 72\,\text{W}$.

DE 4.15 $20 = (i_a - 30)1 + 2\left(i_a + \frac{v_\phi}{4}\right), \quad v_\phi = 2\left(i_a + \frac{v_\phi}{4}\right)$

Therefore $v_\phi = 4i_a, \quad i_a = 10\,\text{A}$

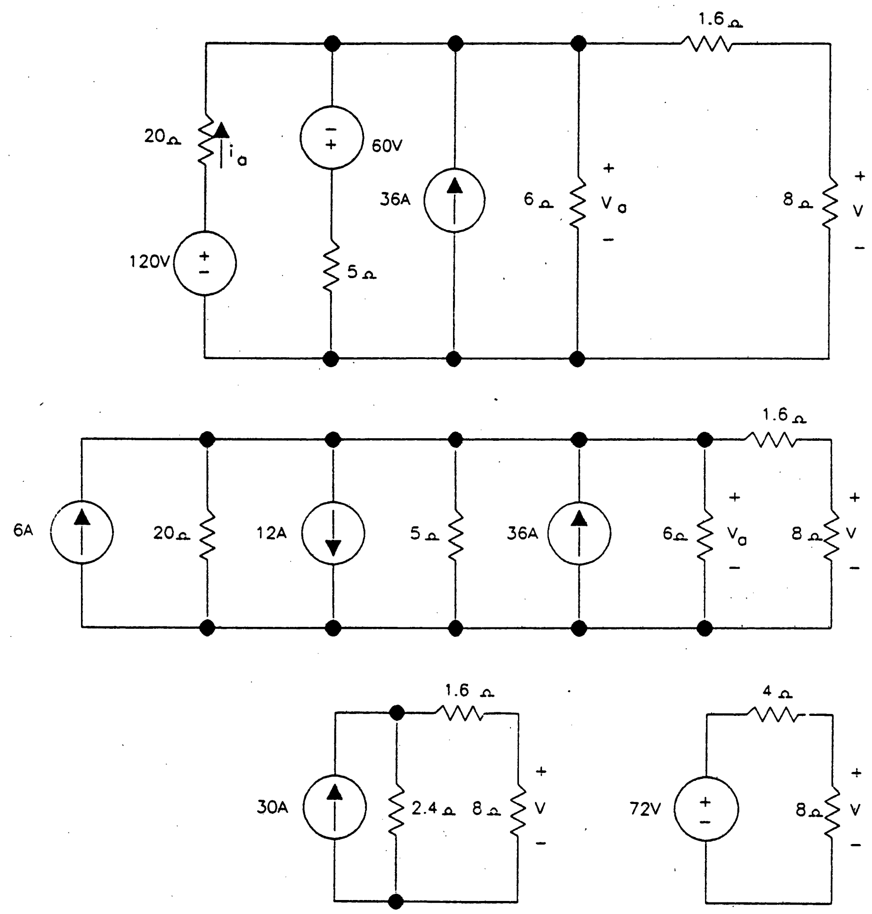
DE 4.16 Let v_1 denote the voltage across the 2-A source. Let v_1 be a voltage rise in the direction of the 2-A current.

$$\frac{v_1 - 20}{15} - 2 + \frac{v_1 - 25}{10} = 0, \qquad v_1 = 35\,\text{V}$$

$$p_{2A} = -35(2) = -70\,\text{W}, \qquad p_{2A}(\text{del}) = 70\,\text{W}$$

DE 4.17 Define two clockwise mesh currents i_1 and i_2 in the lower left and lower right windows. The two mesh equations are $128 = 12i_1 - 6i_2 - 4(4)$ and $0 = -6i_1 + 14i_2 - 3(4) + 30(i_1 - 4)$. The solutions for i_1 and i_2 are $i_1 = 9$ A and $i_2 = -6$ A. The voltage drop across the 4-A source in the direction of the source current is $v_{4A} = 4(i_1 - 4) + 3(i_2 - 4) = 20 - 30 = -10$ V, therefore $p_{4A} = 4(-10) = -40$ W, therefore $p_{4A}(\text{del}) = 40$ W.

DE 4.18

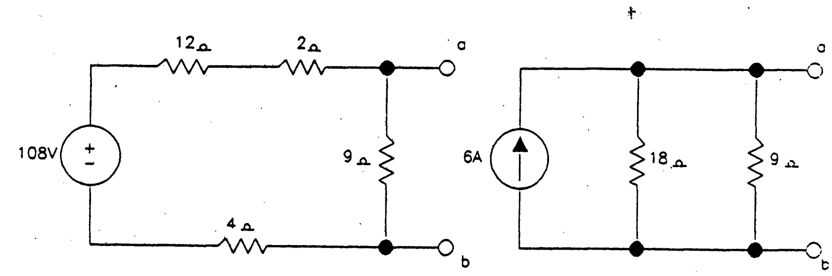


[a] $v = \frac{72}{12}(8) = 48\,\text{V}, \qquad i_a = \frac{120 - 57.6}{20} = 3.12\,\text{A}$

[b] $v_a = 6(9.6) = 57.6\,\text{V}, \qquad p_{120V}(\text{del}) = 120 i_a = 374.40\,\text{W}$

DE 4.19 To find the Thévenin resistance, deactivate the independent voltage source and note that $R_{\rm Th} = 12//[8 + 20//5] = 12//12 = 6\,\Omega$. With the terminals a, b open, the current delivered by the 72-V source is 72/24 or 3 A. The current (left-to-right) in the 5-Ω resistor is $(20/25)(3) = 2.4$ A, and the current (left-to-right) in the 12-Ω resistor is $(5/25)3$ or 0.6 A. The Thévenin voltage $v_{\rm Th} = v_{ab}$ is the drop across the 8-Ω resistor plus the drop across the 20-Ω resistor. Thus $v_{\rm Th} = (8)(0.6) + (20)(3) = 64.8$ V.

DE 4.20 After one source transformation, the circuit becomes



Therefore $I_N = 6\,{\rm A}, \qquad R_N = 18//9 = 6\,\Omega$

DE 4.21 Find the Thévenin equivalent with respect to A, B.

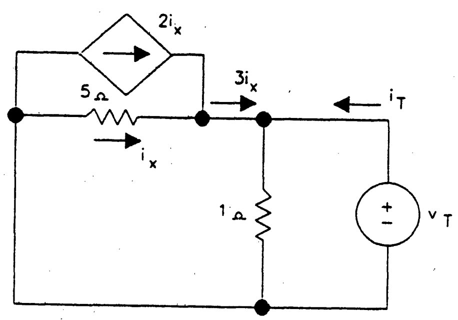
$$\frac{V_{\rm Th} + 36}{12} + \frac{V_{\rm Th}}{60} - 18 = 0, \qquad V_{\rm Th} = 150\,{\rm V}$$

$$R_{\rm Th} = 15 + \frac{(60)(12)}{72} = 25\,{\rm k}\Omega, \qquad \text{therefore} \quad v_{\rm meas} = \frac{150}{125}(100) = 120\,{\rm V}$$

DE 4.22 Summing the currents away from node a, where $v_{\rm Th} = v_{ab}$, we have

$$\frac{v_{\rm Th}}{1} - 8 - 2i_x + \frac{v_{\rm Th} - 40}{5} = 0, \qquad i_x = \frac{40 - v_{\rm Th}}{5}$$

Solving for $v_{\rm Th}$ yields $v_{\rm Th} = 20$ V.



$$v_T = 3i_x + i_T, \qquad i_x = -v_T/5$$

Therefore $i_T = 1.6(v_T)$ and $R_{Th} = v_T/i_T = 0.625\,\Omega$

DE 4.23 Use the bottom node as the reference. Let v_1 be the node voltage across the 60-Ω resistor. Then

$$\frac{v_1}{60} + \frac{v_1 - (v_{Th} + 160i_\Delta)}{20} - 4 = 0, \qquad \frac{v_{Th}}{40} + \frac{v_{Th}}{80} + \frac{v_{Th} + 160i_\Delta - v_1}{20} = 0$$

$$i_\Delta = \frac{v_{Th}}{40}, \qquad \text{therefore} \quad v_{Th} = 30\,\text{V}$$

Let i_T be the test current into terminal a:

$$i_T = \frac{v_T}{80} + \frac{v_T}{40} + \frac{v_T + 160(v_T/40)}{80}, \qquad \frac{i_T}{v_T} = \frac{1}{10}; \qquad \text{therefore} \quad R_{Th} = 10\,\Omega$$

DE 4.24 First find the Thévenin equivalent circuit. To find v_{Th}, use the bottom node as the reference. Let $v_{Th} = v_{ab}$ and v_1 = node voltage across the 20-V − 4-Ω branch. The two node-voltage equations are

$$\frac{v_{Th} - 100 - v_\phi}{4} + \frac{v_{Th} - v_1}{4} = 0, \qquad (v_\phi = v_1 - 20)$$

$$\frac{v_1 - 100}{4} - \frac{v_1 - 20}{4} + \frac{v_1 - v_{Th}}{4} = 0$$

Solving for v_{Th} gives $v_{Th} = 120\,\text{V}$. To find R_{Th}, deactivate the two independent sources and apply a test voltage source across a, b. Let v_T be positive at a and i_T directed into a. Then the two node-voltage equations are

$$\frac{v_T - v_\phi}{4} + \frac{v_{Th} - v_\phi}{4} = i_T, \qquad \frac{v_\phi}{4} + \frac{v_\phi}{4} + \frac{v_\phi - v_T}{4} = 0$$

Therefore $v_\phi = v_T/3$ and $12i_T = 4v_T$

Therefore $R_{Th} = v_T/i_T = 3\,\Omega$

[a] For maximum power transfer, $R_L = R_{Th} = 3\,\Omega$

[b] $p_{max} = (120/6)^2(3) = 1200\,\text{W}$

DE 4.25 When $R_L = 3\,\Omega$, the voltage across R_L is 60 V. As before, let v_1 be the node voltage across the 20 − V = 4 − Ω branch, then $v_\phi = v_1 - 20$ and

$$\frac{60}{3} + \frac{60 - v_1}{4} + \frac{60 - 100 - v_\phi}{4} = 0$$

Therefore $v_1 = 60\,\text{V}$ and $v_\phi = 40\,\text{V}$. The current out of the plus terminal of the 100-V source is

$$i_1 = \frac{100 - 60}{4} + \frac{100 + 40 - 60}{4} = 10 + 20 = 30\,\text{A}$$

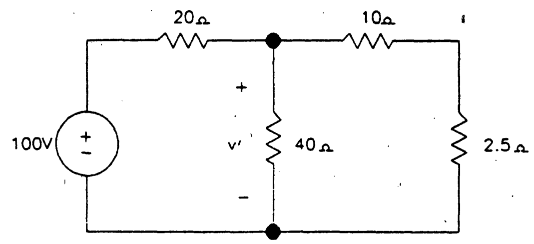
[a] Therefore 100 V is delivering 3000 W to the circuit.

[b] The current out of the plus terminal of the dependent source is 20 A. Therefore the dependent source is delivering 800 W to the circuit.

[c] The load power is (1200/3800)100 or 31.58% of this generated power.

DE 4.26 100-V source acting alone:

$$\frac{v'}{40} + \frac{v' - 100}{20} + \frac{v'}{12.5} = 0, \qquad v' = \frac{100}{3.1} = 32.26\,\text{V}$$



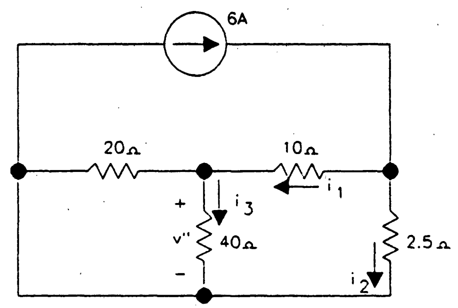
6-A source acting alone:

$$20//40 + 10 = \frac{70}{3}\,\Omega$$

$$\text{Therefore} \quad i_1 = \frac{2.5(6)}{2.5 + 70/3} = \frac{18(2.5)}{77.5}$$

$$i_3 = \frac{20}{60} i_1 = \frac{15}{77.5}$$

$$v'' = 40 i_3 = \frac{600}{77.5} = 7.74\,\text{V}$$



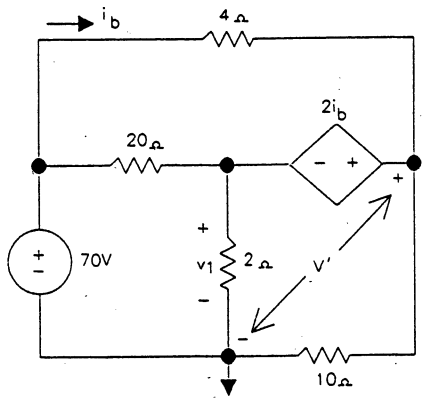
[a] $v = v' = v'' = 32.26 + 7.74 = 40\,\text{V}$

[b] $p_{40\Omega} = \frac{(40)^2}{40} = 40\,\text{W}$

DE 4.27 70-V source acting alone:

$$\frac{v'}{10} + \frac{v' - 70}{4} + \frac{v_1' - 70}{20} + \frac{v_1'}{2} = 0, \qquad i_b = \frac{70 - v'}{4}, \qquad v_1' + 2i_b' = v'$$

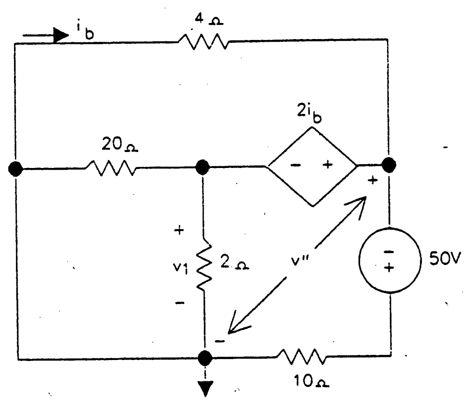
$$\text{Therefore} \quad v' = \frac{805}{23.5} = 34.26\,\text{V}$$



50-V source acting alone:

$$\frac{v_1''}{2} + \frac{v_1''}{20} + \frac{v''}{4} + \frac{v'' + 50}{10} = 0, \qquad v'' = v_1'' + 2i_b'', \qquad i_b'' = \frac{-v''}{4}$$

$$\text{Therefore} \quad v'' = \frac{-100}{23.5} = -4.6\,\text{V} \quad \text{and} \quad v = v' + v'' = 34.26 - 4.26 = 30\,\text{V}$$



Problems

P 4.1 [a] Five.

[b] Three.

[c] Sum the currents at any three of the four essential nodes a, b, c, and d. Using nodes a, b, and c we get

$$-i_g + i_1 + i_2 = 0$$
$$-i_1 - i_4 + i_3 = 0$$
$$i_5 - i_2 - i_3 = 0$$

[d] Two.

[e] Sum the voltages around two independent closed paths, avoiding a path that contains the independent current source since the voltage across the current source is not known. Using the upper and lower meshes formed by the five resistors gives

$$R_1i_1 + R_3i_3 - R_2i_2 = 0$$
$$R_3i_3 + R_5i_5 + R_4i_4 = 0$$

P 4.2 Use the lower terminal of the 25-Ω resistor as the reference node.

$$\frac{v_o}{25} + 0.04 + \frac{v_o + 25}{125} = 0$$
$$5v_o + 5 + v_o + 25 = 0$$
$$6v_o = -30$$
$$v_o = -5\text{ V}$$

P 4.3 $$\frac{v_1}{40} + \frac{v_1 - v_2}{8} = 6$$
$$\boxed{6v_1 - 5v_2 = 240}$$
$$\frac{v_2 - v_1}{8} + \frac{v_2}{80} + \frac{v_2}{120} + 1 = 0$$
$$\boxed{-30v_1 + 35v_2 = -240}$$

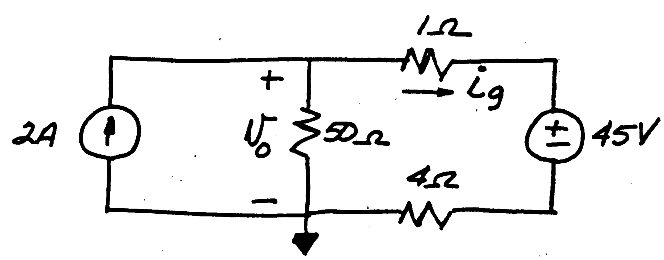
$$\Delta = \begin{vmatrix} 6 & -5 \\ -30 & 35 \end{vmatrix} = 210 - 150 = 60$$

$$N_1 = \begin{vmatrix} 240 & -5 \\ -240 & 35 \end{vmatrix} = 240(35 - 5) = (240)(30)$$

$$N_2 = \begin{vmatrix} 6 & 240 \\ -30 & -240 \end{vmatrix} = 240(-6 + 30) = 240(24)$$

$$v_1 = \frac{N_1}{\Delta} = 120\text{ V}; \qquad v_2 = \frac{N_2}{\Delta} = 96\text{ V}$$

P 4.4



$$\frac{v_o}{50} + \frac{v_o - 45}{5} = 2$$
$$11v_o = 100 + 450 = 550$$

$v_o = 50\,\text{V}$

$i_g = \dfrac{v_o - 45}{5} = 1\,\text{A}$

$p_{45\text{V}} = (45)(1) = 45\,\text{W}$ (absorbing)

P 4.5 $\dfrac{v_1}{10} + \dfrac{v_1 - 144}{4} + \dfrac{v_1 - v_2}{80} = 0$

$\dfrac{v_2 - v_1}{80} - 3 + \dfrac{v_2}{5} = 0$

$\therefore\ 29v_1 - v_2 = 2880; \qquad -v_1 + 17v_2 = 240$

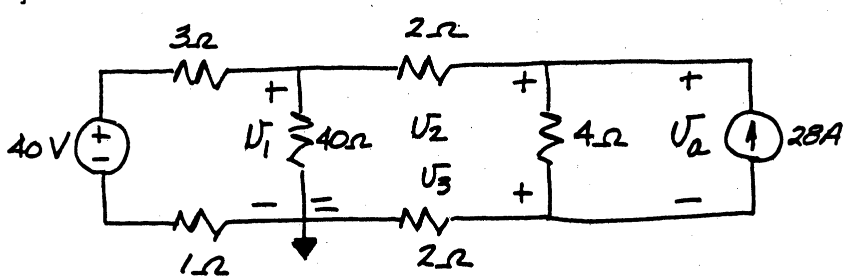
$$\Delta = \begin{vmatrix} 29 & -1 \\ -1 & 17 \end{vmatrix} = 492$$

$$N_1 = \begin{vmatrix} 2880 & -1 \\ 240 & 17 \end{vmatrix} = 49{,}200$$

$$N_2 = \begin{vmatrix} 29 & 2880 \\ -1 & 240 \end{vmatrix} = 9840$$

$v_1 = (N_1/\Delta) = 100\,\text{V}; \qquad v_2 = (N_2/\Delta) = 20\,\text{V}$

P 4.6 [a]



$\dfrac{v_1}{40} + \dfrac{v_1 - 40}{4} + \dfrac{v_1 - v_2}{2} = 0$

$v_1 + 10v_1 + 20v_1 - 20v_2 = 400$

$\boxed{31v_1 - 20v_2 + 0v_3 = 400}$

$\dfrac{v_2 - v_1}{2} + \dfrac{v_2 - v_3}{4} - 28 = 0$

$\boxed{-2v_1 + 3v_2 - v_3 = 112}$

$\dfrac{v_3}{2} + \dfrac{v_3 - v_2}{4} + 28 = 0$

$\boxed{0v_1 - v_2 + 3v_3 = -112}$

$$\Delta = \begin{vmatrix} 31 & -20 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 3 \end{vmatrix} = 31(9-1) + 2(-60) = 128$$

$$N_1 = \begin{vmatrix} 400 & -20 & 0 \\ 112 & 3 & -1 \\ -112 & -1 & 3 \end{vmatrix} = 1(-400 - 2240) + 3(1200 + 2240) = 7680$$

$$N_2 = \begin{vmatrix} 31 & 400 & 0 \\ -2 & 112 & -1 \\ 0 & -112 & 3 \end{vmatrix} = 31(336 - 112) + 2(1200) = 9344$$

$$N_3 = \begin{vmatrix} 31 & -20 & 400 \\ -2 & 3 & 112 \\ 0 & -1 & -112 \end{vmatrix} = 31(-336 + 112) + 2(2640) = -1664$$

$$v_1 = \frac{N_1}{\Delta} = \frac{7680}{128} = 60\,\text{V}$$

$$v_2 = \frac{N_2}{\Delta} = \frac{9344}{128} = 73\,\text{V}$$

$$v_3 = \frac{N_3}{\Delta} = \frac{-1664}{128} = -13\,\text{V}$$

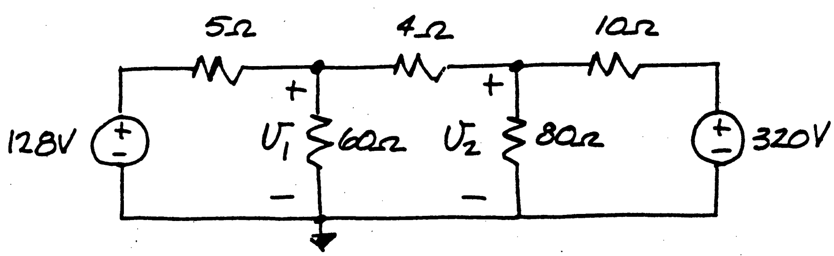
[b] $v_2 = v_a + v_3$

$v_a = v_2 - v_3 = 73 - (-13) = 86\,\text{V}$

$p_{28\text{A}} = v_a(28) = (86)(28)$

$p_{28\text{A}} = 2408\,\text{W}$

P 4.7 [a]



$$\frac{v_1}{60} + \frac{v_1 - 128}{5} + \frac{v_1 - v_2}{4} = 0$$

$$\boxed{28v_1 - 15v_2 = 1536}$$

$$\frac{v_2}{80} + \frac{v_2 - v_1}{4} + \frac{v_2 - 320}{10} = 10$$

$$\boxed{-20v_1 + 29v_2 = 2560}$$

$$\Delta = \begin{vmatrix} 28 & -15 \\ -20 & 29 \end{vmatrix} = 512$$

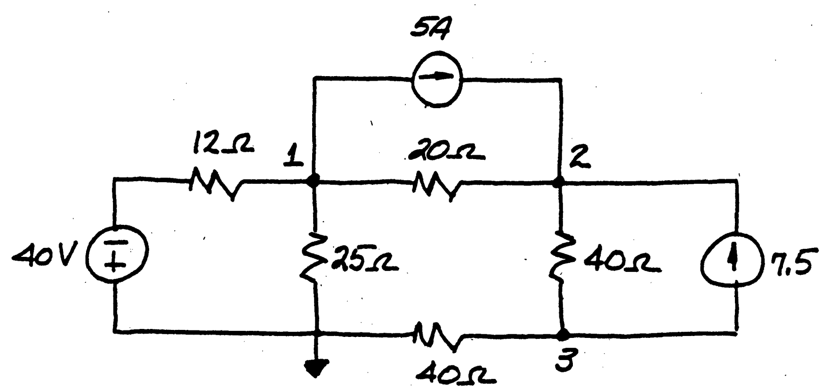
$$N_1 = \begin{vmatrix} 1536 & -15 \\ 2560 & 29 \end{vmatrix} = 82{,}944$$

$$N_2 = \begin{vmatrix} 28 & 1536 \\ -20 & 2560 \end{vmatrix} = 102{,}400$$

$$v_1 = \frac{N_1}{\Delta} = \frac{82{,}944}{512} = 162\,\text{V}$$
$$v_2 = \frac{N_2}{\Delta} = \frac{102{,}400}{512} = 200\,\text{V}$$
$$i_a = \frac{128 - 162}{5} = -6.8\,\text{A}$$
$$i_b = \frac{162}{60} = 2.7\,\text{A}$$
$$i_c = \frac{162 - 200}{4} = -9.5\,\text{A}$$
$$i_d = \frac{200}{80} = 2.5\,\text{A}$$
$$i_e = \frac{200 - 320}{10} = -12\,\text{A}$$

[b] The only source developing power is the 320-V independent voltage source. Hence the total power developed is $P_{\text{dev}} = (320)(12) = 3840\,\text{W}$.

P 4.8



$$\frac{v_1 + 40}{12} + \frac{v_1}{25} + \frac{v_1 - v_2}{20} + 5 = 0$$
$$25v_1 + 1000 + 12v_1 + 15v_1 - 15v_2 + 1500 = 0$$
$$\boxed{52v_1 - 15v_2 + 0v_3 = -2500}$$
$$\frac{v_2 - v_1}{20} + \frac{v_2 - v_3}{40} - 5 - 7.5 = 0$$
$$\boxed{-2v_1 + 3v_2 - v_3 = 500}$$
$$\frac{v_3}{40} + \frac{v_3 - v_2}{40} + 7.5 = 0$$
$$\boxed{0v_1 - v_2 + 2v_3 = -300}$$

$$\Delta = \begin{vmatrix} 52 & -15 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 2 \end{vmatrix} = 52(6-1) + 2(-30) = 200$$

$$N_1 = \begin{vmatrix} -2500 & -15 & 0 \\ 500 & 3 & -1 \\ -300 & -1 & 2 \end{vmatrix} = 1(2500 - 4500) + 2(-7500 + 7500) = -2000$$

$$N_2 = \begin{vmatrix} 52 & -2500 & 0 \\ -2 & 500 & -1 \\ 0 & -300 & 2 \end{vmatrix} = 52(1000 - 300) + 2(-5000) = 26{,}400$$

$$N_3 = \begin{vmatrix} 52 & -15 & -2500 \\ -2 & 3 & 500 \\ 0 & -1 & -300 \end{vmatrix} = 52(-900 + 500) + 2(4500 - 2500)$$

$$= 52(-400) + 4000 = -20{,}800 + 4000 = -16,800$$

$$v_1 = \frac{N_1}{\Delta} = \frac{-2000}{200} = -10\,\text{V}$$
$$v_2 = \frac{N_2}{\Delta} = \frac{26{,}400}{200} = 132\,\text{V}$$
$$v_3 = \frac{N_3}{\Delta} = \frac{-16{,}800}{200} = -84\,\text{V}$$

Now observe all three independent sources are developing power.

The current into the negative terminal of the voltage source is

$$i_g = \frac{-10 + 40}{12} = 2.5\,\text{A}$$
$$p_{40\text{V}} = -(2.5)(40) = -100\,\text{W} \quad \text{(dev)}$$

The voltage rise across the 5-A source is

$$v_{5\text{A}} = 132 - (-10) = 142\,\text{V}$$
$$p_{5\text{A}} = -5(142) = -710\,\text{W} \quad \text{(dev)}$$

The voltage rise across the 7.5-A source is

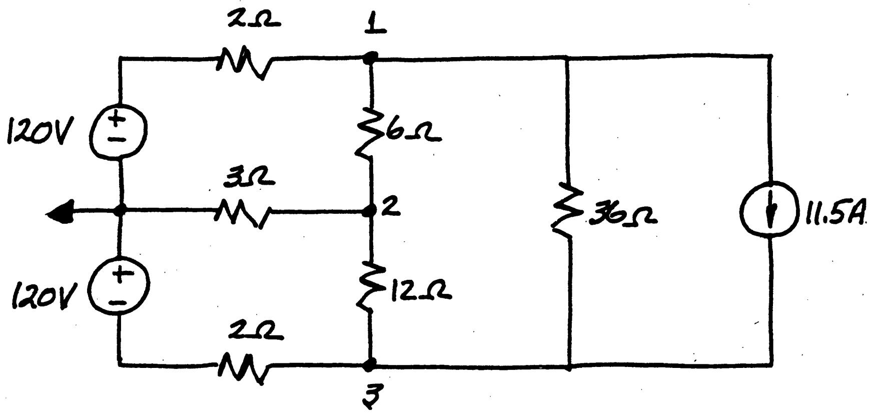
$$v_{7.5\text{A}} = 132 - (-84) = 216\,\text{V}$$
$$p_{7.5\text{A}} = -7.5(216) = -1620\,\text{W} \quad \text{(dev)}$$

$$\sum P_{\text{dev}} = \sum P_{\text{diss}} = 100 + 710 + 1620 = 2430\,\text{W}$$

Check:

$$p_{12\Omega} = (2.5)^2(12) = 75\,\text{W}$$
$$p_{25\Omega} = \frac{(10)^2}{25} = 4\,\text{W}$$
$$p_{20\Omega} = \frac{(142)^2}{20} = 1008.2\,\text{W}$$
$$p_{40\Omega} = \frac{(216)^2}{40} = 1166.4\,\text{W}$$
$$p_{40\Omega} = \frac{(84)^2}{40} = 176.4\,\text{W}$$
$$\sum P_{\text{diss}} = 2430\,\text{W}$$

P 4.9 [a]



$$\frac{v_1 - 120}{2} + \frac{v_1 - v_2}{6} + \frac{v_1 - v_3}{36} + 11.5 = 0$$
$$18v_1 - 2160 + 6v_1 - 6v_2 + v_1 - v_3 + 414 = 0$$
$$25v_1 - 6v_2 - v_3 = 1746$$
$$\frac{v_2 - v_1}{6} + \frac{v_2}{3} + \frac{v_2 - v_3}{12} = 0$$
$$2v_2 - 2v_1 + 4v_2 + v_2 - v_3 = 0$$
$$-2v_1 + 7v_2 - v_3 = 0$$
$$\frac{v_3 + 120}{2} + \frac{v_3 - v_2}{12} + \frac{v_3 - v_1}{36} - 11.5 = 0$$
$$18v_3 + 2160 + 3v_3 - 3v_2 + v_3 - v_1 - 414 = 0$$
$$-v_1 - 3v_2 + 22v_3 = -1746$$

$$\Delta = \begin{vmatrix} 25 & -6 & -1 \\ -2 & 7 & -1 \\ -1 & -3 & 22 \end{vmatrix} = 25(151) + 2(-135) - 1(13) = 3492$$

$$N_1 = \begin{vmatrix} 1746 & -6 & -1 \\ 0 & 7 & -1 \\ -1746 & -3 & 22 \end{vmatrix} = 1746(151) - 1746(13) = 240{,}948$$

$$N_2 = \begin{vmatrix} 25 & 1746 & -1 \\ -2 & 0 & -1 \\ -1 & -1746 & 22 \end{vmatrix} = -1746(-45) + 1746(-27) = 31{,}428$$

$$N_3 = \begin{vmatrix} 25 & -6 & 1746 \\ -2 & 7 & 0 \\ -1 & -3 & -1746 \end{vmatrix} = 1746(13) - 1746(163) = -261{,}900$$

$$v_1 = \frac{N_1}{\Delta} = \frac{240{,}948}{3492} = 69\text{ V}$$
$$v_2 = \frac{N_2}{\Delta} = \frac{31{,}428}{3492} = 9\text{ V}$$
$$v_3 = \frac{N_3}{\Delta} = \frac{-261{,}900}{3492} = -75\text{ V}$$

$$i_1 = \frac{120 - v_1}{2} = \frac{120 - 69}{2} = 25.50\,\text{A} \qquad i_4 = \frac{v_1 - v_2}{6} = \frac{69 - 9}{6} = 10\,\text{A}$$

$$i_2 = \frac{v_2}{3} = \frac{9}{3} = 3\,\text{A} \qquad i_5 = \frac{v_2 - v_3}{12} = \frac{9 + 75}{12} = 7\,\text{A}$$

$$i_3 = \frac{v_3 + 120}{2} = \frac{-75 + 120}{2} = 22.5\,\text{A} \qquad i_6 = \frac{v_1 - v_3}{36} = \frac{69 + 75}{36} = 4\,\text{A}$$

[b] $P_{120\text{V upper}} = -120(25.5) = -3060\,\text{W}$ (dev)

$P_{120\text{V lower}} = -120(22.5) = -2700\,\text{W}$ (dev)

$p_{11.5\text{A}} = (36)(4)(11.5) = 1656\,\text{W}$ (diss)

$p_{2\Omega}(\text{upper}) = (25.5)^2(2) = 1300.5\,\text{W}$ (diss)

$p_{3\Omega} = (3)^2(3) = 27\,\text{W}$ (diss)

$p_{2\Omega}(\text{lower}) = (22.5)^2(2) = 1012.50\,\text{W}$ (diss)

$p_{6\Omega} = (10)^2(6) = 600\,\text{W}$

$p_{12\Omega} = (7)^2(12) = 588\,\text{W}$

$p_{36\Omega} = (4)^2(36) = 576\,\text{W}$

$\sum P_{\text{dev}} = 5760\,\text{W}$

$\sum P_{\text{diss}} = 5760\,\text{W}$

P 4.10 Let v_2 be the node voltage across the 150-Ω resistor, positive at the upper terminal.

Then $2 + \frac{v_1}{50} + \frac{v_2}{150} + \frac{v_2}{75} = 0$ Note we have created a super node in writing this expression.

$300 + 3v_1 + v_2 + 2v_2 = 0$

$3v_1 + 3v_2 = -300$

$v_1 + v_2 = -100$

$v_1 = v_2 + 25$

$\therefore\ 2v_2 = -125$

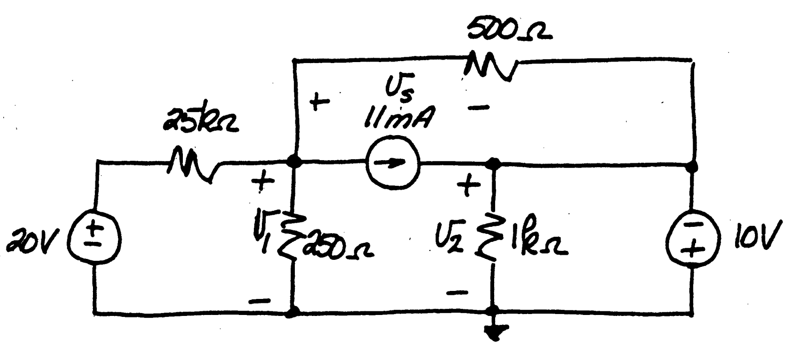
$v_2 = -62.5\,\text{V}$

$v_1 = -37.5\,\text{V}$

$p_{2\text{A}} = 2v_1 = -75\,\text{W}$

$\therefore\ p_{2\text{A}}(\text{del}) = 75\,\text{W}$

P 4.11 [a]



$$\frac{v_1}{0.25} + \frac{v_1 - 20}{25} + \frac{v_1 + 10}{0.50} + 11 = 0$$
$$100v_1 + v_1 - 20 + 50v_1 + 500 + 275 = 0$$
$$151v_1 = -755$$
$$v_1 = -5\,\text{V}$$
$$i_1 = \frac{20 - v_1}{25} = \frac{20 + 5}{25} = 1\,\text{mA}$$
$$i_2 = \frac{v_1}{0.25} = \frac{-5}{0.25} = -20\,\text{mA}$$
$$i_3 + \frac{v_2}{1} - 11 + \frac{v_2 - v_1}{0.5} = 0$$
$$i_3 = 11 - v_2 + \frac{v_1 - v_2}{0.5}$$
$$i_3 = 11 - (-10) + \frac{(-5 + 10)}{0.5} = 31\,\text{mA}$$

[b] $p_{20\text{V}} = -20i_1 = -20\,\text{mW}$ (del)

$p_{10\text{V}} = -10i_3 = -310\,\text{mW}$ (del)

$v_1 = v_s + v_2, \qquad v_s = v_1 - v_2 = -5 + 10 = 5\,\text{V}$

$p_{11\,\text{mA}} = 11v_s = 55\,\text{mW}$ (abs)

$p_{1\,\text{k}\Omega} = 10^2/1000 = 100\,\text{mW}$

$p_{25\,\text{k}\Omega} = (10^{-3})^2(25 \times 10^3) = 25\,\text{mW}$

$p_{250\Omega} = (20 \times 10^{-3})^2(0.25 \times 10^3) = 100\,\text{mW}$

$$p_{500\Omega} = \frac{v_s^2}{500} = \frac{25}{0.50} = 50\,\text{mW}$$

$\sum P_{\text{dev}} = 330\,\text{mW}$

$\sum P_{\text{diss}} = 55 + 25 + 200 + 50 = 330\,\text{mW}$

P 4.12 [a] From the solution to Problem 4.2 we know $v_o = -5$ V, therefore

$p_{40\,\text{mA}} = 40 \times 10^{-3}v_o = -200\,\text{mW}$

$\therefore\ p_{40\,\text{mA}}(\text{developed}) = 200\,\text{mW}$

[b] The current into the negative terminal of the 25-V source is

$$i_g = \frac{v_o + 25}{125} = \frac{20}{125} = 160\,\text{mA}$$

$p_{25\text{V}} = -25i_g = -4000\,\text{mW}$

$\therefore\ p_{25\text{V}}(\text{developed}) = 4000\text{mW}$

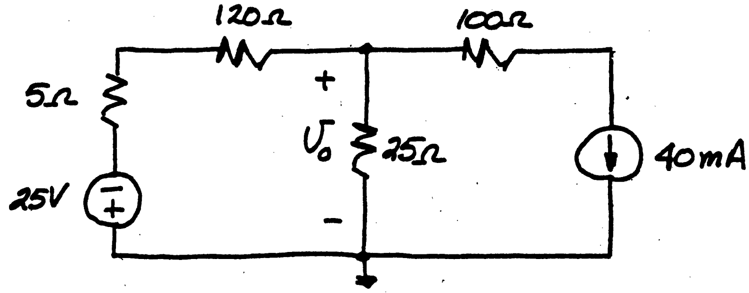
[c] $$p_{25\Omega} = \frac{(-5)^2}{25} = 1\,\text{W} = 1000\,\text{mW}$$

$p_{120\Omega} = 120i_g^2 = 3072\,\text{mW}$

$p_{5\Omega} = 5i_g^2 = 128\,\text{mW}$

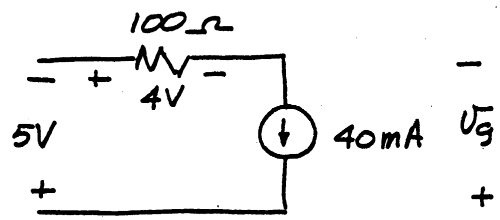
$\sum P_{\text{diss}} = 4200\,\text{mW} = \sum P_{\text{dev}}$

P 4.13 [a]



$$\frac{v_o}{25} + \frac{v_o + 25}{125} + 0.04 = 0$$
$$5v_o + v_o + 25 + 5 = 0$$
$$6v_o = -30; \qquad v_o = -5\,\text{V}$$

[b]



$$v_g = +5 + 4 = 9\,\text{V}$$
$$p_{40\,\text{mA}} = -40 \times 10^{-3} v_g = -360\,\text{mW}$$
$$\therefore\; p_{40\,\text{mA}}(\text{developed}) = 360\,\text{mW}$$

[c] Since v_o is identical to its value in Problem 4.2, the 25-V voltage source develops the same power as calculated in Problem 4.12[b]:

$$P_{25-\text{V}}(\text{developed}) = 4000\,\text{mW}$$

[d] The power dissipated in the 5-Ω, 120-Ω, and 25-Ω is the same as in Problem 4.12[c]; thus

$$p_{5\Omega} = 128\,\text{mW}$$
$$p_{120\Omega} = 3072\,\text{mW}$$
$$p_{25\Omega} = 1000\,\text{mW}$$
$$p_{100\Omega} = (40 \times 10^{-3})^2 100 = 160\,\text{mW}$$

$$\sum P_{\text{diss}} = 4360\,\text{mW}$$
$$\sum P_{\text{dev}} = 4360\,\text{mW}$$
$$\therefore\; \sum P_{\text{diss}} = \sum P_{\text{dev}}$$

[e] None, since the node-voltage equation is not affected by such a resistance.

P 4.14 Use d as the reference node

$$\frac{v_b}{10} + \frac{v_b - 33}{20} + \frac{v_b - v_c}{8} = 0$$
$$\frac{v_c - v_b}{8} - 4.95 + \frac{v_c}{26.4} + \frac{v_c - 33}{198} = 0$$

$$4v_b + 2v_b - 66 + 5v_b - 5v_c = 0$$

$$24.75v_c - 24.75v_b + 7.5v_c + v_c - 33 = 980.10$$

$$11v_b - 5v_c = 66$$

$$-24.75v_b + 33.25v_c = 980.10 + 33 = 1013.10$$

$$\Delta = \begin{vmatrix} 11 & -5 \\ -24.75 & 33.25 \end{vmatrix} = 242$$

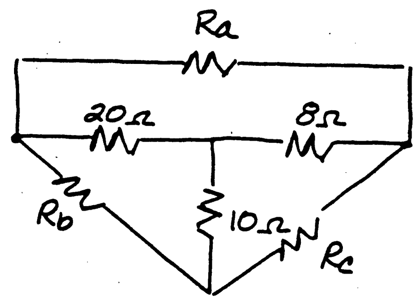
$$N_c = \begin{vmatrix} 11 & 66 \\ -24.75 & 1013.10 \end{vmatrix} = 12{,}777.60$$

$$v_c = \frac{N_c}{\Delta} = \frac{12{,}777.60}{242} = 52.80\,\text{V}$$

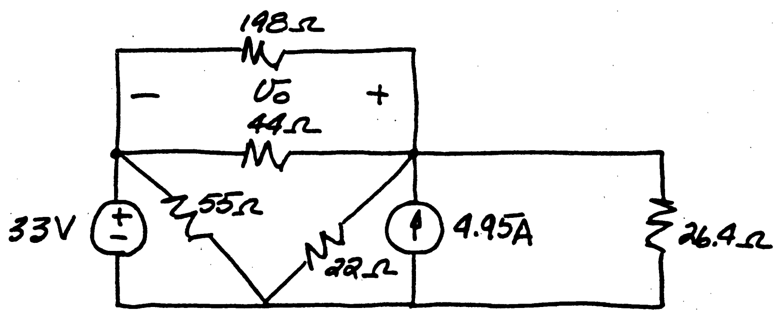
$$33 + v_o = 52.80$$

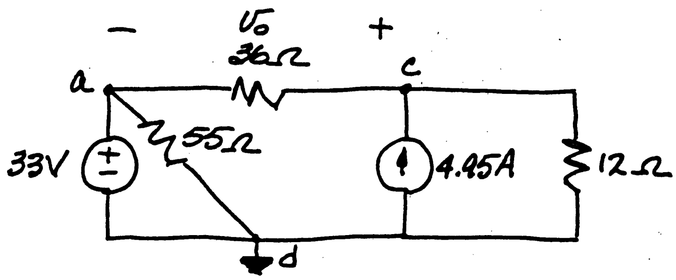
$$v_o = 19.80\,\text{V}$$

P 4.15



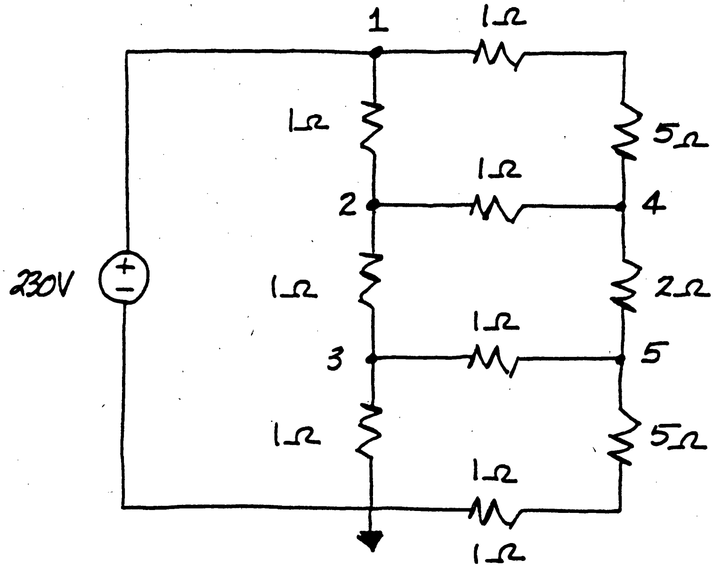
$$R_a = \frac{(20)(8) + 8(10) + 10(20)}{10} = 44\,\Omega; \qquad R_b = \frac{440}{8} = 55\,\Omega; \qquad R_c = \frac{440}{20} = 22\,\Omega$$





$$\frac{v_c}{12} - 4.95 + \frac{v_c - 33}{36} = 0$$
$$3v_c + v_c = 33 + 178.20$$
$$4v_c = 211.20$$
$$v_c = 52.80\,\text{V}$$
$$\therefore\; v_o = 52.80 - 33 = 19.8\,\text{V}$$

