7

Inductance and Capacitance

Drill Exercises

DE 7.1 [a] $v = L\frac{di}{dt} = -2e^{-200t} + 8e^{-800t}, \qquad t > 0^+$

$v(0^+) = -2 + 8 = 6\text{ V}$

[b] $v = 0$ when $8e^{-800t} = 2e^{-200t}$ or $t = (\ln 4)/600 = 2.31\text{ ms}$

[c] $p = vi = 50e^{-1000t} - 10e^{-400t} - 40e^{-1600t}\text{ W}$

[d] $\frac{dp}{dt} = 0$ when $e^{1200t} - 12.5e^{600t} + 16 = 0$

Let $x = e^{600t}$ and solve the quadratic $x^2 - 12.5x + 16 = 0$

$x = 1.45, \quad x = 11.05, \quad t = \frac{\ln 1.45}{600} = 616.58\,\mu\text{s}, \quad t = \frac{\ln 11.05}{600} = 4\text{ ms}$

p is maximum at $t = 616.58\,\mu\text{s}$

[e] $p_{\text{max}} = 50e^{-0.61658} - 10e^{-0.4(0.61658)} - 40e^{-1.6(0.61658)} = 4.26\text{ W}$

[f] $i_{\text{max}} = 5[e^{-0.2(2.31)} - e^{-0.8(2.31)}] = 2.36\text{ A}$

$w_{\text{max}} = (1/2)(2 \times 10^{-3})(2.36)^2 = 5.58\text{ mJ}$

[g] W is max when i is max, i is max when di/dt is zero.

When $di/dt = 0$, $v = 0$, therefore $t = 2.31\text{ ms}$.

DE 7.2 [a] $i = C\frac{dv}{dt} = 50 \times 10^{-6} d[e^{-20{,}000t}\sin 40{,}000t]$

$= [2\cos 40{,}000t - \sin 40{,}000t]e^{-20{,}000t}\text{ A}, \qquad i(0^+) = 2\text{ A}$

[b] $i\left(\frac{\pi}{80}\text{ ms}\right) = -455.94\text{ mA}, \quad v\left(\frac{\pi}{80}\text{ ms}\right) = 45.59\text{ V}, \quad p = vi = -20.79\text{ W}$

[c] $w = \left(\frac{1}{2}\right)Cv^2 = 519.70\,\mu\text{J}$

DE 7.3 [a] $v = \left(\frac{1}{C}\right)\int_{0^-}^{t} i\,dx + v(0^-)$

$= 2 \times 10^6 \int_{0^-}^{t} 2\cos 50{,}000x\,dx = 80 \sin 50{,}000t \text{ V}$

[b] $p(t) = vi = [160\cos 50{,}000t]\sin 50{,}000t$

$= 80 \sin 100{,}000t \text{ W}, \qquad p_{(\max)} = 80 \text{ W}$

[c] $w_{(\max)} = \left(\frac{1}{2}\right) C v_{\max}^2 = 0.25(80)^2 = 1600\,\mu\text{J} = 1.6 \text{ mJ}$

DE 7.4 [a] $L_{eq} = \frac{5(20)}{25} = 4 \text{ H}$

[b] $i(0^+) = -2 + 4 = 2 \text{ A}$, down

[c] $i = 0.25 \int_{0^+}^{t} (-40e^{-5x})\,dx + 2 = 2e^{-5t} \text{ A}$

[d] $i_1 = 0.2 \int_{0^+}^{t} (-40e^{-5x})\,dx - 2 = 1.6e^{-5t} - 3.6 \text{ A}$

$i_2 = 0.05 \int_{0^+}^{t} (-40e^{-5x})\,dx + 4 = 0.4e^{-5t} + 3.6 \text{ A}$

$i_1 + i_2 = i$

DE 7.5 $v_1 = 0.5 \times 10^6 \int_{0^+}^{t} 240 \times 10^{-6} e^{-10x}\,dx - 10 = -12e^{-10t} + 2 \text{ V}$

$v_2 = 0.125 \times 10^6 \int_{0^+}^{t} 240 \times 10^{-6} e^{-10x}\,dx - 5 = -3e^{-10t} - 2 \text{ V}$

$v_1(\infty) = 2 \text{ V}, \qquad v_2(\infty) = -2 \text{ V}, \qquad W = \frac{1}{2}(2)(4) + \frac{1}{2}(8)(4) = 20\,\mu\text{J}$

Problems

P 7.1 $p = vi = 40t[e^{-10t} - 10te^{-20t} - e^{-20t}]$

$\text{W} = \int_0^{\infty} p\,dx = \int_0^{\infty} 40x[e^{-10x} - 10xe^{-20x} - e^{-20x}]\,dx = 0.2 \text{ J}$

This is energy stored in the inductor at $t = \infty$.

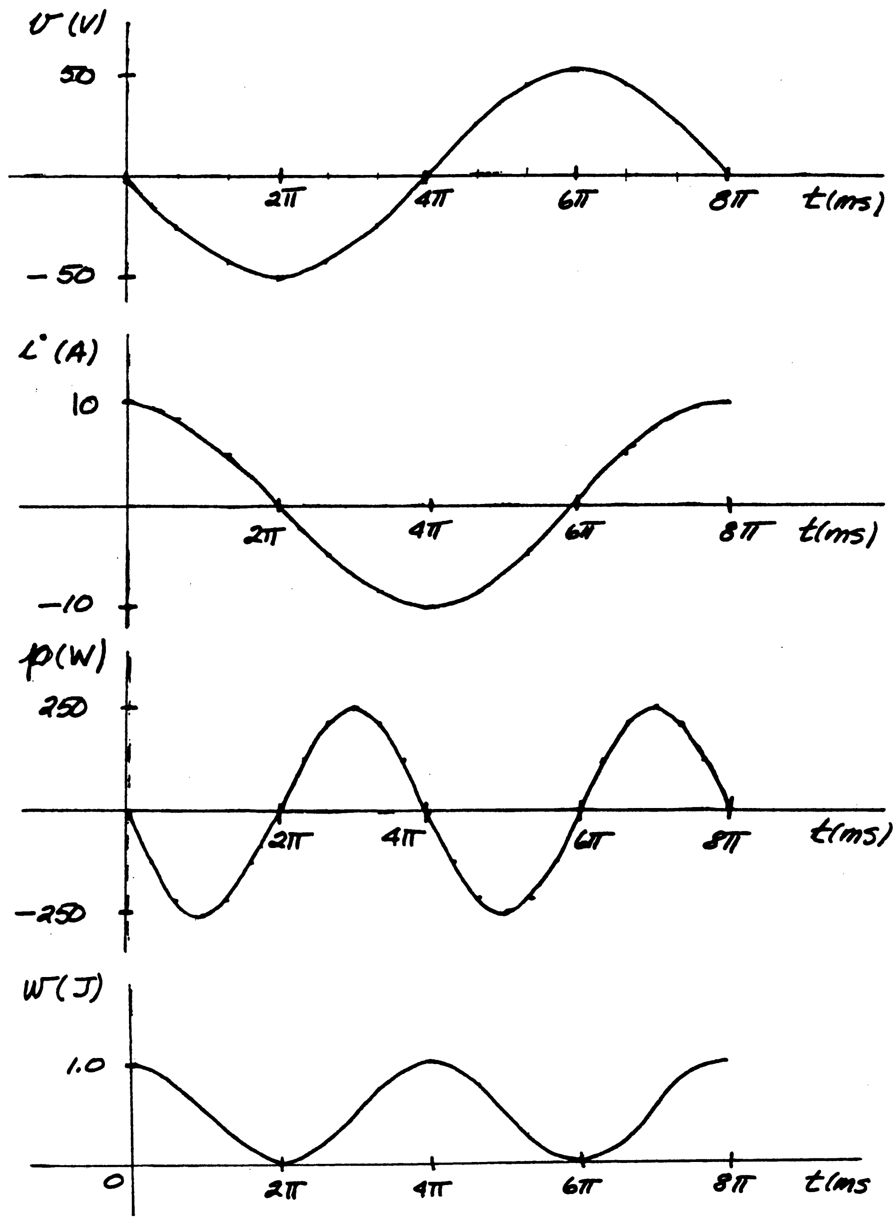
P 7.2 [a] $i = 0, \qquad t \le 0$

$i = \frac{400 \times 10^{-3}}{5 \times 10^{-3}} t = 80t \text{ A}, \qquad 0 \le t \le 5 \text{ ms}$

$i(t) = 0.8 - 80t \text{ A}, \qquad 5 \text{ ms} \le t \le 10 \text{ ms}$

$i(t) = 0, \qquad 10 \text{ ms} \le t \le \infty$

[b]



[c] Absorbing power: $2\pi - 4\pi$ ms, $6\pi - 8\pi$ ms   Delivering power: $0 - 2\pi$ ms, $4\pi - 6\pi$ ms

P7.6 [a] $v = L\frac{di}{dt}$

$$v = -25 \times 10^{-3}\frac{d}{dt}[10\cos 400t + 5\sin 400t]e^{-200t}$$

$$= -25 \times 10^{-3}\{-200e^{-200t}[10\cos 400t + 5\sin 400t]$$

$$+e^{-200t}[-4000\sin 400t + 2000\cos 400t]\}$$

$$v = -25 \times 10^{-3} e^{-200t}\{-1000 \sin 400t - 4000 \sin 400t\}$$
$$= -25 \times 10^{-3} e^{-200t}\{-5000 \sin 400t\}$$
$$= 125e^{-200t} \sin 400t \text{ V}$$

$$\frac{dv}{dt} = 125\{e^{-200t} \cdot 400 \cos 400t - 200e^{-200t} \sin 400t\}$$
$$= 25{,}000e^{-200t}\{2 \cos 400t - \sin 400t\} \frac{\text{V}}{\text{s}}$$

$$\frac{dv}{dt} = 0 \quad \text{when} \quad 2 \cos 400t = \sin 400t$$

$\therefore$ $\tan 400t = 2$, $\quad 400t = 1.11$, $\quad t = 2.77$ ms

[b] $v(2.77 \text{ ms}) = 125e^{-0.55} \sin 1.11 = 64.27$ V

P7.7 [a] $v_L = L\dfrac{di_L}{dt} = 50 \times 10^{-6}[18]\{e^{-10t} - 10te^{-10t}\}$

$$= 900 \times 10^{-6} e^{-10t}(1 - 10t) = 900e^{-10t}(1 - 10t)\,\mu\text{V}$$

[b] $i(200 \text{ ms}) = 18(0.2)(e^{-2}) = 487.21$ mA

$v(200 \text{ ms}) = 900e^{-2}(1 - 2) = -121.80\,\mu$V

$p(200 \text{ ms}) = vi = -59{,}342.67 \times 10^{-9} = -59.34\,\mu$W

[c] Delivering

[d] $w = \frac{1}{2}Li^2$; $\quad w(200 \text{ ms}) = \frac{1}{2}(50 \times 10^{-6})(0.48721^2) = 5.93\,\mu$J

[e] Energy stored is maximum when i is maximum.

$$\frac{di_L}{dt} = 18\left[t(-10)e^{-10t} + e^{-10t}\right] = 18e^{-10t}(1 - 10t)$$

$$\frac{di_L}{dt} = 0, \qquad t = 0.1 \text{ s}$$

$i_L(\text{max}) = 18(0.1)e^{-1} = 662.18$ mA

$w(\text{max}) = \frac{1}{2}(50 \times 10^{-6})(0.66218)^2 = 10.96\,\mu$J

P7.8 [a] $v = -100t, \qquad 0 \le t \le 1$ s

$$i = \frac{1}{5}\int_0^t -100x\,dx + 0 = (-20)\frac{x^2}{2}\Big|_0^t + 0 = -10t^2$$

$i = -10t^2, \qquad 0 \le t \le 1$ s

$i(1) = -10$ A

$v = -200 + 100t$ V, $\qquad 1 \text{ s} \le t \le 3$ s

$$i = \frac{1}{5}\int_1^t (-200 + 100x)\,dx - 10 = 20\int_1^t (-2 + x)\,dx - 10$$

$i = 10t^2 - 40t + 20$ A, $\qquad 1 \text{ s} \le t \le 3$ s

$i(3) = 90 - 120 + 20 = -10$ A

$v = 100\,\text{V}, \qquad 3\,\text{s} \le t \le 5\,\text{s}$

$i = \frac{1}{5}\int_3^t 100\,dx - 10 = 20(t-3) - 10$

$i = 20t - 70\,\text{A}, \qquad 3\,\text{s} \le t \le 5\,\text{s}$

$i(5) = 100 - 70 = 30\,\text{A}$

$v = 600 - 100t\,\text{V}, \qquad 5\,\text{s} \le t \le 6\,\text{s}$

$i = \frac{1}{5}\int_5^t (600 - 100x)\,dx + 30$

$i = 20\int_5^t (6 - x)\,dx + 30 = 120(t-5) - 20\frac{x^2}{2}\Big|_5^t + 30$

$= 120t - 600 - 10t^2 + 250 + 30$

$i = 120t - 10t^2 - 320\,\text{A}, \qquad 5\,\text{s} \le t \le 6\,\text{s}$

$i(6\,\text{s}) = 720 - 360 - 320 = 40\,\text{A}$

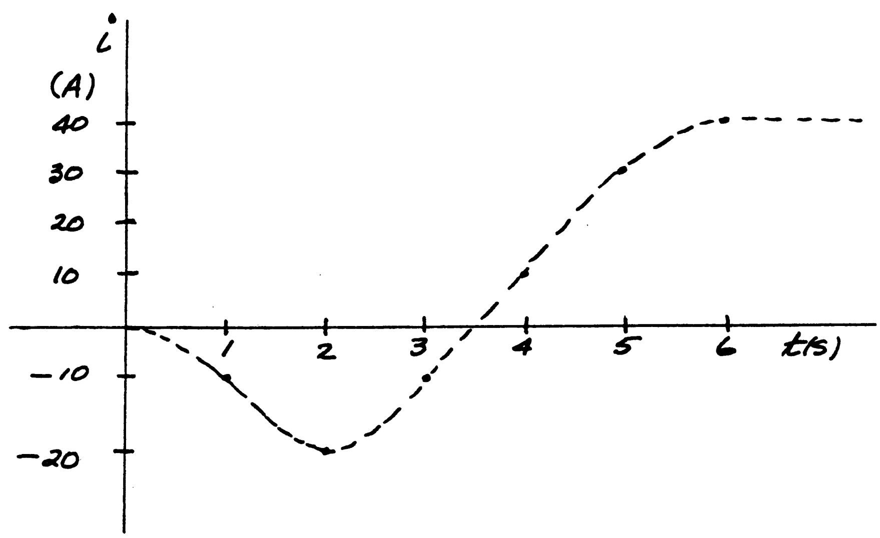
$i = 40\,\text{A}, \qquad 6\,\text{s} \le t \le \infty$

[b] $v = 0$ at $t = 2\,\text{s}$ and $t = 6\,\text{s}$

$i(2) = 10(4) - 40(2) + 20 = -20\,\text{A}$

$i(6) = 40\,\text{A}$

[c]



P7.9 $i(0) = B_1 = 10\,\text{A}$

$\frac{di}{dt} = -0.5e^{-0.5t}(B_1\cos 4t + B_2\sin 4t)^{-0.5t} + (-4B_1\sin 4t + 4B_2\cos 4t)$

$\frac{di}{dt}(0) = -0.5B_1 + 4B_2$

$4\frac{di}{dt}(0) = v(0) = 60 = -2B_1 + 16B_2$

$\therefore\ 16B_2 = 60 + 2(10) = 80; \qquad B_2 = 5$

$\therefore\ i = (10\cos 4t + 5\sin 4t)e^{-0.5t}\,\text{A}, \qquad t \geq 0$

$\therefore\ v = 4\left[-0.5e^{-0.5t}(10\cos 4t + 5\sin 4t) + e^{-0.5t}(-40\sin 4t + 20\cos 4t)\right]$

$= (60\cos 4t - 170\sin 4t)e^{-0.5t}\,\text{V}$

$\therefore\ i(1) = -6.26\,\text{A}, \qquad v(1) = 54.25\,\text{V}, \qquad p(1) = -339.57\,\text{W}$ (delivering)

P7.10 [a] $v = 20\times 10^{-3}\dfrac{di}{dt} = 20\times 10^{-3}[-10{,}000A_1e^{-10{,}000t} - 40{,}000A_2e^{-40{,}000t}]$

$= -200A_1e^{-10{,}000t} - 800A_2e^{-40{,}000t}$

$v(0) = -200A_1 - 800A_2 = 28$

$\therefore\ 200A_1 + 800A_2 = -28$

$50A_1 + 200A_2 = -7$

Also $i(0) = 40\times 10^{-3} = A_1 + A_2$

$$\therefore\ \Delta = \begin{vmatrix} 50 & 200 \\ 1 & 1 \end{vmatrix} = 50 - 200 = -150$$

$$N_1 = \begin{vmatrix} -7 & 200 \\ 40\times 10^{-3} & 1 \end{vmatrix} = -7 - 8 = -15$$

$$N_2 = \begin{vmatrix} 50 & -7 \\ 1 & 40\times 10^{-3} \end{vmatrix} = 2 + 7 = 9$$

$$\therefore\ A_1 = \frac{-15}{-150} = 0.1 = 100\times 10^{-3}\,\text{A}$$

$$A_2 = \frac{9}{-150} = -0.06 = -60\times 10^{-3}\,\text{A}$$

$\therefore\ v = -200\times 100\times 10^{-3}e^{-10{,}000t} - 800(-60\times 10^{-3})e^{-40{,}000t}$

$v = -20e^{-10{,}000t} + 48e^{-40{,}000t}\,\text{V}, \qquad t > 0$

[b] From part [a] we have

$i = 100e^{-10{,}000t} - 60e^{-40{,}000t}\,\text{mA}, \qquad t \geq 0$

Now note i cannot equal zero for $t > 0$, i.e.,

$i = 0$ requires $100e^{-10{,}000t} = 60e^{-40{,}000t}$

or $e^{30{,}000t} = 0.6$ or $t = -17.03\,\mu\text{s}$

From our solution for v we see that v will equal zero when

$48e^{-40{,}000t} = 20e^{-10{,}000t}$ or $e^{30{,}000t} = 2.4$

$\therefore\ t = 29.18\,\mu\text{s}$

Therefore the power at the terminals is zero when $t = 29.18\,\mu\text{s}$.

P7.11 [a] From the solution to Problem 7.10 we can write $\boxed{A_1 + A_2 = 40\times 10^{-3}}$

and $-200A_1 - 800A_2 = -68$ or $\boxed{50A_1 + 200A_2 = 17}$

$$\Delta = \begin{vmatrix} 1 & 1 \\ 50 & 200 \end{vmatrix} = 150$$

$$N_1 = \begin{vmatrix} 40 \times 10^{-3} & 1 \\ 17 & 200 \end{vmatrix} = 8 - 17 = -9$$

$$N_2 = \begin{vmatrix} 1 & 40 \times 10^{-3} \\ 50 & 17 \end{vmatrix} = 17 - 2 = 15$$

$$\therefore\ A_1 = \frac{N_1}{\Delta} = -60 \times 10^{-3}\,\text{A}, \qquad A_2 = \frac{N_2}{\Delta} = 100 \times 10^{-3}\,\text{A}$$

Therefore $i = (-60e^{-10,000t} + 100e^{-40,000t})\,\text{mA}, \qquad t \geq 0$

$$v = L\frac{di}{dt} = 20 \times 10^{-3}[600 \times 10^3 e^{-10,000t} - 4000 \times 10^3 e^{-40,000t}] \times 10^{-3}$$

$$v = 12e^{-10,000t} - 80e^{-40,000t}\,\text{V}, \qquad t \geq 0$$

[b] Note that $i = 0$ when

$$60e^{-10,000t} = 100e^{-40,000t}$$

$$e^{30,000t} = \frac{100}{60} = \frac{5}{3}; \qquad t = 17.03\,\mu\text{s}$$

Therefore i is positive for $0 \leq t \leq 17.03\,\mu$s, and i is negative for $17.03\,\mu\text{s} \leq t \leq \infty$.

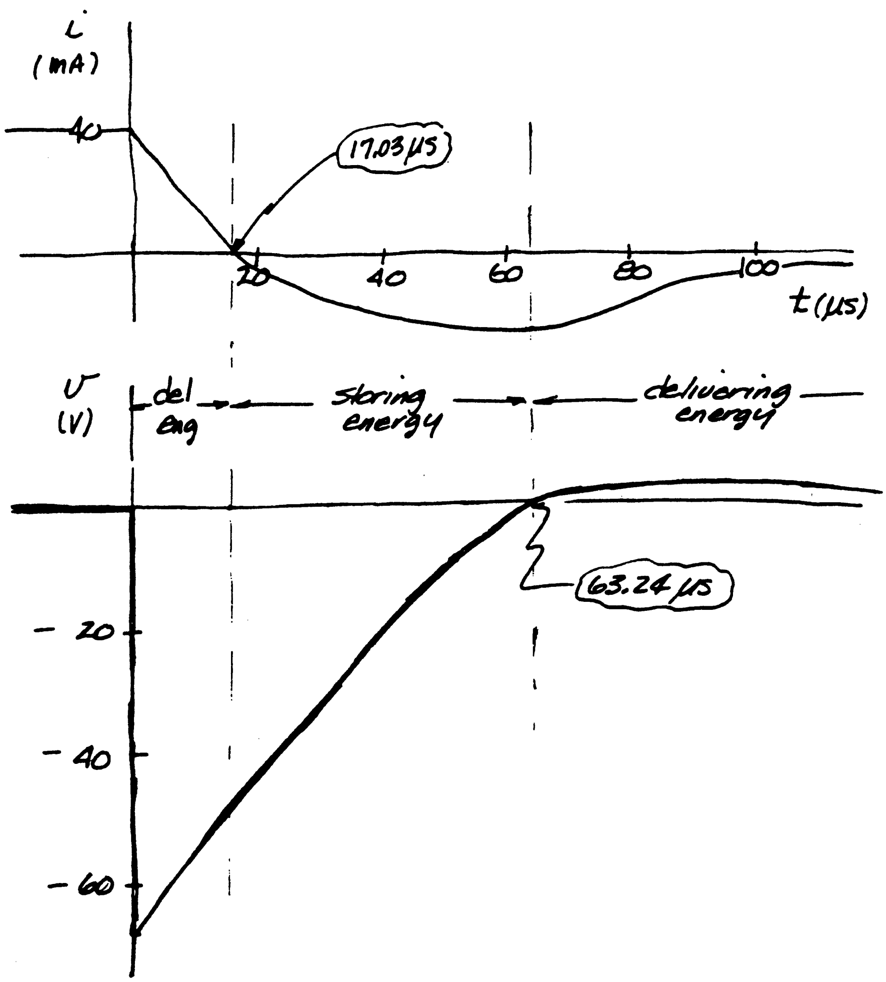
Note that $v = 0$ when

$$12e^{-10,000t} = 80e^{-40,000t}$$

$$e^{30,000t} = \frac{80}{12} = \frac{20}{3}; \qquad t = 63.24\,\mu\text{s}$$

Therefore v is negative for $0 \leq t \leq 63.24\,\mu$s, and v is positive for $63.24\,\mu\text{s} \leq t \leq 0$. See the following sketches of i and v. From the sketches we conclude

- energy is being delivered by the inductor between 0 and $17.03\,\mu$s and between $63.24\,\mu$s and infinity.
- the inductor is storing energy between $17.03\,\mu$s and $63.24\,\mu$s.



[c] The energy stored at $t = 0$ is

$$w(0) = \frac{1}{2}L[i(0)]^2 = \frac{1}{2}(20 \times 10^{-3})(40 \times 10^{-3})^2 = 16 \times 10^{-6} = 16\,\mu\text{J}$$

The energy for $t > 0$ is

$$w = \int_0^\infty p\,dt$$

$$p = vi = 6e^{-50{,}000t} - 8e^{-80{,}000t} - 0.72e^{-20{,}000t}\text{W}$$

$$w = \int_0^\infty 6e^{-50{,}000t}\,dt - \int_0^\infty 8e^{-80{,}000t}\,dt - \int_0^\infty 0.72e^{-20{,}000t}\,dt$$

$$= \frac{6e^{-5\times10^4 t}}{-5 \times 10^4}\Big|_0^\infty - \frac{8e^{-8\times10^4 t}}{-8 \times 10^4}\Big|_0^\infty - \frac{0.72e^{-2\times10^4 t}}{-2 \times 10^4}\Big|_0^\infty$$

$$= (1.2 - 1 - 0.36) \times 10^{-4} = -0.16 \times 10^{-4}\,\text{J} = -16\,\mu\text{J}$$

Therefore $16\,\mu$J is extracted from the inductor, hence the energy extracted equals the energy stored.

P7.12 $0 \le t \le 1.6\,\text{s}$

$$i_L = \frac{1}{5}\int_0^t 3 \times 10^{-3}\,dx + 0 = 0.6 \times 10^{-3}t$$

$$i_L(1.6) = (0.6)(1.6) \times 10^{-3} = 0.96\,\text{mA}$$

$$v_{\text{meter}}(1.6\,\text{s}) = (0.96 \times 10^{-3})(20 \times 10^3) = 19.2\,\text{V}$$

P7.13 [a] $w(0) = \frac{1}{2}C[v(0)]^2 = \frac{1}{2}(0.2) \times 10^{-6}(150)^2 = 2250 \times 10^{-6} = 2.25\,\text{mJ}$

[b] $v = (A_1t + A_2)e^{-5000t}$

$$v(0) = A_2 = 150\,\text{V}$$

$$\frac{dv}{dt} = -5000e^{-5000t}(A_1t + A_2) + e^{-5000t}(A_1)$$

$$= (-5000A_1t - 5000A_2 + A_1)e^{-5000t}$$

$$\frac{dv}{dt}(0) = A_1 - 5000A_2$$

$$i = C\frac{dv}{dt}, \qquad i(0) = C\frac{dv(0)}{dt}$$

$$\therefore \frac{dv(0)}{dt} = \frac{i(0)}{C} = \frac{250 \times 10^{-3}}{0.2 \times 10^{-6}} = 1250 \times 10^3$$

$$\therefore 1.25 \times 10^6 = A_1 - 5000(150)$$

$$A_1 = 1.25 \times 10^6 + 75 \times 10^4 = 2.0 \times 10^6\,\frac{\text{V}}{\text{s}}$$

[c] $v = (2 \times 10^6t + 150)e^{-5000t}$

$$i = C\frac{dv}{dt} = 0.2 \times 10^{-6}\frac{d}{dt}(2 \times 10^6t + 150)e^{-5000t}$$

$$i = \frac{d}{dt}\left[(0.4t + 30 \times 10^{-6})e^{-5000t}\right]$$

$$= (0.4t + 30 \times 10^{-6})(-5000)e^{-5000t} + e^{-5000t}(0.4)$$

$$= (-2000t - 150 \times 10^{-3} + 0.4)e^{-5000t}$$

$$= (0.25 - 2000t)e^{-5000t}\,\text{A}, \qquad t \ge 0$$

P7.14 [a] $i = C\frac{dv}{dt} = 0, \qquad t < 0$

[b] $i = (0.4 \times 10^{-6})(-40)\{(-2000e^{-2000t})(3\cos 1000t + \sin 1000t)$
$+e^{-2000t}(-3000\sin 1000t + 1000\cos 1000t)\}$

$$= -16 \times 10^{-3}e^{-2000t}\{-6\cos 1000t - 2\sin 1000t - 3\sin 1000t + \cos 1000t\}$$

$$i = 80e^{-2000t}[\cos 1000t + \sin 1000t]\,\text{mA}, \qquad t > 0$$

[c] $v(0) = 100 - 40(1)(3) = -20\,\text{V}$

$\therefore v = -20\,\text{V}$ for $t \le 0$

$v = -20\,\text{V}$ when $t = 0$

$\therefore$ No instantaneous change in v at $t = 0$.

[d] $i = 0, \qquad t < 0$

$i = 80\,\text{mA}, \qquad t = 0^+$

$\therefore$ The current jumps instantaneously from 0 to 80 mA at $t = 0$.

[e] $v(\infty) = 100\,\text{V}$

$$w(\infty) = \frac{1}{2}(0.4)(10^{-6})(100)^2 = 2000\,\mu\text{J}$$

P7.15 [a] $$v = \frac{1}{C}\int_0^t -50\times10^{-3}\,dx + 15 = (10^7)(-50\times10^{-3})\,x\Big|_0^t + 15$$

$v = -50\times10^4 t + 15, \qquad 0 \le t \le 10\,\mu\text{s}$

$v(10\,\mu\text{s}) = -50\times10^4(10\times10^{-6}) + 15 = -5 + 15 = 10\,\text{V}$

[b] $$v = 10^7\int_{10\times10^{-6}}^t 0.1\,dx + 10 = 10^6\,x\Big|_{10\times10^{-6}}^t + 10 = 10^6 t - 10 + 10$$

$v = 10^6 t, \qquad 10\,\mu\text{s} \le t \le 20\,\mu\text{s}$

$v(20\,\mu\text{s}) = 10^6(20\times10^{-6}) = 20\,\text{V}$

[c] $$v = 10^7\int_{20\times10^{-6}}^t 0.16\,dx + 20 = 10^7(0.16)[t - 20\times10^{-6}] + 20$$

$= 1.6\times10^6 t - 32 + 20$

$v = 1.6\times10^6 t - 12, \qquad 20\,\mu\text{s} \le t \le 40\,\mu\text{s}$

[d] $v(40\,\mu\text{s}) = 64 - 12 = 52\,\text{V}$

$v(t) = 52, \qquad 40\,\mu\text{s} \le t \le \infty$

P7.16 [a] $$q = \int_0^{5\times10^{-6}} 8\times10^4 x\,dx + \int_{5\times10^{-6}}^{15\times10^{-6}} 0.4\,dx$$

$$= 8\times10^4\frac{x^2}{2}\Big|_0^{5\times10^{-6}} + 0.4(15\times10^{-6} - 5\times10^{-6})$$

$$= 4\times10^4(25\times10^{-12}) + 0.4(10\times10^{-6})$$

$q = 10^{-6} + 4\times10^{-6} = 5\,\mu\text{C}$

[b] $$v = 4\times10^6\int_0^{5\times10^{-6}} 8\times10^4 x\,dx + 4\times10^6\int_{5\times10^{-6}}^{20\times10^{-6}} 0.4\,dx$$

$$+ 4\times10^6\int_{20\times10^{-6}}^{30\times10^{-6}} (10^4 x - 0.5)\,dx$$

$$= 4\times10^6\left\{8\times10^4\frac{x^2}{2}\Big|_0^{5\times10^{-6}} + 0.4x\Big|_{5\times10^{-6}}^{20\times10^{-6}}\right.$$

$$\left. + 10^4\frac{x^2}{2}\Big|_{20\times10^{-6}}^{30\times10^{-6}} - 0.5x\Big|_{20\times10^{-6}}^{30\times10^{-6}}\right\}$$

$$v = 4 \times 10^6 \{4 \times 10^4(25 \times 10^{-12}) + 0.4(15 \times 10^{-6}) + 5000(900 \times 10^{-12} - 400 \times 10^{-12}) - 0.5(10 \times 10^{-6})\}$$

$$= 4 \times 10^6\{10^{-6} + 6 \times 10^{-6} + 2.5 \times 10^{-6} - 5 \times 10^{-6}\}$$

$$= 4 \times 10^6\{4.5 \times 10^{-6}\} = 18\,\text{V}$$

$$v(30\,\mu\text{s}) = 18\,\text{V}$$

[c] $v(50\,\mu\text{s}) = 4 \times 10^6 \{10^{-6} + 6 \times 10^{-6} + 5000(2500 \times 10^{-12} - 400 \times 10^{-12}) - 0.5(30 \times 10^{-6})\}$

$$= 4 \times 10^6\{7 \times 10^{-6} + 10.5 \times 10^{-6} - 15 \times 10^{-6}\}$$

$$v(50\,\mu\text{s}) = 10\,\text{V}$$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}(0.25 \times 10^{-6})(10)^2 = 12.5 \times 10^{-6}\,\text{J}$$

$$w = 12.5\,\mu\text{J}$$

P 7.17 [a] $v(400\,\mu\text{s}) = \frac{10^6}{5}\left\{\int_0^{100\times10^{-6}} 500 \times 10^{-3}\,dx + \int_{100\times10^{-6}}^{300\times10^{-6}} -250 \times 10^{-3}\,dx + \int_{300\times10^{-6}}^{400\times10^{-6}} 400 \times 10^{-3}\,dx\right\} - 4$

$$= \frac{10^3}{5}\{500(100 \times 10^{-6}) - 250(200 \times 10^{-6}) + 400(100 \times 10^{-6})\}$$

$$= \frac{10^3}{5}\{5 \times 10^{-2} - 5 \times 10^{-2} + 4 \times 10^{-2}\} - 4 = 8 - 4 = 4\,\text{V}$$

$$w = \frac{1}{2}(5 \times 10^{-6})(16) = 40\,\mu\text{J}$$

[b] $v(600\,\mu\text{s}) = \frac{10^3}{5}\{400(200 \times 10^{-6})\} - 4 = 16 - 4 = 12\,\text{V}$

$$w = \frac{1}{2}(5 \times 10^{-6})(144) = 360\,\mu\text{J}$$

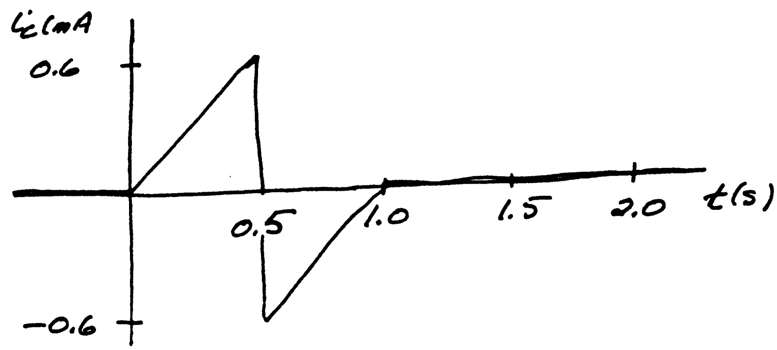
P 7.18 $i_C = C\frac{dv_C}{dt} = 20 \times 10^{-6}60t = 1200 \times 10^{-6}t = 1.2t\,\text{mA}, \qquad 0 \le t \le 0.5^-$

$\therefore\ i_C(0.5^-) = 0.6\,\text{mA}$

$i_C = 20 \times 10^{-6}60(t - 1) = 1.2(t - 1)\,\text{mA}, \qquad 0.5^+ \le t \le 1$

$\therefore\ i_C(0.5^+) = -0.6\,\text{mA}$

$i_C = 0$ elsewhere



P 7.19 [a] $v(20\,\mu s) = 12.5 \times 10^9(20 \times 10^{-6})^2 = 5\,V$ (end of first interval)

$v(20\,\mu s) = 10^6(20 \times 10^{-6}) - (12.5)(400) \times 10^{-3} - 10 = 5\,V$
(start of second interval)

$v(40\,\mu s) = 10^6(40 \times 10^{-6}) - 12.5)(1600) \times 10^{-3} - 10 = 10\,V$
(end of second interval)

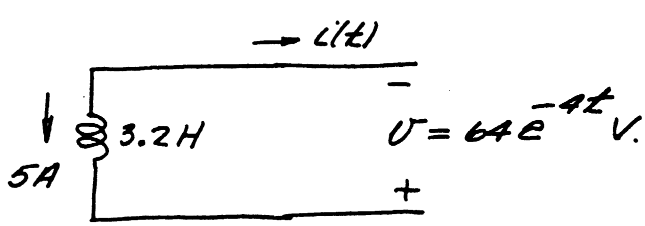
[b] $p(10\mu s) = 62.5 \times 10^{12}(10^{-5})^3 = 62.5\,mW$, $v(10\,\mu s) = 1.25\,V$,
$i(10\mu s) = 50\,mA$, $p(10\,\mu s) = vi = 62.5\,mW$,
$p(30\,\mu s) = 437.50\,mW$, $v(30\,\mu s) = 8.75\,V$, $i(30\,\mu s) = 0.05\,A$

[c] $w(10\,\mu s) = 15.625 \times 10^{12}(10 \times 10^{-6})^4 = 0.15625\,\mu J$
$w = 0.5Cv^2 = 0.5(0.2 \times 10^{-6})(1.25)^2 = 0.15625\,\mu J$
$w(30\,\mu s) = 7.65625\,\mu J$
$w(30\,\mu s) = 0.5(0.2 \times 10^{-6})(8.75)^2 = 7.65625\,\mu J$

P 7.20

$16//24 = 9.6\,H$	$12 + 8 = 20\,H$	$10//15 = 6\,H$
$9.6 + 10.4 = 20\,H$	$20//5 = 4\,H$	$L_{ab} = 2 + 6 = 8\,H$
$20//30 = 12\,H$	$4 + 6 = 10\,H$	

P 7.21 [a]



$$3.2\frac{di}{dt} = 64e^{-4t}; \qquad \frac{di}{dt} = 20e^{-4t}$$

$$i = 20\int_0^t e^{-4x}\,dx - 5 = 20\left[\frac{e^{-4x}}{-4}\right|_0^t - 5$$

$$i = -5e^{-4t}\,\text{A}, \qquad t \geq 0$$

[b] $$4\frac{di_1}{dt} = 64e^{-4t}$$

$$\therefore\ i_1 = 16\int_0^t e^{-4x}\,dx - 10 = 16\left[\frac{e^{-4x}}{-4}\right|_0^t - 10$$

$$i_1 = -4e^{-4t} - 6\,\text{A}, \qquad t \geq 0$$

[c] $$16\frac{di_2}{dt} = 64e^{-4t}$$

$$\therefore\ i_2 = 4\int_0^t e^{-4x}\,dx + 5$$

$$i_2 = -e^{-4t} + 6\,\text{A}, \qquad t \geq 0$$

[d] $$p = -vi = -64e^{-4t}(-5e^{-4t}) = 320e^{-8t}\,\text{W}, \qquad t \geq 0$$

$$w = 320\int_0^\infty e^{-8t}\,dt = 40\,\text{J}$$

[e] $$w(0) = \frac{1}{2}(4)(-10)^2 + \frac{1}{2}(16)(5)^2$$

$$w(0) = 400\,\text{J}$$

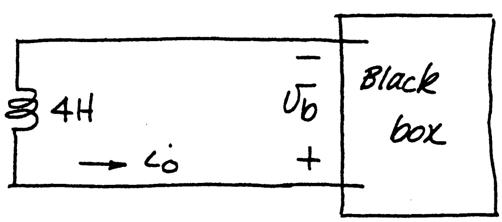
[f] $$i_1(\infty) = -6\,\text{A}, \qquad i_2(\infty) = 6\,\text{A}$$

$$w(\infty) = \frac{1}{2}(4)(-6)^2 + \frac{1}{2}(16)(6)^2 = 360\,\text{J}$$

[g] $$w_{\text{del}} = w_{\text{init}} - w_{\text{trapped}} = 400 - 360 = 40\,\text{J} \quad \text{(yes)}$$

P 7.22 [a] $$i_o(0) = -i_1(0) - i_2(0) = 6 - 1 = 5\,\text{A}$$

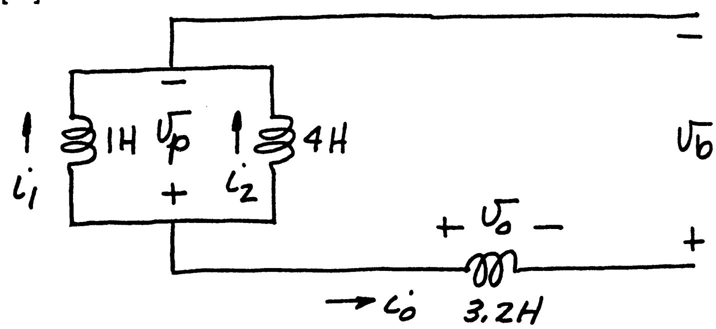
[b]



$$i_o = -\frac{1}{4}\int_0^t 2000e^{-100x}\,dx + 5 = -500\left[\frac{e^{-100x}}{-100}\right|_0^t + 5$$

$$= 5(e^{-100t} - 1) + 5 = 5e^{-100t}\,\text{A}, \qquad t \geq 0$$

[c]



$v_o = 3.2(-500e^{-100t}) = -1600e^{-100t}\,\text{V}$

$v_p = v_b + v_o = 400e^{-100t}\,\text{V}$

$i_1 = \frac{1}{1}\int_0^t 400e^{-100x}\,dx - 6 = -4(e^{-100t} - 1) - 6$

$i_1 = -4e^{-100t} - 2\,\text{A}, \qquad t \geq 0$

[d] $i_2 = \frac{1}{4}\int_0^t 400e^{-100x}\,dx + 1$

$i_2 = -e^{-100t} + 2\,\text{A}, \qquad t \geq 0$

[e] $w(0) = \frac{1}{2}(1)(6)^2 + \frac{1}{2}(4)(1)^2 + \frac{1}{2}(3.2)(5)^2 = 60\,\text{J}$

[f] $w_{\text{del}} = \frac{1}{2}(4)(5)^2 = 50\,\text{J}$

or $w_{\text{del}} = \int_0^\infty 10{,}000e^{-200t}\,dt = -50\left(e^{-200t}\big|_0^\infty\right) = 50\,\text{J}$

[g] $w_{\text{trapped}} = 60 - 50 = 10\,\text{J}$

or $w_{\text{trapped}} = \frac{1}{2}(1)(2)^2 + \frac{1}{2}(4)(2)^2 = 10\,\text{J}$

P 7.23 $w_{\text{del}} = \int_0^{t_o} p\,dt = \int_0^{t_o} 10^4 e^{-200t}\,dt = 10{,}000\left[\frac{e^{-200t}}{-200}\bigg|_0^{t_o}\right] = 50(1 - e^{-200t_o})$

$\%\text{ delivered} = \frac{50(1 - e^{-200t_o})}{50} \times 100 = 100(1 - e^{-200t_o})$

$\therefore \quad 100(1 - e^{-200t_o}) = 80 \qquad 200t_o = \ln 5$

$e^{-200t_o} = 0.2 \qquad t_o = 5\ln 5\,\text{ms}$

$e^{200t_o} = 5 \qquad t_o = 8.05\,\text{ms}$

P 7.24 From Figure 7.17(a) we have

$v = \frac{1}{C_1}\int_0^t i + v_1(0) + \frac{1}{C_2}\int_0^t i\,dx + v_2(0) + \cdots$

$v = \left[\frac{1}{C_1} + \frac{1}{C_2} + \cdots\right]\int_0^t i\,dx + v_1(0) + v_2(0) + \cdots$

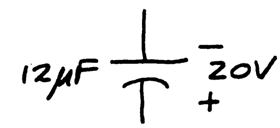
Therefore $\frac{1}{C_{eq}} = \left[\frac{1}{C_1} + \frac{1}{C_2} + \cdots\right]$, $\quad v_{eq}(0) = v_1(0) + v_2(0) + \cdots$

P 7.25 From Fig. 7.18(a)

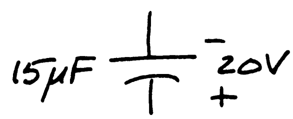
$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + \cdots = [C_1 + C_2 + \cdots]\frac{dv}{dt}$$

Therefore $C_{eq} = C_1 + C_2 + \cdots$. Because the capacitors are in parallel, the initial voltage on every capacitor must be the same. This initial voltage would appear on C_{eq}.

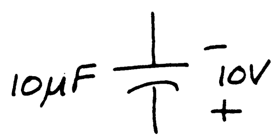
P 7.26 48 μF in series with 16 μF is 12 μF with an initial voltage of 20 V, i.e.,



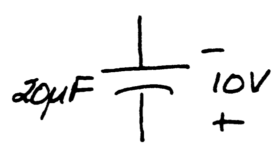
12 μF in parallel with 3 μF is 15 μF charged to 20 V, i.e.,



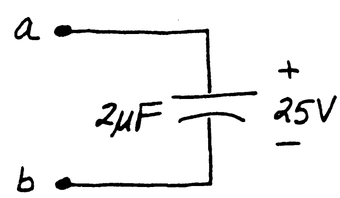
15 μF in series with 30 μF is 10 μF with a net charge of 10 V, i.e.,



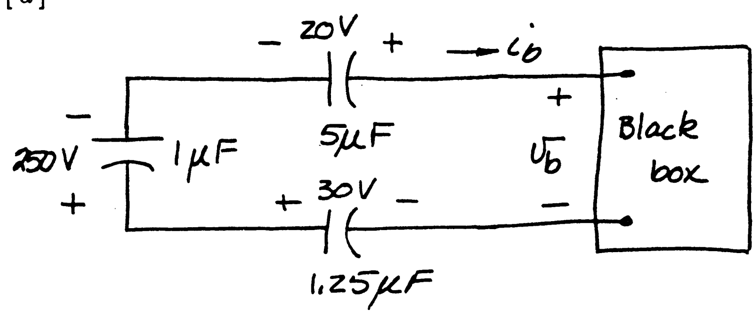
10 μF in parallel with 10 μF is 20 μF with a charge of 10 V, i.e.,



$5\,\mu$F in series with $20\,\mu$F in series with $4\,\mu$F is $2\,\mu$F with a net charge of 25 V.

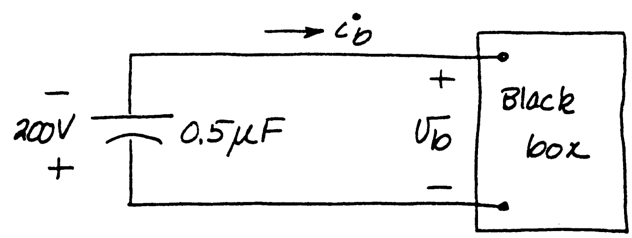


P 7.27 [a]



$$\frac{1}{C_e} = \frac{1}{1} + \frac{1}{5} + \frac{1}{1.25} = \frac{10}{5} = 2, \qquad \therefore\ C_e = 0.5\,\mu\text{F}$$

$$v_b(0) = 20 - 250 + 30 = -200\text{ V}$$



$$v_b = -\frac{10^6}{0.5}\int_0^t (-5\times 10^{-3})e^{-50x}\,dx - 200 = 10^4\frac{e^{-50x}}{-50}\bigg|_0^t - 200$$

$$= -200e^{-50t}\,\text{V}, \qquad t \geq 0$$

[b] $$v_a = \frac{10^6}{5}\int_0^t (-5\times 10^{-3})e^{-50x}\,dx - 20 = 20(e^{-50t} - 1) - 20$$

$$v_a = 20e^{-50t} - 40\,\text{V}, \qquad t \geq 0$$

[c] $$v_c = \frac{10^6}{1.25}\int_0^t (-5\times 10^{-3})e^{-50x}\,dx - 30 = 80(e^{-50t} - 1) - 30$$

$$v_c = 80e^{-50t} - 110\,\text{V}, \qquad t \geq 0$$

[d] $v_d = 10^6 \int_0^t (-5 \times 10^{-3})e^{-50x}\,dx + 250 = 100(e^{-50t} - 1) + 250$

$v_d = 100e^{-50t} + 150\,\text{V}, \qquad t \geq 0$

Check: $v_b = -v_c - v_d - v_a = -200e^{-50t}\,\text{V}, \qquad t \geq 0 \quad \text{(ok)}$

[e] $i_1 = (0.2 \times 10^{-6})\frac{dv_d}{dt} = (0.2 \times 10^{-6})[-5000e^{-50t}]$

$i_1 = -e^{-50t}\,\text{mA}, \qquad t \geq 0$

[f] $i_2 = (0.8 \times 10^{-6})\frac{dv_d}{dt}$

$i_2 = -4e^{-50t}\,\text{mA}, \qquad t \geq 0$

Check: $i_b = i_1 + i_2 = -5e^{-50t}\,\text{mA}, \qquad t \geq 0 \quad \text{(ok)}$

P 7.28 [a] $w(0) = \frac{1}{2}(0.2 \times 10^{-6})(250)^2 + \frac{1}{2}(0.8 \times 10^{-6})(250)^2 + \frac{1}{2}(5 \times 10^{-6})(20)^2 + \frac{1}{2}(1.25 \times 10^{-6})(30)^2$

$w(0) = 32{,}812.50\,\mu\text{J}$

[b] $w(\infty) = \frac{1}{2}(5 \times 10^{-6})(1600) + \frac{1}{2}(1.25 \times 10^{-6})(110)^2 + \frac{1}{2}(0.2 \times 10^{-6})(150)^2 + \frac{1}{2}(0.8 \times 10^{-6})(150)^2$

$w(\infty) = 22{,}812.50\,\mu\text{J}$

[c] $w_{\text{del}} = \frac{1}{2}(0.5 \times 10^{-6})(200)^2 = 10,000\,\mu\text{J}$

Check: $w(0) - w(\infty) = 10{,}000\,\mu\text{J}$

[d] $\%\text{ delivered} = \frac{10{,}000}{32{,}812.50} \times 100 = 30.48\%$

[e] $w = \int_0^{t_o} p\,dt$

$p = v_b i_b = (-200e^{-50t})(-5 \times 10^{-3})e^{-50t} = e^{-100t}\,\text{W}$

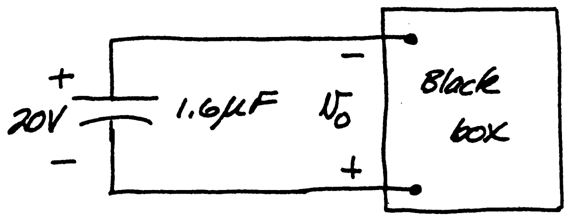
$w = \int_0^{t_o} e^{-100t}\,dt = 10^4(1 - e^{-100t_o})\,\mu\text{J}$

$10{,}000(1 - e^{-100t_o}) = 7500$

$e^{-100t_o} = 0.25, \qquad e^{100t_o} = 4$

$t_o = 10\ln 4\,\text{ms}, \qquad t_o = 13.86\,\text{ms}$

P 7.29 [a]



$$v_o = \frac{10^6}{1.6}\int_0^t 800 \times 10^{-6}e^{-25x}\,dx - 20 = 500\left[\frac{e^{-25x}}{-25}\right|_0^t - 20$$
$$= -20(e^{-25t} - 1) - 20$$
$$v_o = -20e^{-25t}\text{ V}$$

[b] $$v_1 = \frac{10^6}{2}\int_0^t 800 \times 10^{-6}e^{-25x}\,dx + 5 = 400\left[\frac{e^{-25x}}{-25}\right|_0^t + 5$$
$$= -16(e^{-25t} - 1) + 5$$
$$v_1 = -16e^{-25t} + 21\text{ V}$$

[c] $$v_2 = \frac{10^6}{8}\int_0^t 800 \times 10^{-6}e^{-25x}\,dx - 25 = -4(e^{-25t} - 1) - 25$$
$$v_2 = -4e^{-25t} - 21\text{ V}$$

Note: $v_1 + v_2 = v_o = -20e^{-25t}$ V

[d] $$p = -v_o i = -(-20e^{-25t})(800 \times 10^{-6}e^{-25t}) = 16 \times 10^{-3}e^{-50t}\text{ W}$$
$$w = \int_0^\infty 16 \times 10^{-3}e^{-50t}\,dt = 16 \times 10^{-3}\frac{e^{-50t}}{-50}\Big|_0^\infty$$
$$= -0.32 \times 10^{-3}(0 - 1) = 320 \times 10^{-6}\text{ J}$$
$$w = 320\,\mu\text{J}$$

[e] $$w(0) = \frac{1}{2}(2 \times 10^{-6})(5)^2 + \frac{1}{2}(8 \times 10^{-6})(25)^2 = 2525\,\mu\text{J}$$

[f] $$w_{\text{trapped}} = \frac{1}{2}(2 \times 10^{-6})(21)^2 + \frac{1}{2}(8 \times 10^{-6})(-21)^2 = 2205\,\mu\text{J}$$

[g] Yes; $w(0) - w_{\text{trapped}} = 2525 - 2205 = 320\,\mu\text{J}$

P 7.30 Let v_L equal the voltage drop across the 5-H inductor in the direction of i_o, then

$$v_L = 5\frac{di_o}{dt} = 5[60e^{-80t}\cos 60t - 80e^{-80t}\sin 60t]$$
$$= [300\cos 60t - 400\sin 60t]e^{-80t}\text{ V}$$

$$v_C = \frac{10^6}{20}\int_0^t e^{-80x}\sin 60x\,dx - 300$$
$$= 5 \times 10^4\left\{\frac{e^{-80x}}{(6400 + 3600)}[-80\sin 60x - 60\cos 60x]_0^t\right\} - 300$$
$$= -5\{e^{-80t}[80\sin 60t + 60\cos 60t] - 60\} - 300$$

$$v_C = -[400 \sin 60t + 300 \cos 60t]e^{-80t}\,\text{V}$$
$$v_o = -(v_C + v_L) = -(-800e^{-80t} \sin 60t)$$
$$v_o = 800e^{-80t} \sin 60t\,\text{V}$$

P 7.31 $$\frac{di_o}{dt} = 5\left\{e^{-2000t}[-8000 \sin 4000t + 4000 \cos 4000t] -2000e^{-2000t}[2 \cos 4000t + \sin 4000t]\right\}$$
$$\frac{di_o}{dt}(0^+) = 5\{1(4000) - 2000(2)\} = 0$$
$$\therefore\ 10 \times 10^{-3}\frac{di_o}{dt}(0^+) = 0, \qquad \therefore\ v_2(0^+) = 0$$
$$v_1(0^+) = 40i_o(0^+) + v_2(0^+)$$
$$v_1(0^+) = 40(10) = 400\,\text{V}$$

8

Response of First-Order *RL* and *RC* Circuits

Drill Exercises

DE 8.1 [a] $i = \left(\frac{100}{1+4}\right)\left(\frac{-20}{25}\right) = -16\,\text{A}$

[b] $w = 0.5(10^{-2})(16)^2 = 1.28\,\text{J}$

[c] $\tau = \frac{L}{R} = \frac{10^{-2}}{4} = 2.5\,\text{ms}$

[d] $i = -16e^{-400t}\,\text{A}, \qquad t \geq 0$

[e] $i(5\,\text{ms}) = -2.17\,\text{A}, \qquad w\,(5\,\text{ms}) = (0.5)(10)(2.17)^2 = 23.44\,\text{mJ}$

$w\,(\text{diss}) = 1280 - 23.44 = 1256.56\,\text{mJ}$

$\%\ \text{dissipated} = \left(\frac{1256.56}{1280}\right)100 = 98.17\%$

DE 8.2 [a] $i_L(0^-) = 6.4\left(\frac{10}{16}\right) = 4\,\text{A} = i_L(0^+), \qquad t > 0$

$R_{eq} = \frac{(4)(16)}{20} = 3.2\,\Omega, \qquad \tau = \frac{0.32}{3.2} = 0.1\,\text{s}$

Therefore $\frac{1}{\tau} = 10, \qquad i_L = 4e^{-10t}\,\text{A}$

Let i_1 equal the current in the $10\,\Omega$ resistor.

Let the reference direction for i_1 be up. Then

$$i_1 = \left(\frac{4}{20}\right)i_L = 0.8e^{-10t}\,\text{A}, \qquad v_o = -10i_1 = -8e^{-10t}\,\text{V}, \qquad t \geq 0$$

[b] $v_{S4\Omega} = L\frac{di_L}{dt} = 0.32(-40)e^{-10t} = -12.8e^{-10t}\,\text{V}, \qquad t \geq 0$

$p_{4\Omega} = \frac{v_{4\Omega}^2}{4} = 40.96e^{-20t}\,\text{W}, \qquad t \geq 0$

$w_{4\Omega} = \int_0^{\infty} 40.96e^{-20t}dt = 2.048\,\text{J}$

$w_i = \frac{1}{2}Li^2 = \frac{1}{2}(0.32)(16) = 2.56\,\text{J}$

$\%\ \text{dissipated} = \left(\frac{2.048}{2.56}\right)100 = 80\%$

DE 8.3 [a] $v(0) = \left[\frac{15(60)}{90}\right]20 = 200\,\text{V}$

[b] $\tau = RC = (20 \times 10^3)(0.5 \times 10^{-6}) = 10\,\text{ms}$

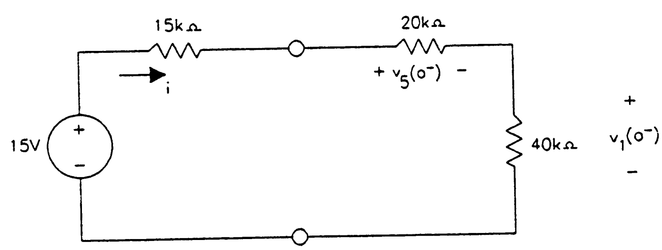
[c] $v = 200e^{-100t}\,\text{V}$

[d] $w(0) = 0.5(0.5 \times 10^{-6})(200)^2 = 10\,\text{mJ}$

[e] $w(t) = 0.5(0.5 \times 10^{-6})(4 \times 10^4)e^{-200t} = 10e^{-200t}\,\text{mJ}$

$10e^{-200t} = 2.5, \qquad t = (\ln 4)/200 = 6.93\,\text{ms}$

DE 8.4 [a] For $t < 0$:



$i = \frac{15}{75} = \frac{1}{5}\,\text{mA}, \qquad v_5(0^-) = 4\,\text{V}, \qquad v_1(0^-) = 8\,\text{V}$

$\tau_5 = (20 \times 10^3)(5 \times 10^{-6}) = 100\,\text{ms}, \qquad 1/\tau_5 = 10$

$\tau_1 = (40 \times 10^3)(1 \times 10^{-6}) = 40\,\text{ms}, \qquad 1/\tau_1 = 25$

Therefore $v_5 = 4e^{-10t}\,\text{V}, \quad t \geq 0; \qquad v_1 = 8e^{-25t}\,\text{V}, \quad t \geq 0;$

$v_o = v_1 + v_5 = [8e^{-25t} + 4e^{-10t}]\,\text{V}, \qquad t \geq 0$

[b] $v_1(60\,\text{ms}) \cong 1.79\,\text{V}, \qquad v_5(60\,\text{ms}) \cong 2.20\,\text{V}$

$w_1(60\,\text{ms}) = (1/2)(1)(1.79)^2 \cong 1.59\,\mu\text{J}$

$w_5(60\,\text{ms}) = (1/2)(5)(2.20)^2 \cong 12.05\,\mu\text{J}$

$w_1(0) = (1/2)(1)(64) + (1/2)(5)(16) = 72\,\mu\text{J}$

$w_{\text{diss}} = 72 - 13.64 = 58.36\,\mu\text{J}$

$\%\ \text{dissipated} = (58.36/72)(100) = 81.05\,\%$

DE 8.5 [a] $i(0^+) = 24/2 = 12\,\text{A}$

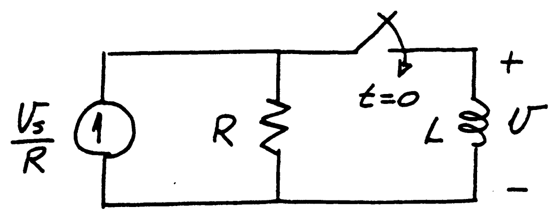
[b] $v(0^+) = -10(8+12) = -200\,\text{V}$

[c] $\tau = L/R = (200/10) \times 10^{-3} = 20\,\text{ms}$

[d] $i = -8 + [12 - (-8)]e^{-50t} = [-8 + 20e^{-50t}]\,\text{A}, \qquad t \geq 0^+$

[e] $v = 0 + [-200 - 0]e^{-50t}\,\text{V} = -200e^{-50t}\,\text{V}, \qquad t \geq 0^+$

DE 8.6 [a]



$$\frac{v}{R} + \frac{1}{L}\int_0^t v\,dx = \frac{V_s}{R}$$

$$\frac{1}{R}\frac{dv}{dt} + \frac{v}{L} = 0$$

$$\frac{dv}{dt} + \frac{R}{L}v = 0$$

[b]

$$\frac{dv}{dt} = -\frac{R}{L}v$$

$$\frac{dv}{dt}\,dt = -\frac{R}{L}v\,dt$$

$$\therefore\ \frac{dv}{v} = -\frac{R}{L}\,dt$$

$$\int_{v(0^+)}^{v(t)} \frac{dy}{y} = -\frac{R}{L}\int_{0^+}^{t} dx$$

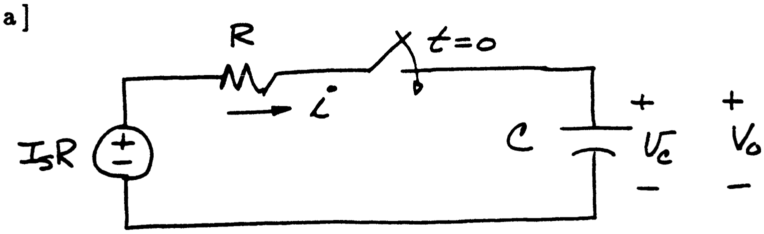
$$\ln y \Big|_{v(0^+)}^{v(t)} = -\left(\frac{R}{L}\right)t$$

$$\ln\left[\frac{v(t)}{v(0^+)}\right] = -\left(\frac{R}{L}\right)t$$

$$v(t) = v(0^+)e^{-(R/L)t}; \qquad v(0^+) = \left(\frac{V_s}{R} - I_o\right)R = V_s - I_oR$$

$$\therefore\ v(t) = (V_s - I_oR)e^{-(R/L)t}$$

DE 8.7 [a]



$$I_s R = Ri + \frac{1}{C}\int_{0+}^{t} i\,dx + V_o$$

$$0 = R\frac{di}{dt} + \frac{i}{C} + 0$$

$$\therefore \frac{di}{dt} + \frac{i}{RC} = 0$$

[b] $$\frac{di}{dt} = -\frac{i}{RC}; \qquad \frac{di}{i} = -\frac{dt}{RC}$$

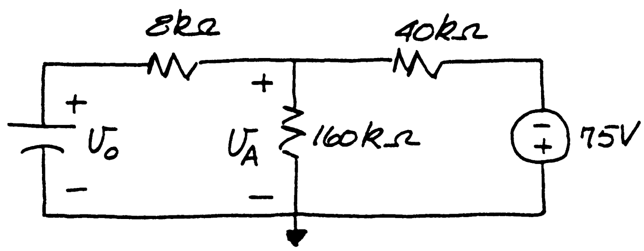
$$\int_{i(0+)}^{i(t)} \frac{dy}{y} = -\frac{1}{RC}\int_{0+}^{t} dx$$

$$\ln\frac{i(t)}{i(0^+)} = \frac{-t}{RC}$$

$$i(t) = i(0^+)e^{-t/RC}; \qquad i(0^+) = \frac{I_s R - V_o}{R} = \left(I_s - \frac{V_o}{R}\right)$$

$$\therefore i(t) = \left(I_s - \frac{V_o}{R}\right)e^{-t/RC}$$

DE 8.8 [a]



$$\frac{v_A - v_o}{8} + \frac{v_A}{160} + \frac{v_A + 75}{40} = 0$$

$$20v_A - 20v_o + v_A + 4v_A + 300 = 0$$

$$25v_A = 20v_o - 300$$

$$v_A = 0.8v_o - 12$$

$$v_A = -48 + 72e^{-100t} - 12 = -60 + 72e^{-100t}\,\text{V}, \qquad t \geq 0^+$$

DE 8.9 [a] $v_c(0^+) = 90\,\text{V}$

[b] $v_c(\infty) = \left(-\frac{40}{80}\right)60 = -30\,\text{V}$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$v_{\text{Th}} = -30\,\text{V}, \qquad R_{\text{Th}} = 20//60 = 15\,\Omega,$

Therefore $\tau = 15(0.5) = 7.5\,\mu\text{s}$

[d] $i(0^+) = -\frac{90 + 30}{15} = -8\,\text{A}$

[e] $v_c = -30 + [90 - (-30)]e^{-t/\tau} = -30 + 120e^{-(400{,}000/3)t}\,\text{V}, \qquad t \geq 0^+$

[f] $i = -8e^{-t/\tau} = -8e^{-(400{,}000/3)t}\,\text{A}, \qquad t \geq 0$

Therefore $e^{100t_r} = 15{,}000/10 = 1500$

$100t_r = \ln 1500; \qquad t_r = 0.01(\ln 1500); \qquad t_r = 73.13\,\text{ms}$

DE 8.10 [a] $v_c(0^+) = (60/15)(11.5) = 46\,\text{V}$

[b] $v_c(\infty) = -8(6.75) = -54\,\text{V}$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor,

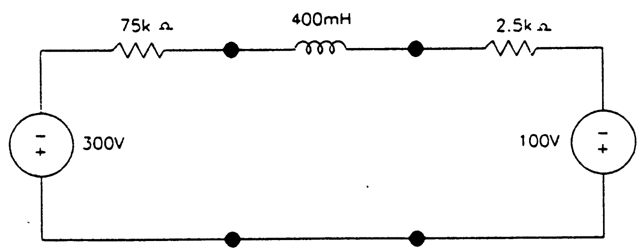
$v_{\text{Th}} = -54\,\text{V}, \qquad R_{\text{Th}} = 8\,\text{k}\Omega$

$\tau = R_{\text{Th}}C = 400\,\mu\text{s}$

[d] $v_c = -54 + (46 + 54)e^{-2500t} = -54 + 100e^{-2500t}$

Therefore $t = \dfrac{\ln(100/54)}{2500} = 246.47\,\mu\text{s}$

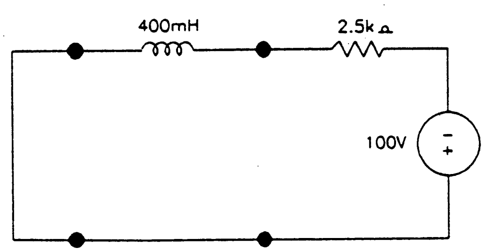
DE 8.11 [a] For $t < 0$, calculate the Thévenin equivalent for the circuit to the left and right of the 400-mH inductor. We get



$i(0^-) = -200/10 = -20\,\text{mA}$

$i(0^-) = i(0^+) = -20\,\text{mA}$

[b] For $t > 0$, the circuit reduces to



Therefore $i(\infty) = 100/2.5 = 40\,\text{mA}$

[c] $\tau = (400/2.5) \times 10^{-6} = 160\,\mu s$

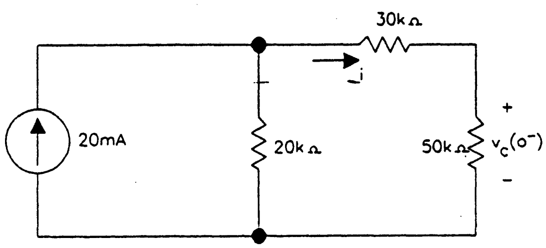
[d] $i(t) = 40 + [-20 - 40]e^{-6250t}\,\text{mA}, \qquad t > 0^+$

DE 8.12 [a] For $t < 0$:

$$i = \left(\frac{20}{100}\right) 20 = 4\,\text{mA}$$

$v_c(0^-) = (4)(50) = 200\,\text{V}$

$v_c(0^+) = 200\,\text{V}$

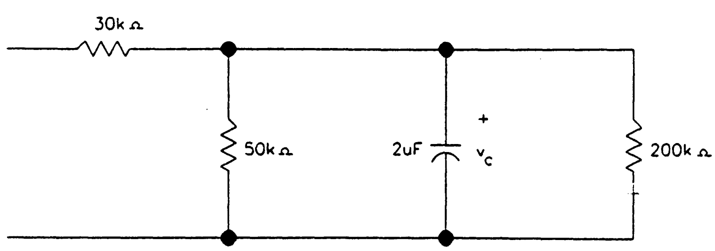


For $0 \le t \le 50\,\text{ms}$:

$\tau = (50)(2.0)10^{-3} = 0.10\,\text{s}$

$1/\tau = 10$

$v_c = 200e^{-10t}\,\text{V}$



[b] $v_c(0.05) = 200e^{-0.5} = 121.31\,\text{V}$. For $t \ge 50\,\text{ms}$:

$R_{eq} = \frac{50(200)}{250} = 40\,\text{k}\Omega, \qquad \tau = (40)(2)10^{-3} = 0.08\,\text{s}$

$\frac{1}{\tau} = 12.5$

Therefore $v_c = 121.31e^{-12.5(t-0.05)}\,\text{V}, \qquad t \geq 0.05\,\text{s}$

[c] $$w_{50\,\text{k}} = \int_0^{0.05} \frac{[200e^{-10t}]^2}{50{,}000}dt + \int_{0.05}^{\infty} \frac{[200e^{-0.5}e^{-12.5(t-0.05)}]^2}{50{,}000}dt = 37.06\,\text{mJ}$$

[d] $$w_{200\,\text{k}\Omega} = \int_{0.05}^{\infty} \frac{[200e^{-0.5}e^{-12.5(t-0.05)}]^2}{200{,}000}dt = 2.94\,\text{mJ}$$

Check: $w_{\text{stored}} = (1/2)(2 \times 10^{-6})(200)^2 = 40\,\text{mJ}$

$w_{\text{diss}} = 37.06 + 2.94 = 40\,\text{mJ}$

DE 8.13 [a] At the instant A is closed, $i(0^+) = 0$. For $0 \leq t \leq 1\,\text{s}$, the Thévenin equivalent with respect to the terminals of the 2-H inductor is $v_{\text{Th}} = 3\,\text{V}$, $R_{\text{Th}} = 1\,\Omega$; therefore in this interval $i(0^+) = 0$ and $i(\infty) = 3\,\text{A}$, also $\tau = 2/1 = 2\,\text{s}$. Thus $i = 3 - 3e^{-0.5t}\,\text{A}$, $0^+ \leq t \leq 1$, and $i(1) = 3 - 3e^{-0.5} = 1.18\,\text{A}$.

