

- An inductor *does* permit an instantaneous change in its terminal voltage.
- A capacitor *does* permit an instantaneous change in its terminal current.
- An inductor appears as a short circuit in relation to a constant terminal current.
- A capacitor appears as an open circuit in relation to a constant terminal voltage.

Table 7.1 summarizes the mathematical equations, based on the passive sign convention, that describe the terminal behavior of the inductor and capacitor.

TABLE 7.1

## TERMINAL EQUATIONS FOR IDEAL INDUCTORS AND CAPACITORS\*

INDUCTORS		CAPACITORS	
1. $v = L \frac{di}{dt}$	(V)	1. $v = \frac{1}{C} \int_{t_0}^t i \, d\tau + v(t_0)$	(V)
2. $i = \frac{1}{L} \int_{t_0}^t v \, d\tau + i(t_0)$	(A)	2. $i = C \frac{dv}{dt}$	(A)
3. $p = vi = Li \frac{di}{dt}$	(W)	3. $p = vi = Cv \frac{dv}{dt}$	(W)
4. $w = \frac{1}{2} Li^2$	(J)	4. $w = \frac{1}{2} Cv^2$	(J)

\*The equations in this table are based on the passive sign convention.

## PROBLEMS

7.1 Evaluate the integral

$$\int_0^{\infty} p \, dt$$

for Example 7.2. Comment on the significance of the result.

7.2 The triangular current pulse shown in Fig. P7.2 is applied to a 25-mH inductor.

- Write the expressions that describe  $i(t)$  in the four intervals  $t < 0$ ,  $0 \leq t \leq 5$  ms,  $5 \text{ ms} \leq t \leq 10$  ms, and  $t > 10$  ms.
- Derive the expressions for the inductor voltage, power, and energy. Use the passive sign convention.

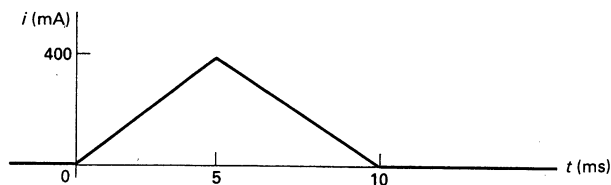


FIGURE P7.2

7.3 The voltage at the terminals of the 200- $\mu\text{H}$  inductor in Fig. P7.3(a) is shown in Fig. P7.3(b). The inductor current  $i$  is known to be zero for  $t \leq 0$ .

- Derive the expressions for  $i$  for  $t \geq 0$ .
- Sketch  $i$  vs.  $t$  for  $0 \leq t \leq \infty$ .

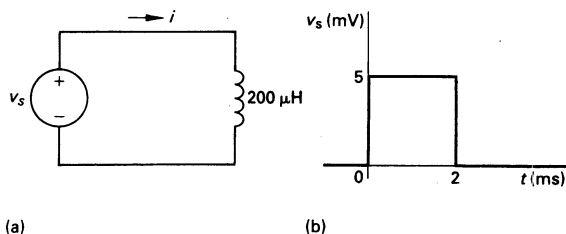


FIGURE P7.3

7.4 The current in the 2.5-mH inductor in Fig. P7.4 is known to be 1 A for  $t \leq 0$ . The inductor voltage for  $t \geq 0^+$  is given by the expressions

$$v_L(t) = 3e^{-4t} \text{ mV}, \quad 0^+ \leq t < 2\text{ s}$$

$$v_L(t) = -3e^{-4(t-2)} \text{ mV}, \quad 2^+ \text{ s} \leq t \leq \infty.$$

Sketch  $v_L(t)$  and  $i_L(t)$  for  $0 \leq t \leq \infty$ .