P 4.16 [a]



$$\frac{v_2 - 230}{1} + \frac{v_2 - v_4}{1} + \frac{v_2 - v_3}{1} = 0$$
$$\frac{v_3 - v_2}{1} + \frac{v_3}{1} + \frac{v_3 - v_5}{1} = 0$$
$$\frac{v_4 - v_2}{1} + \frac{v_4 - 230}{6} + \frac{v_4 - v_5}{2} = 0$$
$$\frac{v_5 - v_3}{1} + \frac{v_5}{6} + \frac{v_5 - v_4}{2} = 0$$
$$3v_2 - v_3 - v_4 + 0v_5 = 230$$
$$-\,v_2 + 3v_3 + 0v_4 - v_5 = 0$$
$$-\,6v_2 + 0v_3 + 10v_4 - 3v_5 = 230$$
$$0v_2 - 6v_3 - 3v_4 + 10v_5 = 0$$

$$\Delta = \begin{vmatrix} 3 & -1 & -1 & 0 \\ -1 & 3 & 0 & -1 \\ -6 & 0 & 10 & -3 \\ 0 & -6 & 3 & 10 \end{vmatrix}$$

$$= 3\begin{vmatrix} 3 & 0 & -1 \\ 0 & 10 & -3 \\ -6 & -3 & 10 \end{vmatrix} + 1\begin{vmatrix} -1 & -1 & 0 \\ 0 & 10 & -3 \\ -6 & -3 & 10 \end{vmatrix} - 6\begin{vmatrix} -1 & -1 & 0 \\ 3 & 0 & -1 \\ -6 & -3 & 10 \end{vmatrix}$$

$$= 3\,[(3)(91) - (6)(10)] + 1\,[(-1)(91) - (6)(3)] - 6\,[(1)(-3) + (10)(3)]$$
$$= (3)(213) - 109 - (6)(27) = 368$$

$$N_2 = \begin{vmatrix} 230 & -1 & -1 & 0 \\ 0 & 3 & 0 & -1 \\ 230 & 0 & 10 & -3 \\ 0 & -6 & -3 & 10 \end{vmatrix}$$

$$= 230(213) + 230(27) = (230)(240) = 55{,}200$$

$$N_3 = \begin{vmatrix} 3 & 230 & -1 & 0 \\ -1 & 0 & 0 & -1 \\ -6 & 230 & 10 & -3 \\ 0 & 0 & -3 & 10 \end{vmatrix} = -230 \begin{vmatrix} -1 & 0 & -1 \\ -6 & 10 & -3 \\ 0 & -3 & 10 \end{vmatrix} - 230 \begin{vmatrix} 3 & -1 & 0 \\ -1 & 0 & -1 \\ 0 & -3 & 10 \end{vmatrix}$$

$$= -230\,[(-1)(91) + (6)(-3)] - 230\,[(3)(-3) + 1(-10)]$$

$$= -230(-109) - 230(-19) = 230(128) = 29{,}440$$

$$N_4 = \begin{vmatrix} 3 & -1 & 230 & 0 \\ -1 & 3 & 0 & -1 \\ -6 & 0 & 230 & -3 \\ 0 & -6 & 0 & 10 \end{vmatrix} = 230 \begin{vmatrix} -1 & 3 & -1 \\ -6 & 0 & -3 \\ 0 & -6 & 10 \end{vmatrix} + 230 \begin{vmatrix} 3 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -6 & 10 \end{vmatrix}$$

$$= 230\,[(-1)(-18) + (6)(24)] + 230\,[(3)(24) + (1)(-10)]$$

$$= 230(162) + 230(62) = 230(224) = 51{,}520$$

$$N_5 = \begin{vmatrix} 3 & -1 & -1 & 230 \\ -1 & 3 & 0 & 0 \\ -6 & 0 & 10 & 230 \\ 0 & -6 & -3 & 0 \end{vmatrix} - 230 \begin{vmatrix} -1 & 3 & 0 \\ -6 & 0 & 10 \\ 0 & -6 & -3 \end{vmatrix} - 230 \begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & 0 \\ 0 & -6 & -3 \end{vmatrix}$$

$$= -230\,[(-1)(60) + (6)(-9)] - 230\,[(3)(-9) + (1)(-3)]$$

$$= -230(-114) - 230(-30) = 230(144) = 33{,}120$$

Therefore

$$v_2 = \frac{N_2}{\Delta} = \frac{55{,}200}{368} = 150\,\text{V}$$

$$v_3 = \frac{N_3}{\Delta} = \frac{29{,}440}{368} = 80\,\text{V}$$

$$v_4 = \frac{N_4}{\Delta} = \frac{51{,}520}{368} = 140\,\text{V}$$

$$v_5 = \frac{N_5}{\Delta} = \frac{33{,}120}{368} = 90\,\text{V}$$

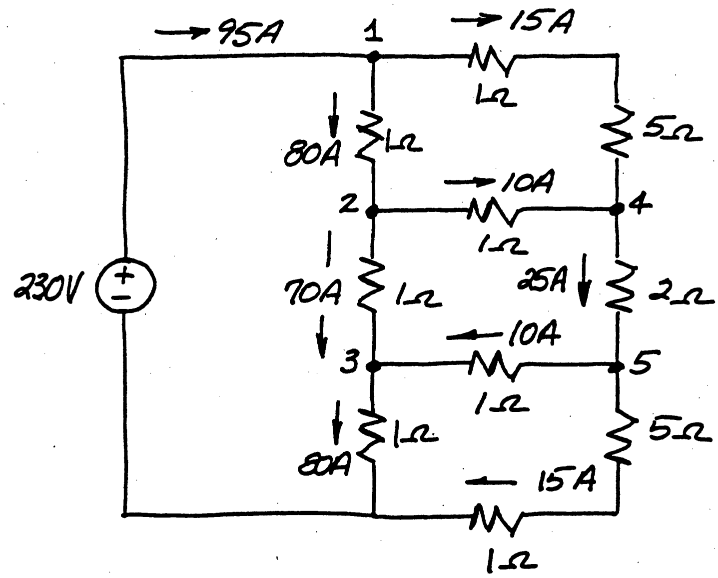
$$i_{2\Omega} = \frac{v_4 - v_5}{2} = \frac{140 - 90}{2} = 25\,\text{A}$$

$$p_{2\Omega} = (25)^2(2) = 1250\,\text{W}$$

[b] $$i_{230\text{V}} = \frac{v_1 - v_2}{1} + \frac{v_1 - v_4}{6} = \frac{230 - 150}{1} + \frac{230 - 140}{6} = 80 + 15 = 95$$

$$p_{230\text{V}} = 21{,}850\,\text{W}$$

Check:



$$\sum P_{\text{diss}} = (80)^2(1) + (70)^2(1) + (80)^2(1) + (15)^2(6) + (10)^2(1) + (25)^2(2) + (10)^2(1) + (15)^2(6)$$

$$= 6400 + 4900 + 6400 + 1350 + 100 + 1250 + 100 + 1350 = 21{,}850\text{ W}$$

P 4.17 $\frac{v_o - 186}{22} + \frac{v_o + 1.6i_\Delta}{14} + \frac{v_o + 43}{2} = 0$

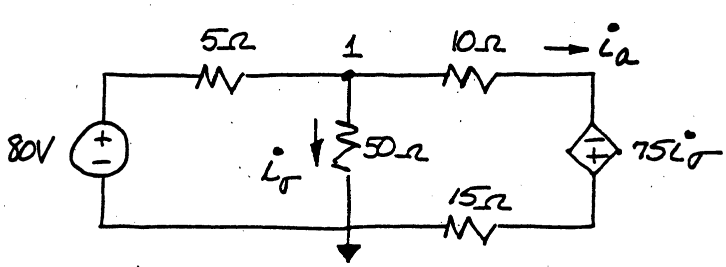
$i_\Delta = \frac{v_o + 43}{2}, \qquad 1.6i_\Delta = 0.8v_o + 34.40$

$14v_o - 2604 + 22v_o + 17.6v_o + 756.80 + 154v_o + 6622 = 0$

$207.60v_o = -4774.8$

$v_o = -23\text{ V}$

P 4.18



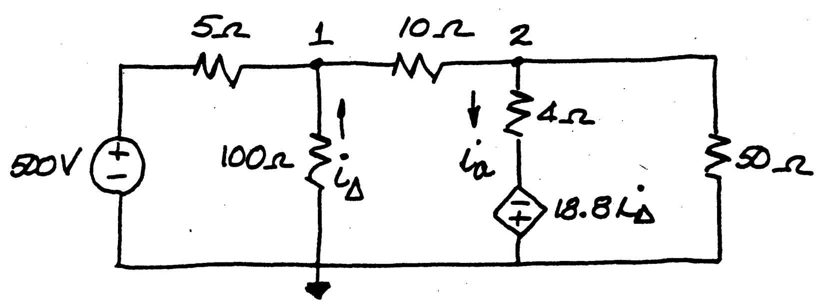
$\frac{v_1 - 80}{5} + \frac{v_1}{50} + \frac{v_1 + 75i_\sigma}{25} = 0$

$i_\sigma = \frac{v_1}{50}, \qquad 75i_\sigma = 1.5v_1$

$$10v_1 - 800 + v_1 + 5v_1 = 0$$
$$16v_1 = 800$$
$$v_1 = 50\,\text{V}$$
$$i_\sigma = \frac{50}{50} = 1\,\text{A}$$
$$i_a = \frac{v_1 + 75i_\sigma}{25} = \frac{125}{25} = 5\,\text{A}$$
$$p_{75i_\sigma} = -75i_\sigma i_a = -(75)(5) = -375\,\text{W}$$

$\therefore$ The dependent voltage source delivers 375 W to the circuit.

P 4.19



$$\frac{v_1 - 500}{5} + \frac{v_1}{100} + \frac{v_1 - v_2}{10} = 0$$
$$\boxed{31v_1 - 10v_2 = 10{,}000}$$
$$\frac{v_2 - v_1}{10} + \frac{v_2 + 18.8(-v_1/100)}{4} + \frac{v_2}{50} = 0$$
$$\boxed{-14.7v_1 + 37v_2 = 0}$$
$$\Delta = \begin{vmatrix} 31 & -10 \\ -14.7 & 37 \end{vmatrix} = 1147 - 147 = 1000$$
$$N_1 = \begin{vmatrix} 10{,}000 & -10 \\ 0 & 37 \end{vmatrix} = 37 \times 10^4$$
$$N_2 = \begin{vmatrix} 31 & 10{,}000 \\ -14.7 & 0 \end{vmatrix} = 147 \times 10^3$$
$$v_1 = \frac{N_1}{\Delta} = \frac{37 \times 10^4}{1000} = 370\,\text{V}$$
$$v_2 = \frac{N_2}{\Delta} = \frac{147 \times 10^3}{1000} = 147\,\text{V}$$
$$i_\Delta = \frac{-v_1}{100} = -3.7\,\text{A}$$
$$18.8i_\Delta = -69.56\,\text{V}$$
$$i_a = \frac{v_2 + 18.8i_\Delta}{4} = \frac{147 - 69.56}{4} = 19.36\,\text{A}$$
$$p_{18.8i_\Delta} = -(18.8i_\Delta)(i_a) = -(-69.56)(19.36) = 1346.6816\,\text{W}$$

Therefore the dependent voltage source dissipates 1346.6818 W, or delivers −1346.6816 W.

P 4.20 Place $5v_\Delta$ inside a supernode and use the lower node as a reference. Then

$$\frac{v_\Delta}{2} + \frac{v_\Delta - 15}{10} + \frac{v_1}{20} + \frac{v_1}{40} = 0$$
$$20v_\Delta + 4v_\Delta - 60 + 2v_1 + v_1 = 0$$
$$v_\Delta = 5v_\Delta + v_1, \qquad \therefore\ v_1 = -4v_\Delta$$
$$\therefore\ 24v_\Delta + 3(-4v_\Delta) = 60$$
$$12v_\Delta = 60; \qquad v_\Delta = 5\,\text{V}$$

P 4.21 [a]
$$-5\left(\frac{v_2}{40}\right) + \frac{v_1}{20} + \frac{v_1 - v_2}{5} = 0$$
$$-5v_2 + 2v_1 + 8v_1 - 8v_2 = 0$$
$$\boxed{10v_1 - 13v_2 + 0v_3 = 0}$$
$$\frac{v_2}{40} + \frac{v_2 - v_1}{5} + \frac{v_2 - v_3}{10} = 0$$
$$v_2 + 8v_2 - 8v_1 + 4v_2 - 4v_3 = 0$$
$$\boxed{-8v_1 + 13v_2 - 4v_3 = 0}$$
$$\frac{v_3 - v_2}{10} + \frac{v_3 - 11.5(v_2/40)}{5} + \frac{v_3 - 96}{4}$$
$$20v_3 - 20v_2 + 40v_3 - 11.5v_2 + 50v_3 = 4800$$
$$\boxed{0v_1 - 63v_2 + 220v_3 = 9600}$$

$$\Delta = \begin{vmatrix} 10 & -13 & 0 \\ -8 & 13 & -4 \\ 0 & -63 & 220 \end{vmatrix} = 10(2860 - 252) + 8(-2860) = 3200$$

$$N_1 = \begin{vmatrix} 0 & -13 & 0 \\ 0 & 13 & -4 \\ 9600 & -63 & 220 \end{vmatrix} = 9600(52)$$

$$N_2 = \begin{vmatrix} 10 & 0 & 0 \\ -8 & 0 & -4 \\ 0 & 9600 & 220 \end{vmatrix} = -9600(-40) = 9600(40)$$

$$N_3 = \begin{vmatrix} 10 & -13 & 0 \\ -8 & 13 & 0 \\ 0 & -63 & 9600 \end{vmatrix} = 9600(130 - 104) = 9600(26)$$

$$v_1 = \frac{N_1}{\Delta} = 156\,\text{V}; \qquad v_2 = \frac{N_2}{\Delta} = 120\,\text{V}; \qquad v_3 = \frac{N_3}{\Delta} = 78\,\text{V}$$

[b] From the numerical values of v_1, v_2, and v_3 we note the independent voltage source and the dependent current source will generate power. All other elements dissipate power. Thus

$$\sum P_{\text{gen}} = \left(\frac{96 - 78}{4}\right)(96) + 5\left(\frac{120}{40}\right)(156) = (4.5)(96) + 15(156)$$
$$= 432 + 2340 = 2772\,\text{W}$$
$$\sum P_{\text{diss}} = \sum P_{\text{gen}} = 2772\,\text{W}$$

P 4.22 For the given values of v_3 and v_4 we have

$$v_\Delta = 120 - v_3 = 120 - 108 = 12\,\text{V}$$
$$1.75 v_\Delta = 21\,\text{A}$$
$$i_\phi = \frac{v_4 - v_3}{8} = \frac{81.60 - 108}{8} = -3.3\,\text{A}$$
$$\left(\frac{40}{3}\right) i_\phi = -44\,\text{V}$$
$$v_1 = v_4 + \left(\frac{40}{3}\right) i_\phi = 81.60 - 44 = 37.60\,\text{V}$$

Let i_a be the current in the dependent voltage source. The reference direction for i_a is from right to left. Then

$$i_a = \frac{v_1}{20} + \frac{v_1 - v_2}{4} = \frac{37.60}{20} + \frac{37.60 - 120}{4} = 1.88 - 20.60 = -18.72\,\text{A}$$

Let i_b be the current supplied by the 120-V source, then

$$i_b = \frac{120 - 37.60}{4} + \frac{120 - 108}{2} = 20.60 + 6 = 26.60\,\text{A}$$

The power developed by the independent voltage source is

$$p_{120\text{V}} = (120)(26.60) = 3192\,\text{W}$$

The power developed by the dependent voltage source is

$$p_{(40/3)i_\phi} = (44)(18.72) = 823.68\,\text{W}$$

The power dissipated by the dependent current source is

$$p_{1.75v_\Delta} = (81.60)(21) = 1713.60\,\text{W}$$

The total power dissipated in the resistors is

$$p_R = \frac{(37.6)^2}{20} + \frac{(82.40)^2}{4} + \frac{(12)^2}{2} + \frac{(108)^2}{40} + (3.3)^2(8) + \frac{(81.60)^2}{80} = 2302.08\,\text{W}$$

Therefore

$$\sum P_{\text{dev}} = 3192 + 823.68 = 4015.68\,\text{W}$$
$$\sum P_{\text{diss}} = 2302.08 + 1713.60 = 4015.68$$

Her solution checks; yes, I agree.

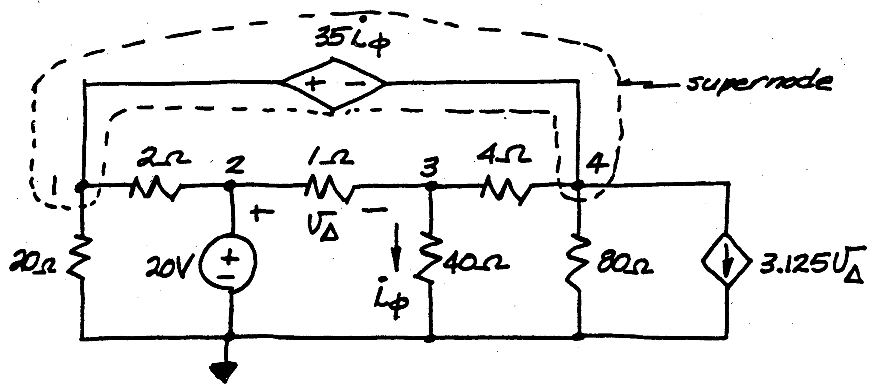
P 4.23 From Eq. 4.16, $i_B = v_c/(1+\beta)R_E$

From Eq. 4.17, $i_B = (v_b - V_o)/1 + \beta)R_E$

From Eq. 4.19,

$$i_B = \frac{1}{(1+\beta)R_E}\left[\frac{V_{CC}(1+\beta)R_E R_2 + V_o R_1 R_2}{R_1R_2 + (1+\beta)R_E(R_1+R_2)} - V_o\right]$$
$$= \frac{V_{CC}R_2 - V_o(R_1+R_2)}{R_1R_2 + (1+\beta)R_E(R_1+R_2)} = \frac{[V_{CC}R_2/(R_1+R_2)] - V_o}{[R_1R_2/(R_1+R_2)] + (1+\beta)R_E}$$

P 4.24



$$\frac{v_1}{20} + \frac{v_1 - 20}{2} + \frac{v_4}{80} + \frac{v_4 - v_3}{4} + 3.125v_\Delta = 0$$

$$\frac{v_3}{40} + \frac{v_3 - 20}{1} + \frac{v_3 - v_4}{4} = 0$$

$$v_\Delta = 20 - v_3; \qquad i_\phi = \frac{v_3}{40}$$

$$v_1 = 35i_\phi + v_4 = \frac{35}{40}v_3 + v_4$$

$$4v_1 + 40v_1 - 800 + v_4 + 20v_4 - 20v_3 + 250(20 - v_3) = 0$$

$$44v_1 - 270v_3 + 21v_4 = -4200$$

$$44\left(\frac{35}{40}v_3 + v_4\right) - 270v_3 + 21v_4 = -4200$$

$$\boxed{-231.50v_3 + 65v_4 = -4200}$$

$$v_3 + 40v_3 - 800 + 10v_3 - 10v_4 = 0$$

$$\boxed{51v_3 - 10v_4 = 800}$$

$$\Delta = \begin{vmatrix} -231.50 & 65 \\ 51 & -10 \end{vmatrix} = -1000$$

$$N_3 = \begin{vmatrix} -4200 & 65 \\ 800 & -10 \end{vmatrix} = -10{,}000$$

$$N_4 = \begin{vmatrix} -231.50 & -4200 \\ 51 & 800 \end{vmatrix} = 29{,}000$$

$$v_3 = \frac{N_3}{\Delta} = \frac{-10{,}000}{-1000} = 10\,\text{V}$$

$$v_4 = \frac{N_4}{\Delta} = \frac{29{,}000}{-1000} = -29\,\text{V}$$

$$v_1 = \frac{350}{40} - 29 = -20.25\,\text{V}$$

Let i_g be the current delivered by the 20-V source, then

$$i_g = \frac{20 - v_1}{2} + \frac{20 - v_3}{1} = \frac{40.25}{2} + 10 = 30.125\,\text{A}$$

$$p_g(\text{delivered}) = 20(30.125) = 602.50\,\text{W}$$

Check:

$$i_\phi = \frac{10}{40} = 0.25\,\text{A}, \qquad 35i_\phi = 8.75\,\text{V}$$

$$v_\Delta = 10\,\text{V}, \qquad 3.125v_\Delta = 31.25\,\text{A}$$

$$p_{3.125v_\Delta} = v_4(31.25) = -906.25\,\text{W}$$

$\therefore$ Dependent current source develops 906.25 W.

Let i_a represent the current left to right in the dependent voltage source. Then

$$i_a = \frac{v_4}{80} + \frac{v_4 - v_3}{4} + 3.125v_\Delta = 21.1375\,\text{A}$$

$$p_{35i_\phi} = (8.75)(21.1375) = 184.953125\,\text{W}$$

$\therefore$ Dependent voltage source absorbs power.

$$\sum P_{\text{diss}} = 184.953125 + \frac{(20.25)^2}{20} + (20.125)^2(2) + \frac{10^2}{1} + \frac{10^2}{40}$$
$$+ \left(\frac{39}{4}\right)^2(4) + \frac{(29)^2}{80}$$
$$= 1508.75\,\text{W}$$

$$\sum P_{\text{dev}} = 906.25 + 602.50 = 1508.75\,\text{W}$$

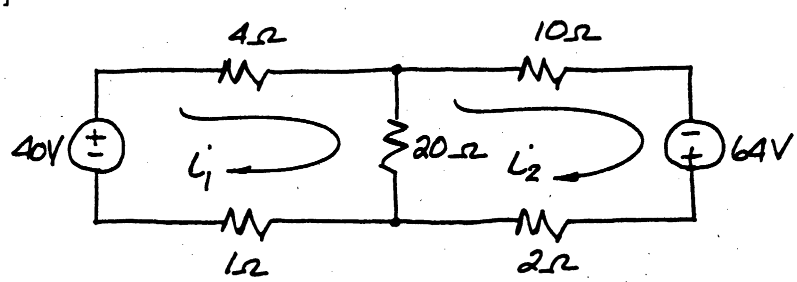
P 4.25 [a] $$\frac{v_o - v_1}{R} + \frac{v_o - v_2}{R} + \frac{v_o - v_3}{R} + \cdots + \frac{v_o - v_n}{R} = 0$$

$$\therefore\ nv_o = v_1 + v_2 + v_3 + \cdots + v_n$$

$$v_o = \frac{1}{n}[v_1 + v_2 + v_3 + \cdots + v_n] = \frac{1}{n}\sum_{R=1}^{n} v_k$$

[b] $$v_o = \frac{1}{3}(100 + 80 - 60) = 40\,\text{V}$$

P 4.26 [a]



$$40 = 25i_1 - 20i_2$$

$$64 = -20i_1 + 32i_2$$

$$\Delta = \begin{vmatrix} 25 & -20 \\ -20 & 32 \end{vmatrix} = 800 - 400 = 400$$

$$N_1 = \begin{vmatrix} 40 & -20 \\ 64 & 32 \end{vmatrix} = 1280 + 1280 = 2560$$

$$N_2 = \begin{vmatrix} 25 & 40 \\ -20 & 64 \end{vmatrix} = 1600 + 800 = 2400$$

$$i_1 = \frac{N_1}{\Delta} = \frac{2560}{400} = 6.4\,\text{A}$$

$$i_2 = \frac{N_2}{\Delta} = \frac{2400}{400} = 6.0\,\text{A}$$

$i_a = i_1 = 6.4\,\text{A}$, $\quad i_b = i_1 - i_2 = 0.4\,\text{A}$, $\quad i_c = -i_2 = -6.0\,\text{A}$

[b] If the polarity of the 64-V source is reversed, we have

$$40 = 25i_1 - 20i_2$$

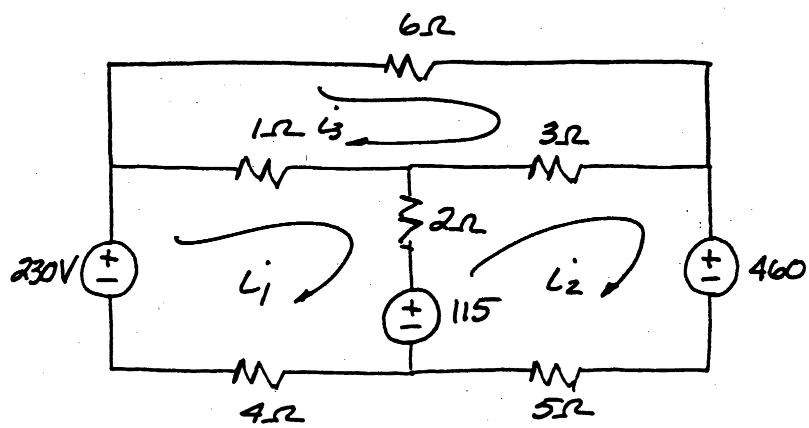
$$-64 = -20i_1 + 32i_2$$

$\Delta = 400$, $\quad N_1 = 0$, $\quad N_2 = -800$

$i_1 = 0$ and $i_2 = -2\,\text{A}$

$i_a = i_1 = 0\,\text{A}$, $\quad i_b = i_1 - i_2 = 2\,\text{A}$, $\quad i_c = -i_2 = 2\,\text{A}$

P 4.27 [a]



$$115 = 7i_1 - 2i_2 - i_3$$

$$-345 = -2i_1 + 10i_2 - 3i_3$$

$$0 = -i_1 - 3i_2 + 10i_3$$

$$\Delta = \begin{vmatrix} 7 & -2 & -1 \\ -2 & 10 & -3 \\ -1 & -3 & 10 \end{vmatrix} = 7(100-9) + 2(-20-3) - 1(6+10) = 575$$

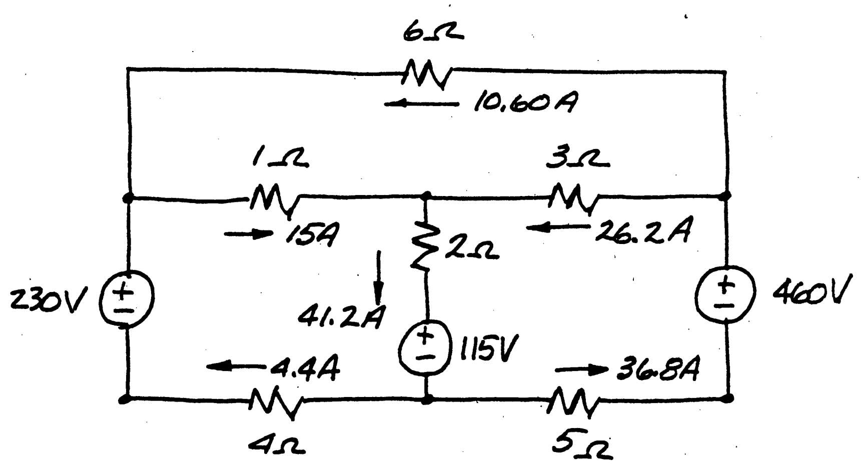
$$N_1 = \begin{vmatrix} 115 & -2 & -1 \\ -345 & 10 & -3 \\ 0 & -3 & 10 \end{vmatrix} = 115(91) + 345(-23) = 2530$$

$$N_2 = \begin{vmatrix} 7 & 115 & -1 \\ -2 & -345 & -3 \\ -1 & 0 & 10 \end{vmatrix} = -115(-23) - 345(69) = -21{,}160$$

$$N_3 = \begin{vmatrix} 7 & -2 & 115 \\ -2 & 10 & -345 \\ -1 & -3 & 0 \end{vmatrix} = 115(16) + 345(-23) = -6095$$

$$i_1 = \frac{N_1}{\Delta} = \frac{2530}{575} = 4.40\,\text{A}$$
$$i_2 = \frac{N_2}{\Delta} = \frac{-21{,}160}{575} = -36.80\,\text{A}$$
$$i_3 = \frac{N_3}{\Delta} = \frac{-6095}{575} = -10.60\,\text{A}$$

The branch currents corresponding to these mesh currents are shown on the following diagram.

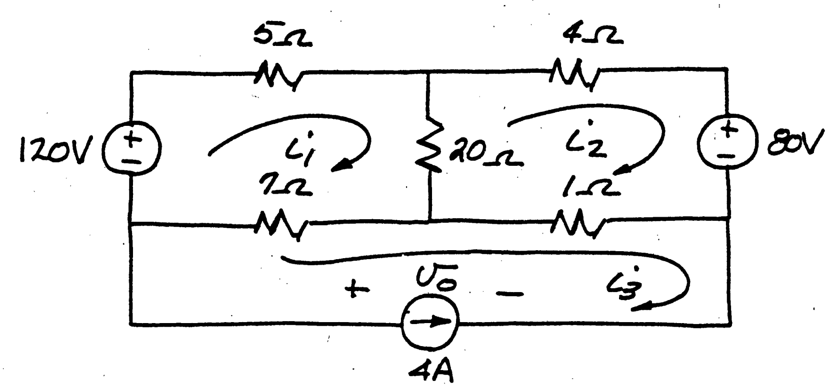


The 230-V and 460-V sources are developing power. Therefore

$$\sum P_{\text{dev}} = (230)(4.4) + 460(36.8) = 17{,}940\,\text{W}$$

[b] $P_{\text{diss}} = (15)^2(1) + (26.2)^2(3) + (10.6)^2(6) + (41.2)^2(2) + (4.4)^2(4) + (36.8)^2(5) + 41.2(115) = 17{,}940\,\text{W}$

P 4.28 [a]



$$120 = 32i_1 - 20i_2 - 7(-4)$$
$$0 = -20i_1 + 25i_2 + 80 - 1(-4)$$

Therefore

$$92 = 32i_1 - 20i_2$$
$$-84 = -20i_1 + 25i_2$$

$$\Delta = \begin{vmatrix} 32 & -20 \\ -20 & 25 \end{vmatrix} = 800 - 400 = 400$$

$$N_1 = \begin{vmatrix} 92 & -20 \\ -84 & 25 \end{vmatrix} = 620$$

$$N_2 = \begin{vmatrix} 32 & 92 \\ -20 & -84 \end{vmatrix} = -848$$

$$i_1 = \frac{N_1}{\Delta} = \frac{620}{400} = 1.55\text{ A}$$
$$i_2 = \frac{N_2}{\Delta} = \frac{-848}{400} = -2.12\text{ A}$$
$$v_o = 7(i_3 - i_1) + 1(i_3 - i_2) = 7(-5.55) + 1(-1.88) = -40.73\text{ V}$$
$$p_{4A} = (-40.73)(4) = -162.92\text{ W}$$

Therefore the 4-A source is delivering 162.92 W to the circuit.

[b] $p_{120V} = -(120)(1.55) = -186\text{ W}$ (dev)

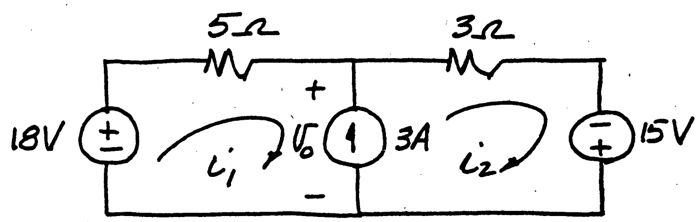
$p_{80V} = 80(-2.12) = -169.60\text{ W}$ (dev)

$\therefore \sum P_{dev} = 162.92 + 186 + 169.60 = 518.52\text{ W}$

[c] $$\sum P_{diss} = i_1^2(5) + i_2^2(4) + (i_1 - i_2)^2(20) + (i_1 - i_3)^2(7) + (i_2 - i_3)^2(1)$$
$$= (1.55)^2(5) + (2.12)^2(4) + (3.67)^2(20) + (5.55)^2(7) + (1.88)^2(1)$$
$$= 518.52\text{ W}$$

Therefore $\sum P_{dev} = \sum P_{diss} = 518.52\text{ W}$

P 4.29



Summing around the supermesh gives

$$18 = 5i_1 + 3i_2 - 15$$
$$33 = 5i_1 + 3i_2$$
$$i_1 - i_2 = -3$$
$$\therefore\ i_2 = i_1 + 3$$

$$33 = 5i_1 + 3i_1 + 9$$
$$\therefore\ 8i_1 = 24$$
$$i_1 = 3\text{ A}$$
$$i_2 = 6\text{ A}$$

$p_{18\text{V}} = -18(3) = -54\,\text{W}$ (dev)
$p_{15\text{V}} = -15(6) = -90\,\text{W}$ (dev)
$v_o = 18 - 5i_1 = 18 - 15 = 3\,\text{V}$
$p_{3\text{A}} = -3(+3) = -9\,\text{W}$ (dev)
$\sum P_{\text{dev}} = 54 + 90 + 9 = 153\,\text{W}$
$\therefore \sum P_{\text{diss}} = 153\,\text{W}$

Check: $\sum P_{\text{diss}} = 5(9) + 36(3) = 45 + 108 = 153\,\text{W}$

P 4.30 Summing around the supermesh used in the solution to Problem 4.29 gives

$6 = 5i_1 + 3i_2 - 15$
$21 = 5i_1 + 3(i_1 + 3)$
$\therefore\ 8i_1 = 12$
$i_1 = 1.5\,\text{A}$
$i_2 = 4.5\,\text{A}$
$p_{6\text{V}} = -6(1.5) = -9\,\text{W}$ (dev)
$p_{15\text{V}} = -15(4.5) = -67.5\,\text{W}$ (dev)
$p_{3\text{A}} = -3v_o$
$v_o = 6 - 5i_1 = 6 - 7.5 = -1.5\,\text{V}$
$p_{3\text{A}} = -3(-1.5) = 4.5\,\text{W}$ (diss)
$p_{5\Omega} = (1.5)^2(5) = 11.25\,\text{W}$
$p_{3\Omega} = (4.5)^2(3) = 60.75\,\text{W}$
$\sum P_{\text{diss}} = 4.5 + 11.25 + 60.75 = 76.5\,\text{W}$

Check: $\sum P_{\text{dev}} = 9 + 67.5 = 76.5\,\text{W}$

P 4.31 [a] Summing around the supermesh used in the solution to Problem 4.29 gives

$10 = 5i + 3i_2 - 15$
$25 = 5i_1 + 3(i_1 + 3)$
$\therefore\ 16 = 8i_1, \quad i_1 = 2\,\text{A}, \quad i_2 = 5\,\text{A}$
$\therefore\ v_o = 10 - 5(2) = 0\,\text{V}$
$\therefore\ \sum P_{\text{dev}} = \sum P_{\text{diss}} = (10)(2) + 15(5) = 95\,\text{W}$

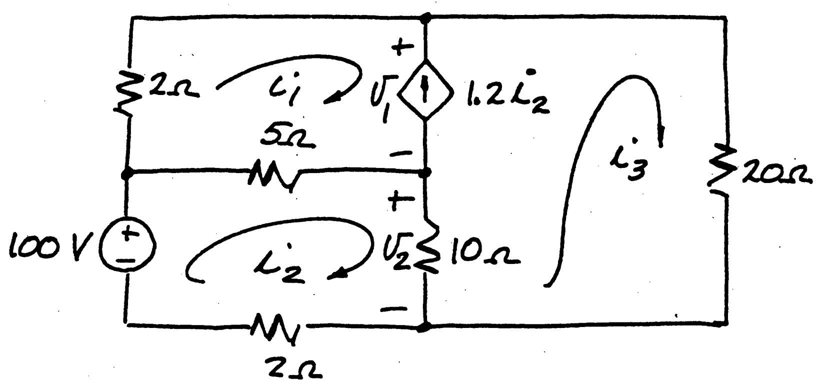
Check: $\sum P_{\text{diss}} = 4(5) + 25(3) = 95\,\text{W}$

[b] With 3-A current source replaced with a short circuit

$i_1 = 10/5 = 2\,\text{A}; \quad i_2 = 15/3 = 5\,\text{A}$
$\therefore\ \sum P_{\text{diss}} = (4)(5) + (25)(3) = 95\,\text{W}$

[c] A 3-A source with zero terminal voltage is equivalent to a short circuit carrying 3 A.

P 4.32 [a]



$100 = 2i_1 + 20i_3 + 2i_2$

$100 = 5(i_2 - i_1) + 10(i_2 - i_3) + 2i_2$

$100 = 2i_1 + 2i_2 + 20i_3$

$100 = -5i_1 + 17i_2 - 10i_3$

$1.2i_2 = i_3 - i_1$

$\therefore \quad i_3 = 1.2i_2 + i_1$

$20i_3 = 20i_1 + 24i_2$

$10i_3 = 10i_1 + 12i_2$

$\therefore \; 100 = 2i + 2i_2 + 20i_1 + 24i_2$

$100 = -5i_1 + 17i_2 - 10i_1 - 12i_2$

$\therefore \; 100 = 22i_1 + 26i_2$

$100 = -15i_1 + 5i_2$

$$\Delta = \begin{vmatrix} 22 & 26 \\ -15 & 5 \end{vmatrix} = 500$$

$$N_1 = \begin{vmatrix} 100 & 26 \\ 100 & 5 \end{vmatrix} = -2100$$

$$N_2 = \begin{vmatrix} 22 & 100 \\ -15 & 100 \end{vmatrix} = 3700$$

$\therefore \; i_1 = \dfrac{N_1}{\Delta} = -4.20\,\text{A}$

$i_2 = \dfrac{N_2}{\Delta} = 7.40\,\text{A}$

$i_3 = 1.2i_2 + i_1 = 4.68\,\text{A}$

$i_a = i_1 = -4.20\,\text{A}$

$i_b = i_2 = 7.40\,\text{A}$

$i_c = i_3 = 4.68\,\text{A}$

$i_d = i_2 - i_1 = 7.4 + 4.2 = 11.6\,\text{A}$

$i_e = i_2 - i_3 = 7.4 - 4.68 = 2.72\,\text{A}$

[b] $v_1 + v_2 = 20i_3 = 93.60\text{ V}$

$v_2 = 10i_e = 27.20\text{ V}$

$\therefore\ v_1 = 93.60 - 27.20 = 66.40\text{ V}$

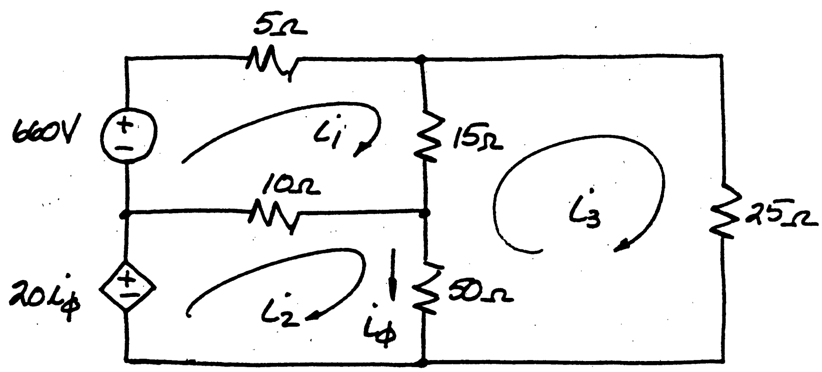
$\sum P_{\text{dev}} = 100i_b + 1.2i_bv_1 = 740 + 589.632 = 1329.632\text{ W}$

$\sum P_{\text{diss}} = 2i_a^2 + 5i_d^2 + 2i_b^2 + 10i_e^2 + 20i_c^2$

$= 2(4.2)^2 + 5(11.6)^2 + 2(7.4)^2 + 10(2.72)^2 + 20(4.68)^2 = 1329.632\text{ W}$

$\therefore\ \sum P_{\text{dev}} = \sum P_{\text{diss}} = 1329.632\text{ W}$

P 4.33



$660 = 30i_1 - 10i_2 - 15i_3$

$\boxed{132 = 6i_1 - 2i_2 - 3i_3}$

$0 = -20(i_2 - i_3) - 10i_1 + 60i_2 - 50i_3$

$0 = -10i_1 + 40i_2 - 30i_3$

$\boxed{0 = -i_1 + 4i_2 - 3i_3}$

$0 = -15i_1 - 50i_2 + 90i_3$

$\boxed{0 = -3i_1 - 10i_2 + 18i_3}$

$$\Delta = \begin{vmatrix} 6 & -2 & -3 \\ -1 & 4 & -3 \\ -3 & -10 & 18 \end{vmatrix} = 6(72 - 30) + 1(-36 - 30) - 3(6 + 12) = 132$$

$$N_1 = \begin{vmatrix} 132 & -2 & -3 \\ 0 & 4 & -3 \\ 0 & -10 & 18 \end{vmatrix} = 132(42)$$

$$N_2 = \begin{vmatrix} 6 & 132 & -3 \\ -1 & 0 & -3 \\ -3 & 0 & 18 \end{vmatrix} = 132(27)$$

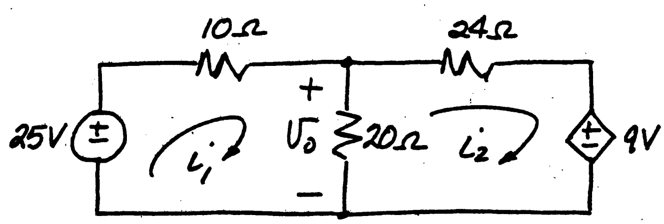
$$N_3 = \begin{vmatrix} 6 & -2 & 132 \\ -1 & 4 & 0 \\ -3 & -10 & 0 \end{vmatrix} = 132(22)$$

$$i_1 = \frac{N_1}{\Delta} = 42\,\text{A}, \qquad i_2 = \frac{N_2}{\Delta} = 27\,\text{A}, \qquad i_3 = \frac{N_3}{\Delta} = 22\,\text{A}$$

$$p_{20i_\phi} = -27(20)(27 - 22) = -2700\,\text{W}$$

$$p_{20i_\phi}(\text{developed}) = 2700\,\text{W}$$

P 4.34 [a] $i_\Delta = \frac{21}{14} = 1.5\,\text{A}, \qquad 6i_\Delta = 9\,\text{V}$



$$25 = 30i_1 - 20i_2$$

$$-9 = -20i_1 + 44i_2$$

$$\Delta = \begin{vmatrix} 30 & -20 \\ -20 & 44 \end{vmatrix} = 920$$

$$N_1 = \begin{vmatrix} 25 & -20 \\ -9 & 44 \end{vmatrix} = 920$$

$$N_2 = \begin{vmatrix} 30 & 25 \\ -20 & -9 \end{vmatrix} = 230$$

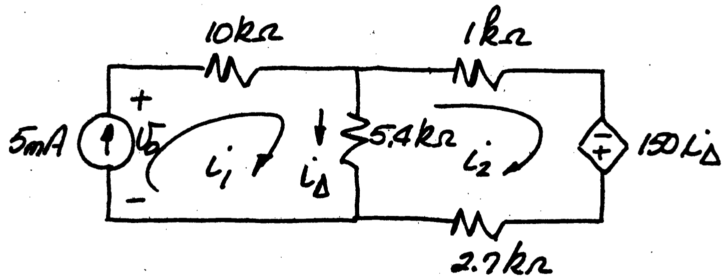
$$i_1 = \frac{N_1}{\Delta} = 1\,\text{A}, \qquad i_2 = \frac{N_2}{\Delta} = 0.25\,\text{A}$$

$$v_o = 20(i_1 - i_2) = 20(1.0 - 0.25) = 15\,\text{V}$$

[b] $p_{9V} = 9i_2 = 2.25\,\text{W} \quad \text{(abs)}$

$\therefore\ p_{9V}(\text{delivered}) = -2.25\,\text{W}$

P 4.35 [a]



$$i_1 = 5\,\text{mA}, \qquad i_\Delta = (i_1 - i_2)$$

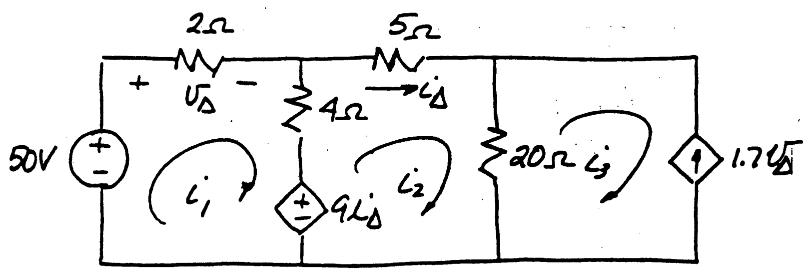
$$5400(i_2 - i_1) + 3700i_2 - 150(i_1 - i_2) = 0$$

$$9250i_2 - 5550i_1 = 0$$
$$i_2 = \frac{(5550)(5 \times 10^{-3})}{9250} = 3\,\text{mA}$$
$$\therefore\ i_\Delta = 5 - 3 = 2\,\text{mA}$$

[b] $p_{5\,\text{mA}} = -v_o i_1$
$$v_o = 10i_1 + 5.4(i_1 - i_2) = 50 + 10.8 = 60.8\,\text{V}$$
$$p_{5\,\text{mA}} = -60.8(5) = -304\,\text{mW}$$
$$p_{5\,\text{mA}}(\text{del}) = 304\,\text{mW}$$

[c] $p_{150i_\Delta} = -150i_\Delta i_2 = -150(2) \times 10^{-3}(3 \times 10^{-3}) = -900 \times 10^{-6} = 900\,\mu\text{W}$
$$p_{150i_\Delta}(\text{del}) = 900\,\mu\text{W}$$

P 4.36



$$50 = 2i_1 + 4(i_1 - i_2) + 9i_2$$
$$0 = -9i_2 + 4(i_2 - i_1) + 5i_2 + 20(i_2 + 1.7v_\Delta)$$
$$50 = 6i_1 + 5i_2$$
$$0 = -5i_2 - 4i_1 + 5i_2 + 20i_2 + 34(2i_1)$$
$$0 = 64i_1 + 20i_2$$

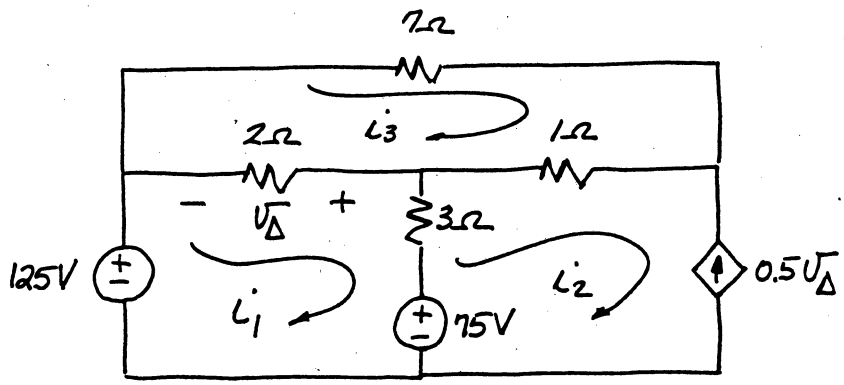
$$\Delta = \begin{vmatrix} 6 & 5 \\ 64 & 20 \end{vmatrix} = 120 - 320 = -200$$

$$N_1 = \begin{vmatrix} 50 & 5 \\ 0 & 20 \end{vmatrix} = 1000$$

$$N_2 = \begin{vmatrix} 6 & 50 \\ 64 & 0 \end{vmatrix} = -3200$$

$$i_1 = \frac{N_1}{\Delta} = -5\,\text{A}, \qquad i_2 = \frac{N_2}{\Delta} = 16\,\text{A}, \qquad i_a = i_1 = -5\,\text{A}$$
$$p_{50\text{V}} = -50i_a = 250\,\text{W} \quad (\text{abs})$$
$$i_b = i_1 - i_2 = -5 - 16 = -21\,\text{A}$$
$$p_{9i_\Delta} = 9i_\Delta i_b = 9(16)(-21) = -3024\,\text{W} \quad (\text{del})$$
$$v_{1.7v_\Delta} = 20(i_2 - i_3)$$
$$i_3 = -1.7(2)(i_1) = -3.4(-5) = 17\,\text{A}$$
$$v_{1.7v_\Delta} = 20(16 - 17) = -20\,\text{V}$$
$$p_{1.7v_\Delta} = -(1.7v_\Delta)v_{1.7v_\Delta} = -(-17)(-20) = 20(-17) = -340\,\text{W} \quad (\text{del})$$

P 4.37



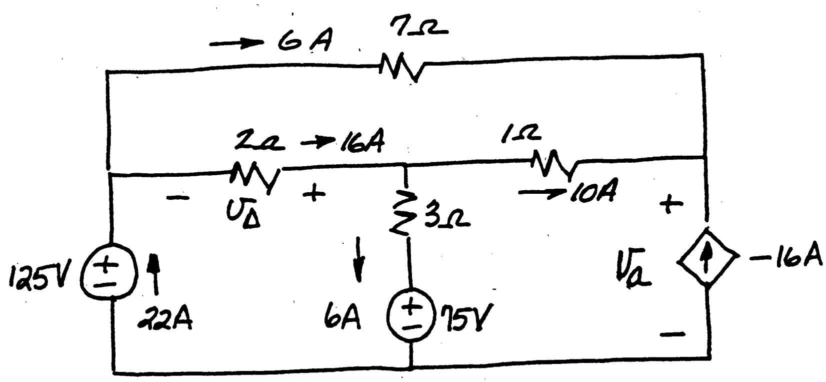
$$125 - 75 = 5i_1 - 3(-0.5)(2)(i_3 - i_1) - 2i_3$$
$$0 = -2i_1 - i_2 + 10i_3$$
$$50 = 2i_1 + i_3$$
$$0 = -3i_1 + 11i_3$$

$$\Delta = \begin{vmatrix} 2 & 1 \\ -3 & 11 \end{vmatrix} = 22 + 3 = 25$$

$$N_1 = \begin{vmatrix} 50 & 1 \\ 0 & 11 \end{vmatrix} = 550$$

$$N_3 = \begin{vmatrix} 2 & 50 \\ -3 & 0 \end{vmatrix} = 150$$

$$i_1 = \frac{N_1}{\Delta} = 22\,\text{A}$$
$$i_3 = \frac{N_3}{\Delta} = 6\,\text{A}$$
$$v_\Delta = 2(i_3 - i_1) = 2(-16) = -32\,\text{V}$$
$$i_2 = -0.5v_\Delta = 16\,\text{A}$$
$$i_1 - i_3 = 16\,\text{A}; \qquad i_1 - i_2 = 6\,\text{A}; \qquad i_2 - i_3 = 10\,\text{A}$$



$v_a = 75 + 18 - 10 = 83\,\text{V}$

Therefore the only source developing power is the 125-V source.