[b] When $t > 1\,\text{s}$, the Thévenin equivalent w.r.t. the 2-H inductor is $v_{\text{Th}} = -12\,\text{V}$, $R_{\text{Th}} = 2.5\,\Omega$. Therefore $i(\infty) = -12/2.5 = -4.8\,\text{A}$ and $\tau = 2/2.5 = 0.8$, thus $i = 4.8 + (1.18 + 4.8)e^{-1.25(t-1)}\,\text{A}$, $1 \leq t \leq \infty$.

DE 8.14 Derivation of Eq. (8.74): Substituting Eq. (8.73) into Eq. (8.72) gives

$$v_o = \frac{V_m R_f}{R_s} + \left\{-\frac{V_m R_f}{R_s}(1 - e^{-t_1/R_f C_f}) - \frac{V_m R_t}{R_s}\right\} e^{-(t-t_1)/R_f C_f}$$
$$= \frac{V_m R_f}{R_s}\left\{1 - [1 - e^{-t_1/R_f C_f} + 1]e^{-(t-t_1)/R_f C_f}\right\}$$
$$= \frac{V_m R_f}{R_s}\left\{1 - [2 - e^{-t_1/R_f C_f}]e^{-(t-t_1)/R_f C_f}\right\}$$
$$v_o = \frac{V_m R_f}{R_s} - \frac{V_m R_f}{R_s}(2 - e^{-t_1/R_f C_f})e^{-(t-t_1)/R_f C_f}, \qquad t_1 \leq t \leq 2t_1$$

Derivation of Eq. (8.75):

$$v_o(2t_i) = \frac{V_m R_f}{R_s} - \frac{V_m R_f}{R_s}(2 - e^{-t_1/R_f C_f})e^{-t_1/R_f C_f}$$
$$= \frac{V_m R_f}{R_s}[1 - 2e^{-t_1/R_f C_f} + e^{-2t_1/R_f C_f}]$$

Let $x = e^{-t_1/R_f C_f}; \qquad x^2 = e^{-2t_1/R_f C_f}$

$$v_o(2t_1) = \frac{V_m R_f}{R_s}(1 - 2x + x^2) = \frac{V_m R_f}{R_s}(1 - x)^2 = \frac{V_m R_f}{R_s}[1 - e^{-t_1/R_f C_f}]^2$$

Derivation of Eq. (8.76):

$v_o(t_o) = 0$

$$\frac{V_m R_f}{R_s} = \frac{V_m R_f}{R_s}(2 - e^{-t_1/R_f C_f})e^{-(t_o-t_1)/R_f C_f}$$

$$1 = (2 - e^{-t_1/R_fC_f})e^{-(t_o - t_1)/R_fC_f}$$

$$e^{+(t_o - t_1)/R_fC_f} = 2 - e^{-t_1/R_fC_f}$$

$$e^{+t_o/R_fC_f} = 2e^{t_1/R_fC_f} - 1$$

$$\frac{t_o}{R_fC_f} = \ln[2e^{t_1/R_fC_f} - 1]$$

$$t_o = R_fC_f \ln[2e^{t_1/R_fC_f} - 1]$$

DE 8.15 [a] From Eqs. (8.70), (8.74), and (8.79):

$$v_o = -\frac{R_f}{R_s}V_m(1 - e^{-t/R_fC_f}), \qquad 0 \le t \le t_1$$

$$v_o = \frac{R_f}{R_s}V_m - \frac{R_f}{R_s}V_m(2 - e^{-t_1/R_fC_f})e^{-(t-t_1)/R_fC_f}, \qquad t_1 \le t \le 2t_1$$

$$v_o = \frac{V_mR_f}{R_s}(1 - e^{-t_1/R_fC_f})^2 e^{-(t-2t_1)/R_fC_f}, \qquad 2t_1 \le t \le \infty$$

$$t_o = R_fC_f \ln(2e^{t_1/R_fC_f} - 1)$$

$$\frac{R_f}{R_s} = \frac{50 \times 10^6}{100 \times 10^3} = 500; \qquad \frac{R_f}{R_s}V_m = (500)(50 \times 10^{-3}) = 25$$

$$\frac{1}{R_fC_f} = \frac{1}{(50)(0.1)} = 0.20$$

$$v_o = -25(1 - e^{-0.2t})\,\text{V}, \qquad 0 \le t \le 1$$

$$v_o = 25 - 25(2 - e^{-0.2})e^{-0.2(t-1)}$$

$$v_o = 25 - 29.53e^{-0.2(t-1)}\,\text{V}, \qquad 1 \le t \le 2$$

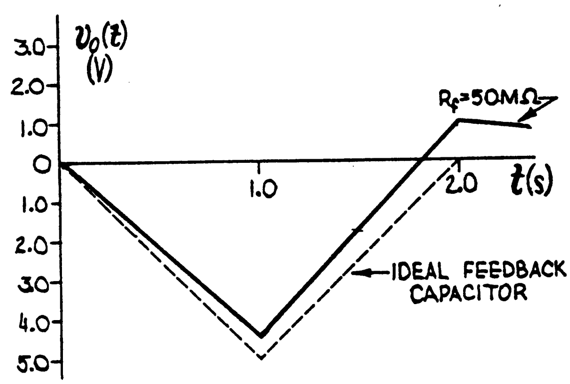
$$v_o = 25(1 - e^{-0.2})^2 e^{-0.2(t-2)}$$

$$v_o = 0.82e^{-0.2(t-2)}\,\text{V}, \qquad 2 \le t \le \infty$$

[b] $v_o(t_1) = v_o(1) = -25(1 - e^{-0.2}) = -4.53\,\text{V}$

$v_o(2t_1) = v_o(2) = 0.82$

$t_o = 5\ln(2e^{0.2} - 1) = 1.83\,\text{s}$



Problems

P 8.1 [a] $R = \frac{v}{i} = 25\,\Omega$

[b] $\frac{1}{\tau} = 10, \qquad \tau = 0.10\,\text{s} = 100\,\text{ms}$

[c] $\frac{1}{\tau} = \frac{R}{L} = 10. \qquad L = \frac{R}{10} = 2.5\,\text{H}$

[d] $i(0) = 6.4\,\text{A}; \qquad w(0) = \frac{1}{2}(2.5)(6.4)^2 = 51.20\,\text{J}$

[e] $w = \int_0^{t_o} vi\,dt = \int_0^{t_o} 1024e^{-20x}\,dx = 1024\left[\frac{e^{-20t}}{-20}\Big|_0^{t_o}\right]$

$= -51.20(e^{-20t_o} - 1) = 51.20(1 - e^{-20t_o})$

$\%\ \text{dissipated} = \frac{51.20(1 - e^{20t_o})}{51.20} \times 100 = 100(1 - e^{-20t_o})$

$\therefore\ 100(1 - e^{-20t_o}) = 60; \qquad e^{-20t_o} = 0.40; \qquad e^{20t_o} = 2.5$

$t_o = \frac{1}{20}\ln 2.5 = 50\ln 2.5\,\text{ms} = 45.81\,\text{ms}$

P 8.2 [a] $i(0) = \frac{24}{12} = 2\,\text{A}$

[b] $\tau = \frac{L}{R} = \frac{1.6}{80} = 0.02\,\text{s} = 20\,\text{ms}$

[c] $i = 2e^{-50t}\,\text{A}, \qquad t \geq 0$

$v_1 = 1.6\frac{d}{dt}(2e^{-50t}) = -160e^{-50t}\,\text{V}, \qquad t \geq 0$

$v_2 = -72i = -144e^{-50t}\,\text{V}, \qquad t \geq 0$

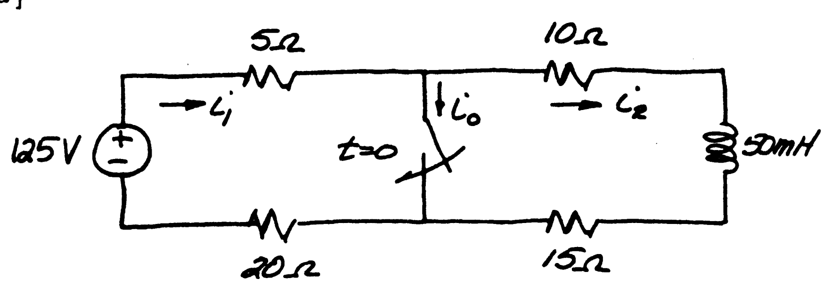
[d] $w(0) = \frac{1}{2}(1.6)(2)^2 = 3.2\,\text{J}$

$w_{72\Omega} = \int_0^{t_o} 72(4e^{-100x})\,dx = 288\left[\frac{e^{-100x}}{-100}\Big|_0^{t_o}\right] = 2.88(1 - e^{-100t_o})$

$w_{72\Omega}(15\,\text{ms}) = 2.88(1 - e^{-1.5}) = 2.24\,\text{J}$

$\%\ \text{dissipated} = \frac{2.24}{3.2} \times 100 = 69.92$

P 8.3 [a]



Before the switch is closed, $i_2 = 125/50 = 2.5\,\text{A}$.

Therefore $i_2(0^-) = i_2(0^+) = 2.5\,\text{A}$

$i_1(0^-) = 2.5\,\text{A}$, but $i_1(0^+) = 125/25 = 5\,\text{A}$

$\therefore\ i_o(0^+) = i_1(0^+) - i_2(0^+) = 5 - 2.5 = 2.5\,\text{A}$

$i_o(\infty) = 125/25 = 5\,\text{A}$

[b] $i_2 = 2.5e^{-t/\tau}\,\text{A}, \qquad \tau = \frac{L}{R} = \frac{50 \times 10^{-3}}{25} = 2\,\text{ms}$

$\therefore\ i_2 = 2.5e^{-500t}\,\text{A}, \qquad t \geq 0$

$i_1 = 5\,\text{A}, \qquad t \geq 0$

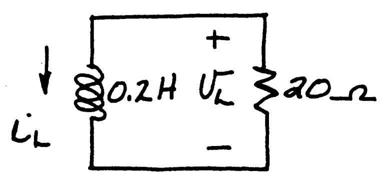
$i_o = i_1 - i_2 = 5 - 2.5e^{-500t}\,\text{A}, \qquad t \geq 0$

[c] $5 - 2.5e^{-500t} = 3 \qquad t = (1/500)\ln 1.25$

$2.5e^{-500t} = 2 \qquad = 2000\ln 1.25\,\mu\text{s}$

$e^{500t} = 1.25 \qquad t = 446.29\,\mu\text{s}$

P 8.4 $t > 0$: $\quad i_L(0) = \left(\frac{80}{20}\right)\left(\frac{50}{80}\right) = 2.5\,\text{A}$



$\tau = \frac{0.2}{20} = 0.01\,\text{s}$

$\frac{1}{\tau} = 100$

$i_L = 2.5e^{-100t}\,\text{A}, \qquad t \geq 0^+$

$v_L = -20i_L = -50e^{-100t}\,\text{V}, \qquad t \geq 0^+$

$v_o = -15i_L = -37.5e^{-100t}\,\text{V}, \qquad t \geq 0^+$

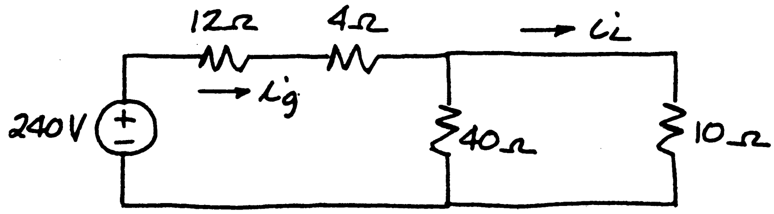
P 8.5 $w(0) = \frac{1}{2}(0.2)(2.5)^2 = 625\,\text{mJ}$

$p_{2\Omega} = 2i_L^2 = 12.5e^{-200t}\,\text{W}$

$$w_{2\Omega} = \int_0^{0.02} 12.5e^{-200t}dt = 12.5\frac{e^{-200t}}{(-200)}\Bigg|_0^{0.02} = 62.5 \times 10^{-3}(1 - e^{-4}) = 61.36\,\text{mJ}$$

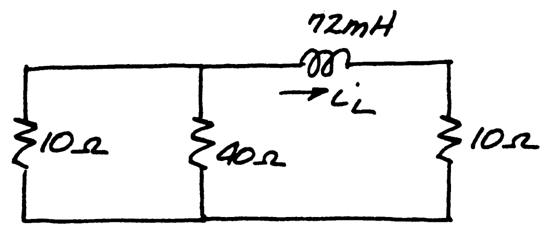
$$\%\ \text{dissipated} = \frac{61.36}{625} \times 100 = 9.82\%$$

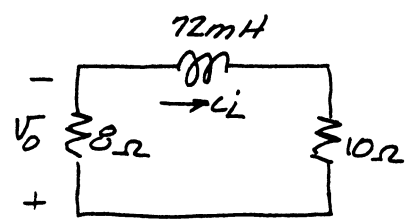
P 8.6 For $t < 0$:



$$i_g = \frac{240}{16+8} = 10\,\text{A}; \qquad \therefore\ i_L(0^-) = (10)\frac{40}{50} = 8\,\text{A}$$

For $t \geq 0^+$:





$$\tau = \frac{72 \times 10^{-3}}{18} = 4\,\text{ms}$$

$$\frac{1}{\tau} = 250$$

$$\therefore\ i_L = 8e^{-250t}\,\text{A}, \qquad t \geq 0^+$$

$$v_o = -\left[10i_L + 0.072\frac{di_L}{dt}\right] = -\left[80e^{-250t} - 144e^{-250t}\right] = 64e^{-250t}\,\text{V}, \qquad t \geq 0^+$$

or $v_o = 8i_L = 64e^{-250t}\,\text{V}, \qquad t \geq 0^+$

P 8.7 $w(0) = \frac{1}{2}(72 \times 10^{-3})(8)^2 = 2304\,\text{mJ}$

$$p_{40\Omega} = \frac{v_o^2}{40} = \frac{(64)^2}{40}e^{-500t} = 102.4e^{-500t}\,\text{W}$$

$$w_{40\Omega} = \int_0^\infty 102.4e^{-500t}dt = 204.8\,\text{mJ}$$

$$\%\ \text{dissipated} = \frac{204.8}{2304} \times 100 = 8.89\%$$

P 8.8 [a] When the switch is opened, v_o is the natural response of the inductor voltage, hence

$$v_o = v_o(0^+)e^{-t/\tau}, \qquad t \geq 0^+$$

where $\tau = L/R$

By hypothesis $\quad v_o(10^{-3}) = v_o(0^+)e^{-10^{-3}/\tau} = 0.5v_o(0^+)$

It follows that $\quad e^{10^{-3}/\tau} = 2, \qquad \frac{10^{-3}}{\tau} = \ln 2$

$$\therefore\ L = \frac{10}{\ln 2} \times 10^{-3} = 14.43\,\text{mH}$$

[b] $v_o(0^+) = -10i_L(0^+) = -10\left(\frac{1}{10}\right)(30 \times 10^{-3}) = -30\,\text{mV} = -0.03\,\text{V}$

$$\therefore\ v_o = -0.03e^{-t/\tau}\,\text{V}, \qquad t \geq 0^+$$

$$p_{10\Omega} = \frac{v_o^2}{10} = 9 \times 10^{-5}e^{-2t/\tau}$$

$$w_{10\Omega} = \int_{0^+}^{10^{-3}} 9 \times 10^{-5}e^{-2t/\tau}dt = 4.5\tau \times 10^{-5}(1 - e^{-2\times10^{-3}/\tau})$$

$$\tau = \frac{1}{1000\ln 2}$$

$$\therefore\ w_{10\Omega} = 48.69\,\text{nJ}$$

$$w_L(0) = \frac{1}{2}Li_L^2(0) = \frac{1}{2}(14.43) \times 10^{-3}(3 \times 10^{-3})^2 = 64.92\,\text{nJ}$$

$$\%\ \text{dissipated in}\ 1\,\text{ms} = \frac{48.69}{64.92} \times 100 = 75\%$$

Alternate solution:

$$i_L = 3e^{-t/\tau}\,\text{mA}, \qquad t \geq 0$$

$$i_L(1\,\text{ms}) = 1.5\,\text{mA}$$

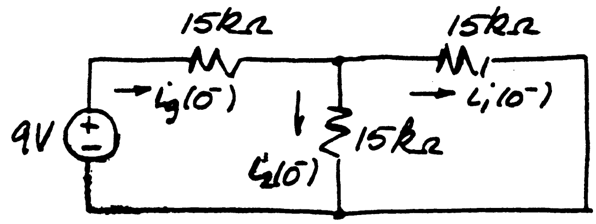
Therefore the energy stored in the inductor at 1 ms is

$$w_L(1\,\text{ms}) = \frac{1}{2}(14.43 \times 10^{-3})(2.25 \times 10^{-6}) = 16.23\,\text{nJ}$$

$$\therefore\ w_L(\text{diss}) = w_L(0) - w_L(1\,\text{ms}) = 64.92 - 16.23 = 48.69\,\text{nJ}$$

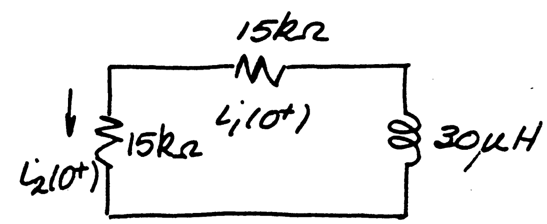
$$\%\ \text{dissipated in}\ 1\,\text{ms} = \frac{48.69}{64.92} \times 100 = 75\%$$

P 8.9 [a] For $t < 0$:



$$i_g(0^-) = \frac{9 \times 10^{-3}}{(15 + 7.5)} = 400 \times 10^{-6} = 400\,\mu\text{A}$$
$$i_1(0^-) = \frac{1}{2}(400) = 200\,\mu\text{A}$$
$$i_2(0^-) = \frac{1}{2}(400) = 200\,\mu\text{A}$$

[b] For $t \geq 0^+$:



Since i_1 is the inductor branch current, it cannot change instantaneously, thus
$$i_1(0^+) = i_1(0^-) = 200\,\mu\text{A}$$
For $t \geq 0^+$: $\quad i_2 = -i_1$, hence $i_2(0^+) = -200\,\mu\text{A}$

[c] $\tau = \dfrac{L}{R} = \dfrac{30 \times 10^{-6}}{30 \times 10^3} = 10^{-9}\,\text{s}$

$\therefore\ i_1(t) = 200e^{-10^9 t}\,\mu\text{A}, \qquad t \geq 0$

[d] $i_2(t) = -200e^{-10^9 t}\,\mu\text{A}, \qquad t \geq 0^+$

[e] i_1 cannot change instantaneously with time. The switching operation forces
$$i_2(t) = -i_1(t) \quad \text{when} \quad t \geq 0^+$$
Therefore $i_2(0^-) \neq i_2(0^+)$

P 8.10 $w(0) = \dfrac{1}{2}(10) \times 10^{-3}(25) = 125\,\text{mJ}$

$0.9w(0) = 112.5\,\text{mJ}$

$w(t) = \dfrac{1}{2}(10 \times 10^{-3})i(t)^2$

$i(t) = 5e^{-t/\tau}\,\text{A}$

$\therefore\ w(t) = 5 \times 10^{-3}(25e^{-2t/\tau}) = 125e^{-2t/\tau}\,\text{mJ}$

$\quad w(10\,\mu\text{s}) = 125e^{-20\times10^{-6}/\tau}\,\text{mJ}$

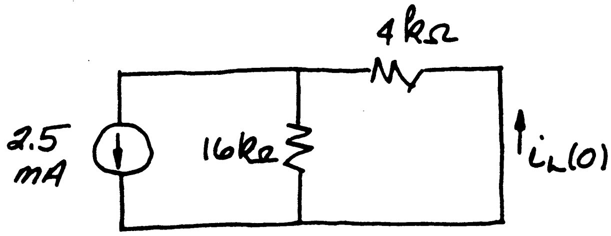
$\therefore\ 125e^{-20\times10^{-6}/\tau} = 112.5; \qquad e^{20\times10^{-6}/\tau} = \dfrac{1}{0.9} = \dfrac{10}{9}$

$$\frac{20 \times 10^{-6}}{\tau} = \ln\left(\frac{10}{9}\right)$$

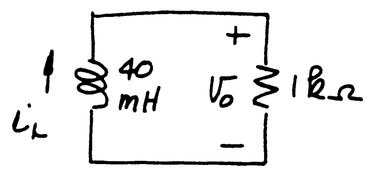
$$\tau = \frac{20 \times 10^{-6}}{\ln(10/9)} = \frac{L}{R}$$

$$\therefore\ R = \frac{10 \times 10^{-3}\ln(10/9)}{20 \times 10^{-6}} = 500\ln(10/9) \cong 52.68\,\Omega$$

P 8.11 [a] For $t < 0$: $i_L(0) = (2.5)\frac{16}{20} = 2.0\,\text{mA}$



For $t > 0$: $L_e = \frac{(120)(60)}{180} = 40\,\text{mH}$



$i_L(0) = 2\,\text{mA}$
$i_L = 2e^{-t/\tau}\,\text{mA},\quad t \geq 0$
$\tau = L/R = 40\,\mu\text{s}$
$1/\tau = 25{,}000$

$\therefore\ i_L = 2e^{-25{,}000t}\,\text{mA},\qquad t \geq 0$
$\therefore\ v_o = 2e^{-25{,}000t}\,\text{V},\qquad t \geq 0^+$

[b] $p_{1\,\text{k}\Omega} = \frac{v_o^2}{1000} = 4 \times 10^{-3}e^{-50{,}000t}\,\text{W}$

$$w_{1\text{k}\Omega} = \int_0^{t_o} 4 \times 10^{-3}e^{-50{,}000t}dt = 4 \times 10^{-3}\left[\frac{e^{-50{,}000t}}{-50{,}000}\right]_0^{t_o}$$
$$= 80 \times 10^{-9}(1 - e^{-50{,}000t_o}) = 80(1 - e^{-50{,}000t_o})\,\text{nJ}$$

$w(0) = \frac{1}{2}(40 \times 10^{-3})(4 \times 10^{-6}) = 80\,\text{nJ}$

% dissipated $= 100(1 - e^{-50{,}000t_o})$

$\therefore\ 1 - e^{-50{,}000t_o} = 0.98;\qquad e^{-50{,}000t_o} = 0.02;\qquad e^{50{,}000t_o} = 50$

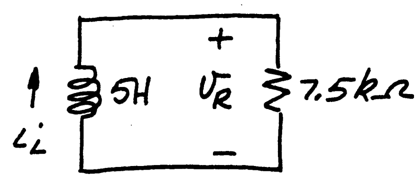
$t_o = \frac{1}{50{,}000}\ln 50 = 20\ln 50\,\mu\text{s}$

$t_o = 78.24\,\mu\text{s}$

$\therefore$ Number of time constants $= t_o/\tau = 1.96$

$\therefore$ Number of time constants < 2

P 8.12 [a] $L_e = 1.25 + \frac{60}{16} = 5.0\,\text{H}$



$i_L = i_L(0)e^{-t/\tau}$
$i_L(0) = 2\,\text{A}$
$\frac{1}{\tau} = \frac{7500}{5} = 1500$

$i_L(t) = 2e^{-1500t}\,\text{A}, \qquad t \geq 0$
$v_R = 15{,}000e^{-1500t}\,\text{V}, \qquad t \geq 0^+$
$v_o = -3.75\frac{di_L}{dt} = -3.75(-3000e^{-1500t})$
$v_o = 11{,}250e^{-1500t}\,\text{V}, \qquad t \geq 0^+$

[b] $i_o = -\frac{1}{6}\int_0^t 11{,}250e^{-1500x}dx + 0 = -1875\frac{e^{-1500x}}{-1500}\Bigg|_0^t$
$i_o = 1.25e^{-1500t} - 1.25\,\text{A}, \qquad t \geq 0$

P 8.13 [a] $w_{\text{diss}} = \frac{1}{2}L_e[i_L(0)]^2 = \frac{1}{2}(5)(2)^2 = 10\,\text{J}$

Alternate solution:

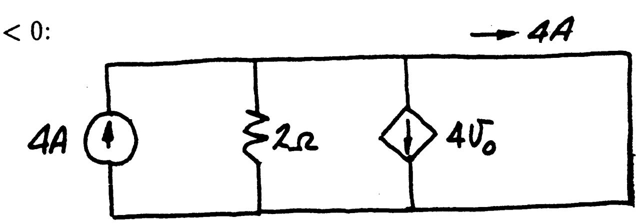
$w_{\text{diss}} = \int_0^\infty [2e^{-1500t}]^2 7500\,dt = 30{,}000\int_0^\infty e^{-3000t}\,dt = -10(0-1) = 10\,\text{J}$

[b] $w_{\text{trapped}} = \frac{1}{2}(10)(1.25)^2 + \frac{1}{2}(6)(1.25)^2 = (5+3)(1.25)^2 = 12.5\,\text{J}$

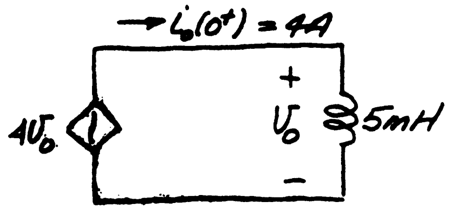
Alternate solution:

$w(0) = \frac{1}{2}(10)(4) + \frac{1}{2}(1.25)(4) = 20 + 2.5 = 22.50\,\text{J}$
$w_{\text{trapped}} = w(0) - w_{\text{diss}} = 22.5 - 10 = 12.5\,\text{J}$

P 8.14 For $t < 0$:



For $t > 0$:



The Thévenin resistance seen by the 5-mH inductor is

$$R_{\text{Th}} = \frac{v_o}{4v_o} = 0.25\,\Omega$$

$$\tau = \frac{L}{R} = 20\,\text{ms}; \qquad \frac{1}{\tau} = 50$$

$$\therefore\ v_o(0^+) = -4(0.25) = -1\,\text{V}$$

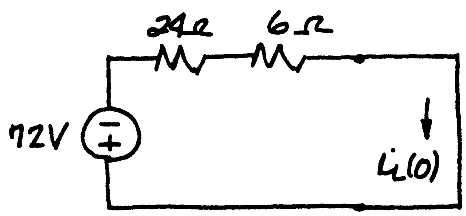
$$\therefore\ v_o = -e^{-50t}\,\text{V}, \qquad t \geq 0^+$$

Alternate solution:

$$i_o = 4e^{-50t}\,\text{A}, \qquad t \geq 0$$

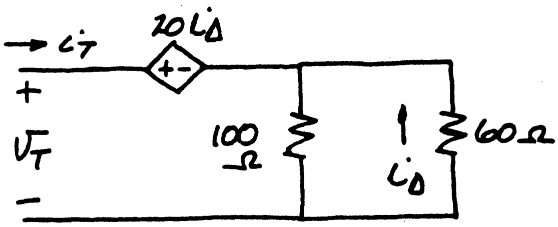
$$v_o = L\frac{di_o}{dt} = (5 \times 10^{-3})(-200)e^{-50t} = -e^{-50t}\,\text{V}, \qquad t \geq 0^+$$

P 8.15 [a] For $t < 0$:



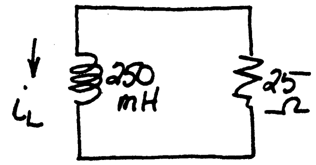
$$i_L(0) = \frac{-72}{30} = -2.4\,\text{A}$$

For $t > 0$:



$$v_T = 20i_\Delta + 37.5i_T = 20\left(-\frac{100}{160}i_T\right) + 37.5i_T = (-12.5 + 37.5)i_T = 25i_T$$

$$\frac{v_T}{i_T} = 25\,\Omega$$



$$\frac{1}{\tau} = \frac{25}{250} \times 10^3 = 100$$

$$i_L(0) = -2.4\,\text{A}$$

$$i_L(t) = -2.4e^{-100t}\,\text{A}, \qquad t \geq 0$$

[b] $v_L = (250 \times 10^{-3})\frac{d}{dt}(-2.4e^{-100t})$

$v_L = 60e^{-100t}\,\text{V}, \qquad t \geq 0^+$

[c] $i_\Delta = 0.625i_L = -1.5e^{-100t}\,\text{A}, \qquad t \geq 0^+$

P 8.16 $p_{diss} = 37.5i_L^2 = 216e^{-200t}\,\text{W}$

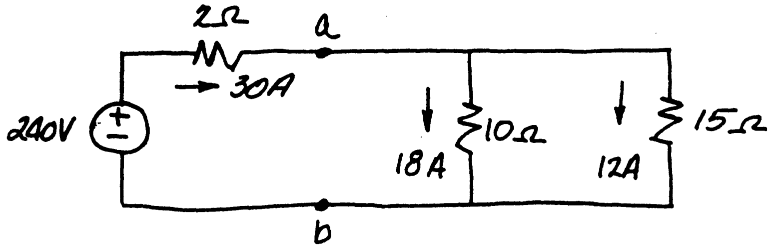
$$w_{diss} = \int_0^\infty 216e^{-200t}\,dt = 216\left[\frac{e^{-200t}}{-200}\Bigg|_0^\infty\right] = 1080\,\text{mJ}$$

$$p_{20i_\Delta} = -(20i_\Delta)i_L = -72e^{-200t}$$

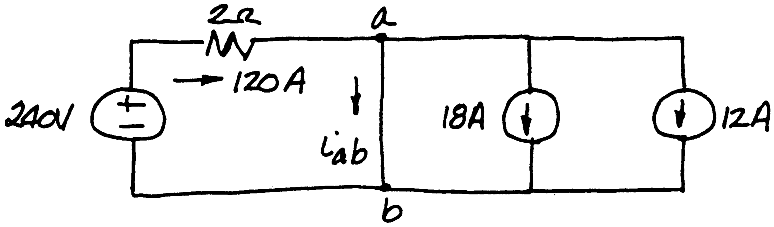
$$w_{20i_\Delta} = -72\left[\frac{e^{-200t}}{-200}\Bigg|_0^\infty\right] = -360\,\text{mJ}; \qquad \therefore \text{ dependent source delivers } 360\,\text{mJ}$$

$$\%\text{ supplied by dependent source} = \frac{360}{1080} \times 100 = 33.33\%$$

P 8.17 [a] $t < 0$:

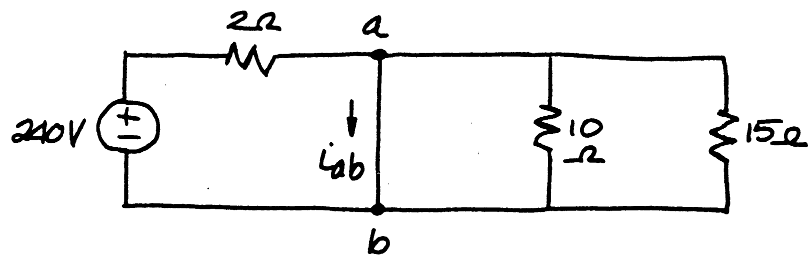


$t = 0^+$:



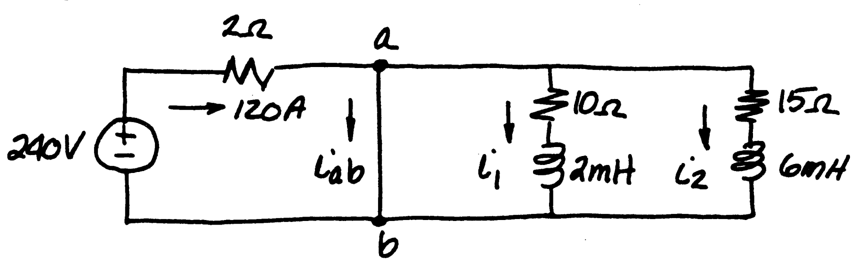
$$120 = i_{ab} + 18 + 12, \qquad i_{ab} = 90\,\text{A}, \quad t = 0^+$$

[b] At $t = \infty$:



$i_{ab} = 240/2 = 120\,\text{A}, \quad t = \infty$

[c]



$$i_1(0) = 18, \qquad \tau_1 = \frac{2}{10} \times 10^{-3} = 0.2\,\text{ms}$$
$$i_2(0) = 12, \qquad \tau_2 = \frac{6}{15} \times 10^{-3} = 0.4\,\text{ms}$$
$$i_1(t) = 18e^{-5000t}\,\text{A}, \quad t \geq 0$$
$$i_2(t) = 12e^{-2500t}\,\text{A}, \quad t \geq 0$$
$$i_{ab} = 120 - 18e^{-5000t} - 12e^{-2500t}\,\text{A}, \quad t \geq 0$$
$$120 - 18e^{-5000t} - 12e^{-2500t} = 114$$
$$6 = 18e^{-5000t} + 12e^{-2500t}$$
$$e^{-2500t} = x, \qquad \therefore\ e^{-5000t} = x^2$$
$$\therefore\ 6 = 18x^2 + 12x$$
$$x^2 + \frac{2}{3}x - \frac{6}{18} = 0$$
$$x = -\frac{1}{3} \pm \sqrt{\frac{1}{9} + \frac{3}{9}}; \qquad x = -\frac{1}{3} \pm \frac{2}{3} = \frac{1}{3}$$
$$e^{-2500t} = \frac{1}{3}; \qquad e^{2500t} = 3$$
$$t = \frac{1}{2500}\ln 3 = 400\ln 3\,\mu\text{s}; \qquad t = 439.44\,\mu\text{s}$$

P 8.18 [a] $R = \frac{v}{i} = 8000\,\Omega = 8\,\text{k}\Omega$

[b] $\frac{1}{\tau} = \frac{1}{RC} = 500; \qquad C = \frac{1}{(500)(8000)} = 0.25\,\mu\text{F}$

[c] $\tau = \frac{1}{500} = 2\,\text{ms}$

[d] $w(0) = \frac{1}{2}(0.25 \times 10^{-6})(72)^2 = 648\,\mu\text{J}$

[e] $W_{\text{diss}} = \int_0^{t_o} \frac{v^2}{R} dt = \int_0^{t_o} \frac{(72)^2 e^{-1000t}}{(8000)} dt = 648 \times 10^{-3} \frac{e^{-1000t}}{-1000}\Big|_0^{t_o}$

$= 648 \times 10^{-6}(1 - e^{-1000t_o}) = 648(1 - e^{-1000t_o})\,\mu\text{J}$

$\therefore\ \%\ \text{dissipated} = 100(1 - e^{-1000t_o})$

$\therefore\ 100(1 - e^{-1000t_o}) = 68$

$e^{-1000t_o} = 0.32; \qquad e^{1000t_o} = 3.125$

$t_o = \frac{1}{1000}\ln 3.125; \qquad t_o \cong 1.14\,\text{ms}$

P 8.19 [a] $v_c(0) = (40)10^{-3}\frac{(2.7)(3.3)}{6} \times 10^3 = 59.4\,\text{V}$

$\tau = \left[\frac{(3)(6)}{9} \times 10^3\right](0.5 \times 10^{-6}) = 1\,\text{ms}$

$v_c(t) = 59.4e^{-1000t}\,\text{V}, \qquad t \geq 0$

$i_o(t) = \left(\frac{59.4}{6} \times 10^{-3}\right) e^{-1000t}$

$i_o(t) = 9.90e^{-1000t}\,\text{mA}, \qquad t \geq 0^+$

[b] $W_{\text{diss}} = \int_0^{t_o} \frac{(59.4)^2}{3000} e^{-2000t} dt$

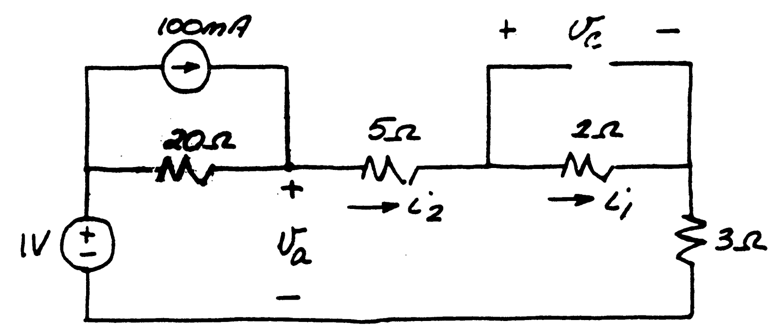
$W_{\text{diss}} = 1176.12 \times 10^{-3}\left[\frac{e^{-2000t}}{-2000}\Big|_0^{t_o}\right] = 588.06 \times 10^{-6}(1 - e^{-2000t_o})$

$w(500\,\mu\text{s}) = (588.06)(1 - e^{-1})\,\mu\text{J} = 371.72\,\mu\text{J}$

$w(0) = \frac{1}{2}(0.5 \times 10^{-6})(59.4)^2 = 882.09\,\mu\text{J}$

$\%\ \text{dissipated} = \frac{371.72}{882.09} \times 100 = 42.14\%$

P 8.20 [a] $t < 0$:

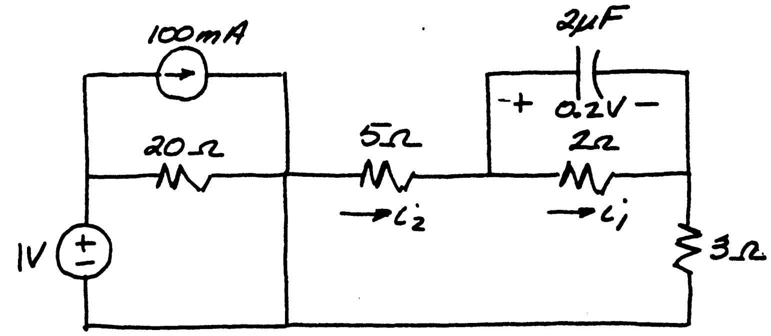


$$\frac{v_a - 1}{20} - 0.1 + \frac{v_a}{10} = 0$$

$$v_a - 1 - 2 + 2v_a = 0, \qquad 3v_a = 3, \qquad v_a = 1\,\text{V}$$

$$\therefore\ i_1(0^-) = i_2(0^-) = \frac{1}{10} = 0.1\,\text{A} = 100\,\text{mA}$$

[b] First note that $v_c(0^-) = v_c(0^+) = 2i_1 = 0.2\,\text{V}$; then for $t = 0^+$



$$i_1(0^+) = \frac{0.2}{2} = 0.1 = 100\,\text{mA}; \qquad i_2(0^+) = \frac{-0.2}{8} = -25\,\text{mA}$$

[c] The voltage across the capacitor cannot change instantaneously; therefore the current in the resistor across the terminals of the capacitor cannot change between $t = 0^-$ and $t = 0^+$.

[d] After the switch closes, i_2 is governed by the capacitor voltage since closing the switch places i_2 in a branch parallel to the 2-μF capacitor. Thus between $t = 0^-$ and $t = 0^+$ i_2 changes instantaneously from 100 mA to −25 mA.

[e] For $t > 0^+$ the time constant of the circuit is R_eC where

$$R_e = (2)(8)/10 = 1.6\,\Omega$$

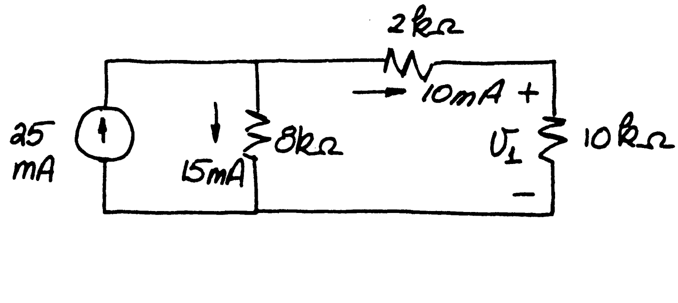
$$C = 2\,\mu\text{F}$$

$$\therefore\ \tau = 3.2\,\mu\text{s} \quad \text{and} \quad \frac{1}{\tau} = 312{,}500$$

$$\therefore\ i_1(t) = 100e^{-312{,}500t}\,\text{mA}, \qquad t \geq 0$$

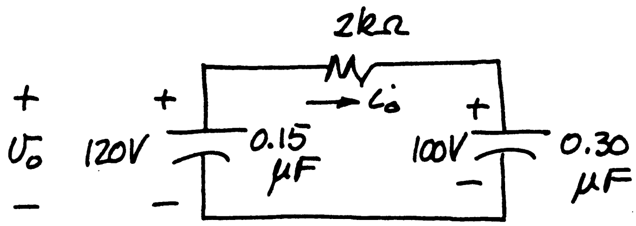
[f] $i_2(t) = -25e^{-312{,}500t}\,\text{mA}, \qquad t \geq 0^+$

P 8.21 [a] For $t < 0$:

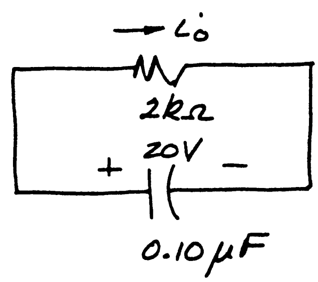


$\therefore\ v_o(0) = 120\,\text{V}; \qquad v_1(0) = 100\,\text{V}$

For $t > 0$:



$$\frac{1}{C_e} = \frac{1}{0.15} + \frac{1}{0.30} = \frac{1}{0.10}; \qquad C_e = 0.10\,\mu\text{F}$$



$$i_o(0^+) = \frac{20}{2} = 10\,\text{mA}$$

$$\tau = (2000)(0.10)10^{-6} = 200\,\mu\text{s}; \qquad \frac{1}{\tau} = 5000$$

$$i_o(t) = 10e^{-5000t}\,\text{mA}, \qquad t \geq 0^+$$

[b] $$v_o = \frac{-10^6}{0.15}\int_0^t (10 \times 10^{-3})e^{-5000x}dx + 120 = \frac{-10{,}000}{0.15}\left[\frac{e^{-5000x}}{-5000}\right|_0^t + 120$$

$$= \frac{40}{3}e^{-5000t} - \frac{40}{3} + 120$$

$$v_o = \frac{40}{3}e^{-5000t} + \frac{320}{3}\,\text{V}, \qquad t \geq 0$$

[c] $$v_o(\infty) = \frac{320}{3}\,\text{V}; \qquad \therefore\ v_1(\infty) = \frac{320}{3}\,\text{V}$$

$$w_{\text{trapped}} = \left[\frac{1}{2}(0.15)\left(\frac{320}{3}\right)^2 + \frac{1}{2}(0.30)\left(\frac{320}{3}\right)^2\right]\,\mu\text{J} = 2560\,\mu\text{J}$$

Check:

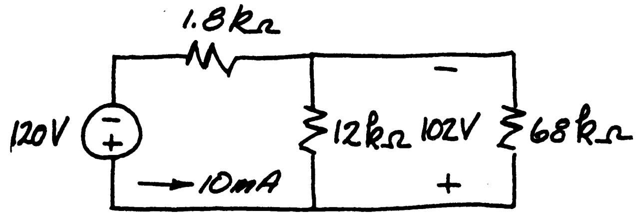
$$w(0) = \left[\frac{1}{2}(0.15)(120)^2 + \frac{1}{2}(0.30)(100)^2\right]\,\mu\text{J} = 2580\,\mu\text{J}$$

$$w_{\text{diss}} = \int_0^{\infty} (10 \times 10^{-3})^2 e^{-10{,}000t}(2000)\,dt$$

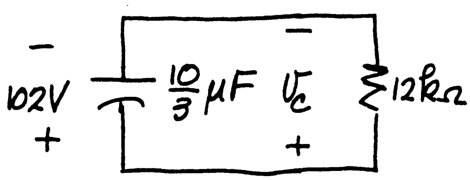
$$= 2000 \times 10^{-4} \left[\frac{e^{-10{,}000t}}{-10{,}000}\right|_0^{\infty} = 20\,\mu\text{J}$$

$$w(0) - w_{\text{diss}} = 2560\,\mu\text{J} = w_{\text{trapped}}$$

P 8.22 [a] $t < 0$:



$t > 0$:



$$\tau = RC$$

$$= 12 \times 10^3 \times \frac{10}{3} \times 10^{-6}$$

$$= 40\,\text{ms}$$

$$1/\tau = 25$$

$$v_C = 102e^{-25t}\,\text{V}, \quad t \geq 0$$

$$W_{\text{diss}} = \int_0^{t_o} \frac{(102)^2 e^{-50t}}{12{,}000}\,dt = \frac{10{,}404}{12{,}000}\left[\frac{e^{-50t}}{-50}\right|_0^{t_o} = 17.34 \times 10^{-3}(1 - e^{-50t_o})$$

$$W_{\text{diss}}(12\,\text{ms}) = (17.34)(1 - e^{-0.6}) \times 10^{-3} = 7.82\,\text{mJ}$$

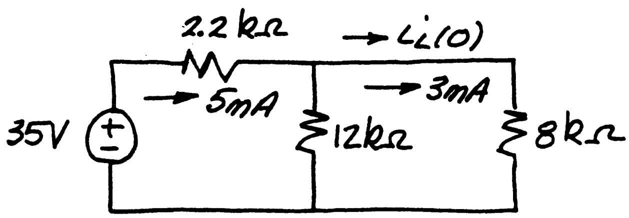
[b] $\%\ \text{dissipated} = 100(1 - e^{-50t_o}) = 75$

$$\therefore\ 1 - e^{-50t_o} = 0.75$$

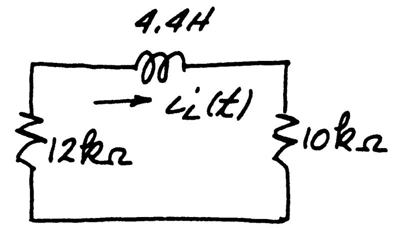
$$e^{-50t_o} = 0.25; \qquad e^{50t_o} = 4$$

$$t_o = \frac{1}{50}\ln 4 = 20\ln 4\,\text{ms}; \qquad t_o = 27.73\,\text{ms}$$

P 8.23 [a] $t < 0$:



$t > 0$:



$\tau = L/R = 4.4/22$
$= 0.2\,\text{ms}$
$1/\tau = 5000$
$i_L(t) = 3e^{-5000t}\,\text{mA}, \quad t \geq 0$

$$w(0) = \frac{1}{2}(4.4)(3 \times 10^{-3})^2 = 19.8\,\mu\text{J}$$

$$w_{10\text{k}} = \int_0^{t_o} (9 \times 10^{-6} e^{-10{,}000t})(10{,}000)\,dt = 9(1 - e^{-10{,}000t_o})\,\mu\text{J}$$

$$\therefore\ 9(1 - e^{-10{,}000t_o}) = 0.2(19.8)$$

$$1 - e^{-10{,}000t_o} = 0.44$$

$$e^{10{,}000t_o} = \frac{1}{0.56} = \frac{25}{14}$$

$$t_o = 100 \ln\left(\frac{25}{14}\right)\,\mu\text{s} = 57.98\,\mu\text{s}$$

[b] $$W_{\text{diss}}(\text{total}) = \int_0^{t_o} (9 \times 10^{-6} e^{-10{,}000t})(22 \times 10^3)\,dt$$

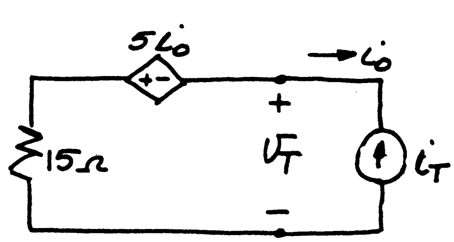
$$= 198 \times 10^{-3} \left[\frac{e^{-10{,}000t}}{-10^4}\Bigg|_0^{t_o}\right] = 19.8(1 - e^{-10{,}000t_o})\,\mu\text{J}$$

$$W_{\text{diss}}(\text{total})[57.98\,\mu\text{s}] = 8.712\,\mu\text{J}$$

$$\%\text{ dissipated } @57.98\,\mu\text{s} = \left(\frac{8.712}{19.8}\right)100 = 44$$

P 8.24 For $t < 0$, i_o is zero; therefore the capacitor charges to 15 V, positive at the upper terminal.

For $t > 0$ the Thévenin resistance seen from the terminals of the capacitor is 20 Ω, i.e.,

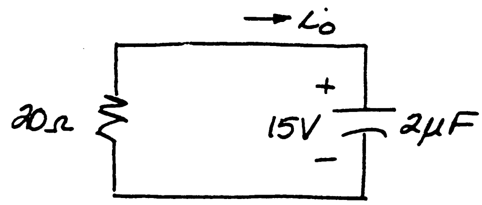


Now $i_o = -i_T$, therefore

$$v_T = 15i_T - 5(-i_T) = 20i_T, \qquad R_{\text{Th}} = \frac{v_T}{i_T} = 20\,\Omega$$

Therefore the time constant is $\tau = (2 \times 10^{-6})(20) = 40\,\mu s$ and $\frac{1}{\tau} = 25{,}000$.

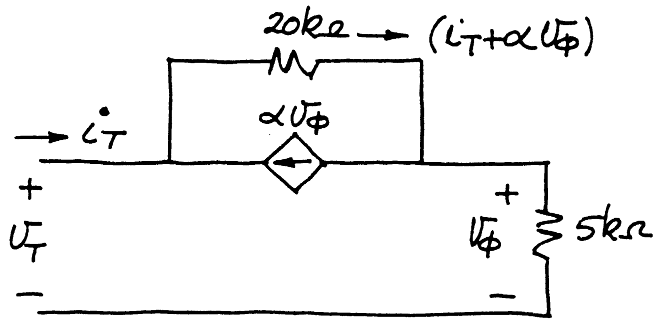
$t > 0^+$:



$$i_o(0^+) = -\frac{15}{20} = -0.75 = -750\,\text{mA}$$

$$\therefore\ i_o(t) = -750e^{-25{,}000t}\,\text{mA}, \qquad t \geq 0^+$$

P 8.25 [a]



$$v_T = 20 \times 10^3(i_T + \alpha v_\phi) + 5000i_T = 25{,}000i_T + 20 \times 10^3(\alpha)(5000i_T)$$

$$\frac{v_T}{i_T} = R_{\text{Th}} = 25{,}000 + 100 \times 10^6\alpha$$

$$\tau = R_{\text{Th}}C = 40 \times 10^{-3} = R_{\text{Th}}(0.8 \times 10^{-6})$$

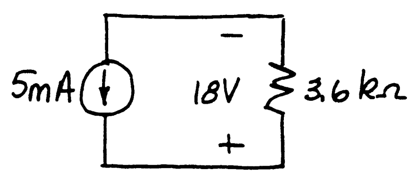
$$\therefore\ R_{\text{Th}} = 50\,\text{k}\Omega$$

$$\therefore\ 25 \times 10^3 + 100 \times 10^6\alpha = 50 \times 10^3$$

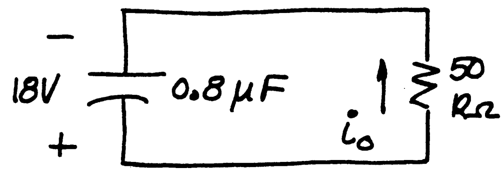
$$\therefore\ \alpha = \frac{25{,}000}{100 \times 10^6} = 0.25 \times 10^{-3}$$

$$\alpha = 250 \times 10^{-6}\,\text{A/V}$$

[b] $t < 0$:



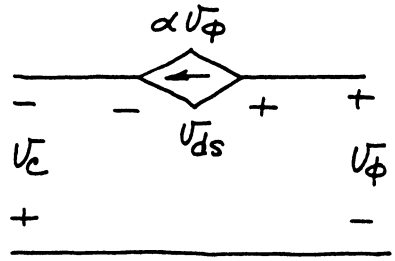
$t > 0$:



$\tau = 40$ ms
$1/\tau = 25$
$i_o(0^+) = 0.36$ mA

$i_o = 0.36e^{-25t}$ mA, $\quad t \geq 0^+$
$v_\phi = -5000i_o = -1.8e^{-25t}$ V, $\quad t \geq 0^+$

P 8.26 [a]



$v_c = 18e^{-25t}$ V, $\quad v_\phi = -1.8e^{-25t}$ V, $\quad v_{ds} = v_c + v_\phi = 16.2e^{-25t}$ V

$i_{ds} = \alpha v_\phi = (250 \times 10^{-6})(-1.8e^{-25t}) = -450e^{-25t}\,\mu$A

$p_{ds} = v_{ds}i_{ds} = -7290e^{-50t}\,\mu$W

$$w_{ds} = \int_0^\infty -7290 \times 10^{-6}e^{-50t}\,dt = -145.80\,\mu\text{J}$$

$\therefore$ dependent source delivers 145.80 μJ

[b] $$w_{5\text{k}\Omega} = \int_0^\infty [0.36 \times 10^{-3}e^{-25t}]^2 5000\,dt = 648 \times 10^{-6}\int_0^\infty e^{-50t}\,dt = 12.96\,\mu\text{J}$$

$$w_{20\text{k}\Omega} = \int_0^\infty \frac{(16.2e^{-25t})^2}{20{,}000}\,dt = 13{,}122 \times 10^{-6}\int_0^\infty e^{-50t}\,dt = 262.44\,\mu\text{J}$$

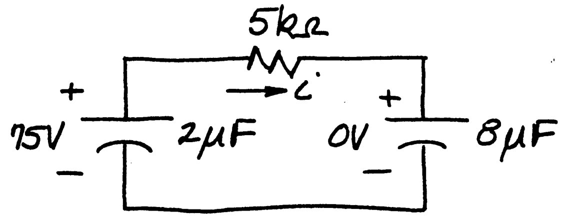
$$w(0) = \frac{1}{2}(0.8 \times 10^{-6})(18)^2 = 129.60\,\mu\text{J}$$

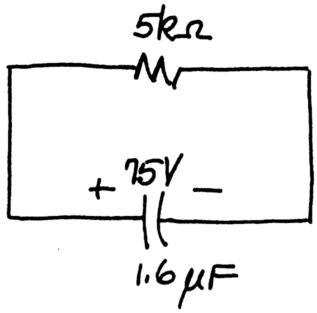
$$\sum w_{\text{diss}} = 12.96 + 262.44 = 275.40\,\mu\text{J}$$

$$\sum w_{\text{del}} = 129.60 + 145.80 = 275.40\,\mu\text{J}$$

$$\therefore \sum w_{\text{diss}} = \sum w_{\text{del}}$$

P 8.27 [a] $t > 0$:





$$\tau = 8 \times 10^{-3} = 8\,\text{ms}$$
$$1/\tau = 125$$
$$i(0^+) = \frac{75}{5} \times 10^{-3} = 15\,\text{mA}$$
$$i(t) = 15e^{-125t}\,\text{mA}, \quad t \geq 0^+$$

$$v_1 = -\frac{10^6}{2}\int_0^t 15 \times 10^{-3}e^{-125x}\,dx + 75$$
$$= 60(e^{-125t} - 1) + 75 = 60e^{-125t} + 15\,\text{V}, \qquad t \geq 0$$
$$v_2 = \frac{10^6}{8}\int_0^t 15 \times 10^{-3}e^{-125x}\,dx + 0$$
$$= -15(e^{-125t} - 1) = -15e^{-125t} + 15\,\text{V}, \qquad t \geq 0$$

[b] $w(0) = \frac{1}{2}(2 \times 10^{-6})(75)^2 = 5625\,\mu\text{J}$

[c] $W_{\text{trapped}} = \frac{1}{2}(2 \times 10^{-6})(15)^2 + \frac{1}{2}(8 \times 10^{-6})(15)^2 = 225[10^{-6} + 4 \times 10^{-6}]$

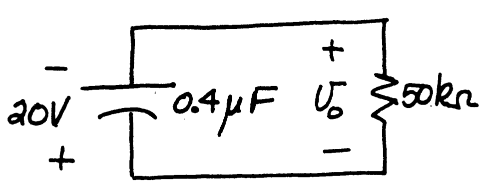
$W_{\text{trapped}} = 1125\,\mu\text{J}$

$W_{\text{diss}} = w(0) - W_{\text{trapped}} = 4500\,\mu\text{J}$

Check: $W_{\text{diss}} = \int_0^\infty 225 \times 10^{-6}e^{-250t}(5000)\,dt$

$$= 1125 \times 10^{-3}\frac{e^{-250t}}{-250}\Bigg|_0^\infty = 4500\,\mu\text{J}$$

P 8.28 [a] $C_e = \dfrac{(0.6)(1.2)}{1.8} = 0.4\,\mu\text{F}$



$$\tau = (0.4)(50) \times 10^{-3}$$
$$= 20\,\text{ms}$$
$$1/\tau = 50$$

$v_o = -20e^{-50t}\,\text{V}, \qquad t \geq 0$

[b] $$w_{\text{diss}} = \frac{1}{2}(0.4)(400) = 80\,\mu\text{J}$$

$$w(0) = \frac{1}{2}(0.4)(30)^2 + \frac{1}{2}(0.2)(30^2) + \frac{1}{2}(1.2)(10)^2 = 330\,\mu\text{J}$$

$$\therefore\ \%\ \text{dissipated} = \frac{80}{330} \times 100 = 24.24\%$$

[c] $$v_1(t) = \frac{-10^6}{1.2}\int_0^t \frac{-20}{50 \times 10^3}e^{-50x}\,dx + 10 = \frac{(20)(1000)}{60}\left[\frac{e^{-50x}}{-50}\right]_0^t + 10$$

$$= \frac{-20}{3}e^{-50t} + \frac{20}{3} + 10 = -\frac{20}{3}e^{-50t} + \frac{50}{3}\,\text{V}, \qquad t \geq 0$$

[d] $$v_2 = \frac{-10^6}{0.6}\int_0^t \frac{-20e^{-50x}}{50 \times 10^3}\,dx - 30 = -\frac{40}{3}(e^{-50t} - 1) - 30$$

$$= \frac{-40}{3}e^{-50t} - \frac{50}{3}\,\text{V}, \qquad t \geq 0$$

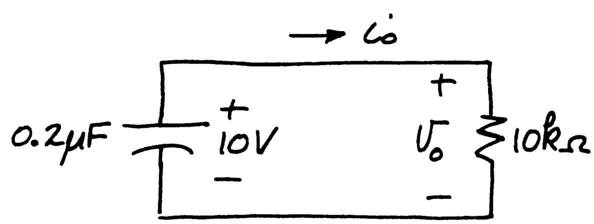
Note: $v_1 + v_2 = v_o$ as it should

[e] $$w_{\text{trapped}} = \frac{1}{2}(1.2)\left(\frac{50}{3}\right)^2 + \frac{1}{2}(0.6)\left(\frac{50}{3}\right)^2 = 250\,\mu\text{J}$$

Check: $w_{\text{trapped}} + w_{\text{diss}} = w(0)$

$250 + 80 = 330$ (ok)

P 8.29 [a]



$\tau = 2\,\text{ms}$

$1/\tau = 500$

$v_o = 10e^{-500t}\,\text{V}, \qquad t \geq 0$

$i_o = e^{-500t}\,\text{mA}, \qquad t \geq 0^+$

$$i_{24\text{k}\Omega} = e^{-500t}\left(\frac{16}{40}\right) = 0.40e^{-500t}\,\text{mA}, \qquad t \geq 0^+$$

$$p_{24\text{k}\Omega} = (0.16 \times 10^{-6}e^{-1000t})(24{,}000) = 3840 \times 10^{-6}e^{-1000t}\,\text{W}$$

$$w_{24\text{k}\Omega} = \int_0^\infty 3840 \times 10^{-6}e^{-1000t}\,dt = -3.84 \times 10^{-6}(0 - 1) = 3.84\,\mu\text{J}$$

$$w(0) = \frac{1}{2}(0.25)(1600) + \frac{1}{2}(1)(2500)\,\mu\text{J} = 1450\,\mu\text{J}$$

$$\%\ \text{diss}\,(24\,\text{k}\Omega) = \frac{384}{1450} = 0.26\%$$

[b] $p_{400\Omega} = 400 \times 10^{-6}e^{-1000t}\,\text{W}$

$$w_{400\Omega} = \int_0^\infty p_{400}\,dt = 0.40\,\mu\text{J}$$

$$\%\ \text{diss}\,(400\Omega) = \frac{40}{1450} = 0.03\%$$

$$p_{16k\Omega} = (0.36 \times 10^{-6}e^{-1000t})(16{,}000) = 5760 \times 10^{-6}e^{-1000t}\,\text{W}$$

$$w_{16k\Omega} = \int_0^\infty 5760 \times 10^{-6}e^{-1000t}\,dt = 5.76\,\mu\text{J}$$

$$\%\ \text{diss}\,(16k\Omega) = 0.40$$

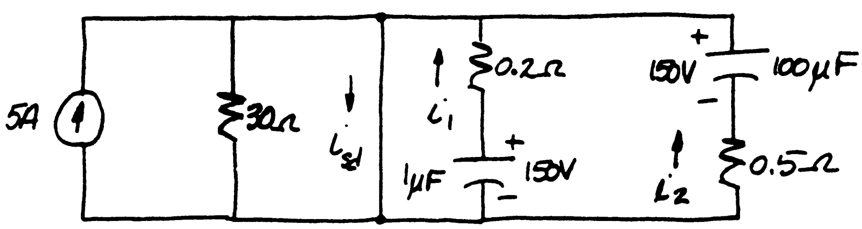
[c] $\sum w_{\text{diss}} = 3.84 + 5.76 + 0.40 = 10\,\mu\text{J}$

$$w_{\text{trapped}} = w(0) - \sum w_{\text{diss}} = 1450 - 10 = 1440\,\mu\text{J}$$

$$\%\ \text{trapped} = \frac{1440}{1450} \times 100 = 99.31\%$$

Check: $0.26 + 0.03 + 0.40 + 99.31 = 100\%$

P 8.30 [a] At $t = 0^-$ the voltage on each capacitor will be 150 V(5 × 30), positive at the upper terminal. Hence at $t \geq 0^+$ we have



$$\therefore\ i_{sd}(0^+) = 5 + \frac{150}{0.2} + \frac{150}{0.5} = 1055\,\text{A}$$

At $t = \infty$, both capacitors will have completely discharged.

$$\therefore\ i_{sd}(\infty) = 5\,\text{A}$$

[b] $i_{sd}(t) = 5 + i_1(t) + i_2(t)$

$$\tau_1 = (0.2)(1) \times 10^{-6} = 0.2\,\mu\text{s}$$

$$\tau_2 = (0.5)(100 \times 10^{-6}) = 50\,\mu\text{s}$$

$$\therefore\ i_1(t) = 750e^{-5\times10^6 t}\,\text{A}, \qquad t \geq 0^+$$

$$i_2(t) = 300e^{-20{,}000t}\,\text{A}, \qquad t \geq 0$$

$$\therefore\ i_{sd} = 5 + 750e^{-5\times10^6 t} + 300e^{-20{,}000t}\,\text{A}, \qquad t \geq 0^+$$

P 8.31 [a] $v = L\frac{di}{dt} = L(10{,}000)e^{-400t}$

$\therefore\ 10^4 L = 100$

$L = 10\,\text{mH}$

$\frac{1}{\tau} = \frac{R}{L} = 400, \qquad \therefore\ R = 4\,\Omega$

$i(\infty) = 25 = \frac{V_s}{R}, \qquad \therefore\ V_s = 25R = 100\,\text{V}$

[b] $w(\infty) = \frac{1}{2}(10 \times 10^{-3})(25)^2 = 3125\,\text{mJ}$

$w(t) = \frac{1}{2}(10 \times 10^{-3})[625 - 1250e^{-400t} + 625e^{-800t}]$

$= 3125 - 6250e^{-400t} + 3125e^{-800t}\,\text{mJ}$

$\therefore\ 3125 - 6250e^{-400t} + 3125e^{-800t} = 0.25(3125)$

$1 - 2e^{-400t} + e^{-800t} = 0.25$

$x = e^{-400t}$

$1 - 2x + x^2 = 0.25$

$x^2 - 2x + 0.75 = 0$

$x = 1 \pm \sqrt{1 - 0.75} = 1 \pm 0.5$

$x_1 = 1.5, \qquad x_2 = 0.5$

Only x_2 has physical significance, i.e., $t > 0$.