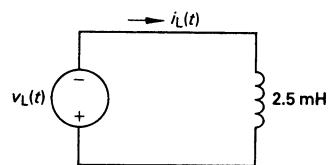


FIGURE P7.4

- Find the inductor current in the circuit in Fig. P7.5 if  $v = -50 \sin 250t$  V,  $L = 20$  mH, and  $i(4\pi \text{ ms}) = -10$  A.
- Sketch  $v$ ,  $i$ ,  $p$ , and  $w$ , versus time. In making these sketches, use the format used in Fig. 7.8. Plot over one complete cycle of the voltage waveform.
- Describe the subintervals in the time interval between 0 and  $4\pi$  ms when power is being

absorbed by the inductor. Repeat for the subintervals when power is being delivered by the inductor.

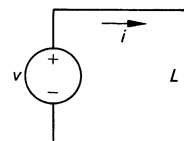


FIGURE P7.5

7.6 The current in a 25-mH inductor is known to be  $-10$  A for  $t \leq 0$  and  $i = -[10 \cos 400t + 5 \sin 400t]e^{-200t}$  A for  $t \geq 0$ . Assume the passive sign convention.

- At what instant of time is the voltage across the inductor maximum?
- What is the maximum voltage?

7.7 The current in a 50- $\mu\text{H}$  inductor is known to be

$$i_L = 18t e^{-10t} \text{ A for } t \geq 0.$$

- Find the voltage across the inductor for  $t > 0$ . (Assume the passive sign convention.)
- Find the power (in  $\mu\text{W}$ ) at the terminals of the inductor when  $t = 200$  ms.

- Is the inductor absorbing or delivering power at 200 ms?
- Find the energy (in  $\mu\text{J}$ ) stored in the inductor at 200 ms.
- Find the maximum energy (in  $\mu\text{J}$ ) stored in the inductor, and the time (in ms) when it occurs.

**7.8** The current in and the voltage across a 5-H inductor are known to be zero for  $t \leq 0$ . The voltage across the inductor is given by the graph in Fig. P7.8 for  $t \geq 0$ .

- Derive the expressions for the current as a function of time in the intervals  $0 \leq t \leq 1$  s,  $1 \leq t \leq 3$  s,  $3 \leq t \leq 5$  s,  $5 \leq t \leq 6$  s, and  $6 \leq t \leq \infty$ .
- For  $t > 0$  what is the current in the inductor when the voltage is zero?
- Sketch  $i$  vs  $t$  for  $0 \leq t \leq \infty$ .

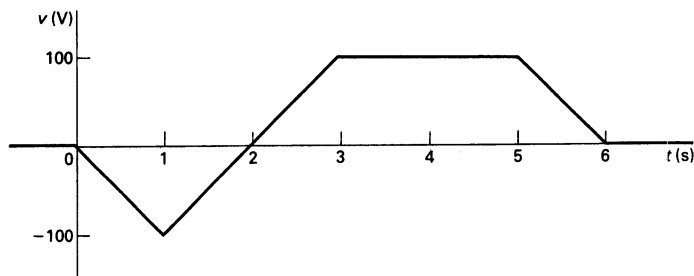


FIGURE P7.8

**7.9** The current in a 4-H inductor is

$$i = 10 \text{ A}, t \leq 0;$$

$$i = (B_1 \cos 4t + B_2 \sin 4t)e^{-0.5t} \text{ A}, t \geq 0.$$

The voltage across the inductor (passive-sign convention) is 60 V at  $t = 0$ . Calculate the power at the terminals of the inductor at  $t = 1$  s. State whether the inductor is absorbing or delivering power.

**7.10** The current in a 20-mH inductor is known to be

$$i = 40 \text{ mA}, t \leq 0$$

$$i = A_1 e^{-10,000t} + A_2 e^{-40,000t} \text{ A}, t \geq 0.$$

The voltage across the inductor (passive-sign convention) is 28 V at  $t = 0$ .

- Find the expression for the voltage across the inductor for  $t > 0$ .
- Find the time, greater than zero, when the power at the terminals of the inductor is zero.

**7.11** Assume in Problem 7.10 that the value of the voltage across the inductor at  $t = 0$  is  $-68$  V instead of 28 V.