$P_{\text{dev}} = (125)(22) = 2750\,\text{W}$

Check:

$p_{0.5v_\Delta} = (83)(16) = 1328\,\text{W}$ (absorbed)

$p_{75\text{V}} = 6(75) = 450\,\text{W}$ (absorbed)

$p_{7\Omega} = 36(7) = 252\,\text{W}$ (absorbed)

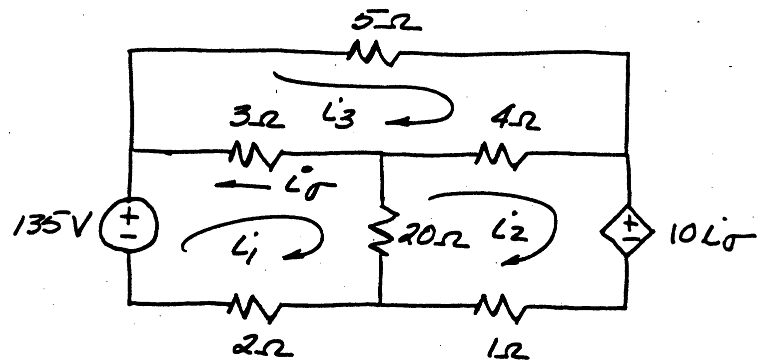
$p_{2\Omega} = 256(2) = 512\,\text{W}$ (absorbed)

$p_{1\Omega} = 100(1) = 100\,\text{W}$ (absorbed)

$p_{3\Omega} = 36(3) = 108\,\text{W}$ (absorbed)

$\sum P_{\text{abs}} = 2750\,\text{W}$

P 4.38



$135 = 25i_1 - 20i_2 - 3i_3$

$0 = -20i_1 + 25i_2 - 4i_3 + 10(i_3 - i_1)$

$0 = -3i_1 - 4i_2 + 12i_3$

$135 = 25i_1 - 20i_2 - 3i_3$

$0 = -30i_1 + 25i_2 + 6i_3$

$0 = -3i_1 - 4i_2 + 12i_3$

$$\Delta = \begin{vmatrix} 25 & -20 & -3 \\ -30 & 25 & 6 \\ -3 & -4 & 12 \end{vmatrix} = 25(300 + 24) + 30(-240 - 12) - 3(-120 + 75) = 675$$

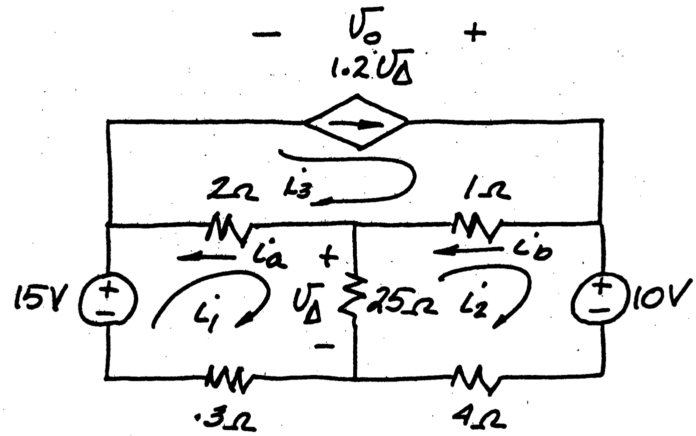
$$N_1 = \begin{vmatrix} 135 & -20 & -3 \\ 0 & 25 & 6 \\ 0 & -4 & 12 \end{vmatrix} = (324)(135) = 43{,}740$$

$$N_2 = \begin{vmatrix} 25 & 135 & -3 \\ -30 & 0 & 6 \\ -3 & 0 & 12 \end{vmatrix} = -135(-360 + 18) = (342)(135) = 46{,}170$$

$i_1 = \dfrac{N_1}{\Delta} = 64.80\,\text{A}; \qquad i_2 = \dfrac{N_2}{\Delta} = 68.40\,\text{A}; \qquad i_2 - i_1 = 36\,\text{A}$

$p_{20\Omega} = (3.6)^2(20) = 259.20\,\text{W}$

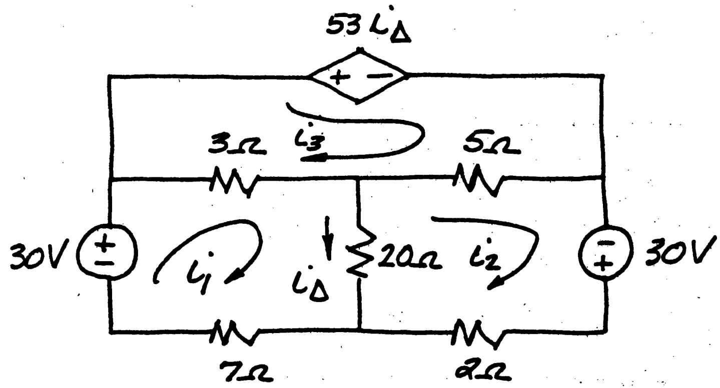
P 4.39 [a]



$$15 = 30i_1 - 25i_2 - 2(1.2)(25)(i_1 - i_2)$$
$$15 = 30i_1 - 25i_2 - 60i_1 + 60i_2$$
$$\boxed{15 = -30i_1 + 35i_2}$$
$$0 = -25i_1 + 30i_2 + 10 - 1.2(25)(i_1 - i_2)$$
$$-10 = -25i_1 + 30i_2 - 30i_1 + 30i_2$$
$$-10 = -55i_1 + 60i_2$$
$$\Delta = \begin{vmatrix} -30 & 35 \\ -55 & 60 \end{vmatrix} = -1800 + 1925 = 125$$
$$N_1 = \begin{vmatrix} 15 & 35 \\ -10 & 60 \end{vmatrix} = 900 + 350 = 1250$$
$$N_2 = \begin{vmatrix} -30 & 15 \\ -55 & -10 \end{vmatrix} = 300 + 825 = 1125$$
$$i_1 = \frac{N_1}{\Delta} = \frac{1250}{125} = 10\text{ A}$$
$$i_2 = \frac{N_2}{\Delta} = \frac{1125}{125} = 9.0\text{ A}$$
$$i_1 - i_2 = 10 - 9 = 1\text{ A}$$
$$p_{25\Omega} = (1)^2(25) = 25\text{ W}$$

[b]
$$1.2v_\Delta = (1.2)(25) = 30\text{ A} = i_3$$
$$i_a = i_3 - i_1 = 20\text{ A}$$
$$i_b = i_3 - i_2 = 30 - 9 = 21\text{ A}$$
$$v_o = 2i_a + 1i_b = 40 + 21 = 61\text{ V}$$
$$p_{1.2v_\Delta} = (1.2v_\Delta)v_o = i_3 v_o = 30(61) = 1830\text{ W}$$
$$p_{15V} = 15i_1 = 15(10) = 150\text{ W}$$
$$\sum p_{\text{dev}} = 1830 + 150 = 1980\text{ W}$$
$$\%\text{delivered} = \frac{25}{1980} \times 100 = 1.26\%$$

P 4.40



$$\boxed{30 = 30i_1 - 20i_2 - 3i_3}$$

$$\boxed{30 = -20i_1 + 27i_2 - 5i_3}$$

$$0 = -3i_1 - 5i_2 + 8i_3 + 53(i_1 - i_2)$$

$$\boxed{0 = 50i_1 - 58i_2 + 8i_3}$$

$$\Delta = \begin{vmatrix} 30 & -20 & -3 \\ -20 & 27 & -5 \\ 50 & -58 & 8 \end{vmatrix} = 30(216 - 290) + 20(-160 - 174) + 50(100 + 81)$$

$$= 30(-74) - 20(334) + 50(181) = 150$$

$$N_1 = \begin{vmatrix} 30 & -20 & -3 \\ 30 & 27 & -5 \\ 0 & -58 & 8 \end{vmatrix} = 30(-74) - 30(-334) = 7800$$

$$N_2 = \begin{vmatrix} 30 & 30 & -3 \\ -20 & 30 & -5 \\ 50 & 0 & 8 \end{vmatrix} = -30(-160 + 250) + 30(240 + 150)$$

$$N_3 = \begin{vmatrix} 30 & -20 & 30 \\ -20 & 27 & 30 \\ 50 & -58 & 0 \end{vmatrix} = 30(1160 - 1350) - 30(-1740 + 1000)$$

$$i_1 = \frac{N_1}{\Delta} = \frac{7800}{150} = 52\,\text{A}$$

$$i_2 = \frac{N_2}{\Delta} = \frac{9000}{150} = 60\,\text{A}$$

$$i_3 = \frac{N_3}{\Delta} = \frac{16{,}500}{150} = 110\,\text{A}$$

$$i_\Delta = i_1 - i_2 = 52 - 60 = -8\,\text{A}$$

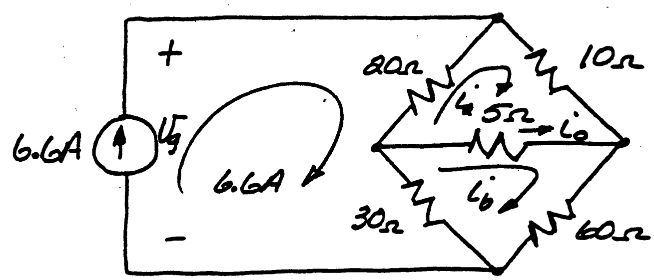
$$p_{53i_\Delta} = (53i_\Delta)(i_3) = (53(-8)(110) = -46{,}640\,\text{W}$$

Therefore the dependent voltage source develops 46,640 W or 46.64 kW.

P 4.41 [a] There are three unknown node voltages and only two unknown mesh currents. Use the mesh current method to minimize the number of simultaneous equations.

[b] See following figure for mesh currents

$$20(i_a - 6.6) + 10i_a + 5(i_a - i_b) = 0$$
$$30(i_b - 6.6) + 5(i_b - i_a) + 60i_b = 0$$



Our equations simplify to

$$7i_a - i_b = 26.4$$
$$-i_a + 19i_b = 39.6$$

$$\therefore \Delta = \begin{vmatrix} 7 & -1 \\ -1 & 19 \end{vmatrix} = 132$$

$$N_a = \begin{vmatrix} 26.4 & -1 \\ 39.6 & 19 \end{vmatrix} = 541.20$$

$$N_b = \begin{vmatrix} 7 & 26.4 \\ -1 & 39.6 \end{vmatrix} = 303.60$$

$$i_a = \frac{N_a}{\Delta} = 4.1\text{ A}; \qquad i_b = \frac{N_b}{\Delta} = 2.3\text{ A}; \qquad i_o = i_b - i_a = -1.8\text{ A}$$

$$p_{5\Omega} = (-1.8)^2(5) = 16.20\text{ W}$$

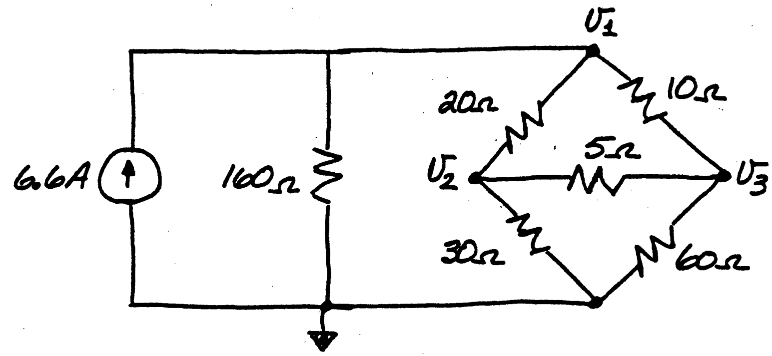
[c] No, the voltage across the 6.6-A current source is readily available from the mesh currents, and solving two simultaneous mesh-current equations is less work than solving three node voltage equations.

[d] $v_g = 10i_a + 60i_b = 41 + 138 = 179\text{ V}$

$p_{6.6\text{A}}(\text{developed}) = (179)(6.6) = 1181.40\text{ W}$

P 4.42 [a] There are three unknown node voltages and three unknown mesh currents, so the number of simultaneous equations required are the same for both methods. The node-voltage method has the advantage of having to solve the three simultaneous equations for one unknown voltage provided the connection at either the top or bottom of the circuit is used as the reference node. Therefore recommend the node-voltage method.

[b]



$$-6.6+\frac{v_1}{160}+\frac{v_1-v_2}{20}+\frac{v_1-v_3}{10}=0 \quad \text{or} \quad \boxed{25v_1-8v_2-16v_3=1056}$$

$$\frac{v_2-v_1}{20}+\frac{v_2-v_3}{5}+\frac{v_3}{30}=0 \quad \text{or} \quad \boxed{-3v_1+17v_2-12v_3=0}$$

$$\frac{v_3-v_2}{5}+\frac{v_3}{60}+\frac{v_3-v_1}{10}=0 \quad \text{or} \quad \boxed{-6v_1-12v_2+19v_3=0}$$

$$\Delta=\begin{vmatrix}25 & -8 & -16\\ -3 & 17 & -12\\ -6 & -12 & 19\end{vmatrix}=1235$$

$$N_1=\begin{vmatrix}1056 & -8 & -16\\ 0 & 17 & -12\\ 0 & -12 & 19\end{vmatrix}=189{,}024$$

$$v_1=\frac{N_1}{\Delta}=153.06\,\text{V}$$

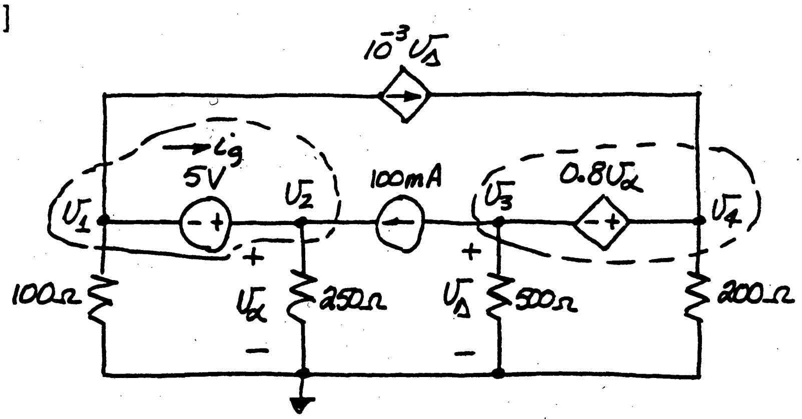
$$p_{\text{dev}}=(6.6)(153.06)=1010.17\,\text{W}$$

P 4.43 [a] The node-voltage method requires summing the currents at two supernodes in terms of four node voltages and using two constraint equations to reduce the system of equations to two unknowns. If the connection at the bottom of the circuit is used as the reference node, then the voltages controlling the dependent sources are node voltages. This makes it easy to formulate the constraint equations. The current in the 5-V source is obtained by summing the currents at either terminal of the source.

The mesh-current method requires summing the voltages around the two meshes not containing current sources in terms of four mesh currents. In addition the voltages controlling the dependent sources must be expressed in terms of the mesh currents. Thus the constraint equations are more complicated, and the reduction to two equations and two unknowns involves more algebraic manipulation. The current in the 5-V source is found by subtracting two mesh currents.

Because the constraint equations are easier to formulate in the node-voltage method, it is the preferred approach.

[b]



$v_\Delta = v_3; \qquad v_\alpha = v_2$

$$\frac{v_1}{100} + \frac{v_2}{250} - 0.1 + 10^{-3}v_3 = 0 \quad \text{or} \quad 5v_1 + 2v_2 - 50 + 0.5v_3 = 0$$

$$v_1 = v_2 - 5, \qquad \therefore\ 5v_1 = 5v_2 - 25$$

$$\therefore\ \boxed{7v_2 + 0.5v_3 = 75}$$

$$\frac{v_3}{500} + 0.1 + \frac{v_4}{200} - 10^{-3}v_3 = 0$$

$$\therefore\ 2v_3 + 100 + 5v_4 - v_3 = 0$$

$$v_4 = v_3 + 0.8v_2; \qquad \therefore\ 5v_4 = 5v_3 + 4v_2$$

$$\therefore\ 4v_2 + 6v_3 = -100 \quad \text{or} \quad \boxed{2v_2 + 3v_3 = -50}$$

$$\therefore\ \Delta = \begin{vmatrix} 7 & 0.5 \\ 2 & 3 \end{vmatrix} = 21 - 1 = 20$$

$$N_2 = \begin{vmatrix} 75 & 0.5 \\ -50 & 3 \end{vmatrix} = 225 + 25 = 250$$

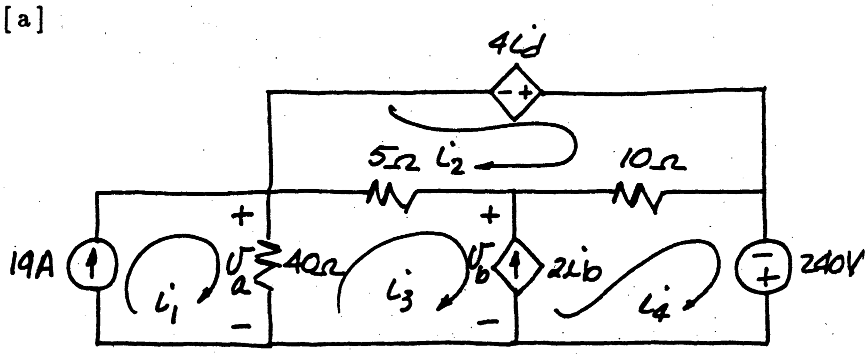
$$\therefore\ v_2 = \frac{250}{20} = 12.5\,\text{V}$$

$$i_g = \frac{v_2}{250} - 0.1 = \frac{12.5}{250} - 0.1 = -0.05\,\text{A} = -50\,\text{mA}$$

$$p_{5\text{V}} = -5i_g = -5(-50) = 250\text{mW}$$

$$\therefore\ p_{5\text{V}}(\text{absorbed}) = 250\,\text{mW}$$

P 4.44 [a]

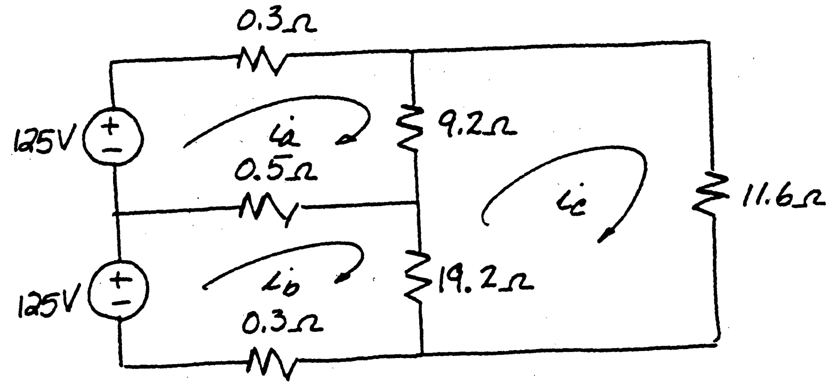


$$40(i_3 - 19) - 4i_4 - 240 = 0$$
$$40i_3 - 4i_4 = 1000$$
$$\boxed{10i_3 - i_4 = 250}$$
$$40(i_3 - 19) + 5(i_3 - i_2) + 10(i_4 - i_2) - 240 = 0$$
$$45i_3 - 15i_2 + 10i_4 = 1000$$
$$i_b = i_2 - i_3$$
$$2i_b = i_4 - i_3$$
$$2i_2 - 2i_3 = i_4 - i_3; \qquad i_2 = 0.5i_4 + 0.5i_3$$
$$15i_2 = 7.5i_4 + 7.5i_3$$
$$45i_3 - 7.5i_4 - 7.5i_3 + 10i_4 = 1000$$
$$37.5i_3 + 2.5i_4 = 1000$$
$$\boxed{15i_3 + i_4 = 400}$$
$$\Delta = \begin{vmatrix} 10 & -1 \\ 15 & 1 \end{vmatrix} = 25$$
$$N_3 = \begin{vmatrix} 250 & -1 \\ 400 & 1 \end{vmatrix} = 650$$
$$N_4 = \begin{vmatrix} 10 & 250 \\ 15 & 400 \end{vmatrix} = +250$$
$$i_3 = \frac{N_3}{\Delta} = 26\text{ A}; \qquad i_2 = 18\text{ A}$$
$$i_4 = \frac{N_4}{\Delta} = 10\text{ A}; \qquad i_1 = 19\text{ A}$$
$$i_a = 19 - 26 = -7\text{ A}$$
$$i_b = 18 - 26 = -8\text{ A}$$
$$i_c = 18 - 10 = 8\text{ A}$$
$$i_d = 10\text{ A}$$
$$i_e = i_c + i_d = 8 + 10 = 18\text{ A}$$

[b] $v_a = 40i_a = -280\text{ V}; \qquad v_b = -10i_c - 240 = -320\text{ V}$

$$p_{19A} = -19v_a = -19(-280) = 5320\text{ W} \quad \text{(diss)}$$
$$p_{4i_d} = -4i_d i_e = -4(10)(18) = -720\text{ W} \quad \text{(gen)}$$
$$p_{2i_b} = -2i_b v_b = -2(-8)(-320) = -5120\text{ W} \quad \text{(gen)}$$
$$p_{240V} = -240i_d = -240(10) = -2400\text{ W} \quad \text{(gen)}$$
$$p_{40\Omega} = (-7)^2(40) = 1960\text{ W} \quad \text{(diss)}$$
$$p_{5\Omega} = (-8)^2(5) = 320\text{ W(diss)}$$
$$p_{10\Omega} = (8)^2(10) = 640\text{ W} \quad \text{(diss)}$$
$$\sum P_{gen} = 720 + 5120 + 2400 = 8240\text{ W}$$
$$\sum P_{diss} = 5320 + 1960 + 320 + 640 = 8240\text{ W}$$
$$\therefore \sum P_{gen} = \sum P_{diss} = 8240\text{ W}$$

P 4.45 [a]



$$125 = 10i_a - 0.5i_b - 9.2i_c$$
$$125 = -0.5i_a + 20i_b - 19.2i_c$$
$$0 = -9.2i_a - 19.2i_b + 40i_c$$

$$\Delta = \begin{vmatrix} 10 & -0.5 & -9.2 \\ -0.5 & 20 & -19.2 \\ -9.2 & -19.2 & 40 \end{vmatrix} = 2434.16$$

$$N_a = \begin{vmatrix} 125 & -0.5 & -9.2 \\ 125 & 20 & -19.2 \\ 0 & -19.2 & 40 \end{vmatrix} = (125)(628)$$

$$N_b = \begin{vmatrix} 10 & 125 & -9.2 \\ -0.5 & 125 & -19.2 \\ -9.2 & 0 & 40 \end{vmatrix} = (125)(512)$$

$$N_c = \begin{vmatrix} 10 & -0.5 & 125 \\ -0.5 & 20 & 125 \\ -9.2 & -19.2 & 0 \end{vmatrix} = 125(390.2)$$

$$i_a = \frac{N_a}{\Delta} = 32.25\text{ A}; \qquad i_b = \frac{N_b}{\Delta} = 26.29\text{ A}; \qquad i_c = \frac{N_c}{\Delta} = 20.04\text{ A}$$

$$v_1 = 9.2(i_a - i_c) = 112.35\text{ V}$$
$$v_2 = 19.2(i_b - i_c) = 120.09\text{ V}$$
$$v_c = 11.6i_c = 232.44\text{ V}$$

[b] $p_1 = (i_a - i_c)^2(9.2) = 1371.93\text{ W}$

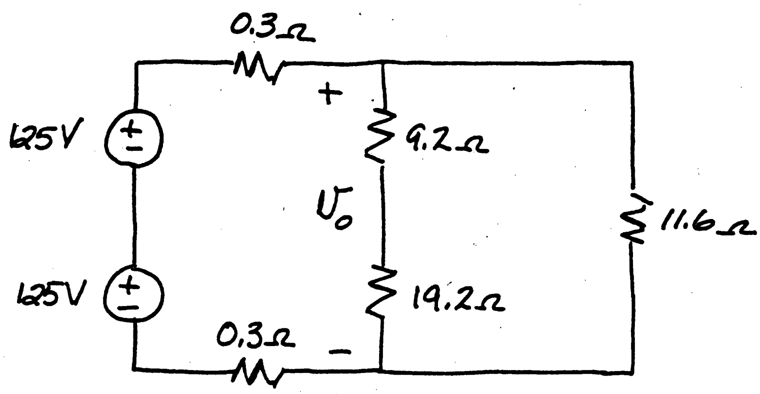
$p_2 = (i_b - i_c)^2(19.2) = 751.13\text{ W}$

$p_3 = i_c^2(11.6) = 4657.52\text{ W}$

[c] $\sum P_{dev} = 125(32.25) + 125(26.29) = 4031.16 + 3286.55 = 7317.72\text{ W}$

$$\%\text{ delivered} = \left(\frac{p_1 + p_2 + p_3}{\sum P_{dev}}\right)100 = \frac{6780.58}{7317.72} \times 100 = 92.66\%$$

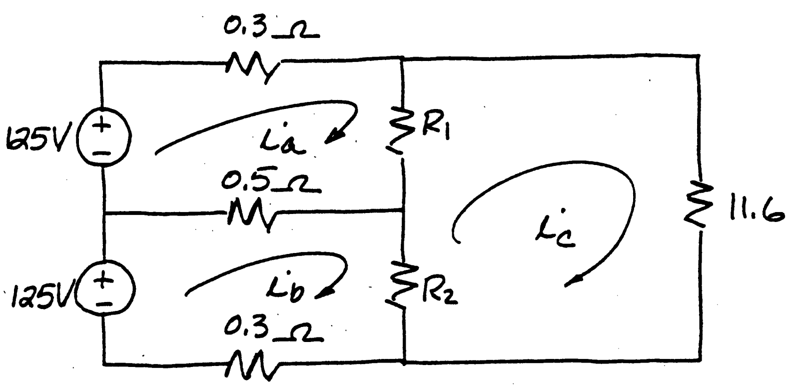
[d]



$$\frac{v_o - 250}{0.6} + \frac{v_o}{28.4} + \frac{v_o}{11.6} = 0$$
$$\therefore\ v_o = 233.02\,\text{V}$$
$$\therefore\ v_1 = \frac{233.02}{28.4}(9.2) = 75.49\,\text{V}$$
$$v_2 = \frac{233.02}{28.4}(19.2) = 157.54\,\text{V}$$

Voltage across load R_2 is excessive and could cause damage to the appliance.

P 4.46



$$125 = (R_1 + 0.8)i_a - 0.5i_b - R_1 i_c$$
$$125 = -0.5i_a + (R_2 + 0.8)i_b - R_2 i_c$$
$$0 = -R_1 i_a - R_2 i_b + (R_1 + R_2 + 11.6)i_c$$

$$\Delta = \begin{vmatrix} (R_1 + 0.8) & 0.5 & -R_1 \\ -0.5 & (R_2 + 0.8) & -R_2 \\ -R_1 & -R_2 & (R_1 + R_2 + 11.6) \end{vmatrix}$$

When $R_1 = R_2$, Δ reduces to $\Delta = 12.2R_1^2 + 19.34R_1 + 4.524$.

$$N_a = \begin{vmatrix} 125 & -0.5 & -R_1 \\ 125 & (R_2 + 0.8) & -R_2 \\ 0 & -R_2 & (R_1 + R_2 + 11.6) \end{vmatrix}$$
$$= 125\left[(R_2 + 1.3)(R_1 + R_2 + 11.6) + R_1 R_2 - R_2^2\right]$$

$$N_b = \begin{vmatrix} (R_1+0.8) & 125 & -R_1 \\ -0.5 & 125 & -R_2 \\ -R_1 & 0 & (R_1+R_2+11.6) \end{vmatrix}$$
$$= 125\left[(R_1+1.3)(R_1+R_2+11.6)+R_1R_2-R_1^2\right]$$

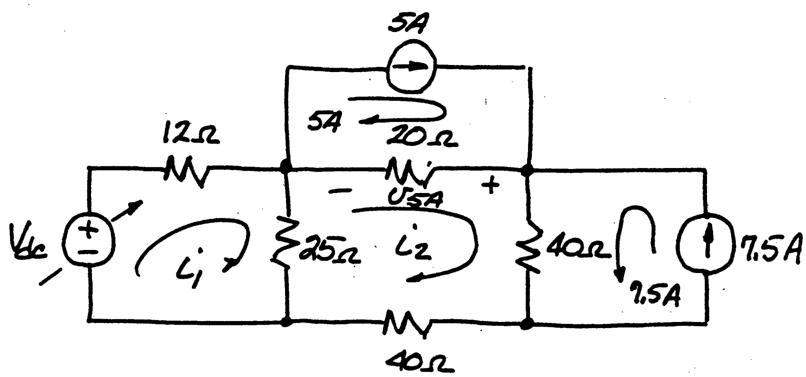
$$i_a = \frac{N_a}{\Delta}, \qquad i_b = \frac{N_b}{\Delta}$$

$$i_{neutral} = i_a - i_b = \frac{N_a - N_b}{\Delta} = \frac{125[(R_1+R_2+11.6)(R_2-R_1)+R_1^2-R_2^2]}{\Delta}$$

Now note that when $R_1 = R_2$, $i_{neutral}$ reduces to

$$i_{neutral} = \frac{0}{12.2R_1^2 + 19.34R_1 + 4.524} = 0$$

P 4.47 The power developed by the 5-A source is zero when the voltage across the source is zero. Only two mesh-current equations are needed to find the voltage across the 5-A source, whereas the node-voltage approach requires three equations.



$$V_{dc} = 37i_1 - 25i_2$$
$$0 = -25i_1 + 125i_2 - 100 + 300$$
$$-200 = -25i_1 + 125i_2$$

$$\Delta = \begin{vmatrix} 37 & -25 \\ -25 & 125 \end{vmatrix} = 4000$$

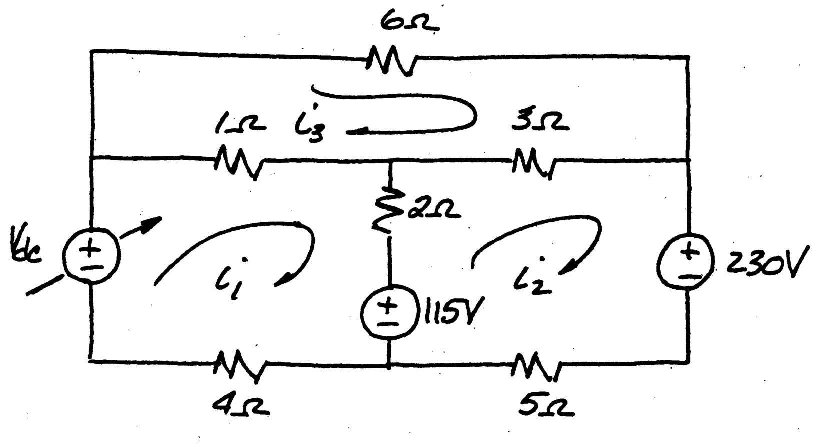
$$N_2 = \begin{vmatrix} 37 & V_{dc} \\ -25 & -200 \end{vmatrix} = 25V_{dc} - 7400$$

$$i_2 = \frac{N_2}{\Delta} = \frac{25V_{dc} - 7400}{4000}$$

$$v_{5A} = (5 - i_2)20 = 100 - 20i_2 = 100 - \frac{(25V_{dc} + 7400)}{200}$$
$$= \frac{20{,}000 - 25V_{dc} + 7400}{200} = \frac{27{,}400 - 25V_{dc}}{200}$$

$v_{5A} = 0$ when $25V_{dc} = 27{,}400$ or $V_{dc} = 1096\text{ V}$

P 4.48 Using the mesh-current method we have



$V_{dc} - 115 = 7i_1 - 2i_2 - i_3$

$-115 = -2i_1 + 10i_2 - 3i_3$

$0 = -i_1 - 3i_2 + 10i_3$

By hypothesis $i_2 = 0$, therefore we find V_{dc} such that $N_2 = 0$.

$$N_2 = \begin{vmatrix} 7 & V_{dc} - 115 & -1 \\ -2 & -115 & -3 \\ -1 & 0 & 10 \end{vmatrix} = -(V_{dc} - 115)(-20 - 3) - 115(70 - 1)$$

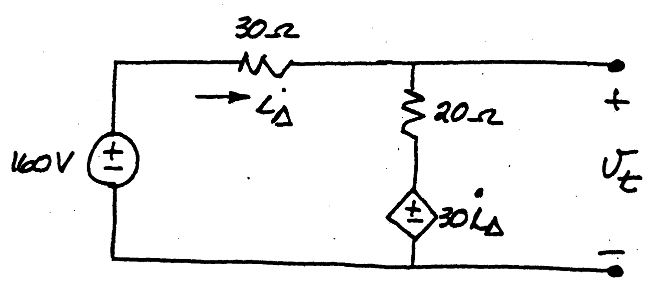
$$= 23(V_{dc} - 115) - 69(115)$$

$$\therefore\ 23(V_{dc} - 115) - 69(115) = 0$$

$$V_{dc} - 115 - 345 = 0$$

$$V_{dc} = 460\text{ V}$$

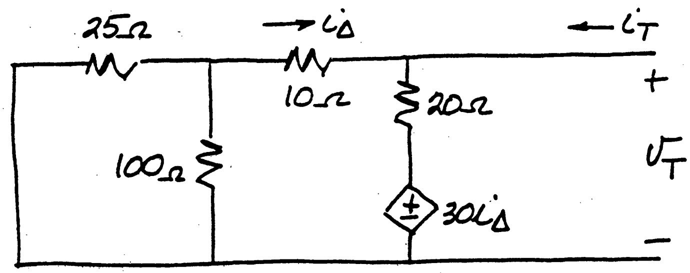
P 4.49 We begin by finding the Thévenin equivalent with respect to R_o. After making a couple of source transformations the circuit simplifies to



$$i_\Delta = \frac{160 - 30i_\Delta}{50}; \qquad i_\Delta = 2\text{ A}$$

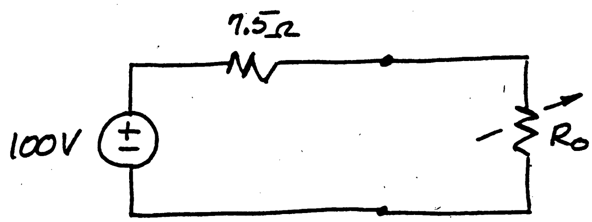
$$v_{\text{Th}} = 20i_\Delta + 30i_\Delta = 50i_\Delta = 100\text{ V}$$

Using the test-source method to find the Thévenin resistance gives



$$i_T = \frac{v_T}{30} + \frac{v_T - 30(-v_T/30)}{20}$$
$$\frac{i_T}{v_T} = \frac{1}{30} + \frac{1}{10} = \frac{4}{30} = \frac{2}{15}$$
$$R_{\mathrm{Th}} = \frac{v_T}{i_T} = \frac{15}{2} = 7.5\,\Omega$$

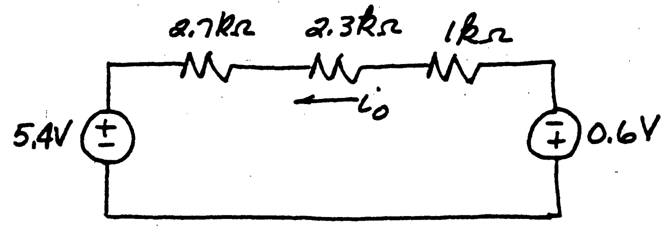
Thus our problem is reduced to analyzing the circuit shown below.



$$p = \left(\frac{100}{7.5 + R_o}\right)^2 R_o = 250$$
$$\frac{10^4}{R_o^2 + 15R_o + 56.25} R_o = 250$$

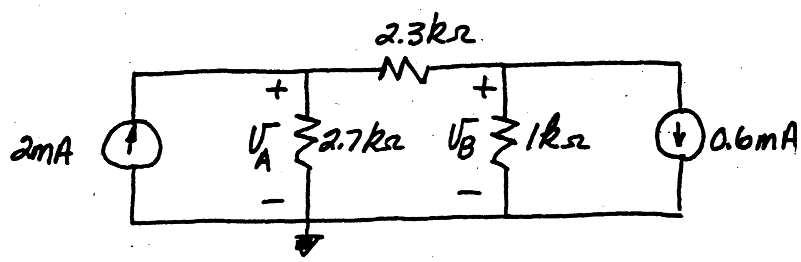
$$\frac{10^4 R_o}{250} = R_o^2 + 15R_o + 56.25$$
$$40R_o = R_o^2 + 15R_o + 56.25$$
$$R_o^2 - 25R_o + 56.25 = 0$$
$$R_o = 12.5 \pm \sqrt{156.25 - 56.25} = 12.5 \pm 10$$
$$R_o = 22.5\,\Omega$$
$$R_o = 2.5\,\Omega$$

P 4.50 [a] Apply source transformations to both current sources to get



$$i_o = \frac{-6}{6} = -1\,\text{mA}$$

[b]



$$-2 \times 10^{-3} + \frac{v_A}{2700} + \frac{v_A - v_B}{2300} = 0$$

$$\frac{v_B - v_A}{2300} + \frac{v_B}{1000} + 0.6 \times 10^{-3} = 0$$

or $2.3v_A + 2.7v_A - 2.7v_B = (2)(2.7)(2.3)$

$$\boxed{5v_A - 2.7v_B = 12.42}$$

$$v_B - v_A + 2.3v_B = -1.38$$

$$\boxed{-v_A + 3.3v_B = -1.38}$$

$$\Delta = \begin{vmatrix} 5 & -2.7 \\ -1 & 3.3 \end{vmatrix} = 13.8$$

$$N_A = \begin{vmatrix} 12.42 & -2.7 \\ -1.38 & 3.3 \end{vmatrix} = 37.26$$

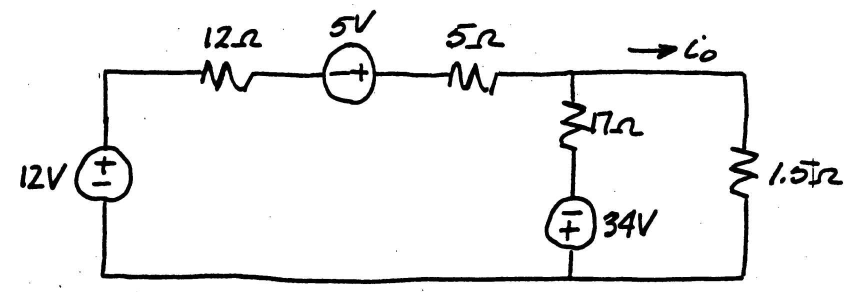
$$N_B = \begin{vmatrix} 5 & 12.42 \\ -1 & -1.38 \end{vmatrix} = 5.52$$

$$\therefore\ v_A = \frac{N_A}{\Delta} = 2.7\,\text{V}$$

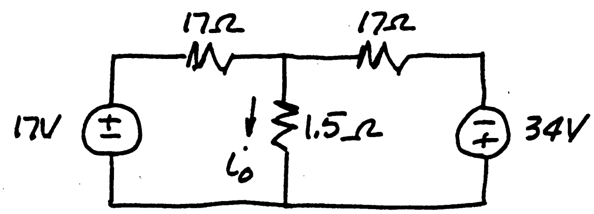
$$\therefore\ v_B = \frac{N_B}{\Delta} = \frac{5.52}{13.8} = 0.4\,\text{V}$$

$$\therefore\ i_o = \frac{v_B - v_A}{2.3} = \frac{-2.3}{2.3} = -1\,\text{mA}$$

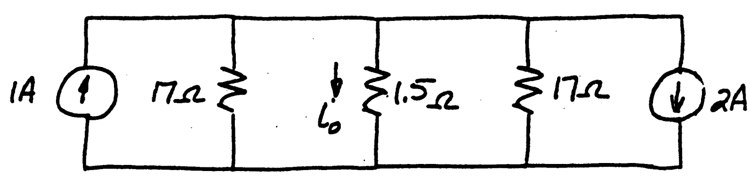
P 4.51 [a] Applying a source transformation to each current source yields



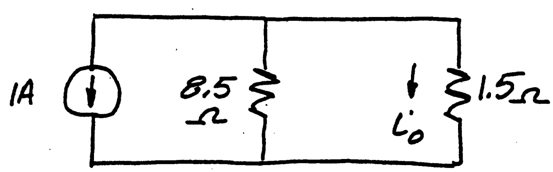
Now combine the 12-V and 5-V sources into a single voltage source and the 12-Ω and 5-Ω resistors into a single resistor, to get



Now use a source transformation on each voltage source, thus

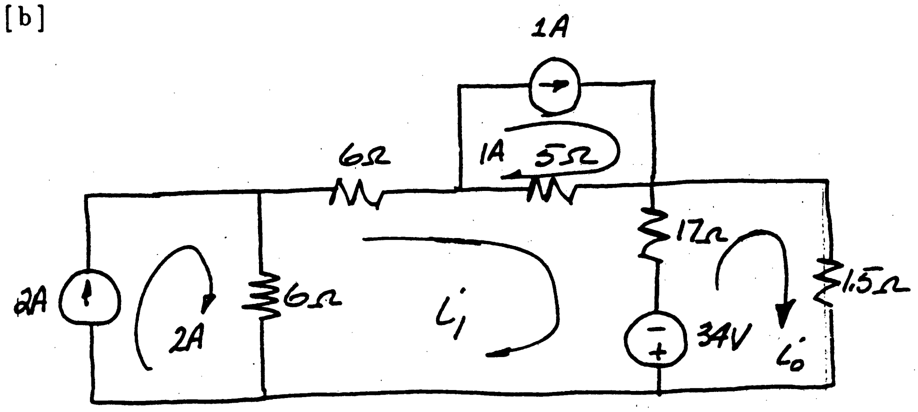


which can be reduced to



$$\therefore\; i_o = \frac{-1(8.5)}{10} = -0.85\,\text{A}$$

[b]



$$6(i_1 - 2) + 6i_1 + 5(i_1 - 1) + 17(i_1 - i_o) - 34 = 0$$

$$34i_1 - 17i_o = 51 \quad \text{or} \quad \boxed{2i_1 - i_o = 3}$$

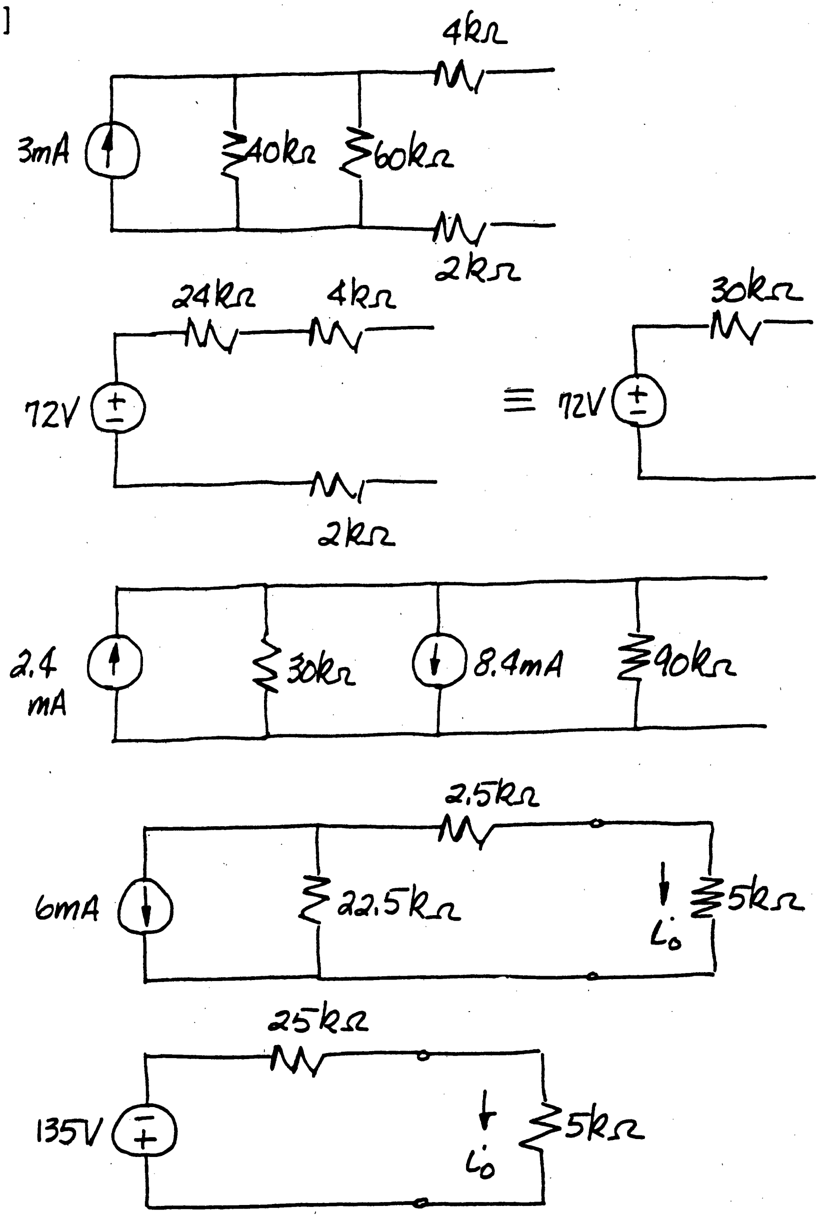
$$\text{or} \quad 34 + 17(i_o - i_1) + 1.5i_o = 0 \quad \text{or} \quad \boxed{-17i_1 + 18.5i_o = -34}$$

$$\Delta = \begin{vmatrix} 2 & -1 \\ -17 & 18.5 \end{vmatrix} = 37 - 17 = 20$$

$$N_o = \begin{vmatrix} 2 & 3 \\ -17 & -34 \end{vmatrix} = -68 + 51 = -17$$

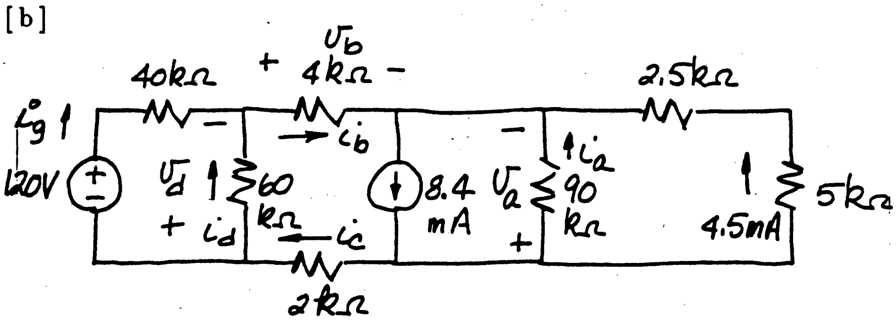
$$i_o = \frac{N_o}{\Delta} = \frac{-17}{20} = -0.85\,\text{A}$$

P 4.52 [a]



$$i_o = \frac{-135}{30} = -4.5\,\text{mA}$$

[b]



$v_a = (7.5)(4.5) = 33.75\,\text{V}$

$i_a = \frac{v_a}{90} = 0.375\,\text{mA}$

$i_b = 8.4 - 0.375 - 4.5 = 3.525\,\text{mA}$

$i_c = i_b = 3.525\,\text{mA}$

$v_b = 4i_b = 4(3.525) = 14.10\,\text{V}$

$v_c = 2i_c = 2(3.525) = 7.05\,\text{V}$

$v_d + 14.10 - 33.75 + 7.05 = 0$

$v_d = 33.75 - 21.15 = 12.60\,\text{V}$

$i_d = \frac{v_d}{60} = 0.21\,\text{mA}$

$i_g = i_b - i_d = 3.525 - 0.21 = 3.315\,\text{mA}$

$p_{120\text{V}}(\text{developed}) = (120)(3.315) = 397.80\,\text{mW}$

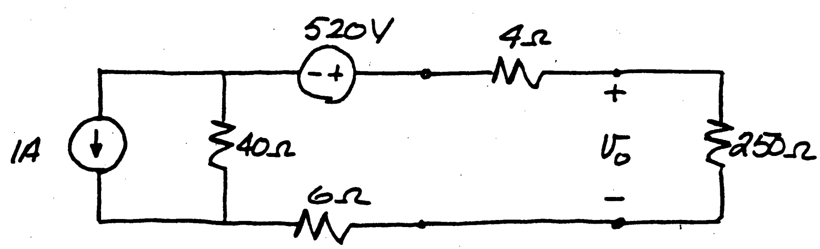
Check:

$p_{8.4\,\text{mA}}(\text{developed}) = (33.75)(8.4) = 283.50\,\text{mW}$

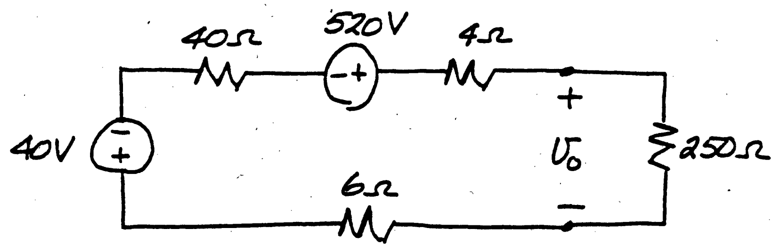
$\sum P_{\text{dev}} = 397.80 + 283.50 = 681.30\,\text{mW}$

$$\sum P_{\text{diss}} = (3.315)^2(40) + (0.21)^2(60) + (3.525)^2(6) + (0.375)^2(90) + (4.5)^2(7.5) = 681.30\,\text{mW}$$

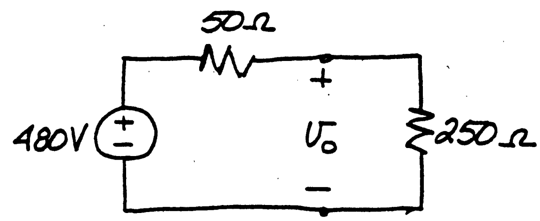
P 4.53 [a] First remove the 16-Ω and 260-Ω resistors.