$e^{-400t} = 0.5; \qquad e^{400t} = 2$

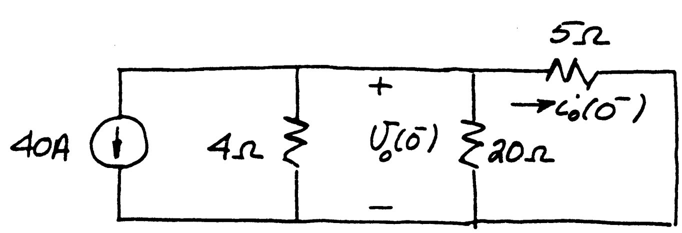
$t = 2.5 \ln 2\,\text{ms} \cong 1.73\,\text{ms}$

A better way:

$w = \frac{1}{2}Li^2 = \frac{1}{2}L(25)^2(1 - e^{-400t})^2 = w(\infty)(1 - e^{-400t})^2$

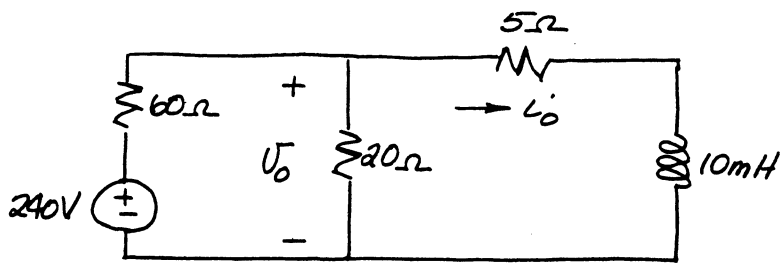
$\therefore\ (1 - e^{-400t})^2 = 0.25; \qquad 1 - e^{-400t} = 0.5; \qquad t \cong 1.73ms$

P 8.32 [a] $t < 0$:



$$\frac{v_o(0^-)}{4} + \frac{v_o(0^-)}{20} + \frac{v_o(0^-)}{5} = -40$$
$$10v_o(0^-) = -800; \qquad v_o(0^-) = -80\,\text{V}$$
$$\therefore\ i_o(0^-) = -80/5 = -16\,\text{A}$$

$t > 0$:



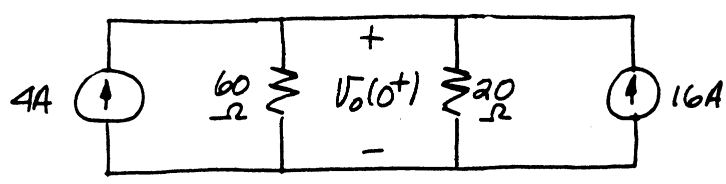
$$\frac{v_o}{20} + \frac{v_o - 240}{60} + \frac{v_o}{5} = 0, \qquad t = \infty$$
$$\therefore\ 16v_o = 240, \qquad v_o(\infty) = 15\,\text{V}$$
$$i_o(\infty) = 15/5 = 3\,\text{A}$$

Resistance seen from the terminals of the 10-mH inductor:

$$R_{\text{Th}} = 5 + \frac{(60)(20)}{80} = 20\,\Omega$$
$$\therefore\ \tau = \frac{10}{20} \times 10^{-3} = 0.5\,\text{ms} = 500\,\mu\text{s}; \qquad 1/\tau = 2000$$
$$\therefore\ i_o(t) = 3 + (-16 - 3)e^{-2000t} = 3 - 19e^{-2000t}\,\text{A}, \qquad t \geq 0$$

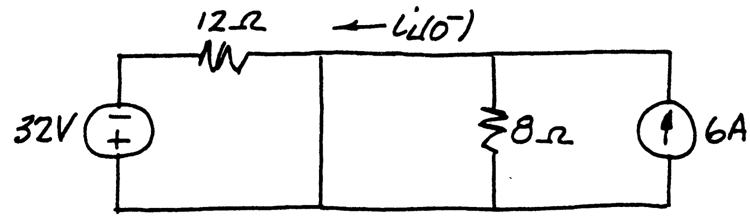
[b] $$v_o = 5i_o(t) + 10 \times 10^{-3}\frac{di_o}{dt}(t) = 15 - 95e^{-2000t} + 10 \times 10^{-3}(38{,}000)e^{-2000t}$$
$$= 15 - 95e^{-2000t} + 380e^{-2000t} = 15 + 285e^{-2000t}\,\text{V}, \qquad t \geq 0$$

Check: $v_o(0^+) = 15 + 285 = 300\,\text{V}$; at $t = 0^+$ the circuit is



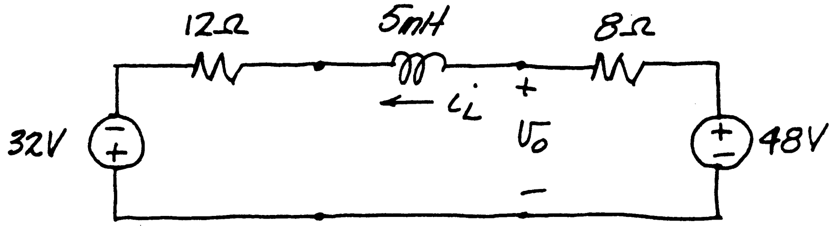
$$\frac{v_o(0^+)}{60} + \frac{v_o(0^+)}{20} = 20$$
$$4v_o(0^+) = 1200$$
$$v_o(0^+) = 300\,\text{V} \quad \text{(Checks)}$$

P 8.33 [a] $t < 0$:



$i_L(0^-) = 6A$

$t > 0$:



$$i_L(\infty) = \frac{80}{20} = 4\,A$$

$$\tau = \frac{L}{R} = \frac{5 \times 10^{-3}}{20} = 0.25 \times 10^{-3} = 250\,\mu s; \qquad \frac{1}{\tau} = 4000$$

$$i_L = 4 + (6-4)e^{-4000t}\,A = 4 + 2e^{-4000t}\,A, \qquad t \geq 0$$

$$v_o = 48 - 8i_L = 48 - 32 - 16e^{-4000t} = 16 - 16e^{-4000t}\,V, \qquad t \geq 0^+$$

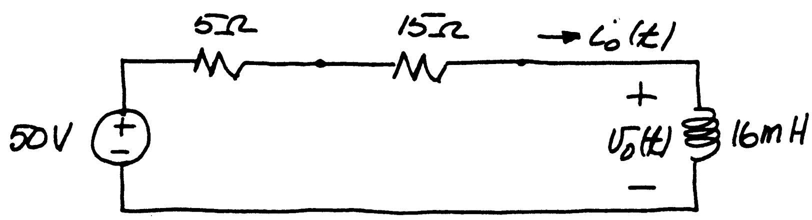
[b] $$v_L = L\frac{di_L}{dt} = (5 \times 10^{-3})(-8000e^{-4000t}) = -40e^{-4000t}\,V, \qquad t \geq 0^+$$

$\therefore\ v_L(0^+) = -40\,V$

$v_o(0^+) = 0\,V$

P 8.34 $i_o(0^-) = i_o(0^+) = 10\,A$

$t > 0$:



$$i_o(\infty) = \frac{50}{20} = 2.5\,\text{A}$$

$$\tau = \frac{16}{20} \times 10^{-3} = 0.8\,\text{ms} = 800\,\mu\text{s}; \qquad \frac{1}{\tau} = \frac{1000}{0.8} = 1250 \text{ nepers/s}$$

$$i_o = 2.5 + (10 - 2.5)e^{-1250t}$$

$$i_o(t) = 2.5 + 7.5e^{-1250t}\,\text{A}, \qquad t \geq 0$$

$$v_o = L\frac{di_o(t)}{dt} = 16 \times 10^{-3}(-9375)e^{-1250t} = -150e^{-1250t}\,\text{V}, \qquad t \geq 0$$

P 8.35 [a] $v_o(0^+) = -I_g R_2; \qquad \tau = \dfrac{L}{R_1 + R_2}$

$$v_o(\infty) = 0$$

$$v_o(t) = -I_g R_2 e^{-[(R_1+R_2)/L]t}\,\text{V}, \qquad t \geq 0$$

[b] $v_o = -12(20)e^{-(20/8)\times 10^3 t} = -240e^{-2500t}\,\text{V}, \qquad t \geq 0$

[c] $v_o(0^+) \rightarrow \infty$, and the duration of $v_o(t) \rightarrow$ zero

[d] $v_{sw} = R_2 i_o; \qquad \tau = \dfrac{L}{R_1 + R_2}$

$$i_o(0^+) = I_g; \qquad i_o(\infty) = I_g\frac{R_1}{R_1 + R_2}$$

Therefore $i_o(t) = \dfrac{I_g R_1}{R_1 + R_2} + \left[I_g - \dfrac{I_g R_1}{R_1 + R_2}\right] e^{-[(R_1+R_2)/L]t}$

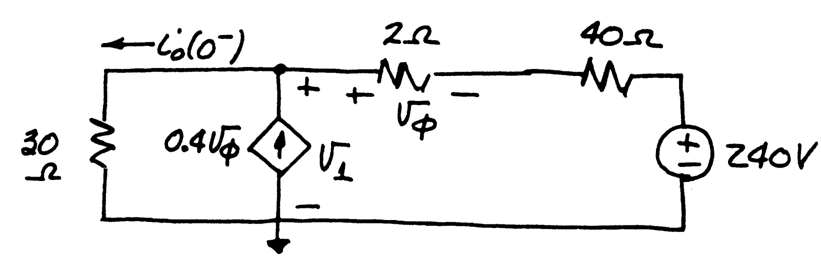
$$i_o(t) = \frac{R_1 I_g}{(R_1 + R_2)} + \frac{R_2 I_g}{(R_1 + R_2)}e^{-[(R_1+R_2)/L]t}$$

Therefore $v_{sw} = \dfrac{R_1 I_g}{(1 + R_1/R_2)} + \dfrac{R_2 I_g}{(1 + R_1/R_2)}e^{-[(R_1+R_2)/L]t}, \qquad t \geq 0$

[e] $|v_{sw}(0^+)| \rightarrow \infty; \qquad$ duration $\rightarrow 0$

P 8.36 Opening the inductive circuit causes a very large voltage to be induced across the inductor L. This voltage also appears across the switch (part [e] of Problem 8.35) causing the switch to arc over. At the same time, the large voltage across L damages the meter movement.

P 8.37 $t < 0$:



$$\frac{v_1}{30} - 0.4v_\phi + \frac{v_1 - 240}{42} = 0$$

$$v_\phi = \left(\frac{v_1 - 240}{42}\right)(2) = \frac{v_1 - 240}{21}$$

$$\frac{v_1}{30} - 0.4\left(\frac{v_1 - 240}{21}\right) + \frac{v_1 - 240}{42} = 0$$

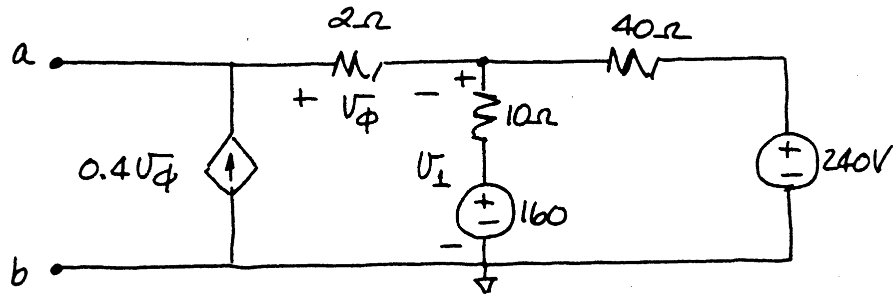
$$1.4v_1 - 0.8(v_1 - 240) + (v_1 - 240) = 0$$

$$1.4v_1 + 0.2(v_1 - 240) = 0$$

$$1.6v_1 = 48; \qquad v_1 = 30\,\text{V}$$

$$i_o(0^-) = \frac{30}{30} = 1\,\text{A} = i_o(0^+)$$

For $t < 0$: First find the Thévenin equivalent with respect to the terminals a, b.



$$-0.4v_\phi + \frac{v_{ab} - v_1}{2} = 0$$

$$v_{ab} = v_\phi + v_1, \qquad v_\phi = v_{ab} - v_1$$

$$\therefore \quad -0.8(v_{ab} - v_1) + (v_{ab} - v_1) = 0$$

$$0.2(v_{ab} - v_1) = 0; \qquad \therefore\ v_{ab} = v_1, \quad v_\phi = 0$$

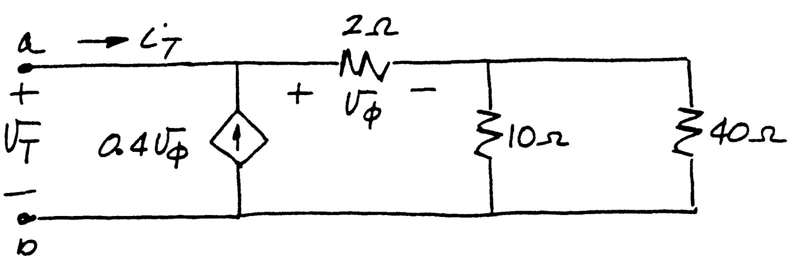
$$\therefore \quad \frac{v_1 - 160}{10} + \frac{v_1 - 240}{40} = 0$$

$$4v_1 - 640 + v_1 - 240 = 0$$

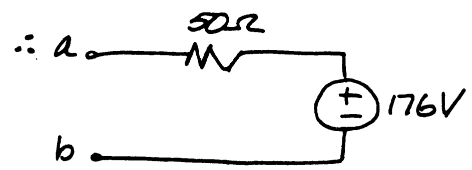
$$5v_1 = 880, \qquad v_1 = 176\,\text{V}$$

$$\therefore\ v_{ab} = 176\,\text{V}$$

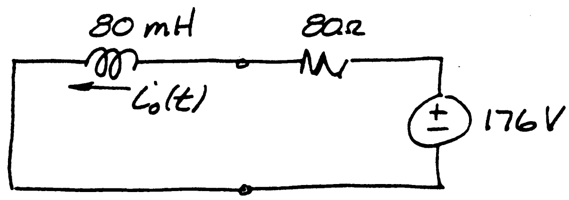
Find Thévenin resistance looking into the terminals a, b.



$$v_T = (i_T + 0.4v_\phi)(2+8) = 10i_T + 4v_\phi$$
$$v_\phi = (i_T + 0.4v_\phi)2 = 2i_T + 0.8v_\phi$$
$$0.2v_\phi = 2i_T, \qquad v_\phi = 10i_T$$
$$\therefore\ v_T = 10i_T + 40i_T = 50i_T$$
$$R_{\text{Th}} = v_T/i_T = 50\,\Omega$$

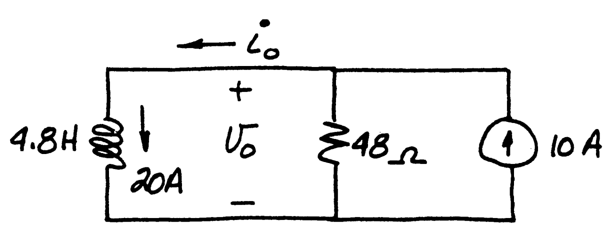


It follows that the Thévenin equivalent with respect to the terminals of the 80-mH inductor is



$$\therefore\ i_o(\infty) = \frac{176}{80} = 2.2\,\text{A}$$
$$\tau = \frac{80}{80} \times 10^{-3} = 1\,\text{ms}; \qquad \frac{1}{\tau} = 1000$$
$$\therefore\ i_o(t) = 2.2 + (1-2.2)e^{-1000t} = 2.2 - 1.2e^{-1000t}\,\text{A}, \qquad t \geq 0$$

P 8.38 [a] For $t > 0$:



$$i_o(0^-) = i_o(0^+) = 20\,\text{A}$$
$$\tau = \frac{4.8}{4.8} = 0.1\,\text{s}; \qquad \frac{1}{\tau} = 10$$

$$i_o(\infty) = 10\,\text{A}$$

$$i_o = 10 + (20 - 10)e^{-10t} = 10 + 10e^{-10t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 4.8\frac{di_o}{dt} = 4.8(-100e^{-10t}) = -480e^{-10t}\,\text{V}, \qquad t \geq 0^+$$

[b] $$i_1(t) = \frac{1}{12}\int_0^t -480e^{-10x}\,dx + i_1(0)$$

$$i_1(0) = (20)\left(\frac{8}{20}\right) = 8\,\text{A}$$

$$i_1(t) = -40\left[\frac{e^{-10x}}{-10}\bigg|_0^t\right] + 8$$

$$\therefore\; i_1(t) = 4e^{-10t} + 4\,\text{A}, \qquad t \geq 0$$

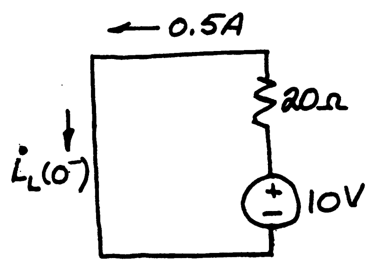
[c] $$i_2(t) = \frac{1}{8}\int_0^t -480e^{-10x}\,dx + i_2(0)$$

$$i_2(0) = 20\left(\frac{12}{20}\right) = 12\,\text{A}$$

$$i_2(t) = -60\left[\frac{e^{-10x}}{-10}\bigg|_0^t\right] + 12$$

$$\therefore\; i_2(t) = 6e^{-10t} + 6\,\text{A}, \qquad t \geq 0$$

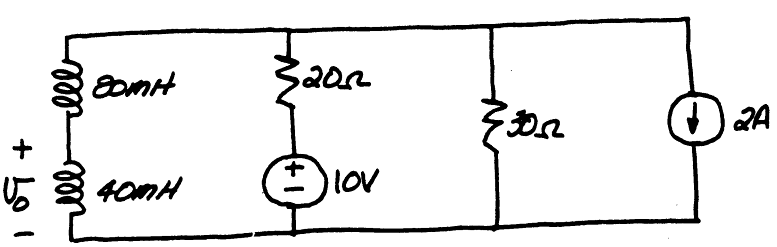
P 8.39 $t < 0$:



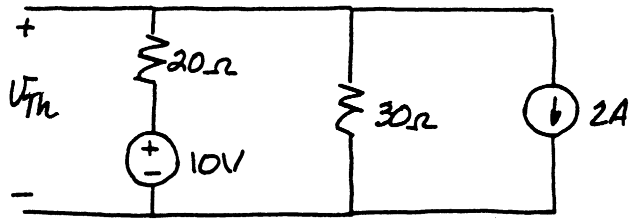
$$i_L(0^-) = 10/20 = 0.5\,\text{A}$$

$$i_L(0^+) = i_L(0^-) = 0.5\,\text{A}$$

For $t > 0$:

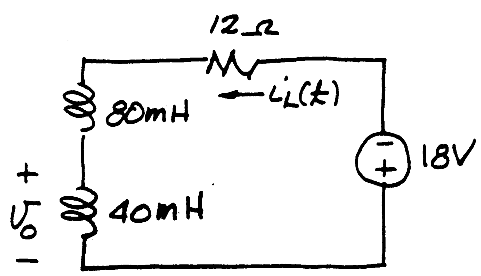


Find the Thévenin equivalent with respect to the terminals of the inductors.



$$\frac{v_{Th}-10}{20}+\frac{v_{Th}}{30}+2=0$$
$$3v_{Th}-30+2v_{Th}+120=0$$
$$5v_{Th}=-90$$
$$v_{Th}=-18\,\text{V}$$

$$R_{Th}=20//30=12\,\Omega$$
$$\tau=\frac{120\times10^{-3}}{12}=10\,\text{ms}$$
$$\frac{1}{\tau}=100$$

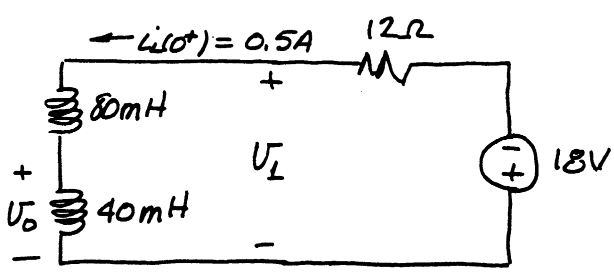


$$i_L(\infty)=\frac{-18}{12}=-1.5\,\text{A}$$

$$\therefore\ i_L(t)=-1.5+(0.5+1.5)e^{-100t}\,\text{A},\qquad t\geq0$$
$$=-1.5+2e^{-100t}\,\text{A},\qquad t\geq0$$

$$v_o=40\times10^{-3}\frac{di_L}{dt}=(40\times10^{-3})[-200e^{-100t}]=-8e^{-100t}\,\text{V},\qquad t\geq0^+$$

Alternate solution:



$v_1(0^+) = -18 - 12(0.5) = -24\,\text{V}$

$v_o = \frac{40}{120}v_1 = \frac{1}{3}v_1$

$v_1 = -24e^{-100t}\,\text{V}, \qquad t \geq 0^+$

$\therefore\ v_o = -8e^{-100t}\,\text{V}, \qquad t \geq 0^+$

P 8.40 [a] Let v be the voltage drop across the parallel branches, positive at the top node, then

$$-I_g + \frac{v}{R_g} + \frac{1}{L_1}\int_0^t v\,dx + \frac{1}{L_2}\int_0^t v\,dx = 0$$

$$\frac{v}{R_g} + \left(\frac{1}{L_1} + \frac{1}{L_2}\right)\int_0^t v\,dx = I_g$$

$$\frac{v}{R_g} + \frac{1}{L_e}\int_0^t v\,dx = I_g$$

$$\frac{1}{R_g}\frac{dv}{dt} + \frac{v}{L_e} = 0$$

$$\frac{dv}{dt} + \frac{R_g}{L_e}v = 0$$

Therefore $v = I_g R_g e^{-t/\tau}; \qquad \tau = L_e/R_g$

Therefore $i_1 = \frac{1}{L_1}\int_0^t I_g R_g e^{-x/\tau}\,dx = \frac{I_g R_g}{L_1}\frac{e^{-x/\tau}}{(-1/\tau)}\Big|_0^t = \frac{I_g L_e}{L_1}(1 - e^{-t/\tau})$

$$i_1 = \frac{I_g L_2}{L_1 + L_2}(1 - e^{-t/\tau}) \quad \text{and} \quad i_2 = \frac{I_g L_1}{L_1 + L_2}(1 - e^{-t/\tau})$$

[b] $i_1(\infty) = \frac{L_2}{L_1 + L_2}I_g; \qquad i_2(\infty) = \frac{L_1}{L_1 + L_2}I_g$

P 8.41 [a] $i = C\frac{dv}{dt} = 10^5 Ce^{-500t}$

$\therefore\ 10^5 C = 25 \times 10^{-3}$

$C = 25 \times 10^{-8} = 250 \times 10^{-9} = 250\,\text{nF}$

$\tau = RC = \frac{1}{500} = 2\,\text{ms}$

$R = \frac{1}{500C} = 800\,\text{k}\Omega$

$v(\infty) = 200\,\text{V} = I_g R$

$\therefore\ I_g = \frac{200}{800} \times 10^{-3} = 0.25\,\text{mA}$

[b] $w(\infty) = \frac{1}{2}(250 \times 10^{-9})(4 \times 10^4) = 5000\,\mu\text{J}$

$w(t) = \frac{1}{2}(250 \times 10^{-9})[4 \times 10^4 - 8 \times 10^4 e^{-500t} + 4 \times 10^4 e^{-1000t}]$

$= 5000[e^{-1000t} - 2e^{-500t} + 1]\,\mu\text{J}$

$$\therefore\ 5000[e^{-1000t} - 2e^{-500t} + 1] = 0.36(5000)$$
$$e^{-1000t} - 2e^{-500t} + 1 = 0.36$$

Let $x = e^{-500t}$ $\qquad x = 1 \pm \sqrt{1 - 0.64} = 1 \pm 0.6$

$x^2 - 2x + 0.64 = 0$ $\qquad x_1 = 1.6, \quad x_2 = 0.4$

Only x_2 has physical significance:

$e^{-500t} = 0.4$ $\qquad 500t = \ln 2.5$

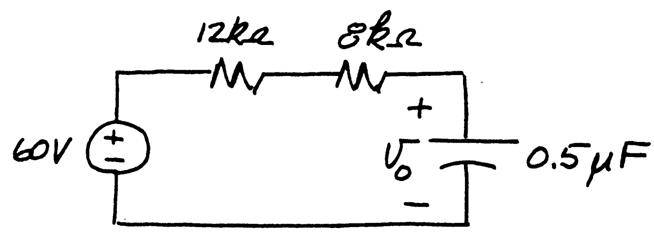
$e^{500t} = 2.5$ $\qquad t = 2\ln 2.5\,\text{ms} \cong 1.83\,\text{ms}$

A better way:

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}Cv^2(\infty)[1 - e^{-500t}]^2 = w(\infty)[1 - e^{-500t}]^2$$
$$w(\infty)(1 - e^{-500t})^2 = 0.36w(\infty)$$
$$\therefore\ 1 - e^{-500t} = 0.6; \qquad e^{-500t} = 0.4; \qquad t \cong 1.83\,\text{ms}$$

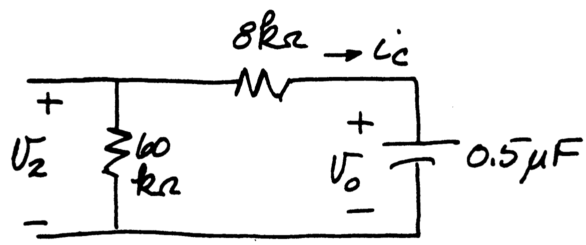
P 8.42 [a] $v_o(0^-) = v_o(0^+) = 0$

$t > 0$:



$$C\frac{dv_o}{dt} + \frac{v_o - 60}{20 \times 10^3} = 0$$
$$\frac{dv_o}{dt} + \frac{v_o - 60}{10 \times 10^{-3}} = 0$$
$$\frac{dv_o}{dt} = \frac{-v_o}{10 \times 10^{-3}} + \frac{60}{10 \times 10^{-3}}$$
$$\frac{dv_o}{v_o - 60} = \frac{-dt}{10^{-2}} = -100\,dt$$
$$\int_{v_o(0)}^{v_o(t)} \frac{dx}{x - 60} = \int_0^t -100\,dy$$
$$\ln[v_o(t) - 60] - \ln[v_o(0) - 60] = -100t$$
$$\ln\frac{[v_o(t) - 60]}{[v_o(0) - 60]} = -100t$$
$$\frac{v_o(t) - 60}{v_o(0) - 60} = e^{-100t}$$
$$v_o(t) - 60 = (v_o(0) - 60)e^{-100t}$$
$$v_o(t) = 60 - 60e^{-100t}\,\text{V}, \qquad t \geq 0^+$$

[b]



$$i_C = C\frac{dv_o}{dt} = (0.5 \times 10^{-6})(6000e^{-100t}) = 3e^{-100t}\,\text{mA}, \qquad t \geq 0^+$$

$$\therefore\ v_2 = v_o + 8000i_C = 60 - 60e^{-100t} + 24e^{-100t} = 60 - 36e^{-100t}\,\text{V}, \qquad t \geq 0^+$$

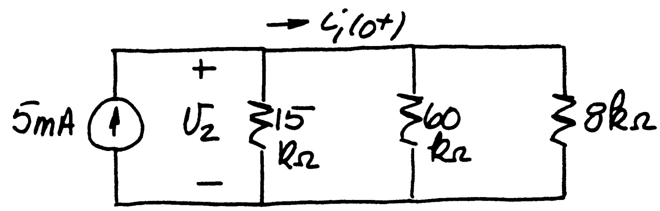
$$i_o(t) = \frac{v_2}{60} = 1 - 0.6e^{-100t}\,\text{mA}, \qquad t \geq 0^+$$

[c] $i_1(t) = i_o(t) + i_C = 1 - 0.6e^{-100t} + 3e^{-100t} = 1 + 2.4e^{-100t}\,\text{mA}, \qquad t \geq 0^+$

[d] $i_2(t) = \dfrac{v_2(t)}{15} = 4i_o(t) = 4 - 2.4e^{-100t}\,\text{mA}, \qquad t \geq 0^+$

[e] $i_1(0^+) = 3.4\,\text{mA}$

Check: at $t = 0^+$ we have:



$$\frac{1}{R_e} = \frac{1}{15} + \frac{1}{60} + \frac{1}{8} = \frac{25}{120}\,\text{mmhos}$$

$$\therefore\ R_e = \frac{120}{25} = 4.8\,\text{k}\Omega$$

$$\therefore\ v_2(0^+) = (5)(4.8) = 24\,\text{V}$$

$$\therefore\ i_1(0^+) = \frac{24}{60} + \frac{24}{8} = 0.4 + 3 = 3.4\,\text{mA} \quad \text{(ok)}$$

P 8.43 [a] $v_o(0^-) = v_o(0^+) = 120\,\text{V}; \qquad v_o(\infty) = -4(37.5) = -150\,\text{V}$

$$\tau = (37.5 + 12.5)10^3(0.04 \times 10^{-6}) = 2\,\text{ms}; \qquad \frac{1}{\tau} = 500$$

$$\therefore\ v_o(t) = -150 + 270e^{-500t}\,\text{V}, \qquad t \geq 0^+$$

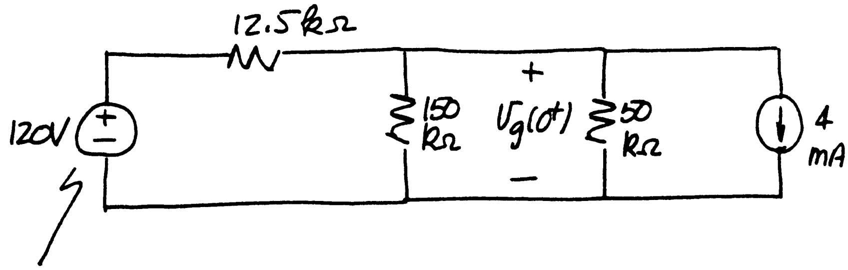
[b] $i_o(t) = -C\dfrac{dv_o}{dt} = -0.04 \times 10^{-6}(-135{,}000e^{-500t}) = 5.4e^{-500t}\,\text{mA}, \qquad t \geq 0^+$

[c] $v_g(t) = v_o - 12.5i_o(t) = -150 + 270e^{-500t} - 67.5e^{-500t}$

$$= -150 + 202.5e^{-500t}\,\text{V}, \qquad t \geq 0^+$$

[d] $v_g(0^+) = 52.50\,\text{V}$

We can check $v_g(0^+)$ as follows. At $t = 0^+$ we have:



Capacitor acts like a voltage source at $t = 0^+$.

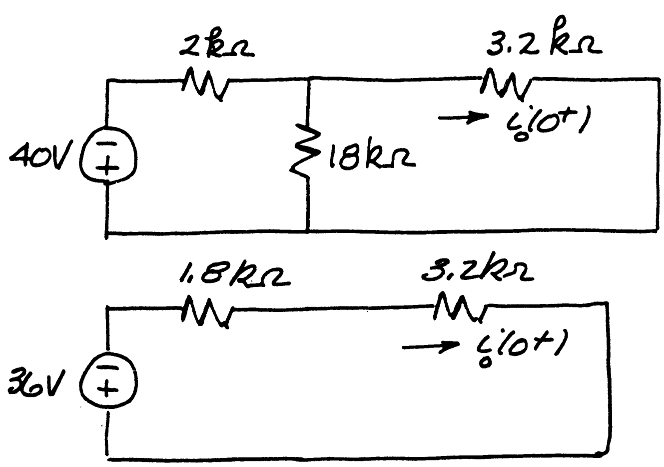
$$\frac{v_g(0^+) - 120}{12.5} + \frac{v_g(0^+)}{150} + \frac{v_g(0^+)}{50} + 4 = 0$$

$$12v_g(0^+) - 1440 + v_g(0^+) + 3v_g(0^+) + 600 = 0$$

$$16v_g(0^+) = 840$$

$$v_g(0^+) = 52.50\,\text{V} \quad \text{(ok)}$$

P 8.44 [a] $t = 0^+$:



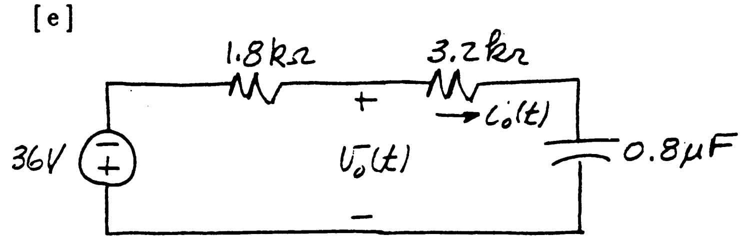
$$i_o(0^+) = \frac{-36}{5} = -7.2\,\text{mA}$$

[b] $i_o(\infty) = 0$

[c] $\tau = (5 \times 10^3)(0.8 \times 10^{-6}) = 4\,\text{ms}$

[d] $i_o(t) = 0 - 7.2e^{-250t}\,\text{mA}, \qquad t \geq 0^+$

[e]

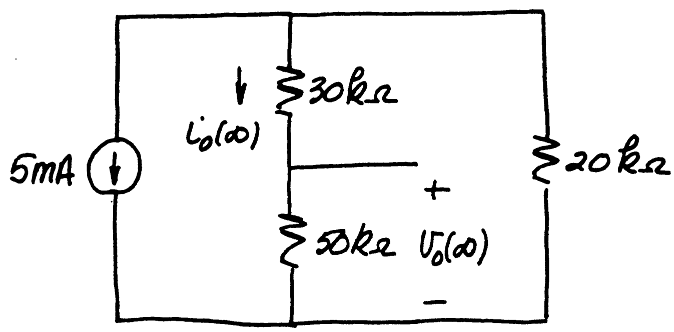


$$v_o(t) = -36 - 1.8i_o(t) = -36 - 1.8(-7.2)e^{-250t}$$
$$= -36 + 12.96e^{-250t}\,\text{V}, \qquad t \geq 0^+$$

P 8.45 $t < 0$:

$$i_o(0^-) = (10)\frac{20}{100} = 2\,\text{mA}; \qquad v_o(0^-) = (2)(50) = 100\,\text{V}$$

$t = \infty$:

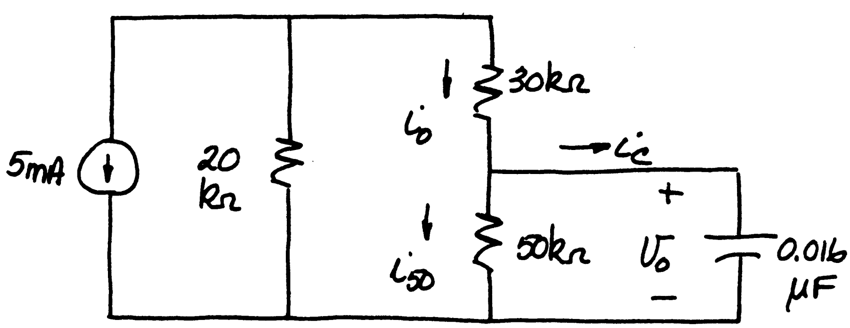


$$i_o(\infty) = -5\left(\frac{20}{100}\right) = -1\,\text{mA}; \qquad v_o(\infty) = i_o(\infty)(50) = -50\,\text{V}$$

$$R_{\text{Th}} = 50\,\text{k}\Omega // 50\,\text{k}\Omega = 25\,\text{k}\Omega; \qquad C = 0.016\,\mu\text{F}$$

$$\tau = (25)(0.016) = 0.4\,\text{ms}; \qquad \frac{1}{\tau} = 2500$$

$$\therefore\ v_o(t) = -50 + 150e^{-2500t}\,\text{V}, \qquad t \geq 0^+$$

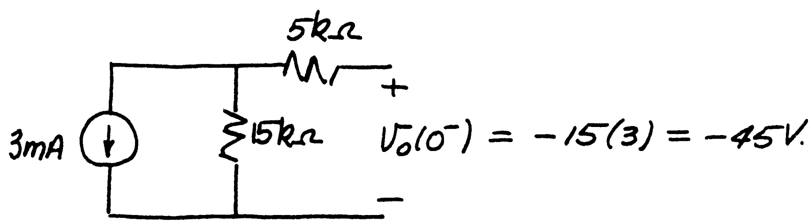


$$i_C = C\frac{dv_o}{dt} = -6e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$$

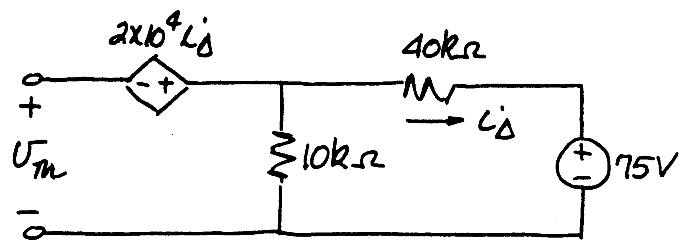
$$i_{50} = \frac{v_o}{50} = -1 + 3e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_o = i_C + i_{50} = -(1 + 3e^{-2500t})\,\text{mA}, \qquad t \geq 0^+$$

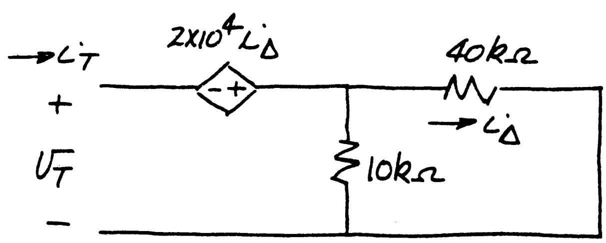
P 8.46 $t < 0$:



$t > 0$: Find the Thévenin equivalent with respect to the capacitor.



$$v_{\text{Th}} = -2 \times 10^4 i_\Delta + \frac{75}{50}(10) = -2 \times 10^4\left(\frac{-75}{50 \times 10^3}\right) + 15 = 30 + 15 = 45\,\text{V}$$

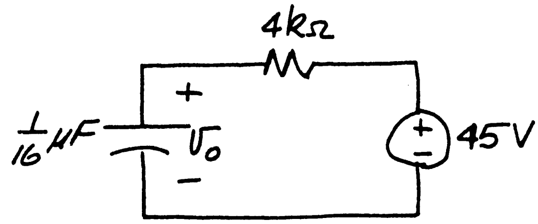


$$v_T = -2 \times 10^4 i_\Delta + 8000 i_T$$

$$i_\Delta = i_T\frac{(10)}{(50)} = 0.2 i_T$$

$$v_T = (-0.4 \times 10^4 + 8000) i_T = 4000 i_T$$

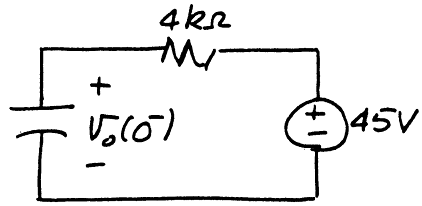
$$\therefore\ R_{\text{Th}} = \frac{v_T}{i_T} = 4\,\text{k}\Omega$$



$$\therefore\ v_o(\infty) = 45\,\text{V}; \qquad \tau = \left(\frac{1}{16}\right)(4)10^{-3} = 0.25ms; \qquad \frac{1}{\tau} = 4000$$

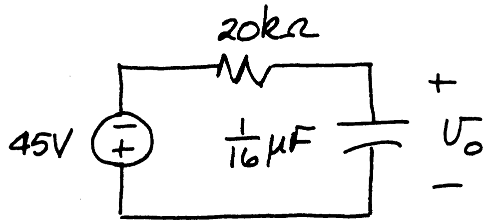
$$v_o(t) = 45 + (-45 - 45)e^{-4000t} = 45 - 90e^{-4000t}\,\text{V}, \qquad t \geq 0$$

P 8.47 $t < 0$:



$$v_o(0^-) = 45\,\text{V}$$
$$v_o(0^+) = 45\,\text{V}$$

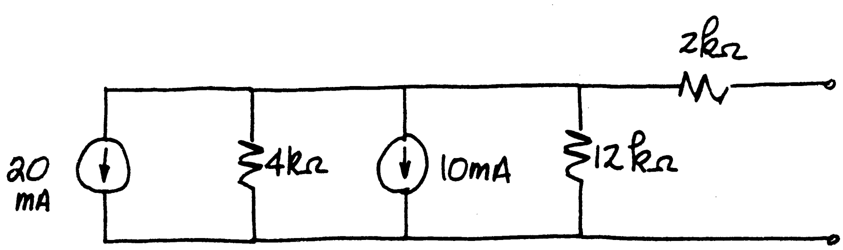
$t > 0$:

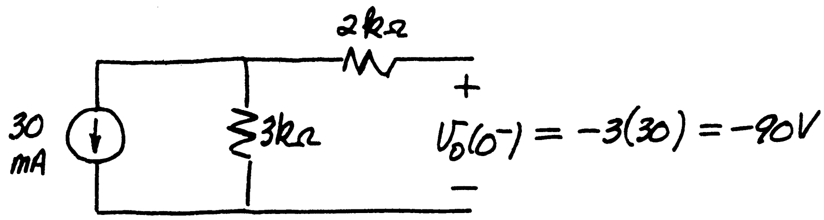


$$v_o(\infty) = -45\,\text{V}$$
$$\tau = (20)\left(\frac{1}{16}\right) \times 10^{-3}$$
$$= 1.25\,\text{ms}$$
$$\frac{1}{\tau} = 800$$

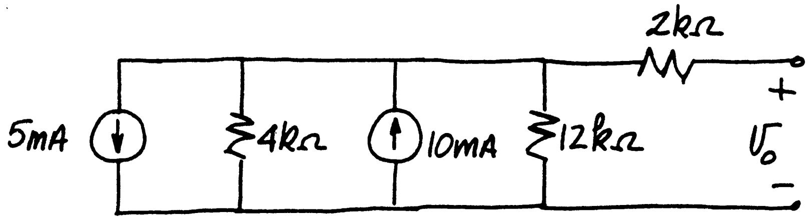
$$\therefore\ v_o = -45 + (45 + 45)e^{-800t} = -45 + 90e^{-800t}\,\text{V}, \qquad t \geq 0$$

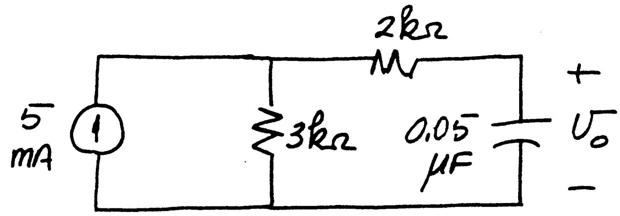
P 8.48 $t < 0$:

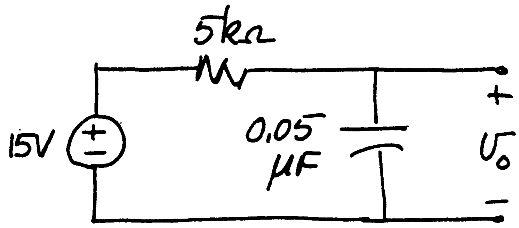




$t > 0$:







$v_o(\infty) = 15\,\text{V}; \qquad \tau = 0.25\,\text{ms}; \qquad \frac{1}{\tau} = 4000$

$v_o = 15 + (-90 - 15)e^{-4000t}$

$v_o = 15 - 105e^{-4000t}\,\text{V}, \qquad t \geq 0$

P 8.49 [a] Let i be the current in the clockwise direction around the circuit. Then

$$V_g = iR_g + \frac{1}{C_1}\int_0^t i\,dx + \frac{1}{C_2}\int_0^t i\,dx$$

$$= iR_g + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int_0^t i\,dx = iR_g + \frac{1}{C_e}\int_0^t i\,dx$$

Now differentiate the equation

$$0 = R_g\frac{di}{dt} + \frac{i}{C_e} \quad \text{or} \quad \frac{di}{dt} + \frac{1}{R_gC_e}i = 0$$

Therefore $i = \dfrac{V_g}{R_g}e^{-t/R_gC_e} = \dfrac{V_g}{R_g}e^{-t/\tau}; \qquad \tau = R_gC_e$

$$v_1(t) = \frac{1}{C_1}\int_0^t \frac{V_g}{R_g}e^{-x/\tau}\,dx = \frac{V_g}{R_gC_1}\frac{e^{-x/\tau}}{-1/\tau}\Bigg|_0^t = -\frac{V_gC_e}{C_1}(e^{-t/\tau} - 1)$$

$$v_1(t) = \frac{V_gC_2}{C_1 + C_2}(1 - e^{-t/\tau}); \qquad \tau = R_gC_e$$

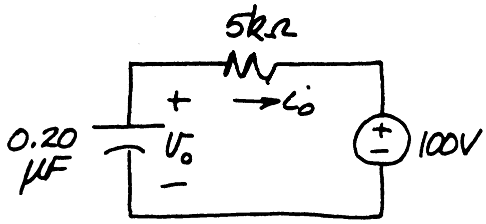
$$v_2(t) = \frac{V_gC_1}{C_1 + C_2}(1 - e^{-t/\tau}); \qquad \tau = R_gC_e$$

[b] $v_1(\infty) = \dfrac{C_2}{C_1 + C_2}V_g; \qquad v_2(\infty) = \dfrac{C_1}{C_1 + C_2}V_g$

P 8.50 [a] It follows from Problem 8.49 that

$$v_1(0) = \left(\frac{0.6}{0.9}\right)(60) = 40\,\text{V}; \qquad v_2(0) = \left(\frac{0.3}{0.9}\right)(60) = 20\,\text{V}$$

We also have $C_e = \dfrac{(0.3)(0.6)}{0.90} = 0.2\,\mu\text{F}$



$$\tau = (0.2)(5) \times 10^{-3} = 1 ms$$
$$1/\tau = 1000$$
$$v_o(0) = 60\,\text{V}$$
$$v_o(\infty) = 100\,\text{V}$$

$$v_o = 100 + (60 - 100)e^{-1000t}$$
$$v_o(t) = 100 - 40e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[b] $i_o(t) = -C_e\dfrac{dv_o}{dt} = -0.2 \times 10^{-6}(40{,}000e^{-1000t}) = -8e^{-1000t}\,\text{mA}, \qquad t \geq 0^+$

[c]
$$v_1(t) = -\frac{10^6}{0.3}\int_0^t (-8 \times 10^{-3})e^{-1000x}\,dx + 40 = \frac{8000}{0.3}\left[\frac{e^{-1000x}}{-1000}\Bigg|_0^t\right] + 40$$
$$= \frac{-80}{3}[e^{-1000t} - 1] + 40 = \frac{-80}{3}e^{-1000t} + \frac{200}{3}\,\text{V}, \qquad t \geq 0$$

[d]
$$v_2(t) = \frac{-10^6}{0.6}\int_0^t (-8 \times 10^{-3})e^{-1000x}\,dx + 20 = \frac{8000}{0.6}\left[\frac{e^{-1000x}}{-1000}\Bigg|_0^t\right] + 20$$
$$= \frac{-80}{6}[e^{-1000t} - 1] + 20 = \frac{-80}{6}e^{-1000t} + \frac{200}{6}$$
$$= \frac{-40}{3}e^{-1000t} + \frac{100}{3}\,\text{V}, \qquad t \geq 0$$

$$\text{Check:}\quad v_1 + v_2 = \frac{-120}{3}e^{-1000t} + \frac{300}{3} = -40e^{-1000t} + 100 \quad \text{(ok)}$$

[e] $v_1(\infty) = \frac{200}{3}\,\text{V}; \qquad v_2(\infty) = \frac{100}{3}\,\text{V}$

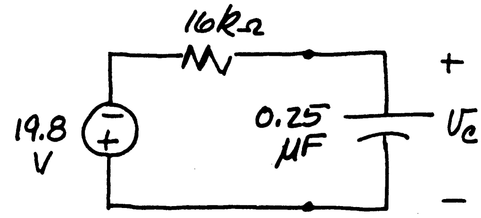
$$w(\infty) = \frac{1}{2}(0.3\times 10^{-6})\left(\frac{4\times 10^4}{9}\right) + \frac{1}{2}(0.6\times 10^{-6})\left(\frac{10^4}{9}\right)$$

$$= \frac{(2\times 10^4)(10^{-6})}{30} + \frac{10^4(10^{-6})}{30} = \frac{10^{-2}}{30}(3) = 10^{-3} = 1\,\text{mJ}$$

P 8.51 $i_b = \frac{120(33)}{80} = 49.5\,\mu\text{A}$

$v_{\text{Th}} = -25i_b(16\times 10^3) - 400\times 10^3 i_b = -19{,}800\times 10^{-3} = -19.8\,\text{V}$

$R_{\text{Th}} = 16\,\text{k}\Omega$



$\tau = (16)(0.25)\times 10^{-3} = 4\,\text{ms}$

$1/\tau = 250$

$v_C(0) = 0$

$v_C(\infty) = -19.8\,\text{V}$

$v_C = -19.8 + 19.8e^{-250t}\,\text{V}, 0 \le t \le \infty$

$w = \frac{1}{2}Cv_C^2 = 0.125\times 10^{-6}[392.04 - 784.08e^{-250t} + 392.04e^{-500t}]\,\text{J}$

$w(\infty) = 0.125\times 10^{-6}(392.04)$

$\therefore\ 0.125\times 10^{-6}v_C^2 = 0.36[0.125\times 10^{-6}]392.04$

$\therefore\ 392.04 - 784.08e^{-250t} + 392.04e^{-500t} = 0.36(392.04)$

$\therefore\ 1 - 2e^{-250t} + e^{-500t} = 0.36$

$\therefore\ e^{-500t} - 2e^{-250t} + 0.64 = 0$

$x^2 - 2x + 0.64 = 0, \qquad x = e^{-250t}$

$x = 1 \pm \sqrt{1 - 0.64} = 1 \pm 0.6$

$x_1 = 1.6, \qquad x_2 = 0.4$

Only x_2 has physical significance.

$0.4 = e^{-250t}; \qquad e^{250t} = 2.5$

$250t = \ln 2.5; \qquad t = 4\ln 2.5\,\text{ms} \cong 3.67\,\text{ms}$

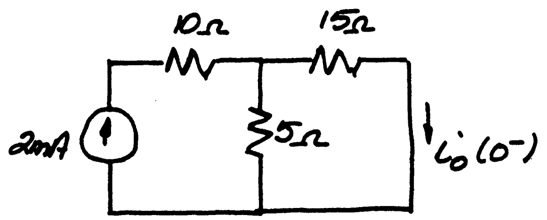
A better way:

$w(t) = w(\infty)[1 - e^{-250t}]^2$

$\therefore\ (1 - e^{-250t})^2 = 0.36; \qquad 1 - e^{-250t} = 0.6$

$t \approx 3.67\,\text{ms}$

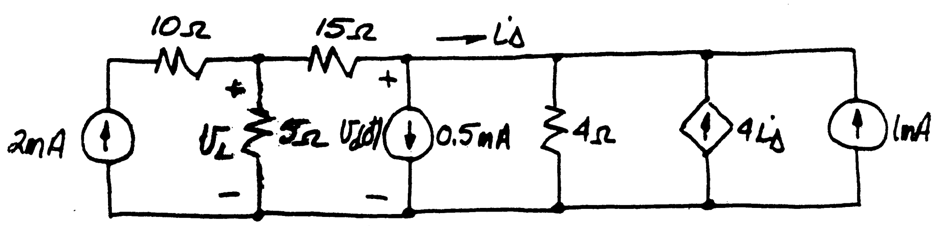
P 8.52 $t < 0$:



$$i_o(0^-) = 2\left(\frac{5}{20}\right) = 0.5\,\text{mA}$$

$$i_o(0^+) = i_o(0^-) = 0.5\,\text{mA}$$

$t = 0^+$:



$$0.5 \times 10^{-3} + \frac{v_o(0^+) - v_1}{15} + \frac{v_o(0^+)}{4} - 4i_\Delta - 10^{-3} = 0$$

or $\quad 19v_o(0^+) - 4v_1 - 240i_\Delta = 30 \times 10^{-3}$

$$\frac{v_1}{5} + \frac{v_1 - v_o(0^+)}{15} - 2 \times 10^{-3} = 0$$

or $\quad 8v_1 - 2v_o(0^+) = 60 \times 10^{-3}; \qquad \therefore\ v_1 = 0.25v_o(0^+) + 7.5 \times 10^{-3}$

$$i_\Delta = \frac{v_1 - v_o(0^+)}{15} - 0.5 \times 10^{-3}$$

$\therefore\ 240i_\Delta = 16v_1 - 16v_o(0^+) - 120 \times 10^{-3}$

$\therefore\ 19v_o(0^+) - 4v_1 - 16v_1 + 16v_o(0^+) + 120 \times 10^{-3} = 30 \times 10^{-3}$

$\therefore\ 35v_o(0^+) - 20v_1 = -90 \times 10^{-3}$

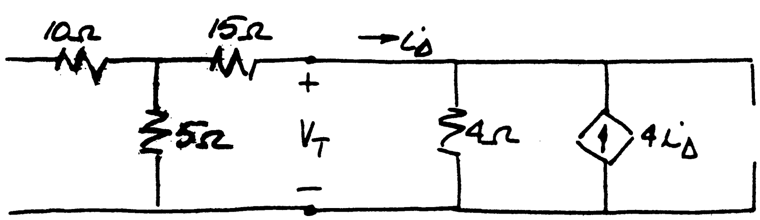
$35v_o(0^+) - 5v_o(0^+) - 150 \times 10^{-3} = -90 \times 10^{-3}$

$30v_o(0^+) = 60 \times 10^{-3}$

$v_o(0^+) = 2\,\text{mV}$

We know $\quad v_o(\infty) = 0\,\text{V}$

To find the time constant find the Thévenin resistance with respect to the terminals of the inductor.



$$i_T = \frac{V_T}{20} + \frac{V_T}{4} - 4i_\Delta$$

$$i_\Delta = \frac{V_T}{4} - 4i_\Delta$$

$$\therefore\ 5i_\Delta = \frac{V_T}{4}; \qquad i_\Delta = \frac{V_T}{20}; \qquad 4i_\Delta = \frac{V_T}{5}$$

$$\therefore\ i_T = \frac{V_T}{20} + \frac{V_T}{4} - \frac{V_T}{5} = \left(\frac{1}{20} + \frac{5}{20} - \frac{4}{20}\right) V_T$$

$$\frac{i_T}{V_T} = \frac{1}{10}$$

$$\therefore\ R_{\text{Th}} = \frac{V_T}{i_T} = 10\,\Omega$$

$$\tau = \frac{L}{R_{\text{Th}}} = 0.2\,\text{ms}; \qquad \frac{1}{\tau} = 5000$$

$$\therefore\ v_o(t) = 0 + (2 - 0)e^{-5000t}\,\text{mV} = 2e^{-5000t}\,\text{mV}, \qquad t \geq 0^+$$

P 8.53 [a] $i(0^+) = \left[\frac{240}{(1.6 + 6.4)}\right]\left(\frac{8}{40}\right) = 6\,\text{A}$

[b] $0 \leq t \leq 800\,\mu\text{s}$

$$\tau = \frac{L}{R} = \frac{20 \times 10^{-3}}{32 + 4.8} = \frac{20 \times 10^{-3}}{36.8}; \qquad \frac{1}{\tau} = 1840$$

$$i = 6e^{-1840t}\,\text{A}, \qquad 0 \leq t \leq 800\,\mu\text{s}$$

$$i(300\,\mu\text{s}) = 6e^{-0.552} \cong 3.45\,\text{A}$$

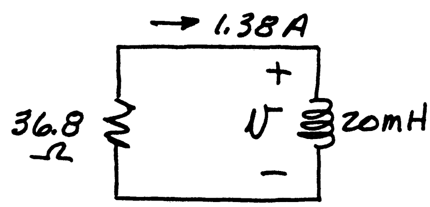
[c] $i(800\,\mu\text{s}) = 6e^{-1.472} \approx 1.38\,\text{A}, \qquad 800\,\mu\text{s} \leq t \leq \infty$

$$i = 1.38e^{-(t-0.0008)/\tau}; \qquad \tau = \frac{L}{R} = \frac{20 \times 10^{-3}}{32}; \qquad \frac{1}{\tau} = 1600$$

$$i = 1.38e^{-1600(t-0.0008)}$$

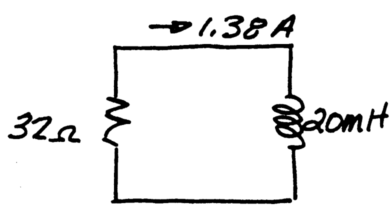
$$i(1\,\text{ms}) = 1.38e^{-1600(2\times10^{-4})} = 1.38e^{-0.32} \cong 1.0\,\text{A}$$

[d] $t = 800^-\,\mu\text{s}$



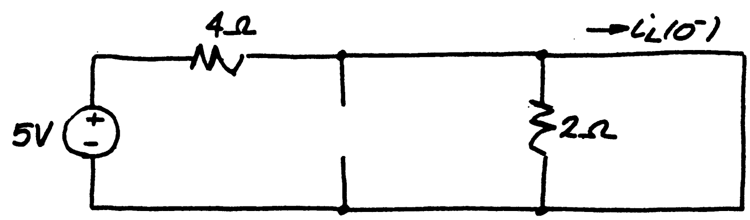
$$v(800^-\,\mu\text{s}) = -36.8(1.38) = -50.67\,\text{V}$$

[e] $t = 800^+\,\mu\text{s}$



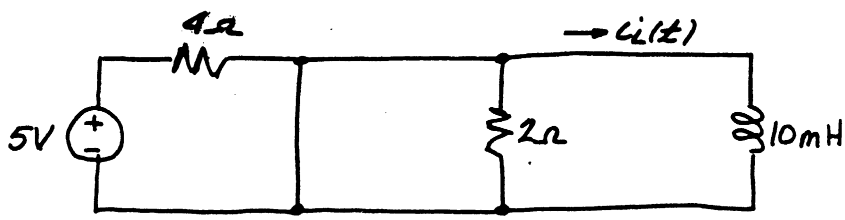
$$v(800^+\,\mu\text{s}) = -32(1.38) = -44.06\,\text{V}$$

P 8.54 For $t < 0$:



$$i_L(0^-) = \frac{5}{4} = 1.25\,\text{A}; \qquad \therefore\ i_L(0^+) = 1.25\,\text{A}$$

$0 \le t \le 1\text{s}$:

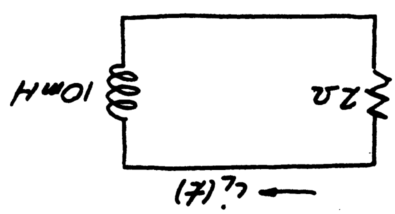


In this interval the voltage across the inductor is zero, hence $i_L(t)$ stays at 1.25 A. Also note the Thévenin resistance seen from the terminals of the inductor is zero, hence the time constant is infinite and the exponential function is unity, that is

$$e^{-t/\infty} = 1 \quad \text{for all finite values of } t.$$

Thus $i_L = 1.25e^0 = 1.25\,\text{A}, \qquad 0 \le t \le 1\text{s}$

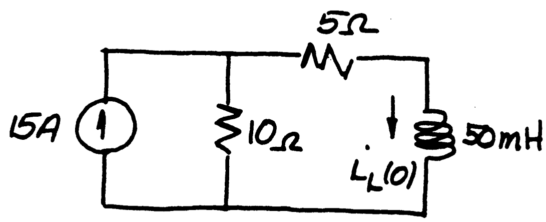
For $1 \le t \le \infty$:



$\tau = L/R = 5\,\text{ms}$
$1/\tau = 200$

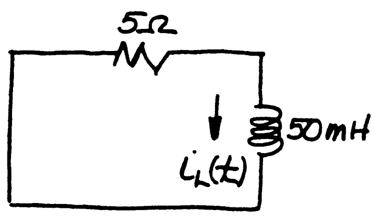
$$\therefore\ i_L(t) = 1.25e^{-200(t-1)}\,\text{A}, \qquad 1 \le t \le \infty$$

P 8.55 $t < 0$:



$$i_L(0) = \frac{15(10)}{15} = 10\,\text{A}$$

$0 \leq t \leq 10\,\text{ms}$:



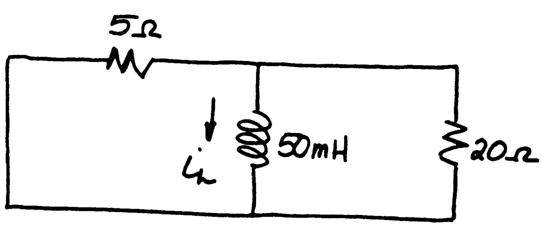
$$\tau = \frac{50}{5} \times 10^{-3} = 10\,\text{ms}$$

$1/\tau = 100$

$i_L(t) = 10e^{-100t}\,\text{A}, \qquad 0 \leq t \leq 10\,\text{ms}$

$i_L(10\,\text{ms}) = 10e^{-1}\,\text{A}$

$10\,\text{ms} \leq t \leq 20\,\text{ms}$:



$$R_e = \frac{(20)(5)}{25} = 4\,\Omega$$

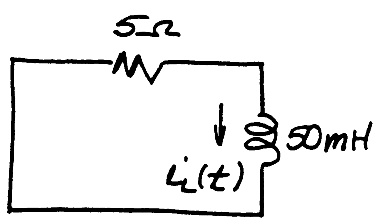
$$\tau = \frac{50}{4} = 12.5\,\text{ms}$$

$1/\tau = 80$

$i_L(t) = [10e^{-1}]e^{-80(t-0.01)}\,\text{A}, \qquad 10\,\text{ms} \leq t \leq 20\,\text{ms}$

$i_L(20\,\text{ms}) = (10e^{-1})(e^{-80(0.01)}) = 10e^{-1.8}$

$20\,\text{ms} \leq t \leq \infty$:



$\tau = 10\,\text{ms}$

$1/\tau = 100$

$i_L(t) = (10e^{-1.8})e^{-100(t-20\times10^{-3})}$

$i_L(25\,\text{ms}) = (10e^{-1.8})e^{-0.5} = 10e^{-2.3}\,\text{A}$

$v_o = -5i_L = -50e^{-2.3} = -5.01\,\text{V}$

P 8.56 $w(0) = \frac{1}{2}(50 \times 10^{-3})(10)^2 = 2.5\,\text{J}$

$0.04w(0) = 0.10\,\text{J}$

$\therefore\ \frac{1}{2}50 \times 10^{-3}i_L^2 = 0.10; \qquad i_L^2 = 4; \qquad i_L = 2\,\text{A}$

From the solution to Problem 8.55 we note t must lie between 10 and 20 ms, i.e.,

$i_L(10\,\text{ms}) = 10e^{-1} \cong 3.68\,\text{A}$

$i_L(20\,\text{ms}) = 10e^{-1.8} \cong 1.65\,\text{A}$

$\therefore\ (10e^{-1})e^{-80(t-0.01)} = 2$

$e^{-80(t-0.01)} = 0.2e^1$

$80(t-0.01) = \ln[0.2e^1]^{-1}$

$t = \frac{1}{80}\ln[(0.2e^1)]^{-1} + 0.01$

$t = 17.62\,\text{ms}$

Note $[0.2e^1]^{-1} = 5e^{-1}$

P 8.57 $0 \le t \le 5\,\text{s}$:

$v_o(0^-) = v_o(0^+) = -5\,\text{V}$

Since the capacitor does not discharge while the switch is in position (b), v_o is constant at $-5\,\text{V}$:

$v_o = -5\,\text{V}, \qquad 0 \le t \le 5\,\text{s}$

$5\,\text{s} \le t \le \infty$:

$\tau = RC = 100 \times 10^3 \times 100 \times 10^{-6} = 10\,\text{s}; \qquad 1/\tau = 0.1$

$\therefore\ v_o = -5e^{-0.1(t-5)}\,\text{V}, \qquad 5\,\text{s} \le t \le \infty$

P 8.58 $0 \le t \le 3\,\mu\text{s}$:

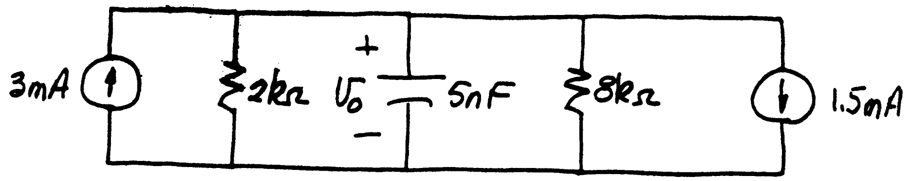
$v_o(0) = 0 \qquad \tau = (2000)(5 \times 10^{-9}) = 10\,\mu\text{s}$

$v_o(\infty) = 6\,\text{V} \qquad 1/\tau = 100{,}000$

$v_o = 6 - 6e^{-100{,}000t}\,\text{V}, \qquad 0 \le t \le 3\,\mu\text{s}$

$v_o(3\,\mu\text{s}) = 6 - 6e^{-0.3} = 1.56\,\text{V}$

$3\,\mu s \le t \le \infty$:



$2\,k\Omega // 8\,k\Omega = 1.6\,k\Omega$

$v_o(\infty) = (1.5)(1.6) = 2.40\,V$

$\tau = (1.6)(5) \times 10^{-6} = 8\,\mu s; \qquad 1/\tau = 125{,}000$

$\therefore\ v_o = 2.40 + (1.56 - 2.40)e^{-125{,}000(t-3\times10^{-6})}$

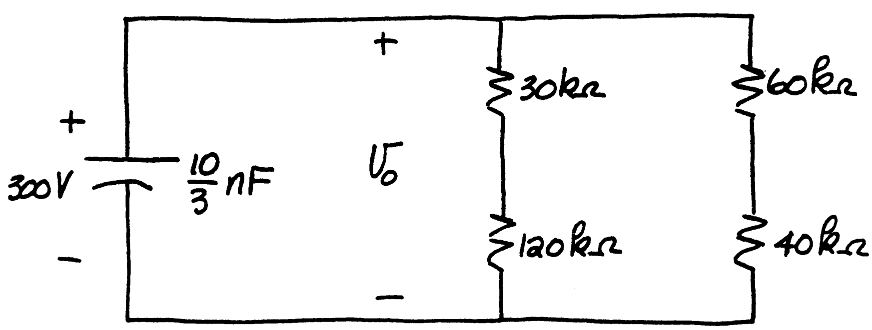
$v_o = 2.4 - 0.84e^{-125{,}000(t-3\times10^{-6})}\,V, \qquad 3\,\mu s \le t \le \infty$

Summary:

$v_o = 6 - 6e^{-100{,}000t}\,V, \qquad 0 \le t \le 3\,\mu s$

$v_o = 2.4 - 0.84e^{-125{,}000(t-3\times10^{-6})}\,V, \qquad 3\,\mu s \le t \le \infty$

P 8.59 $0 \le t \le 200\,\mu s$:



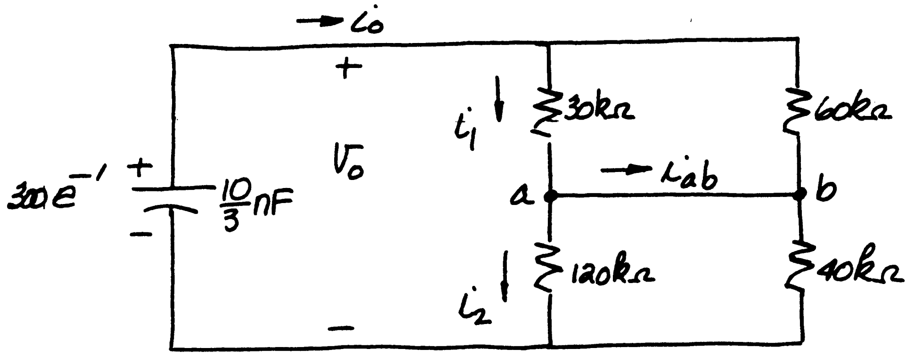
$R = 150\,k\Omega // 100\,k\Omega = 60\,k\Omega$

$\tau = (60{,}000)\left(\frac{10}{3}\right) \times 10^{-9} = 2 \times 10^{-4}\,s; \qquad \frac{1}{\tau} = 5000$

$v_o = 300e^{-5000t}\,V, \qquad 0 \le t \le 200\,\mu s$

$v_o(200) = 300e^{-5000(200)10^{-6}} = 300e^{-1}$

$200\,\mu s \leq t \leq \infty$:



$R = 30\,k\Omega//60\,k\Omega + 120\,k\Omega//40\,k\Omega = 20\,k\Omega + 30\,k\Omega = 50\,k\Omega$

$\tau = \left(\frac{10}{3} \times 10^{-9}\right)(50{,}000) = \frac{5}{3} \times 10^{-4}\,\text{s}; \qquad \frac{1}{\tau} = 6000$

$v_o = (300e^{-1})e^{-6000(t-200\times10^{-6})}\,\text{V}, \qquad 200\,\mu s \leq t \leq \infty$

$v_o(300\,\mu s) = 300e^{-1}e^{-0.6} = (300e^{-1.6})$

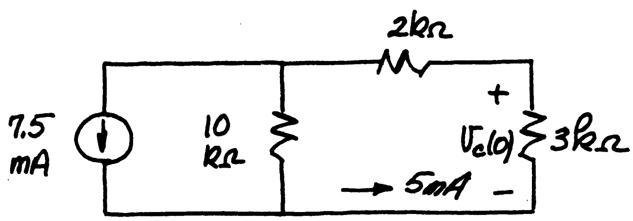
$i_o(300\,\mu s) = \frac{300e^{-1.6}}{50} \times 10^{-3} = 6e^{-1.6}\,\text{mA}$

$i_1 = \frac{60}{90}i_o = \frac{2}{3}i_o; \qquad i_2 = \frac{40}{160}i_o = \frac{1}{4}i_o$

$i_{ab} = i_1 - i_2 = \left(\frac{2}{3} - \frac{1}{4}\right)i_o = \frac{5}{12}i_o = \left(\frac{5}{12}\right)(6e^{-1.6})\,\text{mA} = 2.5e^{-1.6}\,\text{mA}$

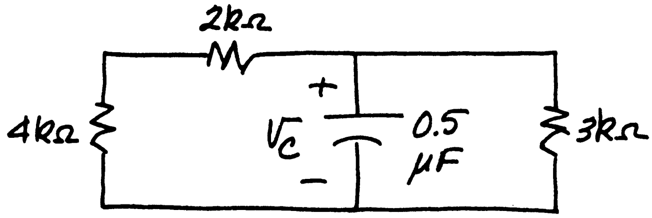
$i_{ab} = 504.74\,\mu\text{A}$ (left-to-right in the switch)

P 8.60 $t < 0$:



$v_C(0) = -5(3) = -15\,\text{V}$

$0 \le t \le 800\,\mu s$:



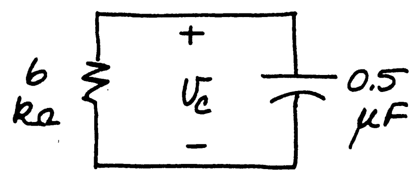
$R_e = 3\,k\Omega // 6\,k\Omega = 2\,k\Omega$

$\tau = 2000(0.5 \times 10^{-6}) = 10^{-3} = 1\,\text{ms}; \qquad 1/\tau = 1000$

$v_C = -15e^{-1000t}\,\text{V}, \qquad 0 \le t \le 800\,\mu s$

$v_C(800\,\mu s) = -15e^{-0.8}\,\text{V}$

$800\,\mu s \le t \le 1100\,\mu s$:



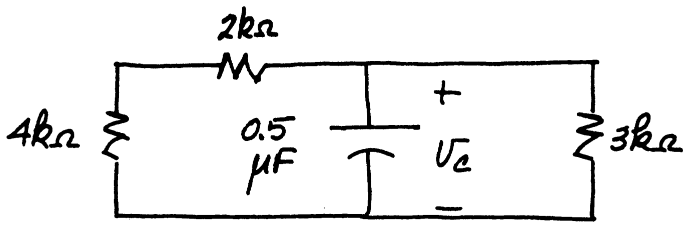
$\tau = 3000 \times 10^{-6} = 3\,\text{ms}$

$1/\tau = 1000/3$

$v_C = (-15e^{-0.8})e^{-(1000/3)(t - 800\times10^{-6})}\,\text{V}, \qquad 800\,\mu s \le t \le 1100\,\mu s$

$v_C(1100\,\mu s) = (-15e^{-0.8})(e^{(-1000/3)(300\times10^{-6})}) = -15e^{-0.8}e^{-0.1} = (-15e^{-0.90})\,\text{V}$

$1100\,\mu s \le t \le \infty$:



$\tau = 1\,\text{ms}; \qquad 1/\tau = 1000$

$v_C = (-15e^{-0.9})e^{-1000(t - 1100\mu s)}; \qquad v_C(1500\,\mu s) = -15e^{-0.9}e^{-0.4} = -15e^{-1.3}\,\text{V}$

$v_o = \frac{2}{3}v_C; \qquad v_o(1500\,\mu s) = -10e^{-1.3} \cong -2.73\,\text{V}$

P 8.61 $0 \le t \le 800\,\mu$s:

$$p_{3k} = \frac{v_C^2}{3000} = \frac{225e^{-2000t}}{3000} = 75 \times 10^{-3}e^{-2000t}\,\text{W}$$

$$w_{3k} = 75 \times 10^{-3}\int_0^{800\times10^{-6}} e^{-2000t}\,dt = (75 \times 10^{-3})\frac{e^{-2000t}}{-2000}\Bigg|_0^{800\times10^{-6}}$$

$$= 37.5 \times 10^{-6}(1 - e^{-1.6})\,\text{J} = 37.5(1 - e^{-1.6})\,\mu\text{J}$$

$800\,\mu\text{s} \le t \le 1100\,\mu$s:

$w_{3k} = 0$

$1100\,\mu\text{s} \le t \le \infty$:

$$p = \frac{225e^{-1.8}}{3000}e^{-2000(t-1100\times10^{-6})}$$

$$w_{3k} = 75 \times e^{-1.8} \times 10^{-3}\int_{1100\times10^{-6}}^{\infty} e^{-2000(t-1100\times10^{-6})}\,dt$$

$$= 37.5e^{-1.8} \times 10^{-6} = 37.5e^{-1.8}\,\mu\text{J}$$

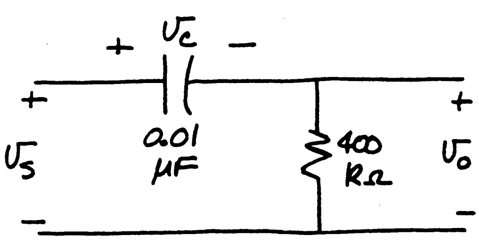
$$w_{3k} = w_{3k}(0 - 800\,\mu\text{s}) + w_{3k}(1100\,\mu\text{s} - \infty)$$

$$= 37.5(1 - e^{-1.6}) + 37.5e^{-1.8} = 37.5(1 + e^{-1.8} - e^{-1.6})\,\mu\text{J}$$

$$w(0) = \frac{1}{2}(0.5)(15)^2 = 56.25\,\mu\text{J}$$

$$\%\text{ dissipated} = \frac{37.5}{56.25}(1 + e^{-1.8} - e^{-1.6})(100) = 64.23\%$$

P 8.62 [a]



$v_s = v_C + v_o; \qquad \therefore\ v_o = v_s - v_C$

$0 \le t \le 1$ ms: :

$v_C(0) = 0\,\text{V}; \qquad v_C(\infty) = 50\,\text{V}$

$\tau = (0.01)(400) \times 10^{-3} = 4\,\text{ms}; \qquad 1/\tau = 250$

$v_C(t) = 50 - 50e^{-250t}\,\text{V}, \qquad 0 \le t \le 1\,\text{ms}$

$\therefore\ v_o(t) = 50e^{-250t}\,\text{V}, \qquad 0 \le t \le 1\,\text{ms}$

$1\,\text{ms} \le t \le \infty; \qquad v_s = 0\,\text{V}$

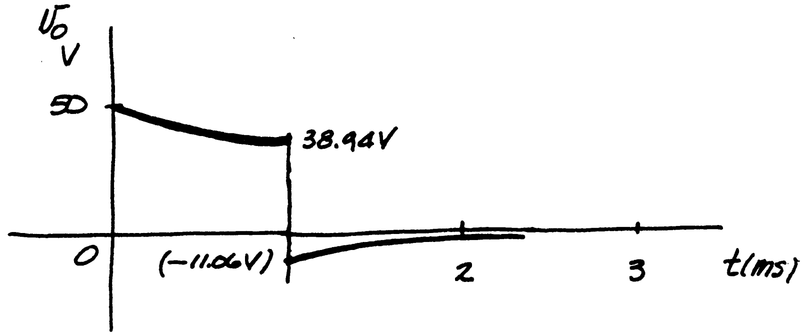
$v_C(1\,\text{ms}) = 50 - 50e^{-0.25} \cong 11.06\,\text{V}$

$v_C(\infty) = 0; \qquad \tau = 4\,\text{ms}; \qquad 1/\tau = 250$

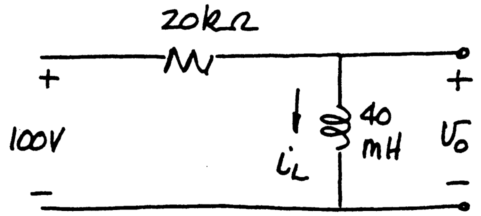
$v_C = 0 + (11.06 - 0)e^{-250(t-0.001)} = 11.06e^{-250(t-0.001)}\,\text{V}, \qquad 1\,\text{ms} \le t \le \infty$

$\therefore\ v_o = 0 - 11.06e^{-250(t-0.001)} = -11.06e^{-250(t-0.001)}\,\text{V}, \qquad 1\,\text{ms} \le t \le \infty$

[b] $v_o(1\text{ ms}) = 50e^{-0.25} \cong 38.94\text{ V}$



P 8.63 [a] $0 \le t \le 2\,\mu s$:



$i_L(0) = 0$
$i_L(\infty) = 5\text{ mA}$
$\tau = (40/20) \times 10^{-6}$
$= 2\,\mu s$

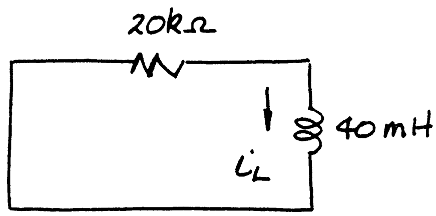
$\therefore\ i_L = 5 - 5e^{-500{,}000t}\text{ mA}, \qquad 0 \le t \le 2\,\mu s$

$v_o = 40 \times 10^{-3}[+2{,}500{,}000 \times 10^{-3}e^{-500{,}000t}] = (40)(2.5)e^{-500{,}000t}$

$= 100e^{-500{,}000t}\text{ V}, \qquad 0^+ \le t < 2\,\mu s$

$2\,\mu s \le t \le \infty$

$i_L(2\,\mu s) = 5 - 5e^{-1} \cong 3.16\text{ mA}$



$i_L(\infty) = 0; \qquad \tau = 2\,\mu s; \qquad 1/\tau = 500{,}000$

$i_L = 0 + (3.16 - 0)e^{-500{,}000(t-2\times10^{-6})}\text{ mA} = 3.16e^{-500{,}000(t-2\times10^{-6})}\text{ mA},$

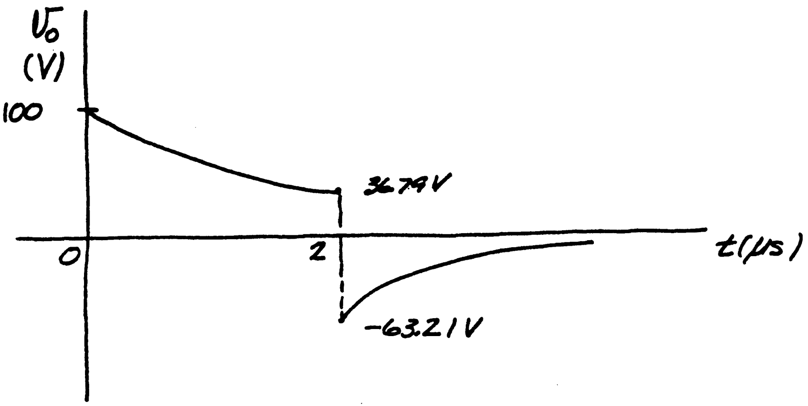
$2\,\mu s \le t \le \infty$

$v_o = L\frac{di_L}{dt} = 40 \times 10^{-3}(3.16 \times 10^{-3})(-5 \times 10^5)e^{-500{,}000(t-2\,\mu s)}$

$= (-5)(4)(3.16)e^{-500{,}000(t-2\times10^{-6})}\text{ V}$

$= -63.21e^{-500{,}000(t-2\times10^{-6})}\text{ V}, \qquad 2\,\mu s < t \le \infty$

[b]



P 8.64 [a] $t < 0; \qquad v_o = 0; \qquad 0 < t < 75\,\mu s$:

$$i_o(0) = 0 \qquad \tau = 125\,\mu s$$
$$i_o(\infty) = 25\,\text{mA} \qquad 1/\tau = 8000$$
$$i_o = 25 - 25e^{-8000t}\,\text{mA}, \qquad 0 \le t \le 75\,\mu s$$
$$v_o = L\frac{di_o}{dt} = 0.25 \times (+200{,}000 \times 10^{-3})e^{-8000t}$$
$$= 50e^{-8000t}\,\text{V}, \qquad 0 < t < 75\,\mu s$$

$75\,\mu s < t < \infty$:

$$i_o(75\,\mu s) = 25(1 - e^{-0.6}) \cong 11.28\,\text{mA}$$
$$i_o(\infty) = 0$$
$$\therefore\ i_o = 11.28e^{-8000(t-75\times10^{-6})}\,\text{mA}, \qquad 75\,\mu s \le t \le \infty$$
$$v_o = 0.25(-90.24)e^{-8000(t-75\times10^{-6})}$$
$$= -22.56e^{-8000(t-75\times10^{-6})}\,\text{V}, \qquad 75\,\mu s < t < \infty$$

[b] $v_o(75^-) = 50e^{-0.6} = 27.44\,\text{V}$

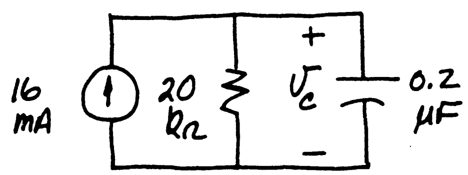
$v_o(75^+) = -22.56\,\text{V}$

[c] $i_o(75^-) = 11.28\,\text{mA}$

$i_o(75^+) = 11.28\,\text{mA}$

P 8.65 [a] $t < 0; \qquad i_o(t) = 0; \qquad v_o(t) = 0$

$0 \le t \le 2\,\text{ms}$:



$$v_C(0^-) = 0$$
$$v_C(\infty) = 320\,\text{V}$$
$$\tau = 4\,\text{ms}$$
$$1/\tau = 250$$

$$\therefore\ v_C = 320 - 320e^{-250t}\,\mathrm{V}, \qquad 0 \le t \le 2\,\mathrm{ms}$$

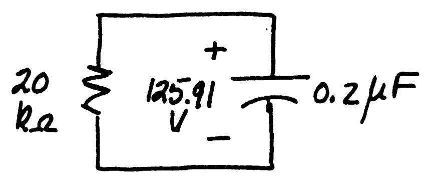
$$i_o = C\frac{dv_C}{dt} = (0.2\times10^{-6})[80{,}000e^{-250t}] = 16e^{-250t}\,\mathrm{mA}, \qquad 0 < t < 2\,\mathrm{ms}$$

$$v_o = 4i_o + v_C = 64e^{-250t} + 320 - 320e^{-250t}$$

$$= 320 - 256e^{-250t}\,\mathrm{V}, \qquad 0 < t < 2\,\mathrm{ms}$$

$2\,\mathrm{ms} \le t < \infty$:

$$v_C(2\,\mathrm{ms}) = 320 - 320e^{-0.5} \cong 125.91\,\mathrm{V}$$



$$v_C = 125.91e^{-250(t-0.002)}\,\mathrm{V}$$

$$i_o = C\frac{dv_C}{dt} = (0.2\times10^{-6})(-31{,}477.55)e^{-250(t-0.002)}$$

$$= -6.30e^{-250(t-0.002)}\,\mathrm{mA}, \qquad 2\,\mathrm{ms} < t < \infty$$

$$v_o = 4i_o + v_C = (-25.18 + 125.91)e^{-250(t-0.002)}$$

$$= 100.73e^{-250(t-0.002)}\,\mathrm{V}, \qquad 2\,\mathrm{ms} < t < \infty$$

[b] $i_o(0^-) = 0; \qquad i_o(0^+) = 16\,\mathrm{mA}$

$i_o(0.002^-) = 16e^{-0.5} = 9.7\,\mathrm{mA}$

$i_o(0.002^+) = -6.30\,\mathrm{mA}$

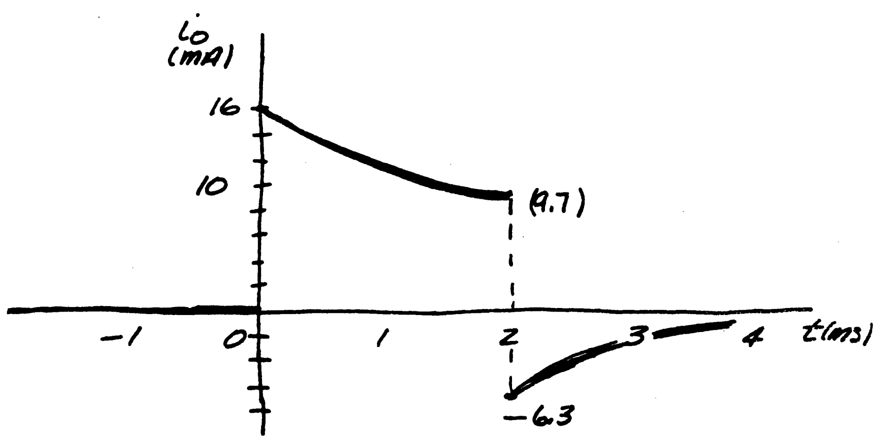
[c] $v_o(0^+) = (16\times10^3)(4\times10^{-3}) = 64\,\mathrm{V}$

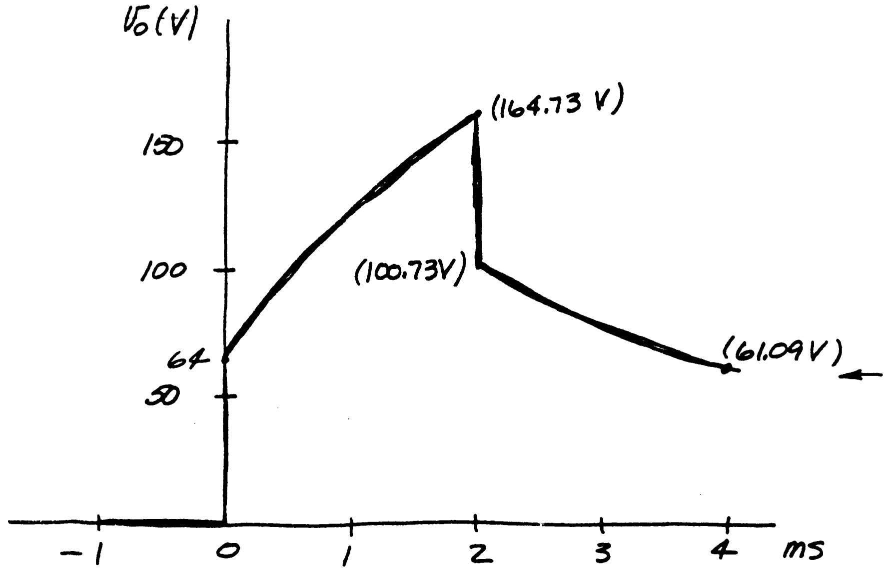
$v_o(0^-) = 0\,\mathrm{V}$

$v_o(0.002^-) = 320 - 256e^{-0.5} = 164.73\,\mathrm{V}$

$v_o(0.002^+) = 100.73\,\mathrm{V}$

[d]





P 8.66 [a] $t < 0$: $\quad v_o = 0$

$0 \le t \le 4\,\text{ms}$:

$\tau = (200)(0.025) \times 10^{-3} = 5\,\text{ms}; \qquad 1/\tau = 200$

$v_o = 100 - 100e^{-200t}\,\text{V}, \qquad 0 \le t \le 4\,\text{ms}$

$v_o(4\,\text{ms}) = 100(1 - e^{-0.8}) = 55.07\,\text{V}$

$4\,\text{ms} \le t \le 8\,\text{ms}$:

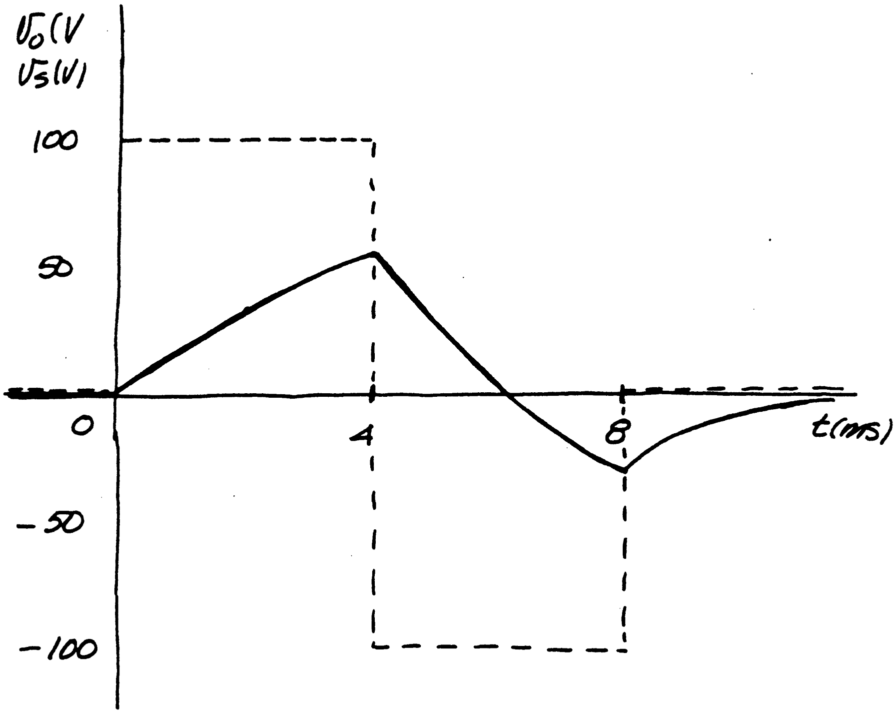
$v_o = -100 + 155.07e^{-200(t-0.004)}\,\text{V}$

$v_o(8\,\text{ms}) = -100 + 155.07e^{-0.8} = -30.32\,\text{V}$

$8\,\text{ms} \le t \le \infty$:

$v_o = -30.32e^{-200(t-0.008)}\,\text{V}$

[b]



[c] $t \leq 0$: $\quad v_o = 0$

$0 \leq t \leq 4$ ms:

$$v_o = 100 - 100e^{-800t}\,\text{V}, \qquad 0 \leq t \leq 4\,\text{ms}$$

$$v_o(4\,\text{ms}) = 100 - 100e^{-3.2} = 95.92\,\text{V}$$

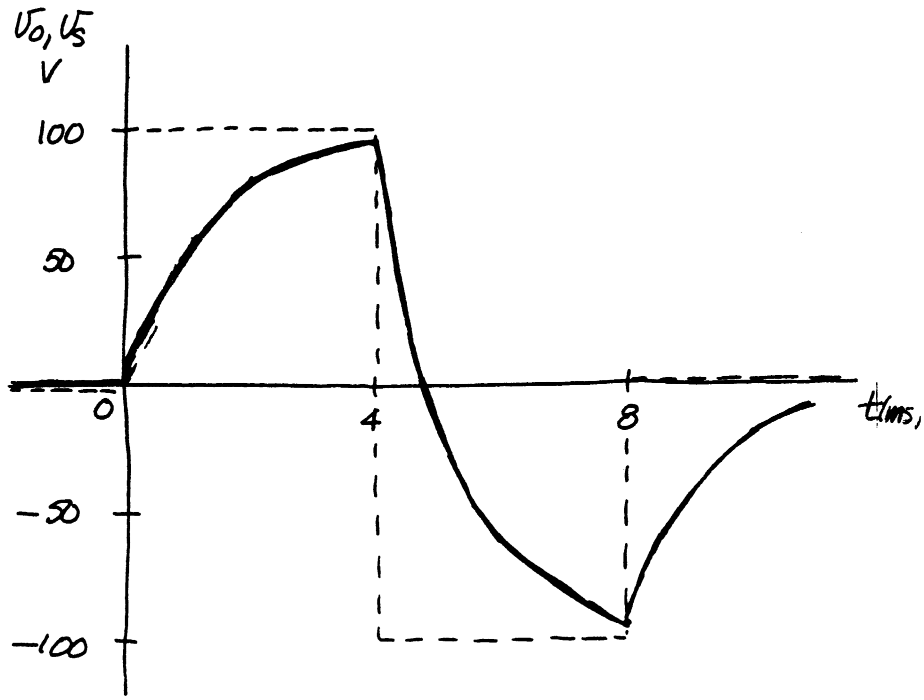
$4\,\text{ms} \leq t \leq 8$ ms:

$$v_o = -100 + 195.92e^{-800(t-0.004)}\,\text{V}, \qquad 4\,\text{ms} \leq t \leq 8\,\text{ms}$$

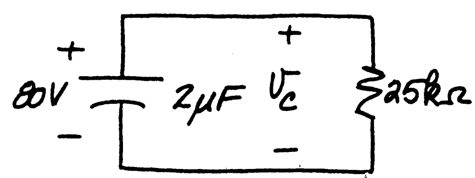
$$v_o(8\,\text{ms}) = -100 + 195.92e^{-3.2} = -92.01\,\text{V}$$

$8\,\text{ms} \leq t \leq \infty$:

$$v_o = -92.01e^{-800(t-0.008)}\,\text{V}, \qquad 8\,\text{ms} \leq t \leq \infty$$



P 8.67 [a]



$\tau = (25)(2) \times 10^{-3}$
$= 50\,\text{ms}$
$1/\tau = 20$
$v_C(0^+) = 80\,\text{V}$
$v_C(\infty) = 0$

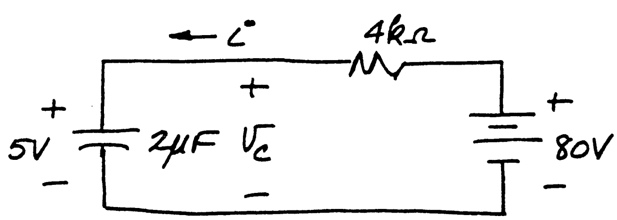
$v_C = 80e^{-20t}\,\text{V}$

$\therefore\ 80e^{-20t} = 5; \qquad e^{20t} = 16; \qquad t = 50\ln 16\,\text{ms} = 138.63\,\text{ms}$

[b] $0^+ < t < 138.63\,\text{ms}$:

$i = (2 \times 10^{-6})(-1600e^{-20t}) = -3.2e^{-20t}\,\text{mA}, \qquad 0^+ \leq t < 138.63\,\text{ms}$

$138.63^+\,\text{ms ms} < t \leq \infty$:



$\tau = (2)(4) \times 10^{-3} = 8\,\text{ms}; \qquad 1/\tau = 125$

$v_C(138.63^+\,\text{ms}) = 5\,\text{V}; \qquad v_C(\infty) = 80\,\text{V}$

$v_C = 80 - 75e^{-125(t-0.13863)}\,\text{V}, \qquad 138.63^+\,\text{ms} \le t \le \infty$

$i = 2 \times 10^{-6}(9375)e^{-125(t-0.13863)}$

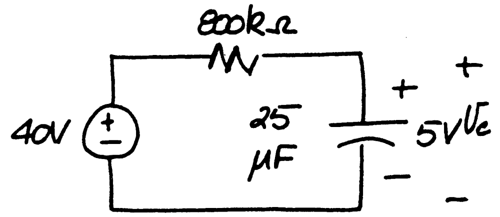
$\quad = 18.75e^{-125(t-0.13863)}\text{mA}, \qquad 138.63^+\,\text{ms} \le t \le \infty$

[c] $80 - 75e^{-125\Delta t} = 0.85(80) = 68$

$80 - 68 = 75e^{-125\Delta t} = 12$

$e^{125\Delta t} = 6.25; \qquad \Delta t = 8\ln 6.25\,\text{ms} \cong 14.66\,\text{ms}$

P 8.68 [a] At $t = 0$ we have



$\tau = (800)(25) \times 10^{-3}$

$= 20\,\text{sec}$

$1/\tau = 0.05$

$v_C(\infty) = 40\,\text{V}; \qquad v_C(0) = 5\,\text{V}$

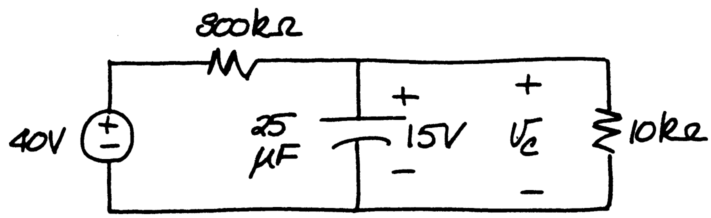
$v_C = 40 - 35e^{-0.05t}\,\text{V}, \qquad 0 \le t \le t_o$

$40 - 35e^{-0.05t_o} = 15$

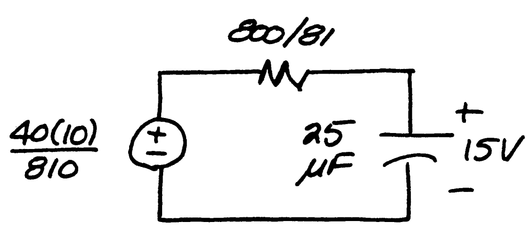
$\therefore\ e^{0.05t_o} = 1.4$

$t_o = 20\ln 1.4\,\text{s} = 6.73\,\text{s}$

At $t = t_o$ we have



The Thévenin equivalent with respect to the capacitor is



$$\tau = \left(\frac{800}{81}\right)(25) \times 10^{-3} = \frac{20}{81}\,\text{s}; \qquad \frac{1}{\tau} = \frac{81}{20} = 4.05$$

$$v_C(t_o) = 15\,\text{V}; \qquad v_C(\infty) = \frac{40}{81}\,\text{V}$$

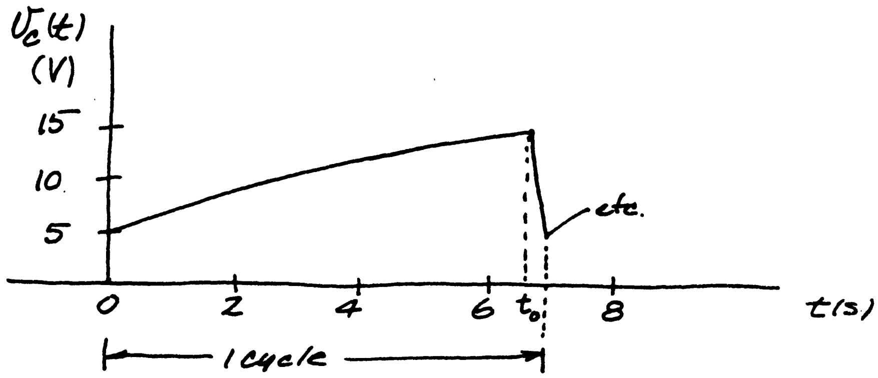
$$v_C(t) = \frac{40}{81} + \left(15 - \frac{40}{81}\right)e^{-4.05(t-t_o)}\,\text{V} = \frac{40}{81} + \frac{1175}{81}e^{-4.05(t-t_o)}$$

$$\therefore \frac{40}{81} + \frac{1175}{81}e^{-4.05(t-t_o)} = 5$$

$$\frac{1175}{81}e^{-4.05(t-t_o)} = \frac{365}{81}$$

$$e^{4.05(t-t_o)} = \frac{1175}{365} = 3.22$$

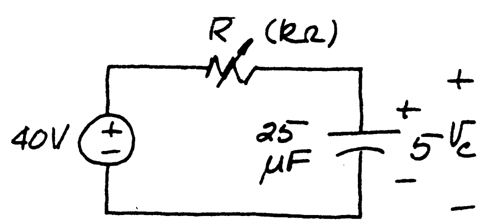
$$t - t_o = \frac{1}{4.05}\ln 3.22 \cong 0.29\,\text{s}$$



One cycle = 7.02 seconds.

[b] $N = 60/7.02 = 8.55$ flashes per minute

[c] At $t = 0$ we have



$\tau = 25R \times 10^{-3}$

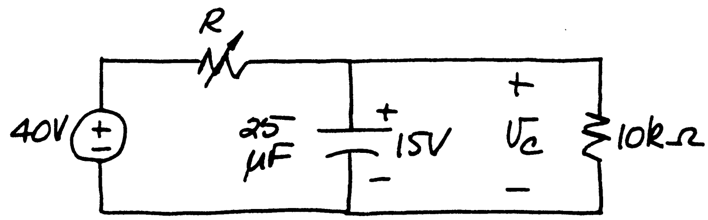
$1/\tau = 40/R$

$$v_C = 40 - 35e^{-(40/R)t}$$

$$40 - 35e^{-(40/R)t_o} = 15$$

$$\therefore t_o = \frac{R}{40}\ln 1.4, \qquad R \text{ in } \text{k}\Omega$$

At $t = t_o$:



$$v_{\rm Th} = \frac{10}{R+10}(40) = \frac{400}{R+10}; \qquad R_{\rm Th} = \frac{10R}{R+10}\,\text{k}\Omega$$

$$\tau = \frac{(25)(10R) \times 10^{-3}}{R+10} = \frac{0.25R}{R+10}; \qquad \frac{1}{\tau} = \frac{4(R+10)}{R}$$

$$v_C = \frac{400}{R+10} + \left(15 - \frac{400}{R+10}\right) e^{-\frac{4(R+10)}{R}(t-t_o)}$$

$$\therefore \frac{400}{R+10} + \left[\frac{15R-250}{R+10}\right] e^{-\frac{4(R+10)}{R}(t-t_o)} = 5$$

or $$\left(\frac{15R-250}{R+10}\right) e^{-\frac{4(R+10)}{R}(t-t_o)} = \frac{5R-350}{(R+10)}$$

$$\therefore e^{\frac{4(R+10)}{R}(t-t_o)} = \frac{3R-50}{R-70}$$

$$\therefore t - t_o = \frac{R}{4(R+10)} \ln\left(\frac{3R-50}{R-70}\right)$$

At 12 flashes per minute $\quad t_o + (t - t_o) = 5\,\text{s}$

$$\therefore \underbrace{\frac{R}{40}\ln 1.4}_{\text{dominant term}} + \frac{R}{4(R+10)} \ln\left(\frac{3R-50}{R-70}\right) = 5$$

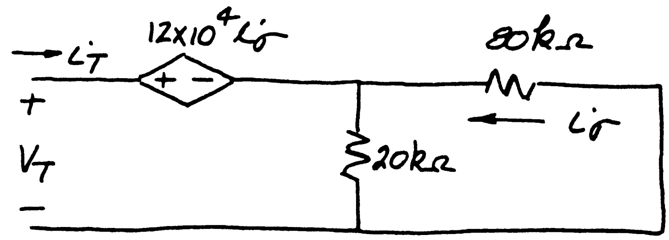
Start the trial-and-error procedure by setting $(R/40)\ln 1.4 = 5$, then $R = 200/(\ln 1.4)$ or $594.40\,\text{k}\Omega$. If $R = 594.40\,\text{k}\Omega$ then $t - t_o \cong 0.29\,\text{s}$.

Second trial set $(R/40)\ln 1.4 = 4.7\,\text{s}$ or $R = 558.74\,\text{k}\Omega$.

With $\quad R = 558.74\,\text{k}\Omega, \quad t - t_o \cong 0.30\,\text{s}$

$\therefore\ R = 558.74\,\text{k}\Omega$

P 8.69



$v_T = 12 \times 10^4 i_\sigma + 16 \times 10^3 i_T; \qquad i_\sigma = -\frac{20}{100} i_T = -\frac{1}{5} i_T$

$v_T = -2.4 \times 10^4 i_T + 16 \times 10^3 i_T$

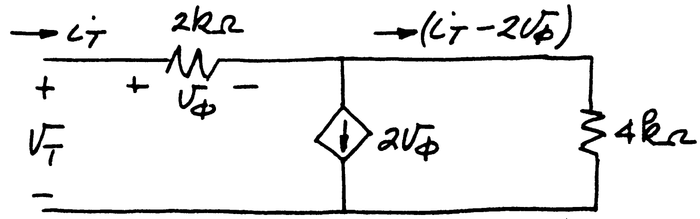
$\frac{v_T}{i_T} = R_{\text{Th}} = 16 \times 10^3 - 24 \times 10^3 = -8000\,\Omega$

$\tau = (-8000)(2.5 \times 10^{-6}) = -20 \times 10^{-3}; \qquad \frac{1}{\tau} = \frac{-1000}{20} = -50$

$v_C = 20e^{50t}\,\text{V}, \qquad t \geq 0$

$20e^{50t} = 20{,}000; \qquad e^{50t} = 1000; \qquad t = 20 \ln 1000\,\text{ms} = 138.16\,\text{ms}$

P 8.70



$v_T = 2i_T + 4(i_T - 2v_\phi) = 6i_T - 8v_\phi = 6i_T - 8(2i_T) = -10i_T$

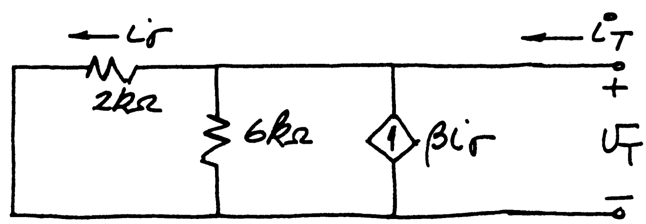
$R_{\text{Th}} = \frac{v_T}{i_T} = -10\,\text{k}\Omega$

$\tau = \frac{-10}{10 \times 10^3} = -10^{-3} = -1\,\text{ms}; \qquad \frac{1}{\tau} = -1000$

$i_L(t) = 25e^{1000t}\,\text{mA}, \qquad t \geq 0$

$25e^{1000t} \times 10^{-3} = 5; \qquad e^{1000t} = 200; \qquad 1000t = \ln 200; \qquad t = \ln 200\,\text{ms} = 5.30\,\text{ms}$

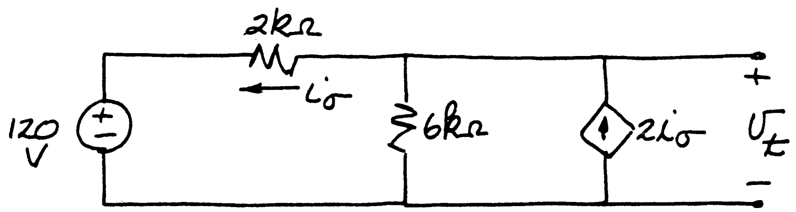
P 8.71



$$i_T = -\beta i_\sigma + \frac{v_T}{6} + \frac{v_T}{2} = -\beta\frac{v_T}{2} + \frac{v_T}{6} + \frac{v_T}{2} = \left[\left(\frac{1-\beta}{2}\right) + \frac{1}{6}\right] v_T = \left(\frac{4-3\beta}{6}\right) v_T$$

$\therefore \frac{v_T}{i_T} = \frac{6}{4-3\beta} = -3$

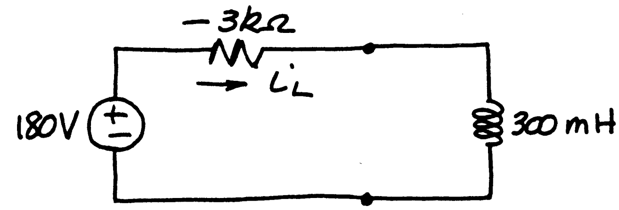
$6 = -12 + 9\beta, \qquad 9\beta = 18, \qquad \beta = 2$



$$\frac{v_T - 120}{2} + \frac{v_T}{6} - 2\left(\frac{v_T - 120}{2}\right) = 0$$

$$3v_T - 360 + v_T - 6v_T + 720 = 0$$

$$-2v_T + 360 = 0; \qquad v_T = 180\,\text{V}$$



$$180 = -3000 i_L + 0.3\frac{di_L}{dt}$$

$$\frac{di_L}{dt} - 10^4 i_L = 600$$

$$\frac{di_L}{dt} = 10^4(i_L + 0.06)$$

$$\frac{di_L}{i_L + 0.06} = 10^4\,dt$$

$$\int_{i_L(0)}^{i_L(t)} \frac{dx}{x + 0.06} = \int_0^t 10^4\,dy$$

$$\ln(x + 0.06)\Big|_{i_L(0)}^{i_L(t)} = 10^4 t$$

$$\ln\frac{[i_L(t) + 0.06]}{[i_L(0) + 0.06]} = 10^4 t$$

$$[i_L(t) + 0.06] = [i_L(0) + 0.06]e^{10^4 t}$$

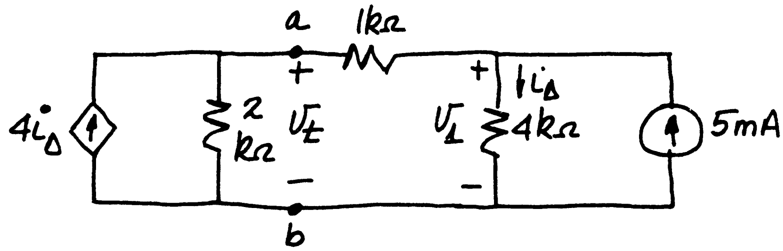
$$i_L(t) = -0.06 + 0.06e^{10^4 t}$$

$$v_L = 0.3\frac{di_L}{dt} = 0.3[600e^{10^4 t}] = 180e^{10^4 t}\,\text{V}, \qquad 0^+ \le t \le \infty$$

$$180e^{10^4 t} = 36{,}000; \qquad e^{10^4 t} = 200; \qquad 10^4 t = \ln 200;$$

$$t = 10^{-4}\ln 200 = 100\ln 200\,\mu\text{s} \cong 529.83\,\mu\text{s}$$

P 8.72 First find the Thévenin equivalent with respect to the capacitor terminals.



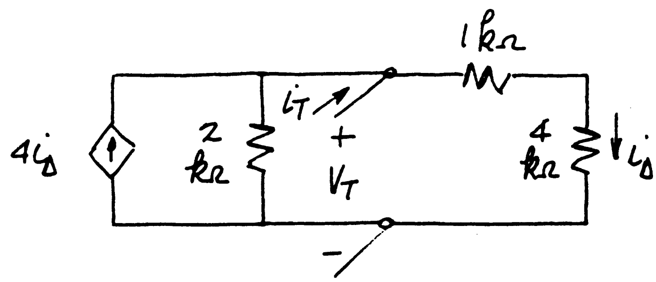
$$-4\left(\frac{v_1}{4}\right)+\frac{v_{Th}}{2}+\frac{v_{Th}-v_1}{1}=0$$

$$-4v_1+2v_{Th}+4v_{Th}-4v_1=0$$

$$\boxed{6v_{Th}-8v_1=0} \qquad \Delta=\begin{vmatrix}6 & -8\\ -4 & 5\end{vmatrix}=30-32=-2$$

$$\frac{v_1}{4}+\frac{v_1-v_{Th}}{1}=5 \qquad N_{Th}=\begin{vmatrix}0 & -8\\ 20 & 5\end{vmatrix}=160$$

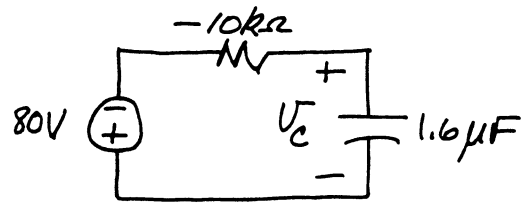
$$\boxed{-4v_{Th}+5v_1=20} \qquad v_{Th}=\frac{N_{Th}}{\Delta}=-80\text{ V}$$



$$i_T=\frac{v_T}{2}+\frac{v_T}{5}-4\frac{v_T}{5}$$

$$\frac{i_T}{v_T}=\frac{1}{2}+\frac{1}{5}-\frac{4}{5}=\frac{1}{2}-\frac{3}{5}=\frac{5}{10}-\frac{6}{10}=-\frac{1}{10}\text{ millimho}$$

$$R_{Th}=\frac{v_T}{i_T}=-10\text{ k}\Omega$$



$$1.6\times 10^{-6}\frac{dv_C}{dt}+\frac{v_C+80}{-10{,}000}=0$$

$$\frac{dv_C}{dt}-62.5v_C-5000=0$$

$$\frac{dv_C}{dt}=62.5(v_C+80)$$

$$\frac{dv_C}{v_C+80}=62.5\,dt$$

$$\therefore\ v_C=-80+80e^{62.5t}\text{ V},\qquad 0\le t\le\infty$$

$$\therefore\ -80+80e^{62.5t}=14{,}400;\qquad 80e^{62.5t}=14{,}400;\qquad e^{62.5t}=181;$$

$$62.5t=\ln 181;\qquad t=16\ln 181\text{ ms}\cong 83.18\text{ ms}$$

P 8.73 $v_o = -\frac{10^6}{0.5R}\int_0^t 4\,dx + 0 = (-10^6)\frac{8}{R}t$

$\therefore\ -10 = \frac{-8\times 10^6}{R}(15)\times 10^{-3};\qquad \therefore\ R = (8)(15)\times 10^2 = 12\,\text{k}\Omega$

P 8.74 $v_o = \frac{-8\times 10^6 t}{R} - 6$

$\therefore\ -10 = \frac{-8\times 10^6(40)\times 10^{-3}}{R} - 6;\qquad \therefore\ \frac{32\times 10^4}{R} = 4;\qquad R = 8\times 10^4 = 80\,\text{k}\Omega$

P 8.75 $v_o = \frac{-10^3}{200(0.2)}\int_0^t -10\,dx + 0 = 250t$

$v_o(32\,\text{ms}) = (250)(32)\times 10^{-3} = 8\,\text{V}$

$v_o = \frac{-10^3}{(250)(0.2)}\int_{32\times 10^{-3}}^t 5\,dx + 8 = -100t + 11.2\,\text{V}$

$\therefore\ -15 = -100t + 11.2;\qquad t = 262\,\text{ms}$

P 8.76 $v_2 = \frac{-45}{100}(80) = -36\,\text{V} = v_1$

$\frac{-36-14}{80\times 10^3} + 2.5\times 10^{-6}\frac{d}{dt}(-36 - v_o) = 0$

$\therefore\ -2.5\times 10^{-6}\frac{dv_o}{dt} = 6.25\times 10^{-4};\qquad \frac{dv_o}{dt} = -2.5\times 10^2 = -250;\qquad dv_o = -250\,dt$

$\int_{v_o(0)}^{v_o(t)} dx = -250\int_0^t dy;\qquad v_o(t) - v_o(0) = -250t;\qquad v_o(0) = -36 + 56 = 20\,\text{V}$

$\therefore\ v_o(t) = -250t + 20;\qquad 0 = -250t + 20;\qquad 250t = 20;\qquad t = 80\,\text{ms}$

P 8.77 [a] $\frac{v_1}{10} + \frac{v_1 - v_o}{40} = 0$

$5v_1 = v_o = 5v_2;\qquad v_2 = -2 + 2e^{-625t}\,\text{V}$

$\therefore\ v_o = -10 + 10e^{-625t}\,\text{V},\qquad 0 \le t \le t_{\text{sat}}$

$\therefore\ -5 = -10 + 10e^{-625t_{\text{sat}}}$

$\therefore\ e^{625t_{\text{sat}}} = 2;\qquad t_{\text{sat}} = 1.6\ln 2\,\text{ms} = 1.11\,\text{ms}$

[b] $v_2 = -2 + 3e^{-625t}$

$v_o = 5v_2 = -10 + 15e^{-625t},\qquad 0 \le t \le t_{\text{sat}}$

$-5 = -10 + 15e^{-625t_{\text{sat}}}$

$t_{\text{sat}} = 1.6\ln 3\,\text{ms} = 1.76\,\text{ms}$

P 8.78 [a] $\frac{Cdv_2}{dt} + \frac{v_2 - v_b}{R} = 0;\qquad \text{therefore}\quad \frac{dv_2}{dt} + \frac{1}{RC}v_2 = \frac{v_b}{RC}$

$\frac{v_1 - v_a}{R} + C\frac{d(v_1 - v_o)}{dt} = 0;\qquad \text{therefore}\quad \frac{dv_o}{dt} = \frac{dv_1}{dt} + \frac{v_1}{RC} - \frac{v_a}{RC}$

But $v_1 = v_2$

Therefore $\frac{dv_1}{dt} + \frac{v_1}{RC} = \frac{dv_2}{dt} + \frac{v_2}{RC} = \frac{v_b}{RC}$

Therefore $\frac{dv_o}{dt} = \frac{1}{RC}(v_b - v_a); \qquad v_o = \frac{1}{RC}\int_0^t (v_b - v_a)\, dy$

[b] The output is the integral of the difference between v_b and v_a and then scaled by a factor of $1/RC$.

[c] $v_o = \frac{1}{RC}\int_0^t (v_b - v_a)\, dx$

$RC = (40) \times 10^3(25) \times 10^{-9} = 1000 \times 10^{-6} = 10^{-3}$

$v_b - v_a = (60 - 10)10^{-3} = 50 \times 10^{-3}$

$v_o = 50\int_0^t dx = 50t; \qquad 50t_{sat} = 12; \qquad t_{sat} = 0.24\,\text{s} = 240\,\text{ms}$

P 8.79 $RC = (100 \times 10^3)(0.05 \times 10^{-6}) = 5 \times 10^{-3}; \qquad \frac{1}{RC} = 200; \qquad v_o(0) = -4 + 12 = 8\,\text{V}$

$v_o = 200\int_0^t (-15 + 7)\, dx + v_o(0) = -1600t + 8\,\text{V}, \qquad 0 \le t \le t_{sat}$

$v_2 = -15 + (-4 + 15)e^{-200t} = -15 + 11e^{-200t}\,\text{V}, \qquad 0 \le t \le t_{sat}$

$v_f + v_2 = v_o$

$v_f = v_o - v_2 = -1600t + 8 + 15 - 11e^{-200t}$

$v_f = 23 - 11e^{-200t} - 1600t\,\text{V}, \qquad 0 \le t \le t_{sat}$

Note: $-1600t_{sat} + 8 = -20; \qquad \therefore\ t_{sat} = \frac{-28}{-1600} = 17.5\,\text{ms}$

P 8.80 $0 \le t \le 20\,\text{ms}$:

$RC = (25 \times 10^3)(0.8 \times 10^{-6}) = 20 \times 10^{-3}; \qquad \frac{1}{RC} = 50; \qquad v_o(0) = 0$

$v_o = -50\int_0^t -10\, dx + 0 = 500t, \qquad 0 \le t \le 20\,\text{ms}$

$v_o(20\,\text{ms}) = (500)(20) \times 10^{-3} = 10\,\text{V}$

$20\,\text{ms} \le t \le t_{sat}$:

$RC = (100)(0.80) \times 10^{-3} = 80\,\text{ms}; \qquad \frac{1}{RC} = 12.5$

$v_o = -12.5\int_{20\,\text{ms}}^t 20\, dx + 10 = -250(t - 0.02) + 10$

$= -250t + 15\,\text{V}, \qquad 20\,\text{ms} \le t \le t_{sat}$

$-250t_{sat} + 15 = -20; \qquad -250t_{sat} = -35; \qquad t_{sat} = 140\,\text{ms}$

$v_o = -250t + 15\,\text{V}, \qquad 20\,\text{ms} \le t \le 140\,\text{ms}$

P 8.81 $R_fC_f = (24)\left(\frac{1}{3}\right) \times 10^{-3} = 8\,\text{ms};\qquad \frac{1}{R_fC_f} = 125$

$v_o(0) = -5\,\text{V};\qquad v_o(\infty) = -\frac{24}{3}(-4) = 32\,\text{V}$

$v_o(t) = 32 - 37e^{-125t}\,\text{V},\qquad 0 \le t \le t_{\text{sat}}$

$12 = 32 - 37e^{-125t_{\text{sat}}}$

$\therefore\ e^{125t_{\text{sat}}} = \frac{37}{20} = 1.85;\qquad t_{\text{sat}} = 8\ln 1.85\,\text{ms} = 4.92\,\text{ms}$

P 8.82 [a] $RC = 25(0.4) \times 10^{-3} = 10\,\text{ms};\qquad \frac{1}{RC} = 100;\qquad v_o = 0,\quad t < 0$

[b] $0 \le t \le 250\,\text{ms}$:

$$v_o = -100\int_0^t -0.20\,dx = 20t\,\text{V}$$

[c] $250\,\text{ms} \le t \le 500\,\text{ms}$:

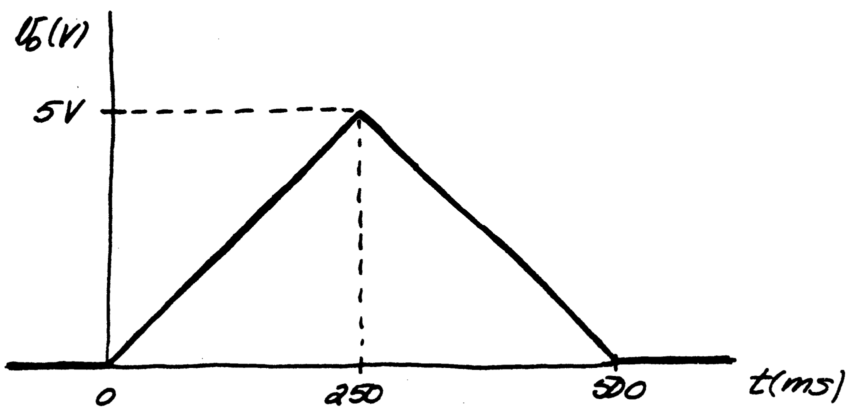
$v_o(0.25) = 20(0.25) = 5\,\text{V}$

$$v_o(t) = -100\int_{0.25}^t 0.20\,dx + 5 = -20(t - 0.25) + 5 = -20t + 10\,\text{V}$$

[d] $500\,\text{ms} \le t \le \infty$:

$v_o(0.5) = -10 + 10 = 0\,\text{V}$

$v_o(t) = 0\,\text{V}$



P 8.83 [a] $v_o = 0,\qquad t < 0$

[b] $R_fC_f = (5 \times 10^6)(0.4 \times 10^{-6}) = 2;\qquad \frac{1}{R_fC_f} = 0.5$

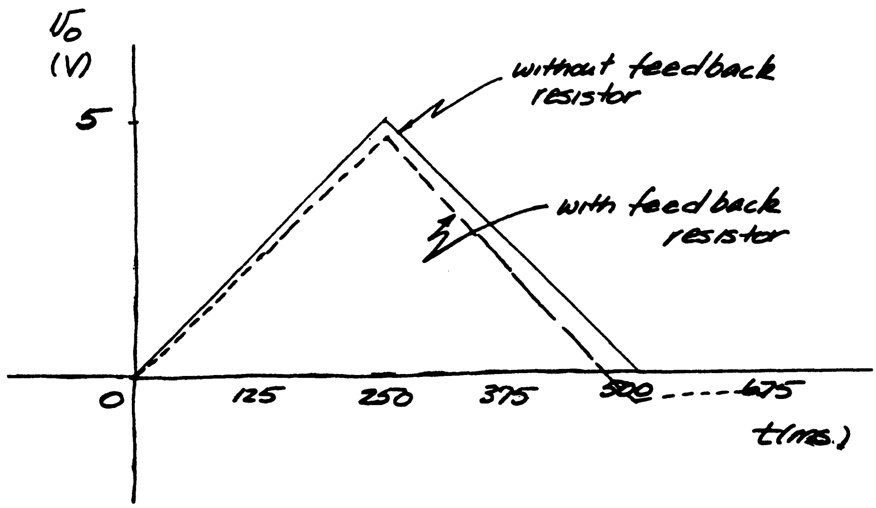
$$v_o = \frac{-5 \times 10^6}{25 \times 10^3}(-0.2)[1 - e^{-0.5t}] = 40(1 - e^{-0.5t})\,\text{V},\qquad 0 \le t \le 250\,\text{ms}$$

[c] $v_o(0.25) = 40(1 - e^{-0.125}) \cong 4.70\,\text{V}$

$$v_o = \frac{-V_m R_f}{R_s} + \frac{V_m R_f}{R_s}(2 - e^{-0.125})e^{-0.5(t-0.25)}$$
$$= -40 + 40(2 - e^{-0.125})e^{-0.5(t-0.25)}$$
$$= -40 + 44.70e^{-0.5(t-0.25)}\,\text{V}, \qquad 250\,\text{ms} \le t \le 500\,\text{ms}$$

[d] $v_o(0.50) = -40 + 44.70e^{-0.125} \cong -0.55\,\text{V}$

$$v_o = -0.55e^{-0.5(t-0.5)}\,\text{V}, \qquad 500\,\text{ms} \le t \le \infty$$



P 8.84 [a] $R_f C_f = (400 \times 10^3)(0.05 \times 10^{-6}) = 20\,\text{ms}; \qquad \frac{1}{R_f C_f} = 50; \qquad \frac{R_f}{R_s} = 12.5$

$0 \le t \le 2\,\text{ms}$:

$$v_o = -125(1 - e^{-50t})\,\text{V}, \qquad 0 \le t \le 2\,\text{ms}$$
$$v_o(2\,\text{ms}) = -125(1 - e^{-0.10}) \cong -11.90\,\text{V}$$

$2\,\text{ms} \le t \le \infty$:

$$v_o(\infty) = -(-8)(12.5) = 100\,\text{V}$$
$$\therefore\ v_o = 100 - 111.90e^{-50(t-0.002)}\,\text{V}$$
$$\therefore\ v_o = 0 \quad \text{when} \quad 100 = 111.90e^{-50(t_o-0.002)}$$
$$\text{or} \quad e^{50(t_o-0.002)} = 1.119$$
$$50(t_o - 0.002) = \ln 1.119$$
$$t_o = 0.002 + 0.02\ln 1.119$$
$$t_o = 2 + 20\ln 1.119\,\text{ms} = 4.25\,\text{ms}$$

[b] Saturation will occur when $v_o = 15\,\text{V}$.

$$\therefore\ 100 - 111.90e^{-50(t_{sat}-0.002)} = 15$$
$$\therefore\ 111.90e^{-50(t_{sat}-0.002)} = 85; \qquad e^{50(t_{sat}-0.002)} = \frac{111.90}{85}; \qquad t_{sat} = 7.5\,\text{ms}$$

P 8.85 [a] $0 \leq t \leq 1\,\mu$s:

$$RC = (1000)(800) \times 10^{-12} = 8 \times 10^{-7}$$

$$\frac{1}{RC} = 1{,}250{,}000 = 125 \times 10^4$$

$$v_g = 2 \times 10^6 t, \qquad 0 \leq t \leq 1us$$

$$\therefore\ v_o = -125 \times 10^4 \int_0^t 2 \times 10^6 x\,dx + 0 = -250 \times 10^{10} \frac{x^2}{2}\Big|_0^t$$

$$= -125 \times 10^{10} t^2, \qquad 0 \leq t \leq 1\,\mu\text{s}$$

$1\,\mu$s $\leq t \leq 3\,\mu$s:

$$v_g = 4 - 2 \times 10^6 t\,\text{V}, \qquad 1\,\mu\text{s} \leq t \leq 3\,\mu\text{s}$$

$$v_o(1\,\mu\text{s}) = -125 \times 10^{10} \times 10^{-12} = -1.25\,\text{V}$$

$$\therefore\ v_o = -125 \times 10^4 \int_{10^{-6}}^t (4 - 2 \times 10^6 x)\,dx - 1.25$$

$$= -125 \times 10^4 \left[4x\Big|_{10^{-6}}^t - 2 \times 10^6 \frac{x^2}{2}\Big|_{10^{-6}}^t\right] - 1.25$$

$$= -125 \times 10^4 \left[4t - 4 \times 10^{-6} - 10^6(t^2 - 10^{-12})\right] - 1.25$$

$$= -500 \times 10^4 t + 500 \times 10^{-2} + 125 \times 10^{10} t^2 - 1.25 - 1.25$$

$$= 125 \times 10^{10} t^2 - 5 \times 10^6 t + 2.5\,\text{V}, \qquad 1\,\mu\text{s} \leq t \leq 3\,\mu\text{s}$$

$3\,\mu$s $\leq t \leq 4\,\mu$s:

$$v_g = -8 + 2 \times 10^6 t\,\text{V}, \qquad 3\,\mu\text{s} \leq t \leq 4\,\mu\text{s}$$

$$v_o(3\,\mu\text{s}) = 125 \times 10^{10}(9 \times 10^{-12}) - 5 \times 10^6(3 \times 10^{-6}) + 2.5 = -1.25\,\text{V}$$

$$\therefore\ v_o = -125 \times 10^4 \int_{3\times10^{-6}}^t [-8 + 2 \times 10^6 x]\,dx - 1.25$$

$$= -125 \times 10^4 \left[-8x\Big|_{3\times10^{-6}}^t + 10^6 x^2\Big|_{3\times10^{-6}}^t\right] - 1.25$$

$$= -125 \times 10^4 \left[-8(t - 3 \times 10^{-6}) + 10^6(t^2 - 9 \times 10^{-12})\right] - 1.25$$

$$= 1000 \times 10^4 t - 30 - 125 \times 10^{10} t^2 + 11.25 - 1.25$$

$$v_o = -125 \times 10^{10} t^2 + 10^7 t - 20, \qquad 3\,\mu\text{s} \leq t \leq 4\,\mu\text{s}$$

[b]

$v_o(0) = 0$

$v_o(0.5\,\mu\text{s}) = -0.3125\,\text{V}$

$v_o(1.0\,\mu\text{s}) = -1.25\,\text{V}$

$v_o(1.5\,\mu\text{s}) = -2.1875\,\text{V}$

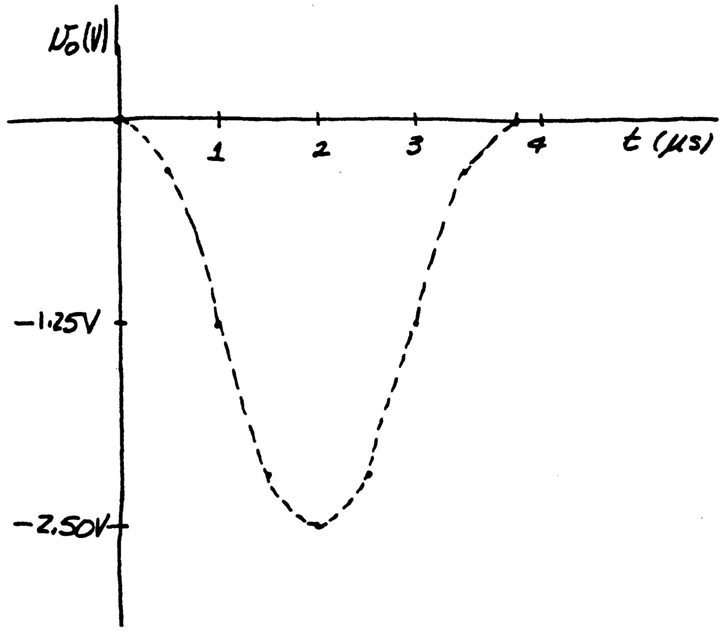
$v_o(2.0\,\mu\text{s}) = -2.50\,\text{V}$

$v_o(2.5\,\mu\text{s}) = -2.1875\,\text{V}$

$v_o(3\,\mu\text{s}) = -1.25\,\text{V}$

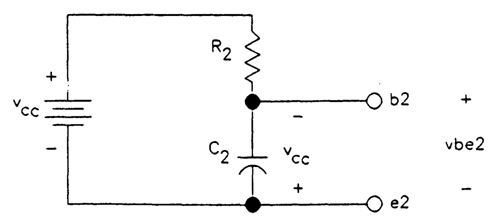
$v_o(3.5\,\mu\text{s}) = -0.3125\,\text{V}$

$v_o(4.0\,\mu\text{s}) = 0$



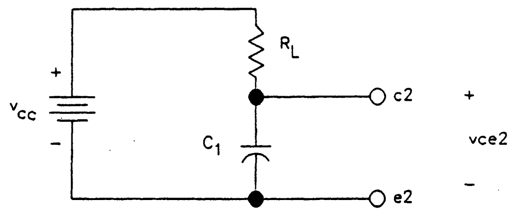
[c] The output voltage will also repeat. This follows from the observation that at $t = 4\,\mu s$ the output voltage is zero, hence there is no energy stored in the capacitor. This means the circuit is in the same state at $t = 4\,\mu s$ as it was at $t = 0$, thus as v_g repeats itself so will v_o.

P 8.86 [a] While T_2 has been ON, C_2 is charged to V_{CC}, positive on the left terminal. At the instant T_1 turns ON the capacitor C_2 is connected across $b_2 - e_2$, thus $v_{be_2} = -V_{CC}$. This negative voltage snaps T_2 OFF. Now the polarity of the voltage on C_2 starts to reverse, that is, the right-hand terminal of C_2 starts to charge toward $+V_{CC}$. At the same time, C_1 is charging toward V_{CC}, positive on the right. At the instant the charge on C_2 reaches zero, v_{be2} is zero, T_2 turns ON. This makes $v_{be1} = -V_{CC}$ and T_1 snaps OFF. Now the capacitors C_1 and C_2 start to charge with the polarities to turn T_1 ON and T_2 OFF. This switching action repeats itself over and over as long as the circuit is energized. At the instant T_1 turns ON, the voltage controlling the state of T_2 is governed by the following circuit:



It follows that $v_{be2} = V_{CC} - 2V_{CC}e^{-t/R_2C_2}$.

[b] While T_2 is OFF and T_1 is ON, the output voltage v_{ce2} is the same as the voltage across C_1, thus



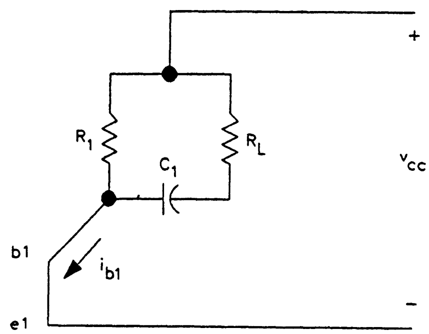
It follows that $v_{ce2} = V_{CC} - V_{CC}e^{-t/R_L C_1}$.

[c] T_2 will be OFF until v_{be2} reaches zero. As soon as v_{be2} is zero, i_{b2} will become positive and turn T_2 ON. $v_{be2} = 0$ when $V_{CC} - 2V_{CC}e^{-t/R_2C_2} = 0$, or when $t = R_2C_2 \ln 2$.

[d] When $t = R_2C_2 \ln 2$, we have

$$v_{ce2} = V_{CC} - V_{CC}e^{-[(R_2C_2 \ln 2)/(R_L C_1)]} = V_{CC} - V_{CC}e^{-10 \ln 2} \cong V_{CC}$$

[e] Before T_1 turns ON, i_{b1} is zero. At the instant T_1 turns ON, we have



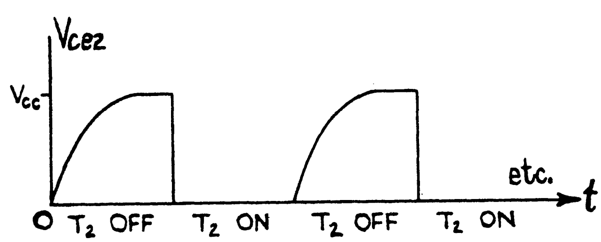
$$i_{b1} = \frac{V_{CC}}{R_1} + \frac{V_{CC}}{R_L}e^{-t/R_L C_1}$$

[f] At the instant T_2 turns back ON, $t = R_2C_2 \ln 2$; therefore

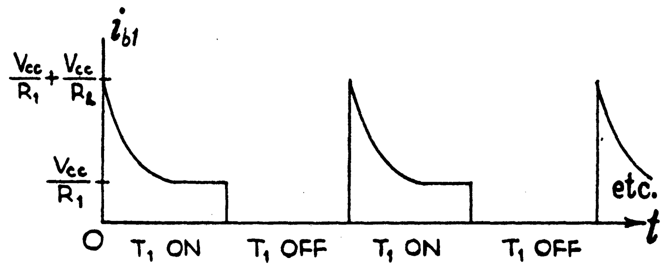
$$i_{b1} = \frac{V_{CC}}{R_1} + \frac{V_{CC}}{R_L}e^{-10 \ln 2} \cong \frac{V_{CC}}{R_1}$$

When T_2 turns OFF, i_{b1} drops to zero instantaneously.

[g]



[h]



P 8.87 [a] $t_{OFF2} = R_2C_2 \ln 2 = 14.43 \times 10^3(10^{-9}) \ln 2 \cong 10\,\mu s$

[b] $t_{ON2} = R_1C_1 \ln 2 \cong 10\,\mu s$

[c] $t_{OFF1} = R_1C_1 \ln 2 \cong 10\,\mu s$

[d] $t_{ON1} = R_2C_2 \ln 2 \cong 10\,\mu s$

[e] $i_{b1} = \frac{10}{1} + \frac{10}{14.43} = 10.693\,\text{mA}$

[f] $i_{b1} = \frac{10}{14.43} + \frac{10}{1}e^{-10} \cong 0.693\,\text{mA}$

[g] $v_{ce2} = 10 - 10e^{-10} \cong 10\,\text{V}$

P 8.88 [a] $t_{OFF2} = R_2C_2 \ln 2 = (14.34 \times 10^3)(0.8 \times 10^{-9}) \ln 2 \cong 8\,\mu s$

[b] $t_{ON2} = R_1C_1 \ln 2 \cong 10\,\mu s$

[c] $t_{OFF1} = R_1C_1 \ln 2 \cong 10\,\mu s$

[d] $t_{ON1} = R_2C_2 \ln 2 = 8\,\mu s$

[e] $i_{b1} = 10.693\,\text{mA}$

[f] $i_{b1} = \frac{10}{14.43} + 10e^{-8} \cong 0.693\,\text{mA}$

[g] $v_{ce2} = 10 - 10e^{-8} \cong 10\,\text{V}$

Note in this circuit T_2 is OFF $8\,\mu s$ and ON $10\,\mu s$ of every cycle, whereas T_1 is ON $8\,\mu s$ and OFF $10\,\mu s$ every cycle.