Next use a source transformation to convert the 1-A current source and 40-Ω resistor

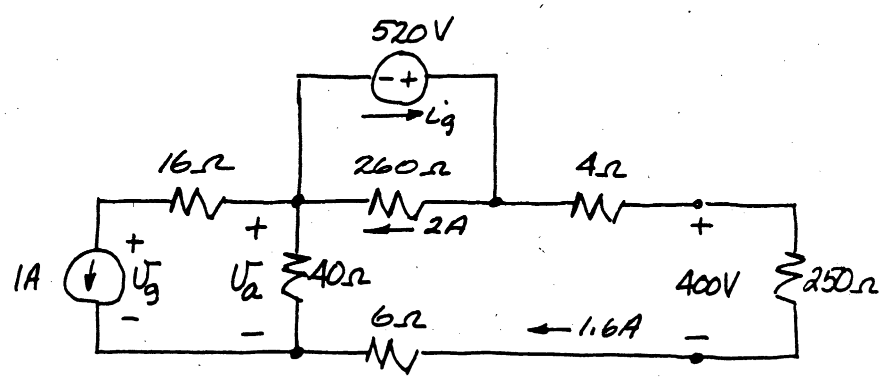


which simplifies to



It follows that $v_o = \frac{480}{300}(250) = 400\,\text{V}$

[b] Return to the original circuit with $v_o = 400\,\text{V}$



$$i_g = \frac{520}{260} + 1.6 = 3.6\,\text{A}$$

$p_{520V}(\text{developed}) = (520)(3.6) = 1872\,\text{W}$

[c] $v_a + 520 - 1.6(250 + 4 + 6) = 0$

$v_a = -104\text{ V}$

$16(1) + v_g = -104$

$v_g = -120\text{ V}$

$p_{1\text{A}}(\text{developed}) = (120)(1) = 120\text{ W}$

[d] $\sum p_{\text{dev}} = 1872 + 120 = 1992\text{ W}$

$p_{16\Omega} = 16\text{ W}$

$$p_{40\Omega} = \frac{(104)^2}{40} = 270.40\text{ W}$$

$p_{260\Omega} = (2)^2(260) = 1040\text{ W}$

$p_{4\Omega} = (1.6)^2(4) = 10.24\text{ W}$

$$p_{250\Omega} = \frac{(400)^2}{250} = 640\text{ W}$$

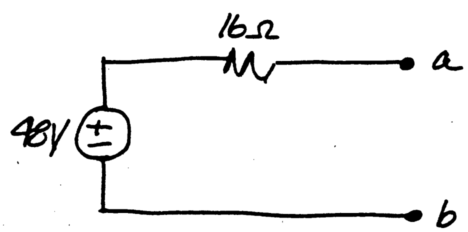
$p_{6\Omega} = (1.6)^2(6) = 15.36\text{ W}$

$\sum p_{\text{diss}} = 1992\text{ W}$

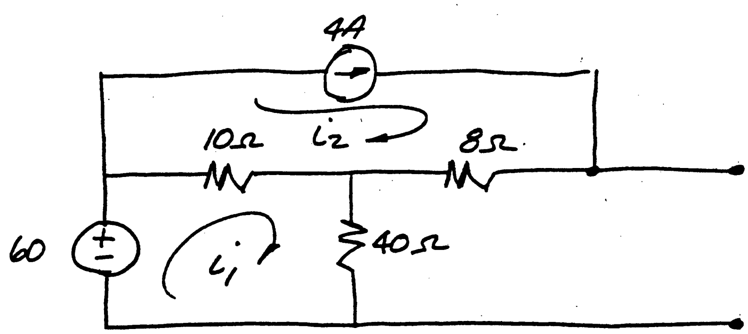
$\therefore \sum p_{\text{dev}} = \sum p_{\text{diss}} = 1992\text{ W}$

P 4.54 $$v_{\text{Th}} = \frac{60}{50} \times 40 = 48\text{ V}$$

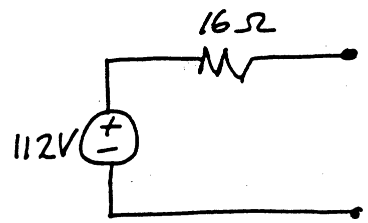
$$R_{\text{Th}} = 8 + \frac{(40)(10}{50} = 16\,\Omega$$



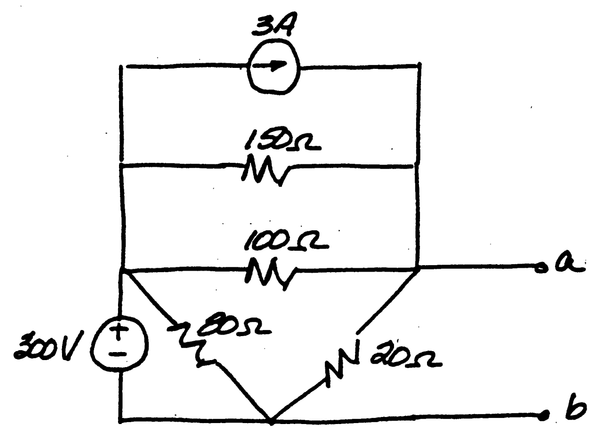
P 4.55



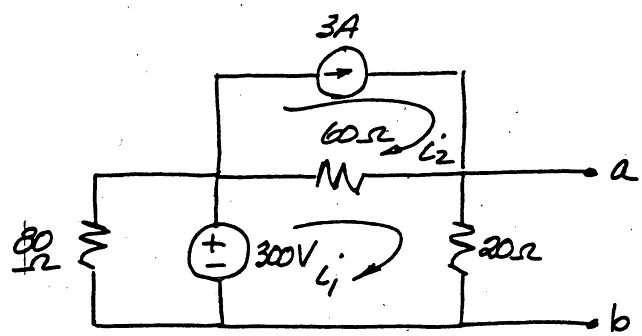
$$60 = 10(i_1 - 4) + 40i_1$$
$$100 = 50i_1$$
$$i_1 = 2\,\text{A}$$
$$v_{\text{Th}} = 8i_2 + 40i_1 = 32 + 80 = 112\,\text{V}$$
$$R_{\text{Th}} = 8 + \frac{(10)(40)}{50} = 16\,\Omega$$



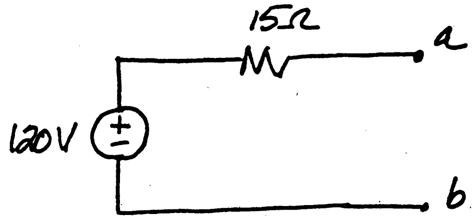
P 4.56 After replacing the 40Ω–10Ω–8Ω wye with an equivalent delta the circuit becomes



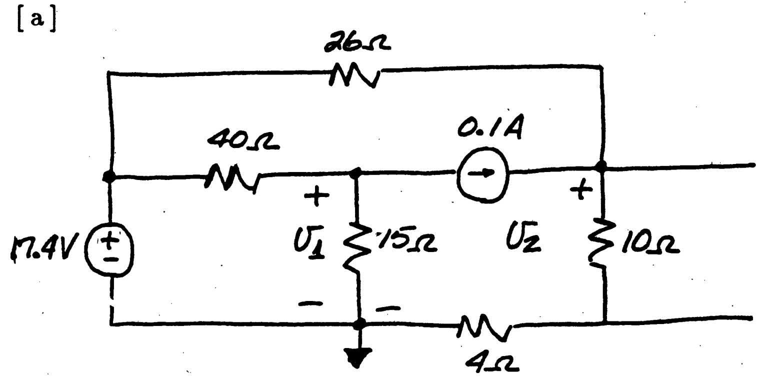
Now the circuit can be further simplified to



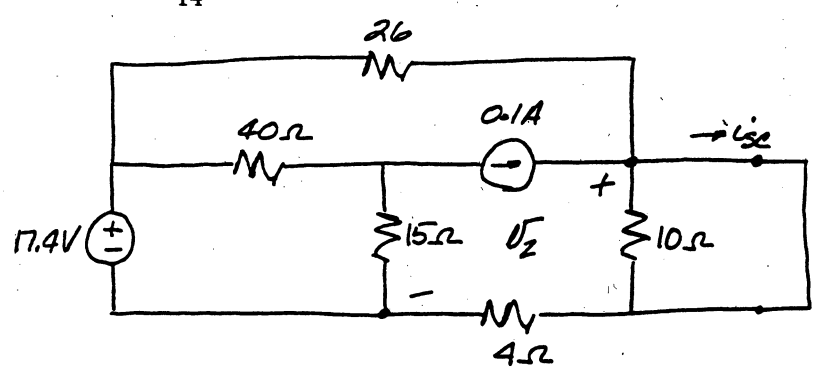
$$300 = 60(i_1 - 3) + 20i_1$$
$$300 = 80i_1 - 180$$
$$i_1 = \frac{480}{80} = 6\text{ A}$$
$$v_{Th} = (6)(20) = 120\text{ V}; \qquad R_{Th} = \frac{(60)(20)}{80} = 15\,\Omega$$



P 4.57 [a]



$$\frac{v_2 - 17.4}{26} + \frac{v_2}{14} - 0.1 = 0$$
$$14v_2 - 243.6 + 26v_2 = 36.4$$
$$40v_2 = 280; \qquad v_2 = 7\text{ V}$$
$$v_{Th} = \frac{v_2}{14}(10) = 5\text{ V}$$



$$\frac{v_2 - 17.4}{26} - 0.1 + \frac{v_2}{4} = 0$$

$$4v_2 - 69.6 + 26v_2 - 10.4 = 0$$

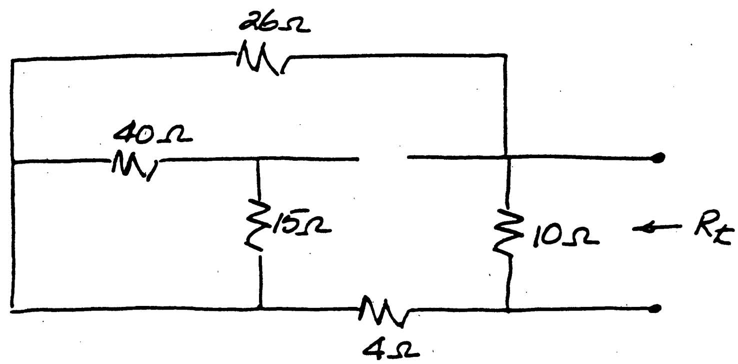
$$30v_2 = 80$$

$$v_2 = \frac{8}{3}\text{ V}$$

$$i_{sc} = \frac{v_2}{4} = \frac{2}{3}\text{ A}$$

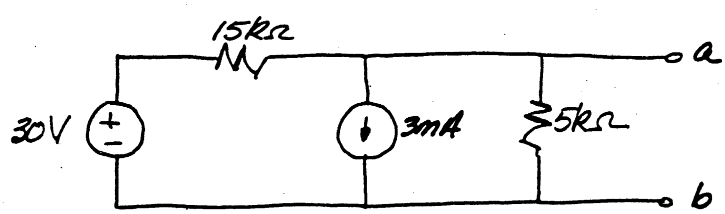
$$R_{\text{Th}} = \frac{5}{(2/3)} = 7.5\,\Omega$$

[b]

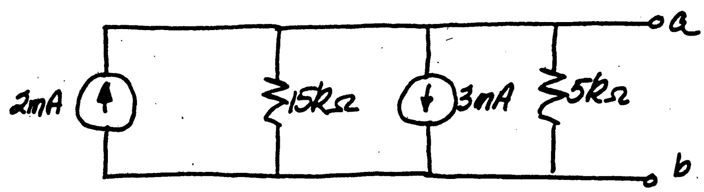


$$R_{\text{Th}} = 30//10 = 7.5\,\Omega$$

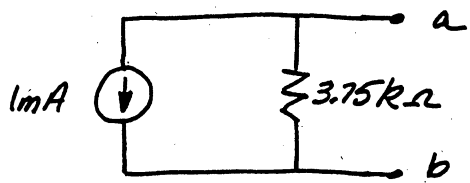
P 4.58 First we make the observation that the 10-mA current source and the 10-kΩ resistor will have no influence on the behavior of the circuit with respect to the terminals a,b. This follows because they are in parallel with an ideal voltage source. Hence our circuit can be simplified to



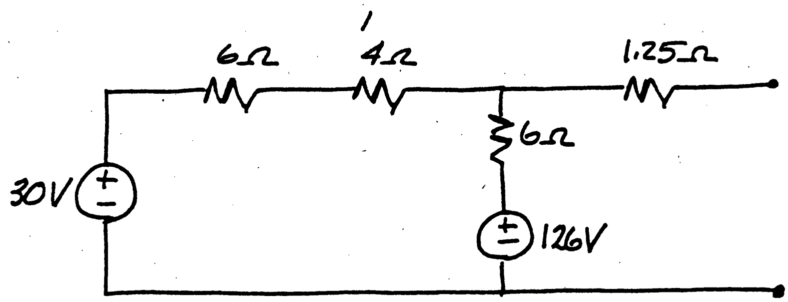
or

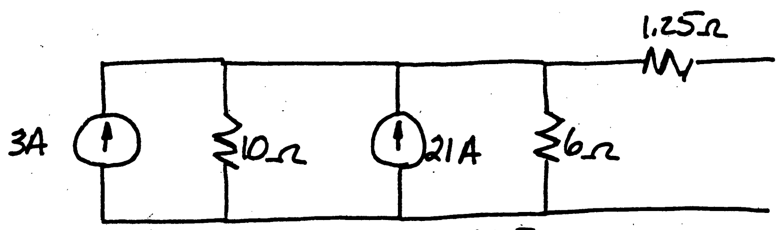


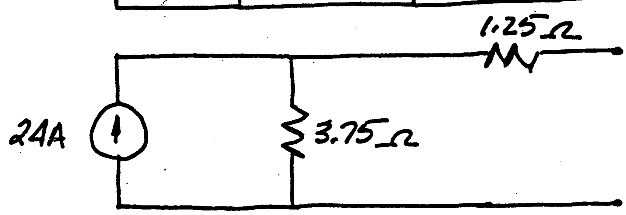
Therefore the Norton equivalent is

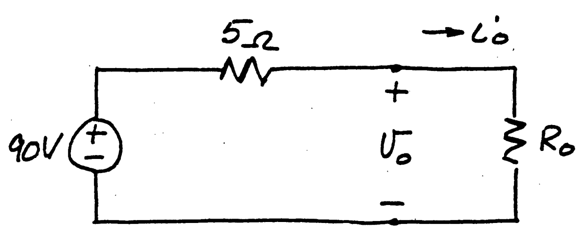


P 4.59 First, find the Thévenin equivalent with respect to R_o.
$15//10 = 6\,\Omega$



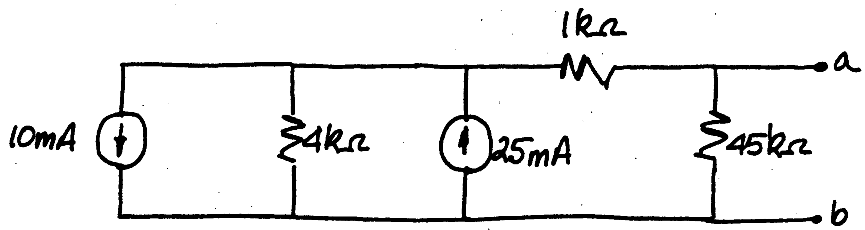


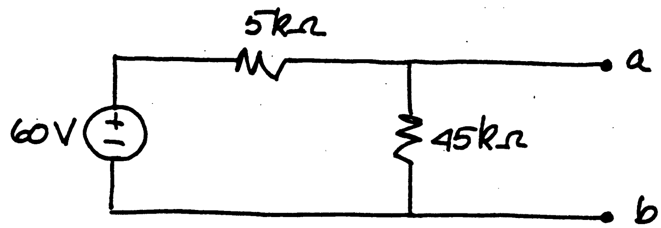




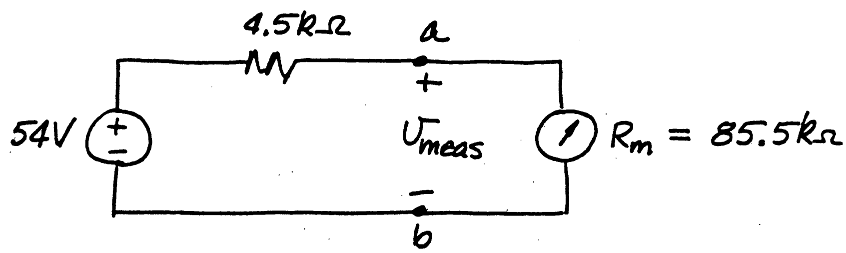
R_o	i_o	v_o	R_o	i_o	v_o
0	18	0	25	3	75
1	15	15.00	40	2	80
3	11.25	33.75	55	1.5	82.50
5	9.00	45.00	70	1.2	84.00
10	6.00	60.00	85	1.0	85.00
15	4.50	67.50	95	0.9	85.50

P 4.60 [a] First, find the Thévenin equivalent with respect to a,b using a succession of source transformations.





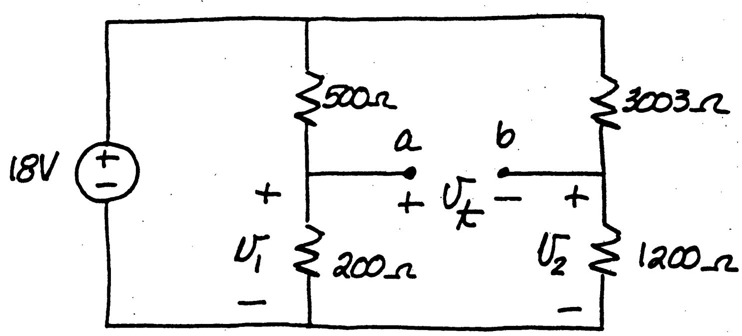
$$\therefore\ v_{Th} = \frac{45}{50}(60) = 54\,V; \qquad R_{Th} = \frac{(5)(45)}{50} = 4.5\,k\Omega$$



$$v_{meas} = \frac{54}{90}(85.5) = 51.30\,V$$

[b] $$\% \text{ error} = \left(\frac{51.30 - 54}{54}\right) \times 100 = -5.00\%$$

P 4.61

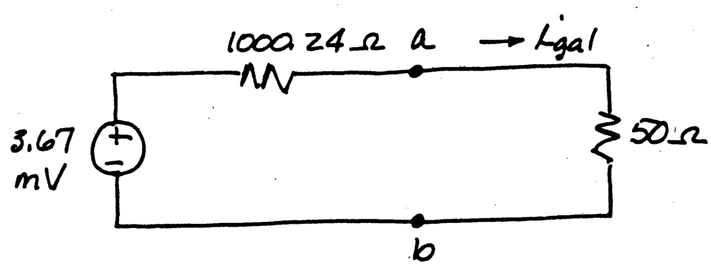


$$v_1 = \frac{18}{700}(200) = \frac{36}{7}\ \text{V}$$

$$v_2 = \frac{18}{4203}(1200)\ \text{V}$$

$$v_{\text{Th}} = v_1 - v_2 = \frac{36}{7} - \frac{21{,}600}{4203} = \frac{108}{29{,}421} = 3.67\,\text{mV}$$

$$R_{\text{Th}} = \frac{(500)(200)}{700} + \frac{(3003)(1200)}{4203} = \frac{29{,}428{,}200}{29{,}421} = 1000.24\,\Omega$$



$$i_{\text{gal}} = \frac{3.67 \times 10^{-3}}{1.05024 \times 10^{3}} = 3.50\,\mu\text{A}$$

P 4.62 $v_{\text{Th}} = -80i_b(50 \times 10^3) = -40 \times 10^5 i_b$

$(500 \times 10^{-6})(100) = 1410i_b + (4 \times 10^{-5})(-40 \times 10^5 i_b)$

$0.05 = (1410 - 160)i_b = 1250i_b$

$i_b = 40\,\mu\text{A}$

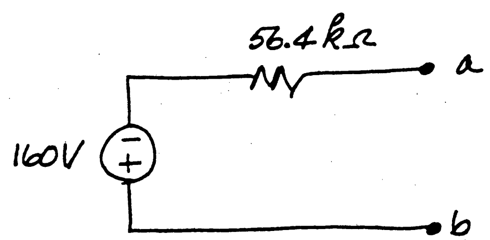
$\therefore\ v_{\text{Th}} = (-40 \times 10^5)(40 \times 10^{-6}) = -160\,\text{V}$

$i_{sc} = -80i_b$

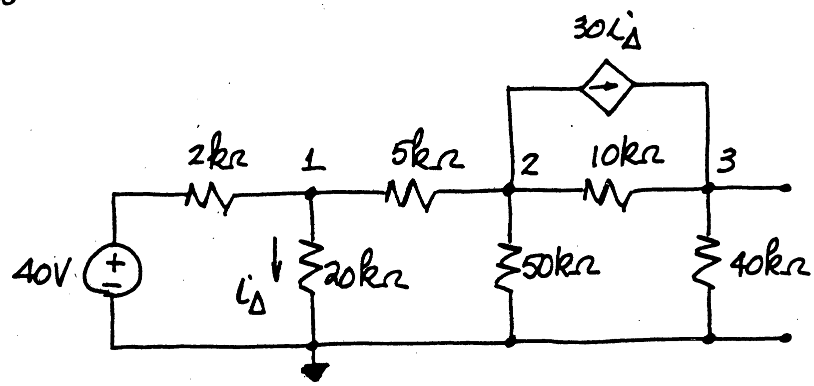
$$i_b = \frac{(500)(100)}{1410} = \frac{5000}{141}\,\mu\text{A}$$

$$i_{sc} = \frac{-80(5000)}{141} \times 10^{-6}$$

$$R_{\text{Th}} = \frac{-160(141)}{-80(5000) \times 10^{-6}} = 56.40\,\text{k}\Omega$$



P 4.63



$$\frac{v_1 - 40}{2} + \frac{v_1}{20} + \frac{v_1 - v_2}{5} = 0$$

$$\boxed{15v_1 - 4v_2 + 0v_3 = 400}$$

$$\frac{v_2}{50} + \frac{v_2 - v_1}{5} + \frac{v_2 - v_3}{10} + 30\left(\frac{v_1}{20}\right) = 0$$

$$\boxed{130v_1 + 32v_2 - 10v_3 = 0}$$

$$\frac{v_3}{40} + \frac{v_3 - v_2}{10} - 30\left(\frac{v_1}{20}\right) = 0$$

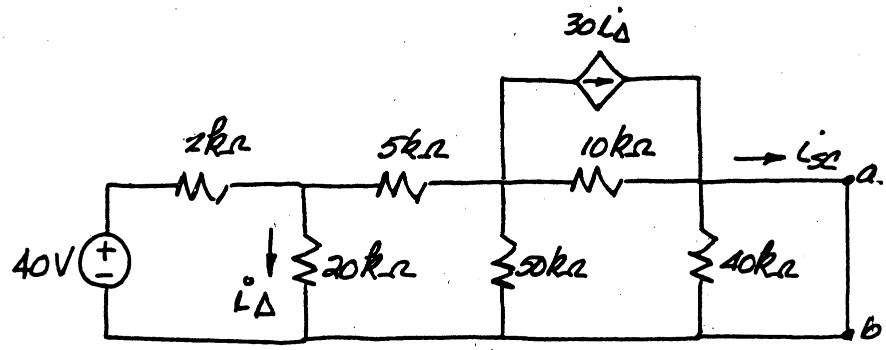
$$\boxed{-60v_1 - 4v_2 + 5v_3 = 0}$$

$$\Delta = \begin{vmatrix} 15 & -4 & 0 \\ 130 & 32 & -10 \\ -60 & -4 & 5 \end{vmatrix} = 10(-60 - 240) + 5(480 + 520) = 2000$$

$$N_3 = \begin{vmatrix} 15 & -4 & 400 \\ 130 & 32 & 0 \\ -60 & -4 & 0 \end{vmatrix} = 400(-520 + 1920) = 560{,}000$$

$$v_3 = \frac{N_3}{\Delta} = 280\text{ V}$$

$$\therefore\ v_{\text{Th}} = 280\text{ V}$$



$$\frac{v_1}{20} + \frac{v_1 - 40}{2} + \frac{v_1 - v_2}{5} = 0$$

$$\boxed{15v_1 - 4v_2 = 400}$$

$$\frac{v_2}{50} + \frac{v_2 - v_1}{5} + \frac{v_2}{10} + 30\left(\frac{v_1}{20}\right) = 0$$

$$\boxed{130v_1 + 32v_2 = 0}$$

$$\therefore\ \Delta = \begin{vmatrix} 15 & -4 \\ 130 & 32 \end{vmatrix} = 1000$$

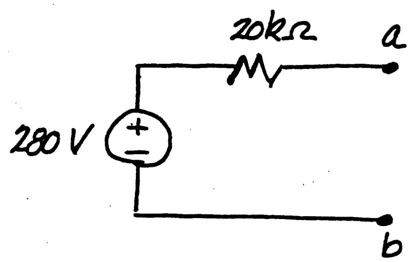
$$N_1 = \begin{vmatrix} 400 & -4 \\ 0 & 32 \end{vmatrix} = 12{,}800$$

$$N_2 = \begin{vmatrix} 15 & 400 \\ 130 & 0 \end{vmatrix} = -52{,}000$$

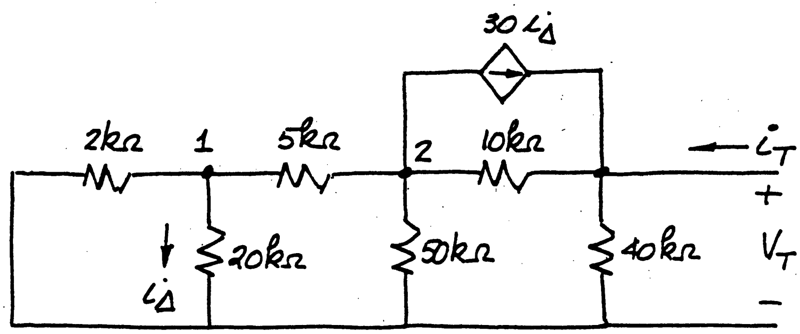
$$\therefore\ v_1 = \frac{N_1}{\Delta} = 12.8\,\text{V}; \qquad v_2 = \frac{N_2}{\Delta} = -52\,\text{V}$$

$$i_{sc} = 30i_\Delta + \frac{v_2}{10} = 30\left(\frac{12.8}{20}\right) - \frac{52}{10} = 14\,\text{mA}$$

$$R_{\text{Th}} = \frac{v_{\text{Th}}}{i_{sc}} = \left(\frac{280}{14}\right)10^3 = 20\,\text{k}\Omega$$



Alternate calculation of R_{Th} :



$$i_T = \frac{v_T}{40} + \frac{v_T - v_2}{10} - 30\left(\frac{v_1}{20}\right)$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{50} + \frac{v_2 - v_T}{10} + 30\left(\frac{v_1}{20}\right) = 0$$

$$\boxed{130v_1 + 32v_2 = 10v_T}$$

$$\frac{v_1}{20} + \frac{v_1}{2} + \frac{v_1 - v_2}{5} = 0$$

$$\boxed{15v_1 - 4v_2 = 0}$$

$$\Delta = \begin{vmatrix} 130 & 32 \\ 15 & -4 \end{vmatrix} = -1000$$

$$N_1 = \begin{vmatrix} 10v_T & 32 \\ 0 & -4 \end{vmatrix} = -40v_T$$

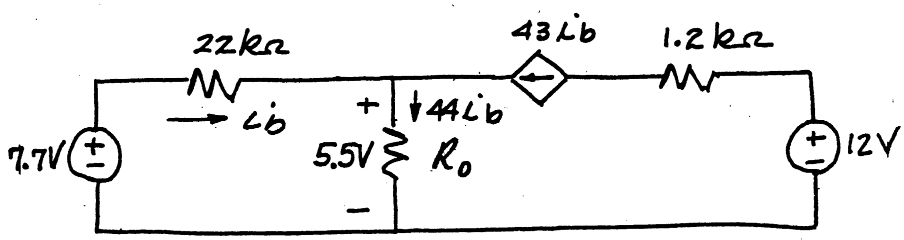
$$N_2 = \begin{vmatrix} 130 & 10v_T \\ 15 & 0 \end{vmatrix} = -150v_T$$

$$\therefore\ v_1 = 0.04v_T \qquad \text{and} \qquad v_2 = 0.15v_T$$

$$\therefore\ i_T = \frac{v_T}{40} + \frac{v_T - 0.15v_T}{10} - 1.5(0.04)v_T$$

$$\frac{i_T}{v_T} = 0.05\,\text{m℧}; \qquad R_{\text{Th}} = \frac{v_T}{i_T} = 20\,\text{k}\Omega$$

P 4.64 [a] Use source transformations to simplify the left side of the circuit.



$$i_b - \frac{7.7-5.5}{22} = 0.10\,\text{mA}$$

Let $R_o = R_{\text{meter}}//1.3\,\text{k}\Omega$

$$R_o = \frac{5.5}{4.4} = 1.25\,\text{k}\Omega$$

$$\therefore\ \frac{(R_{\text{meter}})(1.3)}{R_{\text{meter}}+1.3} = 1.25; \qquad R_{\text{meter}} = \frac{(1.3)(1.25)}{0.05} = 32.50\,\text{k}\Omega$$

[b] Actual value of v_e:

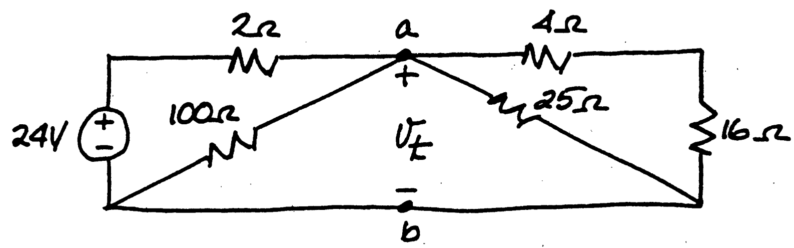
$$i_b = \frac{7.7}{22+44(1.3)} \cong 0.09722\,\text{mA}$$

$$v_e = 44i_b(1.3) \cong 5.56\,\text{V}$$

$$\%\ \text{error} = \left(\frac{5.5-5.56}{5.56}\right)100 = -1.10\%$$

P 4.65 [a] Find the Thévenin equivalent with respect to the terminals of the ammeter. This is most easily done by first finding the Thévenin with respect to the terminals of the 4.8 resistor.

Thévenin voltage: note i_ϕ is zero.

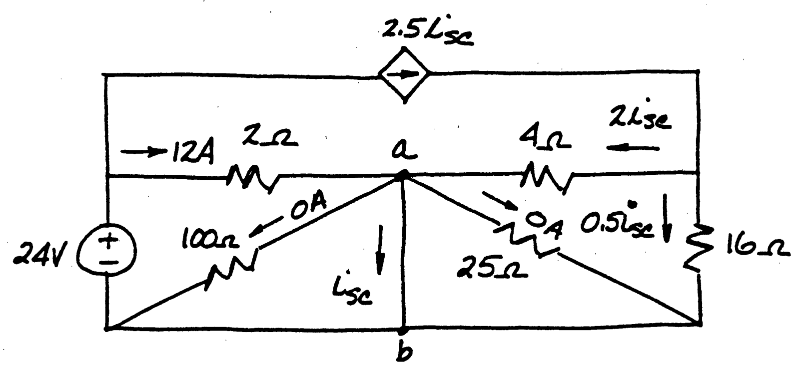


$$\frac{v_{\text{Th}}}{100} + \frac{v_{\text{Th}}}{25} + \frac{v_{\text{Th}}}{20} + \frac{v_{\text{Th}}-24}{2} = 0$$

$$v_{\text{Th}} + 4v_{\text{Th}} + 5v_{\text{Th}} + 50v_{\text{Th}} - 1200 = 0$$

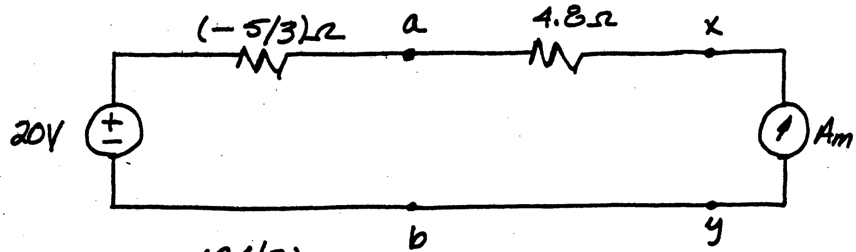
$$60v_{\text{Th}} = 1200; \qquad v_{\text{Th}} = 20\,\text{V}$$

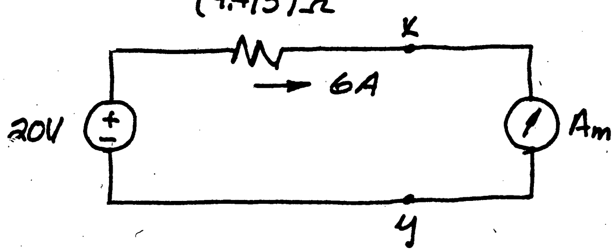
Short-circuit current:



$$i_{sc} = 12 + 2i_{sc}, \qquad \therefore\ i_{sc} = -12\,\text{A}$$

$$R_{\text{Th}} = \frac{20}{-12} = -\frac{5}{3}\,\Omega$$

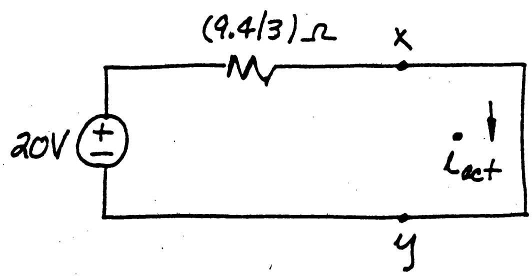




$$R_{\text{total}} = \frac{20}{6} = \frac{10}{3}\,\Omega$$

$$R_{\text{meter}} = \frac{10}{3} - \frac{9.4}{3} = \frac{0.6}{3} = 0.20\,\Omega$$

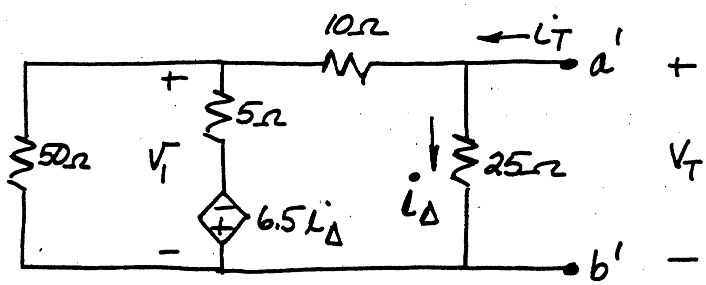
[b] Actual current:



$$i_{\text{actual}} = \frac{20}{9.4}(3) = \frac{60}{9.4} = \frac{600}{94} \approx 6.38\,\text{A}$$

$$\%\ \text{error} = \frac{(6 - 30/4.7)}{30/4.7} \times 100 = \left(\frac{28.2 - 30}{30}\right) \times 100 = -6\%$$

P 4.66 V_{Th}, since there are no independent sources in the circuit. To find R_{Th} we first find $R_{a'b'}$.



$$i_T = \frac{V_T}{25} + \frac{V_T - v_1}{10} = \frac{V_T}{25} + \frac{V_T}{10} - \frac{1}{10}v_1$$

$$\frac{v_1}{50} + \frac{v_1 + 6.5(V_T/25)}{5} + \frac{v_1 - V_T}{10} = 0$$

$$v_1 + 10v_1 + \frac{65}{25}V_T + 5v_1 - 5V_T = 0$$

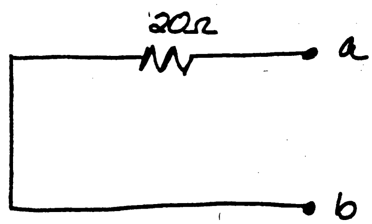
$$16v_1 = 5V_T - \frac{13}{5}V_T$$

$$v_1 = \frac{5}{16}V_T - \frac{13}{80}V_T$$

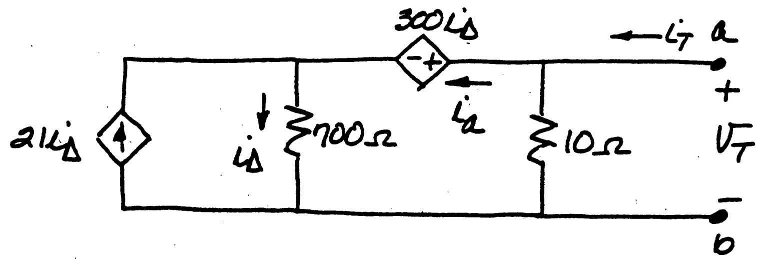
$$\therefore\; i_T = \frac{V_T}{25} + \frac{V_T}{10} - \frac{5}{160}V_T + \frac{13}{800}V_T$$

$$\frac{i_T}{V_T} = \frac{32 + 80 - 25 + 13}{800} = \frac{100}{800} = \frac{1}{8}\mho$$

$$R_{a'b'} = \frac{V_T}{i_T} = 8\,\Omega; \qquad R_{ab} = R_{a'b'} + 12 = 20\,\Omega; \qquad R_{\text{Th}} = 20\,\Omega$$



P 4.67 $V_{\text{Th}} = 0$, since circuit contains no independent sources.



$$i_T = \frac{V_T}{10} + i_a$$

$$i_a = i_\Delta - 21 i_\Delta = -20 i_\Delta$$

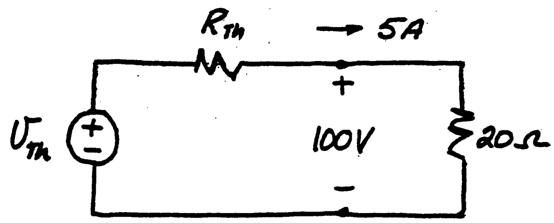
$$i_\Delta = \frac{v_T - 300 i_\Delta}{700}, \qquad 1000 i_\Delta = v_T$$

$$\therefore \ i_T = \frac{V_T}{10} - 20\left(\frac{v_T}{1000}\right) = \frac{v_T}{10} - \frac{v_T}{50}$$

$$\frac{i_T}{V_T} = \frac{1}{10} - \frac{1}{50} = \frac{4}{50}$$

$$R_{\text{Th}} = \frac{V_T}{i_T} = \frac{50}{4} = 12.5\,\Omega$$

P 4.68

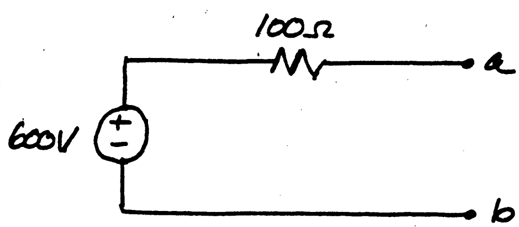


$$100 = v_{\text{Th}} - 5R_{\text{Th}} \qquad \text{and} \qquad 200 = v_{\text{Th}} - 4R_{\text{Th}}$$

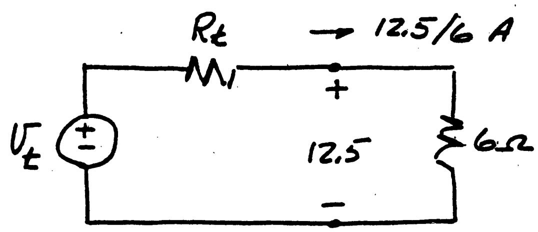
$$\therefore \ 100 = 1R_{\text{Th}}, \qquad R_{\text{Th}} = 100\,\Omega$$

$$\therefore \ v_{\text{Th}} = 100 + 5(100) = 600\,\text{V} \qquad \text{or} \qquad v_{\text{Th}} = 200 + 4(100) = 600\,\text{V}$$

$$v_{\text{Th}} = 600\,\text{V}, \qquad R_{\text{Th}} = 100\,\Omega$$



P 4.69



$$12.5 = v_{\text{Th}} - \frac{12.5}{6}R_{\text{Th}}; \qquad 11.8 = v_{\text{Th}} - \frac{11.8}{0.75}R_{\text{Th}}; \qquad 0.7 = \left(\frac{11.8}{0.75} - \frac{12.5}{6}\right)R_{\text{Th}}$$

$R_{\text{Th}} = 51.28\,\text{m}\Omega$

$$v_{\text{Th}} = 12.5 + \frac{12.5}{6}(51.28) \times 10^{-3}) = 12.61\,\text{V}$$

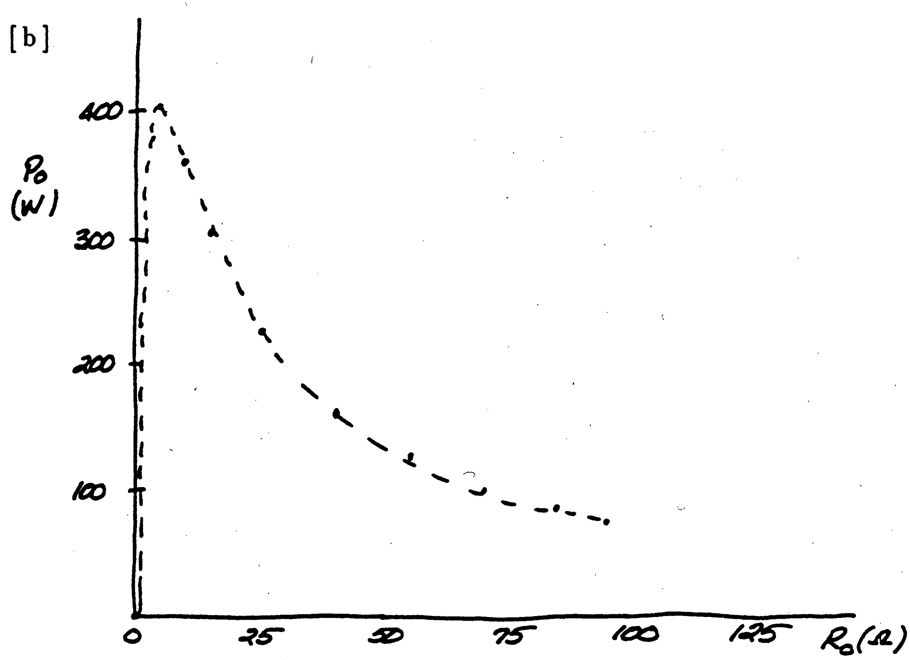
$$i_N = \frac{12.61}{0.05128} = 245.83\,\text{A}$$

$R_N = R_{\text{Th}} = 51.28\,\text{m}\Omega$

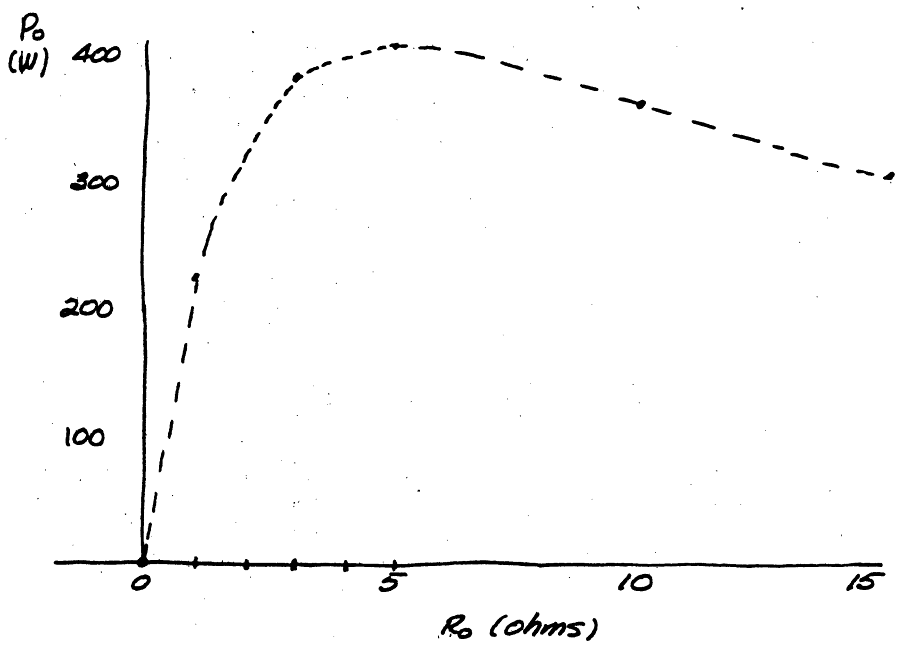
P 4.70 [a]

$R_o(\Omega)$	$P_o(\text{W})$	$R_o(\Omega)$	$P_o(\text{W})$
0	0	25	225.00
1	225.00	40	160.00
3	379.69	55	123.75
5	405.00	70	100.80
10	360.00	85	85.00
15	303.75	95	76.95

[b]



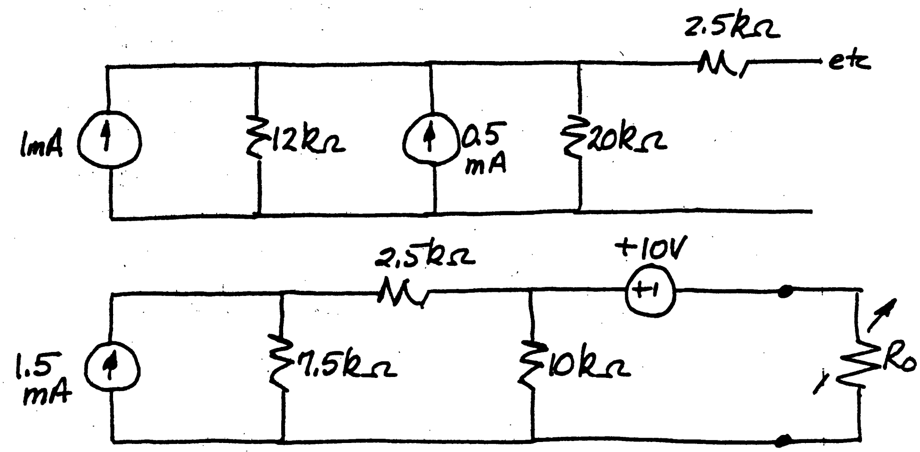
Magnify lower range of R_o scale.