P 8.89 If $R_1 = R_2 = 50R_L = 100\,\text{k}\Omega$, then

$$C_1 = \frac{48 \times 10^{-6}}{100 \times 10^3 \ln 2} = 692.49\,\text{pF}; \qquad C_2 = \frac{36 \times 10^{-6}}{100 \times 10^3 \ln 2} = 519.37\,\text{pF}$$

If $R_1 = R_2 = 6R_L = 12\,\text{k}\Omega$, then

$$C_1 = \frac{48 \times 10^{-6}}{12 \times 10^3 \ln 2} = 5.77\,\text{nF}; \qquad C_2 = \frac{36 \times 10^{-6}}{12 \times 10^3 \ln 2} = 4.33\,\text{nF}$$

Therefore $692.49\,\text{pF} \le C_1 \le 5.77\,\text{nF}$ and $519.37\,\text{pF} \le C_2 \le 4.33\text{nF}$

P 8.90 [a] T_2 is normally ON since its base current i_{b2} is greater than zero, i.e., $i_{b2} = V_{CC}/R$ when T_2 is ON. When T_2 is ON, $v_{ce2} = 0$, therefore $i_{b1} = 0$. When $i_{b1} = 0$, T_1 is OFF. When T_1 is OFF and T_2 is ON, the capacitor C is charged to V_{CC}, positive at the left terminal. This is a stable state; there is nothing to disturb this condition if the circuit is left to itself.

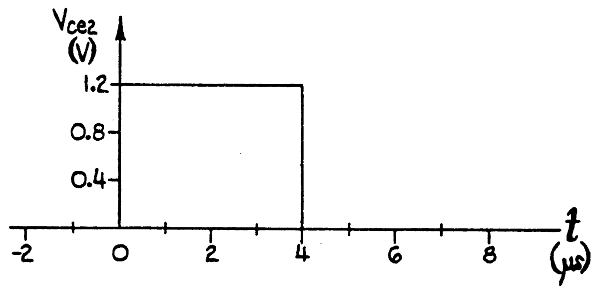
[b] When S is closed momentarily, v_{be2} is changed to $-V_{CC}$ and T_2 snaps OFF. The instant T_2 turns OFF, v_{ce2} jumps to $V_{CC}R_1/(R_1 + R_L)$ and i_{b1} jumps to $V_{CC}/(R_1 + R_L)$, which turns T_1 ON.

[c] As soon as T_1 turns ON, the charge on C starts to reverse polarity. Since v_{be2} is the same as the voltage across C, it starts to increase from $-V_{CC}$ toward $+V_{CC}$. However, T_2 turns ON as soon as $v_{be2} = 0$. The equation for v_{be2} is $v_{be2} = V_{CC} - 2V_{CC}e^{-t/RC}$. $v_{be2} = 0$ when $t = RC\ln 2$, therefore T_2 stays OFF for $RC\ln 2$ seconds.

P 8.91 [a] For $t < 0$, $v_{ce2} = 0$. When the switch is momentarily closed, v_{ce2} jumps to

$$v_{ce2} = \left(\frac{V_{CC}}{R_1 + R_L}\right) R_1 = \frac{6(5)}{25} = 1.2\,\mathrm{V}$$

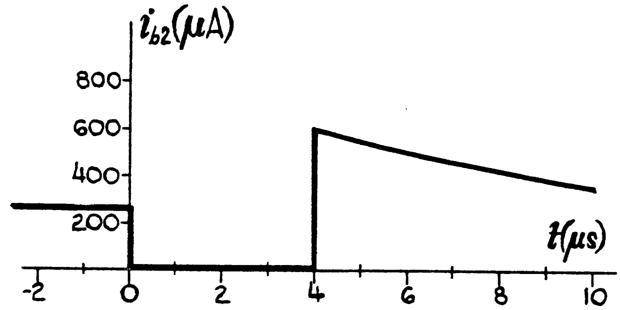
T_2 remains open for $(23{,}083)(250) \times 10^{-12} \ln 2 \cong 4\,\mu s$.



[b] $i_{b2} = \dfrac{V_{CC}}{R} = 259.93\,\mu A, \qquad -5 \le t \le 0\,\mu s$

$i_{b2} = 0, \qquad 0 < t < RC\ln 2$

$$i_{b2} = \frac{V_{CC}}{R} + \frac{V_{CC}}{R_L} e^{-(t-RC\ln 2)/R_L C}$$
$$= 259.93 + 300e^{-0.2\times 10^6 (t - 4\times 10^{-6})}\,\mu A, \qquad RC\ln 2 < t$$



9

Natural and Step Responses of *RLC* Circuits

Drill Exercises

DE 9.1 [a] $\frac{1}{(2RC)^2} = \frac{1}{LC},$ therefore $C = 62.5\,\text{nF}$

[b] $\alpha = 10^4 = \frac{1}{2RC},$ therefore $C = 0.25\,\mu\text{F} = 250\,\text{nF}$

$$s_{1,2} = -10^4 \pm \sqrt{10^8 - \frac{(10^3)(10^6)}{2.5}} = (-10^4 \pm j17{,}320.51)\ \text{rad/s}$$

[c] $\frac{1}{\sqrt{LC}} = 50{,}000,$ therefore $C = 0.04\,\mu\text{F} = 40\,\text{nF}$

$$s_{1,2} = \left[-62.5 \pm \sqrt{(62.5)^2 - 50^2}\right] 10^3,$$

$$s_1 = -25\,\text{krad/s}, \qquad s_2 = -100\,\text{krad/s}$$

DE 9.2
$$i_L = \frac{1000}{50}\int_0^t [-14e^{-5000x} + 26e^{-20{,}000x}]\,dx + 30 \times 10^{-3}$$
$$= 20\left\{\frac{-14e^{-5000x}}{-5000}\Bigg|_0^t + \frac{26e^{-20{,}000t}}{-20{,}000}\Bigg|_0^t\right\} + 30 \times 10^{-3}$$
$$= 56 \times 10^{-3}(e^{-5000t} - 1) - 26 \times 10^{-3}(e^{-20{,}000t} - 1) + 30 \times 10^{-3}$$
$$= [56e^{-5000t} - 56 - 26e^{-20{,}000t} + 26 + 30]\,\text{mA}$$
$$= 56e^{-5000t} - 26e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0$$

DE 9.3 From the given values of R, L, and C, $s_1 = -10\,\text{krad/s}$ and $s_2 = -40\ \text{krad/s}$.

[a] $v(0^-) = v(0^+) = 0,$ therefore $i_R(0^+) = 0$

[b] $i_C(0^+) = 4\,\text{A}$

[c] $C\dfrac{dv_C(0^+)}{dt} = 4,\qquad$ therefore $\dfrac{dv_C(0^+)}{dt} = 8 \times 10^7\,\text{V/s}$

[d] $v = [A_1e^{-10{,}000t} + A_2e^{-40{,}000t}]\,\text{V},\qquad t \geq 0^+$

$v(0^+) = A_1 + A_2,\qquad \dfrac{dv(0^+)}{dt} = -10{,}000A_1 - 40{,}000A_2$

Therefore $A_1 + A_2 = 0,\qquad -A_1 - 4A_2 = 8000,\qquad A_1 = 8000/3$

[e] $A_2 = -8000/3$

[f] $v = [8000/3][e^{-10{,}000t} - e^{-40{,}000t}]\,\text{V},\qquad t \geq 0^+$

DE 9.4 [a] $\dfrac{1}{2RC} = 8000,\qquad$ therefore $R = 62.5\,\Omega$

[b] $i_R(0^+) = \dfrac{10}{62.5} = 160\,\text{mA}$

$i_C(0^+) = -80 - 160 = -240\,\text{mA},\qquad i_C(0^+) = C\dfrac{dv(0^+)}{dt}$

Therefore $\dfrac{dv(0^+)}{dt} = -240\,\text{kV/s}$

[c] $B_1 = v(0^+) = 10\,\text{V},\qquad \dfrac{dv_C(0^+)}{dt} = \omega_d B_2 - \alpha B_1$

Therefore $6000B_2 - 8000B_1 = -240{,}000,\qquad B_2 = (-80/3)\,\text{V}$

[d] $i_L = -(i_R + i_C);\qquad i_R = v/R;\qquad i_C = C\dfrac{dv}{dt}$

$v = e^{-8000t}[10\cos 6000t - \dfrac{80}{3}\sin 6000t]\,\text{V}$

Therefore $i_R = e^{-8000t}[160\cos 6000t - \dfrac{1280}{3}\sin 6000t]\,\text{mA}$

$i_C = e^{-8000t}[-240\cos 6000t + \dfrac{460}{3}\sin 6000t]\,\text{mA}$

$i_L = 10e^{-8000t}[8\cos 6000t + \dfrac{82}{3}\sin 6000t]\,\text{mA},\qquad t \geq 0^+$

DE 9.5 [a] $\left(\dfrac{1}{2RC}\right)^2 = \dfrac{1}{LC} = \dfrac{10^6}{4},\qquad$ therefore $\dfrac{1}{2RC} = 500,\quad R = 100\,\Omega$

[b] $0.5CV_0^2 = 12.5 \times 10^{-3},\qquad$ therefore $V_0 = 50\,\text{V}$

[c] $0.5LI_0^2 = 12.5 \times 10^{-3},\qquad I_0 = 250\,\text{mA}$

[d] $D_2 = v(0^+) = 50,\qquad \dfrac{dv(0^+)}{dt} = D_1 - \alpha D_2$

$i_R(0^+) = \dfrac{50}{100} = 500\,\text{mA}$

Therefore $i_C(0^+) = -(500 + 250) = -750\,\text{mA}$

Therefore $\frac{dv(0^+)}{dt} = -750 \times \frac{10^{-3}}{C} = -75{,}000\,\text{V/s}$

Therefore $D_1 - \alpha D_2 = -75{,}000;\qquad \alpha = \frac{1}{2RC} = 500,\quad D_1 = -50{,}000$

[e] $v = [50e^{-500t} - 50{,}000te^{-500t}]\,\text{V}$

$i_R = \frac{v}{R} = [0.5e^{-500t} - 500te^{-500t}]\,\text{A},\qquad t \geq 0^+$

DE 9.6 [a] $i_R(0^+) = \frac{V_0}{R} = \frac{80}{250} = 0.32\,\text{A}$

[b] $i_C(0^+) = I - i_R(0^+) - i_L(0^+) = -1.5 - 0.32 - 0.5 = -2.32\,\text{A}$

[c] $\frac{di_L(0^+)}{dt} = \frac{80}{.32} = 250\,\text{A/s}$

[d] $\alpha = \frac{1}{2RC} = 1000;\qquad \frac{1}{LC} = 1{,}562{,}500;\qquad s_{1,2} = -1000 \pm j750$

[e] $i_L = i_f + D_1e^{-\alpha t}\cos\omega_d t + D_2e^{-\alpha t}\sin\omega_d t,\qquad i_f = -1.5\,\text{A}$

$i_L(0^+) = 0.5 = i_f + D_1,\qquad$ therefore $D_1 = 2\,\text{A}$

$\frac{di_L(0^+)}{dt} = 250 = -\alpha D_1 + \omega_d D_2,\qquad$ therefore $D_2 = 3\,\text{A}$

Therefore $i_L(t) = -1.5 + 2e^{-1000t}[\cos 750t + 1.5\sin 750t]\,\text{A},\qquad t \geq 0^+$

DE 9.7 [a] $i(0^+) = 0$

[b] $v_C(0^+) = v_C(0^-) = \left(\frac{50}{32}\right)(12.8) = 20\,\text{V}$

[c] $20 + L\frac{di(0^+)}{dt} = 100,\qquad \frac{di(0^+)}{dt} = 8000\,\text{A/s}$

[d] $\alpha = 5000;\qquad \frac{1}{LC} = 5 \times 10^6;\qquad s_{1,2} = -5000 \pm j5000$

[e] $i = i_f + e^{-\alpha t}[D_1\cos\omega_d t + D_2\sin\omega_d t];\qquad i_f = 0,\quad i(0^+) = 0$

Therefore $D_1 = 0;\qquad \frac{di(0^+)}{dt} = 8000 = -\alpha D_1 + \omega_d D_2$

Therefore $D_2 = 1.6\,\text{A};\qquad i = 1.6e^{-5000t}\sin 5000t\,\text{A},\qquad t \geq 0^+$

DE 9.8 $v_C(t) = v_f + e^{-\alpha t}[D_1\cos\omega_d t + D_2\sin\omega_d t],\qquad v_f = 100\,\text{V}$

$v_C(0^+) = 20\,\text{V};\qquad \frac{dv_C(0^+)}{dt} = 0;\qquad$ therefore $20 = 100 + D_1$

$D_1 = -80\,\text{V};\qquad 0 = -\alpha D_1 + \omega_d D_2$

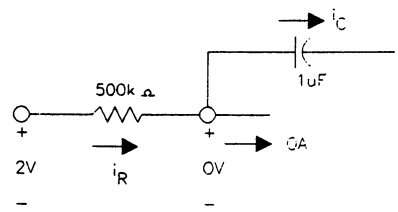
Therefore $D_2 = \frac{\alpha}{\omega_d} D_1 = \left(\frac{5000}{5000}\right)(-80) = -80\text{ V}$

Therefore $v_C(t) = 100 - 80e^{-5000t}[\cos 5000t + \sin 5000t]\text{ V}, \qquad t \geq 0^+$

DE 9.9 At $t = 0$ the voltage across each capacitor is zero. It follows that since the operational amplifiers are ideal, the current in the 500-kΩ is zero. Therefore there cannot be an instantaneous change in the current in the 1-μF capacitor. Since the capacitor current equals $C(dv_o/dt)$, the derivative must be zero.

DE 9.10 [a] From Example 9.13 $\frac{d^2v_o}{dt^2} = 2;$ therefore $\frac{dg(t)}{dt} = 2, \quad g(t) = \frac{dv_o}{dt}$

$g(t) - g(0) = 2t; \qquad g(t) = 2t + g(0); \qquad g(0) = \frac{dv_o(0)}{dt}$



$$i_R = \frac{2}{500} \times 10^{-3} = 4\,\mu\text{A} = i_c = -C\frac{dv_o(0)}{dt}$$

$$\frac{dv_o(0)}{dt} = \frac{-4 \times 10^{-6}}{1 \times 10^{-6}} = -4 = g(0)$$

$$\frac{dv_o}{dt} = 2t - 4$$

$$dv_o = 2t\,dt = 4\,dt$$

$$v_o - v_o(0) = t^2 - 4t; \qquad v_o(0) = 8\text{ V}$$

$$v_o = t^2 - 4t + 8, \qquad 0 \leq t \leq (2 + \sqrt{5})\text{ s}$$

[b] $t^2 - 4t + 8 = 9$

$t^2 - 4t - 1 = 0$

$t = 2 \pm \sqrt{4+1} = 2 \pm \sqrt{5}$

$= 4.24\text{ s}$ (Negative value has no physical significance.)

$t \cong 4.24\text{ s}$

DE 9.11 [a] $R_a = 100\text{ k}\Omega; \quad C_1 = 0.1\,\mu\text{F}; \quad R_b = 25\text{ k}\Omega; \quad C_2 = 1\,\mu\text{F}$

$$\frac{d^2v_o}{dt^2} = \left(\frac{1}{R_aC_1}\right)\left(\frac{1}{R_bC_2}\right)v_g; \qquad \frac{1}{R_aC_1} = 100 \qquad \frac{1}{R_bC_2} = 40$$

$v_g = 250 \times 10^{-3}; \qquad$ therefore $\frac{d^2v_o}{dt^2} = 1000$

[b] Since $v_o(0) = 0 = \frac{dv_o(0)}{dt}$, our solution is $v_o = 500t^2$

The second op-amp will saturate when

$$v_o = 6\,\text{V}, \quad \text{or} \quad t_{sat} = \sqrt{6/500} \cong 0.1095\,\text{s}$$

[c] $\frac{dv_{o1}}{dt} = -\frac{1}{R_aC_1}v_g = -25$

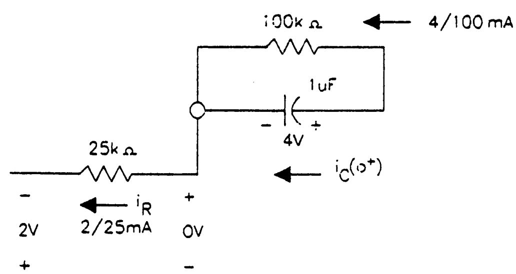
[d] Since $v_{o1}(0) = 0, \quad v_{o1} = -25t\,\text{V}$

At $t = 0.1095\,\text{s}, \quad v_{o1} \cong -2.74\,\text{V}$

Therefore the second amplifier saturates before the first amplifier saturates. Our expressions are valid for $0 \le t \le 0.1095\,\text{s}$. Once the second op-amp saturates, our linear model is no longer valid.

DE 9.12 [a] Initial conditions will not change the differential equation; hence the equation is the same as Example 9.14.

[b] $v_o = 5 + A_1'e^{-10t} + A_2'e^{-20t}$ (from Example 9.14)
$v_o(0) = 4 = 5 + A_1' + A_2'$



$$\frac{4}{100} + i_C(0^+) - \frac{2}{25} = 0$$
$$i_C(0^+) = \frac{4}{100}\,\text{mA} = C\frac{dv_o(0^+)}{dt}$$
$$\frac{dv_o(0^+)}{dt} = \frac{0.04 \times 10^{-3}}{10^{-6}} = 40\,\text{V/s}$$
$$\frac{dv_o}{dt} = -10A_1'e^{-10t} - 20A_2'e^{-20t}$$
$$\frac{dv_o}{dt}(0^+) = -10A_1' - 20A_2' = 40; \quad \text{therefore} \quad \left.\begin{array}{r} -A_1' - 2A_2' = 4 \\ A_1' + A_2' = -1 \end{array}\right\} \begin{array}{l} A_1' = 2 \\ A_2' = -3 \end{array}$$
$$v_o = 5 + 2e^{-10t} - 3e^{-20t}\,\text{V}$$

[c] Same as Example (9.14): $\frac{dv_{o1}}{dt} + 20v_{o1} = -25$

[d] From Example (9.14): $v_{o1}(\infty) = -1.25\,\text{V}, v_1(0) = -2\,\text{V}$ (given)

Therefore $v_{o1} = -1.25 + (-2 + 1.25)e^{-20t}\,\text{V}$

$$v_{o1} = -1.25 - 0.75e^{-20t}\,\text{V}$$

Problems

P 9.1 [a] $s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$

$$\frac{1}{2RC} = \frac{10^6}{(2000)(2)} = 250; \qquad \frac{1}{LC} = \frac{10^6}{25} = 40{,}000$$

$$s_{1,2} = -250 \pm \sqrt{62{,}500 - 40{,}000} = -250 \pm 150$$

$$s_1 = -100\,\text{rad/s}; \qquad s_2 = -400\,\text{rad/s}$$

[b] Overdamped

[c] $\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2} = 120$

$$40{,}000 - \left(\frac{1}{2RC}\right)^2 = 14{,}400$$

$$\left(\frac{1}{2RC}\right)^2 = 25{,}600; \qquad \frac{1}{2RC} = 160; \qquad R = \frac{10^6}{(4)(160)} = 1562.50\,\Omega$$

[d] $\frac{1}{2RC} = \frac{10^6}{4(1562.50)} = 160$

$$\therefore\ s_{1,2} = -160 \pm \sqrt{(160)^2 - 40{,}000} = -160 \pm j120\,\text{rad/s}$$

$$s_1 = -160 + j120\,\text{rad/s}; \qquad s_2 = -160 - j120\,\text{rad/s}$$

[e] $\left(\frac{1}{2RC}\right)^2 = \frac{1}{LC} = 40{,}000; \qquad \frac{1}{2RC} = 200$

$$\therefore\ R = \frac{10^6}{800} = 1250\,\Omega$$

P 9.2 [a] $-\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} = -250$

$$-\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} = -1000$$

$$\therefore\ \frac{-1}{RC} = -1250; \qquad R = \frac{1}{1250C} = \frac{10^6}{125} = 8000\,\Omega$$

$$2\sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} = 750; \qquad 4\left[\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}\right] = (750)^2; \qquad \frac{1}{2RC} = 625$$

$$\therefore\ 1{,}562{,}500 - \frac{4}{LC} = 562{,}500; \qquad \frac{4}{LC} = 10^6; \qquad L = 40\,\text{H}$$

$$\alpha = \frac{1}{2RC} = 625\,\text{neper/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{4}; \qquad \omega_o = 500\,\text{rad/s}$$

[b] $$i_R = \frac{v(t)}{R} = -e^{-250t} + 4e^{-1000t}\,\text{mA}$$

$$i_C = C\frac{dv(t)}{dt} = (0.1 \times 10^{-6})[2000e^{-250t} - 32{,}000e^{-1000t}]$$

$$= 0.2e^{-250t} - 3.2e^{-1000t}\,\text{mA}$$

$$i_L(t) = -i_R(t) - i_C(t) = e^{-250t} - 4e^{-1000t} - 0.2e^{-250t} + 3.2e^{-1000t}$$

$$= 0.8e^{-250t} - 0.8e^{-1000t}\,\text{mA}$$

P 9.3 [a] $i_C(0) = -1.2\,\text{mA}; \qquad i_R(0) = 0; \qquad i_L(0) = 1.2\,\text{mA}$

[b] $$\alpha = \frac{1}{2RC} = \frac{10^6}{2(20{,}000)(0.02)} = \frac{100}{0.08} = 1250\,\text{neper/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{(50)(0.02)} = 10^6$$

$$\omega_o = 1000\,\text{rad/s}$$

$$s_{1,2} = -1250 \pm \sqrt{1{,}562{,}500 - 10^6} = -1250 \pm \sqrt{562{,}500} = -1250 \pm 750$$

$$s_1 = -500\,\text{rad/s}; \qquad s_2 = -2000\,\text{rad/s}$$

$$v(t) = A_1e^{-500t} + A_2e^{-2000t}$$

$$v(0) = 0 = A_1 + A_2; \qquad \therefore\ A_1 = -A_2$$

$$C\frac{dv(0)}{dt} = 0.02 \times 10^{-6}[-500A_1 - 2000A_2]$$

$$10^{-6}(-10A - 40A_2) = -1.2 \times 10^{-3}$$

$$10A_1 + 40A_2 = 1200$$

$$30A_2 = 1200$$

$$A_2 = 40, \qquad A_1 = -40$$

$$\therefore\ v(t) = -40e^{-500t} + 40e^{-2000t}\,\text{V}, \qquad t \geq 0$$

[c] $$i_R(t) = \frac{v}{R} = -2e^{-500t} + 2e^{-2000t}\,\text{mA}, \qquad t \geq 0$$

$$i_C(t) = (0.02 \times 10^{-6})[20{,}000e^{-500t} - 80{,}000e^{-2000t}]$$

$$= 0.4e^{-500t} - 1.6e^{-2000t}\,\text{mA}, \qquad t \geq 0$$

$$i_L(t) = -i_R(t) - i_C(t) = 2e^{-500t} - 2e^{-2000t} - 0.4e^{-500t} + 1.6e^{-2000t}$$

$$i_L(t) = 1.6e^{-500t} - 0.4e^{-2000t}\,\text{mA}, \qquad t \geq 0$$

P 9.4 $$-\alpha + \sqrt{\alpha^2 - \omega_o^2} = -100; \qquad -\alpha - \sqrt{\alpha^2 - \omega_o^2} = -900$$

$$2\alpha = -1000; \qquad \alpha = 500$$

$$\frac{1}{2RC} = 500; \qquad R = \frac{1}{1000C} = \frac{1000}{2.5} = 400\,\Omega$$

$$v(0) = 3(1+1) = 6\,\text{V}$$

$$i_C(0) = C\frac{dv(0)}{dt} = 2.5 \times 10^{-6}(-300 - 2700) = -7.5\,\text{mA}$$

$$i_R(0) = \frac{v(0)}{R} = \frac{6}{400} = 15\,\text{mA}$$

$$i_L(0^+) = -i_R(0^+) - i_C(0^+) = -15 + 7.5 = -7.5\,\text{mA}$$

P 9.5 [a] $\alpha = -20{,}000 = -\frac{1}{2RC}$; $\quad \therefore\ R = \frac{1}{4 \times 10^4 \times 0.04 \times 10^{-6}} = 625\,\Omega$

[b] $\omega_o^2 = \frac{1}{LC}, \qquad L = \frac{1}{\omega_o^2 C}$

$$\omega_o^2 - \alpha^2 = 225 \times 10^6$$

$$\omega_o^2 = 2.25 \times 10^8 + 4 \times 10^8 = 6.25 \times 10^8$$

$$L = \frac{1}{(6.25 \times 10^8)(0.04) \times 10^{-6}} = \frac{1}{25}$$

$$L = 40\,\text{mH}$$

[c] $V_0 = v(0) = 100\,\text{V}$

[d] $I_0 = i_L(0^+) = -i_R(0^+) - i_C(0^+)$

$$i_R(0^+) = \frac{100}{625} = 0.16 = 160\,\text{mA}$$

$$i_C(0^+) = C\frac{dv(0^+)}{dt}$$

$$\frac{dv}{dt} = 100\,\{e^{-20{,}000t}(-15{,}000\sin 15{,}000t - 30{,}000\cos 15{,}000t)$$

$$+(\cos 15{,}000t - 2\sin 15{,}000t)(-20{,}000e^{-20{,}000t})\}$$

$$= 100e^{-20{,}000t}[25{,}000\sin 15{,}000t - 50{,}000\cos 15{,}000t]$$

$$= 25 \times 10^5 e^{-20{,}000t}[\sin 15{,}000t - 2\cos 15{,}000t]$$

$$\frac{dv}{dt}(0) = -50 \times 10^5 = -5 \times 10^6$$

$$i_C(0^+) = (0.04 \times 10^{-6})(-5 \times 10^6) = -0.20 = -200\,\text{mA}$$

$$\therefore\ i_L(0^+) = I_0 = -160 + 200 = 40\,\text{mA}$$

[e] $i_R(t) = \frac{v}{R} = 160e^{-20{,}000t}[\cos 15{,}000t - 2\sin 15{,}000t]\,\text{mA}$

$$i_C = C\frac{dv}{dt} = 100e^{-20{,}000t}[\sin 15{,}000t - 2\cos 15{,}000t]\,\text{mA}$$

$$i_L(t) = -i_R(t) - i_C(t) = -e^{-20{,}000t}[160\cos 15{,}000t - 320\sin 15{,}000t]$$

$$- e^{-20{,}000t}[100\sin 15{,}000t - 200\cos 15{,}000t]$$

$$i_L(t) = e^{-20{,}000t}[220\sin 15{,}000t + 40\cos 15{,}000t]\,\text{mA}, \qquad t \geq 0$$

P9.6 [a] $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} = -40; \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_o^2} = -160$

$\therefore \; -2\alpha = -200; \qquad \alpha = 100\,\text{rad/s}$

$s_1 - s_2 = 2\sqrt{\alpha^2 - \omega_o^2} = 120$

$\therefore \; \sqrt{\alpha^2 - \omega_o^2} = 60; \qquad \alpha^2 - \omega_o^2 = 3600$

$\omega_o^2 = 6400; \qquad \omega_o = 80\,\text{rad/s}$

$\alpha = \frac{1}{2RC}; \qquad C = \frac{1}{2R\alpha} = 25\,\mu\text{F}$

$\omega_o^2 = \frac{1}{LC}; \qquad L = \frac{1}{\omega_o^2 C} = 6.25\,\text{H}$

$i_C(0^+) = A_1 + A_2 = 15 \times 10^{-3}$

$i_L + i_C + i_R = 0; \qquad \frac{di_L}{dt} + \frac{di_R}{dt} + \frac{di_C}{dt} = 0$

At $t = 0^+$ $\quad \frac{di_L}{dt}(0^+) = 0$ since $v(0^+) = 0$

$\therefore \; \frac{di_C}{dt}(0^+) = -\frac{di_R(0^+)}{dt} = -\frac{1}{R}\frac{dv}{dt}(0^+)$

$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C}$

$\therefore \; \frac{di_C}{dt}(0^+) = -\frac{1}{R}\frac{i_C(0^+)}{C} = \frac{-15 \times 10^{-3}}{(200)(25)} \times 10^6 = -3\,\text{A/s}$

$\frac{di_C}{dt}(0^+) = -160A_1 - 40A_2$

$\therefore \; 160A_1 + 40A_2 = 3; \qquad 4A_1 + A_2 = 75 \times 10^{-3}$

Also $A_1 + A_2 = 15 \times 10^{-3}; \qquad \therefore \; A_1 = 20\,\text{mA}, \quad A_2 = -5\,\text{mA}$

$\therefore \; i_C = 20e^{-160t} - 5e^{-40t}\,\text{mA}, \qquad t \geq 0^+$

[b] We know that the solution for v will also be of the form

$v = A_3 e^{-160t} + A_4 e^{-40t}, \qquad t \geq 0$

By hypothesis

$v(0^+) = A_3 + A_4 = 0$

$\frac{dv(0^+)}{dt} = -160A_3 - 40A_4 = \frac{+15 \times 10^{-3}}{25 \times 10^{-6}}$

$\therefore \; 4A_3 + A_4 = -15; \qquad \therefore \; A_3 = -5\,\text{V}, \quad A_4 = 5\,\text{V}$

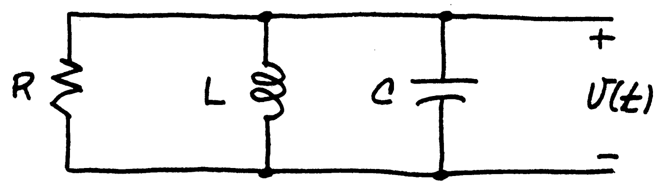
$v = -5e^{-160t} + 5e^{-40t}\,\text{V}, \qquad t \geq 0$

[c] $i_R = \frac{v}{R} = \frac{v}{200} = -25e^{-160t} + 25e^{-40t}\,\text{mA}, \qquad t \geq 0$

[d] $i_L = -i_R - i_C = 25e^{-160t} - 25e^{-40t} - 20e^{-160t} + 5e^{-40t}$

$= 5e^{-160t} - 20e^{-40t}\,\text{mA}, \qquad t \geq 0$

P 9.7 [a]



$$\alpha = 500 = \frac{1}{2RC}$$

$$\omega_o = 500, \qquad \omega_o^2 = 25 \times 10^4 = \frac{1}{LC}$$

$$\therefore\ C = \frac{1}{4(25 \times 10^4)} = 1\,\mu\text{F}$$

$$R = \frac{1}{1000C} = 1000\,\Omega$$

$$v(0) = D_2 = 8\,\text{V}$$

$$i_R(0^+) = \frac{8}{1000} = 8\,\text{mA}; \qquad i_L(0^+) = -10\,\text{mA}$$

$$\therefore\ i_C(0^+) = -i_R(0^+) - i_L(0^+) = 2\,\text{mA}$$

$$\frac{dv}{dt}(0^+) = \frac{2 \times 10^{-3}}{1 \times 10^{-6}} = 2000\,\text{V/s}$$

$$\frac{dv}{dt} = D_1 e^{-500t} - 500e^{-500t}[D_1 t + D_2]$$

$$\frac{dv}{dt}(0^+) = D_1 - 500D_2 = 2000$$

$$\therefore\ D_1 = 2000 + 500(8) = 6000\,\text{V/s}$$

$$\therefore\ v = (6000t + 8)e^{-500t}\,\text{V}, \qquad t \geq 0$$

[b] $$i_C = C\frac{dv}{dt}$$

$$\therefore\ i_C = 10^{-6}\left[(6000t + 8)(-500e^{-500t}) + 6000e^{-500t}\right]$$

$$= -3te^{-500t} + 2 \times 10^{-3}e^{-500t}$$

$$i_C = -3000te^{-500t} + 2e^{-500t}\,\text{mA}, \qquad t \geq 0^+$$

P 9.8 [a] $$\alpha = \frac{1}{2RC} = \frac{1}{2} = 0.5$$

$$\omega_o^2 = \frac{1}{LC} = \frac{101}{4} = 25.25$$

$$\omega_d = \sqrt{25.25 - 0.25} = 5\,\text{rad/s}$$

$$\therefore\ v = B_1 e^{-0.5t}\cos 5t + B_2 e^{-0.5t}\sin 5t$$

$$v(0) = B_1 = 0; \qquad v = B_2 e^{-0.5t}\sin 5t$$

$$\frac{dv}{dt} = B_2\left[e^{-0.5t}(5\cos 5t) - 0.5e^{-0.5t}\sin 5t\right] = 0.5B_2 e^{-0.5t}[10\cos 5t - \sin 5t]$$

$$C\frac{dv}{dt}(0) = 0.04B_2(10) = 0.4B_2$$

$$\therefore\ 0.4B_2 = -(-4) = 4; \qquad B_2 = 10$$

$$\therefore\ v = 10e^{-0.5t}\sin 5t\,\text{V}, \qquad t \geq 0$$

[b] $\frac{dv}{dt} = 5e^{-0.5t}(10\cos 5t - \sin 5t)$

$\frac{dv}{dt} = 0$ when $10\cos 5t = \sin 5t$ or $\tan 5t = 10$

$\therefore\ 5t_1 = 1.47, \quad t_1 = 294.23\,\text{ms}$

$5t_2 = 1.47 + \pi, \quad t_2 = 922.54\,\text{ms}$

$5t_3 = 1.47 + 2\pi, \quad t_3 = 1550.86\,\text{ms}$

[c] $t_3 - t_1 = 1256.64\,\text{ms}; \quad T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{5} = 1256.64\,\text{ms}$

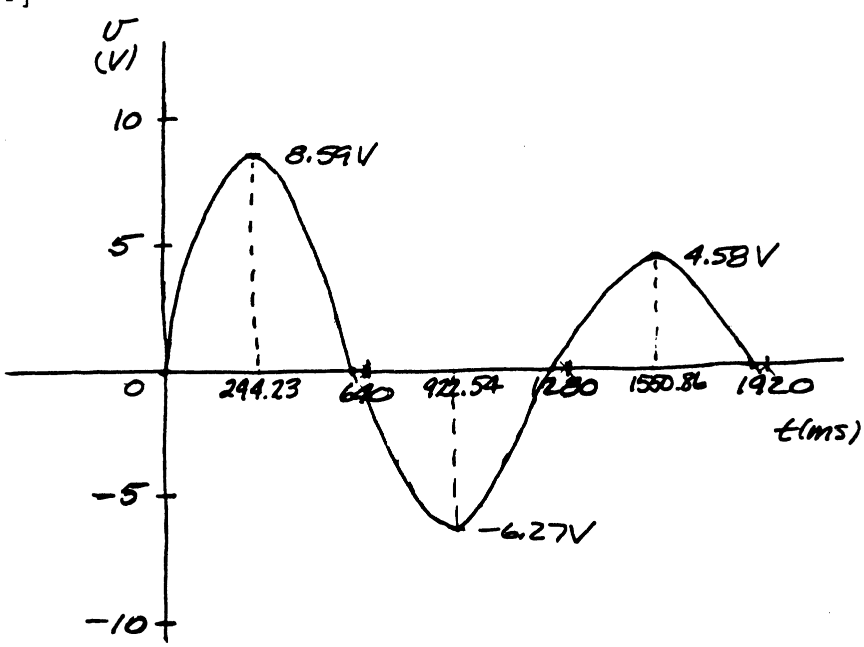
[d] $t_2 - t_1 = 628.32\,\text{ms}; \quad \frac{T_d}{2} = \frac{1256.64}{2} = 628.32\,\text{ms}$

[e] $v(t_1) = 10e^{-0.5(0.29423)}\sin 5(0.29423) = 8.59\,\text{V}$

$v(t_2) = 10e^{-0.5(0.92254)}\sin 5(0.92254) = -6.27\,\text{V}$

$v(t_3) = 10e^{-0.5(1.55086)}\sin 5(1.55086) = 4.58\,\text{V}$

[f]



P9.9 [a] $\alpha = 0; \quad \omega_d = \omega_o = \sqrt{25.25} = 5.02\,\text{rad/s}$

$v = B_1\cos\omega_o t + B_2\sin\omega_o t; \quad v(0) = B_1 = 0; \quad v = B_2\sin\omega_o t$

$\frac{dv}{dt} = \omega_o B_2\cos\omega_o t; \quad C\frac{dv}{dt}(0) = \omega_o C B_2 = -i_L(0) = 4$

$\therefore\ B_2 = \frac{4}{0.08\sqrt{25.25}} = \frac{50}{\sqrt{25.25}} = 9.95$

$v = 9.95\sin 5.02t\,\text{V}, \quad t \geq 0$

[b] $2\pi f = 5.02; \qquad f = \frac{5.02}{2\pi} \cong 0.80\,\text{Hz}$

[c] 9.95 V

P9.10 [a] $\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(12.5)(3.2)} = 25 \times 10^6$

$$\alpha^2 = 25 \times 10^6, \qquad \alpha = 5000 = \frac{1}{2RC}$$

$$\therefore\ R = \frac{10^9}{10{,}000}(3.2) = \frac{10^5}{3.2} = 31.25\,\text{k}\Omega$$

[b] $v(t) = (D_1 t + D_2)e^{-5000t}$

$$v(0) = D_2 = 100\,\text{V}; \qquad v(t) = (D_1 t + 100)e^{-5000t}$$

$$i_C(0^+) = i_L(0^+) - i_R(0^+); \qquad i_R(0^+) = \frac{100}{31.25} = 3.2\,\text{mA}$$

$$\therefore\ i_C(0^+) = -6.4 - 3.2 = -9.6\,\text{mA}$$

$$\frac{dv}{dt} = (D_1 t + 100)(-5000e^{-5000t}) + D_1 e^{-5000t}$$

$$= (-5000D_1 t - 5 \times 10^5 + D_1)e^{-5000t}$$

$$\frac{dv}{dt}(0) = D_1 - 5 \times 10^5 = \frac{-9.6 \times 10^{-3}}{3.2 \times 10^{-9}}$$

$$\therefore\ D_1 = 5 \times 10^5 - 3 \times 10^6 = (5 - 30)10^5 = -25 \times 10^5$$

$$\therefore\ v(t) = [-25 \times 10^5 t + 100]e^{-5000t}\,\text{V}, \qquad t \geq 0$$

[c] $i_C(t) = 0 \quad \text{when} \quad \frac{dv(t)}{dt} = 0$

$$\frac{dv}{dt} = [-25 \times 10^5 t + 100](-5000e^{-5000t}) - 25 \times 10^5 e^{-5000t}$$

$$= (125 \times 10^8 t - 30 \times 10^5)e^{-5000t}$$

$$\frac{dv}{dt} = 0 \quad \text{when} \quad 125 \times 10^8 t = 30 \times 10^5 \quad \text{or} \quad t = \frac{30 \times 10^{-3}}{125} = 240\,\mu\text{s}$$

$$v(240\,\mu\text{s}) = \left[-25 \times 10^5(240) \times 10^{-6} + 100\right] e^{-1.2}$$

$$= (-600 + 100)e^{-1.2} = -500e^{-1.2} \cong -150.60\,\text{V}$$

[d] $w(0) = \frac{1}{2}(12.5)(6.4 \times 10^{-3})^2 + \frac{1}{2}(3.2 \times 10^{-9})(100)^2 = 256 + 16 = 272\,\mu\text{J}$

When $i_C(t) = 0, \quad t = 240\,\mu\text{s}$

$$\therefore\ i_L(240\,\mu\text{s}) = -i_R(240\,\mu\text{s})$$

$$i_R(240\,\mu\text{s}) = \frac{v(240\,\mu\text{s})}{31.25 \times 10^3} = \frac{-500e^{-1.2}}{31.25 \times 10^3} = -16e^{-1.2}\,\text{mA}$$

$$\therefore\ i_L(240\,\mu\text{s}) = 16e^{-1.2}\,\text{mA}$$

$$\therefore\ w(240\,\mu s) = \frac{1}{2}(3.2\times10^{-9})(25\times10^{4})e^{-2.4} + \frac{1}{2}(12.5)(256\times10^{-6}e^{-2.4})$$
$$= 2000e^{-2.4}\,\mu J$$
$$\% = \frac{2000e^{-2.4}}{272}\times 100 = 66.70\%$$

P9.11 [a] $\alpha = \dfrac{1}{2RC} = 1250,\qquad \omega_o = 10^3,\qquad$ therefore overdamped

$s_1 = -500,\qquad s_2 = -2000,\qquad$ therefore $v = A_1e^{-500t} + A_2e^{-2000t}$

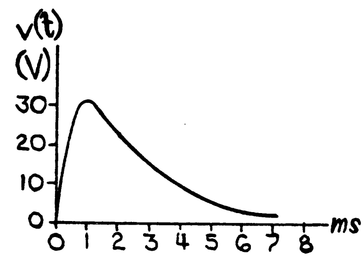
$$v(0^+) = 0 = A_1 + A_2;\qquad \left[\frac{dv(0^+)}{dt}\right] = \frac{i_C(0^+)}{C} = 98{,}000\text{ V/s}$$

Therefore $-500A_1 - 2000A_2 = 98{,}000;\qquad A_1 = \dfrac{+980}{15},\quad A_2 = \dfrac{-980}{15}$

$$v(t) = \left[\frac{980}{15}\right][e^{-500t} - e^{-2000t}]\text{ V},\qquad t \ge 0^+$$

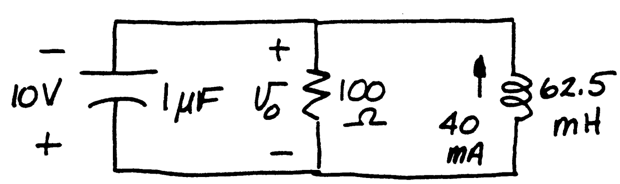
[b]

t(ms)	0	1	2	3	4	5	6	7
$v(t)$(V)	0.0	30.78	22.84	14.42	8.82	5.36	3.25	1.97



Example 9.4: $v_{max} \cong 74$ V at 1.4 ms
Example 9.5: $v_{max} \cong 36$ V at 1.0ms
Problem 9.11: $v_{max} \cong 30.8$ at 0.92 ms

P9.12 For $t > 0$:



$$\alpha = \frac{1}{2RC} = \frac{10^6}{200} = 5000;\qquad \omega_o^2 = \frac{1}{LC} = \frac{10^9}{62.5} = 16\times10^6$$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - 16 \times 10^6} = -5000 \pm 3000$$
$$s_1 = -2000\,\text{rad/s}, \quad s_2 = -8000\,\text{rad/s}; \qquad \therefore \text{ overdamped}$$
$$v_o = A_1 e^{-2000t} + A_2 e^{-8000t}$$
$$v_o(0) = A_1 + A_2 = -10; \qquad \frac{dv_o}{dt}(0) = -2000A_1 - 8000A_2$$
$$i_R(0) = \frac{-10}{100} = -0.10\,\text{A}; \qquad i_L(0) = -40 \times 10^{-3} = -0.04\,\text{A}$$
$$i_C(0) + i_R(0) + i_L(0) = 0$$
$$\therefore\; i_C(0) = 0.14\,\text{A}; \qquad i_C(0) = C\frac{dv_o}{dt}(0)$$
$$\therefore\; \frac{dv_o}{dt}(0) = \frac{0.14}{1} \times 10^6 = -2000A_1 - 8000A_2$$
$$\therefore\; -2A_1 - 8A_2 = 140; \qquad -A_1 - 4A_2 = 70; \qquad A_1 + A_2 = -10$$
$$\therefore\; -3A_2 = 60; \qquad A_2 = -20$$
$$\therefore\; A_1 = -10 - (-20) = 10$$
$$\therefore\; v_o(t) = 10e^{-2000t} - 20e^{-8000t}\,\text{V}, \qquad t \geq 0$$

P 9.13 $\alpha = \frac{1}{2RC} = \frac{10^6}{500} = 2000; \qquad \omega_o^2 = \frac{1}{LC} = 16 \times 10^6$

$$\omega_o^2 > \alpha^2; \qquad \therefore \text{ underdamped}$$
$$s_{1,2} = -2000 \pm j\sqrt{16 \times 10^6 - 4 \times 10^6} = -2000 \pm j2000\sqrt{3}\,\text{rad/s}$$
$$v_o(t) = B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t$$
$$\alpha = 2000; \qquad \omega_d = 2000\sqrt{3}$$
$$v_o(0) = B_1 = -10\,\text{V}$$
$$\frac{dv_o}{dt}(0) = -\alpha B_1 + \omega_d B_2 = \frac{i_C(0)}{C} = 10^6 i_C(0)$$
$$i_R(0) = \frac{-10}{250} = -0.04\,\text{A}; \qquad i_L(0) = -0.04\,\text{A}$$
$$\therefore\; i_C(0) = -(-0.04 - 0.04) = 0.08\,\text{A}$$
$$10^6 i_C(0) = 80{,}000$$
$$\therefore\; -2000B_1 + 2000\sqrt{3}\,B_2 = 80{,}000$$
$$-B_1 + \sqrt{3}\,B_2 = 40; \qquad 10 + \sqrt{3}\,B_2 = 40; \qquad B_2 = \frac{30}{\sqrt{3}} = 10\sqrt{3}$$
$$\therefore\; v_o(t) = -10e^{-2000t}\cos 2000\sqrt{3}t + 10\sqrt{3}\,e^{-2000t}\sin 2000\sqrt{3}t$$
$$v_o(t) = 10\left[\sqrt{3}\sin 2000\sqrt{3}t - \cos 2000\sqrt{3}t\right]e^{-2000t}\,\text{V}, \qquad t \geq 0$$

P 9.14 $\alpha = \frac{10^6}{250} = 4000, \qquad \alpha^2 = 16 \times 10^6$

$$\omega_o^2 = \frac{1}{LC} = 16 \times 10^6; \qquad \alpha^2 = \omega_o^2 \quad \text{critical damping}$$

$\therefore\ v_o(t) = D_1te^{-\alpha t} + D_2e^{-\alpha t}, \qquad \alpha = 4000$

$v_o(0) = D_2 = -10\,\text{V}$

$\frac{dv_o}{dt}(0) = -4000D_2 + D_1 = \frac{i_C(0)}{C} = 10^6 i_C(0)$

$i_R(0) = \frac{-10}{125} = -0.08A_1; \qquad i_L(0) = -0.04\,\text{A}$

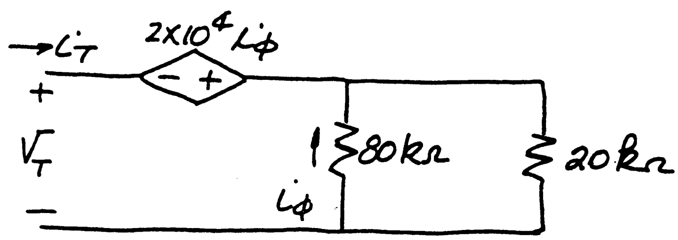
$\therefore\ i_C(0) = 0.12\,\text{A}$

$\therefore\ -4000D_2 + D_1 = 120 \times 10^3; \qquad \therefore\ D_1 = (120 - 40) \times 10^3 = 8 \times 10^4\,\text{V/s}$

$v_o = (8 \times 10^4 t - 10)e^{-4000t}\,\text{V}, \qquad t \geq 0$

P 9.15 $v_o(0) = \frac{3}{5}(50) = 30\,\text{V}$

Thévenin resistance:



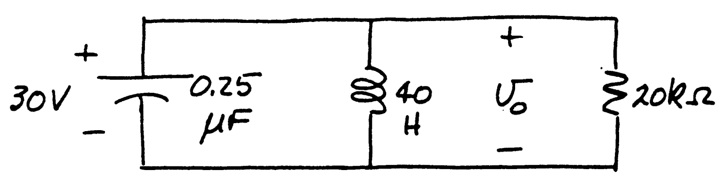
$$V_T = -2 \times 10^4 i_\phi + i_T\frac{(80)(20)}{100} \times 10^3 = -2 \times 10^4 i_\phi + 16 \times 10^3 i_T$$

$$i_\phi = \frac{-i_T(20)}{100} = -\frac{i_T}{5}$$

$$V_T = -20 \times 10^3\left(\frac{-i_T}{5}\right) + 16 \times 10^3 i_T$$

$$R_{\text{Th}} = \frac{V_T}{i_T} = 20 \times 10^3 = 20\,\text{k}\Omega$$

$t \geq 0$:



$$\alpha = \frac{1}{2RC} = \frac{10^6}{40 \times 10^3(0.25)} = 100; \qquad \alpha^2 = 10^4$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{(40)(0.25)} = 10^5$$

$\omega_o^2 > \alpha^2; \qquad \therefore$ underdamped

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{10 \times 10^4 - 10^4} = 300\,\text{rad/s}$$

$$v_o(t) = (B_1 \cos 300t + B_2 \sin 300t)e^{-100t}$$

$$i_R(0) = \frac{30}{20} = 1.5\,\text{mA}; \qquad i_L(0) = 0$$

$$\therefore\ i_C(0) = -1.5\,\text{mA} = C\frac{dv_o}{dt}(0)$$

$$\therefore\ \frac{dv_o}{dt}(0) = \frac{-1.5}{0.25} \times 10^3 = -6 \times 10^3\,\text{V/s}$$

$$\frac{dv_o}{dt} = (B_1 \cos 300t + B_2 \sin 300t)(-100e^{-100t})$$

$$+ (-300B_1 \sin 300t + 300B_2 \cos 300t)e^{-100t}$$

$$\frac{dv_o}{dt}(0) = -100B_1 + 300B_2 = -6 \times 10^3$$

$$v_o(0) = B_1 = 30$$

$$\therefore\ -B_1 + 3B_2 = -6 \times 10$$

$$\therefore\ 3B_2 = -60 + 30 = -30; \qquad B_2 = -10$$

$$\therefore\ v_o = [30 \cos 300t - 10 \sin 300t]e^{-100t}\,\text{V}, \qquad t \geq 0$$

P 9.16 [a] $v = L\left(\frac{di_L}{dt}\right) = 16[e^{-20{,}000t} - e^{-80{,}000t}]\,\text{V}, \qquad t \geq 0^+$

[b] $i_R = \frac{v}{R} = 40[e^{-20{,}000t} - e^{-80{,}000t}]\,\text{mA}, \qquad t \geq 0^+$

[c] $i_C = I - i_L - i_R = [-8e^{-20{,}000t} + 32e^{-80{,}000t}]\,\text{mA}, \qquad t \geq 0^+$

P 9.17 [a] $v = L\left(\frac{di_L}{dt}\right) = 40e^{-32{,}000t} \sin 24{,}000t\,\text{V}, \qquad t \geq 0^+$

[b] $i_C(t) = I - i_R - i_L$

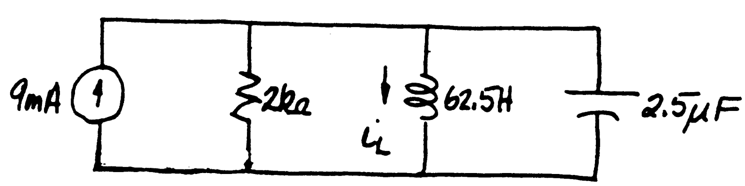
$= [24e^{-32{,}000t} \cos 24{,}000t - 32e^{-32{,}000t} \sin 24{,}000t]\,\text{mA}, \qquad t \geq 0^+$

P 9.18 $v = L\left(\frac{di_L}{dt}\right) = 960{,}000te^{-40{,}000t}\,\text{V}, \qquad t \geq 0^+$

P 9.19 For $t < 0$:

$$i_L(0^-) = \frac{9}{3000} = 3\,\text{mA} = i_L(0^+); \qquad v_C(0^-) = 0 = v_C(0^+)$$

For $t > 0$:



$$\alpha = \frac{1}{2RC} = \frac{10^6}{2(2000)(2.5)} = 100; \qquad \omega_o^2 = \frac{1}{LC} = \frac{10^6}{(62.5)(2.5)} = 6400$$

$\alpha^2 = 10{,}000; \quad \therefore\ \alpha^2 > \omega_o^2$ overdamped

$s_1 = -100 + 60 = -40\,\text{rad/s}; \qquad s_2 = -100 - 60 = -160\,\text{rad/s}$

$\therefore\ i_L = i_F + A_1' e^{-40t} + A_2' e^{-160t}, \qquad i_F = 9\,\text{mA}$

$i_L = 9 + A_1' e^{-40t} + A_2' e^{-160t}\,\text{mA}, \qquad t \ge 0$

$i_L(0) = 3 = 9 + A_1' + A_2'; \qquad \therefore\ A_1' + A_2' = -6 \times 10^{-3}$

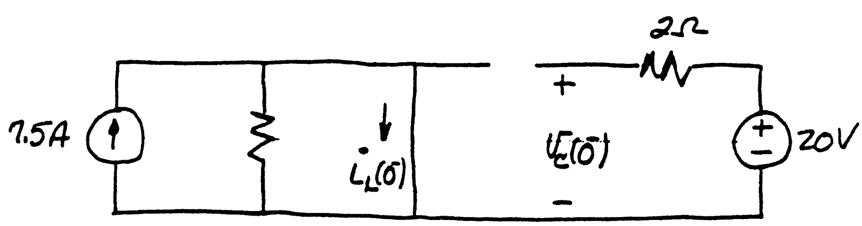
$\frac{di_L}{dt}(0^+) = 0 = -40A_1' - 160A_2'$

$\therefore\ A_1' = -4A_2'$

$-3A_2' = -6\,\text{mA}; \qquad A_2' = 2\,\text{mA}; \qquad A_1' = -8\,\text{mA}$

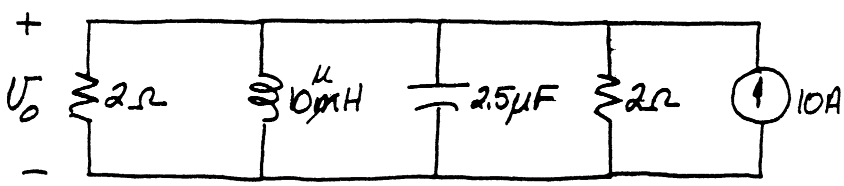
$i_L = 9 - 8e^{-40t} + 2e^{-160t}\,\text{mA}, \qquad t \ge 0$

P 9.20 $t < 0$:



$i_L(0^-) = 7.5\,\text{A} = i_L(0^+); \qquad v_C(0^-) = 20\,\text{V} = v_C(0^+)$

$t > 0$:



$$\alpha = \frac{1}{2RC} = \frac{10^6}{(2)(1)(2.5)} = 20 \times 10^4; \qquad \omega_o^2 = \frac{1}{LC} = \frac{10^{12}}{25} = 4 \times 10^{10}$$

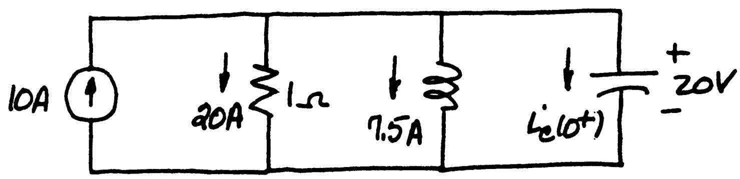
$\alpha^2 = \omega_o^2$ critically damped

$v_o = v_F + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}, \qquad v_F = 0$

$v_o = (D_1' t + D_2')e^{-200{,}000t}$

$v_o(0^+) = 20 = D_2'$

At $t = 0^+$:



$\therefore\ i_C(0^+) = 10 - 20 - 7.5 = -17.5\,\text{A}$

$\frac{dv_o}{dt}(0^+) = \frac{-17.5}{2.5} \times 10^6 = -7 \times 10^6$

$\frac{dv_o}{dt}(0^+) = -2 \times 10^5 D_2' + D_1' = -7 \times 10^6$

$-2 \times 10^5(20) + D_1' = -7 \times 10^6; \qquad D_1' = -3 \times 10^6\,\text{V/s}$

$\therefore\ v_o = -3 \times 10^6 te^{-2\times10^5 t} + 20e^{-2\times10^5 t}\,\text{V}, \qquad t \geq 0^+$

P 9.21 From the solution of Problem 9.20 we know i_L will be of the form

$$i_L = i_F + D_3' te^{-2\times10^5 t} + D_4' e^{-2\times10^5 t}$$

We also have from the Problem 9.20 solution

$$i_L(0^-) = i_L(0^+) = 7.5\,\text{A}$$

and

$$L\frac{di_L}{dt}(0^+) = 20\,\text{V} \quad \text{or} \quad \frac{di_L}{dt}(0^+) = 2 \times 10^6\,\text{A/s}$$

The final value of i_L is 10 A, hence we have

$$i_L = 10 + D_3' te^{-2\times10^5 t} + D_4' e^{-2\times10^5 t}$$

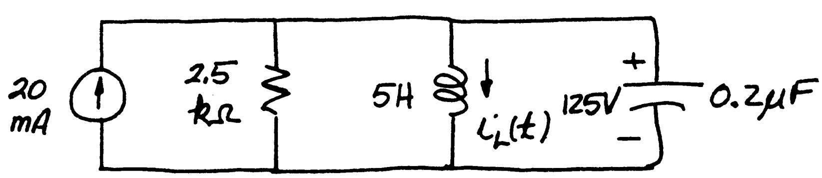
$$i_L(0^+) = 7.5 = 10 + D_4'; \qquad D_4' = -2.5A$$

$$\frac{di_L}{dt}(0^+) = 2 \times 10^6 = -200{,}000 D_4' + D_3'$$

$$\therefore\ D_3' = 2 \times 10^6 - 0.5 \times 10^6 = 1.5 \times 10^6\,\text{A/s}$$

$\therefore\ i_L = 10 + [1.5 \times 10^6 t - 2.5]e^{-200{,}000t}\,\text{A}, \qquad t \geq 0$

P 9.22 For $t > 0$:



$$\alpha = \frac{1}{2RC} = \frac{10^6}{5000(0.2)} = 10^3 = 1000\,\text{rad/s}$$
$$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{(5)(0.2)} = 10^6; \qquad \therefore\ \alpha^2 = \omega_o^2 \quad \text{critical damping}$$
$$i_L(t) = 20 \times 10^{-3} + D_1' te^{-1000t} + D_2' e^{-1000t}$$
$$i_L(0) = 0 = 20 \times 10^{-3} + D_2'; \qquad \therefore\ D_2' = -20 \times 10^{-3}$$
$$L\frac{di_L(0)}{dt} = 125, \qquad \frac{di_L(0)}{dt} = \frac{125}{5} = 25$$
$$\frac{di_L(t)}{dt} = (D_1't + D_2')(-1000e^{-1000t}) + e^{-1000t}(D_1')$$
$$\frac{di_L(0)}{dt} = -1000D_2' + D_1' = 25$$
$$\therefore\ D_1' = 25 + 1000D_2' = 25 + 1000(-20 \times 10^{-3}) = 5$$
$$i_L(t) = 20 \times 10^{-3} + 5te^{-1000t} - 20 \times 10^{-3}e^{-1000t}$$
$$i_L(t) = 20 + (5000t - 20)e^{-1000t}\,\text{mA}, \qquad t \geq 0$$

P 9.23 [a] $i_L(0^+) = 0$

[b] $\frac{di_L}{dt}(0^+) = 0$

[c]
$$\alpha = \frac{1}{2RC} = \frac{1}{(4)(0.25)} = 1\,\text{rad/s}, \qquad \alpha^2 = 1$$
$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(0.8)(0.25)} = 5, \qquad \omega_o^2 > \alpha^2 \quad \text{underdamped}$$
$$s_{1,2} = -1 \pm \sqrt{1-5} = -1 \pm j2\,\text{rad/s}$$
$$\therefore\ i_L(t) = -16 + B_1'e^{-t}\cos 2t + B_2'e^{-t}\sin 2t$$
$$i_L(0) = -16 + B_1' = 0, \qquad B_1' = 16\,\text{A}$$
$$\frac{di_L}{dt} = (B_1'\cos 2t + B_2'\sin 2t)(-e^{-t}) + e^{-t}(-2B_1'\sin 2t + 2B_2\cos 2t)$$
$$\frac{di_L}{dt}(0^+) = -B_1' + 2B_2' = 0$$
$$B_2' = \frac{B_1'}{2} = \frac{16}{2} = 8\,\text{A}$$
$$\therefore\ i_L(t) = -16 + 16e^{-t}\cos 2t + 8e^{-t}\sin 2t\,\text{A}, \qquad t \geq 0$$

P 9.24 [a] Let v_C be the voltage across the capacitor, positive at the top terminal. Then note

$$5\frac{di}{dt} + 20\frac{di}{dt} = v_C$$
$$\frac{1}{4}v_o + v_o = v_C$$
$$1.25v_o = v_C; \qquad v_o = 0.8v_C$$

Now $v_C(0^+) = 0; \qquad \therefore\ v_o(0^+) = 0$

[b] $\frac{dv_o}{dt} = 0.8\frac{dv_C}{dt}$

$$C\frac{dv_C}{dt}(0^+) = -15 \times 10^{-3}; \qquad \frac{dv_C}{dt}(0^+) = \frac{-15 \times 10^{-3}}{0.25 \times 10^{-6}} = -60{,}000\,\text{V/s}$$

$$\therefore\ \frac{dv_o}{dt}(0^+) = 0.8(-60{,}000) = -48{,}000\,\text{V/s}$$

[c] $R = 20\,\text{k}\Omega // 5\,\text{k}\Omega = 4\,\text{k}\Omega$

$$\alpha = \frac{10^6}{8000(0.25)} = \frac{1000}{2} = 500\,\text{rad/s}, \qquad \alpha^2 = 25 \times 10^4$$

$$\omega_o^2 = \frac{10^6}{(25)(0.25)} = \frac{400 \times 10^4}{25} = 16 \times 10^4$$

$\alpha^2 > \omega_o^2$ overdamped

$$\therefore\ v_C = V_f + A_1' e^{s_1 t} + A_2' e^{-s_2 t}; \qquad v_C(\infty) = V_f = 0$$

$$s_{1,2} = -500 \pm \sqrt{(25 - 16)10^4} = -500 \pm 300$$

$$s_1 = -200\,\text{rad/s}, \qquad s_2 = -800\,\text{rad/s}$$

$$\therefore\ v_C = A_1' e^{-200t} + A_2' e^{-800t}$$

$$v_C(0) = 0 = A_1' + A_2'; \qquad \therefore\ A_1' = -A_2'$$

$$\frac{dv_C}{dt}(0) = -200A_1' - 800A_2' = -60{,}000$$

$$-200A_1' + 800A_1' = -60{,}000; \qquad 6A_1' = -600; \qquad A_1' = -100$$

$$v_C(t) = -100e^{-200t} + 100e^{-800t}\,\text{V}$$

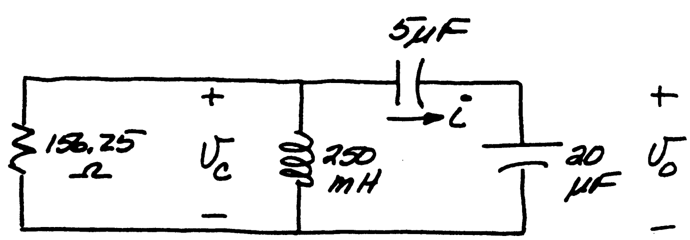
$$\therefore\ v_o(t) = 80e^{-800t} - 80e^{-200t}\,\text{V}, \qquad t \geq 0$$

$$= 80[e^{-800t} - e^{-200t}]\,\text{V}, \qquad t \geq 0$$

P 9.25 [a] $v_o(0^+) = 0$

[b] $20 \times 10^{-6}\frac{dv_o}{dt}(0^+) = 60 \times 10^{-3}; \qquad \frac{dv_o(0^+)}{dt} = \frac{60{,}000}{20} = 3000\,\text{V/s}$

[c]



$$\alpha = \frac{1}{2RC_e} = \frac{10^6}{(312.5)(4)} = 800; \qquad \text{Note: } C_e = \frac{(5)(20)}{25} = 4\,\mu\text{F}$$

$$\omega_o^2 = \frac{1}{LC_e} = \frac{10^6}{(0.250)(4)} = 10^6$$

$$\alpha^2 = 64 \times 10^4 = 0.64 \times 10^6$$

$$\omega_o^2 > \alpha^2 \quad \text{underdamped;} \qquad \omega_d = 600\,\text{rad/s}$$

$$\therefore\ v_C = V_f + B_1' e^{-800t}\cos 600t + B_2' e^{-800t}\sin 600t$$

$$V_f = 0, \qquad v_C(0^+) = 0, \qquad \therefore\ B_1' = 0$$

$$\therefore\ v_C = B_2' e^{-800t}\sin 600t$$

$$\frac{dv_C}{dt} = B_2'\left[e^{-800t}(600)\cos 600t - 800e^{-800t}\sin 600t\right]$$

$$= B_2' e^{-800t}[600\cos 600t - 800\sin 600t]$$

$$\frac{dv_C}{dt}(0^+) = 600B_1'$$

$$C_e\frac{dv_C}{dt}(0^+) = 60 \times 10^{-3}$$

$$\frac{dv_C(0^+)}{dt} = \frac{60{,}000}{4} = 15{,}000\,\text{V/s}$$

$$\therefore\ 600B_2' = 15{,}000, \qquad B_2' = 25$$

$$v_C(t) = 25e^{-800t}\sin 600t\,\text{V}$$

$$v_C = \frac{10^6}{5}\int_0^t i\,dt + \frac{10^6}{20}\int_0^t i\,dt = 4v_o + v_o = 5v_o$$

$$\therefore\ v_o(t) = \frac{1}{5}v_C(t)$$

$$v_o(t) = 5e^{-800t}\sin 600t\,\text{V}, \qquad t \ge 0$$

P 9.26 [a] $\alpha = \dfrac{1}{2RC} = \dfrac{10^6}{20 \times 10^3(2.5)} = 20$

$$\alpha^2 = 400, \qquad \omega_o^2 = \frac{1}{LC} = 20{,}000$$

$$\omega_o^2 > \alpha^2, \qquad \therefore\ \text{underdamped}$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{19{,}600} = 140\,\text{rad/s}$$

$$\therefore\ i_L(t) = I_g + B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t$$

$$i_L(0) = I_g + B_1'$$

$$\frac{di_L(0)}{dt} = -\alpha B_1' + \omega_d B_2'$$

$$\frac{1}{2}(2.5 \times 10^{-6})(56)^2 + \frac{1}{2}(20)i_L(0)^2 = 11.76 \times 10^{-3}$$

$$10i_L(0)^2 = 11.76 \times 10^{-3} - 3.92 \times 10^{-3} = 7.84 \times 10^{-3}$$

$$i_L(0)^2 = 7.84 \times 10^{-4}; \qquad i_L(0) = \pm 2.8 \times 10^{-2} = \pm 28\,\text{mA}$$

$$\therefore\ B_1' = \pm 28 - 7\,\text{mA}$$

$$\frac{di_L(0)}{dt} = \frac{v(0)}{L} = \frac{56}{20} = 2.8$$

$$\therefore\ -20B_1' + 140B_2' = 2.8$$

$$-B_1' + 7B_2' = 0.14$$

$$\therefore\ 7B_2' = 0.14 + B_1'$$

When $B_1' = 21\,\text{mA}$, $7B_2' = 140 + 21$; $B_2' = 20 + 3 = 23\,\text{mA}$

When $B_1' = -35\,\text{mA}$, $B_2' = \dfrac{140 - 35}{7} = 15\,\text{mA}$

$$\therefore\ i_L(t) = 7 + 21e^{-20t}\cos 140t + 23e^{-20t}\sin 140t\,\text{mA}, \qquad t \geq 0$$

$$i_L(t) = 7 - 35e^{-20t}\cos 140t + 15e^{-20t}\sin 140t\,\text{mA}, \qquad t \geq 0$$