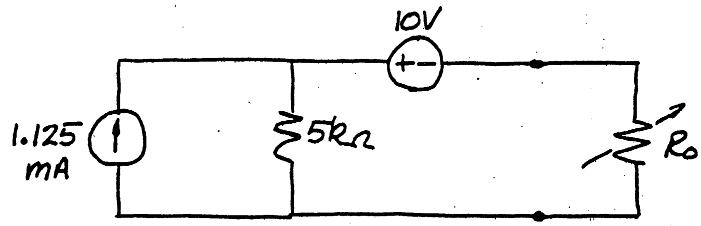
[c] $R_o = 5\,\Omega$, $P_o(\text{max}) = 405\text{ W}$

P 4.71 [a] First find the Thévenin equivalent with respect to the terminals of R_o.

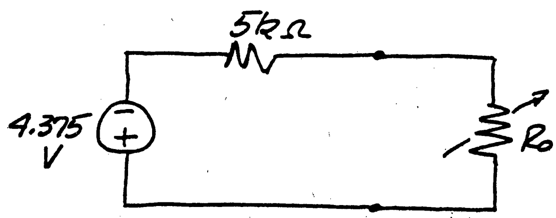
After a series of source transformations we have



Continuing with source transformations leads to



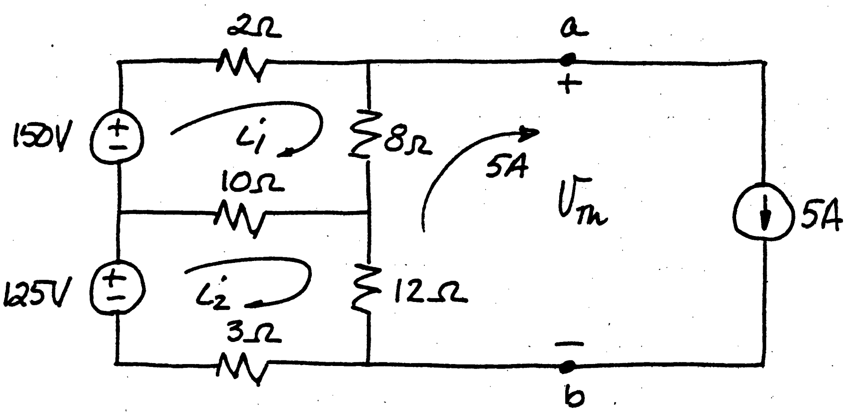
Finally we get



$\therefore\ R_o = R_{\text{Th}} = 5\,\text{k}\Omega$

[b] $P = \left(\frac{4.375}{10^4}\right)^2 (5000) = 957.03\,\mu\text{W}$

P 4.72 First find the Thévenin equivalent with respect to the terminals of R_o.



$150 = 20i_1 - 10i_2 - 40$

$190 = 20i_1 - 10i_2$

$\boxed{19 = 2i_1 - i_2}$

$125 = -10i_1 + 25i_2 - 60$

$\boxed{185 = -10i_1 + 25i_2}$

$$\Delta = \begin{vmatrix} 2 & -1 \\ -10 & 25 \end{vmatrix} = 40$$

$$N_1 = \begin{vmatrix} 19 & -1 \\ 185 & 25 \end{vmatrix} = 660$$

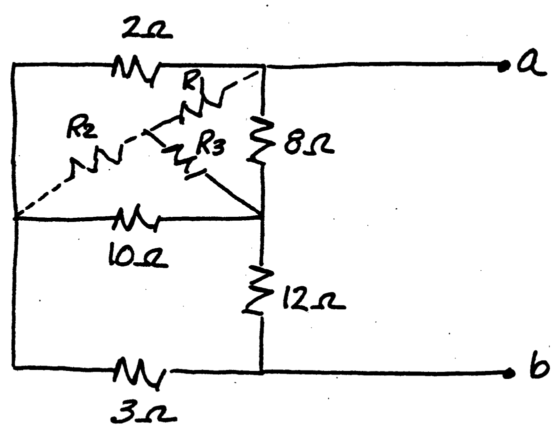
$$N_2 = \begin{vmatrix} 2 & 19 \\ -10 & 185 \end{vmatrix} = 560$$

$$i_1 = \frac{N_1}{\Delta} = \frac{660}{40} = 16.5\,\text{A}$$

$$i_2 = \frac{N_2}{\Delta} = \frac{560}{40} = 14\,\text{A}$$

$v_{Th} = 8(i_1 - 5) + 12(i_2 - 5) = 8(11.5) + 12(9) = 200\text{V}$

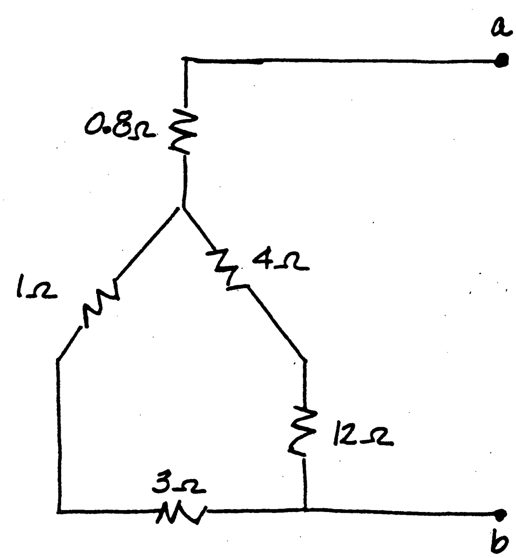
Thévenin resistance:



$$R_1 = \frac{16}{20} = 0.8\,\Omega$$

$$R_2 = \frac{20}{20} = 1.0\,\Omega$$

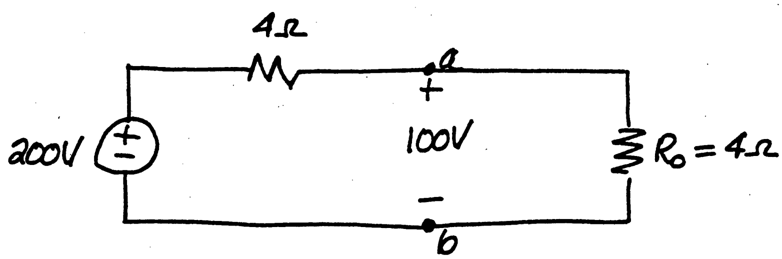
$$R_3 = \frac{80}{20} = 4.0\,\Omega$$



$$R_{Th} = 0.8 + \frac{(16)(4)}{20} = 0.8 + 3.2$$

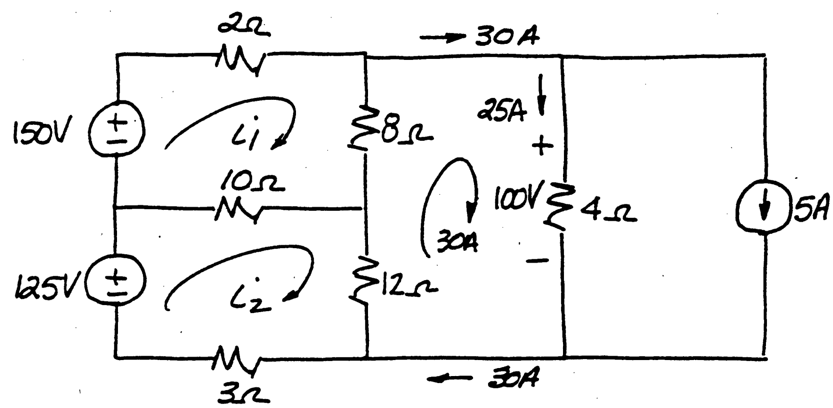
$R_{Th} = 4\,\Omega$

Thévenin equivalent:



$$P_{\max} = \frac{(100)^2}{4} = 2500\,\text{W}$$

Now we return to the original circuit and set $R_o = 4\,\Omega$. We also use the fact that when $R_o = 4\,\Omega$, $v_{ab} = 100\,\text{V}$. Therefore we get the circuit shown below. After studying this circuit we note the current source is absorbing power. Hence we must find i_1 and i_2 to see if both voltage sources are developing power.



$150 = 20i_1 - 10i_2 - 240$

$390 = 20i_1 - 10i_2$

$\boxed{39 = 2i_1 - i_2}$

$125 = -10i_1 + 25i_2 - 360$

$\boxed{485 = -10i_1 + 25i_2}$

As before $\Delta = 40$

$$N_1 = \begin{vmatrix} 39 & -1 \\ 485 & 25 \end{vmatrix} = 1460$$

$$N_2 = \begin{vmatrix} 2 & 39 \\ -10 & 485 \end{vmatrix} = 1360$$

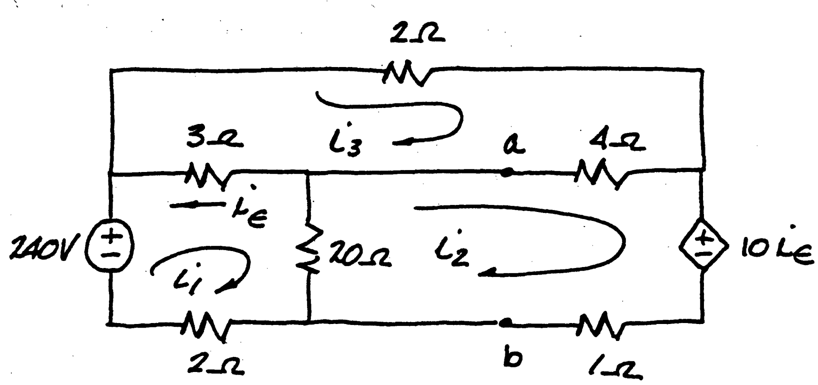
$$i_1 = \frac{N_1}{\Delta} = 36.5\,\text{A}; \qquad i_2 = \frac{N_2}{\Delta} = 34\,\text{A}$$

Thus both voltage sources are developing power.

$$\sum p_{\text{dev}} = 150(36.5) + 125(34) = 9725\,\text{W}$$

$$\%\text{ delivered to } R_o = \frac{2500}{9725} \times 100 = 25.71\%$$

P 4.73 [a] Find the Thévenin equivalent with respect to the terminals of R_L.



$$240 = 25i_1 - 20i_2 - 3i_3$$
$$0 = -20i_1 + 25i_2 + 10(i_3 - i_1) - 4i_3$$
$$0 = -3i_1 - 4i_2 + 9i_3$$
$$240 = 25i_1 - 20i_2 - 3i_3$$
$$0 = -30i_1 + 25i_2 + 6i_3$$
$$0 = -3i_1 - 4i_2 + 9i_3$$

$$\Delta = \begin{vmatrix} 25 & -20 & -3 \\ -30 & 25 & 6 \\ -3 & -4 & 9 \end{vmatrix} = 25(249) + 30(-192) - 3(-45) = 600$$

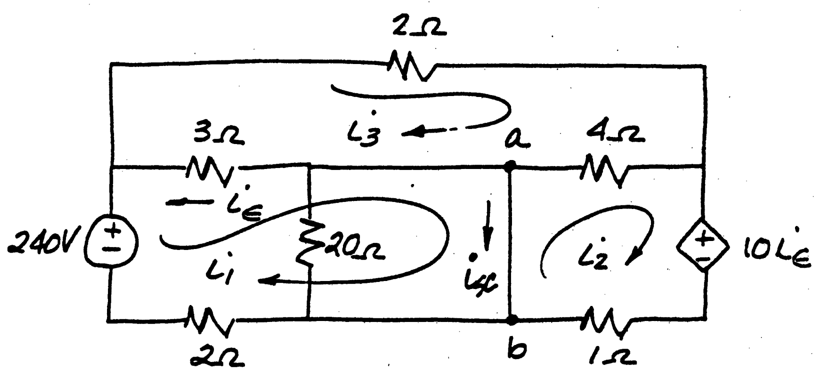
$$N_1 = \begin{vmatrix} 240 & -20 & -3 \\ 0 & 25 & 6 \\ 0 & -4 & 9 \end{vmatrix} = 240(249) = 59{,}760$$

$$N_2 = \begin{vmatrix} 25 & 240 & -3 \\ -30 & 0 & 6 \\ -3 & 0 & 9 \end{vmatrix} = -240(-252) = 60{,}480$$

$$i_1 = \frac{N_1}{\Delta} = 99.60\,\text{A}; \qquad i = \frac{N_2}{\Delta} = 100.80\,\text{A}$$

$$v_{\text{Th}} = v_{\text{ab}} = 20(i_1 - i_2) = -24\,\text{V}$$

Short-circuit current:



$$240 = 5i_1 - 0i_2 - 3i_3$$
$$0 = 0i_1 + 5i_2 - 4i_3 + 10(i_3 - i_1)$$
$$0 = -3i_1 - 4i_2 + 9i_3$$
$$240 = 5i_1 - 0i_2 - 3i_3$$
$$0 = -10i_1 + 5i_2 + 6i_3$$
$$0 = -3i_1 - 4i_2 + 9i_3$$

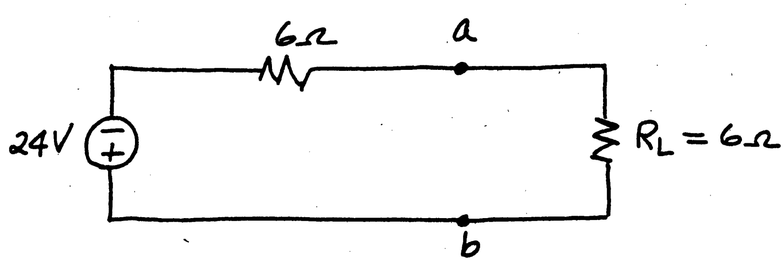
$$\Delta = \begin{vmatrix} 5 & 0 & -3 \\ -10 & 5 & 6 \\ -3 & -4 & 9 \end{vmatrix} = 5(45-9) + 4(30-30) = 180$$

$$N_1 = \begin{vmatrix} 240 & 0 & -3 \\ 0 & 5 & 6 \\ 0 & -4 & 9 \end{vmatrix} = 240(69) = 16{,}560$$

$$N_2 = \begin{vmatrix} 5 & 240 & -3 \\ -10 & 0 & 6 \\ -3 & 0 & 9 \end{vmatrix} = -240(-72) = 17{,}280$$

$$i_1 = \frac{N_1}{\Delta} = \frac{16{,}560}{180} = 92\text{ A}; \qquad i_2 = \frac{N_2}{\Delta} = \frac{17{,}280}{180} = 96\text{ A}$$

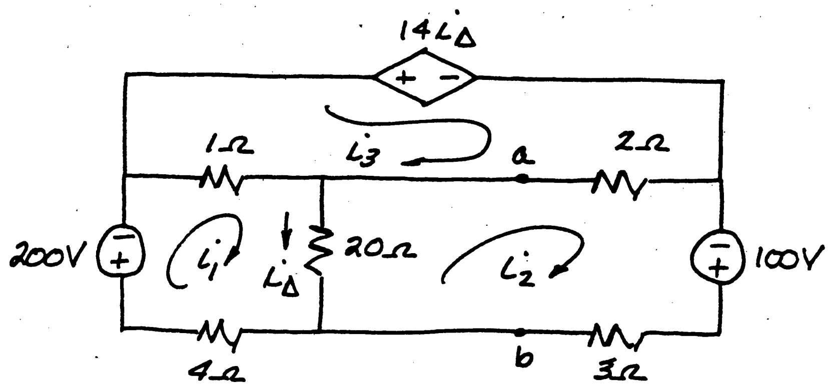
$$i_{sc} = i_1 - i_2 = -4\text{ A}; \qquad R_{\text{Th}} = \frac{v_{\text{Th}}}{i_{sc}} = \frac{-24}{-4} = 6\,\Omega$$



$$R_L = R_{\text{Th}} = 6\,\Omega$$

[b] $P_{\max} = \left(\frac{24}{12}\right)^2 (16) = 24\,\text{W}$

P 4.74 [a] We begin by finding the Thévenin equivalent with respect to the terminals of R_o.



$$200 + 25i_1 - 20i_2 - i_3 = 0$$
$$-20i_1 + 25i_2 - 2i_3 - 100 = 0$$
$$-i_1 - 2i_2 + 3i_3 + 14(i_1 - i_2) = 0$$
$$\therefore\ 25i_1 - 20i_2 - i_3 = -200$$
$$-20i_1 + 25i_2 - 2i_3 = 100$$
$$13i_1 - 16i_2 + 3i_3 = 0$$

$$\Delta = \begin{vmatrix} 25 & -20 & -1 \\ -20 & 25 & -2 \\ 13 & -16 & 3 \end{vmatrix} = 25(75-32) + 20(-60-16) + 13(40+25) = 400$$

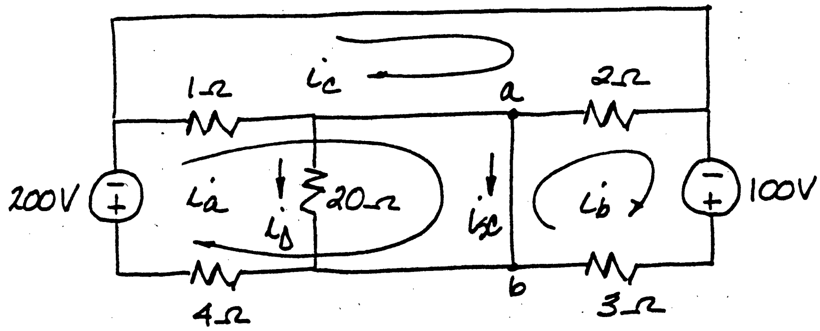
$$N_1 = \begin{vmatrix} -200 & -20 & -1 \\ 100 & 25 & -2 \\ 0 & -16 & 3 \end{vmatrix} = -200(75-32) - 100(-60-16) = -1000$$

$$N_2 = \begin{vmatrix} 25 & -200 & -1 \\ -20 & 100 & -2 \\ 13 & 0 & 3 \end{vmatrix} = 200(-60+26) + 100(75+13) = 2000$$

$$i_1 = \frac{N_1}{\Delta} = \frac{-1000}{400} = -2.5\,\text{A}; \qquad i_2 = \frac{N_2}{\Delta} = \frac{2000}{400} = 5\,\text{A}$$

$$\therefore\ i_\Delta = i_1 - i_2 = -7.5\,\text{A}; \qquad v_{\text{Th}} = 20i_\Delta = -150\,\text{V}$$

Now find the short-circuit current.



Note with the short circuit from a to b that i_Δ is zero, hence $14i_\Delta$ is also zero.

$$-200 = 5i_a + 0i_b - i_c$$
$$100 = 0i_a + 5i_b - 2i_c$$
$$0 = -i_a - 2i_b + 3i_c$$

$$\Delta = \begin{vmatrix} 5 & 0 & -1 \\ 0 & 5 & -2 \\ -1 & -2 & 3 \end{vmatrix} = 5(15-4) - 1(5) = 55 - 5 = 50$$

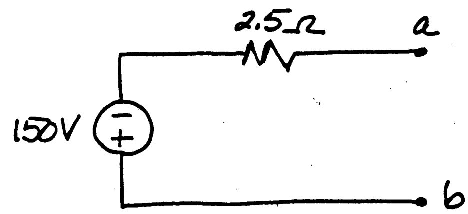
$$\boxed{\Delta = 50}$$

$$N_a = \begin{vmatrix} -200 & 0 & -1 \\ 100 & 5 & -2 \\ 0 & -2 & 3 \end{vmatrix} = -200(15-4) - 100(-2) = -2200 + 200 = -2000$$

$$N_b = \begin{vmatrix} 5 & -200 & -1 \\ 0 & 100 & -2 \\ -1 & 0 & 3 \end{vmatrix} = 200(-2) + 100(15-1) = -400 + 1400 = 1000$$

$$i_a = \frac{N_a}{\Delta} = \frac{-2000}{50} = -40\text{ A}; \qquad i_b = \frac{N_b}{\Delta} = \frac{1000}{50} = 20\text{ A}$$

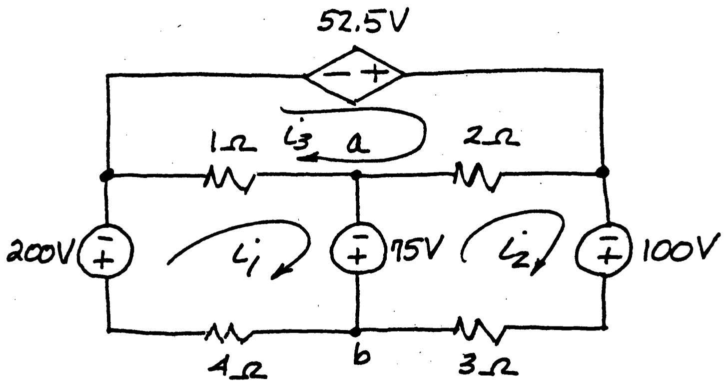
$$i_{sc} = i_a - i_b = -60\text{ A}; \qquad R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{-150}{-60} = 2.5\,\Omega$$



For maximum power transfer $R_o = R_{Th} = 2.5\,\Omega$

[b] $P_{max} = \left(\frac{150}{5}\right)^2 (2.5) = 2{,}250\text{ W}$

P 4.75 From the solution of Problem 4.74 we know that when R_o is 2.5 Ω, the voltage across R_o is 75 V, positive at the lower terminal. Therefore our problem reduces to the analysis of the following circuit. In constructing the circuit we have used the fact that i_Δ is $-75/20$, or -3.75 A, and hence $14i_\Delta$ is -52.50 V.



$$200 + 5i_1 - 0i_2 - i_3 - 75 = 0$$
$$0i_1 + 5i_2 - 2i_3 - 100 + 75 = 0$$
$$- i_1 - 2i_2 + 3i_3 - 52.5 = 0$$

$$5i_1 - 0i_2 - i_3 = -125$$
$$0i_1 + 5i_2 - 2i_3 = 25$$
$$- i_1 - 2i_2 + 3i_3 = 52.5$$

$$\Delta = \begin{vmatrix} 5 & 0 & -1 \\ 0 & 5 & -2 \\ -1 & -2 & 3 \end{vmatrix} = 50$$

$$N_1 = \begin{vmatrix} -125 & 0 & -1 \\ 25 & 5 & -2 \\ 52.5 & -2 & 3 \end{vmatrix} = -1062.50$$

$$N_2 = \begin{vmatrix} 5 & -125 & -1 \\ 0 & 25 & -2 \\ -1 & 52.5 & 3 \end{vmatrix} = 625$$

$$N_3 = \begin{vmatrix} 5 & 0 & -125 \\ 0 & 5 & 25 \\ -1 & -2 & 52.5 \end{vmatrix} = 937.50$$

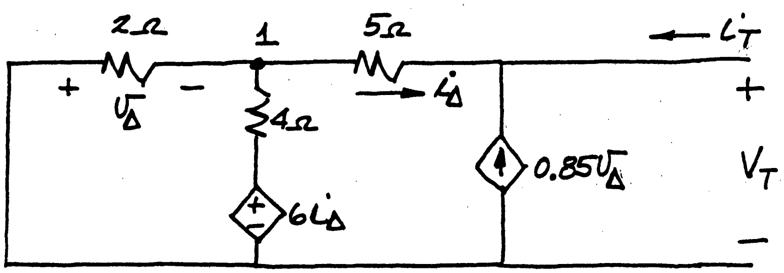
$$i_1 = \frac{N_1}{\Delta} = \frac{-1062.50}{50} = -21.25\text{ A}$$
$$i_2 = \frac{N_2}{\Delta} = \frac{625}{50} = 12.5\text{ A}$$
$$i_3 = \frac{N_3}{\Delta} = \frac{937.50}{50} = 18.75\text{ A}$$

Note that all three sources are developing power. Therefore

$$\sum p_{\text{dev}} = 200(21.25) + 100(12.5) + 52.5(18.75) = 6484.375\text{ W}$$

$$\%\text{ delivered} = \frac{2250}{6484.375} \times 100 = 34.70\%$$

P 4.76 [a] Find the Thévenin resistance



$$i_T = -0.85v_\Delta + \frac{V_T - v_1}{5}, \qquad v_\Delta = -v_1$$

$$i_T = 0.85v_1 + 0.2V_T - 0.2v_1 = 0.65v_1 + 0.2V_T$$

$$\frac{v_1}{2} + \frac{v_1 - 6i_\Delta}{4} + \frac{v_1 - V_T}{5} = 0$$

$$10v_1 + 5v_1 - 30i_\Delta + 4v_1 - 4V_T = 0$$

$$19v_1 - 30\left(\frac{v_1 - V_T}{5}\right) - 4V_T = 0$$

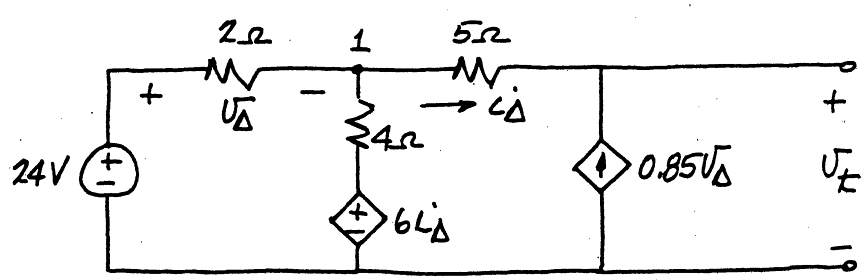
$$13v_1 + 2V_T = 0; \qquad \therefore\ v_1 = \frac{-V_T}{6.5}$$

$$\therefore\ i_T = 0.65\left(\frac{-V_T}{6.5}\right) + 0.2V_T$$

$$\frac{i_T}{V_T} = \frac{1}{R_{\text{Th}}} = 0.1$$

$$\therefore\ R_{\text{Th}} = 10\,\Omega = R_o$$

[b] Find the Thévenin voltage.



$$\frac{v_1 - 24}{2} + \frac{v_1 - 6i_\Delta}{4} + \frac{v_1 - v_{Th}}{5} = 0$$

$$10v_1 - 240 + 5v_1 - 30i_\Delta + 4v_1 - 4v_{Th} = 0$$

$$19v_1 - 30\left(\frac{v_1 - v_{Th}}{5}\right) - 4v_{Th} = 240$$

$$\boxed{13v_1 + 2v_{Th} = 240}$$

$$\frac{v_{Th} - v_1}{5} - 0.85v_\Delta = 0$$

$$v_\Delta = 24 - v_1; \qquad 0.85v_\Delta = 20.4 - 0.85v_1$$

$$\frac{v_{Th} - v_1}{5} - 20.4 + 0.85v_1 = 0$$

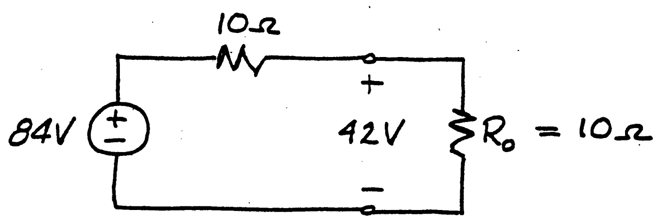
$$\boxed{3.25v_1 + v_{Th} = 102}$$

$$\Delta = \begin{vmatrix} 13 & 2 \\ 3.25 & 1 \end{vmatrix} = 6.5$$

$$N_{Th} = \begin{vmatrix} 13 & 240 \\ 3.25 & 102 \end{vmatrix} = 1326 - 780 = 546$$

$$v_{Th} = \frac{N_{Th}}{\Delta} = \frac{546}{6.5} = 84\,\text{V}$$

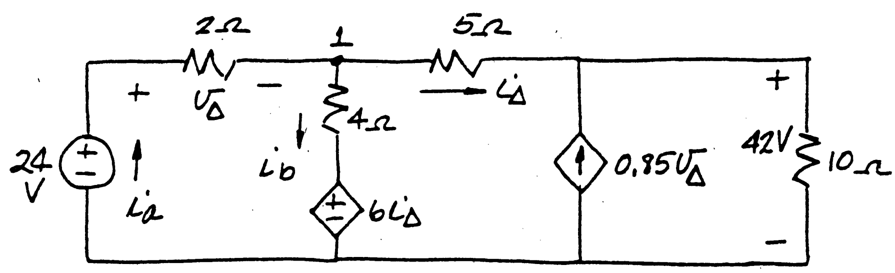
Therefore the Thévenin equivalent is



The maximum power delivered to R_o is

$$P_{\max} = \frac{(42)^2}{10} = 176.40\,\text{W}.$$

When $R_o = 10\,\Omega$ the original circuit becomes



$$\frac{v_1 - 24}{2} + \frac{v_1 - 6i_\Delta}{4} + \frac{v_1 - 42}{5} = 0$$
$$10v_1 - 240 + 5v_1 - 30i_\Delta + 4v_1 - 168 = 0$$
$$19v_1 - 30\left(\frac{v_1 - 42}{5}\right) = 408$$
$$13v_1 = 156, \qquad v_1 = 12\,\text{V}$$
$$i_\Delta = \frac{12 - 42}{5} = -6\,\text{A}$$
$$i_b = \frac{12 - 6(-6)}{4} = 12\,\text{A}$$
$$i_a = \frac{24 - 12}{2} = 6\,\text{A}$$
$$v_\Delta = 24 - 12 = 12\,\text{V}, \qquad 0.85v_\Delta = 10.20\,\text{A}$$
$$p_{24\text{V}} = -24i_a = -144\,\text{W} \quad \text{(dev)}$$
$$p_{6i_\Delta} = (6i_\Delta)(i_b) = -36(12) = -432\,\text{W} \quad \text{(dev)}$$
$$p_{0.85v_\Delta} = -42(10.20) = -428.4\,\text{W} \quad \text{(dev)}$$

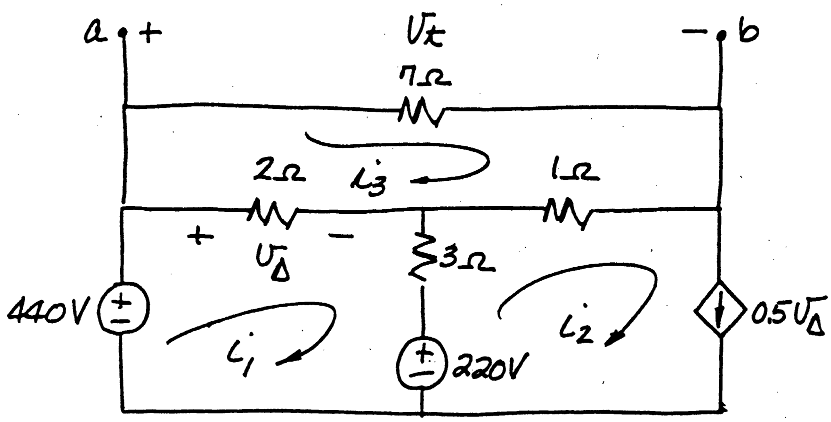
∴ All three sources are developing power.

$$\therefore \sum p_{\text{dev}} = 144 + 432 + 428.40 = 1004.40\,\text{W}$$

∴ Percent delivered to R_o is

$$\%\text{ delivered} = \frac{176.40}{1004.40} \times 100 \cong 17.56\%$$

P 4.77 Find the Thévenin equivalent with respect to the terminals of R_o.

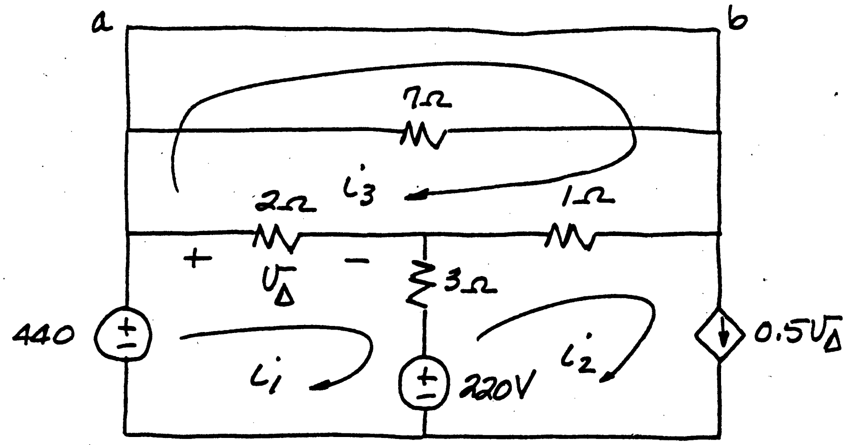


$$440 = 5i_1 - 3(0.5)(2)(i_1 - i_3) + 220 - 2i_3$$
$$0 = -2i_1 - 0.5(2)(i_1 - i_3) + 10i_3$$
$$220 = 2i_1 + i_3$$
$$0 = -3i_i + 11i_3$$

$$\Delta = \begin{vmatrix} 2 & 1 \\ -3 & 11 \end{vmatrix} = 25$$

$$N_3 = \begin{vmatrix} 2 & 220 \\ -3 & 0 \end{vmatrix} = 660$$

$$i_3 = \frac{N_3}{\Delta} = 26.4\,\text{A}; \qquad v_{\text{Th}} = v_{\text{ab}} = 7i_3 = 184.80\,\text{V}$$

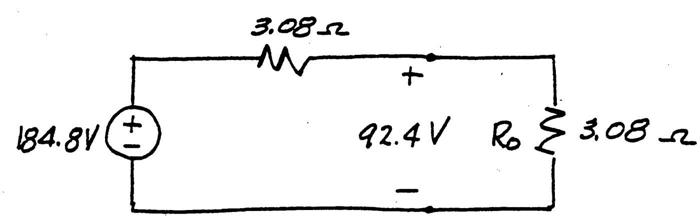


$$220 = 5i_1 - 3(0.5)(2)(i_1 - i_3) - 2i_3$$
$$0 = -2i_1 - 0.5(2)(i_1 - i_3) + 3i_3$$
$$220 = 2i_1 + i_3$$
$$0 = i_1 + 4i_3$$

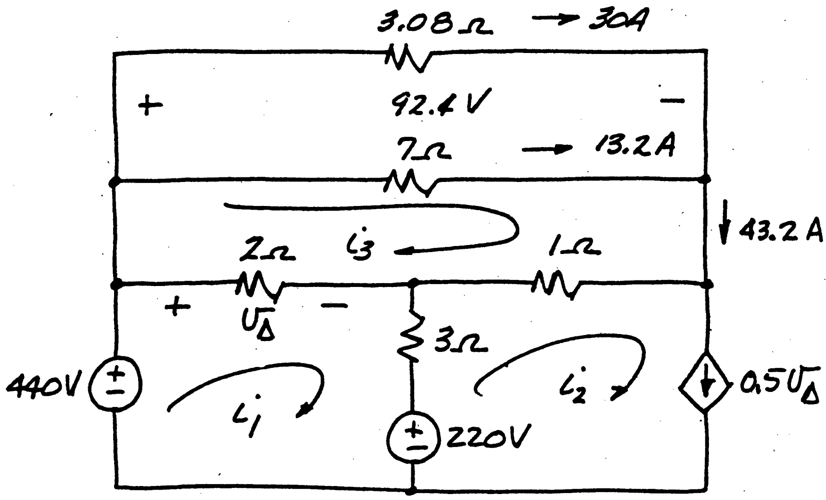
$$\Delta = \begin{vmatrix} 2 & 1 \\ -3 & 4 \end{vmatrix} = 11$$

$$N_3 = \begin{vmatrix} 2 & 220 \\ -3 & 0 \end{vmatrix} = 660$$

$$i_3 = \frac{N_3}{\Delta} = \frac{660}{11} = 60\,\text{A} = i_{sc}; \qquad R_{\text{Th}} = \frac{v_{\text{Th}}}{i_{sc}} = \frac{184.80}{60} = 3.08\,\Omega$$

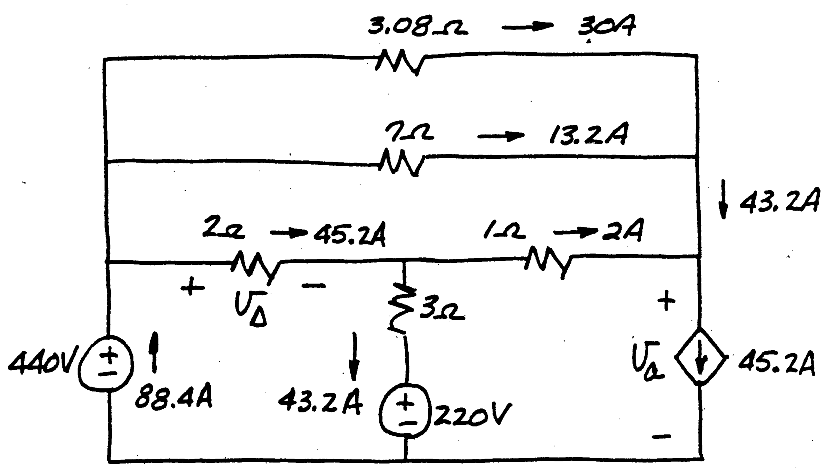


$$P_{\max} = \frac{(92.4)^2}{3.08} = 2772\,\text{W}$$



$$220 = 5i_1 - 3(0.5)(2)(i_1 - i_3) - 2i_3$$
$$220 = 2i_1 + i_3$$
$$\therefore\ 2i_1 = 220 - i_3 = 220 - 43.2$$
$$i_1 = 88.40\text{ A}$$
$$v_\Delta = 2(i_1 - i_3) = 90.40\text{ V}$$
$$i_2 = 45.2\text{ A}$$
$$\therefore\ i_1 - i_2 = 88.4 - 45.2 = 43.2\text{ A}$$
$$i_1 - i_3 = 45.2\text{ A}$$
$$i_2 - i_3 = 2\text{ A}$$

Thus the circuit becomes



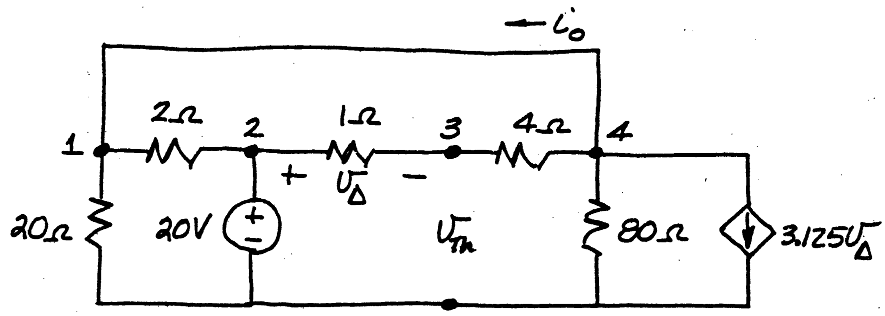
$v_a = 220 + 3(43.2) - 2 = 347.6\text{ V}$

Therefore the only source developing power is the 440-V source.