[b] $v = L\dfrac{di_L}{dt} = 20\dfrac{di_L}{dt}$

$$\frac{di_L}{dt} = \frac{d}{dt}[-35\cos 140t + 15\sin 140t]e^{-20t} \times 10^{-3}$$

$$= 10^{-3}[-35\cos 140t + 15\sin 140t](-20e^{-20t})$$

$$+ 10^{-3}e^{-20t}[4900\sin 140t + 2100\cos 140t]$$

$$= 10^{-3}[700\cos 140t - 300\sin 140t + 4900\sin 140t + 2100\cos 140t]e^{-20t}$$

$$= (4.6\sin 140t + 2.8\cos 140t)e^{-20t}$$

$$v = (92\sin 140t + 56\cos 140t)e^{-20t}\,\text{V}, \qquad t \geq 0$$

$$\frac{di_L}{dt} = \frac{d}{dt}[21\cos 140t + 23\sin 140t][e^{-20t}] \times 10^{-3}$$

$$= 10^{-3}[21\cos 140t + 23\sin 140t][-20e^{-20t}]$$

$$+ 10^{-3}[-2940\sin 140t + 3220\cos 140t]e^{-20t}$$

$$= 10^{-3}[-420\cos 140t - 460\sin 140t - 2940\sin 140t + 3220\cos 140t]e^{-20t}$$

$$= [2.8\cos 140t - 3.4\sin 140t]e^{-20t}$$

$$v = (56\cos 140t - 68\sin 140t)e^{-20t}\,\text{V}, \qquad t \geq 0$$

$\therefore$ When $i = 7 + 21e^{-20t}\cos 140t + 23e^{-20t}\sin 140t\,\text{mA}$

$$v = (56\cos 140t - 68\sin 140t)e^{-20t}\,\text{V}$$

When $i = 7 - 35e^{-20t}\cos 140t + 15e^{-20t}\sin 140t\,\text{mA}$

$$v = (92\sin 140t + 56\cos 140t)e^{-20t}\,\text{V}$$

[c] When $v = (56\cos 140t - 68\sin 140t)e^{-20t}\,\text{V}$

$$\frac{dv}{dt} = -20e^{-20t}(56\cos 140t - 68\sin 140t)$$

$$+ e^{-20t}(-7840\sin 140t - 9520\cos 140t)$$

$$= (-1120\cos 140t + 1360\sin 140t - 9520\cos 140t - 7840\sin 140t)e^{-20t}$$

$$= (-10{,}640\cos 140t - 6480\sin 140t)e^{-20t}$$

$$\frac{dv}{dt} = 0; \qquad 10{,}640 \cos 140t + 6480 \sin 140t = 0$$

$$\tan 140t = -\frac{10{,}640}{6480} = -1.64$$

$$140t = -1.02 + \pi; \qquad t = 15.13 \text{ ms}$$

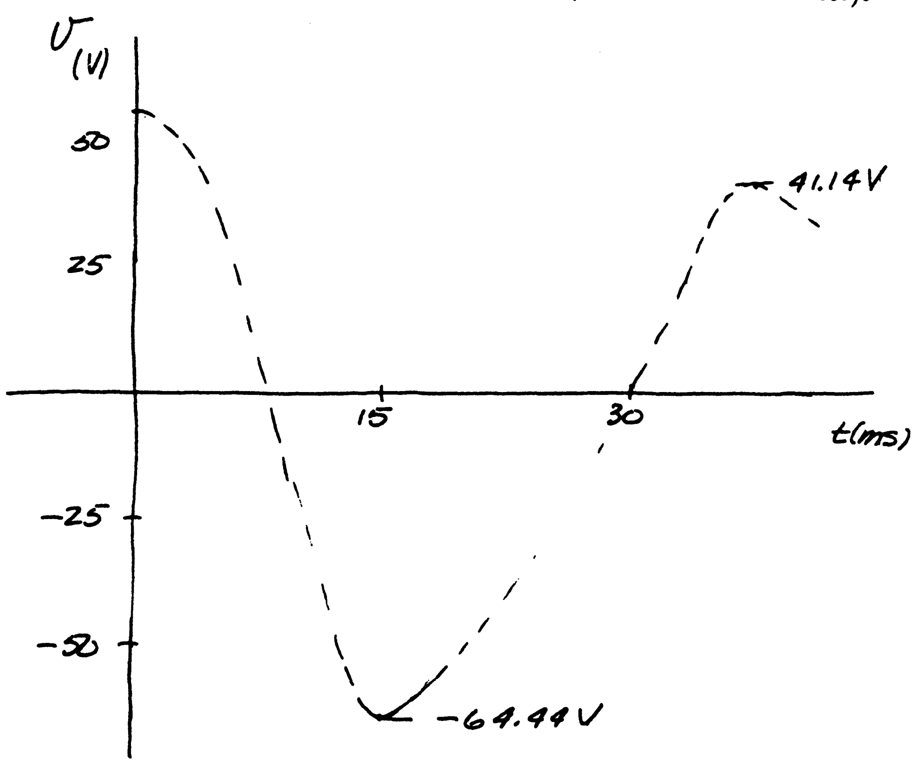
$$v(15.13 \text{ ms}) = [56 \cos(2.12) - 68 \sin(2.12)]e^{-0.30} = -64.44 \text{ V}$$

When $\quad 140t = -1.02 + 2\pi, \quad t = 37.57 \text{ ms}$

$v(37.57) = 41.14 \text{ V}$

$\therefore \; v(t)_{\max} = 56 \text{ V}$

$$v = (56 \cos 140t - 68 \sin 140t)e^{-20t} \text{ V}$$



When $\quad v = (92 \sin 140t + 56 \cos 140t)e^{-20t} \text{ V}$

$$\frac{dv}{dt} = (92 \sin 140t + 56 \cos 140t)(-20e^{-20t})$$

$$+ e^{-20t}(12{,}880 \cos 140t - 7840 \sin 140t)$$

$$= [12{,}880 \cos 140t - 1120 \cos 140t - 1840 \sin 140t - 7840 \sin 140t]e^{-20t}$$

$$= [11{,}760 \cos 140t - 9680 \sin 140t]e^{-20t}$$

$\frac{dv}{dt} = 0$ when $\tan 140t = \frac{11,760}{9680} = 1.21$

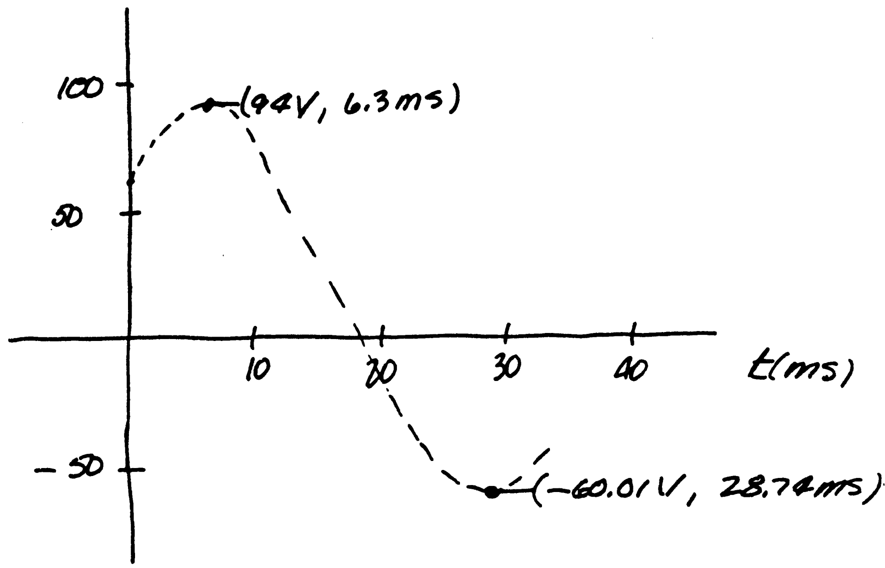
$\therefore\ 140t = 0.88; \qquad t = 6.30\,\text{ms}$

$v(6.30\,\text{ms}) = (92 \sin 0.88 + 56 \cos 0.88)e^{-0.13} = 94\,\text{V}$

When $140t = 0.88 + \pi, \quad t = 28.74\,\text{ms}$

$v(28.74\,\text{ms}) = (92 \sin 4.02 + 56 \cos 4.02)e^{-0.57} = -60.01\,\text{V}$

$\boxed{\therefore\ v_{\text{max}} = 94\,\text{V}}$



P 9.27 [a] $-\alpha + \sqrt{\alpha^2 - \omega_o^2} = -4000; \qquad -\alpha - \sqrt{\alpha^2 - \omega_o^2} = -16,000$

$\therefore\ \alpha = 10,000$

$\therefore\ \omega_o^2 = 64 \times 10^6$

$\therefore\ \frac{1}{LC} = 64 \times 10^6, \qquad L = \frac{10^3}{(64)(31.25)} = 0.5\,\text{H}$

$\frac{R}{2L} = 10,000, \qquad R = (2)(0.5)(10^4) = 10\,\text{k}\Omega$

[b] $i(0^+) = 0$

$\frac{1}{2}(31.25)[v_C(0^+)]^2 = 9 \times 10^{-6} \times 10^9$

$[v_C(0^+)]^2 = \frac{18,000}{31.25} = 576$

$v_C(0^+) = \sqrt{576} = 24\,\text{V}$

$\therefore\ 0.5\frac{di(0^+)}{dt} = 24; \qquad \frac{di(0^+)}{dt} = 48\,\text{A/s}$

[c] $i(t) = A_1e^{-4000t} + A_2e^{-16,000t}$

$\therefore\ i(0^+) = A_1 + A_2 = 0, \qquad A_2 = -A_1$

$i(t) = A_1[e^{-4000t} - e^{-16,000t}]$

$$\frac{di(t)}{dt} = A_1[-4000e^{-4000t} + 16,000e^{-16,000t}]$$

$$\frac{di(0^+)}{dt} = A_1(12,000) = 48$$

$\therefore\ A_1 = 4 \times 10^{-3} = 4\,\text{mA}$

$i(t) = 4[e^{-4000t} - e^{-16,000t}]\,\text{mA}, \qquad t \geq 0$

[d] $\dfrac{di(t)}{dt} = 0$ when $16,000e^{-16,000t} = 4000e^{-4000t}$

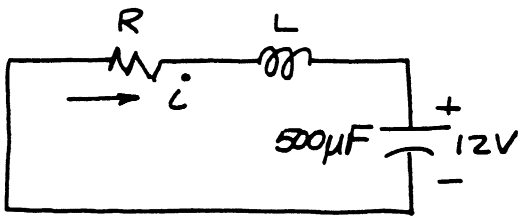
$\therefore\ 4 = e^{12,000t}; \qquad 12,000t = \ln 4; \qquad t = \dfrac{\ln 4}{0.012}\,\mu\text{s} = 115.52\,\mu\text{s}$

[e] $i_{\text{max}} = 4[e^{-0.46} - e^{-1.85}] = 1.89\,\text{mA}$

[f] $v_L(t) = 0.5(4 \times 10^{-3})[-4000e^{-4000t} + 16,000e^{-16,000t}]$

$= 32e^{-16,000t} - 8e^{-4000t}\,\text{V}, \qquad t \geq 0$

P 9.28



$\dfrac{R}{2L} = 800 \qquad \omega_d = 600 = \sqrt{\omega_o^2 - \alpha^2}$

$\therefore\ 36 \times 10^4 = \omega_o^2 - (800)^2$

$\therefore\ \omega_o^2 = 10^6, \qquad \omega_o = 1000\,\text{rad/s}$

$\therefore\ L = \dfrac{1}{10^6 C} = 2\,\text{mH}; \qquad R = 1600(2 \times 10^{-3}) = 3.2\,\Omega$

$i(0^+) = 0 = B_1$

$$\frac{di(0^+)}{dt} = \frac{-12}{2 \times 10^{-3}} = -6000$$

$$\frac{di}{dt} = e^{-800t}[-B_1 600 \sin 600t + 600B_2 \cos 600t]$$

$$- 800e^{-800t}[B_1 \cos 600t + B_2 \sin 600t]$$

$$\frac{di(0^+)}{dt} = 600B_2 - 800B_1 = 600B_2$$

$$\therefore\ B_2 = \frac{-6000}{600} = -10\,\text{A}$$

P 9.29 From the solution to Problem 9.28 we know i is $i = -10e^{-800t}\sin 600t$ A, $t \geq 0$

From the circuit we have $v_C = -iR - L\frac{di}{dt} = -3.2i - 2\times 10^{-3}\frac{di}{dt}$

$$\frac{di}{dt} = [-6000\cos 600t + 8000\sin 600t]e^{-800t}$$

$$\therefore\ v_C = 32e^{-800t}\sin 600t + [12\cos 600t - 16\sin 600t]e^{-800t}$$

$$= [12\cos 600t + 16\sin 600t]e^{-800t}\,\text{V}, \qquad t \geq 0$$

P 9.30 [a] $\omega_o^2 = \frac{1}{LC} = \frac{2\times 10^6}{80\times 10^{-3}} = 25\times 10^6$

$$\therefore\ \alpha^2 = \omega_o^2 = 25\times 10^6$$

$$\alpha = 5000 = \frac{R}{2L}, \qquad R = (5000)(160)\times 10^{-3} = 800\,\Omega$$

[b] $i(0) = 30$ mA

$$v_C(0) = 800i(0) = 80\times 10^{-3}\frac{di(0)}{dt}$$

$$\therefore\ \frac{20 - 800(30\times 10^{-3})}{0.08} = \frac{di(0)}{dt}$$

$$\therefore\ \frac{di(0)}{dt} = \frac{20-24}{0.08} = -50\,\text{A/s}$$

[c] $v_C(t) = 800i + 0.08\frac{di}{dt}$

$$i = D_1te^{-5000t} + D_2e^{-5000t}$$

$$i(0) = D_2 = 30\times 10^{-3} = 30\,\text{mA}$$

$$\frac{di}{dt} = (D_1t + D_2)(-5000e^{-5000t}) + e^{-5000t}(D_1)$$

$$\frac{di}{dt}(0) = -5000D_2 + D_1 = -50$$

$$\therefore\ D_1 = -50 + 5000(30\times 10^{-3}) = 100$$

$$\therefore\ i = 10^5te^{-5000t} + 30e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

$$800i = 8\times 10^7te^{-5000t} + 24{,}000e^{-5000t}\,\text{mV} = 8\times 10^4te^{-5000t} + 24e^{-5000t}\,\text{V}$$

$$0.08\frac{di}{dt} = 0.08\left[(100t + 0.03)(-5000e^{-5000t}) + 100e^{-5000t}\right]$$

$$= -4\times 10^4te^{-5000t} - 4e^{-5000t}\,\text{V}$$

$$\therefore\ v_C(t) = 4\times 10^4te^{-5000t} + 20e^{-5000t}\,\text{V}$$

Alternate solution: Solve for $v_C(t)$ directly.

$$v_C(t) = D_1te^{-5000t} + D_2e^{-5000t}$$

$$v_C(0) = D_2 = 20\,\text{V}$$

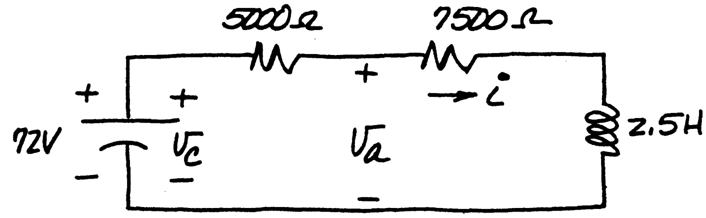
$$\frac{dv_C}{dt}(0) = -5000D_2 + D_1 = \frac{-30\times 10^{-3}}{0.5\times 10^{-6}}$$

$\therefore \ -5000D_2 + D_1 = -60{,}000; \qquad D_1 = -60{,}000 + 100{,}000 = 40{,}000$

$\therefore \ v_C(t) = 4 \times 10^4 te^{-5000t} + 20e^{-5000t}\,\text{V}, \qquad t \geq 0$

P9.31 [a] $v_a(0) = v_c(0) = 72\,\text{V}$

[b] $t > 0$:



$v_a = v_c - 5000i$

$\frac{dv_a}{dt} = \frac{dv_c}{dt} - 5000\frac{di}{dt}$

$\frac{dv_c(0)}{dt} = 0$ since $i(0) = 0; \qquad \therefore \ \frac{dv_a(0)}{dt} = -5000\frac{di(0)}{dt}$

$\frac{di(0)}{dt} = \frac{72}{2.5} = 28.80\,\text{A/s}; \qquad \therefore \ \frac{dv_a(0)}{dt} = -144{,}000\,\text{V/s}$

[c] $\alpha = \frac{R}{2L} = \frac{12{,}500}{5} = 2500\,\text{rad/s}; \qquad \alpha^2 = 625 \times 10^4$

$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{(0.1)(2.5)} = 4 \times 10^6 = 400 \times 10^4$

$\therefore \ \alpha^2 > \omega_o^2$ overdamped

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} = -2500 + \sqrt{225 \times 10^4} = -2500 + 1500 = -1000\,\text{rad/s}$

$s_2 = -2500 - 1500 = -4000\,\text{rad/s}$

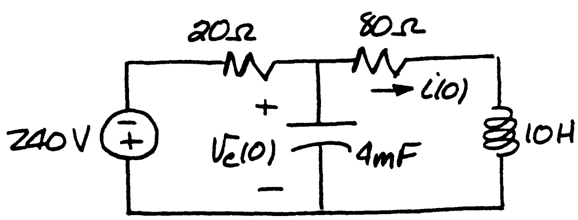
$v_a = A_1e^{-1000t} + A_2e^{-4000t}; \qquad v_a(0) = A_1 + A_2 = 72$

$\frac{dv_a(0)}{dt} = -1000A_1 - 4000A_2 = -144{,}000$

$\therefore \ A_1 + 4A_2 = 144; \qquad \therefore \ 3A_2 = 72, \quad A_2 = 24, \quad A_1 = 48$

$\therefore \ v_a = 48e^{-1000t} + 24e^{-4000t}\,\text{V}, \qquad t \geq 0$

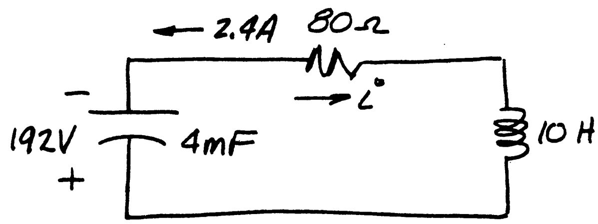
P9.32 $t < 0$:



$i(0) = -240/100 = -2.4\,\text{A}$

$v_C(0) = -240 - 20[i(0)] = -240 - 20(-2.4) = -240 + 48 = -192\,\text{V}$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{80}{20} = 4\,\text{rad/s}, \qquad \alpha^2 = 16$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^3}{(10)(4)} = \frac{100}{4} = 25$$

$\omega_o^2 > \alpha^2$ underdamped

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{25 - 16} = 3\,\text{rad/s}$$

$\therefore\ i(t) = B_1e^{-4t}\cos 3t + B_2e^{-4t}\sin 3t$

$i(0) = B_1 = -2.4\,\text{A}$

$$L\frac{di(0)}{dt} = -192 - 80[i(0)] = -192 - 80(-2.4) = -192 + 192 = 0$$

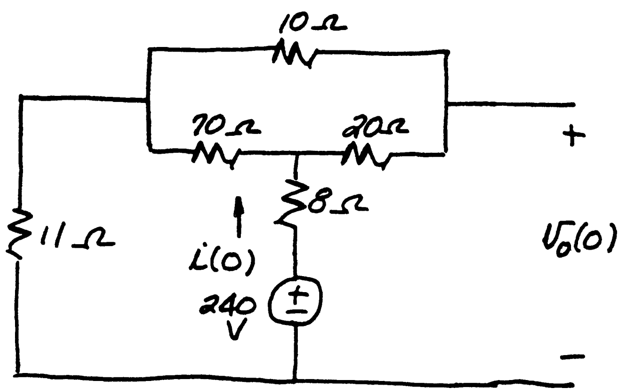
$$\frac{di(0)}{dt} = -4B_1 + 3B_2 = 0$$

$$\therefore\ B_2 = \frac{4}{3}B_1 = \frac{4}{3}(-2.4) = -3.2$$

$\therefore\ i(t) = (-2.4\cos 3t - 3.2\sin 3t)e^{-4t}$

$i(t) = -(2.4\cos 3t + 3.2\sin 3t)e^{-4t}\,\text{A}, \qquad t \geq 0$

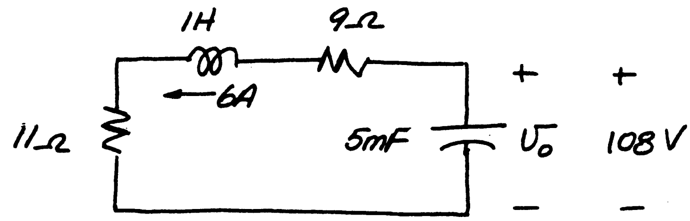
P9.33 $t < 0$:



$$i(0) = \frac{240}{8 + 21 + 11} = \frac{240}{40} = 6\,\text{A}$$

$$v_o(0) = 240 - 6(8) - 20(6)\left(\frac{70}{100}\right) = 108\,\text{V}$$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{20}{2} = 10, \qquad \alpha^2 = 100$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1000}{5} = 200$$

$\omega_o^2 > \alpha^2$ underdamped

$$v_o = B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t; \qquad \omega_d = \sqrt{200 - 100} = 10$$

$$v_o = B_1 e^{-10t}\cos 10t + B_2 e^{-10t}\sin 10t$$

$$v_o(0) = B_1 = 108\,\text{V}$$

$$C\frac{dv_o}{dt}(0) = -6, \qquad \frac{dv_o}{dt} = \frac{-6}{5} \times 10^3 = -1200\,\text{V/s}$$

$$\frac{dv_o}{dt}(0) = -10B_1 + 10B_2 = -1200$$

$$10B_2 = -1200 + 10B_1 = -1200 + 1080$$

$$B_2 = -120/10 = -12\,\text{V}$$

$$\therefore\; v_o = 108e^{-10t}\cos 10t - 12e^{-10t}\sin 10t\,\text{V}, \qquad t \geq 0$$

P 9.34 [a] $i_o(0) = \dfrac{80}{800} = 0.10\,\text{A}; \qquad v_o(0) = 500(0.1) = 50\,\text{V}$

$$L\frac{di_o(0)}{dt} = 0; \qquad \therefore\; \frac{di_o}{dt}(0) = 0$$

$$\alpha = \frac{R}{2L} = \frac{500}{5} \times 10^3 = 10^5; \qquad \alpha^2 = 10^{10}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{(10^3)(10^9)}{(2.5)(40)} = 10^{10}$$

$\therefore\; \omega_o^2 = \alpha^2$ critically damped

$$i_o(t) = D_1 t e^{-10^5 t} + D_2 e^{-10^5 t}$$

$$i_o(0) = D_2 = 0.10\,\text{A}$$

$$\frac{di_o}{dt}(0) = -\alpha D_2 + D_1 = 0; \qquad D_1 = \alpha D_2 = 10^5(0.1) = 10^4$$

$$i_o(t) = 10^4 t e^{-10^5 t} + 0.1e^{-10^5 t} A, \qquad t \geq 0$$

[b] $v_o(t) = D_1te^{-\alpha t} + D_2e^{-\alpha t}$

$v_o(0) = 50 = D_2$

$C\frac{dv_o}{dt}(0) = -0.10; \qquad \frac{dv_o}{dt} = -25 \times 10^5$

$\frac{dv_o}{dt}(0) = -\alpha D_2 + D_1 = -25 \times 10^5; \qquad D_1 = -25 \times 10^5 + 10^5(50) = 25 \times 10^5$

$\therefore \ v_o(t) = 25 \times 10^5te^{-10^5t} + 50e^{-10^5t}\,\text{V}, \qquad t \geq 0$

P 9.35 $\alpha = \frac{R}{2L} = \frac{250 \times 10^3}{(2)(62.5)} = 2000\,\text{rad/s}; \qquad \alpha^2 = 4 \times 10^6$

$\omega_o^2 = \frac{1}{LC} = \frac{(10^3)(10^6)}{(62.5)(6.25)} = 2.56 \times 10^6$

$\alpha^2 > \omega_o^2$ overdamped

$\therefore \ v_o(t) = V_f + A_1'e^{s_1t} + A_2'e^{s_2t}$

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} = -2000 + 1200 = -800$

$s_2 = -2000 - 1200 = -3200$

$V_f = 60\,\text{V}$

$\therefore \ v_o(t) = 60 + A_1'e^{-800t} + A_2'e^{-3200t}\,\text{V}$

$v_o(0) = 0 = 60 + A_1' + A_2'$

$\frac{dv_o}{dt}(0) = 0 = -800A_1' - 3200A_2'$

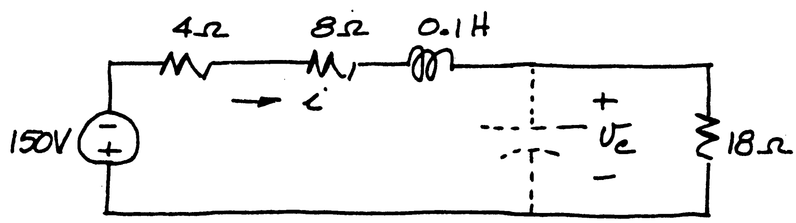
$\therefore \ -800A_1' = 3200A_2'; \qquad A_1' = -4A_2'$

$\therefore \ 60 - 4A_2' + A_2' = 0; \qquad A_2' = 20\,\text{V}$

$\therefore \ A_1' = -80\,\text{V}$

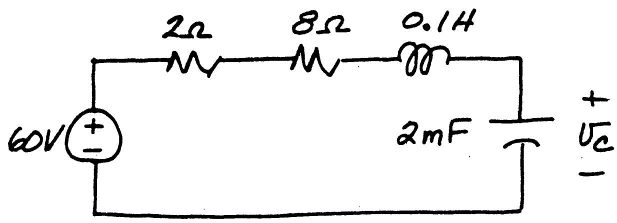
$\therefore \ v_o(t) = 60 - 80e^{-800t} + 20e^{-3200t}\,\text{V}, \qquad t \geq 0$

P 9.36 For $t < 0$:



$i = \frac{-150}{30} = -5\,\text{A}, \qquad v_C(0) = 18i = -90\,\text{V}$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{10}{0.2} = 50\,\text{rad/s}; \qquad \alpha^2 = 2500$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^3}{(0.1)(2)} = 5000$$

$\omega_o^2 > \alpha^2$ underdamped; $\quad \omega_d = 50\,\text{rad/s}$

$\therefore\ v_C = 60 + B_1 e^{-50t}\cos 50t + B_2 e^{-50t}\sin 50t$

$v_C(0) = -90 = 60 + B_1; \qquad B_1 = -150\,\text{V}$

$$C\frac{dv_C}{dt} = -5, \qquad \frac{dv_C}{dt} = \frac{-5}{2} \times 10^3 = -2500$$

$$\frac{dv_C}{dt}(0) = -\alpha B_1 + \omega_d B_2 = -2500$$

$-50B_1 + 50B_2 = -2500$

$-B_1 + B_2 = -50; \qquad \therefore\ B_2 = -50 + B_1 = -50 - 150 = -200\,\text{V}$

$\therefore\ v_C = 60 - 150e^{-50t}\cos 50t - 200e^{-50t}\sin 50t\,\text{V}, \qquad t \geq 0$

P 9.37 $\alpha = \dfrac{10}{0.4} = 25\,\text{rad/s}, \qquad \alpha^2 = 625$

$$\omega_o^2 = \frac{1000}{(0.2)(8)} = 625$$

$\alpha^2 = \omega_o^2$ critical damping

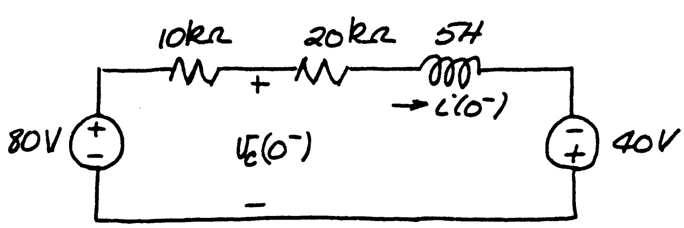
$v_C = 400 + D_1' te^{-25t} + D_2' e^{-25t}$

$v_C(0) = 100 = 400 + D_2', \qquad D_2' = -300$

$$\frac{dv_C}{dt}(0) = 0 = -25D_2' + D_1'; \qquad \therefore\ D_1' = 25D_2' = 25(-300) = -7500$$

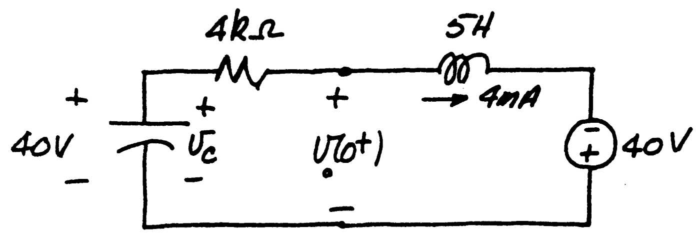
$\therefore\ v_C(t) = 400 - 7500te^{-25t} - 300e^{-25t}\,\text{V}, \qquad t \geq 0$

P 9.38 [a] $t < 0$:



$$i(0^-) = \frac{120}{30} = 4\,\text{mA}; \qquad v_C(0^-) = 80 - 10(4) = 40\,\text{V}$$

$t = 0^+$:



$$v_o(0^+) = 40 - 4(4) = 40 - 16 = 24\,\text{V}$$

[b] $v_o = v_C - 4000i$

$$\frac{dv_o}{dt} = \frac{dv_C}{dt} - 4000\frac{di}{dt}$$

$$\frac{dv_o}{dt}(0^+) = \frac{dv_C}{dt}(0^+) - 4000\frac{di}{dt}(0^+)$$

$$\frac{dv_C}{dt}(0^+) = \frac{-4\times10^{-3}}{(125/64)\times10^{-6}} = \frac{-256{,}000}{125} = -2048\,\text{V/s}$$

$$\frac{di(0^+)}{dt} = \frac{64}{5} = 12.8\,\text{A/s}$$

$$\frac{dv_o(0^+)}{dt} = -2048 - 4000(12.8) = -53{,}248\,\text{V/s}$$

[c] $\alpha = \dfrac{R}{2L} = \dfrac{4000}{10} = 400, \qquad \alpha^2 = 16\times10^4$

$$\omega_o^2 = \frac{64\times10^6}{(5)(125)} = 10.24\times10^4$$

$\alpha^2 > \omega_o^2$ overdamped

$$s_1 = -400 + \sqrt{(16 - 10.24)10^4} = -400 + 240 = -160\,\text{rad/s}$$

$$s_2 = -640\,\text{rad/s}$$

$$v_o = -40 + A_1'e^{-160t} + A_2'e^{-640t}$$

$$v_o(0^+) = 24 = -40 + A_1' + A_2'; \qquad \therefore\; A_1' + A_2' = 64$$

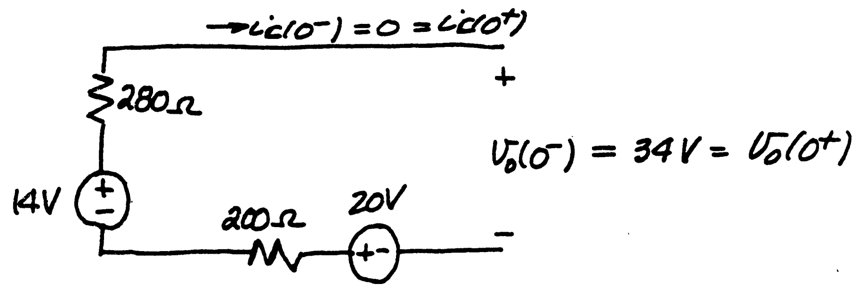
$$\frac{dv_o}{dt}(0^+) = -160A_1' - 640A_2' = -53{,}248$$

$$\therefore\; A_1' + 4A_2' = 332.80$$

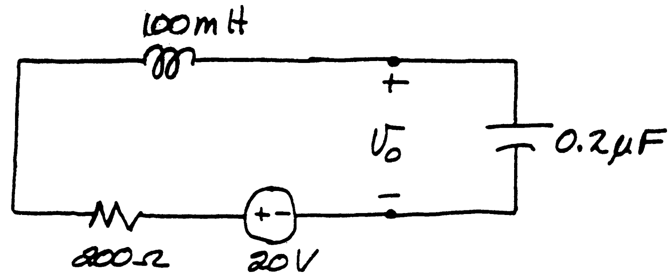
$$\therefore\; 3A_2' = 268.80; \qquad A_2' = 89.60\,\text{V}; \qquad \therefore\; A_1' = -25.60\,\text{V}$$

$$\therefore\; v_o(t) = -40 - 25.60e^{-160t} + 89.60e^{-640t}\,\text{V}, \qquad t \geq 0^+$$

P 9.39 For $t < 0$:



For $t > 0$:



$$\alpha = \frac{R}{2L} = \frac{200}{0.2} = 1000, \qquad \alpha^2 = 10^6$$

$$\omega_o^2 = \frac{1}{LC} = \frac{(10^3)(10^6)}{100(0.2)} = 50 \times 10^6$$

$\omega_o^2 > \alpha^2$ underdamped

$s_{1,2} = -1000 \pm j7000\,\text{rad/s}$

$v_o = v_F + B_1' e^{-1000t} \cos 7000t + B_2' e^{-1000t} \sin 7000t, \qquad v_F = 20\,\text{V}$

$v_o(0) = 20 + B_1' = 34; \qquad B_1' = 34 - 20 = 14\,\text{V}$

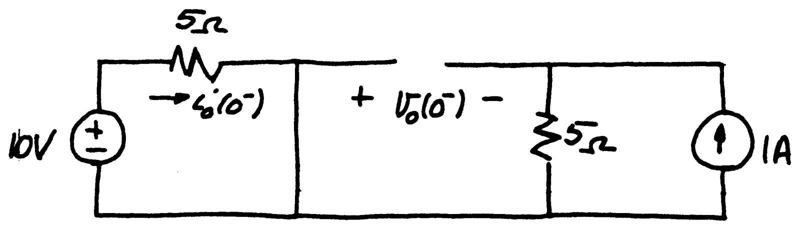
$$\frac{dv}{dt} = e^{-1000t}[-7000B_1' \sin 7000t + 7000B_2' \cos 7000t]$$
$$+ [B_1' \cos 7000t + B_2' \sin 7000t](-1000e^{-1000t})$$

$$\frac{dv}{dt}(0^+) = 7000B_2' - 1000B_1'$$

$$\frac{dv}{dt}(0^+) = 0 \quad \text{since} \quad i_C(0^+) = 0; \qquad \therefore\ B_2' = \frac{1}{7}B_1' = \frac{1}{7}(14) = 2\,\text{V}$$

$$\therefore\ v_o = 20 + 14e^{-1000t} \cos 7000t + 2e^{-1000t} \sin 7000t\,\text{V}, \qquad t \geq 0$$

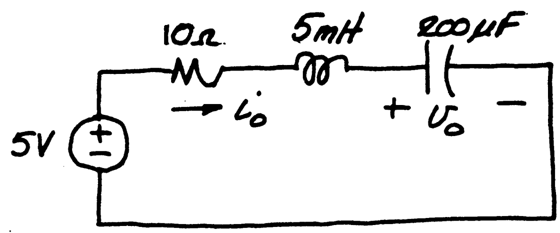
P 9.40 [a] $t < 0$:



$i_o(0^-) = 10/5 = 2\,\text{A} = i_o(0^+)$

$v_o(0^-) = -5(1) = -5\,\text{V} = v_o(0^+)$

$t > 0^+$:



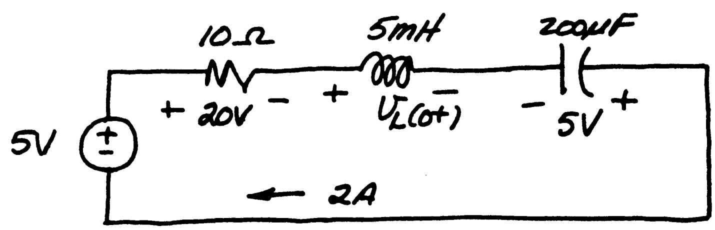
$$\alpha = \frac{R}{2L} = \frac{10}{10} \times 1000 = 1000\,\text{rad/s}; \qquad \omega_o^2 = \frac{1}{LC} = \frac{(10^3)(10^6)}{(5)(200)} = 10^6$$

$\alpha^2 = \omega^2 \quad \therefore$ critically damped

$\therefore\ i_o = i_F + D_1' t e^{-1000t} + D_2' e^{-1000t}; \qquad i_F = 0$

$\therefore\ i_o = D_1' t e^{-1000t} + D_2' e^{-1000t}; \qquad i_o(0^+) = D_2' = 2$

At $t = 0^+$:



$-5 + 20 + v_L(0^+) - 5 = 0, \qquad v_L(0^+) = -10\,\text{V}$

$$\therefore\ \frac{di}{dt}(0^+) = \frac{-10}{5} \times 10^3 = -2000\,\text{A/s}$$

$$\frac{di_o}{dt} = -1000e^{-1000t}(D_1't + D_2') + e^{-1000t}D_1'$$

$$\frac{di_o}{dt}(0^+) = -1000D_2' + D_1' = -2000$$
$$-2000 + D_1' = -2000; \qquad D_1' = 0$$
$$\therefore\ i_o(t) = 2e^{-1000t}\,\text{A}, \qquad t \geq 0$$

[b] $v_o = v_F + D_3'te^{-1000t} + D_4'e^{-1000t}$
$$v_o(0^+) = v_F + D_4' = -5$$
$$v_F = 5\,\text{V}; \qquad \therefore\ D_4' = -10\,\text{V}$$
$$v_o = 5 + D_3'te^{-1000t} - 10e^{-1000t}$$
$$\frac{dv_o}{dt} = -1000e^{-1000t}(D_3't - 10) + D_3'e^{-1000t}$$
$$\frac{dv_o}{dt}(0^+) = 10{,}000 + D_3'$$
$$\frac{dv_o}{dt}(0^+) = \frac{2}{200} \times 10^6 = 10^4 = 10{,}000$$
$$\therefore\ D_3' = 0$$
$$\therefore\ v_o = 5 - 10e^{-1000t}\,\text{V}, \qquad t \geq 0$$

P 9.41 [a] $v_c = V_f + [A_1' \cos\omega_d t + A_2' \sin\omega_d t]\,e^{-\alpha t}$
$$\frac{dv_c}{dt} = \left[(\omega_d A_2' - \alpha A_1')\cos\omega_d t - (\alpha A_2' + \omega_d A_1')\sin\omega_d t\right]e^{-\alpha t}$$

Since the initial stored energy is zero, $v_c(0^+) = 0$ and $\dfrac{dv_c(0^+)}{dt} = 0$

It follows that $A_1' = -V_f$ and $A_2' = \dfrac{\alpha A_1'}{\omega_d}$

When these values are substituted into the expression for $[dv_c/dt]$, we get
$$\frac{dv_c}{dt} = \left(\frac{\alpha^2}{\omega_d} + \omega_d\right) V_f e^{-\alpha t} \sin\omega_d t$$

But $V_f = V$ and $\dfrac{\alpha^2}{\omega_d} + \omega_d = \dfrac{\alpha^2 + \omega_d^2}{\omega_d} = \dfrac{\omega_o^2}{\omega_d}$

Therefore $\dfrac{dv_c}{dt} = \left(\dfrac{\omega_o^2}{\omega_d}\right) Ve^{-\alpha t}\sin\omega_d t$

[b] $\dfrac{dv_c}{dt} = 0$ when $\sin\omega_d t = 0$, or $\omega_d t = n\pi$ where $n = 1, 2, 3, \ldots$

Therefore $t = \dfrac{n\pi}{\omega_d}$

[c] When $t_n = \dfrac{n\pi}{\omega_d}$, $\cos\omega_d t_n = \cos n\pi = (-1)^n$ and $\sin\omega_d t = \sin n\pi = 0$

Therefore $v_c(t_n) = V\left[1 - (-1)^n e^{-\alpha n\pi/\omega_d}\right]$

[d] It follows from [c] that
$$v_(t_1) = V + Ve^{-(\alpha\pi/\omega_d)} \quad \text{and} \quad v_c(t_3) = V + Ve^{-(3\alpha\pi/\omega_d)}$$

Therefore $\dfrac{v_c(t_1) - V}{v_c(t_3) - V} = \dfrac{e^{-(\alpha\pi/\omega_d)}}{e^{-(3\alpha\pi/\omega_d)}} = e^{(2\alpha\pi/\omega_d)}$

But $\dfrac{2\pi}{\omega_d} = t_3 - t_1 = T_d$, thus $\alpha = \dfrac{1}{T_d}\ln\dfrac{[v_c(t_1) - V]}{[v_c(t_3) - V]}$

P 9.42 $\dfrac{1}{T_d}\ln\left\{\dfrac{v_c(t_1) - V}{v_c(t_3) - V}\right\}; \qquad T_d = t_3 - t_1 = \dfrac{3\pi}{7} - \dfrac{\pi}{7} = \dfrac{2\pi}{7}\,\text{ms}$

$$\alpha = \frac{7000}{2\pi}\ln\left[\frac{63.84}{26.02}\right] = 999.91; \qquad \omega_d = \frac{2\pi}{T_d} = 7000\,\text{rad/s}$$

$$\omega_o^2 = \omega_d^2 + \alpha^2 = 49\times10^6 + 0.9998\times10^6 = 49.9998\times10^6 \approx 50$$

$$L = \frac{1}{(49.9998)(0.1)} = 0.20\,\text{H}; \qquad R = 2\alpha L = (0.4)(999.91) = 399.96\,\Omega$$

P 9.43 [a] Let i be the current in the direction of the voltage drop $v_o(t)$. Then by hypothesis

$$i = i_f + B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t$$

$$i_f = i(\infty) = 0, \qquad i(0) = \frac{V_g}{R} = B_1'$$

Therefore $i = B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t$

$$L\frac{di(0)}{dt} = 0, \qquad \text{therefore} \quad \frac{di(0)}{dt} = 0$$

$$\frac{di}{dt} = [(\omega_d B_2' - \alpha B_1')\cos\omega_d t - (\alpha B_2' + \omega_d B_1')\sin\omega_d t]\,e^{-\alpha t}$$

Therefore $\omega_d B_2' - \alpha B_1' = 0; \qquad B_2' = \dfrac{\alpha}{\omega_d}B_1' = \dfrac{\alpha}{\omega_d}\dfrac{V_g}{R}$

Therefore

$$v_L = L\frac{di}{dt} = -\left\{L\left(\frac{\alpha^2 V_g}{\omega_d R} + \frac{\omega_d V_g}{R}\right)\sin\omega_d t\right\}e^{-\alpha t}$$

$$= -\left\{\frac{LV_g}{R}\left(\frac{\alpha^2}{\omega_d} + \omega_d\right)\sin\omega_d t\right\}e^{-\alpha t} = -\frac{V_g L}{R}\left(\frac{\alpha^2+\omega_d^2}{\omega_d}\right)e^{-\alpha t}\sin\omega_d t$$

$$v_L = -\frac{V_g}{RC\omega_d}e^{-\alpha t}\sin\omega_d t$$

[b] $\dfrac{dv_L}{dt} = -\dfrac{V_g}{\omega_d RC}\{\omega_d\cos\omega_d t - \alpha\sin\omega_d t\}e^{-\alpha t}$

$$\frac{dv_L}{dt} = 0 \quad \text{when} \quad \tan\omega_d t = \frac{\omega_d}{\alpha}$$

Therefore $\omega_d t = \tan^{-1}(\omega_d/\alpha)$ (smallest t)

$$t = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

P9.44 [a] $\alpha = \dfrac{R}{2L} = \dfrac{600}{40} \times 10^3 = 15{,}000$

$$\omega_o^2 = \frac{1}{LC} = \frac{(10^3)(10^6)}{(20)(0.08)} = 0.625 \times 10^9 = 625 \times 10^6$$

$$\omega_d = \sqrt{625 \times 10^6 - 225 \times 10^6} = 20{,}000\,\text{rad/s}$$

$$\frac{-V_g}{\omega_d RC} = \frac{+480 \times 10^6}{(2 \times 10^4)(600)(0.08)} = 500\,\text{V}$$

$$v_o = 500e^{-15{,}000t}\sin 20{,}000t\,\text{V}, \qquad t \geq 0^+$$

[b] $t_{\max} = \dfrac{1}{\omega_d}\tan^{-1}\dfrac{\omega_d}{\alpha} = 50\tan^{-1}\left(\dfrac{4}{3}\right)\ \mu\text{s} \cong 46.36\,\mu\text{s}$

[c] $v_o(\max) = 500e^{-0.015(46.36)}\sin(0.02)(46.36) \cong 199.54\,\text{V}$

[d] $R = 60\,\Omega, \qquad \alpha = 1500\,\text{rad/s}$

$$\omega_d = \sqrt{(625 - 2.25)10^6} = 24{,}954.96\,\text{rad/s}$$

$$\frac{-V_g}{\omega_d RC} = \frac{+480}{(24{,}954.96)(60)(0.08 \times 10^{-6})} = 4007.22\,\text{V}$$

$$v_o = 4007.22e^{-1500t}\sin 24{,}954.96t\,\text{V}, \qquad t \geq 0^+$$

$$t_{\max} = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right) = 40.07\tan^{-1}\left(\frac{\omega_d}{\alpha}\right)\ \mu\text{s} \cong 60.54\,\mu\text{s}$$

$$v_o(\max) = 4007.22e^{-0.0015(60.54)}\sin[0.02495496(60.54)] \cong 3652.77\,\text{V}$$

P9.45 [a] $\dfrac{d^2v_o}{dt^2} = \dfrac{1}{R_1C_1R_2C_2}v_g$

$$\frac{1}{R_1C_1R_2C_2} = \frac{10^{-6}}{(100)(400)(0.5)(0.2) \times 10^{-6} \times 10^{-6}} = 250$$

$$\therefore\ \frac{d^2v_o}{dt^2} = 250v_g$$

$0 \leq t \leq 0.5^-$:

$v_g = 80\,\text{mV}$

$$\therefore\ \frac{d^2v_o}{dt^2} = 20, \qquad \text{let } g(t) = \frac{dv_o}{dt}, \qquad \text{then } \frac{dg}{dt} = 20 \quad \text{or} \quad dg = 20\,dt$$

$$\therefore\ \int_{g(0)}^{g(t)} dx = 20\int_0^t dy; \qquad g(t) - g(0) = 20t, \quad g(0) = \frac{dv_o}{dt}(0) = 0$$

$$\therefore\ g(t) = \frac{dv_o}{dt} = 20t$$

$$\therefore\ dv_o = 20t\,dt$$

$$\therefore\ \int_{v_o(0)}^{v_o(t)} dx = 20\int_0^t x\,dx; \qquad v_o(t) - v_o(0) = 10t^2, \quad v_o(0) = 0$$

$$\therefore\ v_o(t) = 10t^2\,\text{V}, \quad 0 \leq t \leq 0.5^-$$

$$\frac{dv_{o1}}{dt} = -\frac{1}{R_1C_1}v_g = -20v_g = -1.6$$

$$\therefore\ dv_{o1} = -1.6\,dt$$

$$\int_{v_{o1}(0)}^{v_{o1}(t)} dx = -1.6\int_0^t dy; \qquad v_{o1}(t) - v_{o1}(0) = -1.6t, \qquad v_{o1}(0) = 0$$

$$v_{o1}(t) = -1.6t\,\text{V}, \qquad 0 \le t \le 0.5^-$$

$0.5^+ \le t \le t_{\text{sat}}$:

$$\frac{d^2v_o}{dt^2} = -10, \qquad \text{let} \quad g(t) = \frac{dv_o}{dt}$$

$$\frac{dg(t)}{dt} = -10$$

$$dg(t) = -10\,dt$$

$$\int_{g(0.5^+)}^{g(t)} dx = -10\int_{0.5}^t dy$$

$$g(t) - g(0.5^+) = -10(t-0.5) = -10t + 5, \qquad g(0.5^+) = \frac{dv_o(0.5^+)}{dt}$$

$$C\frac{dv_o}{dt}(0.5^+) = \frac{0 - v_{o1}(0.5^+)}{400 \times 10^3}$$

$$v_{o1}(0.5^+) = v_o(0.5^-) = -1.6(0.5) = -0.80\,\text{V}$$

$$\therefore\ C\frac{dv_{o1}(0.5^+)}{dt} = \frac{0.80}{0.4 \times 10^6} = 2\,\mu\text{A}$$

$$\frac{dv_{o1}}{dt}(0.5^+) = \frac{2 \times 10^{-6}}{0.2 \times 10^{-6}} = 10\,\text{V/s}$$

$$\therefore\ g(t) = -10t + 5 + 10 = -10t + 15 = \frac{dv_o}{dt}$$

$$\therefore\ dv_o = -10t\,dt + 15\,dt$$

$$\int_{v_o(0.5^+)}^{v_o(t)} dx = \int_{0.5^+}^t -10y\,dy + \int_{0.5^+}^t 15\,dy$$

$$v_o(t) - v_o(0.5^+) = -5y^2\Big|_{0.5}^t + 15y\Big|_{0.5}^t$$

$$v_o(t) = v_o(0.5^+) - 5t^2 + 1.25 + 15t - 7.5$$

$$v_o(0.5^+) = v_o(0.5^-) = 2.5\,\text{V}$$

$$\therefore\ v_o(t) = -5t^2 + 15t - 3.75\,\text{V}, \qquad 0.5^+ \le t \le t_{\text{sat}}$$

$$\frac{dv_{o1}}{dt} = -20(-40 \times 10^{-3}) = 0.8, \qquad 0.5^+ \le t \le t_{\text{sat}}$$

$$dv_{o1} = 0.8\,dt$$

$$\int_{v_{o1}(0.5^+)}^{v_{o1}(t)} dx = 0.8\int_{0.5^+}^t dy$$

$$v_{o1}(t) - v_{o1}(0.5^+) = 0.8t - 0.4; \qquad v_{o1}(0.5^+) = v_{o1}(0.5^-) = -0.8\,\text{V}$$

$$\therefore\ v_{o1}(t) = 0.8t - 1.2\,\text{V}, \qquad 0.5^+ \le t \le t_{\text{sat}}$$

Summary

$0 \le t \le 0.5^-\text{s}:$ $\quad v_{o1} = -1.6t\,\text{V}, \quad v_o = 10t^2\,\text{V}$

$0.5^+\text{s} \le t \le t_{\text{sat}}:$ $\quad v_{o1} = 0.8t - 1.2\,\text{V}, \quad v_o = -5t^2 + 15t - 3.75\,\text{V}$

[b] $-12.5 = -5t_{sat}^2 + 15t_{sat} - 3.75$

$\therefore\ 5t_{sat}^2 - 15t_{sat} - 8.75 = 0$

$t_{sat}^2 - 3t_{sat} - 1.75 = 0$

$t_{sat} = 1.5 \pm \sqrt{2.25 + 1.75} = 1.5 \pm 2$

$\therefore\ t_{sat} = 3.5$ sec

$v_{o1}(t_{sat}) = 0.8(3.5) - 1.2 = 1.6\,\text{V}$

P 9.46 $\tau_1 = (10^6)(0.5 \times 10^{-6}) = 0.50\,\text{s}$

$\frac{1}{\tau_1} = 2;\qquad \tau_2 = (5 \times 10^6)(0.2 \times 10^{-6}) = 1\,\text{s};\qquad \therefore\ \frac{1}{\tau_2} = 1$

$\therefore\ \frac{d^2v_o}{dt^2} + 3\frac{dv_o}{dt} + 2 = 20$

$s^2 + 3s + 2 = 0$

$(s+1)(s+2) = 0;\qquad s_1 = -1,\quad s_2 = -2$

$v_o = V_f + A_1'e^{-t} + A_2'e^{-2t};\qquad V_f = \frac{20}{2} = 10\,\text{V}$

$v_o = 10 + A_1'e^{-t} + A_2'e^{-2t}$

$v_o(0) = 0 = 10 + A_1' + A_2';\qquad \frac{dv_o}{dt}(0) = 0 = -A_1' - 2A_2'$

$\therefore\ A_1' = -2A_2',\qquad A_2' = 10\,\text{V},\qquad A_1' = -20\,\text{V}$

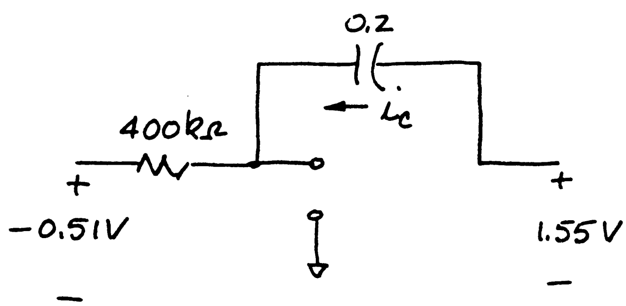
$v_o(t) = 10 - 20e^{-t} + 10e^{-2t}\,\text{V},\qquad 0 \le t \le 0.5\,\text{s}$

$\frac{dv_{o1}}{dt} + 2v_{o1} = -1.6;\qquad \therefore\ v_{o1} = -0.8 + 0.8e^{-2t}\,\text{V},\quad 0 \le t \le 0.5\,\text{s}$

$v_o(0.5) = 10 - 20e^{-0.5} + 10e^{-1} = 1.55\,\text{V}$

$v_{o1}(0.5) = -0.8 + 0.8e^{-1} = -0.51\,\text{V}$

At $t = 0.5\,\text{s}$



$i_C = \frac{0 + 0.51}{0.400 \times 10^6} = 1.26\,\mu\text{A};\qquad C\frac{dv_o}{dt} = 1.26\,\mu\text{A};\qquad \frac{dv_o}{dt} = \frac{1.26}{0.2} = 6.32\,\text{V/s}$

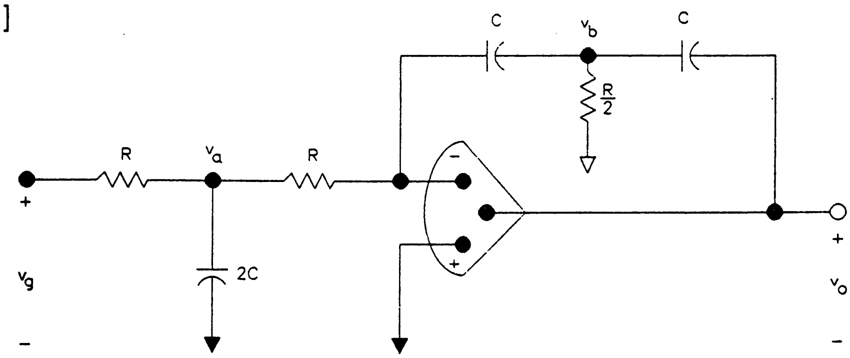
$0.5\,\text{s} \le t \le \infty$

$\frac{d^2v_o}{dt^2} + 3\frac{dv_o}{dt} + 2 = -10$

$v_o(\infty) = -5$

$$\therefore\ v_o = -5 + A_1'e^{-(t-0.5)} + A_2'e^{-2(t-0.5)}$$
$$1.55 = -5 + A_1' + A_2'$$
$$\frac{dv_o}{dt}(0.5) = 6.32 = -A_1' - 2A_2'$$
$$\therefore\ A_1' + A_2' = 6.55; \qquad -A_1' - 2A_2' = 6.32$$
$$-A_2' = 12.87; \qquad A_2' = -12.87; \qquad A_1' = 6.55 - A_2' = 19.42$$
$$\therefore\ v_o = -5 + 19.42e^{-(t-0.5)} - 12.87e^{-2(t-0.5)}\,\text{V}, \qquad 0.5 \le t \le \infty$$
$$\frac{dv_{o1}}{dt} + 2v_{o1} = 0.8$$
$$\therefore\ v_{o1} = 0.4 + (-0.51 - 0.40)e^{-2(t-0.5)} = 0.4 - 0.91e^{-2(t-0.5)}\,\text{V}, \qquad 0.5 \le t \le \infty$$

P 9.47 [a]



$$2C\frac{dv_a}{dt} + \frac{v_a - v_g}{R} + \frac{v_a}{R} = 0$$

(1) Therefore $\dfrac{dv_a}{dt} + \dfrac{v_a}{RC} = \dfrac{v_g}{2RC}$; $\qquad \dfrac{0 - v_a}{R} + C\dfrac{d(0 - v_b)}{dt} = 0$

(2) Therefore $\dfrac{dv_b}{dt} + \dfrac{v_a}{RC} = 0, \quad v_a = -RC\dfrac{dv_b}{dt}$

$$\frac{2v_b}{R} + C\frac{dv_b}{dt} + C\frac{d(v_b - v_o)}{dt} = 0$$

(3) Therefore $\dfrac{dv_b}{dt} + \dfrac{v_b}{RC} = \dfrac{1}{2}\dfrac{dv_o}{dt}$

From (2) we have $\dfrac{dv_a}{dt} = -RC\dfrac{d^2v_b}{dt^2}$ and $v_a = -RC\dfrac{dv_b}{dt}$

When these are substituted into (1) we get

(4) $-RC\dfrac{d^2v_b}{dt^2} - \dfrac{dv_b}{dt} = \dfrac{v_g}{2RC}$

Now differentiate (3) to get

(5) $\dfrac{d^2v_b}{dt^2} + \dfrac{1}{RC}\dfrac{dv_b}{dt} = \dfrac{1}{2}\dfrac{d^2v_o}{dt^2}$

But from (4) we have

(6) $\frac{d^2v_b}{dt^2} + \frac{1}{RC}\frac{dv_b}{dt} = -\frac{v_g}{2R^2C^2}$

Now substitute (6) into (5)

$$\frac{d^2v_o}{dt^2} = -\frac{v_g}{R^2C^2}$$

[b] When $R_1C_1 = R_2C_2 = RC$: $\frac{d^2v_o}{dt^2} = \frac{v_g}{R^2C^2}$

The two equations are the same except for a reversal in algebraic sign.

[c] Two integrations of the input signal with one operational amplifier.

P9.48 [a] $f(t)$ = inertial force + frictional force + spring force

$= M[d^2x/dt^2] + D[dx/dt] + Kx$

[b] $\frac{d^2x}{dt^2} = \frac{f}{M} - \left(\frac{D}{M}\right)\left(\frac{dx}{dt}\right) - \left(\frac{K}{M}\right)x$

Given $v_A = \frac{d^2x}{dt^2}$, then

$$v_B = -\frac{1}{R_1C_1}\int_0^t \left(\frac{d^2x}{dy^2}\right) dy = -\frac{1}{R_1C_1}\frac{dx}{dt}$$

$$v_C = -\frac{1}{R_2C_2}\int_0^t v_B\, dy = \frac{1}{R_1R_2C_1C_2}x$$

$$v_D = -\frac{R_3}{R_4}\cdot v_B = \frac{R_3}{R_4R_1C_1}\frac{dx}{dt}$$

$$v_E = \left[\frac{R_5+R_6}{R_6}\right]v_C = \left[\frac{R_5+R_6}{R_6}\right]\cdot\frac{1}{R_1R_2C_1C_2}\cdot x$$

$$v_F = \left[\frac{-R_8}{R_7}\right]f(t), \qquad v_A = -(v_D + v_E + v_F)$$

Therefore $\frac{d^2x}{dt^2} = \left[\frac{R_8}{R_7}\right]f(t) - \left[\frac{R_3}{R_4R_1C_1}\right]\frac{dx}{dt} - \left[\frac{R_5+R_6}{R_6R_1R_2C_1C_2}\right]x$

Therefore $M = \frac{R_7}{R_8}$, $D = \frac{R_3R_7}{R_8R_4R_1C_1}$ and $K = \frac{R_7(R_5+R_6)}{R_8R_6R_1R_2C_1C_2}$

Box Number	Function
1	inverting and scaling
2	inverting and summing
3	integrating and scaling
4	integrating and scaling
5	inverting and scaling
6	noninverting and scaling

P 9.49 From the form of the solution we have

$$v(0) = A_1 + A_2$$

$$\frac{dv(0^+)}{dt} = -\alpha(A_1 + A_2) + j\omega_d(A_1 - A_2)$$

We know both $v(0)$ and $dv(0^+)/dt$ will be real numbers. To facilitate the algebra we let these numbers be K_1 and K_2, respectively. Then our two simultaneous equations are

$$K_1 = A_1 + A_2$$

$$K_2 = (-\alpha + j\omega_d)A_1 + (-\alpha - j\omega_d)A_2$$

The characteristic determinate is $\Delta = \begin{vmatrix} 1 & 1 \\ (-\alpha + j\omega_d) & (-\alpha - j\omega_d) \end{vmatrix} = -j2\omega_d$

The numerator determinates are $N_1 = \begin{vmatrix} K_1 & 1 \\ K_2 & (-\alpha - j\omega_d) \end{vmatrix} = -(\alpha + j\omega_d)K_1 - K_2$

and $N_2 = \begin{vmatrix} 1 & K_1 \\ (-\alpha + j\omega_d) & K_2 \end{vmatrix} = K_2 + (\alpha - j\omega_d)K_1$

It follows that $A_1 = \dfrac{N_1}{\Delta} = \dfrac{\omega_d K_1 - j(\alpha K_1 + K_2)}{2\omega_d}$

and $A_2 = \dfrac{N_2}{\Delta} = \dfrac{\omega_d K_1 + j(\alpha K_1 + K_2)}{2\omega_d}$

We see from these expressions that $A_1 = A_2^*$

P 9.50 By definition $B_1 = A_1 + A_2$

From the solution to Problem 9.49 we have $A_1 + A_2 = \dfrac{2\omega_d K_1}{2\omega_d} = K_1$

But K_1 is $v(0)$, therefore $B_1 = v(0)$, which is identical to Eq. (9.27).

By definition $B_2 = j(A_1 - A_2)$

From Problem 9.49 we have $B_2 = j(A_1 - A_2) = \dfrac{j[-2j(\alpha K_1 + K_2)]}{2\omega_d} = \dfrac{\alpha K_1 + K_2}{\omega_d}$

It follows that $K_2 = -\alpha K_1 + \omega_d B_2$, but $K_2 = \dfrac{dv(0^+)}{dt}$ and $K_1 = B_1$

Thus we have $\dfrac{dv}{dt}(0^+) = -\alpha B_1 + \omega_d B_2$, which is identical to Eq. (9.28).

10

Sinusoidal Steady-State Analysis

Drill Exercises

DE 10.1 [a] $\omega = 2\pi f = 2513.27\,\text{rad/s}, \quad f = 400\,\text{Hz}$

[b] $T = 1/f = 2.5\,\text{ms}$

[c] $V_m = 40\,\text{V}$

[d] $v(0) = 40(0.8) = 32\,\text{V}$

[e] $\phi = 36.87°$

[f] $2513.27t = 53.13/57.3 = 0.9273\,\text{rad}, \qquad t = 368.96\,\mu\text{s}$

[g] $(dv/dt) = (-40)2513.27\sin(2513.27t + 36.87°)$

$(dv/dt) = 0$ when $2513.27t + 36.87° = 180°$ or $2513.27t = 2.498\,\text{rad}$

Therefore $t = 993.96\,\mu\text{s}$

DE 10.2 $V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^{T/2} V_m^2 \sin^2 \frac{2\pi}{T}t\,dt}$

$$\int_0^{T/2} V_m^2 \sin^2\left(\frac{2\pi}{T}\right) t\,dt = \frac{V_m^2}{2}\int_0^{T/2}\left(1 - \cos\frac{4\pi}{T}t\right)dt = \frac{V_m^2 T}{4}$$

Therefore $V_{\text{rms}} = \sqrt{\frac{1}{T}\frac{V_m^2 T}{4}} = \frac{V_m}{2}$

DE 10.3 [a] The numerical values of the terms in Eq. 10.8 are

$V_m = 100, \qquad R/L = 533.33, \qquad \omega L = 30$

$$\sqrt{R^2 + \omega^2 L^2} = 50$$

$$\phi = 60^\circ, \qquad \theta = \tan^{-1} 30/40, \qquad \theta = 36.87^\circ$$

$$i = \left[-1.84e^{-533.33t} + 2\cos(400t + 23.13^\circ)\right] \text{ A}, \qquad t \geq 0^+$$

[b] Transient component $= -1.84e^{-533.33t}$
Steady-state component $= 2\cos(400t + 23.13^\circ)$

[c] By direct substitution into the expression in [a], $i(1.875\,\text{ms}) = 133.61\,\text{mA}$

[d] 2 A, 400 rad/s, 23.13°

[e] The current lags the voltage by 36.87°.

DE 10.4 [a] $\mathbf{V} = 170\angle{-40^\circ}$

[b] $\mathbf{I} = 10\angle{-70^\circ}$

[c] $\mathbf{I} = 5\angle{36.87^\circ} + 10\angle{-53.13^\circ} = 4 + j3 + 6 - j8 = 10 - j5 = 11.18\angle{-26.6^\circ}$

[d] $\mathbf{V} = 300\angle{45^\circ} - 100\angle{-60^\circ} = 212.13 + j212.13 - (50 - j86.60)$
$= 162.13 + j298.73 = 339.90\angle{61.51^\circ}\,\text{mV}$

DE 10.5 [a] $v = 86.3\cos(\omega t + 26^\circ)\,\text{V}$

[b] $\mathbf{I} = 10\angle{30^\circ} + 25\angle{60^\circ} = 8.66 + j5.00 + 12.50 + j21.65$
$= 21.16 + j26.65 = 34.03\angle{51.55^\circ}$
Therefore $i = 34.03\cos(\omega t + 51.55^\circ)\,\text{mA}$

[c] $\mathbf{V} = 60 + j30 + 100\angle{-28^\circ} = 60 + j30 + 88.29 - j46.95$
$= 148.29 - j16.95 = 149.26\angle{-6.52^\circ}$
$v = 149.26\cos(\omega t - 6.52^\circ)\,\text{V}$

DE 10.6 [a] $\omega L = (4 \times 10^4)(75 \times 10^{-3}) = 3000\,\Omega$

[b] $Z_L = j3000\,\Omega$

[c] $\mathbf{V}_L = \mathbf{I}Z_L = (4\angle{-38^\circ})(3000\angle{90^\circ}) \times 10^{-3} = 12\angle{52^\circ}\,\text{V}$

[d] $v_L = 12\cos(40{,}000t + 52^\circ)\,\text{V}$

DE 10.7 [a] $X_C = \dfrac{-1}{\omega C} = -\dfrac{10^6}{10^5(0.2)} = -50\,\Omega$

[b] $Z_C = jX_C = -j50\,\Omega$

[c] $\mathbf{I} = \dfrac{40\angle{-50^\circ}}{50\angle{-90^\circ}} = 0.8\angle{40^\circ}\,\text{A}$

[d] $i = 0.8\cos(100{,}000t + 40^\circ)\,\text{A}$

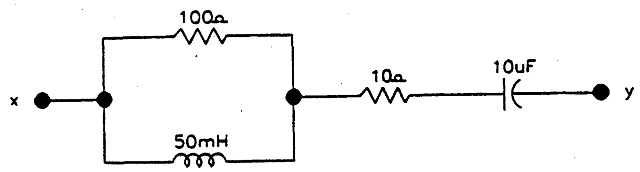
DE 10.8 $\mathbf{I}_1 = 100\angle 25^\circ = 90.63 + j42.26$

$\mathbf{I}_2 = 100\angle 145^\circ = -81.92 + j57.36$

$\mathbf{I}_3 = 100\angle -95^\circ = -8.71 - j99.62$

$\mathbf{I}_4 = -(\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3) = (0 + j0)\,\text{A}, \qquad \text{therefore} \quad i_4 = 0\,\text{A}$

DE 10.9 [a]



$\omega = 1000\,\text{rad/s}$

$\omega L = 50\,\Omega, \qquad \dfrac{-1}{\omega C} = -100\,\Omega$

$$Z_{xy} = \frac{100(j50)}{(100 + j50)} + 10 - j100 = 20 + j40 + 10 - j100 = (30 - j60)\,\Omega$$

[b] $\omega L = 200\,\Omega, \qquad \dfrac{-1}{\omega C} = -25\,\Omega$

$$Z_{xy} = 10 - j25 + \left[\frac{(100)(j200)}{100 + j200}\right] = 10 - j25 + 80 + j40 = (90 + j15)\,\Omega$$

[c] $$Z_{xy} = \left[\frac{100(j\omega L)}{100 + j\omega L}\right] + \left(10 - \frac{j10^5}{\omega}\right) = \frac{100\omega^2 L^2}{10^4 + \omega^2 L^2} + \frac{j10^4 \omega L}{10^4 + \omega^2 L^2} + 10 - \frac{j10^5}{\omega}$$

The impedance will be purely resistive when the j terms cancel, i.e.,

$$\frac{10^4 \omega L}{10^4 + \omega^2 L^2} = \frac{10^5}{\omega}$$

Solving for ω yields $\omega = 2000\,\text{rad/s}$.

[d] $$Z_{xy} = \frac{100\omega^2 L^2}{10^4 + \omega^2 L^2} + 10 = 50 + 10 = 60\,\Omega$$

DE 10.10 $\mathbf{V} = 300\angle 0^\circ, \qquad \mathbf{I}_s = \dfrac{300\angle 0^\circ}{60} = 5\angle 0^\circ\,\text{A}$

$$\mathbf{I}_\text{L} = \frac{5(100)}{100 + j100} = 2.5 - j2.5 = 3.54\angle -45^\circ\,\text{A}$$

$i_\text{L} = 3.54\cos(2000t - 45^\circ)\,\text{A}, \qquad I_m = 3.54\,\text{A}$

DE 10.11 [a] $$Y = \frac{1}{3 + j4} + \frac{1}{16 - j12} + \frac{1}{-j4}$$

$= 0.12 - j0.16 + 0.04 + j0.03 + j0.25 = 0.16 + j0.12$

$= 0.2\angle 36.87^\circ\,\mho \quad = 200\angle 36.87^\circ\,\text{m}\mho$

[b] $G = 0.16\,\mho \quad = 160\,\text{m}\mho$

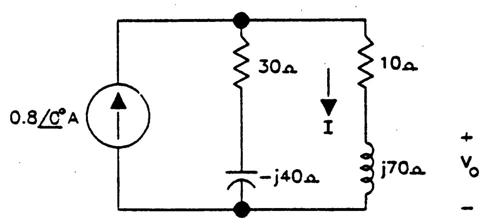
[c] $B = 0.12\mho = 120\,\text{m}\mho$

[d] $\mathbf{I} = 8\underline{/0^\circ}\,\text{A}, \qquad \mathbf{V} = \frac{\mathbf{I}}{Y} = \frac{8}{0.2\underline{/36.87^\circ}} = 40\underline{/-36.87^\circ}\,\text{V}$

$$\mathbf{I}_C = \frac{\mathbf{V}}{Z_C} = \frac{40\underline{/-36.87^\circ}}{4\underline{/-90^\circ}} = 10\underline{/53.13^\circ}\,\text{A}$$

$$i_C = 10\cos(\omega t + 53.13^\circ)\,\text{A}, \qquad I_m = 10\,\text{A}$$

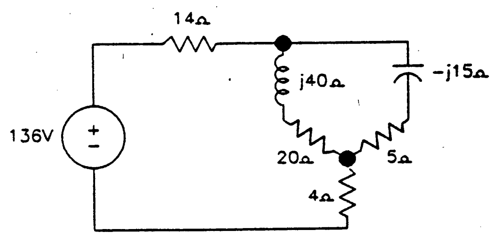
DE 10.12 Construct the phasor domain equivalent circuit:



$$\mathbf{I} = \frac{0.8(30 - j40)}{40 + j40 + j70} = -j0.8\,\text{A}$$

$$\mathbf{V}_o = j70\mathbf{I} = 56\underline{/0^\circ}, \qquad v_o = 56\cos 4000t\,\text{V}$$

DE 10.13 After replacing the delta made up of the 50-Ω, 40-Ω, and 10-Ω resistors with its equivalent wye, the circuit becomes



The circuit is further simplified by combining the parallel branches,

$$(20 + j40)//(5 - j15) = (12 - j16)\,\Omega$$

Therefore $\mathbf{I} = \frac{136\underline{/0^\circ}}{14 + 12 - j16 + 4} = 4\underline{/28.07^\circ}\,\text{A}$

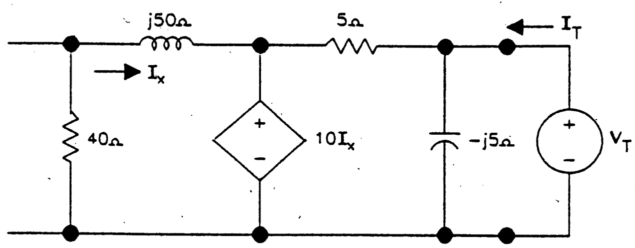
DE 10.14 Use the lower node as the reference node. Let $\mathbf{V}_1$ = node voltage across the 40-Ω resistor and $\mathbf{V}_{\text{Th}}$ = node voltage across the capacitor.

Writing the node voltage equations gives us

$$\frac{\mathbf{V}_1}{40} - 5 + \frac{\mathbf{V}_1 - 10\mathbf{I}_x}{j50} = 0 \quad \text{and} \quad \frac{\mathbf{V}_{\mathrm{Th}}}{-j5} + \frac{\mathbf{V}_{\mathrm{Th}} - 10\mathbf{I}_x}{5} = 0$$

We also have $\mathbf{I}_x = \dfrac{\mathbf{V}_1 - 10\mathbf{I}_x}{j50}$

Solving these equations for $\mathbf{V}_{\mathrm{Th}}$ gives $\mathbf{V}_{\mathrm{Th}} = 20\underline{/-90^\circ}V$. To find the Thévenin impedance, we remove the independent current source and apply a test voltage source at the terminals a, b. Thus



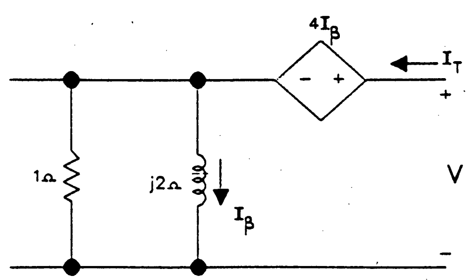
It follows from the circuit that $10\mathbf{I}_x = -(40 + j50)\mathbf{I}_x$

Therefore $\mathbf{I}_x = 0$ and $\mathbf{I}_T = \dfrac{\mathbf{V}_{\mathrm{Th}}}{-j5} + \dfrac{\mathbf{V}_T}{5}$

$Z_{\mathrm{Th}} = \dfrac{\mathbf{V}_{\mathrm{Th}}}{\mathbf{I}_T}$, therefore $Z_{\mathrm{Th}} = (2.5 - j2.5)\,\Omega$

DE 10.15 $\mathbf{V}_{\mathrm{ab}} = \mathbf{V}_{\mathrm{Th}} = j2\mathbf{I}_\beta + 4\mathbf{I}_\beta = 2(2 + j1)\mathbf{I}_\beta$

$\mathbf{I}_\beta = \dfrac{5(1)}{1 + j2} = (1 - j2)\,\mathrm{A}$, therefore $\mathbf{V}_{\mathrm{Th}} = 8 - j6 = 10\underline{/-36.87^\circ}\,\mathrm{V}$



$\mathbf{V}_T = 4\mathbf{I}_\beta + j2\mathbf{I}_\beta = 2(2 + j1)\mathbf{I}_\beta, \qquad \mathbf{I}_\beta = \dfrac{1}{1 + j2}\mathbf{I}_T = \dfrac{1}{5}(1 - j2)\mathbf{I}_T$

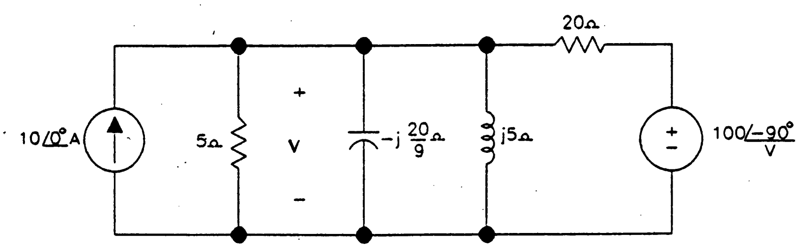
$Z_{\mathrm{Th}} = \dfrac{\mathbf{V}_T}{\mathbf{I}_T} = \dfrac{2}{5}(4 - j3) = 1.6 - j1.2\,\Omega$

DE 10.16 The phasor domain circuit is as shown in the following diagram. The node voltage equation is

$$-10+\frac{\mathbf{V}}{5}+\frac{9\mathbf{V}}{-j20}+\frac{\mathbf{V}}{j5}+\frac{\mathbf{V}-100/\!\underline{-90^\circ}}{20}=0$$

Therefore $\mathbf{V}=10-j30=31.62/\!\underline{-71.57^\circ}$

Therefore $v=31.62\cos(50{,}000t-71.57^\circ)\,\mathrm{V}$



DE 10.17 Let $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$ be the three clockwise mesh currents going from left to right. Summing the voltages around meshes a and b gives $33.8=(1+j2)\mathbf{I}_a+(3-j5)(\mathbf{I}_a-\mathbf{I}_b)$ and $0=(3-j5)(\mathbf{I}_b-\mathbf{I}_a)+2(\mathbf{I}_b-\mathbf{I}_c)$. But $\mathbf{V}_x=-j5(\mathbf{I}_a-\mathbf{I}_b)$, therefore $\mathbf{I}_c=-0.75[-j5(\mathbf{I}_a-\mathbf{I}_b)]$. Solving for $\mathbf{I}=\mathbf{I}_a=29+j2=29.07/\!\underline{3.95^\circ}$ A.

DE 10.18 [a] $\mathbf{I}=\frac{240}{24}+\frac{240}{j32}=(10-j7.5)\,\mathrm{A}$

$\mathbf{V}_s=240/\!\underline{0^\circ}+(0.1+j0.8)(10-j7.5)=247+j7.25=247.11/\!\underline{1.68^\circ}\,\mathrm{V}$

[b] Use the capacitor to eliminate the j component of $\mathbf{I}$, therefore

$$\mathbf{I}_c=j7.5\,\mathrm{A},\qquad Z_c=\frac{240}{j7.5}=-j32\,\Omega$$

$$\mathbf{V}_s=240+(0.1+j0.8)10=241+j8=241.13/\!\underline{1.90^\circ}\,\mathrm{V}$$

[c] Let $\mathbf{I}_c$ denote the magnitude of the current in the capacitor branch. Then

$$\mathbf{I}=(10-j7.5+j\mathbf{I}_c)=10+j(\mathbf{I}_c-7.5)\,\mathrm{A}$$

$$\mathbf{V}_s=240/\!\underline{\alpha}=240+(0.1+j0.8)[10+j(\mathbf{I}_c+7.5)]$$
$$=(247-0.8\mathbf{I}_c)+j(7.25+0.1\mathbf{I}_c)$$

It follows that $240\cos\alpha=(247-0.8\mathbf{I}_c)$ and $240\sin\alpha=(7.25+0.1\mathbf{I}_c)$

Now square each term and then add to generate the quadratic equation $\mathbf{I}_c^2-605.77\mathbf{I}_c+5325.48=0$, $\mathbf{I}_c=302.88\pm293.96$, therefore $\mathbf{I}_c=8.92\,\mathrm{A}$ (smallest value) and $Z_c=240/j8.92$ or $-j26.90\,\Omega$.

Problems

P 10.1 [a] 100 V; [b] $f = 400\pi/2\pi = 200$ Hz; [c] $\omega = 400\pi = 1256.64$ rad/s

[d] $\theta = \pi/3$ or 1.05 radians; [e] $\theta = 60°$; [f] $T = 1/f = 5$ ms

[g] $400\pi t + \frac{\pi}{3} = 2\pi$

$\therefore\ 400\pi t = \frac{6\pi}{3} - \frac{\pi}{3} = \frac{5\pi}{3}; \qquad t = \frac{5}{1200} = \frac{5}{1.2}\text{ ms} = \frac{25}{6}\text{ ms}; \qquad t = 4\frac{1}{6}\text{ ms}$

[h] $v(t) = 100\cos\left[400\pi\left(t + \frac{5}{12}\times 10^{-3}\right) + \frac{\pi}{3}\right] = 100\cos\left(400\pi t + \frac{\pi}{6} + \frac{\pi}{3}\right)$

$= 100\cos\left(400\pi t + \frac{\pi}{2}\right)$ or

$v(t) = -100\sin(400\pi t)$ V

[i] $400\pi(t - t_o) + \frac{\pi}{3} = 400\pi t$

$\therefore\ -400\pi t_o + \frac{\pi}{3} = 0; \qquad 400\pi t_o = \frac{\pi}{3}$

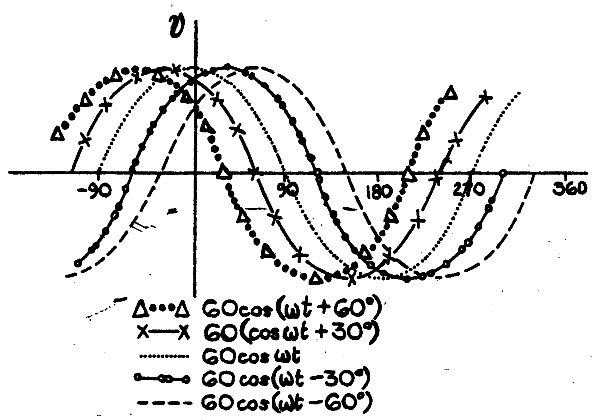
$t_o = \frac{\pi}{1200\pi} = \frac{1}{1200} = \frac{1}{1.2}\text{ ms}; \qquad t_o = \frac{10}{12}\text{ ms} = \frac{5}{6}\text{ ms}$

[j] $400\pi(t + t_o) + \frac{\pi}{3} = 400\pi t + \frac{3\pi}{2}$

$\therefore\ 400\pi t_o = -\frac{\pi}{3} + \frac{3\pi}{2} = \frac{-2\pi}{6} + \frac{9\pi}{6} = \frac{7\pi}{6}$

$t_o = \frac{7\pi}{2400\pi} = \frac{7}{2400} = \frac{7}{2.4}\text{ ms}; \qquad t_o = \frac{70}{24}\text{ ms} = \frac{35}{12}\text{ ms}$

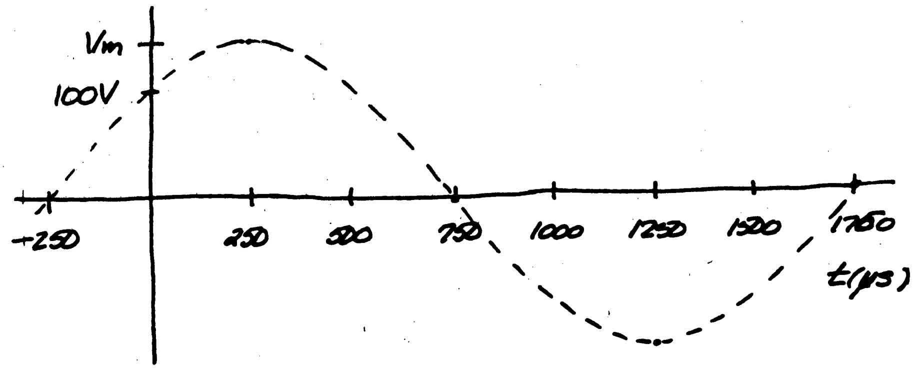
P 10.2



[a] Left as ϕ becomes more positive

[b] Right

P 10.3 [a]



$$T = 1750 + 250 = 2000\,\mu\text{s}; \qquad f = \frac{1}{T} = \frac{10^6}{2000} = 500\text{ Hz}$$

[b] $\omega = 2\pi f = 1000\pi$ rad/s; $\quad 250\,\mu\text{s} \Rightarrow \pi/4$ radians

$\therefore\ v = V_m \sin(1000\pi t + \pi/4)$

$\therefore\ 100 = V_m \sin\pi/4, \qquad V_m = 100\sqrt{2}$

$\therefore\ v = 100\sqrt{2}\sin(1000\pi t + \pi/4) = 100\sqrt{2}\cos(1000\pi t - \pi/4)$

P 10.4 [a] $i = 10\cos(\omega t + \theta); \qquad \dfrac{di}{dt} = -10\omega\sin(\omega t + \theta)$

$i(15\,\mu\text{s}) = 0 = 10\cos(15 \times 10^{-6}\omega + \theta)$

$\therefore\ 15 \times 10^{-6}\omega + \theta = \pm\pi/2$

$-\ 10\omega\sin(\pm\pi/2) = 2 \times 10^5\pi$

Since $\omega > 0$, minus $\pi/2$ must be used.

$\therefore\ 10\omega = 2 \times 10^5\pi; \qquad \omega = 2 \times 10^4\pi = 20{,}000\pi$ rad/s

[b] $15 \times 10^{-6}(20{,}000\pi) + \theta = -\dfrac{\pi}{2}$

$\theta = -\dfrac{\pi}{2} - 0.3\pi = -0.8\pi = \dfrac{-8\pi}{10} = -\dfrac{4\pi}{5}$ radians or $-\ 144^\circ$

$i = 10\cos(2 \times 10^4\pi t - 144^\circ)$ A

P 10.5 [a] From Eq. 10.8 we have

$$L\frac{di}{dt} = \frac{V_m R\cos(\phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}} e^{-(R/L)t} - \frac{\omega L V_m \sin(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}}$$

$$Ri = \frac{-V_m R\cos(\phi - \theta)e^{-(R/L)t}}{\sqrt{R^2 + \omega^2 L^2}} + \frac{V_m R\cos(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}}$$

$$L\frac{di}{dt} + Ri = V_m\left[\frac{R\cos(\omega t + \phi - \theta) - \omega L\sin(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}}\right]$$

But $\dfrac{R}{\sqrt{R^2 + \omega^2 L^2}} = \cos\theta$ and $\dfrac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} = \sin\theta$

Therefore the right-hand side reduces to $V_m \cos(\omega t + \phi)$.

At $t = 0$, Eq. 10.8 reduces to $i(0) = \frac{-V_m \cos(\phi - \theta)}{\sqrt{R^2 - \omega^2 L^2}} + \frac{V_m \cos(\phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}} = 0$

[b] $i_{ss} = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)$

Therefore $L\frac{di_{ss}}{dt} = \frac{-\omega L V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t + \phi - \theta)$ and

$Ri_{ss} = \frac{V_m R}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)$

$$L\frac{di_{ss}}{dt} + Ri_{ss} = V_m \left[\frac{R\cos(\omega t + \phi - \theta) - \omega L \sin(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}}\right] = V_m \cos(\omega t + \phi)$$

P 10.6 [a] $\mathbf{Y} = 100\underline{/30^\circ} + 50\underline{/-45^\circ} = 86.60 + j50 + 35.36 - j35.36$

$= 121.96 + j14.64 = 122.83\underline{/6.85^\circ}$

$\therefore\ y = 122.83\cos(500t + 6.85^\circ)$

[b] $\mathbf{Y} = 100\underline{/-50^\circ} - 50\underline{/200^\circ} = 64.28 - j76.60 - (-46.98 - j17.10)$

$= 111.26 - j59.50 = 126.18\underline{/-28.14^\circ}$

$\therefore\ y = 126.18\cos(377t - 28.14^\circ)$

[c] $\mathbf{Y} = 40\underline{/60^\circ} + 80\underline{/45^\circ} - 100\underline{/270^\circ}$

$= 20 + j34.64 + 56.57 + j56.57 - (0 - j100)$

$= 76.57 + j191.21 = 205.97\underline{/68.18^\circ}$

$\therefore\ y = 205.97\cos(\omega t + 68.18^\circ)$

[d] $\mathbf{Y} = 100\underline{/0^\circ} + 100\underline{/120^\circ} + 100\underline{/-120^\circ}$

$= 100 + j0 + (-50 + j86.60) + (-50 - j86.60)$

$= 100 - 100 + j86.60 - j86.60 = 0$

$\therefore\ y = 0$

P 10.7 $u = \int_{t_o}^{t_o+T} V_m^2 \cos^2(\omega t + \phi)\, dt$

$$= V_m^2 \int_{t_o}^{t_o+T} \frac{1}{2} + \frac{1}{2}\cos(2\omega t + 2\phi)\, dt$$

$$= \frac{V_m^2}{2}\left\{\int_{t_o}^{t_o+T} dt + \int_{t_o}^{t_o+T} \cos(2\omega t + 2\phi)\, dt\right\}$$

$$= \frac{V_m^2}{2}\left\{T + \frac{1}{2\omega}\left[\sin(2\omega t + 2\phi)\ \Big|_{t_o}^{t_o+T}\right]\right\}$$

$$= \frac{V_m^2}{2}\left\{T + \frac{1}{2\omega}[\sin(2\omega t_o + 4\pi + 2\phi) - \sin(2\omega t_o + 2\phi)]\right\}$$

$$= V_m^2\left(\frac{T}{2}\right) + \frac{1}{2\omega}(0) = V_m^2\left(\frac{T}{2}\right)$$

P 10.8 $V_m = \sqrt{2}V_{\text{rms}} = \sqrt{2}(120) = 169.71\,\text{V}$

P 10.9 [a] 1000 Hz; [b] 0°

[c] $\mathbf{I} = \dfrac{\mathbf{V}}{j\omega L} = \dfrac{200\underline{/0^\circ}}{\omega L\underline{/90^\circ}} = \dfrac{200}{\omega L}\underline{/-90^\circ}$; $\therefore\ \theta_i = -90^\circ$

[d] $\dfrac{200}{\omega L} = 25, \qquad \omega L = 8\,\Omega$

[e] $\omega = 2\pi f = 2000\pi\,\text{rad/s}$; $\therefore\ L = \dfrac{8}{2000\pi} = \dfrac{4}{\pi}\,\text{mH} \cong 1.27\,\text{mH}$

[f] $Z_L = j\omega L = j8\,\Omega$

P 10.10 [a] $\omega = 2\pi(50 \times 10^3) = 100\pi\,\text{krad/s}$

[b] $\mathbf{I} = j\omega C\mathbf{V} = j\omega C \times 10 \times 10^{-3}\underline{/0^\circ} = 1000\pi C\underline{/90^\circ}$; $\therefore\ \theta_i = 90^\circ$

[c] $1000\pi C = 628.32 \times 10^{-6}$

$$\therefore\ C = \frac{628.32 \times 10^{-6}}{1000\pi} = 0.2 \times 10^{-6}$$

$$\therefore\ -\frac{1}{\omega C} = \frac{10^6}{(100\pi \times 10^3)(0.2)} = \frac{-10}{0.2\pi} = -\frac{50}{\pi} = -15.92\,\Omega$$

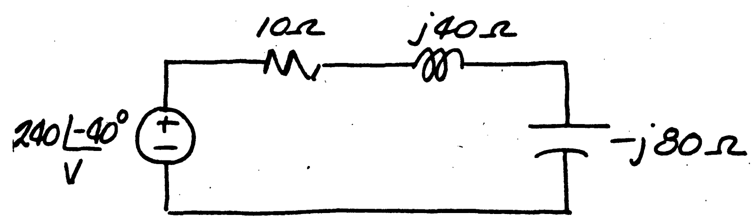
[d] $C = 0.2\,\mu\text{F}$

[e] $Z_C = \dfrac{1}{j\omega C} = -j\dfrac{1}{\omega C} = -j15.92\,\Omega$

P 10.11 [a] $j\omega L = j(5000)(8 \times 10^{-3}) = j40\,\Omega$

$$\frac{1}{j\omega C} = \frac{10^6}{j(5000)(2.5)} = -j\frac{1000}{12.5} = -j80\,\Omega$$

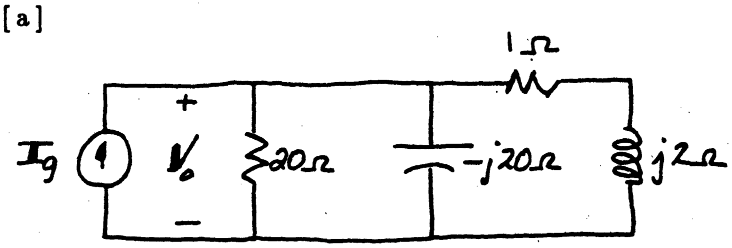
$\mathbf{V} = 240\underline{/-40^\circ}\,\text{V}$



[b] $\mathbf{I} = \dfrac{240\underline{/-40^\circ}}{10 + j40 - j80} = \dfrac{24\underline{/-40^\circ}}{1 - j4}$; $\mathbf{I} = 5.82\underline{/35.96^\circ}\,\text{A}$

[c] $i = 5.82\cos(5000t + 35.96^\circ)\,\text{A}$

P 10.12 [a]



$$\frac{1}{j\omega C} = \frac{10^6}{j50{,}000} = -j20\,\Omega$$

$$j\omega L = j(50{,}000)(40 \times 10^{-6}) = j2\,\Omega$$

[b] $$Y = \frac{1}{20} + \frac{1}{-j20} + \frac{1}{1+j2} = 0.05 + j0.05 + \frac{1-j2}{5}$$

$$= 0.25 + j0.05 - j0.40 = 0.25 - j0.35 = 0.43\angle{-54.46^\circ}\ \mho$$

$$Z = \frac{1}{Y} = 2.32\angle{54.46^\circ}\,\Omega$$

$$\mathbf{V}_o = Z\mathbf{I}_g; \qquad \mathbf{I}_g = 20\angle{-20^\circ}\text{ A}; \qquad \mathbf{V}_o = 46.50\angle{34.46^\circ}\text{ V}$$

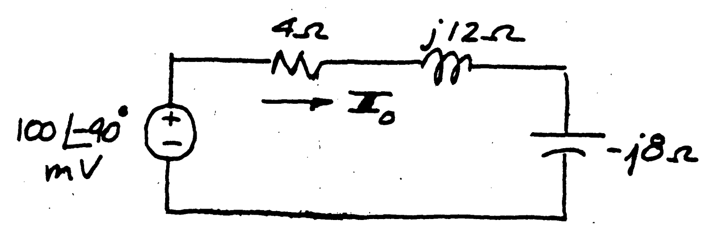
[c] $$v_o(t) = 46.50\cos(50{,}000t + 34.46^\circ)\text{ V}$$

P 10.13 Construct the phasor-domain circuit

$$j\omega L = j50(240 \times 10^{-3}) = j12\,\Omega$$

$$\frac{1}{j\omega C} = \frac{10^3}{j50(2.5)} = -j8\,\Omega$$

$$\mathbf{V}_s = 100\angle{-90^\circ}\text{ mV}$$

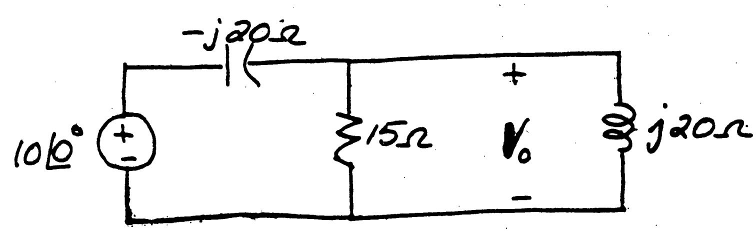


$$\mathbf{I}_o = \frac{100\angle{-90^\circ} \times 10^{-3}}{4 + j12 - j8} = \frac{25\angle{-90^\circ}}{1+j1}\text{ mA} = 17.68\angle{-135^\circ}\text{ mA}$$

$$i_o(t) = 17.68\cos(50t - 135^\circ)\text{ mA}$$

P 10.14 Construct the phasor-domain circuit

$$\frac{1}{j\omega C} = \frac{10^6}{(j100)(500)} = -j20\,\Omega; \qquad j\omega L = j20\,\Omega; \qquad \mathbf{V}_g = 10\angle{0^\circ}\text{ V}$$



Let $Z_1 = -j20\,\Omega$, $\quad Z_2 = \dfrac{(15)(j20)}{15+j20} = \dfrac{j60}{3+j4} = 12\underline{/36.87^\circ}$

$$\mathbf{V}_o = \frac{10\underline{/0^\circ}(Z_2)}{(Z_1+Z_2)} = \frac{120\underline{/36.87^\circ}}{9.6-j12.80} = 7.5\underline{/90^\circ}\,\text{V}$$

$v_o = 7.5\cos(100t + 90^\circ)$ V

P 10.15 Let $Z_1 = 4 - j8\,\Omega$, $\quad Z_2 = 2 + j10\,\Omega$

Then $Z_1 /\!/ Z_2 = \dfrac{(4-j8)(2+j10)}{6+j2} = \dfrac{88+j24}{(6+j2)} = \dfrac{(88+j24)(6-j2)}{40} = 14.4 - j0.8\,\Omega$

$Z_3 = 20\,\Omega$, $\quad Z_4 = -j10\,\Omega$

$$Z_3 /\!/ Z_4 = \frac{-j200}{20-j10} = \frac{-j20}{2-j1} = \frac{-j20(2+j1)}{5} = 4 - j8\,\Omega$$

$Z_{ab} = 21.6 + j38.8 + 14.4 - j0.8 + 4 - j8 = 40 + j30\,\Omega = 50\underline{/36.87^\circ}\,\Omega$

P 10.16 $Y_1 = \dfrac{1}{6-j2} + \dfrac{1}{4+j12} + \dfrac{1}{5} + \dfrac{1}{j10} = \dfrac{6+j2}{40} + \dfrac{4-j12}{160} + \dfrac{32}{160} - j\dfrac{16}{160}$

$$= \frac{24+j8+4-j12+32-j16}{160} = \frac{60-j20}{160} = \frac{3-j1}{8}$$

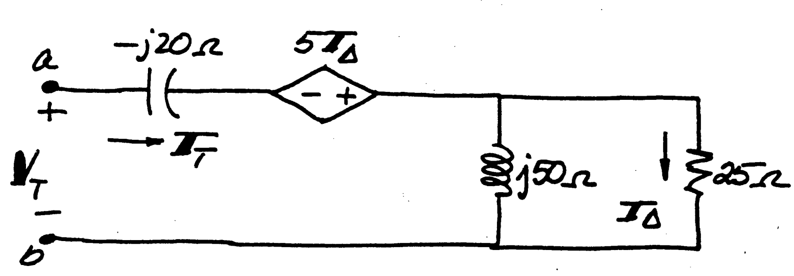
$$Z_1 = \frac{1}{Y_1} = \frac{8}{3-j1} = 2.4 + j0.8$$

$Z_{ab} = 13.6 - j12.8 + 2.4 + j0.8 = 16 - j12 = 20\underline{/-36.87^\circ}$

$$Y_{ab} = \frac{1}{Z_{ab}} = 50\underline{/36.87^\circ}\,\text{m}\mho \quad = 40 + j30\,\text{m}\mho$$

P 10.17 $\dfrac{1}{j\omega C} = \dfrac{10^6}{j2\times 10^4(2.5)} = \dfrac{100}{j5} = -j20\,\Omega$

$j\omega L = j2 \times 10^4(2.5) \times 10^{-3} = j50\,\Omega$



$$\mathbf{V}_T = -j20\mathbf{I}_T - 5\frac{(j50)\mathbf{I}_T}{25+j50} + \mathbf{I}_T\frac{(25)(j50)}{(25+j50)}$$

$$\frac{\mathbf{V}_T}{\mathbf{I}_T} = -j20 - \frac{j250}{25+j50} + j\frac{1250}{25+j50} = -j20 + j\frac{1000}{25+j50} = -j20 + j\frac{40}{1+j2}$$

$$Z_{ab} = -j20 + j8 + 16 = 16 - j12\,\Omega$$

P 10.18 [a] $$Z_{ab} = j\omega 4 + \frac{5000[(4\times10^6)/j\omega]}{5000+[(4\times10^6)/j\omega]} = j\omega 4 + \frac{2\times10^{10}}{5000(j\omega)+4\times10^6}$$

$$= j\omega 4 + \frac{2\times10^7}{j\omega 5 + 4000} = j\omega 4 + \frac{(4000 - j\omega 5)(2\times10^7)}{16\times10^6 + 25\omega^2}$$

$$\therefore\ 4\omega = \frac{10^8\omega}{25\omega^2 + 16\times10^6}$$

$$\therefore\ 25\omega^2 + 16\times10^6 = 25\times10^6$$

$$25\omega^2 = 9\times10^6; \qquad 5\omega = 3000; \qquad \omega = 600\,\text{rad/s}$$

[b] $$Z_{ab} = j2400 + \frac{5000[(4\times10^6)/j600]}{5000+[(4\times10^6)/j600]} = j2400 + \frac{2\times10^{10}}{4\times10^6 + j3\times10^6}$$

$$= j2400 + \frac{20{,}000}{4+j3} = j2400 + \frac{20{,}000(4-j3)}{25}$$

$$= j2400 + 3200 - j2400$$

$$Z_{ab} = 3200\,\Omega$$

P 10.19 [a] $$\frac{1}{j\omega C} = \frac{10^6}{j(500)(2.5)} = -j800\,\Omega$$

$$\therefore\ Z = 400 - j800 + \frac{2000(j\omega L)}{2000 + j\omega L} = 400 - j800 + \frac{j2000L}{4+jL}$$

$$= 400 - j800 + \frac{j2000L(4-jL)}{16+L^2} = 400 - j800 + \frac{2000L^2}{16+L^2} + j\frac{8000L}{16+L^2}$$

$$\therefore\ \frac{8000L}{16+L^2} = 800; \qquad 10L = 16 + L^2$$

$$\therefore\ L^2 - 10L + 16 = 0; \qquad L = 5 \pm \sqrt{25-16} = 5 \pm 3$$

$$L_1 = 2\,\text{H}; \qquad L_2 = 8\,\text{H}$$

[b] When $L_1 = 2\,\text{H}$

$$Z = 400 + \frac{2000(4)}{(16+4)} = 400 + 400 = 800\,\Omega$$

$$\mathbf{I}_g = \frac{200\angle 0^\circ}{800\angle 0^\circ} = 0.25\angle 0^\circ\,\text{A}$$

$$i_g = 0.25\cos 500t\,\text{A}$$

When $L_2 = 8\,\text{H}$

$$Z = 400 + \frac{2000(64)}{(16+64)} = 2000\,\Omega$$

$$\mathbf{I}_g = \frac{200/\underline{0^\circ}}{2000/\underline{0^\circ}} = 0.1/\underline{0^\circ}\text{ A}$$
$$i_g = 0.10\cos 500t\text{ A}$$

P10.20 [a] $$Z = j1000(5) + \frac{12{,}500(1/j1000C)}{12{,}500 + (1/j1000C)}$$
$$= j5000 + \frac{12{,}500}{1 + j12.5\times 10^6 C}$$
$$= j5000 + \frac{12{,}500(1 - j12.5\times 10^6 C)}{1 + 156.25\times 10^{12}C^2}$$
$$= \frac{12{,}500}{1 + 156.25\times 10^{10}C^2} + j5000 - \frac{j156.25\times 10^9 C}{1 + 156.25\times 10^{12}C^2}$$

i_g will be in phase with v_g when the j terms cancel, therefore
$$5000 = \frac{156.25\times 10^9 C}{1 + 156.25\times 10^{12}C^2}$$
$$1 + 156.25\times 10^{12}C^2 = 31{,}250\times 10^3 C$$
$$C^2 - 2\times 10^{-7}C + 6400\times 10^{-18} = 0$$
$$C = 10^{-7} \pm \sqrt{10^{-14} - 0.64\times 10^{-14}} = 10^{-7} \pm \sqrt{0.36\times 10^{-14}}$$
$$= 10^{-7} \pm 0.6\times 10^{-7} = (0.10 \pm 0.06)\times 10^{-6}$$
$$C = 0.16\,\mu\text{F} \quad\text{or}\quad C = 0.04\,\mu\text{F}$$

[b] When $C = 0.16\,\mu$F
$$Z = \frac{12{,}500}{1 + 15625\times 10^{10}(0.16\times 10^{-6})^2} + j0 = \frac{12{,}500}{1+4} = 2500 + j0\,\Omega$$
$$\mathbf{I}_g = \frac{250/\underline{0^\circ}}{2500/\underline{0^\circ}} = 0.10/\underline{0^\circ}\text{ A} = 100/\underline{0^\circ}\text{ mA}$$
$$i_g = 100\cos 1000t\text{ mA}$$

When $C = 0.04\,\mu$F
$$Z = \frac{12{,}500}{1 + 15625\times 10^{10}(0.04\times 10^{-6})^2} + j0 = \frac{12{,}500}{1+0.25} = 10{,}000 + j0\,\Omega$$
$$\mathbf{I}_g = \frac{250/\underline{0^\circ}}{10{,}000/\underline{0^\circ}} = 25\times 10^{-3}/\underline{0^\circ}\text{ A} = 25/\underline{0^\circ}\text{ mA}$$
$$i_g = 25\cos 1000t\text{ mA}$$

P10.21 [a] Let $Y_1 = \frac{11}{2500}\times 10^{-3} = 4.4\times 10^{-6}\,\mho$

Let $$Y_2 = \frac{1}{14{,}000 + j\omega 5} + j\omega 2\times 10^{-9} = \frac{14{,}000 - j5\omega}{196\times 10^6 + 25\omega^2} + j2\omega\times 10^{-9}$$
$$= \frac{14{,}000}{196\times 10^6 + 25\omega^2} + j2\omega\times 10^{-9} - \frac{j5\omega}{196\times 10^6 + 25\omega^2}$$

v_o will be in phase with i_g when Y_2 is pure conductance. Therefore we find ω such that

$$2\omega \times 10^{-9} = \frac{5\omega}{196 \times 10^6 + 25\omega^2}$$

$$\therefore\ 196 \times 10^6 + 25\omega^2 = 2.5 \times 10^9 = 2500 \times 10^6$$

$$\therefore\ 25\omega^2 = 2304 \times 10^6$$

$$\omega^2 = 92.16 \times 10^6; \qquad \omega = 9.6 \times 10^3 = 9600\,\text{rad/s}$$

[b] $Y_2(9600) = \dfrac{14{,}000}{196 \times 10^6 + 25(92.16 \times 10^6)} = \dfrac{14{,}000}{2500} \times 10^{-6} = 5.6 \times 10^{-6}\,\mho$

$$\therefore\ Y = Y_1 + Y_2 = (4.4 + 5.6) \times 10^{-6} = 10^{-5}\,\mho$$

$$Z = \frac{1}{Y} = 10^5\,\Omega$$

$$\mathbf{V}_o = \mathbf{I}_g Z = (0.25 \times 10^{-3}\underline{/0^\circ})10^5\underline{/0^\circ} = 25\underline{/0^\circ}\,\text{V}$$

$$v_o = 25\cos 9600t\ \text{V}$$

P 10.22 [a] Let $Y = j\omega 10 \times 10^{-9} + \dfrac{1}{8000 + j\omega(1)} = j\omega 10^{-8} + \dfrac{8000 - j\omega}{64 \times 10^6 + \omega^2}$

$$= \frac{8000}{64 \times 10^6 + \omega^2} + j10^{-8}\omega - \frac{j\omega}{64 \times 10^6 + \omega^2}$$

i_o will be in phase with v_g when Y is pure conductance, therefore

$$\omega 10^{-8} = \frac{\omega}{\omega^2 + 64 \times 10^6}$$

$$\omega^2 + 64 \times 10^6 = 10^8 = 100 \times 10^6$$

$$\omega^2 = 36 \times 10^6; \qquad w = 6 \times 10^3 = 6000\,\text{rad/s}$$

$$f = \frac{\omega}{2\pi} = \frac{3000}{\pi} = 954.93\,\text{Hz}$$

[b] $Z(6000) = \dfrac{1}{Y(6000)}$

$$Y(6000) = \frac{8000}{64 \times 10^6 + 36 \times 10^6} = \frac{8000}{10^8}$$

$$Z(6000) = \frac{10^8}{8000} = 12{,}500\,\Omega$$

$$\therefore\ Z_g = 3500 + 12{,}500 = 16{,}000\,\Omega$$

$$\mathbf{I}_o = \frac{\mathbf{V}_g}{Z_g} = \frac{80\underline{/0^\circ}}{16{,}000\underline{/0^\circ}} = 5 \times 10^{-3} = 5\,\text{mA}$$

$$i_o = 5\cos 6000t\ \text{mA}$$

P 10.23 $\mathbf{V}_g = 500\underline{/30^\circ}\,\text{V}; \qquad \mathbf{I}_o = 0.10\underline{/83.1301^\circ}$

$$Z = \frac{\mathbf{V}_g}{\mathbf{I}_o} = 5000\underline{/-53.13^\circ} = 3000 - j4000\,\Omega$$

$$\therefore\ j\omega 1 + \frac{10^9}{j\omega(31.25)} = -j4000$$

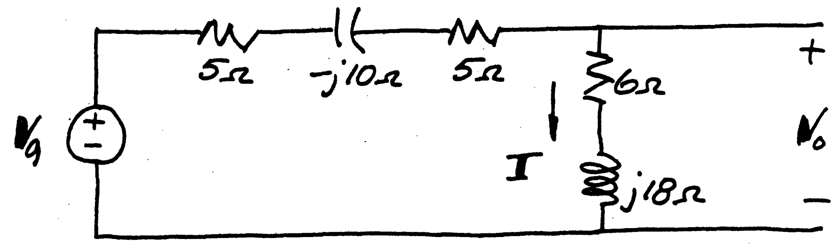
$$\therefore\ j\omega + \frac{32\times10^6}{j\omega} = -j4000$$
$$\omega^2 - 32\times10^6 = -4000\omega$$
$$\omega^2 + 4000\omega - 32\times10^6 = 0$$
$$\omega = -2000 \pm \sqrt{36\times10^6} = -2000 \pm 6000, \qquad \omega > 0$$
$$\therefore\ \omega = 4000\,\text{rad/s}$$

P 10.24 $\frac{1}{j\omega C} = -j\frac{100}{8} = -j12.5\,\Omega$

$j\omega L = j8(2.25) = j18\,\Omega$

$Z_1 = 25\,\Omega // -j12.5\,\Omega = \frac{(25)(-j12.5)}{25 - j12.5} = \frac{-j25}{2 - j1} = -j5(2 + j1) = 5 - j10\,\Omega$

$\mathbf{I}_g = 10\underline{/0^\circ}\,\text{A}; \qquad \mathbf{V}_g = 50 - j100\,\text{V}$



$$\mathbf{I} = \frac{50 - j100}{16 + j8} = \frac{6.25 - j12.5}{2 + j1} = \frac{1}{5}(6.25 - j12.5)(2 - j1) = (1.25 - j2.5)(2 - j1)$$
$$= 2.5 - j1.25 - j5.0 - 2.5 = -j6.25 = 6.25\underline{/-90^\circ}\,\text{A}$$
$$\mathbf{V}_o = (6 + j18)(6.25\underline{/-90^\circ}) = 118.59\underline{/-18.43^\circ}\,\text{V}$$
$$v_o = 118.59\cos(8t - 18.43^\circ)\,\text{V}$$

P 10.25 $Z_1 = 30 + j0.25\times10^{-3}(40{,}000) = 30 + j10\,\Omega$

$Z_2 = 20 - j\frac{10^6}{(40{,}000)(2.5)} = 20 - j10\,\Omega$

$\mathbf{I}_g = 20\underline{/-73.74^\circ}\,\text{A}$

Let $\mathbf{I}_C$ represent the current down through the capacitive branch. Then

$$\mathbf{I}_C = \frac{Z_1}{Z_1 + Z_2}\mathbf{I}_g = \frac{30 + j10}{50}(20\underline{/-73.74^\circ}) = 12.65\underline{/-55.31^\circ}\,\text{A}$$
$$\mathbf{V}_o = 20\mathbf{I}_C = 252.98\underline{/-55.31^\circ}\,\text{V}$$
$$v_o = 252.98\cos(40{,}000t - 55.31^\circ)\,\text{V}$$

P 10.26 [a] $\mathbf{V}_g = 240\underline{/120^\circ}\,\text{V}; \qquad \mathbf{I}_g = 12\underline{/84^\circ}\,\text{A}; \qquad Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = \frac{240\underline{/120^\circ}}{12\underline{/84^\circ}} = 20\underline{/36^\circ}$

[b] $\omega = 2\pi f = 2000\pi; \qquad f = 1000\,\text{Hz}; \qquad T = \frac{1}{1000} = 1\,\text{ms}$

$36^\circ \equiv 1/10$ of the period

$\therefore \left(\frac{1}{10}\right)(10^{-3}) = 100 \times 10^{-6} = 100\,\mu\text{s}$

$\therefore$ Current is out of phase with the voltage by $100\,\mu$s. Note i_g lags v_g.

P10.27 [a] $Z_1 = R_1 - j\frac{1}{\omega C_1}$

$$Z_2 = \frac{R_2/j\omega C_2}{R_2 + (1/j\omega C_2)} = \frac{R_2}{1 + j\omega R_2 C_2} = \frac{R_2 - j\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2}$$

$Z_1 = Z_2$ when $R_1 = \frac{R_2}{1 + \omega^2 R_2^2 C_2^2}$ and

$$\frac{1}{\omega C_1} = \frac{\omega R_2^2 C_2^2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{or} \quad C_1 = \frac{1 + \omega^2 R_2^2 C_2^2}{\omega^2 R_2^2 C_2}$$

[b]
$$R_1 = \frac{1000}{1 + (4 \times 10^4)^2 (0.05 \times 10^{-6})^2 (1000)^2}$$

$$= \frac{1000}{1 + (16 \times 10^8)(0.0025)(10^{-12} \times 10^6)} = \frac{1000}{1 + (16)(0.25)} = \frac{1000}{5}$$

$R_1 = 200\,\Omega$

$$C_1 = \frac{5}{(16 \times 10^8)(10^6)(0.05 \times 10^{-6})}$$

$C_1 = 62.5 \times 10^{-9} = 62.5\,\text{nF}$

P10.28 [a] $Y_2 = \frac{1}{R_2} + j\omega C_2$

$$Y_1 = \frac{1}{R_1 + (1/j\omega C_1)} = \frac{j\omega C_1}{1 + j\omega R_1 C_1} = \frac{\omega^2 R_1 C_1^2 + j\omega C_1}{1 + \omega^2 R_1^2 C_1^2}$$

Therefore $Y_1 = Y_2$ when $R_2 = \frac{1 + \omega^2 R_1^2 C_1^2}{\omega^2 R_1 C_1^2}$ and $C_2 = \frac{C_1}{1 + \omega^2 R_1^2 C_1^2}$

[b]
$$R_2 = \frac{1 + (5 \times 10^4)^2 (10^6)(1600 \times 10^{-18})}{(5 \times 10^4)^2 (1000)(1600 \times 10^{-18})}$$

$$= \frac{1 + (25 \times 10^8)(10^6)(16 \times 10^{-16})}{(25 \times 10^8)(10^3)(16 \times 10^{-16})} = \frac{1 + 400 \times 10^{-2}}{400 \times 10^{-5}} = \frac{5}{4} \times 10^3$$

$R_2 = 1250\,\Omega$

$C_2 = \frac{1}{5}C_1 = 8\,\text{nF}$

P10.29 [a] $Z_1 = R_1 + j\omega L_1$

$$Z_2 = \frac{R_2(j\omega L_2)}{R_2 + j\omega L_2} = \frac{\omega^2 L_2^2 R_2 + j\omega L_2 R_2^2}{R^2 + \omega^2 L_2^2}$$

$Z_1 = Z_2$ when $R_1 = \frac{\omega^2 L_2^2 R_2}{R_2^2 + \omega^2 L_2^2}$ and $L_1 = \frac{R_2^2 L_2}{R_2^2 + \omega^2 L_2^2}$

[b] $R_1 = \dfrac{(64\times10^6)(6.25)(10^4)}{10^8+64\times10^6(6.25)} = \dfrac{400\times10^{10}}{10^8+400\times10^6} = \dfrac{400\times10^2}{5}$

$R_1 = 8000\,\Omega = 8\,\text{k}\Omega$

$L_1 = \dfrac{10^8(2.5)}{5\times10^8} = 0.5\,\text{H}$

P 10.30 [a] $Y_2 = \dfrac{1}{R_2} - \dfrac{j}{\omega L_2}$

$Y_1 = \dfrac{1}{R_1+j\omega L_1} = \dfrac{R_1 - j\omega L_1}{R_1^2+\omega^2L_1^2}$

Therefore $Y_2 = Y_1$ when $R_2 = \dfrac{R_1^2+\omega^2L_1^2}{R_1}$ and $L_2 = \dfrac{R_1^2+\omega^2L_1^2}{\omega^2L_1}$

[b] $R_2 = \dfrac{R_1^2+\omega^2L_1^2}{R_1} = \dfrac{25\times10^6+16\times10^6(0.25)^2}{5000}$

$= \dfrac{25\times10^6+1\times10^6}{5000} = \dfrac{26}{5}\times10^3$

$R_2 = 5200\,\Omega$

$L_2 = \dfrac{26\times10^6}{(16\times10^6)(0.25)} = \dfrac{26}{4} = 6.5\,\text{H}$

P 10.31 [a] $\mathbf{V}_1 = (20+j60)\mathbf{I}_a = 100+j300\,\text{V}$

$\mathbf{I}_b = \dfrac{\mathbf{V}_1}{12-j3.5} = \dfrac{100+j300}{12-j3.5} = 14.40+j20.80 = 25.30\underline{/55.30^\circ}\,\text{A}$

$\mathbf{I}_c = \mathbf{I}_a+\mathbf{I}_b+22.56\underline{/24.07^\circ} = 5+j0+14.40+j20.8+20.6+j9.2$

$= 40+j30\,\text{A} = 50\underline{/36.87^\circ}$

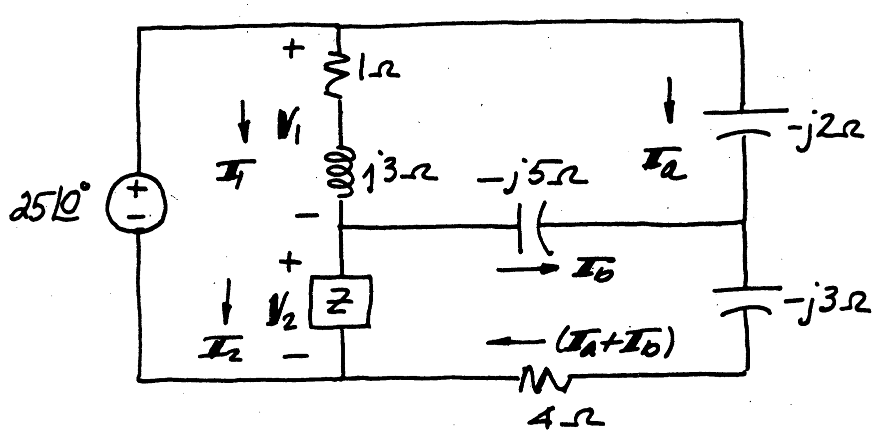
$\mathbf{V}_g = 2\mathbf{I}_c+\mathbf{V}_1 = 80+j60+100+j300 = 180+j360 = 402.49\underline{/63.43^\circ}\,\text{V}$

[b] $i_b = 25.30\cos(5000t+55.30^\circ)\,\text{A}$

$i_c = 50\cos(5000t+36.87^\circ)\,\text{A}$

$v_g = 402.49\cos(5000t+63.43^\circ)\,\text{V}$

P 10.32



$$25\angle 0^\circ = (j5)(-j2) + (4 - j3)(\mathbf{I}_a + \mathbf{I}_b) = 10 + (4 - j3)\mathbf{I}_b + j20 + 15$$

$$\therefore\ \mathbf{I}_b = \frac{-j20}{4 - j3} = \frac{-j20(4 + j3)}{25} = 2.4 - j3.2\ \text{A}$$

$$\mathbf{V}_1 = -j2\mathbf{I}_a - (-j5)\mathbf{I}_b = 10 + j5(2.4 - j3.2) = 26 + j12\ \text{V}$$

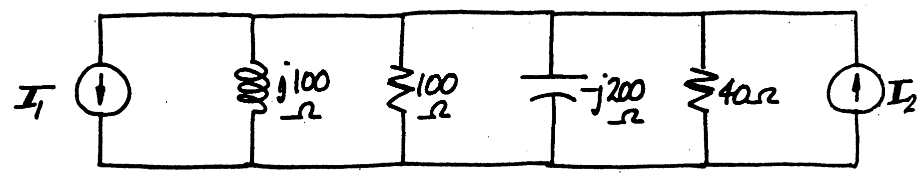
$$\mathbf{I}_1 = \frac{26 + j12}{1 + j3} = \frac{(26 + j12)(1 - j3)}{10} = \frac{26 - j78 + j12 + 36}{10} = 6.2 - j6.6\ \text{A}$$

$$\mathbf{V}_2 = 25 - \mathbf{V}_1 = -1 - j12\ \text{V}$$

$$\mathbf{I}_2 = \mathbf{I}_1 - \mathbf{I}_b = 6.2 - j6.6 - 2.4 + j3.2 = 3.8 - j3.4$$

$$Z = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{-1 - j12}{3.8 - j3.4} = \frac{12.04\angle -94.76^\circ}{5.10\angle -41.82^\circ} = 2.36\angle -52.94^\circ = (1.42 - j1.88)\ \Omega$$

P 10.33 $j\omega L = j5000(20 \times 10^{-3}) = j100\ \Omega;\qquad \frac{1}{j\omega C} = \frac{10^6}{j5000} = -j200\ \Omega$

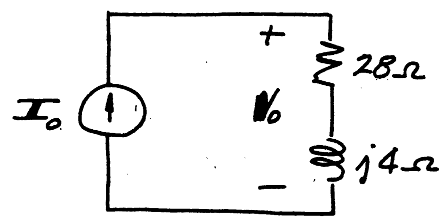


$$\mathbf{I}_1 = \frac{400\angle 36.87^\circ}{j100} = 4\angle -53.13^\circ = 2.4 - j3.2\ \text{A}$$

$$\mathbf{I}_2 = \frac{128\angle -90^\circ}{40} = 3.2\angle -90^\circ = -j3.2\ \text{A}$$

$$Y = \frac{1}{j100} + \frac{1}{100} + \frac{1}{-j200} + \frac{1}{40} = \frac{-j2 + 2 + j + 5}{200} = \frac{7 - j1}{200}$$

$$Z = \frac{200}{7 - j1} = \frac{200}{50}(7 + j1) = 28 + j4\ \Omega$$



$$\mathbf{I}_o = \mathbf{I}_2 - \mathbf{I}_1 = -j3.2 - 2.4 + j3.2 = -2.4 = 2.4\angle 180^\circ$$

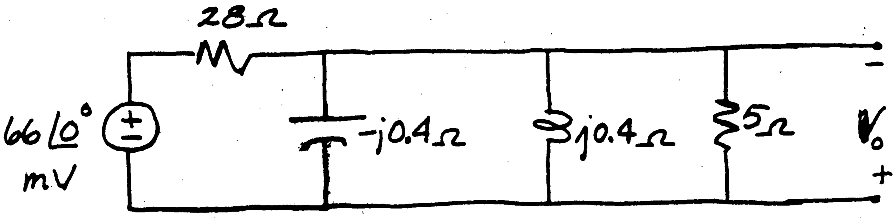
$$\mathbf{V}_o = (28 + j4)\mathbf{I}_o = (28.28\angle 8.13^\circ)(2.4\angle 180^\circ) = 67.88\angle 188.13^\circ$$

$v_o = 67.88\cos(5000t + 188.13^\circ)$ V or $v_o = 67.88\sin(5000t - 81.87^\circ)$ V

P 10.34 Construct a phasor-domain equivalent circuit.

$$\frac{1}{j\omega C} = \frac{1000}{j200(12.5)} = -j0.4\,\Omega; \qquad j\omega L = j200(2 \times 10^{-3}) = j0.40; \qquad \mathbf{I}_s = 3\underline{/0^\circ}\,\text{mA}$$

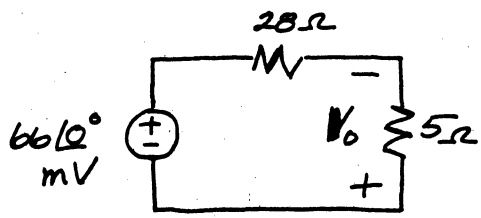
Make a source transformation where $\mathbf{V}_s = 22(3\underline{/0^\circ}) = 66\underline{/0^\circ}\,\text{mV}$



Compute the admittance of the parallel branches.

$$Y_P = \frac{1}{-j0.4} + \frac{1}{j0.4} + \frac{1}{5} = 0.2\,℧ \;; \qquad \therefore\; Z_P = \frac{1}{Y_P} = 5\,\Omega$$

Therefore the circuit simplifies to

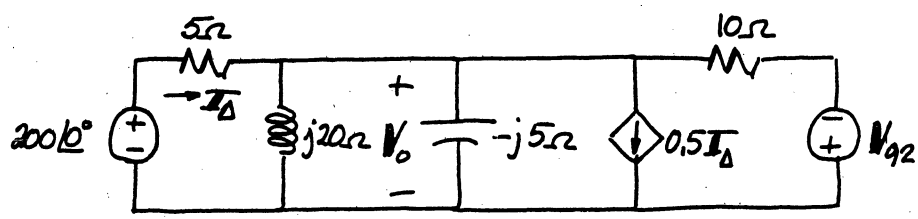


$$\therefore\; \mathbf{V}_o = \frac{-66}{33}(5) = -10\,\text{mV} = 10\underline{/180^\circ}\,\text{mV}$$

$$\therefore\; v_o(t) = 10\cos(200t + 180^\circ)\,\text{mV}$$

P 10.35 $j\omega L = j[40{,}000(0.5) \times 10^{-3}] = j20\,\Omega; \qquad \frac{1}{j\omega C} = \frac{10^6}{j40{,}000(5)} = -j5\,\Omega$

$\mathbf{V}_{g1} = 200\underline{/0^\circ}\,\text{V}; \qquad \mathbf{V}_{g2} = 100\underline{/36.87^\circ}\,\text{V}$



$$\frac{\mathbf{V}_o - 200\underline{/0^\circ}}{5} + \frac{\mathbf{V}_o}{j20} + \frac{\mathbf{V}_o}{-j5} + 0.5\left(\frac{200\underline{/0^\circ} - \mathbf{V}_o}{5}\right) + \frac{\mathbf{V}_o + 100\underline{/36.87^\circ}}{10} = 0$$

$$\mathbf{V}_o\left[\frac{1}{5} - \frac{j}{20} + \frac{j}{5} - 0.1 + 0.1\right] = 40 - 20 - 10\underline{/36.87^\circ}$$

$$\mathbf{V}_o\left[\frac{4 - j1 + 4}{20}\right] = 20 - (8 + j6)$$

$$\mathbf{V}_o(4 + j3) = 20[12 - j6] = 120(2 - j1)$$

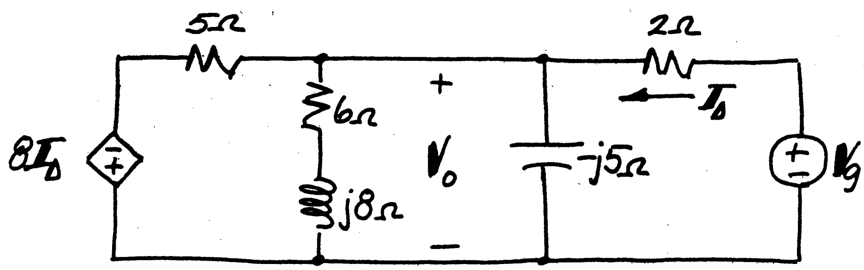
$$\mathbf{V}_o = \frac{120(2 - j1)(4 - j3)}{25} = 4.8(8 - j6 - j4 - 3)$$

$$= 4.8(5 - j10) = 24(1 - j2) = 53.67\underline{/-63.43^\circ}\text{ V}$$

$$\therefore\ v_o(t) = 53.67\cos(40{,}000t - 63.43^\circ)\text{ V}$$

P 10.36 $j\omega L = j50{,}000 \times 160 \times 10^{-6} = j8\,\Omega;\qquad \frac{1}{j\omega C} = \frac{10^6}{j50{,}000 \times 4} = -j5\,\Omega$

$\mathbf{V}_g = 160\underline{/0^\circ}\text{ mV};\qquad \mathbf{I}_\Delta = \frac{\mathbf{V}_g - \mathbf{V}_o}{2} = 0.5\mathbf{V}_g - 0.5\mathbf{V}_o$



$$\frac{\mathbf{V}_o + 8\mathbf{I}_\Delta}{5} + \frac{\mathbf{V}_o}{6 + j8} + \frac{\mathbf{V}_o}{-j5} + \frac{\mathbf{V}_o - \mathbf{V}_g}{2} = 0$$

$$\mathbf{V}_o\left[\frac{1}{5} + \frac{6 - j8}{100} + \frac{j}{5} + \frac{1}{2}\right] + 1.6(0.5\mathbf{V}_g - 0.5\mathbf{V}_o) = \frac{\mathbf{V}_g}{2}$$

$$\mathbf{V}_o[20 + 6 - j8 + j20 + 50] + 80\mathbf{V}_g - 80\mathbf{V}_o = 50\mathbf{V}_g$$

$$\mathbf{V}_o[76 + j12 - 80] = (50 - 80)\mathbf{V}_g$$

$$\mathbf{V}_o(-4 + j12) = -30\mathbf{V}_g$$

$$\mathbf{V}_o = \left(\frac{30}{4 - j12}\right)\mathbf{V}_g = \frac{7.5}{1 - j3}\mathbf{V}_g = 0.75(1 + j3)\mathbf{V}_g$$

$$= 120(1 + j3) = 379.47\underline{/71.57^\circ}\text{ mV}$$

$$\therefore\ v_o = 379.47\cos(50{,}000t + 71.57^\circ)\text{ mV}$$

P 10.37 $\frac{\mathbf{V}_o}{-j8} + \frac{\mathbf{V}_o - 2.4\mathbf{I}_\Delta}{j4} + \frac{\mathbf{V}_o}{5} = 10\sqrt{2}\underline{/+45^\circ}$

$j5\mathbf{V}_o - j10(\mathbf{V}_o - 2.4\mathbf{I}_\Delta) + 8\mathbf{V}_o = 400\sqrt{2}\underline{/+45^\circ}$

$$\mathbf{V}_o[j5 - j10 + 8] + j24\frac{\mathbf{V}_o}{-j8} = 400\sqrt{2}\underline{/+45^\circ}$$

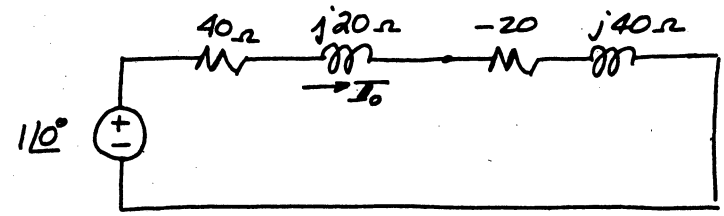
$$\mathbf{V}_o(8 - j5 - 3) = 400\sqrt{2}\underline{/+45^\circ}$$

$$\mathbf{V}_o(5 - j5) = 400\sqrt{2}\underline{/+45^\circ}$$

$$\mathbf{V}_o = \frac{400\sqrt{2}\underline{/+45^\circ}}{5\sqrt{2}\underline{/-45^\circ}} = 80\underline{/90^\circ}\,\text{V}$$

$$\mathbf{V}_o = 0 + j80\,\text{V}$$

P 10.38 $\mathbf{V}_o = -16\mathbf{I}_o\frac{(50)(-j25)}{50 - j25} = (-160 + j320)\mathbf{I}_o;\qquad \frac{\mathbf{V}_o}{8} = (-20 + j40)\mathbf{I}_o$



$$\mathbf{I}_o = \frac{1\underline{/0^\circ}}{20 + j60} = \frac{0.05\underline{/0^\circ}}{1 + j3}$$

$$\mathbf{I}_o = 0.005(1 - j3) = 15.81\underline{/-71.57^\circ}\,\text{mA}$$

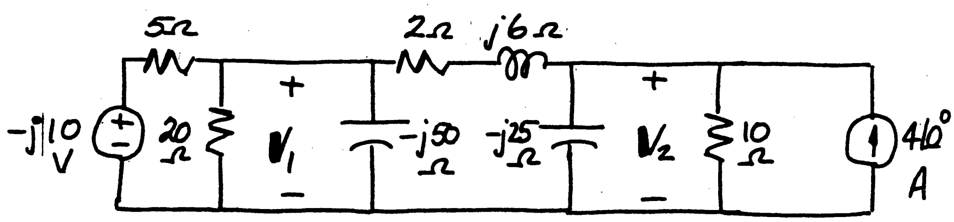
$$\mathbf{V}_o = (-160 + j320)\mathbf{I}_o = 5.66\underline{/45^\circ}\,\text{V}$$

P 10.39 $\mathbf{V}_g = 10\underline{/-90^\circ} = -j10\,\text{V};\qquad \mathbf{I}_g = 4\underline{/0^\circ}\,\text{A}$

$$\frac{1}{j\omega C_1} = \frac{10^6}{j10^4(2)} = -j50\,\Omega$$

$$j\omega L = j10^4(0.6 \times 10^{-3}) = j6\,\Omega$$

$$\frac{1}{j\omega C_2} = \frac{10^6}{j10^4(4)} = -j25\,\Omega$$



$$\frac{\mathbf{V}_1 + j10}{5} + \frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1}{-j50} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{2 + j6} = 0$$

$$\mathbf{V}_1\left[\frac{1}{5} + \frac{1}{20} + \frac{1}{50} + \frac{2 - j6}{40}\right] - \frac{\mathbf{V}_2(2 - j6)}{40} = -j2$$

$$\mathbf{V}_1[20 + 5 + j2 + 5 - j15] - \mathbf{V}_2(5 - j15) = -j200$$

$$\boxed{\mathbf{V}_1(30 - j13) - \mathbf{V}_2(5 - j15) = -j200}$$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{2 + j6} + \frac{\mathbf{V}_2}{-j25} + \frac{\mathbf{V}_2}{10} = 4\underline{/0^\circ}$$

$$-\mathbf{V}_1\left(\frac{1 - j3}{20}\right) + \mathbf{V}_2\left[\frac{1 - j3}{20} + \frac{1}{25} + \frac{1}{10}\right] = 4\underline{/0^\circ}$$

$$-\mathbf{V}_1(5 - j15) + \mathbf{V}_2[5 - j15 + j4 + 10] = 400$$

$$\boxed{-\mathbf{V}_1(5 - j15) + \mathbf{V}_2[15 - j11] = 400}$$

$$\Delta = \begin{vmatrix} 30 - j13 & -(5 - j15) \\ -(5 - j15) & 15 - j11 \end{vmatrix} = 450 - j330 - j195 - 143 - (25 - j75 - j75 - 225)$$

$$= 650 - 143 - j525 + j150 = 507 - j375 = 630.61\underline{/-36.49^\circ}$$

$$N_1 = \begin{vmatrix} -j200 & -(5 - j15) \\ 400 & 15 - j11 \end{vmatrix} = -j3000 - 2200 + 2000 - j6000$$

$$= -200 - j9000 = 9002.22\underline{/-91.27^\circ}$$

$$N_2 = \begin{vmatrix} 30 - j13 & -j200 \\ -(5 - j15) & 400 \end{vmatrix} = 12{,}000 - j5200 - j200(5 - j15)$$

$$= 12{,}000 - j5200 - j1000 - 3000 = 9000 - j6200 = 10{,}928.86\underline{/-34.56^\circ}$$

$$\mathbf{V}_1 = \frac{N_1}{\Delta} = \frac{9002.22\underline{/-91.27^\circ}}{630.61\underline{/-36.49^\circ}} = 14.28\underline{/-54.78^\circ}\text{ V}$$

$$\mathbf{V}_2 = \frac{N_2}{\Delta} = \frac{10{,}928.86\underline{/-34.56^\circ}}{630.61\underline{/-36.49^\circ}} = 17.33\underline{/1.93^\circ}\text{ V}$$

$$\therefore\ v_1 = 14.28\cos(10^4t - 54.78^\circ)\text{ V}; \qquad v_2 = 17.33\cos(10^4t + 1.93^\circ)\text{ V}$$

P 10.40 Let $\mathbf{I}_2$ be the current in the $35 + j20\,\Omega$ branch.

$$400\underline{/0^\circ} = (5 - j10)\mathbf{I}_1 + (35 + j20)\mathbf{I}_2$$

$$\mathbf{I}_1 - \mathbf{I}_2 = 0.1\mathbf{V}_\Delta = 0.1(j20\mathbf{I}_2)$$

$$\mathbf{I}_1 - \mathbf{I}_2 = j2\mathbf{I}_2$$

$$\mathbf{I}_1 = (1 + j2)\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{\mathbf{I}_1}{(1 + j2)} = \frac{(1 - j2)}{5}\mathbf{I}_1$$