$p_{dev} = 440(88.4) = 38{,}896$ W

$\%$ delivered to $R_o = \frac{2772}{38{,}896} \times 100 = 7.13\%$

P 4.78 [a] Find the Thévenin equivalent. With the 40-Ω resistor removed $i_\phi = 0$ and the circuit becomes



$$\frac{v_{Th} - 20}{1} + \frac{v_{Th} - v_4}{4} = 0$$

$$\frac{v_4}{80} + 3.125v_\Delta + \frac{v_4 - 20}{5} + i_o = 0$$

$$\frac{v_4}{20} + \frac{v_4 - 20}{2} - i_o = 0$$

$$v_\Delta = 20 - v_{Th}$$

$$\boxed{5v_{Th} - v_4 = 80}$$

$$\frac{v_4}{80} + 3.125(20 - v_{Th}) + \frac{v_4 - 20}{5} + \frac{v_4}{20} + \frac{v_4 - 20}{2} = 0$$

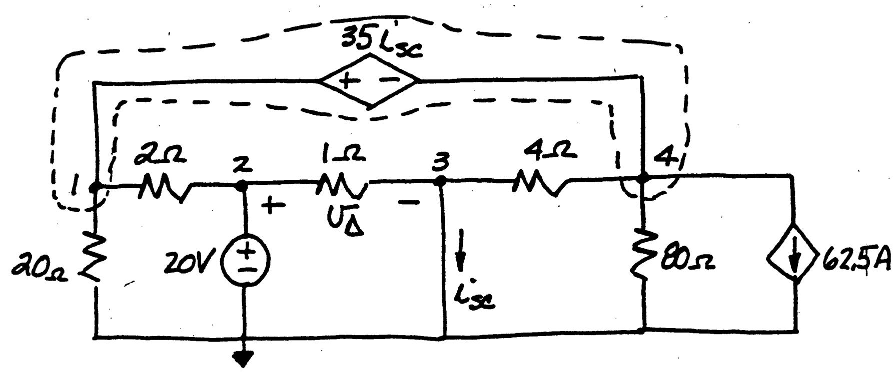
$$\boxed{-250v_{Th} + 61v_4 = -3880}$$

$$\Delta = \begin{vmatrix} 5 & -1 \\ -250 & 61 \end{vmatrix} = 55$$

$$N_{Th} = \begin{vmatrix} 80 & -1 \\ -3880 & 61 \end{vmatrix} = 1000$$

$$v_{Th} = \frac{N_{Th}}{\Delta} = \frac{1000}{55} = \frac{200}{11}\text{ V} \approx 18.18\text{ V}$$

Short-circuit current:



$$v_\Delta = 20\,\text{V}, \qquad 3.125v_\Delta = 62.5\,\text{A}$$

$$\frac{v_1}{20} + \frac{v_1 - 20}{2} + \frac{v_4}{4} + \frac{v_4}{80} + 62.5 = 0$$

$$\boxed{44v_1 + 21v_4 = -4200}$$

$$v_1 = 35i_{sc} + v_4; \qquad i_{sc} = 20 + \frac{v_4}{4}$$

$$v_1 = 35\left[20 + \left(\frac{v_4}{4}\right)\right] + v_4 = 700 + 9.75v_4$$

$$44v_1 = 30{,}800 + 429v_4$$

$$\therefore\ 30{,}800 + 429v_4 + 21v_4 = -4200$$

$$v_4 = -\frac{700}{9}\,\text{V} \approx -77.78\,\text{V}$$

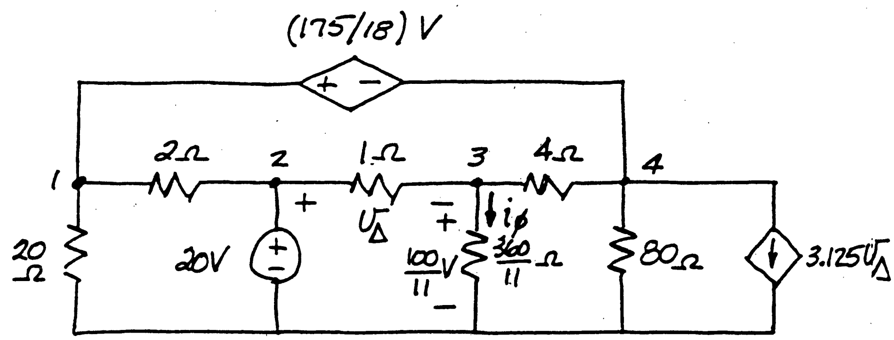
$$i_{sc} = 20 - \frac{700}{36} = \frac{5}{9}\,\text{A}$$

$$R_{\text{Th}} = \frac{v_{\text{Th}}}{i_{sc}} = \frac{200}{11}\cdot\frac{9}{5} = \frac{360}{11}\,\Omega \approx 32.73\,\Omega$$

$$\therefore\ R_o = R_{\text{Th}} = \frac{360}{11}\,\Omega \approx 32.73\,\Omega$$

[b] $P_{\max} = \left(\frac{200}{11}\cdot\frac{11}{720}\right)^2\left(\frac{360}{11}\right) = \left(\frac{1}{3.6}\right)^2\left(\frac{360}{11}\right) = \frac{100}{39.6} = \frac{250}{99} \approx 2.53\,\text{W}$

[c] When $R_o = (360/11)\,\Omega$ the circuit is



$$i_\phi = \frac{100}{11}\cdot\frac{11}{360} = \frac{10}{36} = \frac{5}{18}\text{ A}$$

$$35i_\phi = \frac{175}{18}\text{ V}$$

$$\frac{(100/11)-20}{1} + \frac{(100/11)}{(360/11)} + \frac{(100/11)-v_4}{4} = 0$$

$$\frac{-120}{11} + \frac{5}{18} + \frac{100-11v_4}{44} = 0$$

$$-480 + \frac{220}{18} + 100 - 11v_4 = 0$$

$$11v_4 = \frac{220}{18} - 380 = \frac{-3310}{9}$$

$$v_4 = \frac{-3310}{99}\text{ V}$$

$$v_1 = \frac{175}{18} - \frac{3310}{99} = \frac{-42{,}255}{1782}\text{ V}$$

$$i_{20\text{V}} = \frac{20+(42{,}255/1782)}{2} + \frac{20-(100/11)}{1} = \frac{77{,}895}{3564} + \frac{120}{11}$$

$$= \frac{1{,}284{,}525}{39{,}204} \approx 32.77\text{ A}$$

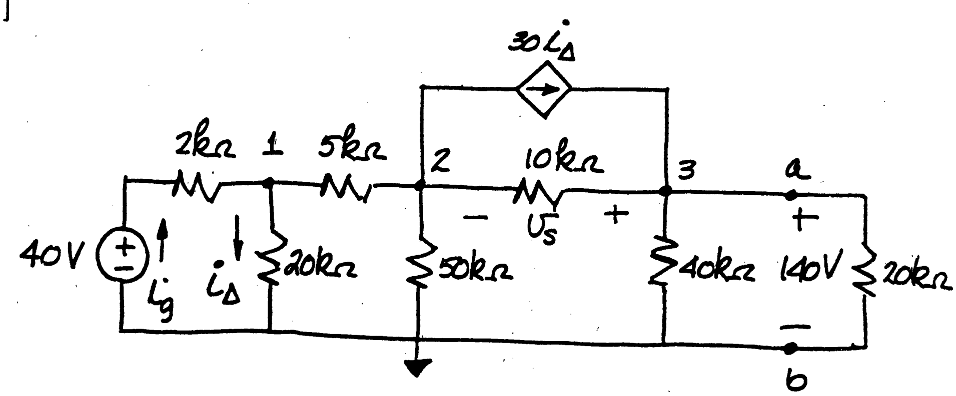
$$p_{20\text{V}} = 20(32.77) \approx 655.30\text{ W}$$

P 4.79 [a] From the solution of Problem 4.63 we have $R_{\text{Th}} = 20\,\text{k}\Omega$ and $v_{\text{Th}} = 280$ V. Therefore

$$R_o = R_{\text{Th}} = 20\,\text{k}\Omega$$

[b] $p = \dfrac{(140)^2}{20} = 980\,\text{mW}$

[c]



$$\frac{v_1}{20} + \frac{v_1-40}{2} + \frac{v_1-v_2}{5} = 0$$

$$\boxed{15v_1 - 4v_2 = 400}$$

$$\frac{v_2}{50} + \frac{v_2-v_1}{5} + \frac{v_2-140}{10} + 30\left(\frac{v_1}{20}\right) = 0$$

$$\boxed{130v_1 + 32v_2 = 1400}$$

$$\Delta = \begin{vmatrix} 15 & -4 \\ 130 & 32 \end{vmatrix} = 1000$$

$$N_1 = \begin{vmatrix} 400 & -4 \\ 1400 & 32 \end{vmatrix} = 18{,}400$$

$$N_2 = \begin{vmatrix} 15 & 400 \\ 130 & 1400 \end{vmatrix} = -31{,}000$$

$\therefore\ v_1 = 18.4\,\text{V}; \qquad v_2 = -31\,\text{V}$

$$i_g = \frac{40 - 18.4}{2} = 10.8\,\text{mA}$$

$p_{40\text{V}}(\text{delivered}) = (40)(10.8) = 432\,\text{mW}$

$v_2 + v_s = 140$

$v_s = 140 - (-31) = 171\,\text{V}$

$$i_\Delta = \frac{v_1}{20} = 0.92\,\text{mA}$$

$30 i_\Delta = 27.60\,\text{mA}$

$p_{30i_\Delta}(\text{delivered}) = (171)(27.6) = 4719.60\,\text{mW}$

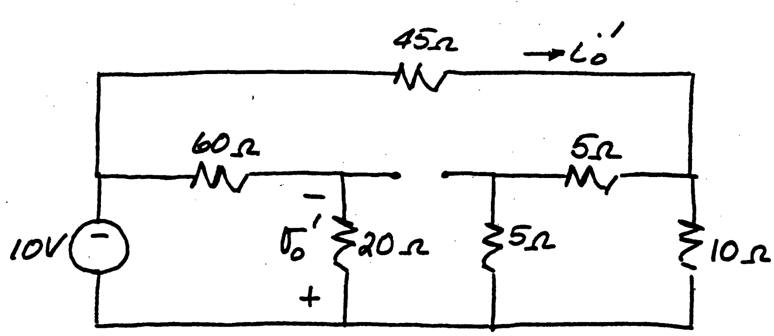
$\sum p_{\text{dev}} = (432 + 4719.6) = 5151.60\,\text{mW}$

$$\%\ \text{delivered} = \frac{980}{5151.60} \times 100 = 19.02\%$$

P 4.80 [a] Since $0 \le R_o \le \infty$ maximum power will be delivered to the 8-Ω resistor when $R_o = 0$.

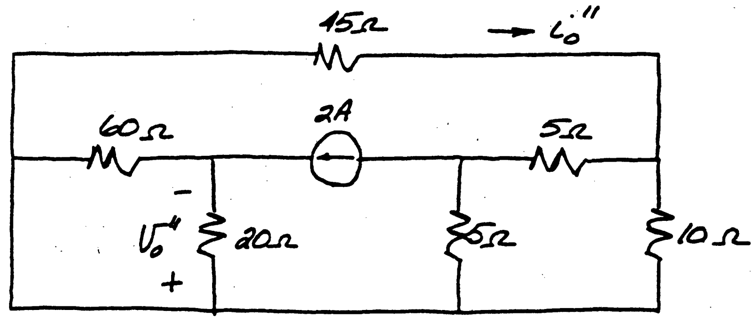
[b] $P = \dfrac{24^2}{8} = 72\,\text{W}$

P 4.81 Voltage source acting alone

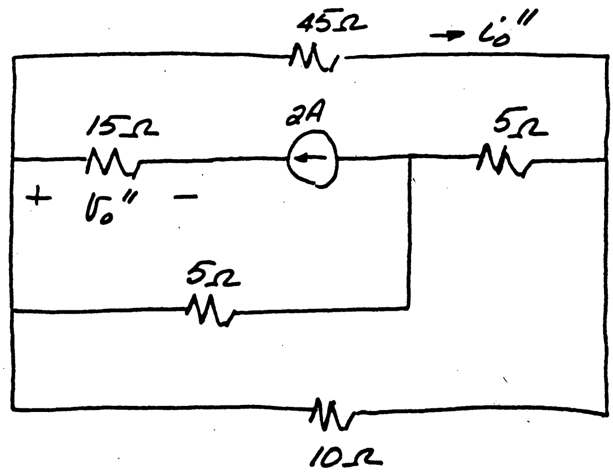


$$v_o' = \frac{-10}{80}(20) = -2.5\,\text{V}; \qquad i_o' = \frac{+10}{45+5} = +0.20\,\text{A}$$

Current source acting alone:



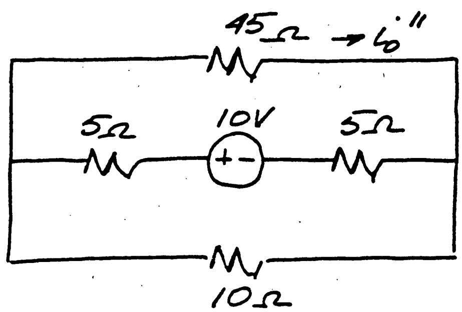
Now the circuit can be redrawn as



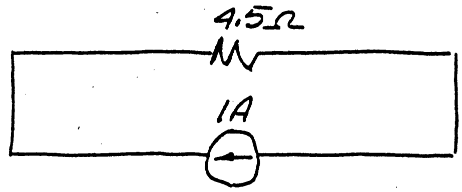
The 15-Ω resistor will have no effect on i_o'' since it is in series with a 2-A source.

$$v_o'' = -15(2) = -30\,\text{V}$$

To find i_o'' we simplify the circuit as follows:



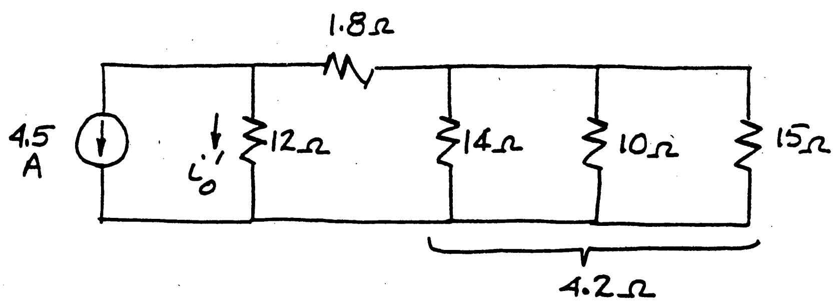
Now the circuit reduces to



Therefore the voltage across the 45-Ω resistor is 4.5 V, hence $i_o'' = \frac{4.5}{45} = 0.10\text{ A}$

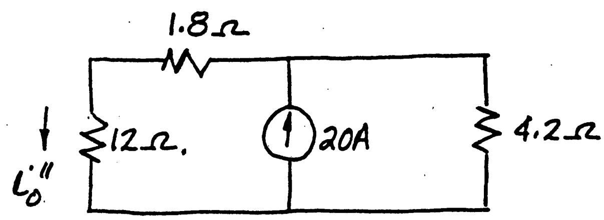
$$\therefore\ v_o = v_o' + v_o'' = -2.5 - 30 = -32.5\text{ V}$$
$$i_o = i_o' + i_o'' = 0.20 + 0.10 = 0.30\text{ A}$$

P 4.82 4.5-A source:



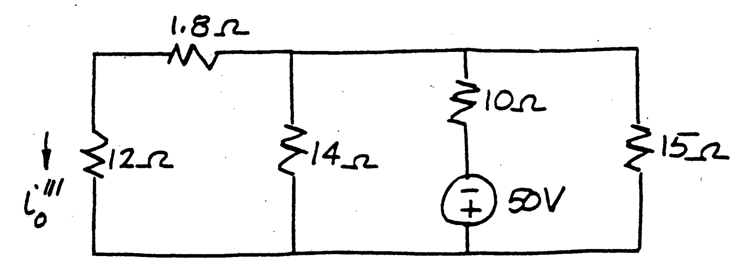
$$i_o' = \frac{(-4.5)(6)}{18} = -\frac{27}{18} = -1.5\text{ A}$$

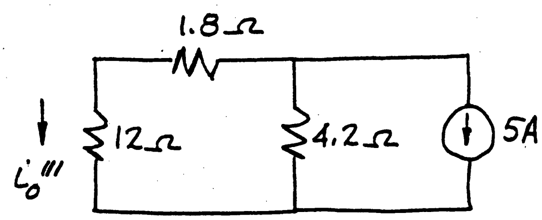
20-A source:



$$i_o'' = \frac{(20)(4.2)}{18} = \frac{84}{18} = \frac{42}{9}\ A$$

50-V source:



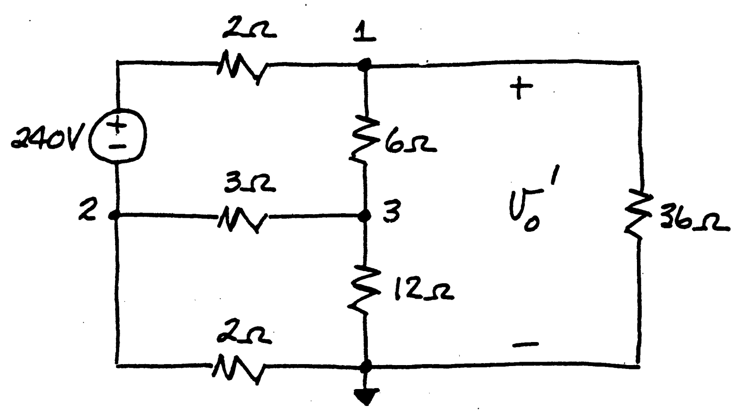


$$i_o''' = \frac{-5(4.2)}{18} = -\frac{21}{18} = -\frac{7}{6}\,\text{A}$$

$$i_o = i_o' + i_o'' + i_o''' = -1.5 + \frac{42}{9} - \frac{7}{6} = \frac{-27 + 84 - 21}{18}$$

$$i_o = \frac{36}{18} = 2\,\text{A}$$

P 4.83



$$\frac{v_1 - (v_2 + 240)}{2} + \frac{v_1 - v_3}{6} + \frac{v_1}{36} = 0$$

$$18v_1 - 18v_2 - 4320 + 6v_1 - 6v_3 + v_1 = 0$$

$$\boxed{25v_1 - 18v_2 - 6v_3 = 4320}$$

$$\frac{v_2}{2} + \frac{v_2 - v_3}{3} + \frac{v_2 - (v_1 - 240)}{2} = 0$$

$$3v_2 + 2v_2 - 2v_3 + 3v_2 - 3v_1 + 720 = 0$$

$$\boxed{-3v_1 + 8v_2 - 2v_3 = -720}$$

$$\frac{v_3}{12} + \frac{v_3 - v_2}{3} + \frac{v_3 - v_1}{6} = 0$$

$$v_3 + 4v_3 - 4v_2 + 2v_3 - 2v_1 = 0$$

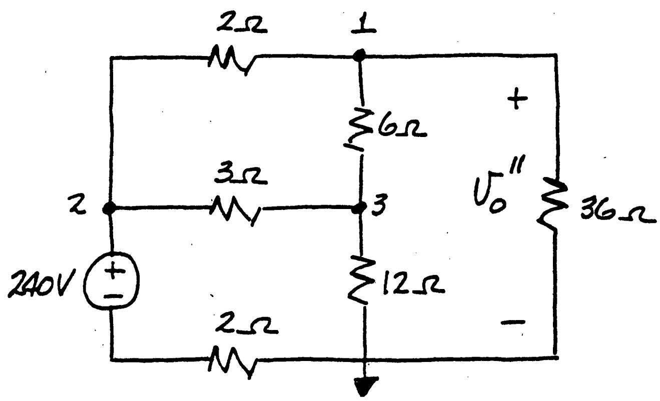
$$\boxed{-2v_1 - 4v_2 + 7v_3 = 0}$$

$$\Delta = \begin{vmatrix} 25 & -18 & -6 \\ -3 & 8 & -2 \\ -2 & -4 & +7 \end{vmatrix} = 25(56 - 8) + 3(-126 - 24) - 2(36 + 48)$$

$$\boxed{\Delta = 582}$$

$$N_1 = \begin{vmatrix} 4320 & -18 & -6 \\ -720 & 8 & -2 \\ 0 & -4 & 7 \end{vmatrix} = 4320(48) + 720(-150) = 99{,}360$$

$$v_o' = \frac{N_1}{\Delta} = \frac{99{,}360}{582} = 170.72\,\text{V}$$



$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{6} + \frac{v_1}{36} = 0$$

$$18v_1 - 18v_2 + 6v_1 - 6v_3 + v_1 = 0$$

$$\boxed{25v_1 - 18v_2 - 6v_3 = 0}$$

$$\frac{v_2 - v_1}{2} + \frac{v_2 - v_3}{3} + \frac{v_2 - 240}{2} = 0$$

$$3v_2 - 3v_1 + 2v_2 - 2v_3 + 3v_2 - 720 = 0$$

$$\boxed{-3v_1 + 8v_2 - 2v_3 = 720}$$

$$\frac{v_3 - v_2}{3} + \frac{v_3 - v_1}{6} + \frac{v_3}{12} = 0$$

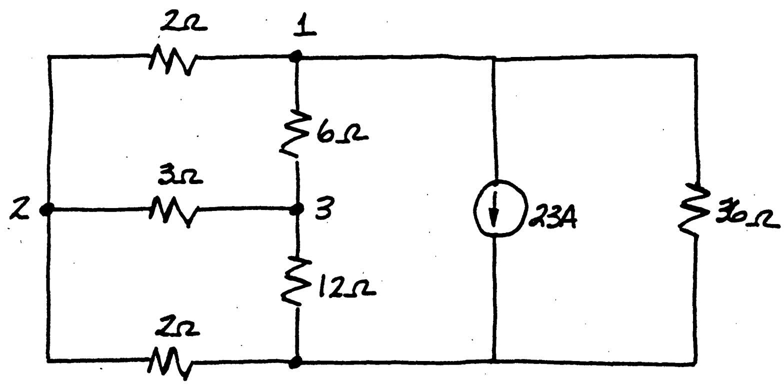
$$4v_3 - 4v_2 + 2v_3 - 2v_1 + v_3 = 0$$

$$\boxed{-2v_1 - 4v_2 + 7v_3 = 0}$$

$$\Delta = \begin{vmatrix} 25 & -18 & -6 \\ -3 & 8 & -2 \\ -2 & -4 & 7 \end{vmatrix} = 582$$

$$N_1 = \begin{vmatrix} 0 & -18 & -6 \\ 720 & 8 & -2 \\ 0 & -4 & 7 \end{vmatrix} = -720(-150) = 108{,}000$$

$$v_o'' = \frac{N_1}{\Delta} = \frac{108{,}000}{582} = 185.57\ V$$



$$\frac{v_1 - v_3}{2} + \frac{v_1 - v_3}{6} + \frac{v_1}{36} + 23 = 0$$

$$18v_1 - 18v_2 + 6v_1 - 6v_3 + v_1 = -828$$

$$\boxed{25v_1 - 18v_2 - 6v_3 = -828}$$

$$\frac{v_2 - v_1}{2} + \frac{v_2 - v_3}{3} + \frac{v_2}{2} = 0$$

$$3v_2 - 3v_1 + 2v_2 - 2v_3 + 3v_2 = 0$$

$$\boxed{-3v_1 + 8v_2 - 2v_3 = 0}$$

$$\frac{v_3}{12} + \frac{v_3 - v_2}{3} + \frac{v_3 - v_1}{6} = 0$$

$$v_3 + 4v_3 - 4v_2 + 2v_3 - 2v_1 = 0$$

$$\boxed{-2v_1 - 4v_2 + 7v_3 = 0}$$

As before $\Delta = 582$

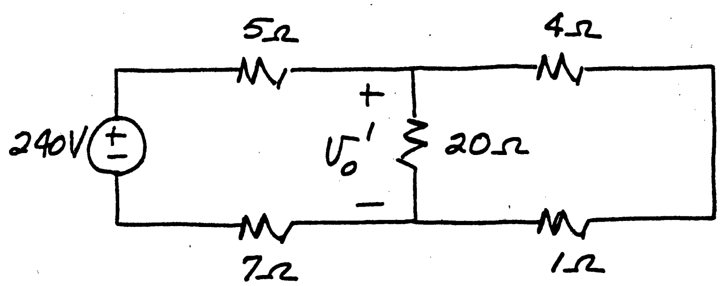
$$N_1 = \begin{vmatrix} -828 & -18 & -6 \\ 0 & 8 & -2 \\ 0 & -4 & 7 \end{vmatrix} = -828(48) = -39{,}744$$

$$v_o'' = \frac{N_1}{\Delta} = \frac{-39{,}744}{582} = -68.29$$

$$v_o = v_o' + v_o'' + v_o''' = 170.72 + 185.57 - 68.29$$

$$v_o = 288\ \text{V}$$

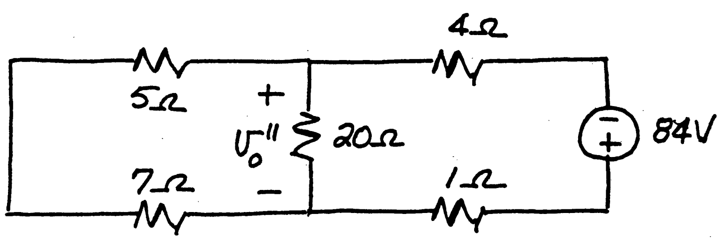
P 4.84 240-V source:



$20\Omega // 5\Omega = 4\Omega$

$$\therefore\ v_o' = \left(\frac{240}{5+7+4}\right)(4) = 60\,\text{V}$$

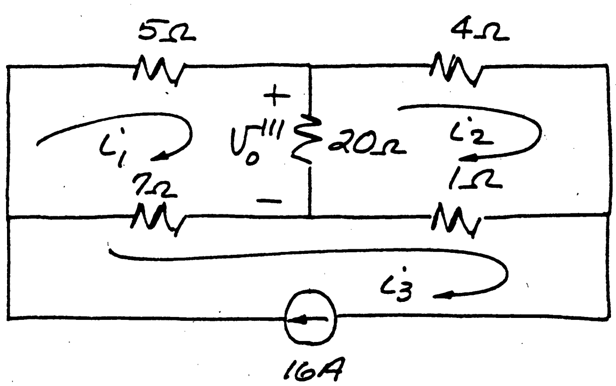
84-V source:



$12\Omega // 20\Omega = 7.5\Omega$

$$\therefore\ v_o'' = \left(\frac{-84}{4+1+7.5}\right)(7.5) = -50.4\,\text{V}$$

16-A source:



$$0 = 32i_1 - 20i_2 - 112 \qquad 112 = 32i_1 - 20i_2$$
$$0 = -20i_1 + 25i_2 - 16 \qquad 16 = -20i_1 + 25i_2$$

$$\Delta = \begin{vmatrix} 32 & -20 \\ -20 & 25 \end{vmatrix} = 400$$

$$N_1 = \begin{vmatrix} 112 & -20 \\ 16 & 25 \end{vmatrix} = 3120$$

$$N_2 = \begin{vmatrix} 32 & 112 \\ -20 & 16 \end{vmatrix} = 2752$$

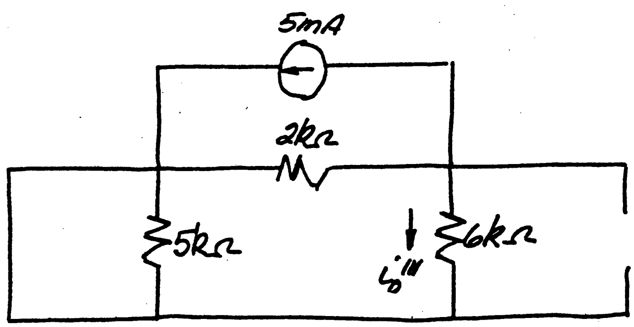
$$i_1 = \frac{N_1}{\Delta} = 7.80\,\text{A}; \qquad i_2 = \frac{N_2}{\Delta} = 6.88\,\text{A}$$

$$i_1 - i_2 = 7.80 - 6.88 = 0.92\,\text{A}$$

$$v_o''' = 20(i_1 - i_2) = 18.4\,\text{V}$$

$$v_o = v_o' + v_o'' + v_o''' = 60 - 50.4 + 18.40 = 28\,\text{V}$$

P 4.85 [a] By hypothesis $i_o' + i_o'' = 3.5\,\text{mA}$.



$$i_o''' = -5\frac{(2)}{(8)} = -1.25\,\text{mA}; \qquad \therefore\ i_o = 3.5 - 1.25 = 2.25\,\text{mA}$$

[b] With all three sources in the circuit write a single node-voltage equation.

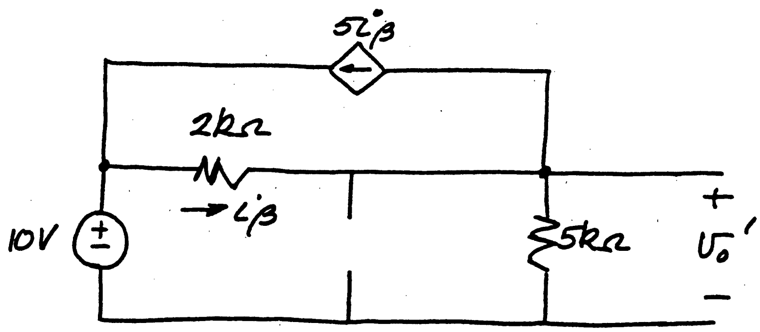
$$\frac{v_b}{6} + \frac{v_b - 8}{2} + 5 - 10 = 0$$

$$v_b + 3v_b - 24 - 30 = 0$$

$$v_b = \frac{54}{4} = 13.5\,\text{V}$$

$$i_o = \frac{v_b}{6} = 2.25\,\text{mA}$$

P 4.86 10-V source acting alone:



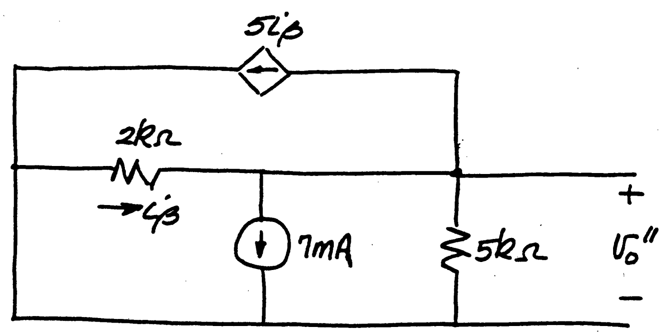
$$\frac{v_o'}{5} + \frac{v_o' - 10}{2} + 5\left(\frac{10 - v_o'}{2}\right) = 0$$

$$2v_o' + 5v_o' - 50 + 250 - 25v_o' = 0$$

$$18v_o' = 200$$

$$v_o' = \frac{100}{9}\text{ V}$$

7-mA source acting alone:



$$\frac{v_o''}{5} + 7 + \frac{v_o''}{2} + 5\left(\frac{-v_o''}{2}\right) = 0$$

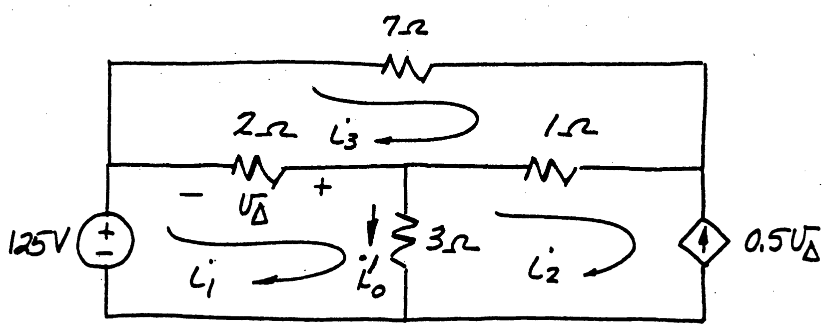
$$2v_o'' + 70 + 5v_o'' - 25v_o'' = 0$$

$$18v_o'' = 70$$

$$v_o'' = \frac{35}{9}\text{ V}$$

$$v_o = v_o' + v_o'' = \frac{135}{9} = 15\text{ V}$$

P 4.87 125-V source acting alone:



$$125 = 5i_1 - 3(-0.5)(2)(i_3 - i_1) - 2i_3$$

$$\boxed{125 = 2i_1 + i_3}$$

$$0 = -2i_1 - 0.5(2)(i_3 - i_1) + 10i_3$$

$$\boxed{0 = -3i_1 + 11i_3}$$

$$\Delta = \begin{vmatrix} 2 & 1 \\ -3 & 11 \end{vmatrix} = 25$$

$$N_1 = \begin{vmatrix} 125 & 1 \\ 0 & 11 \end{vmatrix} = 125(11)$$

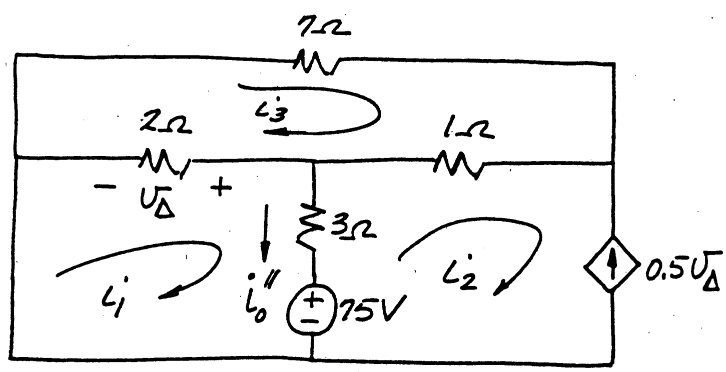
$$N_3 = \begin{vmatrix} 2 & 125 \\ -3 & 0 \end{vmatrix} = 125(3)$$

$$i_1 = \frac{N_1}{\Delta} = 55\text{ A}; \qquad i_3 = \frac{N_3}{\Delta} = 15\text{ A}$$

$$i_2 = -0.5(2)(15 - 55) = 40\text{ A}$$

$$i_o' = i_1 - i_2 = 55 - 40 = 15\text{ A}$$

75-V source acting alone:



$$\boxed{-75 = 2i_1 + 3i_3} \qquad \boxed{0 = -3i_1 + 11i_3}$$

$\Delta = 25; \qquad N_1 = -11(75); \qquad N_3 = -3(75)$

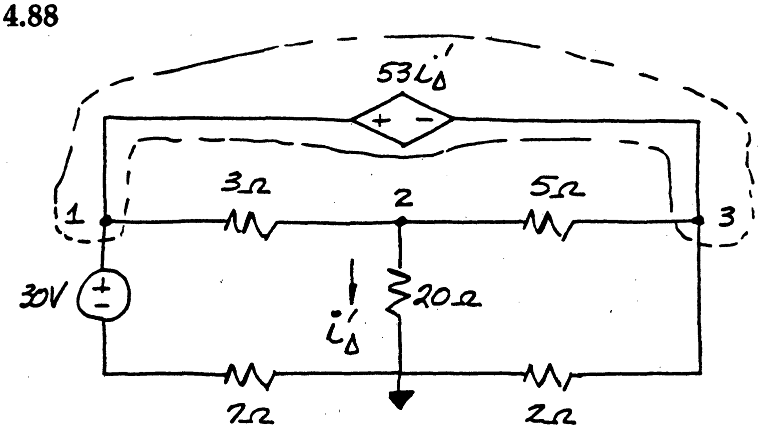
$i_1 = -33\,\text{A}; \qquad i_3 = -9\,\text{A}$

$i_2 = -0.5(2)(-9 + 33) = -24\,\text{A}$

$i_o'' = i_1 - i_2 = -33 + 24 = -9\,\text{A}$

$i_o = i_o'' + i_o' = -9 + 15 = 6\,\text{A}$

P 4.88



$$\frac{v_1 - 30}{7} + \frac{v_1 - v_2}{3} + \frac{v_3 - v_2}{5} + \frac{v_3}{2} = 0$$

$30v_1 - 900 + 70v_1 - 70v_2 + 42v_3 - 42v_2 + 105v_3 = 0$

$100v_1 - 112v_2 + 147v_3 = 900$

$$v_1 = v_3 + 53\left(\frac{v_2}{20}\right)$$

$\therefore\ 100v_1 = 100v_3 + 265v_2$

$100v_3 + 256v_2 - 112v_2 + 147v_3 = 900$

$$\boxed{153v_2 + 247v_3 = 900}$$

$$\frac{v_2}{20} + \frac{v_2 - v_1}{3} + \frac{v_2 - v_3}{5} = 0$$

$3v_2 + 20v_2 - 20v_1 + 12v_2 - 12v_3 = 0$

$-20v_1 + 35v_2 - 12v_3 = 0$

But $20v_1 = 20v_3 + 53v_2$

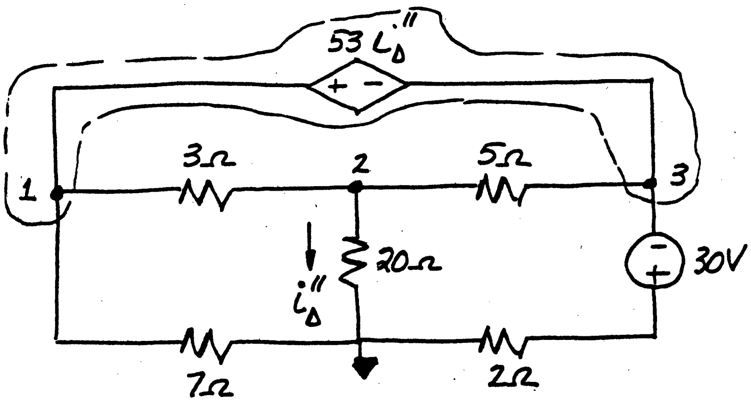
$\therefore\ -20v_3 - 53v_2 + 35v_2 - 12v_3 = 0$

$$\boxed{18v_2 + 32v_3 = 0}$$

$$\Delta = \begin{vmatrix} 153 & 247 \\ 18 & 32 \end{vmatrix} = 450$$

$$N_2 = \begin{vmatrix} 900 & 247 \\ 0 & 32 \end{vmatrix} = 32(900)$$

$$v_2 = \frac{N_2}{\Delta} = \frac{32(900)}{450} = 64\,\text{V}; \qquad i'_\Delta = \frac{v_2}{20} = 3.2\,\text{A}$$



$$\frac{v_1}{7} + \frac{v_1 - v_2}{3} + \frac{v_3 - v_2}{5} + \frac{v_3 + 30}{2} = 0$$

$$30v_1 + 70v_1 - 70v_2 + 42v_3 - 42v_2 + 105v_3 + 3150 = 0$$

$$100v_1 - 112v_2 + 147v_3 = -3150$$

$$v_1 = v_3 + 53\left(\frac{v_2}{20}\right)$$

$$100v_1 = 100v_3 + 265v_2$$

$$100v_3 + 265v_2 - 112v_2 + 147v_3 = -3150$$

$$\boxed{153v_2 + 247v_3 = -3150}$$

$$\frac{v_2 - v_1}{3} + \frac{v_2}{20} + \frac{v_2 - v_3}{5} = 0$$

$$20v_2 - 20v_1 + 3v_2 + 12v_2 - 12v_3 = 0$$

$$-20v_1 + 35v_2 - 12v_3 = 0$$

$$-20v_3 - 53v_2 + 35v_2 - 12v_3 = 0$$

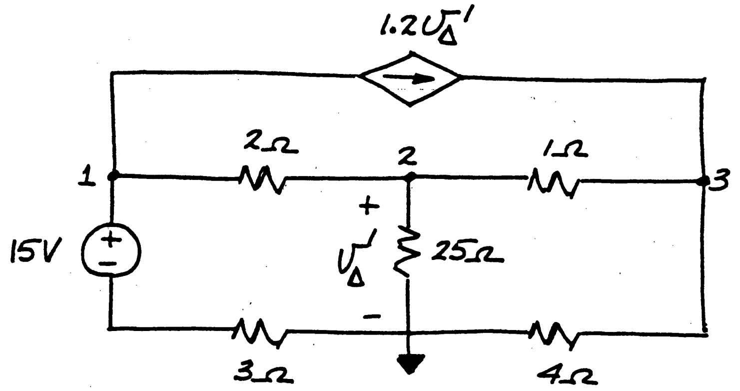
$$-18v_2 - 32v_3 = 0$$

$$\boxed{18v_2 + 32v_3 = 0}$$

$$\Delta = 450; \qquad N_2 = -3150(32); \qquad v_2 = -224\,\text{V}$$

$$i''_\Delta = \frac{-224}{20} = -11.2\,\text{A}; \qquad i_\Delta = i'_\Delta + i''_\Delta = 3.2 - 11.2 = -8\,\text{A}$$

P 4.89



$$\frac{v_1 - 15}{3} + \frac{v_1 - v_2}{2} + 1.2v_2 = 0$$

$2v_1 - 30 + 3v_1 - 3v_2 + 7.2v_2 = 0;$ $\boxed{5v_1 + 4.2v_2 + 0v_3 = 30}$

$$\frac{v_2}{25} + \frac{v_2 - v_1}{2} + \frac{v_2 - v_3}{1} = 0$$

$2v_2 + 25v_2 - 25v_1 + 50v_2 - 50v_3 = 0;$ $\boxed{-25v_1 + 77v_2 - 50v_3 = 0}$

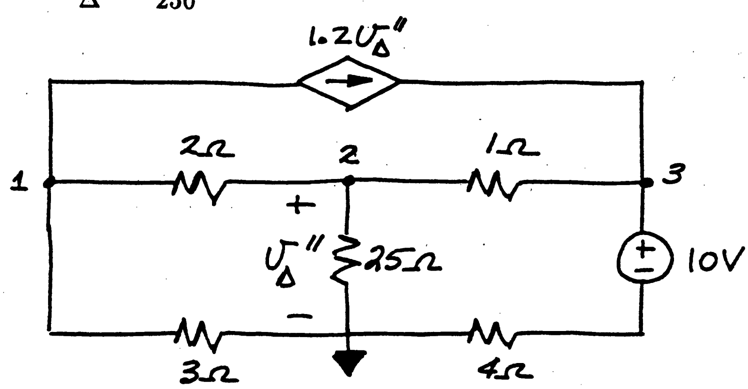
$$\frac{v_3}{4} + \frac{v_3 - v_2}{1} - 1.2v_2 = 0$$

$v_3 + 4v_3 - 4v_2 - 4.8v_2 = 0;$ $\boxed{0v_1 - 8.8v_2 + 5v_3 = 0}$

$$\Delta = \begin{vmatrix} 5 & 4.2 & 0 \\ -25 & 77 & -50 \\ 0 & -8.8 & 5 \end{vmatrix} = 5(-55) + 25(21) = 250$$

$$N_2 = \begin{vmatrix} 5 & 30 & 0 \\ -25 & 0 & -50 \\ 0 & 0 & 5 \end{vmatrix} = -30(-125) = 3750$$

$$v_2 = v'_\Delta = \frac{N_2}{\Delta} = \frac{3750}{250} = 15\,\text{V}$$



$$\frac{v_1}{3} + \frac{v_1 - v_2}{2} + 1.2v_2 = 0; \qquad \boxed{5v_1 + 4.2v_2 + 0v_3 = 0}$$

$$\frac{v_2}{25} + \frac{v_2 - v_1}{2} + \frac{v_2 - v_3}{1} = 0; \qquad \boxed{-25v_1 + 77v_2 - 50v_3 = 0}$$

$$\frac{v_3 - 10}{4} + \frac{v_3 - v_2}{1} - 1.2v_2 = 0; \qquad \boxed{0v_1 - 8.8v_2 + 5v_3 = 10}$$

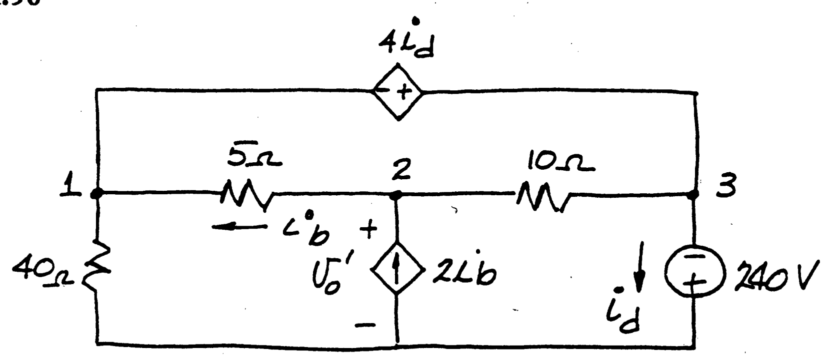
$$\Delta = \begin{vmatrix} 5 & 4.2 & 0 \\ -25 & 77 & -50 \\ 0 & -8.8 & 5 \end{vmatrix} = 250$$

$$N_2 = \begin{vmatrix} 5 & 0 & 0 \\ -25 & 0 & -50 \\ 0 & 10 & 5 \end{vmatrix} = -10(-250)$$

$$v_2 = v''_\Delta = \frac{N_2}{\Delta} = \frac{2500}{250} = 10\,\text{V}$$

$$v_\Delta = v'_\Delta + v''_\Delta = 15 + 10 = 25\,\text{V}$$

P 4.90



$$-2i_b + \frac{v_2 + 240}{10} + \frac{v_2 - v_1}{5} = 0$$

$$-2\left(\frac{v_2 - v_1}{5}\right) + \frac{v_2 + 240}{10} + \frac{v_2 - v_1}{5} = 0$$

$$\therefore \quad \boxed{2v_1 - v_2 = -240}$$

$$v_1 + 4i_d = -240$$

$$v_1 + 4\left(2i_b - \frac{v_1}{40}\right) = -240$$

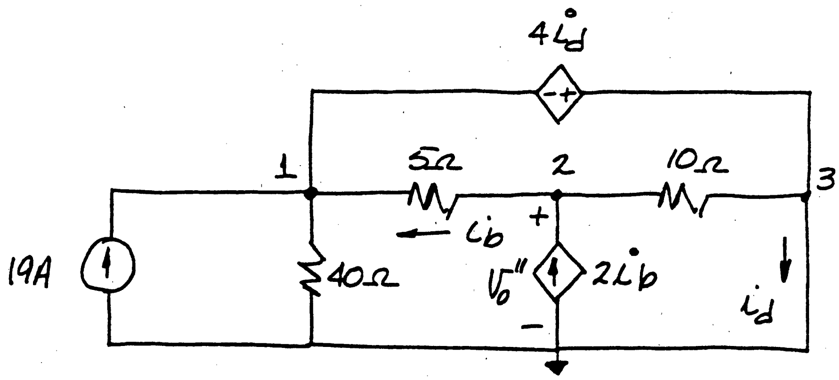
$$v_1 - \frac{v_1}{10} + 8\left(\frac{v_2 - v_1}{5}\right) = -240$$

$$\therefore \quad \boxed{-7v_1 + 16v_2 = -2400}$$

$$\Delta = \begin{vmatrix} 2 & -1 \\ -7 & 16 \end{vmatrix} = 32 - 7 = 25$$

$$N_2 = \begin{vmatrix} 2 & -240 \\ -7 & -2400 \end{vmatrix} = -4800 - 1680 = -6480$$

$$v_2 = \frac{N_2}{\Delta} = \frac{-6480}{25} = -259.20\,\text{V} = v_o'$$



$$\frac{v_2 - v_1}{5} - 2\left(\frac{v_2 - v_1}{5}\right) + \frac{v_2}{10} = 0; \qquad \therefore \quad \boxed{2v_1 - v_2 = 0}$$

$$v_1 + 4i_d = 0$$

$$v_1 + 4\left(2i_b - \frac{v_1}{40} + 19\right) = 0$$

$$v_1 + 8\left(\frac{v_2 - v_1}{5}\right) - \frac{v_1}{10} = -76; \qquad \therefore \quad \boxed{-7v_1 + 16v_2 = -760}$$

$$-7\left(\frac{v_2}{2}\right) + 16v_2 = -760$$

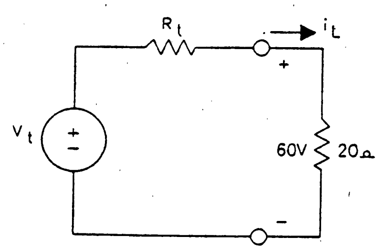
$$12.5v_2 = -760$$

$$v_2 = -60.80\,\text{V} = v_o''$$

$$\therefore \; v_o = v_o' + v_o'' = -259.2 - 60.80$$

$$v_o = -320\,\text{V}$$

P 4.91 [a]



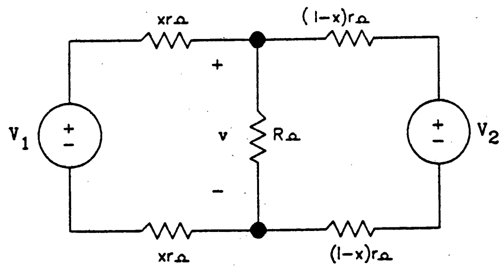
$V_{oc} = V_{Th} = 75\,\text{V}; \qquad i_L = \frac{60}{20} = 3\,\text{A}; \qquad i_L = \frac{75 - 60}{R_{Th}} = \frac{15}{R_{Th}}$

Therefore $R_{Th} = \frac{15}{3} = 5\,\Omega$

[b] $i_L = \dfrac{V_o}{R_L} = \dfrac{V_{Th} - v_o}{R_{Th}}$

Therefore $R_{Th} = \dfrac{V_{Th} - V_o}{V_o/R_L} = \left(\dfrac{V_{Th}}{V_o} - 1\right) R_L$

P 4.92 [a]



$$\frac{v - V_1}{2xr} + \frac{v}{R} + \frac{v - V_2}{2r(\ell - x)} = 0$$

$$v\left[\frac{1}{2xr} + \frac{1}{R} + \frac{1}{2r(\ell - x)}\right] = \frac{V_1}{2xr} + \frac{V_2}{2r(\ell - x)}$$

$$v = \frac{V_1 R\ell + xR(V_2 - V_1)}{R\ell + 2r\ell x - 2rx^2}$$

[b] Let $D = R\ell + 2r\ell x - 2rx^2$

$$\frac{dv}{dx} = \frac{(R\ell + 2r\ell x - 2rx^2)R(V_2 - V_1) - [V_1 R\ell + xR(V_2 - V_1)]2r(\ell - 2x)}{D^2}$$

$\dfrac{dv}{dx} = 0$ when numerator is zero.

The numerator simplifies to $x^2 + \dfrac{2\ell V_1}{(V_2 - V_1)}x + \dfrac{R\ell(V_2 - V_1) - 2rV_1\ell^2}{2r(V_2 - V_1)} = 0$

Solving for the roots of the quadratic yields

$$x = \frac{\ell}{V_2 - V_1}\left\{-V_1 \pm \sqrt{V_1 V_2 - \frac{R}{2r\ell}(V_2 - V_1)^2}\right\}$$

[c] $x = \dfrac{\ell}{V_2 - V_1}\left\{-V_1 \pm \sqrt{V_1 V_2 - \dfrac{R}{2r\ell}(V_1 - V_2)^2}\right\}$

$V_2 = 1200\,\text{V}, \quad V_1 = 1000\,\text{V}, \quad \ell = 16\,\text{km}$

$r = 5 \times 10^{-5}\,\Omega/\text{m}; \quad R = 3.9\,\Omega$

$\dfrac{\ell}{V_2 - V_1} = \dfrac{16{,}000}{1200 - 1000} = 80; \quad V_1 V_2 = 1.2 \times 10^6$

$\dfrac{R}{2r\ell}(V_1 - V_2)^2 = \dfrac{3.9(-200)^2}{(10 \times 10^{-5})(16 \times 10^3)} = 0.975 \times 10^5$

$x = 80\{-1000 \pm \sqrt{1.2 \times 10^6 - 0.0975 \times 10^6}\} = 80\{-1000 \pm 1050\} = 80(50)$

$x = 4000\,\text{m}$

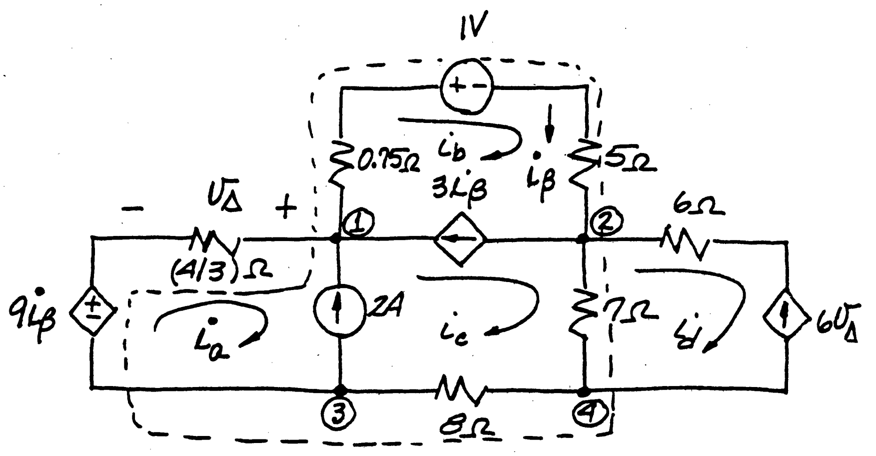
[d] $$v_{\min} = \frac{V_1 R\ell + R(V_2 - V_1)x}{R\ell + 2r\ell x - 2rx^2}$$
$$= \frac{(1000)(3.9)(16 \times 10^3) + 3.9(200)(4000)}{(3.9)(16{,}000) + 10 \times 10^{-5}(16{,}000)(4000) - 10 \times 10^{-5}(16 \times 10^6)}$$
$$= 975\text{ V}$$

P 4.93 [a] In studying the circuit in Fig. P4.93 we note it contains six meshes and six essential nodes. Further study shows that by replacing the parallel resistors with their equivalent values the circuit reduces to four meshes and four essential nodes as shown in the following diagram.

The node-voltage approach will require solving three node-voltage equations along with equations involving v_Δ and i_β.

The mesh-current approach will require writing one supermesh equation plus three constraint equations involving the three current sources. Thus at the outset we know the supermesh equation can be reduced to a single unknown current. Since we are interested in the power developed by the 1-V source, we will retain the mesh current i_b and eliminate the mesh currents i_a, i_c, and i_d.

The supermesh is denoted by the dashed line in the following figure.



[b] Summing the voltages around the supermesh yields

$$-9i_\beta + \frac{4}{3}i_a + 0.75i_b + 1 + 5i_b + 7(i_c - i_d) + 8i_c = 0$$

Note that $i_\beta = i_b$, and multiply the equation by 12:

$$-108i_b + 16i_a + 9i_b + 12 + 60i_b + 84(i_c - i_d) + 96i_c = 0$$

or $16i_a - 39i_b + 180i_c - 84i_d = -12$

Now note: $i_b - i_c = 3i_\beta = 3i_b$; $\therefore\ i_c = -2i_b$

whence $16i_a - 39i_b - 360i_b - 84i_d = -12$

Now use the constraint that

$$i_a - i_c = -2$$

$$i_a = -2 + i_c = -2 - 2i_b$$

Therefore

$$-32 - 32i_b - 399i_b - 84i_d = -12$$

$$-431i_b - 84i_d = 20$$

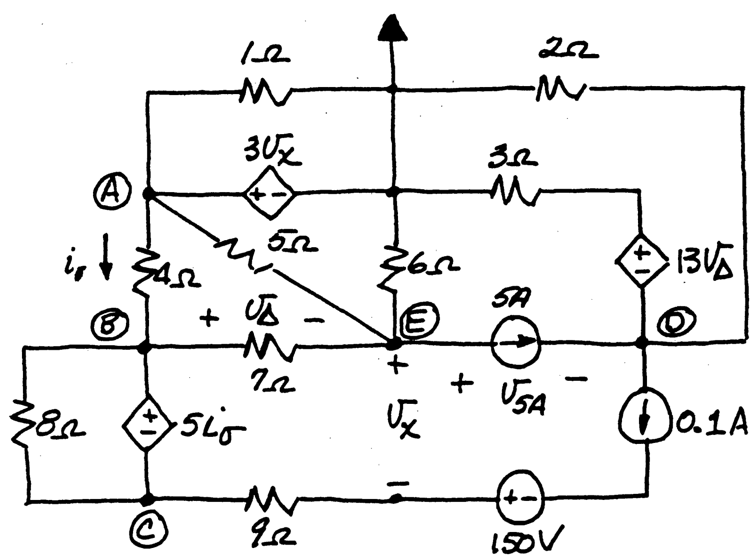
Now use the constraint $i_d = -6v_\Delta = -6\left(\frac{-4}{3}i_a\right) = 8i_a = -16 - 16i_b$

Therefore $-431i_b - 84(-16 - 16i_b) = 20$ or $913i_b = -1324$

$$i_b \approx -1.45\text{ A}$$

$p_{1\text{V}} = 1i_b \cong -1.45\text{ W};$ $\therefore$ $p_{1\text{V}}(\text{developed}) \cong 1.45\text{ W}$

P 4.94



$$\frac{v_B - v_A}{4} + \frac{v_B - v_E}{7} - 0.1 = 0 \qquad (1)$$

$$0.1 + \frac{v_D}{2} + \frac{v_D + 13v_\Delta}{3} - 5 = 0 \qquad (2)$$

$$\frac{v_E - v_B}{7} + \frac{v_E - v_A}{5} + \frac{v_E}{6} + 5 = 0 \qquad (3)$$

Multiply equation (1) by 28, equation (2) by 6, and equation (3) by 42 to get

$$-7v_A + 11v_B - 4v_E = 2.8$$

$$5v_D + 26v_\Delta = 29.4$$

$$-8.4v_A - 6v_B + 21.4v_E = -210$$

Now write the constraint equations

$$v_A = 3v_x; \qquad v_x = v_E - v_C - 0.9; \qquad v_\Delta = v_B - v_E$$

$$i_\sigma = \frac{v_A - v_B}{4} = 0.25v_A - 0.25v_B$$

$$5i_\sigma = v_B - v_C$$

Now use the constraint equations to eliminate v_A, v_B, and v_Δ, reducing the number of equations to three.

$$v_A = 3v_E - 3v_C - 2.7$$

$$v_B = \frac{15}{9}v_E - \frac{11}{9}v_C - 1.5$$

$$v_\Delta = \frac{6}{9}v_E - \frac{11}{9}v_C - 1.5$$

We get

$$68v_C + 0v_D - 60v_E = 3.6$$

$$-286v_C + 45v_D + 156v_E = 615.60$$

$$292.80v_C + 0v_D - 124.2v_E = -2175.12$$

or

$$\begin{bmatrix} 68 & 0 & -60 \\ -286 & 45 & 156 \\ 292.8 & 0 & -124.2 \end{bmatrix} \begin{bmatrix} v_C \\ v_D \\ v_E \end{bmatrix} = \begin{bmatrix} 3.6 \\ 615.60 \\ -2175.12 \end{bmatrix}$$

Solving yields

$$v_C \approx -14.3552\,\text{V}; \qquad v_D \approx -20.9474\,\text{V}; \qquad v_E \approx 16.3293\,\text{V}$$

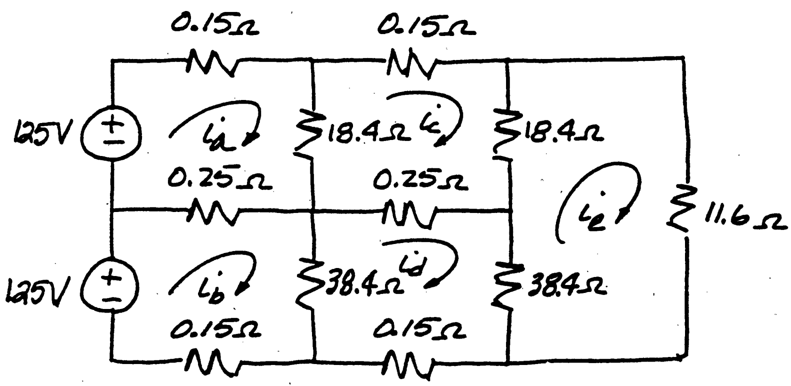
From the circuit diagram

$$p_{5\text{A}} = 5v_{5\text{A}}$$

$$5v_{5\text{A}} = 5(v_E - v_D) \cong 23.09\,\text{W}$$

$$\therefore\ p_{5\text{A}} = 23.09\,\text{W(absorbing)}$$

P 4.95



$$125 = 18.8i_a - 0.25i_b - 18.4i_c + 0i_d + 0i_e$$

$$125 = -0.25i_a + 38.8i_b + 0i_c - 38.4i_d + 0i_e$$

$$0 = -18.4i_a + 0i_b + 37.2i_c - 0.25i_d - 18.4i_e$$
$$0 = 0i_a - 38.4i_b - 0.25i_c + 77.2i_d - 38.4i_e$$
$$0 = 0i_a + 0i_b - 18.4i_c - 38.4i_d + 68.4i_e$$

$$\begin{bmatrix} 18.8 & -0.25 & -18.4 & 0 & 0 \\ -0.25 & 38.8 & 0 & -38.4 & 0 \\ -18.4 & 0 & 37.2 & -0.25 & -18.4 \\ 0 & -38.4 & -0.25 & 77.2 & -38.4 \\ 0 & 0 & -18.4 & -38.4 & 68.4 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_d \\ i_e \end{bmatrix} = \begin{bmatrix} 125 \\ 125 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

A computer solution yields

$i_a = 32.77 = 32.76947\text{ A}$ $i_d = 23.26650\text{ A}$

$i_b = 26.45943\text{ A}$ $i_e = 20.14451\text{ A}$

$i_c = 26.32887\text{ A}$

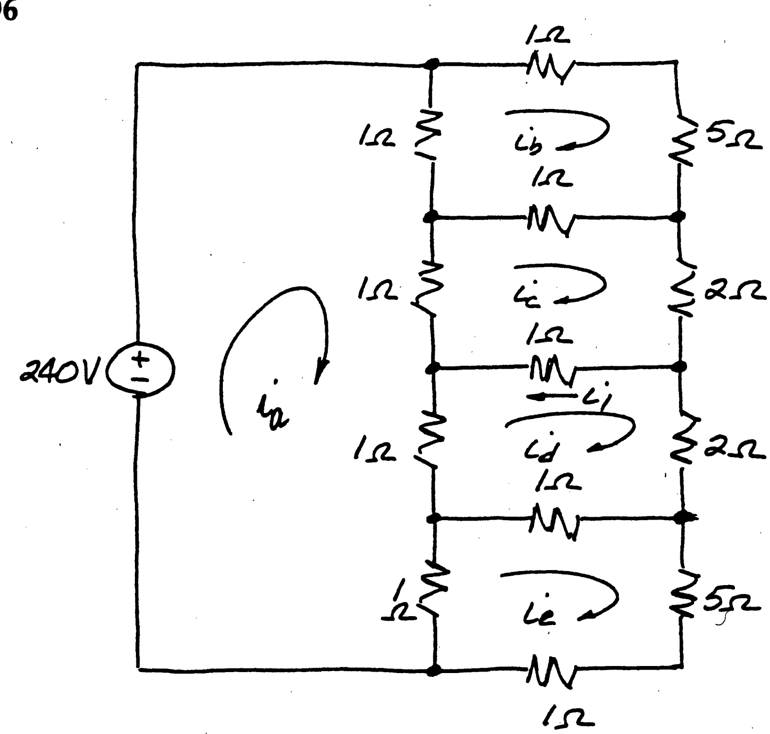
Test:

$i_c - i_e = 6.18436\text{ A}$ $v_2 = 119.88442\text{ V}$

$v_1 = 113.79222\text{ V}$ $v_3 = 233.67632\text{ V}$

$i_d - i_e = 3.12199\text{ A}$ $v_1 + v_2 = 233.67664\text{ V}$

P 4.96



$$240 = 4i_a - i_b - i_c - i_d - i_e$$
$$0 = -i_a + 8i_b - i_c + 0i_d + 0i_e$$
$$0 = -i_a - i_b + 5i_c - i_d + 0i_e$$
$$0 = -i_a + 0i_b - i_c + 5i_d - i_e$$
$$0 = -i_a + 0i_b + 0i_c - i_d + 8i_e$$

$$\begin{bmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 8 & -1 & 0 & 0 \\ -1 & -1 & 5 & -1 & 0 \\ -1 & 0 & -1 & 5 & -1 \\ -1 & 0 & 0 & -1 & 8 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_d \\ i_e \end{bmatrix} = \begin{bmatrix} 240 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

A computer solution yields

$$i_a = 77.50 \qquad i_d = 22.50$$
$$i_b = 12.50 \qquad i_e = 12.50$$
$$i_c = 22.50 \qquad i_1 = i_c - i_d = 0\text{ A}$$

Check:

$$p_{240} = 240i_a = 18{,}600\text{ W}$$
$$i_a - i_b = 77.50 - 12.50 = 65\text{ A}$$
$$p_{1\Omega} = (65)^2 1 = 4225\text{ W}$$
$$i_a - i_c = 77.50 - 22.50 = 55\text{ A}$$
$$p_{1\Omega} = (55)^2 = 3025\text{ W}$$
$$i_a - i_d = 55\text{ A}$$
$$p_{1\Omega} = (55)^2 = 3025\text{ W}$$
$$i_a - i_e = 77.50 - 12.50 = 65\text{ A}$$
$$p_{1\Omega} = (65)^2 = 4225\text{ W}$$
$$p_{6\Omega} = (12.5)^2(6) = 937.50\text{ W}$$
$$p_{6\Omega} = 937.50\text{ W}$$
$$p_{2\Omega} = (22.50)^2(2) = 1012.50\text{ W}$$
$$p_{2\Omega} = 1012.50\text{ W}$$
$$i_b - i_c = -10\text{ A}$$
$$p_{1\Omega} = 100\text{ W}$$
$$p_{1\Omega} = 0\text{ W}$$
$$p_{1\Omega} = 100\text{ W}$$
$$\sum p_{abs} = 4225 + 3025 + 3025 + 4225 + 937.50 + 937.50 + 1012.50 + 1012.50 + 100 + 0 + 100 = 18{,}600\text{ W}$$

6

The Operational Amplifier

Drill Exercises

DE 6.1 [a] $v_o = (-100/8), \quad v_s = -12.5v_s$

v_s(V)	0.40	0.72	2.00	−0.60	−0.80	−2.00
v_o(V)	−5.0	−9.0	−10.00	7.5	10.0	15.0

[b] $-10 = -12.5v_s, \quad v_s = 0.8\,\text{V}; \qquad 15 = -12.5v_s, \quad v_s = -1.2\,\text{V}$

Therefore $-1.2 \leq v_s \leq 0.8\,\text{V}$

DE 6.2 $v_o = (-4_x/8)v_s = (0.64R_x/8) = 15\,\text{V}$

Therefore $R_x = \dfrac{120}{0.64} = 187.50\,\text{k}\Omega, \qquad 0 \leq R_x \leq 187.50\,\text{k}\Omega$

DE 6.3 [a] $v_o = -\dfrac{250}{5}v_a - \dfrac{250}{25}v_b = -50(0.1) - 10(0.25) = -5 - 2.5 = -7.5\,\text{V}$

[b] $v_o = -50v_a - 2.5 = -10\,\text{V}; \qquad \text{therefore} \quad 50v_a = 7.5, \quad v_a = 0.15\,\text{V}$

[c] $v_o = -5 - 10v_b = -10\,\text{V}; \qquad 10v_b = 5, \quad v_b = 0.5\,\text{V}$

[d] $v_o = -50v_a + 10v_b = -5 + 2.5 = -2.5\,\text{V}$

$v_o = -50v_a + 2.5 = -10\,\text{V};$

$50v_a = 12.5, \quad v_a = 0.25\,\text{V}$

$-50v_a + 2.5 = 15, \quad 50v_a = -12.5, \quad v_a = -0.25\,\text{V}$

$v_o = -5 + 10v_b = -10\,\text{V}; \qquad 10v_b = -5; \qquad v_b = -0.5\,\text{V}$

$-5 + 10v_b = 15; \qquad 10v_b = 20; \qquad v_b = 2.0\,\text{V}$

DE 6.4 [a] $\dfrac{v_1}{3.3} + \dfrac{v_1 - v_o}{46.2} = 0,\quad$ therefore $\quad v_o = 15v_1,\quad v_1 = v_2$

Therefore $\quad v_o = 15v_2,\qquad v_2 = \dfrac{0.75R_x}{20 + R_x}$

Therefore when $\quad R_x = 80\,\text{k}\Omega,\qquad v_2 = 0.6\,\text{V},\qquad v_o = 9\,\text{V}$

[b] $\dfrac{15(0.75R_x)}{20 + R_x} = 10,\qquad R_x = 160\,\text{k}\Omega$

DE 6.5 [a] Assume v_a is acting along. Replacing v_b with a short circuit yields $v_2 = 0$, therefore $v_1 = 0$ and we have

$$\frac{0 - v_a}{R_a} + \frac{0 - v_o'}{R_b} + i_1 = 0,\qquad i_1 = 0$$

Therefore $\quad \dfrac{v_o'}{R_b} = -\dfrac{v_a}{R_a},\qquad v_o' = \dfrac{R_b}{R_a}v_a$

Assume v_b is acting alone. Replace v_a with a short circuit. Now

$$v_2 = v_1 = \frac{v_b R_d}{R_c + R_d}$$

$$\frac{v_1}{R_a} + \frac{v_1 - v_o''}{R_b} + i_1 = 0,\qquad i_1 = 0$$

$$\left(\frac{1}{R_a} + \frac{1}{R_b}\right)\left(\frac{R_d}{R_c + R_d}\right)v_b - \frac{v_o''}{R_b} = 0$$

$$v_o'' = \left(\frac{R_b}{R_a} + 1\right)\left(\frac{R_d}{R_c + R_d}\right)v_b = \frac{R_d}{R_a}\left(\frac{R_a + R_b}{R_c + R_d}\right)v_b$$

$$v_o = v_o' + v_o'' = \frac{R_d}{R_a}\left(\frac{R_a + R_b}{R_c + R_d}\right)v_b - \frac{R_b}{R_a}v_a$$

[b] $\dfrac{R_d}{R_a}\left(\dfrac{R_a + R_b}{R_c + R_d}\right) = \dfrac{R_b}{R_a},\quad$ therefore $\quad R_d(R_a + R_b) = R_b(R_c + R_d)$

$R_d R_a = R_b R_c,\quad$ therefore $\quad \dfrac{R_a}{R_b} = \dfrac{R_c}{R_d}$

When $\quad \dfrac{R_d}{R_a}\left(\dfrac{R_a + R_b}{R_c + R_d}\right) = \dfrac{R_b}{R_a}$

Eq. (6.21) reduces to $\quad v_o = \dfrac{R_b}{R_a}v_b - \dfrac{R_b}{R_a}v_a = \dfrac{R_b}{R_a}(v_b - v_a).$

DE 6.6 [a] $v_o = \dfrac{20(50)}{10(25)}v_b - \dfrac{40}{10}v_a = 4(v_b - v_a) = 24 - 4v_a$

$24 - 4v_a = \pm 10\,\text{V}$

$4v_a = 24 \mp 10,\qquad v_a = 3.5\,\text{V},\qquad v_a = 8.5\,\text{V}$

Therefore $\quad 3.5 \le v_a \le 8.5\,\text{V}$

[b] $v_o = \dfrac{5(50)}{10(10)}v_b - 4v_a = 2.5v_b - 4v_a$

$2.5v_b - 4v_a = 15 - 4v_a = \pm 10\,\text{V}$

$15 \mp 10 = 4v_a, \qquad v_a = 1.25\,\text{V}, \qquad v_a = 6.25\,\text{V}$

Therefore $1.25 \le v_a \le 6.25\,\text{V}$

DE 6.7 [a] R_x (balance value) $= 20\,\text{k}\Omega$

Therefore $20{,}001 = \left[10(2) \times \dfrac{10^3}{1}\right](i + \epsilon), \qquad \epsilon = 5 \times 10^{-5}$

Therefore $(v_1 - v_2) \cong (1.5 \times 10^4)(10^3)\dfrac{5 \times 10^{-5}}{(11,000)^2} = 6.2\,\mu\text{V}$

[b] $|\epsilon| \le \dfrac{20(11{,}000)^2}{(1.5 \times 10^7)(2 \times 10^6)} = \left(\dfrac{2420}{3}\right)10^{-7}$

Therefore $R_x = 20{,}000\left[1 \pm \left(\dfrac{2420}{3}\right)10^{-7}\right]$

Thus R_x must lie in the range 20,001.613 to 19,998.387
or $19{,}998.387 \le R_x \le 20{,}001.613\,\Omega$

DE 6.8 [a] $v_c = 0.5(v_1 + v_2) = 0.5(15.6 + 4.4) = 10\,\mu\text{V}$

$v_d = (v_2 - v_1) = 4.4 - 15.6 = -11.2\,\mu\text{V}$

[b] $\text{CMRR} = 20\log_{10}|A_d/A_c| = 20\log_{10}1000 = 60\,\text{dB}$

[c] $A_1 + A_2 = 1000, \qquad -0.5A_1 + 0.5A_2 = 10^6$

Therefore $A_1 = -999{,}500$ and $A_2 = 1{,}000{,}500$

[d] $v_o = A_c v_c + A_d v_d = 1000(10) \times 10^{-6} + 10^6(-11.2 \times 10^{-6})$

$= 0.01 - 11.20 = -11.19\,\text{V}$

[e] $v_1 = v_2 = 15.6\,\mu\text{V}$

Problems

P 6.1 $\dfrac{v_b - v_a}{20} + \dfrac{v_b - v_o}{160} = 0, \qquad$ therefore $v_o = 9v_b - 8v_a$

[a] $v_a = 1.5\,\text{V}, \qquad v_b = 0\,\text{V}, \qquad v_o = -12\,\text{V}$

[b] $v_a = 3.0\,\text{V}, \qquad v_b = 0\,\text{V}, \qquad v_o = -18\,\text{V}$ (sat)

[c] $v_a = 1.0\,\text{V}, \qquad v_b = 2\,\text{V}, \qquad v_o = 10\,\text{V}$

[d] $v_a = 4.0\,\text{V}, \qquad v_b = 2\,\text{V}, \qquad v_o = -14\,\text{V}$

[e] $v_a = 6.0\,\text{V}, \quad v_b = 8\,\text{V}, \quad v_o = 18\,\text{V}$ (sat)

[f] If $v_a = 4.5\,\text{V}, \quad v_o = 9v_b - 36 = \pm 18$

$\therefore\ 9v_b = 36 \pm 18; \quad v_b = 54/9 = 6\,\text{V}; \quad v_b = 18/9 = 2\,\text{V}$

$\therefore\ 2 \leq v_b \leq 6\,\text{V}$

P 6.2 $\dfrac{0-2}{5} + \dfrac{0-v_o}{40} = 0; \qquad v_o = -16\,\text{V}$

$$i_o + \frac{v_o - 0}{40 \times 10^3} + \frac{v_o}{3.2 \times 10^3} = 0$$

$$i_o - 0.4 - 5 = 0$$

$$i_o = 5.4\,\text{mA}$$

P 6.3 Since the current into the inverting input terminal of an ideal op-amp is zero, the voltage across the 2.2 MΩ resistor is (3.5)(2.2) or 7.7 V. Therefore the voltmeter reads 7.7 V.

P 6.4 $v_o = -(10)(0.5) = -5\,\text{V}; \qquad i_b = \dfrac{v_o}{5} \times 10^{-3} = -1\,\text{mA}$

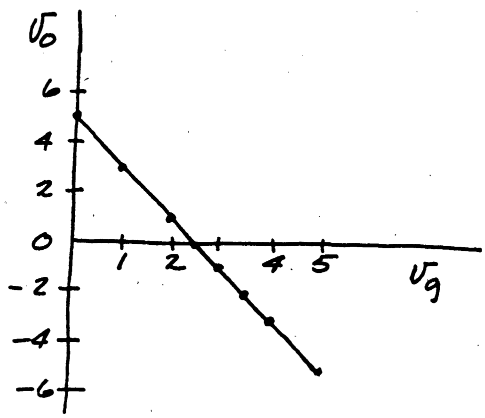
P 6.5 [a] First, note that $v_1 = v_2 = 2.5\,\text{V}$

Let v_{o1} equal the voltage output of the op-amp. Then

$$\frac{2.5 - v_g}{5} + \frac{2.5 - v_{o1}}{10} = 0, \qquad \therefore\ v_{o1} = 7.5 - 2v_g$$

Also note that $v_{o1} - 2.5 = v_o, \qquad \therefore\ v_o = 5 - 2v_g$

v_g	v_o
0	5
1	3
2	1
2.5	0
3	-1
4	-3
5	-5



[b] Yes, the output voltage is bipolar, i.e., $-5\,\text{V} \leq v_o \leq 5\,\text{V}$ and the input voltage is unipolar $0 \leq v_g \leq 5\,\text{V}$.

P 6.6 [a] $\frac{0-0.15}{2}+\frac{0-v_1}{40}=0, \quad \therefore\ v_1=-3\,\text{V}$

[b] $\frac{v_1-0}{40}+\frac{v_1}{20}+\frac{v_1-v_o}{50}=0$

$\therefore\ 19v_1-4v_o=0, \qquad v_o=\frac{19v_1}{4}=-14.25\,\text{V}$

[c] $i_2=\frac{0-v_1}{40\times10^3}=\frac{3}{40\times10^3}=75\,\mu\text{A}$

[d] $i_o+\frac{v_o}{25}+\frac{v_o-v_1}{50}=0$

$i_o+\left(\frac{-14.25}{25}\right)+\left(\frac{-11.25}{50}\right)=0; \qquad i_o=795\,\mu\text{A}$

P 6.7 [a] Let v_Δ be the voltage from the potentiometer contact to ground. Then

$$\frac{0-v_g}{4}+\frac{0-v_\Delta}{20}=0$$

$$-5v_g-v_\Delta=0, \qquad \therefore\ v_\Delta=-800\,\text{mV}$$

$$\frac{v_\Delta}{\alpha R_\Delta}+\frac{v_\Delta-0}{20}+\frac{v_\Delta-v_o}{(1-\alpha)R_\Delta}=0$$

$$\frac{v_\Delta}{\alpha}+6v_\Delta+\frac{v_\Delta-v_o}{1-\alpha}=0$$

$$v_\Delta\left(\frac{1}{\alpha}+6+\frac{1}{1-\alpha}\right)=\frac{v_o}{1-\alpha}$$

$$\therefore\ v_o=-0.80\left[1+6(1-\alpha)+\frac{(1-\alpha)}{\alpha}\right]$$

When $\alpha=0.25, \qquad v_o=-0.80(1+4.5+3)=-6.8\,\text{V}$

When $\alpha=0.80, \qquad v_o=-0.80(1+1.2+0.25)=-1.96\,\text{V}$

$\therefore\ -6.8\,\text{V}\le v_o\le-1.96\,\text{V}$

[b] $-0.8\left[1+6(1-\alpha)+\frac{(1-\alpha)}{\alpha}\right]=-12$

$\alpha+6\alpha(1-\alpha)+(1-\alpha)=15\alpha$

$\alpha+6\alpha-6\alpha^2+1-\alpha=15\alpha$

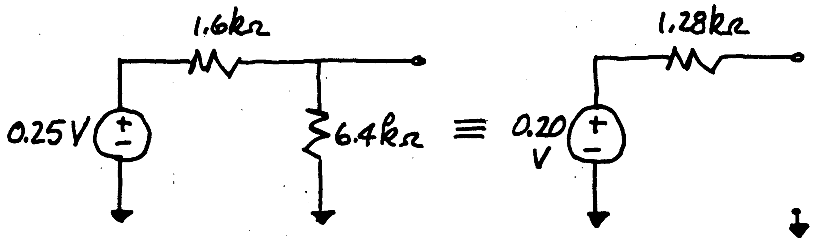
$\therefore\ 6\alpha^2+9\alpha-1=0$

$\alpha^2+1.5\alpha-\frac{1}{6}=0$

$\alpha=-0.75\pm\sqrt{0.72917}$

$\alpha\cong0.1039$

P 6.8 [a] Replace the combination of v_g, 1.6 kΩ, and the 6.4 kΩ resistors with its Thévenin equivalent.



Then $v_o = \dfrac{-[12 + \sigma 50]}{1.28}(0.20)$

At saturation $v_o = -5$ V; therefore

$$-\left(\frac{12 + \sigma 50}{1.28}\right)(0.2) = -5, \quad \text{or} \quad \sigma = 0.4$$

Thus for $0 \leq \sigma < 0.40$ the operational amplifier will not saturate.

[b] When $\sigma = 0.272$, $\quad v_o = \dfrac{-(12 + 13.6)}{1.28}(0.20) = -4\,\text{V}$

Also $\dfrac{v_o}{10} + \dfrac{v_o}{25.6} + i_o = 0$

$$\therefore\ i_o = -\frac{v_o}{10} - \frac{v_o}{25.6} = \frac{4}{10} + \frac{4}{25.6}\,\text{mA} = 556.25\,\mu\text{A}$$

P 6.9 [a] $v_o = -\dfrac{220}{44}v_a - \dfrac{220}{27.5}v_b - \dfrac{220}{80}v_c = -5v_a - 8v_b - 2.75v_c$

$v_o = -5(1) - 8(1.5) - 2.75(-4)$

$v_o = -5 - 12 + 11 = -6\,\text{V}$

[b] $v_o = 6 - 8v_b = \pm 10$

$\therefore\ v_b = -0.5\,\text{V}$ when $v_o = 10\,\text{V}$; $\quad v_b = 2.0\,\text{V}$ when $v_o = -10\,\text{V}$

$\therefore\ -0.5\,\text{V} \leq v_b \leq 2.0\,\text{V}$

P 6.10 [a] $\dfrac{12 - 15}{55} + \dfrac{12 - 10}{66} + \dfrac{12 - 8}{220} + \dfrac{12}{600} + \dfrac{12 - v_o}{330} = 0$

$v_o = -18 + 10 + 6 + 6.6 + 12 = 16.6\,\text{V}$

[b] $-\dfrac{3}{55} + \dfrac{12 - v_b}{66} + \dfrac{4}{220} + \dfrac{12}{600} + \dfrac{12}{330} = \dfrac{v_o}{330}$

$v_o = 66.6 - 5v_b, \quad \therefore\ v_b = 66.6 \pm 20$

$5v_b = 86.6, \quad v_b = 17.32$

$5v_b = 46.6, \quad v_b = 9.32$

$\therefore\ 9.32 \leq v_b \leq 17.32\,\text{V}$

P 6.11 [a] $$-\frac{3}{55}+\frac{2}{66}+\frac{4}{220}+\frac{12}{600}+\frac{12-v_o}{R_f}=0$$

$$\therefore\ v_o = 12+\frac{12R_f}{600}+\frac{4R_f}{220}+\frac{2R_f}{66}-\frac{3R_f}{55}$$

$$\pm 20 = 12+R_f\left(\frac{1}{50}+\frac{1}{55}+\frac{1}{33}-\frac{3}{55}\right)$$

Since $R_f > 0$,

$$8 = R_f\left(\frac{1}{50}+\frac{1}{33}-\frac{2}{55}\right);\qquad 2640 = R_f(4.6);\qquad R_f = 573.91\,\text{k}\Omega$$

[b] $$\frac{20}{20}+\frac{20-12}{573.91}+i_o=0;\qquad i_o=-1.01394\,\text{mA};\qquad i_o=-1013.94\,\mu\text{A}$$

P 6.12 Let v_{o1} = output voltage of the amplifier on the left. Let v_{o2} = output voltage of the amplifier on the right. Then

$$v_{o1}=\frac{-50}{10}(2)=-10\,\text{V};\qquad v_{o2}=\frac{-250}{25}(-1)=10\,\text{V}$$

$$i_a=\frac{v_{o2}-v_{o1}}{5}=\frac{20}{5}=4\,\text{mA}$$

P 6.13 [a] The output voltage of the first op-amp is $v_{o1}=\frac{-80}{20}v_g=-4v_g$

The output voltage of the second op-amp is $v_{o2}=\frac{-16}{10}(-4v_g)=6.4v_g$

When v_g has its largest value, i.e., 1.2 V, $v_{o1}=-4.8\,\text{V}$ and $v_{o2}=7.68\,\text{V}$

Therefore neither op-amp saturates.

The expression for i_g is $i_g=\frac{v_g}{20}+\frac{v_g-6.4v_g}{R_o}=v_g\left[\frac{1}{20}-\frac{5.4}{R_o}\right]$

$i_g=0$ when $\left(\frac{1}{20}-\frac{5.4}{R_o}\right)=0$, or $R_o=108\,\text{k}\Omega$

[b] $$i_{R_o}=\frac{6.4-1.0}{108}=0.05\,\text{mA}$$

$$p_{R_o}=(0.05\times10^{-3})^2(108\times10^3)$$

$$p_{R_o}=270\times10^{-6}=270\,\mu\text{W}$$

P 6.14 [a] Assume the op-amp is operating within its linear range, then

$$i_L=\frac{9}{5}=1.8\,\text{mA}$$

For $R_L=4\,\text{k}\Omega$ $\quad v_o=(4+5)(1.8)=16.2\,\text{V}$

Now since $v_o<18\,\text{V}$ our assumption of linear operation is correct, therefore

$$i_L=1.8\,\text{mA}$$

[b] $i_L = 1.8(5 + R_L)$

$5 + R_L = 10\,\text{k}\Omega, \qquad R_L = 5\,\text{k}\Omega$

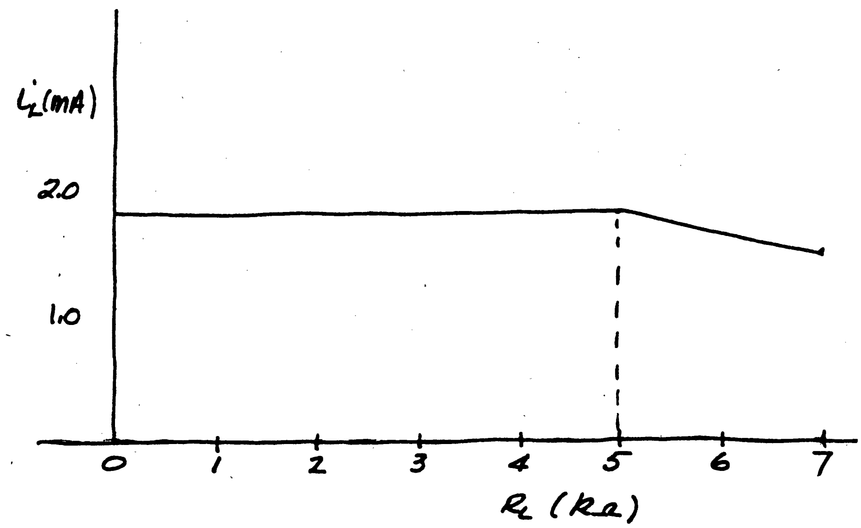
[c] Op-amp saturates for $R_L \geq 5\,\text{k}\Omega$, $\quad \therefore\ i_L = \dfrac{18}{5+7} = 1.5\,\text{mA}$

If we assume $i_1 = i_2 \approx 0$, the voltage across the input terminals of the op-amp is

$$v_{21} \approx 9 - 5(1.5) = 1.5\,\text{V}$$

If we assume $R_{in} = 500\,\text{k}\Omega$, then $i_1 = -i_2 = -(1.5/500) = -3\,\mu\text{A}$, which is small compared to i_L.

[d]



P 6.15 [a] $v_2 = v_1 = 5\,\text{V}$, therefore the current in the left 1-kΩ resistor is $5/1 = 5\,\text{mA}$. This is also the current in the 1-kΩ feedback resistor. Therefore the voltage across the 1-kΩ right-side resistor is 10 V, i.e., $5 + 5 = 10\,\text{V}$. The current i_2 is zero, i.e., the current into the noninverting input terminal. Therefore

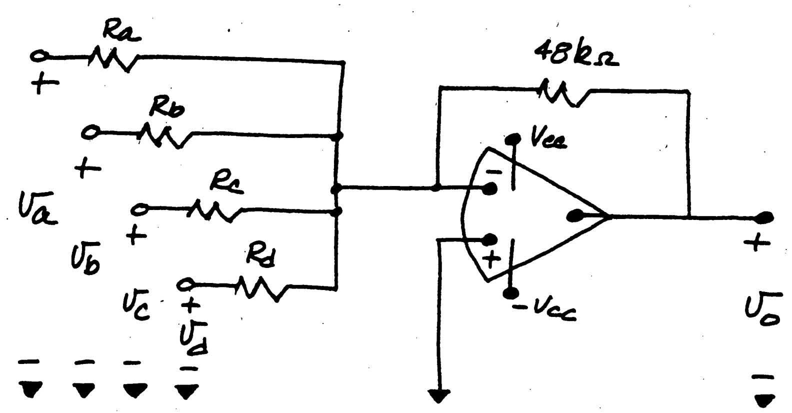
$$i_a = 5 + 10 + 0 = 15\,\text{mA}$$

[b] $v_o = 10 + 15R = 15; \qquad R = \dfrac{5}{15} \times 10^3 = 333.33\,\Omega$

P 6.16 $v_o = -\left[\dfrac{R_f}{3}(0.15) + \dfrac{R_f}{5}(0.10) + \dfrac{R_f}{25}(0.25)\right]$

$-6 = -0.08R_f; \qquad R_f = 75\,\text{k}\Omega; \qquad \therefore\ 0 \leq R_f \leq 75\,\text{k}\Omega$

P 6.17



$$v_o = -\left[\frac{48}{R_a}v_a + \frac{48}{R_b}v_b + \frac{48}{R_c}v_c + \frac{48}{R_d}v_d\right]$$

$\therefore$ $R_a = 48/2 = 24\,\text{k}\Omega$ $\quad$ $R_c = 48/6 = 8\,\text{k}\Omega$

$R_b = 48/4 = 12\,\text{k}\Omega$ $\quad$ $R_d = 48/8 = 6\,\text{k}\Omega$

P 6.18 [a] $v_2 = v_1 = \frac{68}{80}v_g = 0.85v_g$

$\therefore \frac{0.85v_g}{30} + \frac{0.85v_g - v_o}{63} = 0;$ $\quad$ $\therefore v_o = 2.635v_g = 2.635(4),$ $\quad$ $v_o = 10.54\,\text{V}$

[b] $v_o = 2.635v_g = \pm 12$

$v_g = \pm 4.55\,\text{V},$ $\quad$ $-4.55 \leq v_g \leq 4.55\,\text{V}$

[c] $\frac{0.85v_g}{30} + \frac{0.85v_g - v_o}{R_f} = 0$

$\left(\frac{0.85R_f}{30} + 0.85\right)v_g = v_o = \pm 12$

$\therefore 1.7R_f + 51 = \pm 360;$ $\quad$ $1.7R_f = 360 - 51;$ $\quad$ $R_f = 181.76\,\text{k}\Omega$

P 6.19 [a] $v_2 = v_s,$ $\quad$ $v_1 = \frac{R_1 v_o}{R_1 + R_2},$ $\quad$ $v_1 = v_2$

Therefore $v_o = \left(\frac{R_1 + R_2}{R_1}\right)v_s = \left(1 + \frac{R_2}{R_1}\right)v_s$

[b] $v_o = v_s$

[c] Because $v_o = v_s$, thus the output voltage follows the signal voltage.

P 6.20 [a] $p_a = \frac{(320 \times 10^{-3})^2}{(16 \times 10^3)} = 6.4\,\mu\text{W}$

[b] $v_{16\,\mathrm{k}\Omega} = \left(\frac{16}{80}\right)(320) = 64\,\mathrm{mV}$

$$p_b = \frac{(64 \times 10^{-3})^2}{(16 \times 10^3)} = 0.256\,\mu\mathrm{W}$$

[c] $\frac{p_a}{p_b} = \frac{6.4}{0.256} = 25$

[d] Yes, the operational amplifier serves several useful purposes:

- First, it enables the source to control 25 times as much power delivered to the load resistor. When a small amount of power controls a larger amount of power, we refer to it as *power amplification*.
- Second, it allows the full source voltage to appear across the load resistor, no matter what the source resistance. This is the *voltage follower* function of the operational amplifier.
- Third, it allows the load resistor voltage (and thus its current) to be set without drawing any current from the input voltage source. This is the *current amplification* function of the circuit.

P 6.21 [a] $\frac{v_1}{11} + \frac{v_1 - v_o}{110} = 0; \qquad \therefore\ v_o = 11v_1$

$$\frac{v_2 - v_a}{13} + \frac{v_2 - v_b}{27} = 0$$

$$27v_2 - 27v_a + 13v_2 - 13v_b = 0$$

$$40v_2 = 27v_a + 13v_b$$

$$v_2 = 0.675v_a + 0.325v_b$$

$$v_1 = v_2$$

$$\therefore\ v_o = 11(0.675v_a + 0.325v_b) = 7.425v_a + 3.575v_b$$

$$v_o = 5.94 + 1.43 = 7.37\,\mathrm{V}$$

[b] $v_2 = v_1 = \frac{v_o}{11} = 0.67\,\mathrm{V} = 670\,\mathrm{mV}$

$$i_a = \frac{v_a - v_2}{13 \times 10^3} = 10 \times 10^{-6} = 10\,\mu\mathrm{A}$$

$$i_b = \frac{v_b - v_2}{27 \times 10^3} = -10 \times 10^{-6} = -10\,\mu\mathrm{A}$$

[c] The weighting factor for v_a is 7.425. The weighting factor for v_b is 3.575.