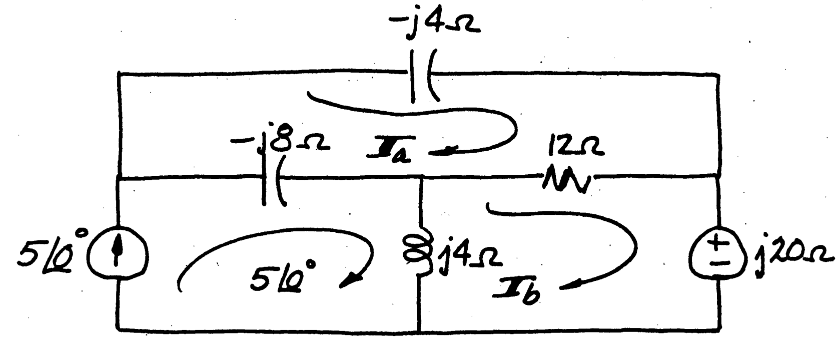
$$\therefore\ 400\underline{/0^\circ} = (5 - j10)\mathbf{I}_1 + (7 + j4)(1 - j2)\mathbf{I}_1$$

$$= [5 - j10 + 7 - j14 + j4 + 8]\mathbf{I}_1 = (20 - j20)\mathbf{I}_1$$

$$\mathbf{I}_1 = \frac{400}{20 - j20} = \frac{20}{1 - j1} = 10(1 + j1)$$

$$\mathbf{I}_1 = 10 + j10 = 14.14\underline{/45^\circ}\text{ A}$$

P 10.41



$-j4\mathbf{I}_a + 12(\mathbf{I}_a - \mathbf{I}_b) - j8(\mathbf{I}_a - 5) = 0;$ $\boxed{\mathbf{I}_a(12 - j12) - 12\mathbf{I}_b = -j40}$

$j4(\mathbf{I}_b - 5) + 12(\mathbf{I}_b - \mathbf{I}_a) + j20 = 0;$ $\boxed{-12\mathbf{I}_a + (12 + j4)\mathbf{I}_b = 0}$

$$\Delta = \begin{vmatrix} 12 - j12 & -12 \\ -12 & 12 + j4 \end{vmatrix} = 144 + j48 - j144 + 48 - 144 = 48 - j96 = 48(1 - j2)$$

$$N_b = \begin{vmatrix} 12 - j12 & -j40 \\ -12 & 0 \end{vmatrix} = -j480$$

$$\mathbf{I}_b = \frac{N_b}{\Delta} = \frac{-j480}{48(1 - j2)} = \frac{-j10(1 + j2)}{5} = -j2(1 + j2) = 4 - j2\,\text{A}$$

$\mathbf{I}_g = 4 - j2 = 4.47\underline{/-26.57^\circ}\,\text{A}$

P 10.42 $\mathbf{I}_1$ and $\mathbf{I}_2$ are clockwise mesh currents.

$$240\underline{/0^\circ} = (3 + j4)\mathbf{I}_1 - (1 - j5)\mathbf{I}_2$$
$$-120\underline{/-90^\circ} = -(1 - j5)\mathbf{I}_1 + (3.92 + j1.44)\mathbf{I}_2$$

$$\Delta = \begin{vmatrix} 3 + j4 & -(1 - j5) \\ -(1 - j5) & 3.92 + j1.44 \end{vmatrix}$$
$$= 11.76 + j4.32 + j15.68 - 5.76 - (+1 - j5 - j5 - 25)$$
$$= 6 + j20 + 24 + j10 = 30 + j30$$

$$N_1 = \begin{vmatrix} 240 & -(1 - j5) \\ j120 & 3.92 + j1.44 \end{vmatrix} = 1540.8 + j465.60$$

$$N_2 = \begin{vmatrix} 3 + j4 & 240 \\ -(1 - j5) & j120 \end{vmatrix} = -240 - j840$$

$$\mathbf{I}_1 = \frac{N_1}{\Delta} = \frac{1540.8 + j465.60}{30 + j30} = 33.44 - j17.92$$

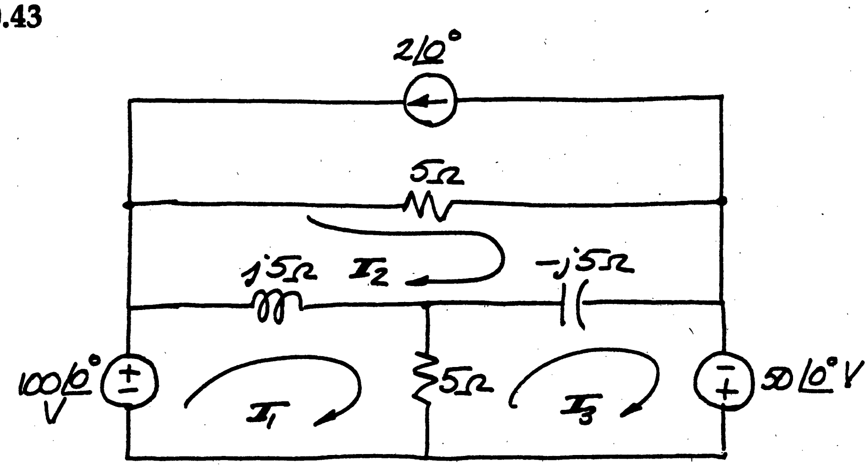
$$\mathbf{I}_2 = \frac{N_2}{\Delta} = \frac{-240 - j840}{30 + j30} = -18 - j10$$

$\mathbf{I}_a = \mathbf{I}_1 = 33.44 - j17.92 = 37.94\underline{/-28.19^\circ}\,\text{A}$

$\mathbf{I}_b = \mathbf{I}_2 = -18 - j10 = 20.59\underline{/-150.95^\circ}\,\text{A}$

$\mathbf{I}_c = \mathbf{I}_1 - \mathbf{I}_2 = 51.44 - j7.92 = 52.05\underline{/-8.75^\circ}\,\text{A}$

P 10.43



$$100\underline{/0^\circ} = (5+j5)\mathbf{I}_1 - j5\mathbf{I}_2 - 5\mathbf{I}_3$$
$$-10\underline{/0^\circ} = -j5\mathbf{I}_1 + 5\mathbf{I}_2 + j5\mathbf{I}_3$$
$$50\underline{/0^\circ} = -5\mathbf{I}_1 + j5\mathbf{I}_2 + (5-j5)\mathbf{I}_3$$
$$20 = (1+j1)\mathbf{I}_1 - j\mathbf{I}_2 - \mathbf{I}_3$$
$$-2 = -j\mathbf{I}_1 + \mathbf{I}_2 + j\mathbf{I}_3$$
$$10 = -\mathbf{I}_1 + j\mathbf{I}_2 + (1-j1)\mathbf{I}_3$$

$$\Delta = \begin{vmatrix} 1+j1 & -j & -1 \\ -j & 1 & j \\ -1 & j & 1-j1 \end{vmatrix}$$
$$= (1+j1)(1-j1+1) + j(-j-1+j) - 1(1+1)$$
$$= (1+j1)(2-j) - j - 2 = 2 - j1 + j2 + 1 - j - 2 = 1$$

$$N_1 = \begin{vmatrix} 20 & -j & -1 \\ -2 & 1 & j \\ 10 & j & (1-j1) \end{vmatrix}$$
$$= 20(1-j1+1) + 2(-j-1+j) + 10(1+1)$$
$$= 20(2-j) + 2(-1) + 20 = 58 - j20$$

$$\therefore\ \mathbf{I}_1 = 58 - j20\text{ A}$$

$$N_2 = \begin{vmatrix} (1+j1) & 20 & -1 \\ -j & -2 & j \\ -1 & 10 & (1-j1) \end{vmatrix}$$
$$= (1+j1)(-2+j2-10j) + j(20-j20+10) - 1(20j-2)$$
$$= (1+j1)(-2-j8) + j(30-j20) - j20 + 2 = 28 + j0$$

$$\therefore\ \mathbf{I}_2 = 28 + j0\text{ A}$$

$$N_3 = \begin{vmatrix} (1+j1) & -j & 20 \\ -j & 1 & -2 \\ -1 & j & 10 \end{vmatrix}$$
$$= 20(1+1) + 2(j-1-j) + 10(1+j+1)$$
$$= 40 - 2 + 20 + j10 = 58 + j10$$

$\therefore\ \mathbf{I}_3 = 58 + j10\,\text{A}$

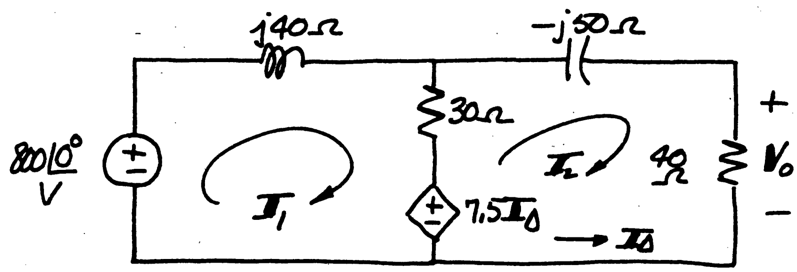
$\mathbf{I}_a = 2 + \mathbf{I}_2 = 30 + j0\,\text{A}$

$\mathbf{I}_b = \mathbf{I}_1 - \mathbf{I}_2 = 58 - j20 - 28 = 30 - j20\,\text{A}$

$\mathbf{I}_c = \mathbf{I}_3 - \mathbf{I}_2 = 58 + j10 - 28 = 30 + j10\,\text{A}$

$\mathbf{I}_d = \mathbf{I}_1 - \mathbf{I}_3 = 58 - j20 - 58 - j10 = -j30\,\text{A}$

P 10.44 $\mathbf{V}_g = 800\underline{/0^\circ}\,\text{V}; \qquad j\omega L = j40\,\Omega; \qquad \dfrac{1}{j\omega C} = -j50\,\Omega$



$800\underline{/0^\circ} = (30 + j40)\mathbf{I}_1 - 30\mathbf{I}_2 + 7.5(-\mathbf{I}_2);$ $\boxed{800\underline{/0^\circ} = (30 + j40)\mathbf{I}_1 - 37.5\mathbf{I}_2}$

$0 = -7.5(-\mathbf{I}_2) + 30(\mathbf{I}_2 - \mathbf{I}_1) + (40 - j50)\mathbf{I}_2;$ $\boxed{0 = -30\mathbf{I}_1 + (77.5 - j50)\mathbf{I}_2}$

$$\Delta = \begin{vmatrix} (30 + j40) & -37.5 \\ -30 & (77.5 - j50) \end{vmatrix} = 2325 - j1500 + j3100 + 2000 - 1125$$

$$= 3200 + j1600 = 1600(2 + j1)$$

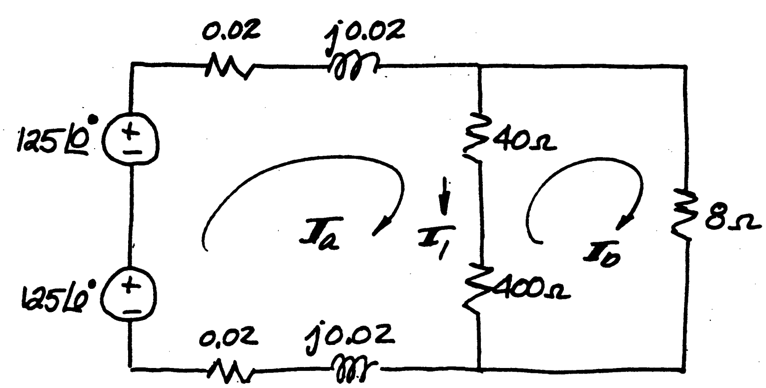
$$N_2 = \begin{vmatrix} (30 + j40) & 800\underline{/0^\circ} \\ -30 & 0 \end{vmatrix} = 24{,}000\underline{/0^\circ}$$

$$\mathbf{I}_2 = \frac{N_2}{\Delta} = \frac{24{,}000}{1600(2 + j1)} = \frac{15}{5}(2 - j1) = 6 - j3\,\text{A}$$

$\mathbf{V}_o = 40\mathbf{I}_2 = 240 - j120\,\text{V} = 268.33\underline{/-26.57^\circ}\,\text{V}$

$v_o = 268.33\cos(8000t - 26.57^\circ)\,\text{V}$

P 10.45 [c]



$$250 = (440.04 + j0.04)\mathbf{I}_a - 440\mathbf{I}_b$$
$$0 = -440\mathbf{I}_a + 448\mathbf{I}_b$$

$$\Delta = \begin{vmatrix} (440.04 + j0.04) & -440 \\ -440 & 448 \end{vmatrix} = 197{,}137.92 + j17.92 - 193{,}600$$
$$= 3537.92 + j17.92 = 3537.97\underline{/0.29^\circ}$$

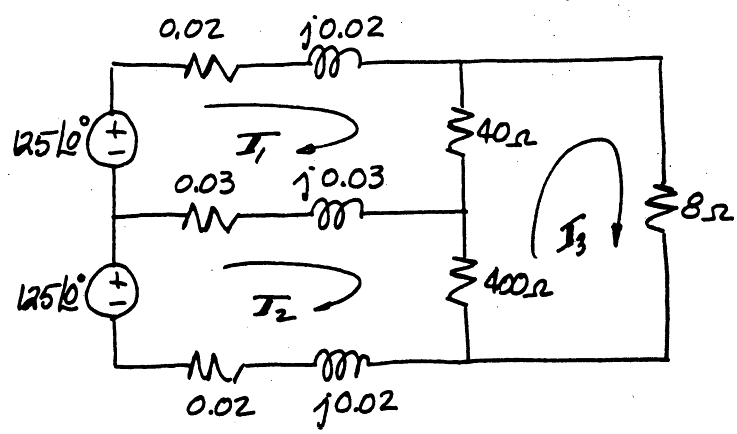
$$N_a = \begin{vmatrix} 250 & -440 \\ 0 & 448 \end{vmatrix} = 112{,}000 + j0$$

$$\mathbf{I}_a = \frac{N_a}{\Delta} = 31.66\underline{/-0.29^\circ} = 31.66 - j0.16$$

$$N_b = \begin{vmatrix} (440.04 + j0.04) & 250 \\ -440 & 0 \end{vmatrix} = 110{,}000\underline{/0^\circ}$$

$$\mathbf{I}_b = \frac{N_b}{\Delta} = 31.09\underline{/-0.29^\circ} = 31.09 - j0.16$$
$$\mathbf{I}_1 = \mathbf{I}_a - \mathbf{I}_b = 0.57 - j0.00286$$
$$\mathbf{V}_1 = 40\mathbf{I}_1 = 22.61 - j0.11 = 22.61\underline{/-0.29^\circ}\text{ V}$$
$$\mathbf{V}_2 = 400\mathbf{I}_1 = 226.12 - j1.15 = 226.12\underline{/-0.29^\circ}\text{ V}$$

[d]



$$125 = (40.05 + j0.05)\mathbf{I}_1 - (0.03 + j0.03)\mathbf{I}_2 - 40\mathbf{I}_3$$
$$125 = -(0.03 + j0.03)\mathbf{I}_1 + (400.05 + j0.05)\mathbf{I}_2 - 400\mathbf{I}_3$$
$$0 = -40\mathbf{I}_1 - 400\mathbf{I}_2 + 448\mathbf{I}_3$$

$$\Delta = \begin{vmatrix} (40.05 + j0.05) & -(0.03 + j0.03) & -40 \\ -(0.03 + j0.03) & (400.05 + j0.05) & -400 \\ -40 & -400 & 448 \end{vmatrix}$$
$$= 128{,}816 + j817.4336 = 128{,}818.59\underline{/0.36^\circ}$$

$$N_1 = \begin{vmatrix} 125 & -(0.03 + j0.03) & -40 \\ 125 & (400.05 + j0.05) & -400 \\ 0 & -400 & 448 \end{vmatrix}$$
$$= 4{,}404{,}480 + j4480 = 4{,}404{,}482.28\underline{/0.06^\circ}$$

$$\mathbf{I}_1 = \frac{N_1}{\Delta} = 34.19\underline{/-0.31^\circ} = 34.19 - j0.18\,\text{A}$$

$$N_2 = \begin{vmatrix} (40.05 + j0.05) & 125 & -40 \\ -(0.03 + j0.03) & 125 & -400 \\ -40 & 0 & 448 \end{vmatrix}$$

$$= 4{,}044{,}480 + j4480 = 4{,}044{,}482.48\underline{/0.06^\circ}$$

$$\mathbf{I}_2 = \frac{N_2}{\Delta} = 31.40\underline{/-0.30^\circ} = 31.40 - j0.16\,\text{A}$$

$$N_3 = \begin{vmatrix} (40.05 + j0.05) & -(0.03 + j0.03) & 125 \\ -(0.03 + j0.03) & (400.05 + j0.05) & 125 \\ -40 & -400 & 0 \end{vmatrix}$$

$$= 4{,}004{,}400 + j4400 = 4{,}004{,}402.42\underline{/0.06^\circ}$$

$$\mathbf{I}_3 = \frac{N_3}{\Delta} = 31.09\underline{/-0.30^\circ} = 31.09 - j0.16\,\text{A}$$

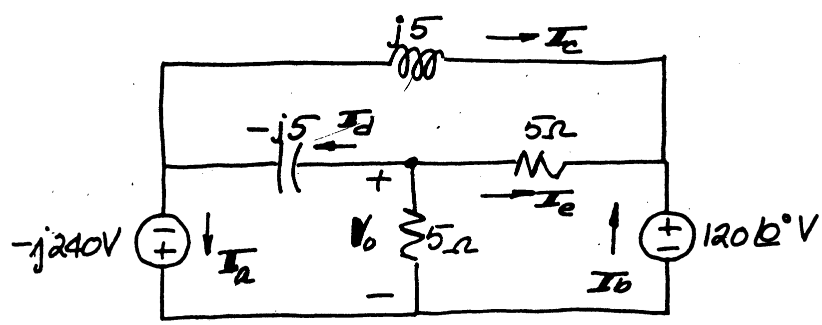
$$\mathbf{V}_1 = 40(\mathbf{I}_1 - \mathbf{I}_3) = 124.23\underline{/-0.35^\circ}\,\text{V}$$

$$\mathbf{V}_2 = 400(\mathbf{I}_2 - \mathbf{I}_3) = 124.45\underline{/-0.25^\circ}\,\text{V}$$

[e] Because an open neutral can result in severely unbalanced voltages across the 125-V loads.

P10.46 $\mathbf{V}_a = 240\underline{/-90^\circ} = -j240\,\text{V}; \qquad \mathbf{V}_b = 120\underline{/0^\circ}\,\text{V}$

$$j\omega L = j10^5(50 \times 10^{-6}) = j5\,\Omega; \qquad \frac{1}{j\omega C} = -j\left(\frac{10^6}{2 \times 10^5}\right) = -j5\,\Omega$$



$$\frac{\mathbf{V}_o}{5} + \frac{\mathbf{V}_o - j240}{-j5} + \frac{\mathbf{V}_o - 120}{5} = 0$$

$$\mathbf{V}_o + (\mathbf{V}_o - j240)j + \mathbf{V}_o - 120 = 0$$

$$(2 + j\mathbf{V}_o) = -120; \qquad \mathbf{V}_o = -48 + j24\,\text{V}$$

$$\mathbf{I}_c = \frac{j240 - 120}{j5} = 48 + j24\,\text{A}$$

$$\mathbf{I}_d = \frac{-48 + j24 - j240}{-j5} = 43.2 - j9.6\,\text{A}$$

$\mathbf{I}_a = \mathbf{I}_d - \mathbf{I}_c = -4.8 - j33.6\text{ A}$

$\mathbf{I}_e = \dfrac{-48 + j24 - 120}{5} = -33.6 + j4.8$

$\mathbf{I}_b = -\mathbf{I}_e - \mathbf{I}_c = 33.6 - j4.8 - 48 - j24 = -14.40 - j28.80\text{ A}$

$\therefore\ \mathbf{I}_a = 33.94\underline{/-98.13^\circ}\text{ A}$

$\mathbf{I}_b = 32.20\underline{/-116.57^\circ}\text{ A}$

$\mathbf{I}_c = 48 + j24 = 53.67\underline{/26.57^\circ}\text{ A}$

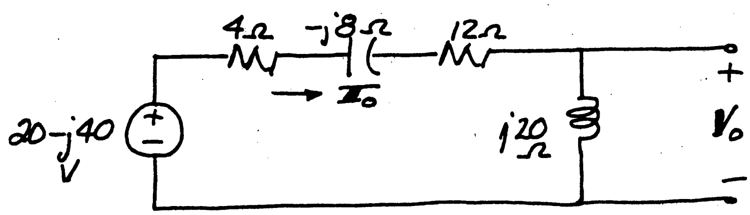
$\therefore\ i_a = 33.94\cos(10^5 t - 98.13^\circ)\text{ A}$

$i_b = 32.20\cos(10^5 t - 116.57^\circ)\text{ A}$

$i_c = 53.67\cos(10^5 t + 26.57^\circ)\text{ A}$

P 10.47 [a] $\mathbf{I}_g = 5\underline{/0^\circ}\text{ A};\qquad \dfrac{1}{j\omega C} = -j10\,\Omega;\qquad j\omega L = j20\,\Omega$

$20\,\Omega // -j10\,\Omega = \dfrac{20(-j10)}{20 - j10} = \dfrac{-j20}{2 - j1} = -j4(2 + j1) = 4 - j8\,\Omega$



$\mathbf{I}_o = \dfrac{20 - j40}{16 + j12} = \dfrac{5 - j10}{4 + j3} = \dfrac{5(1 - j2)(4 - j3)}{25} = 0.2(4 - j3 - j8 - 6)$

$= 0.2(-2 - j11)$

$\mathbf{V}_o = j20\mathbf{I}_o = j20(0.2)(-2 - j11) = 44 - j8 = 44.72\underline{/-10.30^\circ}\text{ V}$

$v_o = 44.72\cos(800{,}000t - 10.30^\circ)\text{ V}$

[b] $T = \dfrac{1}{f} = \dfrac{\pi}{0.8 \times 10^6} = 7.854 \times 10^{-6} = 7854\text{ ns}$

$\dfrac{10.3}{360}(7854) = 224.7\text{ ns}$

P 10.48 $\mathbf{V}_a$ is the voltage across $j5\,\Omega$ impedance.

$\dfrac{\mathbf{V}_a}{j5} + \dfrac{\mathbf{V}_a - 100 + j50}{20} + \dfrac{\mathbf{V}_a - 140 - j30}{12 + j16} = 0$

$-j4\mathbf{V}_a + \mathbf{V}_a - 100 + j50 + \dfrac{5\mathbf{V}_a - 700 - j150}{3 + j4} = 0$

$\mathbf{V}_a\left(1 - j4 + \dfrac{5}{3 + j4}\right) = 100 - j50 + \dfrac{700 + j150}{3 + j4}$

$\mathbf{V}_a(1.6 - j4.8) = 208 - j144$

$$\mathbf{V}_a = 40 + j30\,\text{V}$$

$$\frac{\mathbf{V}_b}{-j10} + \frac{\mathbf{V}_b - \mathbf{V}_a}{12 + j16} - \mathbf{I}_g + \frac{\mathbf{V}_b - \mathbf{V}_g}{Z} = 0$$

$$\frac{140 + j30}{-j10} + \frac{140 + j30 - 40 - j30}{12 + j16} - 30 - j20 + \frac{40 + j80}{Z} = 0$$

$$j(14 + j3) + \frac{100(12 - j16)}{400} - 30 - j20 + \frac{40 + j80}{Z} = 0$$

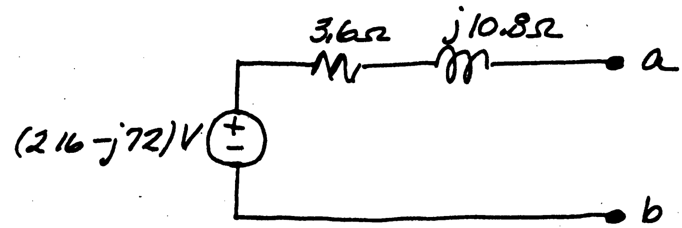
$$j14 - j4 - 30 - j20 + \frac{40 + j80}{Z} - 30 - j10 + \frac{40 + 80}{Z} = 0$$

$$Z = \frac{40 + j80}{30 + j10} = \frac{4 + j8}{3 + j1} = \frac{(4 + j8)(3 - j1)}{10} = \frac{12 - j4 + j24 + 8}{10} = 2 + j2\,\Omega$$

P 10.49 $\mathbf{V}_{Th} = \mathbf{V}_{ab} = \frac{240\angle 0^\circ (36)}{36 + j60 - j48} = \frac{240\angle 0^\circ (36)}{36 + j12} = \frac{720\angle 0^\circ}{3 + j1}$

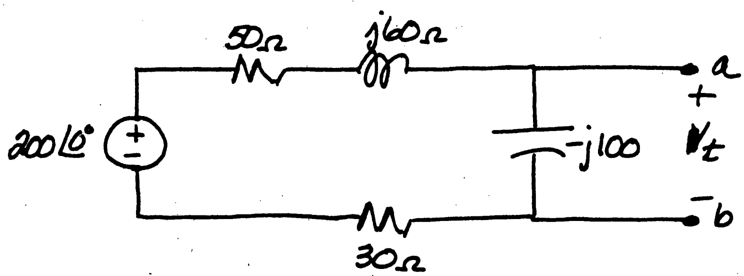
$$= 72(3 - j1) = 216 - j72\,\text{V}$$

$$Z_{Th} = \frac{(36)(j12)}{36 + j12} = \frac{j36}{3 + j1} = j3.6(3 - j1) = 3.6 + j10.8\,\Omega$$



Thevenin Equivalent

P 10.50

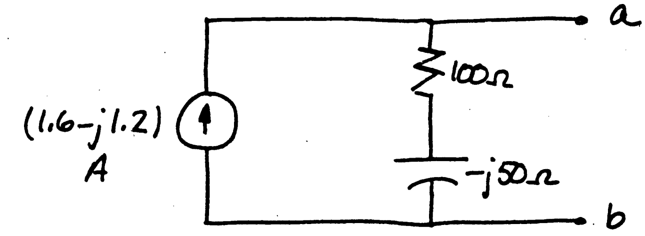


$$\mathbf{V}_{Th} = \left(\frac{200}{80 - j40}\right)(-j100) = \frac{5}{2 - j1}(-j100) = -j100(2 + j1) = 100 - j200\,\text{V}$$

$$Z_{Th} = \frac{(-j100)(80 + j60)}{(80 - j40)} = 100 - j50\,\Omega$$

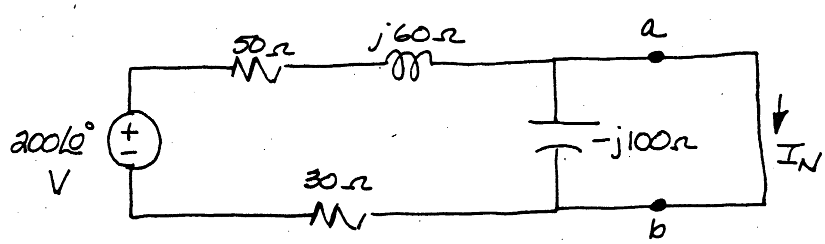
$$\mathbf{I}_N = \frac{\mathbf{V}_{Th}}{Z_{Th}} = \frac{100 - j200}{100 - j50} = \frac{2 - j4}{2 - j1} = 1.6 - j1.2\,\text{A}$$

$$Z_N = Z_{Th} = 100 - j50\,\Omega$$



Norton Equivalent

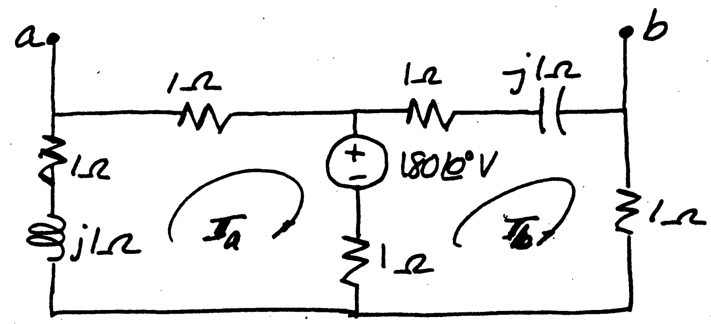
Alternate solution:



$$\mathbf{I}_N = \frac{200}{80 + j60} = 2\underline{/-36.87^\circ} = 1.6 - j1.2\,\text{A}$$

$$Z_N = Z_{\text{Th}} = 100 - j50\,\Omega$$

P 10.51



$$(3 + j1)\mathbf{I}_a - \mathbf{I}_b = -180$$

$$-\mathbf{I}_a + (3 - j1)\mathbf{I}_b = 180$$

$$\Delta = \begin{vmatrix} (3+j1) & -1 \\ -1 & (3-j1) \end{vmatrix} = 9$$

$$N_b = \begin{vmatrix} (3+j1) & -180 \\ -1 & 180 \end{vmatrix} = 180(3 + j1 - 1) = 180(2 + j1)$$

$$N_a = \begin{vmatrix} -180 & -1 \\ 180 & (3-j1) \end{vmatrix} = -180(3 - j1) + 180 = -180(2 - j1)$$

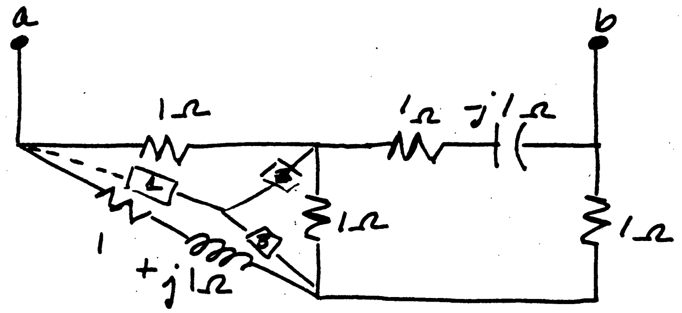
$$\mathbf{I}_a = \frac{N_a}{\Delta} = -20(2 - j1) = -40 + j20\,\text{A}$$

$$\mathbf{I}_b = \frac{N_b}{\Delta} = 20(2 + j1) = 40 + j20\ \text{A}$$

$$\mathbf{V}_{ab} = \mathbf{V}_{Th} = \mathbf{I}_a \cdot 1 + \mathbf{I}_b(1 - j1) = -40 + j20 + (1 - j1)(40 + j20)$$

$$= -40 + j20 + 40 + j20 - j40 + 20$$

$$\mathbf{V}_{Th} = 20\underline{/0^\circ}\ \text{V}$$

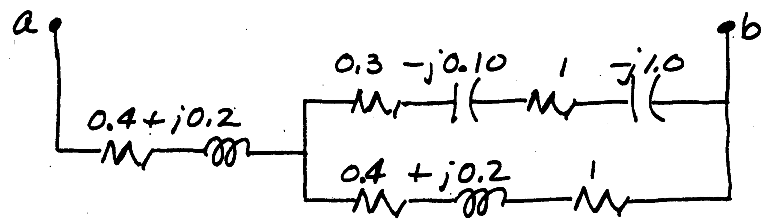


$$Z_1 = \frac{(1)(1 + j1)}{(3 + j1)} = \frac{1}{10}(1 + j1)(3 - j1) = \frac{1}{10}(3 - j1 + j3 + 1)$$

$$= \frac{1}{10}(4 + j2) = 0.4 + j0.2\ \Omega$$

$$Z_2 = \frac{(1)(1)}{3 + j1} = \frac{1}{10}(3 - j1) = 0.3 - j0.1$$

$$Z_3 = Z_1 = 0.4 + j0.2\ \Omega$$



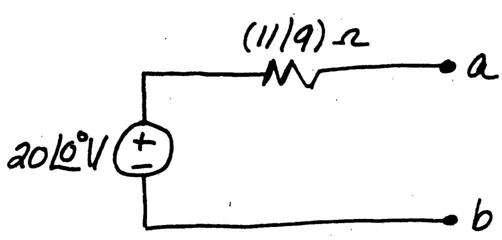
$$Z_{ab} = Z_{Th} = 0.4 + j0.2 + \frac{(1.3 - j1.1)(1.4 + j0.2)}{(2.7 - j0.9)}$$

$$= \frac{3.6 + j1.8 + (1.82 + j0.26 - j1.54 + 0.22)(3 + j1)}{9}$$

$$= \frac{3.6 + j1.8 + (2.04 - j1.28)(3 + j1)}{9}$$

$$= \frac{3.6 + j1.8 + 6.12 + j2.04 - j3.84 + 1.28}{9}$$

$$= \frac{11 + j3.84 - j3.84}{9} = \frac{11}{9}\ \Omega$$



P 10.52

$$-0.03\mathbf{V}_o + \frac{\mathbf{V}_1 - 250}{20 + j10} + \frac{\mathbf{V}_1}{50 - j100} = 0$$

$$\mathbf{V}_o = \frac{\mathbf{V}_1}{50 - j100}(-j100) = \frac{-j2\mathbf{V}_1}{1 - j2} = \frac{(4 - j2)}{5}\mathbf{V}_1$$

$$-0.03\left(\frac{4 - j2}{5}\right)\mathbf{V}_1 + \frac{\mathbf{V}_1}{20 + j10} + \frac{\mathbf{V}_1}{50 - j100} = \frac{250}{20 + j10}$$

$$= \left[\frac{-0.06(2 - j1)}{5} + \frac{20 - j10}{500} + \frac{50 + j100}{12{,}500}\right]\mathbf{V}_1 = \frac{250(20 - j10)}{500}$$

$$[-150(2 - j1) + 25(20 - j10) + 50 + j100]\,\mathbf{V}_1 = (10 - j5)(12{,}500)$$

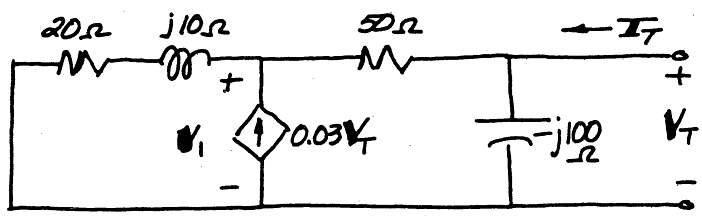
$$[250 + j0]\mathbf{V}_1 = 12{,}500(10 - j5)$$

$$\mathbf{V}_1 = \frac{(12{,}500)(10 - j5)}{250 + j0}$$

$$\mathbf{V}_1 = 500 - j250\text{ V}$$

$$\mathbf{V}_o = \left(\frac{4 - j2}{5}\right)(500 - j250) = (4 - j2)(100 - j50) = 400 - j200 - j200 - 100$$

$$= 300 - j400\text{ V} = 500\underline{/-53.13^\circ}\text{ V}$$



$$\mathbf{I}_T = \frac{\mathbf{V}_T}{-j100} + \frac{\mathbf{V}_T - \mathbf{V}_1}{50}$$

$$-0.03\mathbf{V}_T + \frac{\mathbf{V}_1}{20 + j10} + \frac{\mathbf{V}_1 - \mathbf{V}_T}{50} = 0$$

$$-0.03\mathbf{V}_T + \frac{\mathbf{V}_1(20 - j10)}{500} + \frac{\mathbf{V}_1 - \mathbf{V}_T}{50} = 0$$

$$-15\mathbf{V}_T + (20 - j10)\mathbf{V}_1 + 10\mathbf{V}_1 - 10\mathbf{V}_T = 0$$

$$(30 - j10)\mathbf{V}_1 = 25\mathbf{V}_T$$

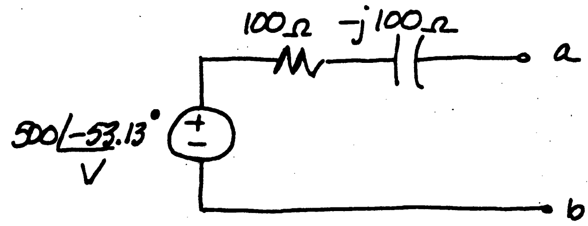
$$\mathbf{V}_1 = \frac{25\mathbf{V}_T}{30 - j10}$$

$$\therefore\ \mathbf{I}_T = \frac{j}{100}\mathbf{V}_T + \frac{1}{50}\mathbf{V}_T - \frac{1}{50}\frac{25\mathbf{V}_T}{(30-j10)}$$

$$\frac{\mathbf{I}_T}{\mathbf{V}_T} = \frac{j}{100} + \frac{1}{50} - \frac{1}{2}\frac{(30+j10)}{1000} = \frac{j}{100} + \frac{1}{50} - \frac{(3+j1)}{200}$$

$$= \frac{2j + 4 - 3 - j1}{200} = \frac{1 + j1}{200}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{200}{1+j1} = 100(1 - j1) = 100 - j100\,\Omega$$



Thevenin equivalent

Alternate solution:

Find I_{sc}: $I_{sc} = \frac{250}{70 + j10} = \frac{250}{5000}(70 - j10) = 3.5 - j0.5$

$$Z_{\text{Th}} = \frac{300 - j400}{3.5 - j0.5} = \frac{600 - j800}{7 - j1} = \frac{(600 - j800)(7 + j1)}{50} = (12 - j16)(7 + j1)$$

$$= 84 + 16 - j112 + j12 = 100 - j100\,\Omega$$

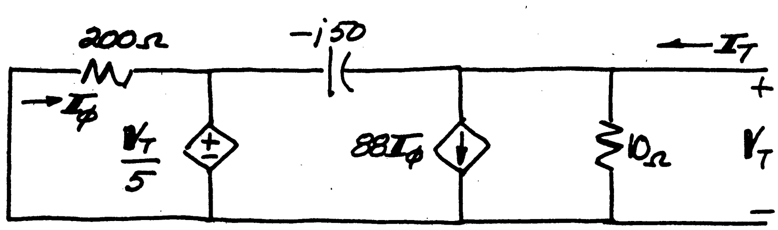
P 10.53 $\frac{\mathbf{V}_2}{10} + 88\mathbf{I}_\phi + \frac{\mathbf{V}_2 - (\mathbf{V}_2/5)}{-j50} = 0$

$$\frac{\mathbf{V}_2}{10} + j\frac{0.8\mathbf{V}_2}{50} + 88\left[\frac{5 - 0.2\mathbf{V}_2}{200}\right] = 0$$

$$20\mathbf{V}_2 + j3.2\mathbf{V}_2 + 440 - 17.6\mathbf{V}_2 = 0$$

$$(2.4 + j3.2)\mathbf{V}_2 = -440$$

$$\mathbf{V}_2 = \mathbf{V}_{\text{Th}} = -110\underline{/-53.13^\circ} = 110\underline{/126.87^\circ}\text{ V}$$

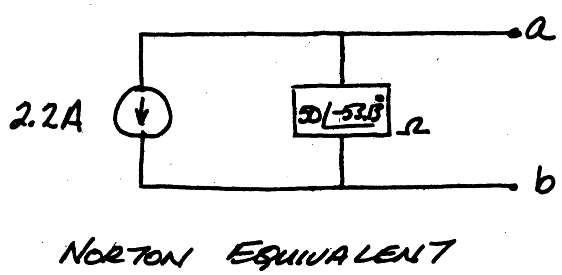


$$\mathbf{I}_T = \frac{\mathbf{V}_T}{10} + 88\left(\frac{-\mathbf{V}_T}{1000}\right) + \frac{\mathbf{V}_T - 0.2\mathbf{V}_T}{-j50}$$

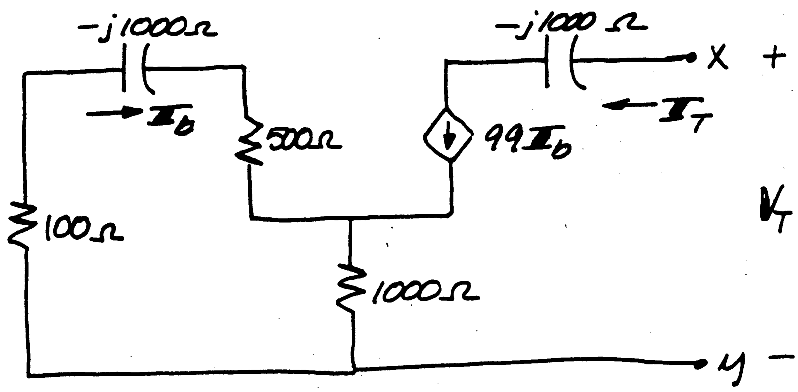
$$\frac{\mathbf{I}_T}{\mathbf{V}_T} = \frac{1}{10} - 0.088 + \frac{0.80}{-j50} = 0.1 - 0.088 + j0.016 = 0.012 + j0.016$$

$$Z_N = Z_{\text{Th}} = \frac{1}{0.012 + j0.016} = \frac{1000}{12 + j16} = 50\underline{/-53.13^\circ}\,\Omega$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{\text{Th}}}{Z_N} = \frac{110\underline{/126.87^\circ}}{50\underline{/-53.13^\circ}} = 2.2\underline{/180^\circ} = -2.2 + j0\,\text{A}$$



P 10.54 [a] $\frac{1}{j\omega C} = -j1000\,\Omega; \qquad \omega = 1000\,\text{rad/s}$



First find Z_{xy}, then $Z_{\text{ab}} = 2000\,\Omega // Z_{xy}$

Now observe $\mathbf{I}_T = 99\mathbf{I}_\text{b}$

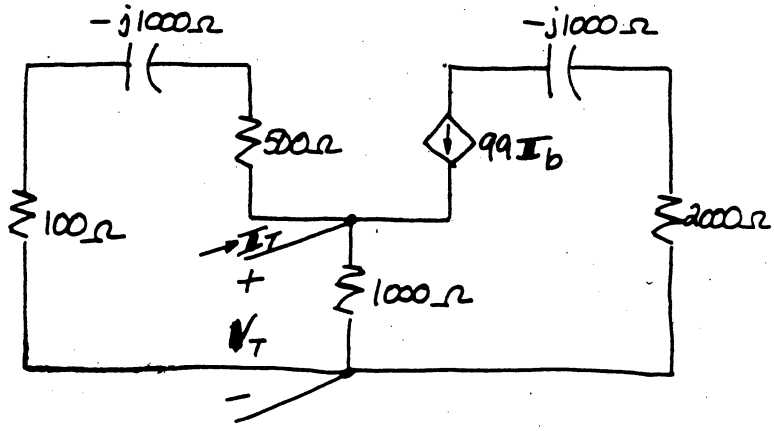
But $\mathbf{I}_\text{b} = -\mathbf{I}_T\left(\frac{1000}{1600 - j1000}\right)$

$\therefore\ \mathbf{I}_T = \frac{-99\mathbf{I}_T(1000)}{1600 - j1000}, \qquad \therefore\ \mathbf{I}_T = 0$

$\therefore\ Z_{xy} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \infty$

$\therefore\ Z_{\text{ab}} = 2000\,\Omega = Z_{\text{Th}}$

[b]



$$\mathbf{I}_T = \frac{\mathbf{V}_T}{1000} + \frac{\mathbf{V}_T}{600 - j1000} - 99\mathbf{I}_b$$

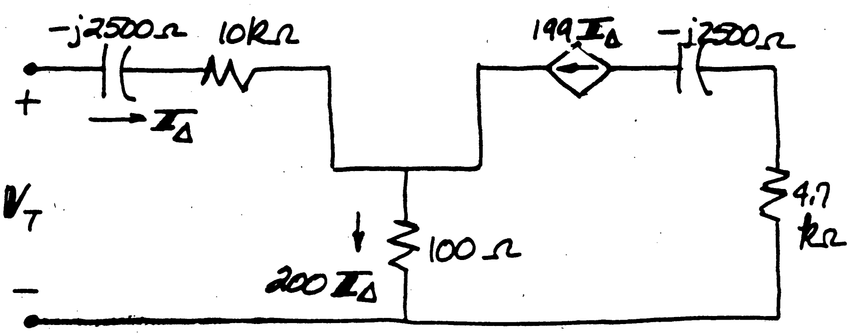
$$\mathbf{I}_b = \frac{-\mathbf{V}_T}{600 - j1000}$$

$$\therefore\ \mathbf{I}_T = \frac{\mathbf{V}_T}{1000} + \frac{\mathbf{V}_T}{600 - j1000} + \frac{99\mathbf{V}_T}{600 - j1000}$$

$$\frac{\mathbf{I}_T}{\mathbf{V}_T} = \frac{1}{1000} + \frac{100}{600 - j1000} = \frac{1}{1000} + \frac{1}{6 - j10} = \frac{1006 - j10}{1000(6 - j10)}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{1000(6 - j10)}{(1006 - j10)} = 11.59\underline{/-58.57^\circ}\ \Omega$$

P 10.55 $\omega = 400\,\text{rad/s}; \qquad \frac{1}{j\omega C} = -j2500\,\Omega$



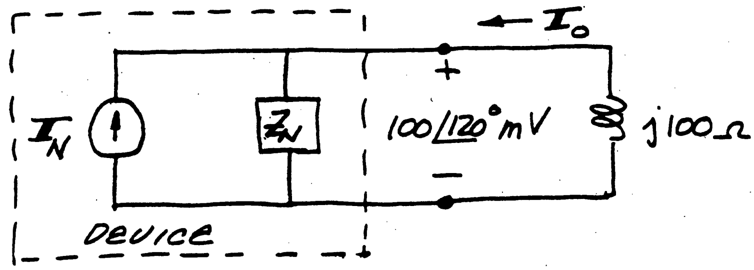
$$\mathbf{I}_T = \mathbf{I}_\Delta$$

$$\mathbf{V}_T = -j2500\mathbf{I}_T + 10{,}000\mathbf{I}_T + 100(200\mathbf{I}_T)$$

$$\frac{\mathbf{V}_T}{\mathbf{I}_T} = 30{,}000 - j2500 = 30{,}103.99\underline{/-4.76^\circ}\ \Omega$$

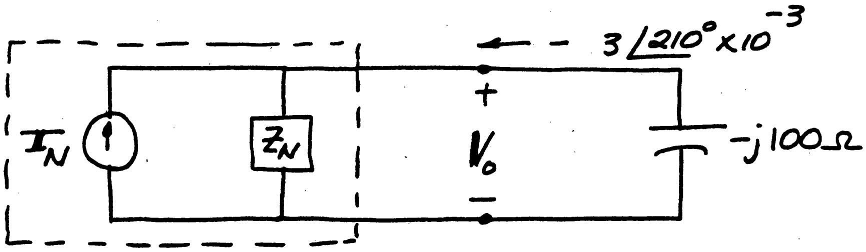
$$Z_{\text{Th}} = Z_{\text{ab}} = 30{,}103.99\underline{/-4.76^\circ}\ \Omega$$

P 10.56



$$\mathbf{I}_N = \frac{100\underline{/120^\circ} \times 10^{-3}}{Z_N} + \frac{100\underline{/120^\circ} \times 10^{-3}}{j100}$$

$$\mathbf{I}_N = \frac{0.1\underline{/120^\circ}}{Z_N} + 10^{-3}\underline{/30^\circ}$$



$$\mathbf{V}_o = -(-j100)(-3\underline{/210^\circ} \times 10^{-3}) = -j300\underline{/210^\circ} \times 10^{-3} = 0.3\underline{/120^\circ}\text{ V}$$

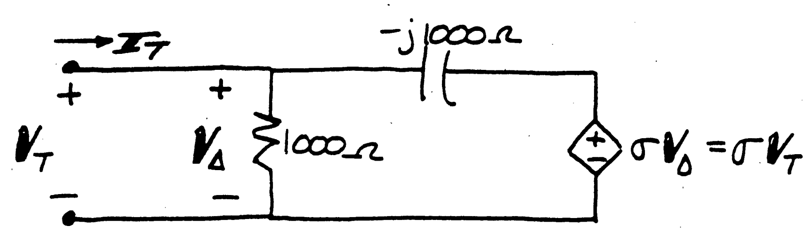
$$\mathbf{I}_N = \frac{0.3\underline{/120^\circ}}{Z_N} - (-3 \times 10^{-3}\underline{/210^\circ}) = \frac{0.3\underline{/120^\circ}}{Z_N} + 3 \times 10^{-3}\underline{/210^\circ}$$

$$\therefore\ \frac{0.1\underline{/120^\circ}}{Z_N} + 10^{-3}\underline{/30^\circ} = \frac{0.3\underline{/120^\circ}}{Z_N} + 3 \times 10^{-3}\underline{/210^\circ}$$

$$\therefore\ \frac{0.2\underline{/120^\circ}}{Z_N} = 10^{-3}\underline{/30^\circ} - 3 \times 10^{-3}\underline{/210^\circ} = 4 \times 10^{-3}\underline{/30^\circ}$$

$$\therefore\ Z_N = 50\underline{/90^\circ}\ \Omega; \qquad \mathbf{I}_N = 3\underline{/30^\circ}\text{ mA}$$

P 10.57 [a] $\dfrac{1}{j\omega C} = \dfrac{10^6}{j10(100)} = -j1000\,\Omega$



$$\mathbf{I}_T = \frac{\mathbf{V}_T}{1000} + \frac{\mathbf{V}_T - \sigma\mathbf{V}_T}{-j1000}$$

$$\frac{\mathbf{I}_T}{\mathbf{V}_T} = \frac{1}{1000} + \frac{j(1-\sigma)}{1000} = \frac{1+j(1-\sigma)}{1000}$$

$$Z_{\text{Th}} = Z_{\text{ab}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{1000}{1+j(1-\sigma)}$$

Z_{Th} is resistive when $\sigma = 1$.

[b] $$Z_{\text{Th}} = \frac{1000}{1+j0} = 1000\,\Omega$$

[c] $$500 - j500 = \frac{1000}{1+j(1-\sigma)} = \frac{1000[1-j(1-\sigma)]}{1+(1-\sigma)^2}$$

$$\therefore\ 500 = \frac{1000}{1+(1-\sigma)^2}$$

$$500 = \frac{1000(1-\sigma)}{1+(1-\sigma)^2}$$

$$\therefore\ 1+(1-\sigma)^2 = 2$$

$$(1-\sigma)^2 = 1$$

$$(1-\sigma) = \pm 1, \qquad \sigma = 1 \mp 1$$

$\sigma = 0$, $\sigma = 2$ makes the real part equal to 500

The imaginary part will equal 500 when

$$1+(1-\sigma)^2 = 2(1-\sigma) \quad \text{or} \quad (1-\sigma)^2 - 2(1-\sigma) + 1 = 0$$

Let $x = (1-\sigma)$

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0; \qquad x = 1$$

$$\therefore\ 1 - \sigma = 1; \qquad \sigma = 0$$

$\therefore$ If $\sigma = 0$, $Z_{\text{Th}} = 500 - j500\,\Omega$

Yes, $\sigma = 0$

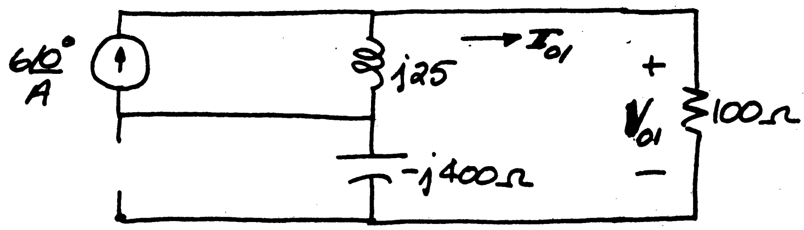
[d] $$Z_{\text{Th}} = \frac{1000}{1+j(1-\sigma)} = \frac{1000}{1+(1-\sigma)^2} - j\frac{1000(1-\sigma)}{1+(1-\sigma)^2}$$

$\therefore$ Z_{Th} will be inductive whenever $\sigma > 1$. For example, if $\sigma = 2$.

$$Z_{\text{Th}} = \frac{1000}{(1+1)} - j\frac{1000(-1)}{(1+1)} = 500 + j500\,\Omega$$

P 10.58 Since the current sources are operating at different frequencies, we must use superposition to find the steady-state expression for v_o.

When $\omega = 25$ rad/s we have

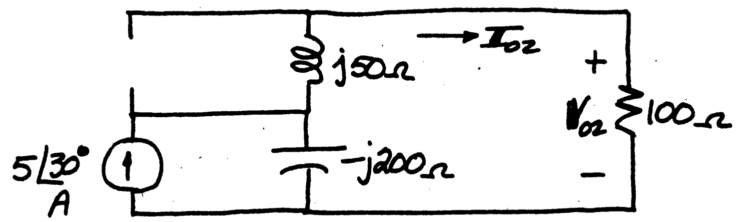


$$\mathbf{I}_{o1} = \frac{6(j25)}{100 - j375} = \frac{j6}{4 - j15}$$

$$\mathbf{V}_{o1} = 100\mathbf{I}_{o1} = \frac{j600}{4 - j15} = 38.65\underline{/165.07^\circ}\text{ V}$$

$$\therefore\ v_{o1} = 38.65\cos(25t + 165.07^\circ)\text{ V}$$

When $\omega = 50$ rad/s



$$\mathbf{I}_{o2} = \frac{5\underline{/30^\circ}(-j200)}{(100 - j150)} = 5.54\underline{/-3.69^\circ}$$

$$\mathbf{V}_{o2} = 100\mathbf{I}_{o2} = 554.70\underline{/-3.69^\circ}\text{ V}$$

$$v_{o2} = 554.70\cos(50t - 3.69^\circ)\text{ V}$$

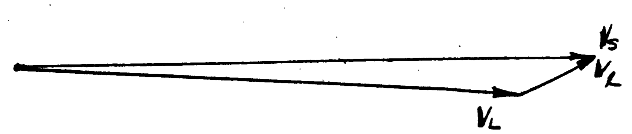
$$v_o = v_{o1} + v_{o2} = 38.65\cos(25t + 165.07^\circ) + 554.70\cos(50t - 3.69^\circ)\text{ V}$$

P 10.59 [a] $\mathbf{I}_\ell = \frac{240\underline{/0^\circ}}{8} + \frac{240\underline{/0^\circ}}{j6} = 30 - j40\text{ A}$

$$\mathbf{V}_\ell = (0.1 + j0.8)(30 - j40) = 3 - j4 + j24 + 32 = 35 + j20\text{ V}$$

$$\mathbf{V}_s = 240\underline{/0^\circ} + \mathbf{V}_\ell = 275 + j20 = 275.73\underline{/4.16^\circ}\text{ V}$$

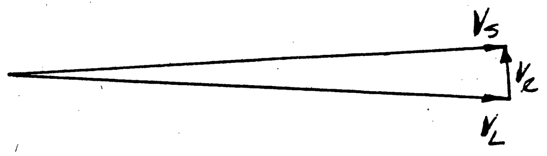
[b]



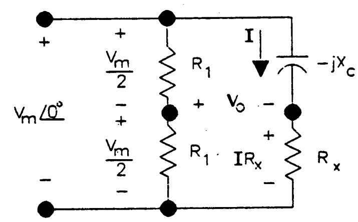
[c] $\mathbf{I}_\ell = 30 - j40 + \dfrac{240\angle 0^\circ}{-j5} = 30 - j40 + j48$

$\mathbf{V}_\ell = (0.1 + j0.8)(30 + j8) = 3 + j0.8 + j24 - 6.4 = -3.4 + j24.80\,\text{V}$

$\mathbf{V}_s = 240\angle 0^\circ - 3.4 + j24.8 = 236.60 + j24.8 = \underline{237.90\angle 5.98^\circ}$

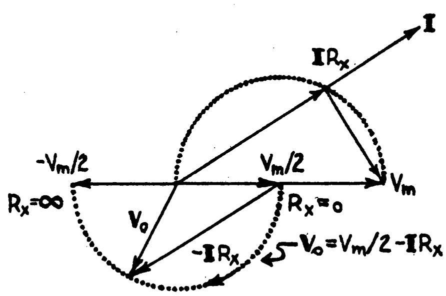


P 10.60 The phasor domain equivalent circuit is



$$V_o = \frac{V_m}{2} - \mathbf{I}R_x; \qquad \mathbf{I} = \frac{V_m}{R_x - jX_c}$$

As R_x varies from 0 to ∞, the amplitude of v_o remains constant and its phase angle increases from 0° to −180°.



P 10.61 [a] $\mathbf{V}_g = 2\angle 0^\circ\,\text{V}; \qquad \mathbf{V}_2 = 0.8\mathbf{V}_g = 1.6\angle 0^\circ\,\text{V}; \qquad \mathbf{V}_1 = \mathbf{V}_2 = 1.6\angle 0^\circ$

$$\frac{1.6\angle 0^\circ}{10^4} + \frac{1.6 - \mathbf{V}_o}{Z} = 0$$

$$1.6\left[\frac{1}{10^4} + \frac{1}{Z}\right] = \frac{\mathbf{V}_o}{Z}$$

$$\therefore\ \mathbf{V}_o = 1.6\left(1 + \frac{Z}{10^4}\right)$$

$$\frac{1}{j\omega C} = \frac{10^{12}}{j10^5(200)} = \frac{10^5}{j2} = -j50{,}000\,\Omega$$

$$Z = \frac{(-j50{,}000)(50{,}000)}{50{,}000 - j50{,}000} = \frac{-j50{,}000}{1 - j1}$$

$$Z = -j25{,}000(1 + j1) = 25{,}000 - j25{,}000\,\Omega$$

$$\mathbf{V}_o = 1.6(1 + 2.5 - j2.5) = 1.6(3.5 - j2.5) = 6.88\underline{/-35.54^\circ}\text{ V}$$

$$v_o = 6.88\cos(10^5 t - 35.54^\circ)\text{ V}$$

[b] $\mathbf{V}_o = 0.8\mathbf{V}_m(3.5 - j2.5) = \mathbf{V}_m(2.8 - j2.0) = 3.44\mathbf{V}_m\underline{/-35.54^\circ}$

$$\therefore\ 3.44\mathbf{V}_m = 15; \qquad \mathbf{V}_m = 4.36\text{ V}$$

P 10.62 $\dfrac{10^{12}}{j2 \times 10^6(100)} = \dfrac{10^4}{j2} = -j5\,\text{k}\Omega$

$$\frac{10^{12}}{j2 \times 10^6(10)} = -j50\,\text{k}\Omega; \qquad \mathbf{V}_g = 2\underline{/0^\circ}\text{ V}$$

$$\therefore\ \frac{\mathbf{V}_a}{-j5} + \frac{\mathbf{V}_a - \mathbf{V}_g}{5} + \frac{\mathbf{V}_a - \mathbf{V}_o}{100} + \frac{\mathbf{V}_a - 0}{20} = 0$$

$$j20\mathbf{V}_a + 20\mathbf{V}_a - 20\mathbf{V}_g + \mathbf{V}_a - \mathbf{V}_o + 5\mathbf{V}_a = 0$$

$$(26 + j20)\mathbf{V}_a - \mathbf{V}_o = 20(2\underline{/0^\circ}) = 40\underline{/0^\circ}$$

$$\frac{0 - \mathbf{V}_a}{20} + \frac{0 - \mathbf{V}_o}{-j50} = 0$$

$$-\frac{\mathbf{V}_a}{20} + \frac{\mathbf{V}_o}{j50} = 0; \qquad \mathbf{V}_a = \frac{20}{j50}\mathbf{V}_o = -j\frac{2}{5}\mathbf{V}_o$$

$$\therefore\ (26 + j20)\left(-j\frac{2}{5}\mathbf{V}_o\right) - \mathbf{V}_o = 40\underline{/0^\circ}$$

$$(7 - j10.40)\mathbf{V}_o = 40\underline{/0^\circ}$$

$$\mathbf{V}_o = 3.19\underline{/-56.06^\circ}\text{ V}$$

$$v_o = 3.19\cos(2 \times 10^6 t - 56.06^\circ)\text{ V}$$

P 10.63 [a] $\dfrac{10^6}{j10^5(0.5)} = -j20\,\Omega$

$$\mathbf{V}_2 = \frac{6\underline{/0^\circ}}{5 + (1/j\omega C_o)}\left(\frac{1}{j\omega C_o}\right) = \frac{6}{1 + j5\omega C_o} = \mathbf{V}_1$$

$$\frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_o}{-j20} = 0; \qquad \mathbf{V}_1(1 + j) = j\mathbf{V}_o$$

$$\mathbf{V}_o = (1 - j1)\mathbf{V}_1 = (1 - j1)\left(\frac{6}{1 + j5\omega C_o}\right)$$

$$|\mathbf{V}_o| = 6$$

$$\therefore\ 6 = \frac{6\sqrt{2}}{\sqrt{1 + 25\omega^2 C_o^2}}$$

$$\therefore\ \sqrt{1 + 25\omega^2 C_o^2} = \sqrt{2}; \qquad 25\omega^2 C_o^2 = 1$$

$$C_o = \frac{1}{5\omega} = \frac{1}{5 \times 10^5} = 2 \times 10^{-6} = 2\,\mu\text{F}$$

[b] $\mathbf{V}_o = \frac{(1-j1)6}{(1+j1)} = \frac{(\sqrt{2}\angle -45^\circ)6}{(\sqrt{2}\angle 45^\circ)} = 6\angle -90^\circ\text{ V}$

$v_o = 6\cos(10^5 t - 90^\circ)\text{ V} = -6\sin 10^5 t\text{ V}$

P10.64 $\mathbf{V}_g = 4\angle 0^\circ\text{ V};\qquad \frac{1}{j\omega C} = -j\frac{10^6}{(200)(0.25)} = -j20\text{ k}\Omega$

$\mathbf{V}_a \equiv$ voltage across the capacitor, positive at the upper terminal, then

$$\frac{\mathbf{V}_a}{-j20} + \frac{\mathbf{V}_a}{20} + \frac{\mathbf{V}_a - 4\angle 0^\circ}{20} = 0$$

$$\mathbf{V}_a(2+j1) = 4\angle 0^\circ$$

$$\frac{0-\mathbf{V}_a}{20} + \frac{0-\mathbf{V}_o}{100} = 0; \qquad \therefore\ \mathbf{V}_a = -0.2\mathbf{V}_o$$

$$-0.2\mathbf{V}_o(2+j1) = 4$$

$$\mathbf{V}_o = \frac{-20}{2+j1} = -4(2-j1) = -8+j4$$

$$\mathbf{V}_o = 8.94\angle 153.43^\circ\text{ V}$$

$$v_o = 8.94\cos(200t + 153.43^\circ)\text{ V}$$

P10.65 [a] $\mathbf{V}_a$ is as defined in the solution of Problem 10.64.

$$\frac{\mathbf{V}_a - 4}{20{,}000} + j200C\mathbf{V}_a + \frac{\mathbf{V}_a}{20{,}000} = 0$$

$$\mathbf{V}_a - 4 + j4\times 10^6 C\mathbf{V}_a + \mathbf{V}_a = 0$$

$$\mathbf{V}_a(2 + j4\times 10^6 C) = 4$$

$$\mathbf{V}_a = -\frac{\mathbf{V}_o}{5}$$

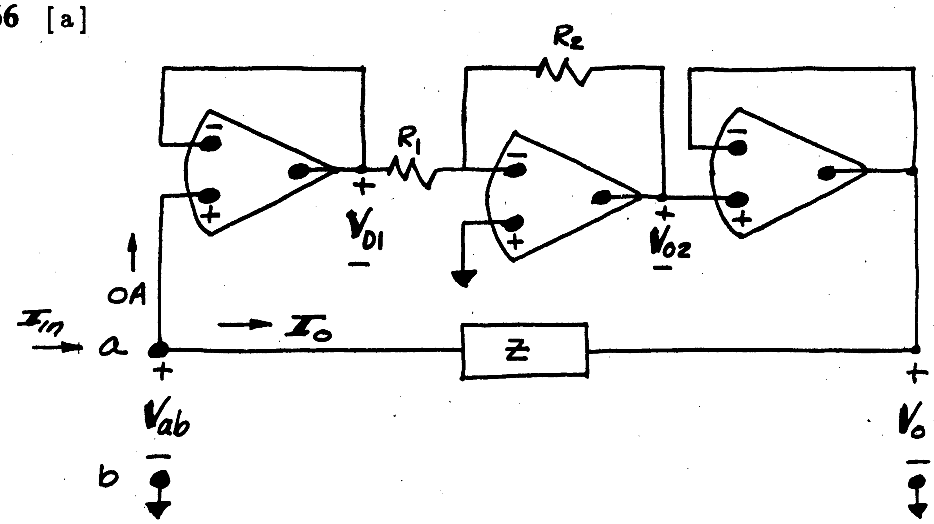
$$\therefore\ (2 + j4\times 10^6 C)\mathbf{V}_o = -20; \qquad \mathbf{V}_o = \frac{20\angle 180^\circ}{2 + j4\times 10^6 C}$$

$$\therefore\ 4\times 10^6 C = 2, \qquad C = \frac{1}{2}\,\mu\text{F}$$

[b] $\mathbf{V}_o = \frac{20\angle 180^\circ}{2+j2} = \frac{10\angle 180^\circ}{2}(1-j1) = 5\sqrt{2}\angle 135^\circ\text{ V} = 7.07\angle 135^\circ\text{ V}$

$v_o = 7.07\cos(200t + 135^\circ)\text{ V}$

P10.66 [a]



Because the op-amps are ideal $\mathbf{I}_{\text{in}} = \mathbf{I}_o$, thus

$$Z_{ab} = \frac{\mathbf{V}_{ab}}{\mathbf{I}_{in}} = \frac{\mathbf{V}_{ab}}{\mathbf{I}_o}; \qquad \mathbf{I}_o = \frac{\mathbf{V}_{ab} - \mathbf{V}_o}{Z}$$

$$\mathbf{V}_{o1} = \mathbf{V}_{ab}; \qquad \mathbf{V}_{o2} = -\left(\frac{R_2}{R_1}\right)\mathbf{V}_{o1} = -K\mathbf{V}_{o1} = -K\mathbf{V}_{ab}$$

$$\mathbf{V}_o = \mathbf{V}_{o2} = -K\mathbf{V}_{ab}$$

$$\therefore\ \mathbf{I}_o = \frac{\mathbf{V}_{ab} - (-K\mathbf{V}_{ab})}{Z} = \frac{(1+K)\mathbf{V}_{ab}}{Z}$$

$$\therefore\ Z_{ab} = \frac{\mathbf{V}_{ab}}{(1+K)\mathbf{V}_{ab}}Z = \frac{Z}{(1+K)}$$

[b] $Z = \frac{1}{j\omega C}; \qquad Z_{ab} = \frac{1}{j\omega C(1+K)}; \qquad \therefore\ C_{ab} = C(1+K)$

P10.67 [a] $\mathbf{I}_1 = \frac{120}{24} + \frac{240}{8.4 + j6} = 23.29 - j13.71 = 27.02\underline{/-30.5^\circ}\text{ A}$

$$\mathbf{I}_2 = \frac{120}{12} - \frac{120}{24} = 5\underline{/0^\circ}\text{ A}$$

$$\mathbf{I}_3 = \frac{120}{12} + \frac{240}{8.4 + j6} = 28.29 - j13.71 = 31.44\underline{/-25.87^\circ}\text{ A}$$

$$\mathbf{I}_4 = \frac{120}{24} = 5\underline{/0^\circ}\text{ A}; \qquad \mathbf{I}_5 = \frac{120}{12} = 10\underline{/0^\circ}\text{ A}$$

$$\mathbf{I}_6 = \frac{240}{8.4 + j6.3} = 18.29 - j13.71 = 22.86\underline{/-36.87^\circ}\text{ A}$$

[b]

$\mathbf{I}_1 = 0$	$\mathbf{I}_3 = 15$ A	$\mathbf{I}_5 = 10$ A
$\mathbf{I}_2 = 10 + 5 = 15$ A	$\mathbf{I}_4 = -5$ A	$\mathbf{I}_6 = 5$ A

[c] The clock and television set were fed from the uninterrupted side of the circuit, that is, the 12-Ω load includes the clock and the TV set.

[d] No, the motor current drops to 5 A, well below its normal running value of 22.86 A.

[e] After fuse A opens, the current in fuse B is only 15 A.