P 6.22 [a] $\frac{v_1}{20} + \frac{v_1 - v_o}{100} = 0$

$$6v_1 - v_o = 0, \qquad \therefore\ v_o = 6v_1 = 6v_2$$

$$\frac{v_2 - v_a}{R_a} + \frac{v_2 - v_b}{R_b} + \frac{v_2 - v_c}{R_c} = 0$$

$$R_bR_cv_2 + R_aR_cv_2 + R_aR_bv_2 = v_aR_bR_c + v_bR_aR_c + v_cR_aR_b$$

$$\therefore\ (R_bR_c + R_aR_c + R_aR_b)v_2 = R_bR_cv_a + R_aR_cv_b + R_aR_bv_c$$

Let $\sum PR = R_bR_c + R_aR_c + R_aR_b$

Then $v_o = \frac{6R_bR_c}{\sum PR}v_a + \frac{6R_aR_c}{\sum PR}v_b + \frac{6R_aR_b}{\sum PR}v_c$

By hypothesis $\frac{6R_bR_c}{\sum PR} = 1;\quad \frac{6R_aR_c}{\sum PR} = 2;\quad \frac{6R_aR_b}{\sum PR} = 3$

Therefore $\frac{R_b}{R_a} = \frac{1}{2},\quad R_a = 2R_b = 30\,\text{k}\Omega;\quad \frac{R_c}{R_b} = \frac{2}{3},\quad R_c = \frac{2}{3}R_b = 10\,\text{k}\Omega$

[b] $v_o = 0.7 + 0.8 + 3.3 = 4.8\,\text{V}$

$\therefore\ v_1 = v_2 = \frac{1}{6}(4.8) = 0.8\,\text{V}$

$\therefore\ i_a = \frac{0.7 - 0.8}{30 \times 10^3} = -3.33\,\mu\text{A}$

$i_b = \frac{0.4 - 0.8}{15 \times 10^3} = -26.67\,\mu\text{A}$

$i_c = \frac{1.1 - 0.8}{10 \times 10^3} = 30\,\mu\text{A}$

Check: $i_a + i_b + i_c = 0$

$-3.33 - 26.67 + 30 = 0$ (checks)

P 6.23 [a] $\frac{v_1}{R_g} + \frac{v_1 - v_o}{180} = 0$

$\therefore\ v_o = \left(1 + \frac{180}{R_g}\right)v_1 = Gv_1 = Gv_2$

$\frac{v_2 - v_a}{8} + \frac{v_2 - v_b}{2} + \frac{v_2 - v_c}{1} + \frac{v_2}{9} = 0$

$\therefore\ 125v_2 = 9v_a + 36v_b + 72v_c$

$\therefore\ v_o = \frac{9Gv_a}{125} + \frac{36Gv_b}{125} + \frac{72Gv_c}{125}$

It follows that $\frac{9G}{125} = 1.8,\quad G = 0.2(125) = 25$

$\therefore\ 1 + \frac{180}{R_g} = 25,\quad R_g = \frac{180}{24} = 7.5\,\text{k}\Omega$

[b] $v_o = 1.8(0.5) + 7.2(0.25) + 14.40(0.15) = 4.86\,\text{V} = 194.40\,\text{mV}$

4.86 V

$V2 = V1 = \frac{4.86(7.5)}{187.5} = 194.4\,\text{mV}$

$i_a = \frac{500 - 194.4}{8} = 38.20\,\mu\text{A}$

$i_b = \frac{250 - 194.4}{2} = 27.80\,\mu\text{A}$

$i_c = \frac{150 - 194.4}{1} = -44.40\,\mu\text{A}$

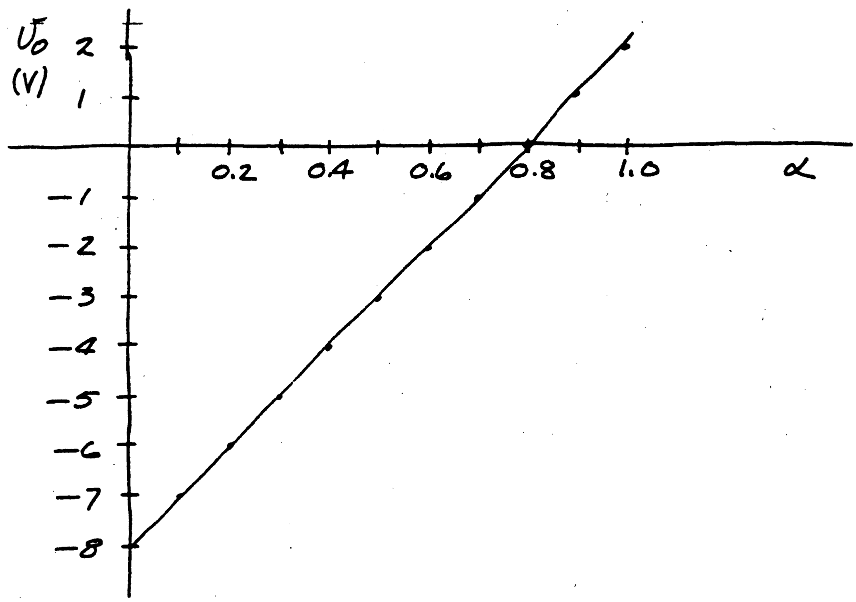
$$i_g = \frac{194.40}{7.5} = 25.92\,\mu\text{A}$$
$$i_h = \frac{194.40}{9} = 21.60\,\mu\text{A}$$
$$i_o = -\frac{4860}{3.6} - 25.92$$
$$i_o = -1375.92\,\mu\text{A}$$

P 6.24 $v_1 = v_2 = \alpha v_g$

$$\frac{v_1 - v_g}{R_1} + \frac{v_1 - v_o}{R_f} = 0$$
$$(v_1 - v_g)\frac{R_f}{R_1} + v_1 - v_o = 0$$

$$v_o = (\alpha v_g - v_g)4 + \alpha v_g$$
$$= [(\alpha - 1)4 + \alpha]v_g$$
$$= (5\alpha - 4)v_g$$
$$= (5\alpha - 4)(2) = 10\alpha - 8$$

α	v_o	α	v_o	α	v_o
0.0	−8 V	0.4	−4 V	0.8	0 V
0.1	−7 V	0.5	−3 V	0.9	1 V
0.2	−6 V	0.6	−2 V	1.0	2 V
0.3	−5 V	0.7	−1 V		



P 6.25 $\frac{v_1 - v_a}{4.7} + \frac{v_1 - v_o}{R_f} = 0$

$$\therefore\ v_o = \left(1 + \frac{R_f}{4.7}\right)v_1 - \frac{R_f}{4.7}v_a, \qquad v_1 = v_2 = \left(\frac{R_b}{R_a + R_b}\right)v_b$$

$$\therefore\ v_o = \left(1 + \frac{R_f}{4.7}\right)\left(\frac{R_b}{R_a + R_b}\right)v_b - \frac{R_f}{4.7}v_a$$

By hypothesis $\frac{R_f}{4.7} = 10, \qquad R_f = 47\,\text{k}\Omega$

$R_a + R_b = 220\,\text{k}\Omega$

Thus $\left(1 + \frac{47}{4.7}\right)\left(\frac{R_b}{220}\right) = 10$

$\therefore\ R_b = 200\,\text{k}\Omega; \qquad \therefore\ R_a = 220 - 200 = 20\,\text{k}\Omega$

P 6.26 [a] $v_o = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)}v_b - \frac{R_b}{R_a}v_a = \frac{120(24 + 75)}{24(130 + 120)} \cdot 5 - \frac{75}{24}(8)$

$v_o = 9.9 - 25 = -15.10\,\text{V}$

[b] $\frac{v_1 - 8}{24} + \frac{v_1 + 15.10}{75} = 0$

$3.125v_1 - 25 + v_1 + 15.10 = 0$

$4.125v_1 = 9.9; \qquad v_1 = 2.4\,\text{V}$

$i_a = \frac{8 - 2.4}{24} = 233\,\mu\text{A}; \qquad R_{\text{in a}} = \frac{8}{i_a} = 34.3\,\text{k}\Omega$

[c] $R_{\text{in b}} = R_c + R_d = 250\,\text{k}\Omega$

P 6.27 $v_o = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)}v_b - \frac{R_b}{R_a}v_a$

By hypothesis: $R_b/R_a = 6; \quad R_c + R_d = 450\,\text{k}\Omega;$ and

$\frac{R_d(R_a + R_b)}{R_a(R_c + R_d)} = 4.2 = \frac{R_d(7R_a)}{R_a(450)}$

$\therefore\ R_d = 0.6(450) = 270\,\text{k}\Omega, \qquad R_c = 450 - 270 = 180\,\text{k}\Omega$

Also, when $v_o = 0$ we have $\frac{v_1 - v_a}{R_a} + \frac{v_1}{R_b} = 0$

$\therefore\ \frac{R_b}{R_a}(v_1 - v_a) + v_1 = 0$ or $7v_1 = 6v_a, \quad v_1 = \left(\frac{6}{7}\right)v_a$

$i_a = \frac{v_a - v_1}{R_a} = \frac{v_a}{7R_a}$

$\therefore\ R_{\text{in a}} = 7R_a = 21\,\text{k}\Omega, \quad R_a = 3\,\text{k}\Omega, \quad R_b = 6R_a = 18\,\text{k}\Omega$

P 6.28 $v_2 = 1000i_b$

$\frac{1000i_b}{R_1} + \frac{1000i_b - v_o}{R_f} - i_a = 0$

$\therefore\ 1000i_b\left[\frac{1}{R_1} + \frac{1}{R_f}\right] - i_a = \frac{v_o}{R_f}$

$\therefore\ 1000i_b\left[1 + \frac{R_f}{R_1}\right] - R_f i_a = v_o$

By hypothesis $v_o = 5000(i_b - i_a)$

$\therefore\ R_f = 5000\,\Omega$

$$1000\left(1 + \frac{R_f}{R_1}\right) = 5000$$

$\therefore\ R_1 = 1250\,\Omega$

P 6.29 [a] $\frac{v_2}{20} + \frac{v_2 - v_c}{30} + \frac{v_2 - v_d}{20} = 0$ or $8v_2 = 2v_c + 3v_d$, $v_2 = 0.25v_c + 0.375v_d$

$\frac{v_1 - v_a}{20} + \frac{v_1 - v_b}{18} + \frac{v_1 - v_o}{180} = 0$ or $20v_1 - 9v_a - 10v_b = v_o$

$v_1 = v_2$ (ideal op-amp)

$\therefore\ 5v_c + 7.5v_d - 9v_a - 10v_b = v_o$

$\therefore\ v_o = 5(3) + 7.5(4) - 9(1) - 10(2) = 15 + 30 - 9 - 20 = 16\,\text{V}$

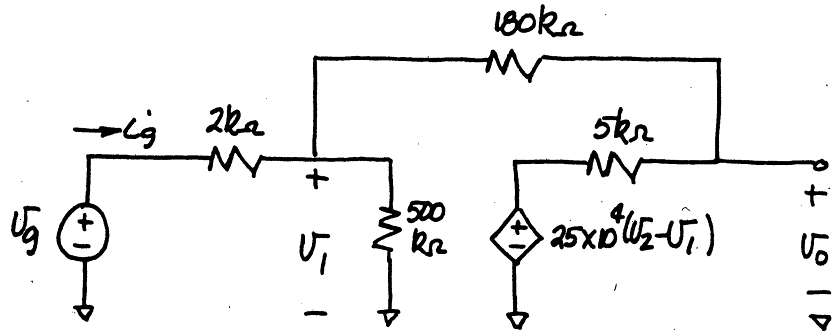
[b] $v_o = 5v_c + 30 - 9 - 20 = 5v_c + 1$

$\therefore\ 5v_c + 1 = \pm 20$

$v_c = 3.8\,\text{V}$ (+ saturation), $v_c = -4.2\,V$ (− saturation)

$-4.2\,\text{V} \le v_c \le 3.8\,\text{V}$

P 6.30 [a]



$$\frac{v_1}{500} + \frac{v_1 - v_g}{2} + \frac{v_1 - v_o}{180} = 0$$

$456.8v_1 - 5v_o = 450v_g$; $\boxed{91.36v_1 - v_o = 90v_g}$

$$\frac{v_o - 250{,}000(-v_1)}{5} + \frac{v_o - v_1}{180} = 0; \quad \boxed{9 \times 10^6 v_1 + 37v_o = 0}$$

$$\Delta = \begin{vmatrix} 91.36 & -1 \\ 9 \times 10^6 & 37 \end{vmatrix} = 9{,}003{,}380.32$$

$$N_o = \begin{vmatrix} 91.36 & 90v_g \\ 9 \times 10^6 & 0 \end{vmatrix} = -81 \times 10^7 v_g$$

$$v_o = \frac{N_o}{\Delta} = -89.966v_g; \quad \boxed{\frac{v_o}{v_g} = -89.966}$$

[b] $N_1 = \begin{vmatrix} 90v_g & -1 \\ 0 & 37 \end{vmatrix} = 3330v_g$

$v_1 = \frac{N_1}{\Delta} = 3.7 \times 10^{-4}v_g$

$v_g = 100\,\text{mV}, \qquad v_1 = 36.986\,\mu\text{V}$

[c] $i_g = \frac{v_g - v_1}{2} = \frac{v_g - 3.7 \times 10^{-4}v_g}{2}$

$R_g = \frac{v_g}{i_g} = \frac{v_g}{v_g - 3.70 \times 10^{-4}v_g} \cdot (2000)$

$R_g = 2000.74\Omega$

[d] $\frac{v_o}{v_g} = 90; \qquad v_1 = 0\,\text{V}; \qquad R_g = 2000\,\Omega$

P 6.31 [a] From the solution of Problem 6.30 we have $\boxed{91.36v_1 - v_o = 90v_g}$

At the output we now have $\frac{v_o + 25 \times 10^4 v_1}{5} + \frac{v_o - v_1}{180} + \frac{v_o}{1.6} = 0$

or $\boxed{9 \times 10^6 v_1 + 149.5v_o = 0}$

$\therefore\ \Delta = \begin{vmatrix} 91.36 & -1 \\ 9 \times 10^6 & 149.5 \end{vmatrix} = 9{,}013{,}658.32$

$N_o = \begin{vmatrix} 91.36 & 90v_g \\ 9 \times 10^6 & 0 \end{vmatrix} = -81 \times 10^7 v_g$

$v_o = \frac{N_o}{\Delta} = -89.86v_g; \qquad \frac{v_o}{v_g} = -89.86$

[b] $N_1 = \begin{vmatrix} 90v_g & -1 \\ 0 & 149.5 \end{vmatrix} = 13{,}455v_g; \qquad v_1 = \frac{N_1}{\Delta} = 1.49 \times 10^{-3}v_g$

When $v_g = 100\,\text{mV}, \quad v_1 = 149.27\,\mu\text{V}$

[c] $R_g = \frac{v_g}{i_g}, \qquad i_g = \frac{v_g - v_1}{2000} = \frac{v_g - 1.49 \times 10^{-3}v_g}{2000}$

$\therefore\ R_g = \frac{1}{1 - 1.49 \times 10^{-3}}(2000) = 2002.99\,\Omega$

[d] $\frac{v_o}{v_g} = -90; \qquad v_1 = 0\,\text{V}; \qquad R_g = 2000\,\Omega$

P 6.32 [a] $\frac{v_1}{16} + \frac{v_1 - v_g}{800} + \frac{v_1 - v_o}{200} = 0$ or $\boxed{55v_1 - 4v_o = v_g}$

$\frac{v_o}{20} + \frac{v_o - v_1}{200} + \frac{v_o - 50{,}000(v_2 - v_1)}{8} = 0$

$36v_o - v_1 - 125 \times 10^4(v_2 - v_1) = 0$

$v_2 = v_g + \frac{(v_1 - v_g)(240)}{800} = 0.7v_g + 0.3v_1$

$36v_o - v_1 - 125 \times 10^4(0.7v_g - 0.7v_1)$

$36v_o + 874{,}999v_1 = 875{,}000v_g$

$$\Delta = \begin{vmatrix} 55 & -4 \\ 874{,}999 & 36 \end{vmatrix} = 3{,}501{,}976$$

$$N_o = \begin{vmatrix} 55 & v_g \\ 874{,}999 & 875{,}000v_g \end{vmatrix} = 47{,}250{,}001v_g$$

$$v_o = \frac{N_o}{\Delta} = 13.49v_g; \qquad \frac{v_o}{v_g} = 13.49$$

[b] $$N_1 = \begin{vmatrix} v_g & -4 \\ 875{,}000v_g & 36 \end{vmatrix} = 3{,}500{,}036v_g$$

$$v_1 = \frac{N_1}{\Delta} = 0.999446v_g; \qquad v_1 = 999.45\,\text{mV}$$

$$v_2 = 0.7(1000) + 0.3(999.45) = 999.83\,\text{mV}$$

[c] $v_2 - v_1 = 387.78\,\mu\text{V}$

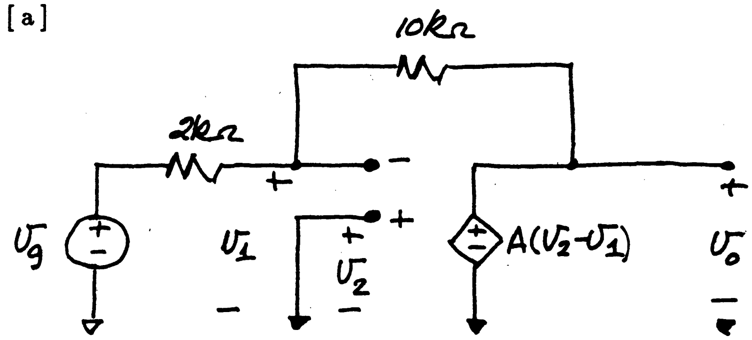
[d] $$i_g = \frac{(1000 - 999.83)10^{-3}}{240 \times 10^3} = 692.47\,p\text{A}$$

[e] $$\frac{v_g}{16} + \frac{v_g - v_o}{200} = 0, \qquad \text{since } v_1 = v_2 = v_g$$

$$\therefore\ v_o = 13.5v_g, \qquad \frac{v_o}{v_g} = 13.5$$

$$v_1 = v_2 = 1\,\text{V}; \qquad v_2 - v_1 = 0\,\text{V}; \qquad i_g = 0\,\text{A}$$

P 6.33 [a]



$$\frac{v_1 - v_g}{2} + \frac{v_1 - v_o}{10} = 0$$

$$\therefore\ 5v_1 - 5v_g + v_1 - v_o = 0$$

$$6v_1 - v_o = 5v_g$$

Also $\quad v_o = A(v_2 - v_1) = -Av_1$

$$\therefore\ v_1 = \frac{-v_o}{A}$$

$$\therefore\ 6\left(\frac{-v_o}{A}\right) - v_o = 5v_g$$

$$v_o = \left(\frac{6}{A_1} + 1\right) = -5v_g$$

$$v_o = \frac{-5Av_g}{(A + 6)}$$

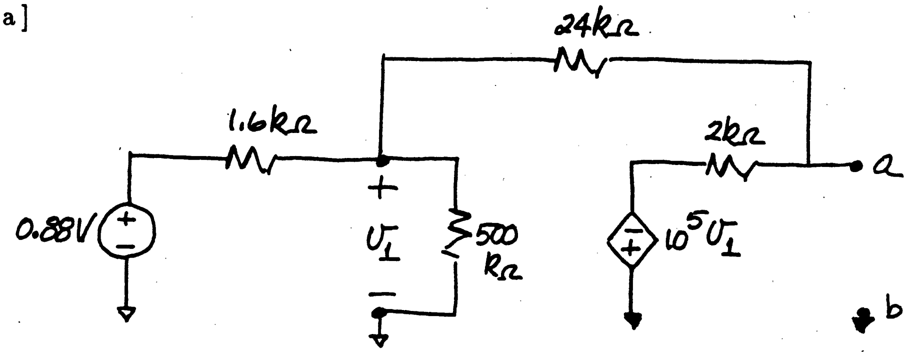
[b] $v_o = \frac{-5(194)}{200}(1) = -4.85\,\text{V}$

[c] $v_o = \frac{-5}{(1+6/A)}(1) = -5\,\text{V}$

[d] $\frac{-5A}{(A+6)}(1) = -0.99(5)$

$\therefore\ -5A = -4.95(A+6); \qquad -0.05A = -29.70; \qquad A = 594$

P 6.34 [a]



$$\frac{v_1 - 0.88}{1.6} + \frac{v_1}{500} + \frac{v_1 - v_o}{24} = 0$$

$300v_1 + 0.96v_1 + 20v_1 - 20v_o = 264;$ $\boxed{320.96v_1 - 20v_o = 264}$

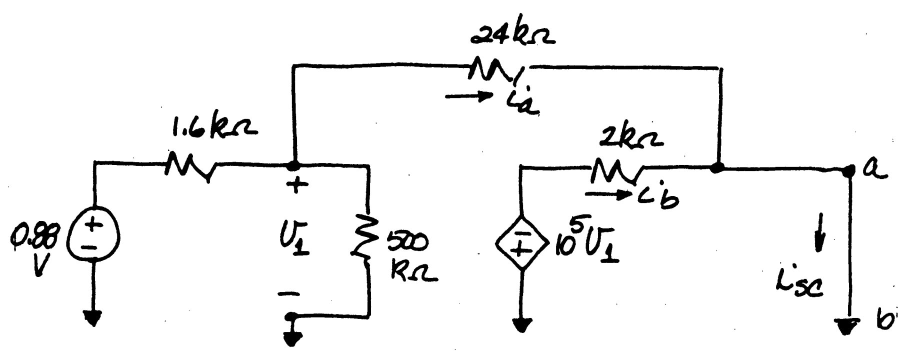
$\frac{v_o + 10^5 v_1}{2} + \frac{v_o - v_1}{24} = 0;$ $\boxed{12 \times 10^5 v_1 + 13v_o = 0}$

$$\Delta = \begin{vmatrix} 320.96 & -20 \\ 12 \times 10^5 & 13 \end{vmatrix} = 24{,}004{,}172.48$$

$$N_o = \begin{vmatrix} 320.96 & 264 \\ 12 \times 10^5 & 0 \end{vmatrix} = -3168 \times 10^5$$

$$v_o = \frac{N_o}{\Delta} = -13.20\,\text{V} = v_{\text{Th}}$$

Short-circuit current calculation:



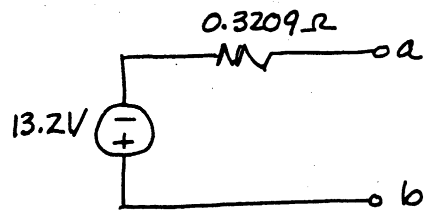
$$i_{sc} = i_a + i_b; \qquad i_a = \frac{v_1}{24}; \qquad i_b = \frac{-10^5 v_1}{2}$$

$$\frac{v_1 - 0.88}{1.6} + \frac{v_1}{500} + \frac{v_1}{24} = 0$$

$$320.96v_1 = 264; \qquad v_1 = 0.8225\,\text{V}$$

$$i_{sc} = \frac{0.8225}{24} + \frac{(-10^5)(0.8225)}{2} = -41{,}126.59\,\text{mA} = -41.13\,\text{A}$$

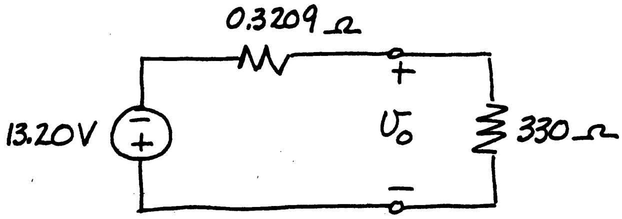
$$R_{Th} = \frac{v_{Th}}{i_{sc}} = \frac{-13.20}{-41.13} = 0.3209\,\Omega$$



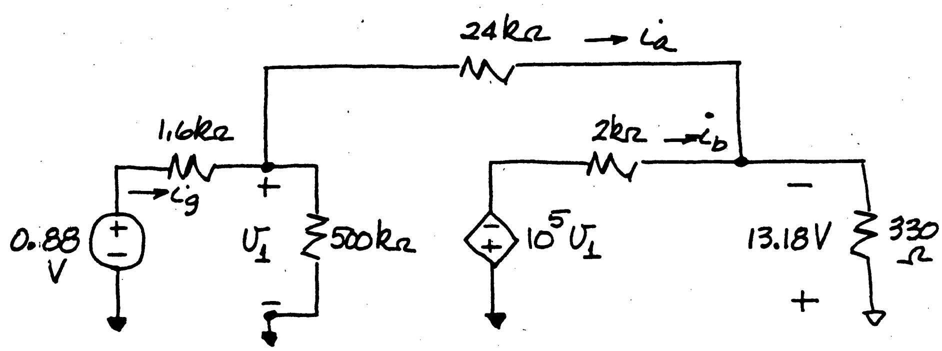
[b] The output resistance of the inverting amplifier is the same as the Thévenin resistance, i.e.,

$$R_o = R_{Th} = 0.3209\,\Omega$$

[c]



$$v_o = \frac{-13.20}{330.3209}(330) = -13.18\,\text{V}$$



$$\frac{v_1 - 0.88}{1.6} + \frac{v_1}{500} + \frac{v_1 + 13.18}{24} = 0$$

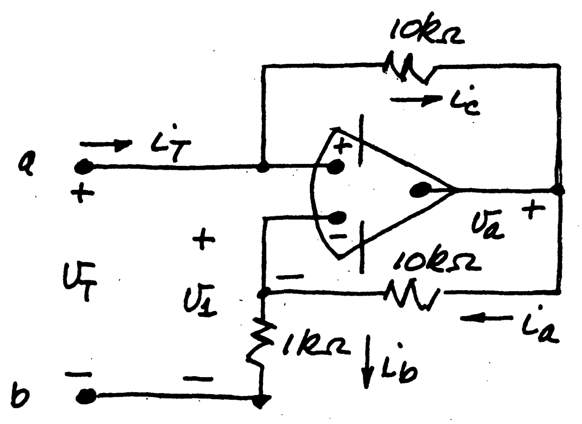
$$320.96v_1 = 0.30; \qquad v_1 = 9.42 \times 10^{-4}\,\text{V}$$

$$i_g = \frac{0.88 - 9.42 \times 10^{-4}}{1.6}$$

$$R_g = \frac{0.88}{0.88 - 9.42 \times 10^{-4}}(1600)$$

$$R_g = 1601.71\,\Omega$$

P 6.35 Since the circuit contains no independent sources, the Thévenin voltage is zero.



Apply a test voltage across a, b and note

$$v_2 = v_T = v_1$$

$$\therefore\ i_b = \frac{v_T}{1}; \qquad i_a = i_b = \frac{v_T}{1}$$

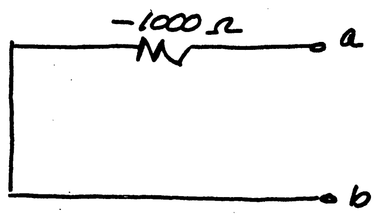
$$v_a = 10i_a = 10v_T$$

$$i_c = \frac{v_T - (v_T + 10v_T)}{10} = \frac{-10v_T}{10} = -v_T(\text{mA})$$

$$i_T = i_c = -10^{-3}v_T$$

$$R_{\text{Th}} = \frac{v_T}{i_T} = -10^3\,\Omega$$

$$R_{\text{Th}} = -1000\,\Omega$$



P 6.36 [a] Since the ideal op-amp draws no current, $v_1 = \dfrac{V_{dc}R_4}{R_1+R_4}$ and $v_2 = \dfrac{V_{dc}R_2}{R_2+R_x}$

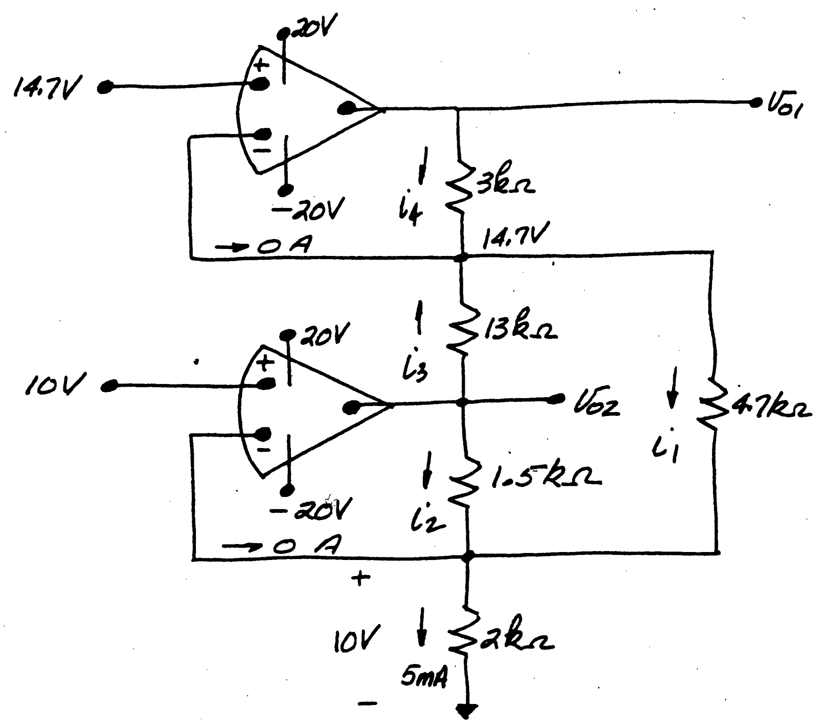
Therefore $v_1 - v_2 = V_{dc}\left[\left(\dfrac{R_4}{R_1+R_4}\right) - \left(\dfrac{R_2}{R_2+R_x}\right)\right]$

When the terms inside the braces [ ] are put over a common denomimator and the relationship $R_4R_x = R_1R_2(1+\epsilon)$ is used to simplify the resulting expression, the result is Eq. 6.37.

[b] $\dfrac{\epsilon}{1+\alpha\epsilon} = \epsilon - \alpha\epsilon^2 + \alpha^2\epsilon^3 - \alpha^3\epsilon^4 \dots$

Since $\alpha < 1$, the righthand side reduces to ϵ for small values of ϵ, and Eq. 6.37 simplifies to Eq. 6.38.

P 6.37



$i_1 = \dfrac{14.7-10}{4.7} = 1\,\text{mA}$

$i_2 + i_1 + 0 = 5; \qquad i_2 = 5 - 1 = 4\,\text{mA}$

$v_{o2} = 10 + 1.5i_2 = 16\,\text{V}$

$i_3 = \dfrac{16-14.7}{13} = \dfrac{1.3}{13} = 0.10\,\text{mA}$

$i_3 + i_4 - i_1 = 0; \qquad i_4 = i_1 - i_3 = 1 - 0.10 = 0.9\,\text{mA}$

$v_{o1} = 14.7 + 3(0.9) = 17.40\,\text{V}$

P 6.38 $v_2 = \frac{-18}{9}(1.5) = -3\,\text{V} = v_1$

$$\frac{v_1 + 18}{1.6} + \frac{v_1 - v_o}{R_f} = 0$$

$$\frac{15}{1.6} - \frac{3}{R_f} = \frac{v_o}{R_f}$$

$$\therefore\ v_o = \frac{15}{1.6}R_f - 3; \qquad v_o = +18\,\text{V} \ \text{or}\ -9\,\text{V}$$

$$\therefore\ \frac{15}{1.6}R_f = 18 + 3 = 21; \qquad R_f = 2.24\,\text{k}\Omega$$

P 6.39 Let the top op-amp be No. 1 and the lower op-amp be No. 2. Then

$$i_a = \frac{-1.32}{2.2} = -0.60\,\text{mA}$$

$$v_{o2} = (6.8 + 2.2)i_a = -5.4\,\text{V}$$

$$i_{27\,\text{k}\Omega} = \frac{v_{o2}}{27\,\text{k}\Omega} = \frac{-5.4}{27\,\text{k}\Omega} = -0.2\,\text{mA}$$

$$v_x = -(-0.2)(20) = 4\,\text{V}$$

$$i_a + i_o + i_{27\,\text{k}\Omega} = 0$$

$$i_o = -i_a - i_{27\,\text{k}\Omega} = +0.6 + 0.2 = 0.8\,\text{mA}$$

P 6.40 Let v_{o1} be the output voltage of the first op-amp. Then

$$\frac{0 - 1.1}{3} + \frac{0 - v_{o1}}{18} + \frac{0 - v_o}{24} = 0 \quad \text{or} \quad \boxed{-4v_{o1} - 3v_o = 26.4}$$

$$\frac{v_{o1}}{27} + \frac{v_{o1} - v_o}{3} = 0; \qquad \boxed{10v_{o1} - 9v_o}$$

$$\Delta = \begin{vmatrix} -4 & -3 \\ 10 & -9 \end{vmatrix} = 36 + 30 = 66$$

$$N_{o1} = \begin{vmatrix} 26.4 & -3 \\ 0 & -9 \end{vmatrix} = -237.6$$

$$N_o = \begin{vmatrix} -4 & 26.4 \\ 10 & 0 \end{vmatrix} = -264$$

$$v_{o1} = \frac{N_{01}}{\Delta} = -3.6\,\text{V}; \qquad v_o = \frac{N_o}{\Delta} = -4.0\,\text{V}$$

$$i_{24\,\text{k}} = \frac{v_o}{24} = \frac{-4}{24} = -\frac{1}{6}\,\text{mA}$$

$$i_{1.6\,\text{k}} = \frac{v_o}{1.6} = \frac{-4}{1.6} = -2.5\,\text{mA}$$

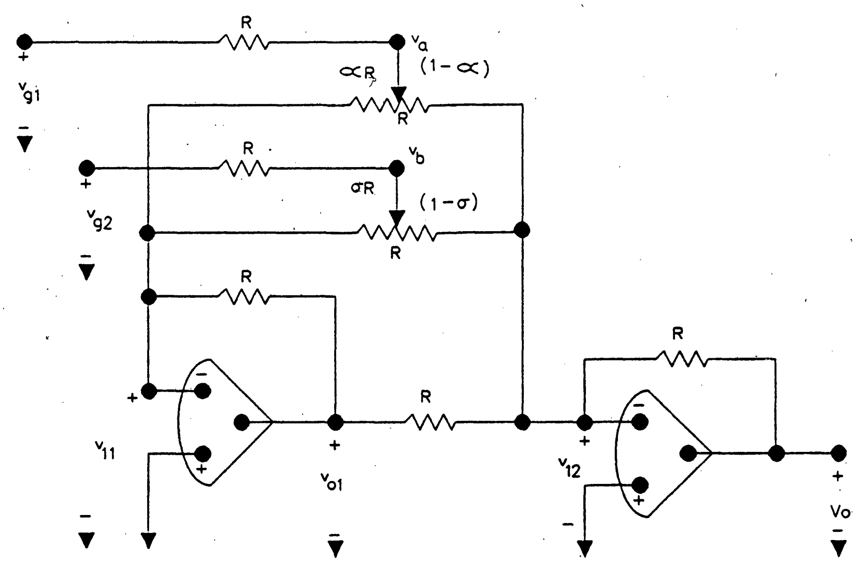
$$i_{3\,\text{k}} = \frac{v_o}{27 + 3} = \frac{-4}{30} = -\frac{2}{15}\,\text{mA}$$

$$i_o + i_{24\,\text{k}} + i_{1.6\,\text{k}} + i_{3\,\text{k}} = 0$$

$$i_o - \frac{1}{6} - 2.5 - \frac{2}{15} = 0$$

$$i_o = \frac{1}{6} + 2.5 + \frac{2}{15} = 2.8\,\text{mA}$$

P 6.41 [a] The circuit of Fig. P6.41 is redrawn with intermediate voltages defined to facilitate the analysis.



$$v_{11} = v_{12} = 0$$

$$\frac{0 - v_{o1}}{R} + \frac{0 - v_b}{\sigma R} + \frac{0 - v_a}{\alpha R} = 0$$

Therefore $v_{o1} = -\frac{v_a}{\alpha} - \frac{v_b}{\sigma}$

$$\frac{0 - v_b}{(1-\sigma)R} + \frac{0 - v_a}{(1-\alpha)R} + \frac{0 - v_{o1}}{R} + \frac{0 - v_o}{R} = 0$$

Therefore $v_o = -v_{o1} - \frac{v_a}{1-\alpha} - \frac{v_b}{1-\sigma}$

$$v_o = \frac{v_a}{\alpha} + \frac{v_b}{\sigma} - \frac{v_a}{1-\alpha} - \frac{v_b}{1-\sigma} = \frac{v_a(1-2\alpha)}{\alpha(1-\alpha)} + v_b\frac{(1-2\sigma)}{\sigma(1-\sigma)}$$

$$\frac{v_a - v_{g1}}{R} + \frac{v_a - 0}{\alpha R} + \frac{v_a - 0}{(1-\alpha)R} = 0$$

$$v_a + \frac{v_a}{\alpha} + \frac{v_a}{1-\alpha} = v_{g1}$$

$$v_a\left(\frac{\alpha(1-\alpha) + (1-\alpha) + \alpha}{\alpha(1-\alpha)}\right) = v_{g1}$$

$$v_a = \frac{v_{g1}\alpha(1-\alpha)}{(\alpha - \alpha^2 + 1)}$$

By symmetry $v_b = \frac{v_{g2}\sigma(1-\sigma)}{\sigma - \sigma^2 + 1)}$

Therefore $v_o = \frac{(1-2\alpha)}{(\alpha - \alpha^2 + 1)}v_{g1} + \frac{(1-2\sigma)}{(\sigma - \sigma^2 + 1)}v_{g2}$

[b] $\alpha = \sigma = 1$:

$v_o = -v_{g1} - v_{g2} = -(v_{g1} + v_{g2})$; inverted summing amplifier

[c] $\alpha = \sigma = 0$:

$v_o = v_{g1} + v_{g2}$; noninverted summing amplifier

P 6.42 It follows directly from the circuit that $v_o = -\dfrac{120}{7.5}v_g = -16v_g$

From the plot of v_g we have $v_g = 0, \quad t < 0$

$v_g = t \qquad 0 \leq t \leq 0.5$

$v_g = 1 - t \qquad 0.5 \leq t \leq 1.5$

$v_g = -2 + t \qquad 1.5 \leq t \leq 2.5$

$v_g = 3 - t \qquad 2.5 \leq t \leq 3.5$

$v_g = -4 + t \qquad 3.5 \leq t \leq 4.5$, etc.

Therefore

$v_o = -16t \qquad 0 \leq t \leq 0.5$

$v_o = -16 + 16t \qquad 0.5 \leq t \leq 1.5$

$v_o = 32 - 16t \qquad 1.5 \leq t \leq 2.5$

$v_o = -48 + 16t \qquad 2.5 \leq t \leq 3.5$

$v_o = 64 - 16t \qquad 3.5 \leq t \leq 4.5$, etc.

These expressions for v_o are valid as long as the op-amp is not saturated. To find the saturation times, we set $v_o = \pm 5$ V.

$-16t = -5; \qquad t = 312.5$ ms

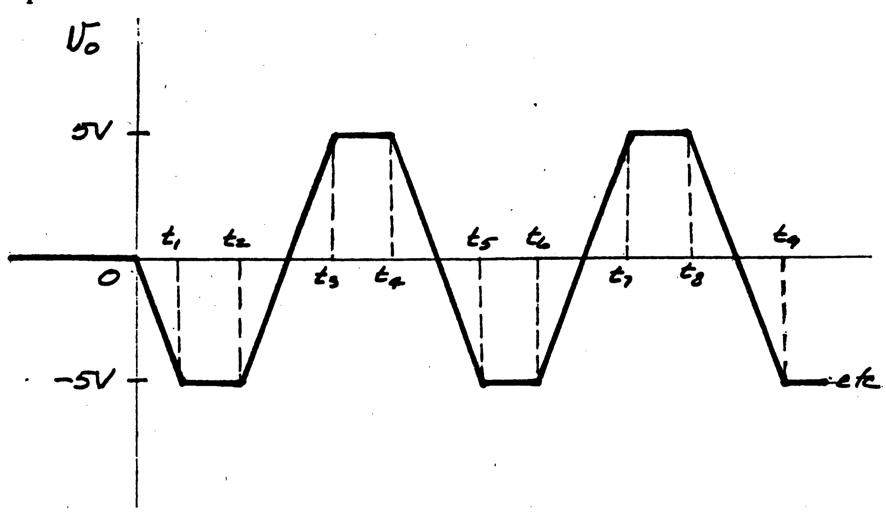
$-16 + 16t = \pm 5; \qquad t = 687.5$ ms, 1312.5 ms

$32 - 16t = \pm 5; \qquad t = 1687.5$ ms, 2312.5 ms

$-48 + 16t = \pm 5; \qquad t = 2687.5$ ms, 3312.5 ms

$64 - 16t = \pm 5; \qquad t = 3687.5$ ms, 4312.5 ms

The plot is shown below.



$t_1 = 312.5\,\text{ms}$ $\quad t_4 = 1687.5\,\text{ms}$ $\quad t_7 = 3312.5\,\text{ms}$

$t_2 = 687.5\,\text{ms}$ $\quad t_5 = 2312.5\,\text{ms}$ $\quad t_8 = 3687.5\,\text{ms}$

$t_3 = 1312.5\,\text{ms}$ $\quad t_6 = 2687.5\,\text{ms}$ $\quad t_9 = 4312.5\,\text{ms},$ etc.

P 6.43 $v_2 = \frac{5.6}{8.0}v_g = 0.7v_g$

$$\frac{0.7v_g}{2} + \frac{0.7v_g - v_o}{18} = 0$$

$$6.3v_g + 0.7v_g = v_o$$

$$v_o = 7v_g = 28\sin\left(\frac{5\pi}{3}\right)t\,\text{V} \qquad 0 \le t \le \infty$$

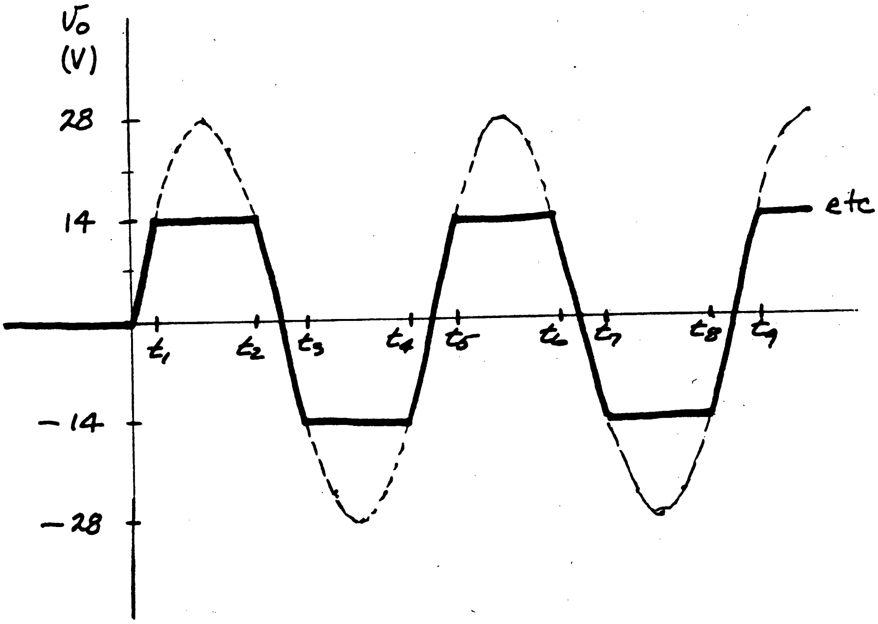
$$v_o = 0 \qquad t \le 0$$

At saturation

$$28\sin\left(\frac{5\pi}{3}\right)t = \pm 14; \qquad \sin\frac{5\pi}{3}t = \pm 0.5$$

$$\therefore\ \frac{5\pi}{3}t = \frac{\pi}{6},\ \frac{5\pi}{6},\ \frac{7\pi}{6},\ \frac{11\pi}{6},\ \text{etc.}$$

$$t = 0.10\,\text{s},\ 0.50\,\text{s},\ 0.70\,\text{s},\ \text{etc.}$$



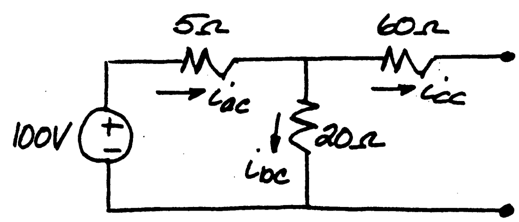
$t_1 = 0.10\,\text{s}$ $\quad t_4 = 1.1\,\text{s}$ $\quad t_7 = 1.9\,\text{s}$

$t_2 = 0.5\,\text{s}$ $\quad t_5 = 1.3\,\text{s}$ $\quad t_8 = 2.3\,\text{s}$

$t_3 = 0.70\,\text{s}$ $\quad t_6 = 1.7\,\text{s}$ $\quad t_9 = 2.5\,\text{s},$ etc.

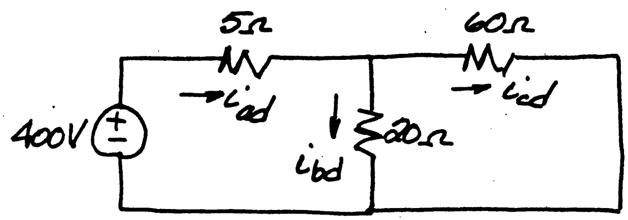
P 6.44 $v_c = \dfrac{300 - 100}{2} = 100\,\text{V}; \qquad v_d = 300 + 100 = 400\,\text{V}$

The common-mode circuit is



$$i_{ac} = i_{bc} = \frac{100}{25} = 4\,\text{A}; \qquad i_{cc} = 0$$

The differential mode circuit is



$$i_{ad} = \frac{400}{5 + 15} = 20\,\text{A}; \qquad i_{bd} = 20\left(\frac{60}{80}\right) = 15\,\text{A}; \qquad i_{cd} = 20\left(\frac{20}{80}\right) = 5\,\text{A}$$

$$\therefore\; i_a = i_{ac} + \frac{1}{2}i_{ad} = 4 + \frac{1}{2}(20) = 14\,\text{A}$$

$$i_b = i_{bc} + \frac{1}{2}i_{bd} = 4 + \frac{1}{2}(15) = 11.5\,\text{A}$$

$$i_c = i_{cc} + \frac{1}{2}i_{cd} = 0 + \frac{1}{2}(5) = 2.5\,\text{A}$$

$$i_d = i_{bc} - \frac{1}{2}i_{bd} = 4 - \frac{1}{2}(15) = -3.5\,\text{A}$$

$$i_e = i_{ac} - \frac{1}{2}i_{ad} = 4 - \frac{1}{2}(20) = -6\,\text{A}$$

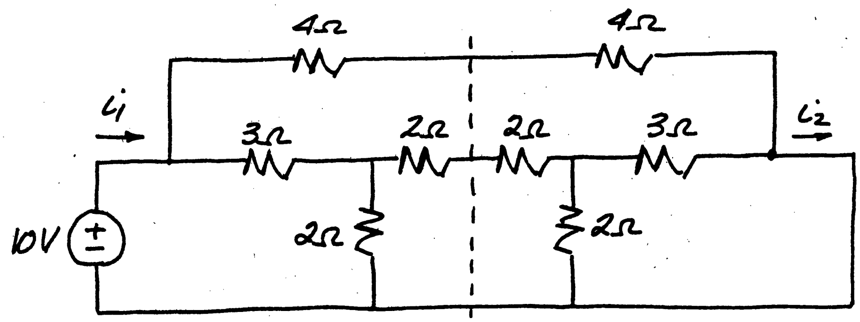
Check:

$$\sum p_{\text{dev}} = 300i_a - 100i_e = 4200 + 600 = 4800\,\text{W}$$

$$\sum p_{\text{diss}} = (14)^2(5) + (11.5)^2(20) + (2.5)^2(120) + (-3.5)^2(20) + (-6)^2(5)$$

$$= 4800\,\text{W}$$

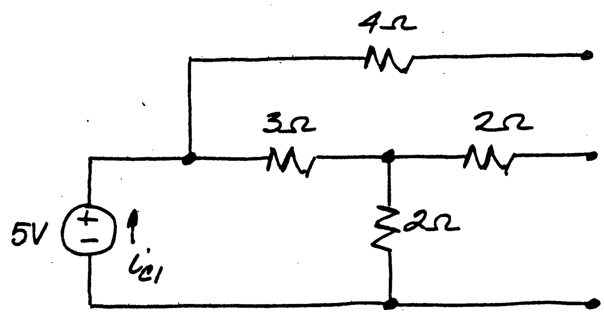
P 6.45 Redraw the circuit to highlight the symmetry.



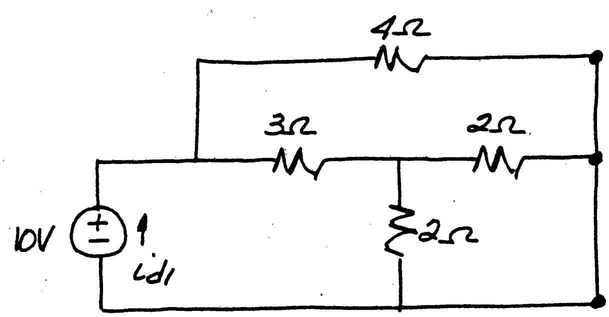
Common-mode voltage $v_c = \frac{1}{2}(10 + 0) = 5\text{ V}$

Differential-mode voltage $v_d = 10 - 0 = 10\text{ V}$

Common-mode circuit $i_{c1} = \frac{5}{5} = 1\text{ A}$



Differential-mode circuit $i_{d1} = \frac{10}{2} = 5\text{ A}$



$$i_1 = i_{c1} + \frac{1}{2}i_{d1} = 1 - \frac{1}{2}(5) = 3.5\,\text{A}$$

$$-i_2 = i_{c1} - \frac{1}{2}i_{d1} = 1 - \frac{1}{2}(5) = -1.5\,\text{A}$$

$$i_2 = 1.5\,\text{A}$$