- Find the numerical expressions for  $i$  and  $v$  for  $t \geq 0$ .
- Specify the time intervals when the inductor

is storing energy and the time intervals when the inductor is delivering energy.

- Show that the total energy extracted from the inductor is equal to the total energy stored.

**7.12** Initially there was no energy stored in the 5-H inductor in the circuit in Fig. P7.12 when it was placed across the terminals of the voltmeter. At  $t = 0$  the inductor was switched instantaneously to position b where it remained for 1.6 s before returning instantaneously to position a. The d'Arsonval voltmeter has a full scale reading of

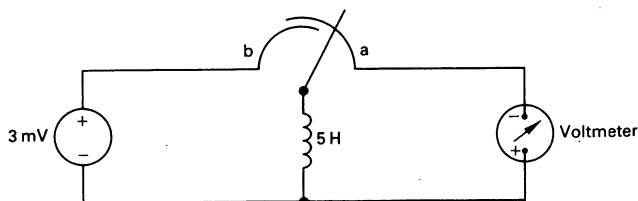


FIGURE P7.12

20 V and a sensitivity of 1000  $\Omega$ /volt. What will the reading of the voltmeter be at the in-

stant the switch returns to position a if the inertia of the d'Arsonval movement is negligible?

- 7.13** The voltage across the terminals of a 0.20- $\mu$ F capacitor is

$$v = 150 \text{ V}, t \leq 0$$

$$v = A_1 t e^{-5000t} + A_2 e^{-5000t} \text{ V}, t \geq 0.$$

The initial current in the capacitor is 250 mA. Assume the passive sign convention.

- What is the initial energy stored in the capacitor?
- Evaluate the coefficients  $A_1$  and  $A_2$ .
- What is the expression for the capacitor current?

- 7.14** The voltage at the terminals of the capacitor in Fig. 7.10 is known to be

$$v = -20 \text{ V}, t \leq 0$$

$$v = 100 - 40e^{-2000t}(3 \cos 1000t + \sin 1000t) \text{ V}, t \geq 0.$$

If  $C = 0.4 \mu\text{F}$ :

- Find the current in the capacitor for  $t < 0$ .
- Find the current in the capacitor for  $t > 0$ .
- Is there an instantaneous change in the voltage across the capacitor at  $t = 0$ ?
- Is there an instantaneous change in the current in the capacitor at  $t = 0$ ?
- How much energy (in  $\mu\text{J}$ ) is stored in the capacitor at  $t = \infty$ ?

- 7.15** The rectangular-shaped current pulse shown in Fig. P7.15 is applied to a 0.1- $\mu\text{F}$  capacitor. The initial voltage on the capacitor is a 15-V drop in the reference direction of the current. Assume the passive sign convention. Derive the expression for the capacitor voltage for each of the following time intervals:

- $0 \leq t \leq 10 \mu\text{s}$
- $10 \mu\text{s} \leq t \leq 20 \mu\text{s}$
- $20 \mu\text{s} \leq t \leq 40 \mu\text{s}$
- $40 \mu\text{s} \leq t \leq \infty$ .

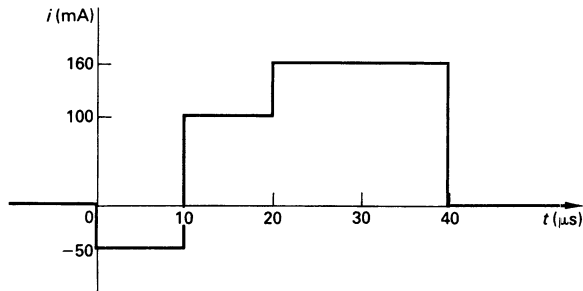


FIGURE P7.15

- 7.16** The current pulse shown in Fig. P7.16 is applied to a 0.25- $\mu\text{F}$  capacitor. The initial voltage on the capacitor is zero.

- Find the charge on the capacitor at  $t = 15 \mu\text{s}$ .
- Find the voltage on the capacitor at  $t = 30 \mu\text{s}$ .
- How much energy is stored in the capacitor by the current pulse?

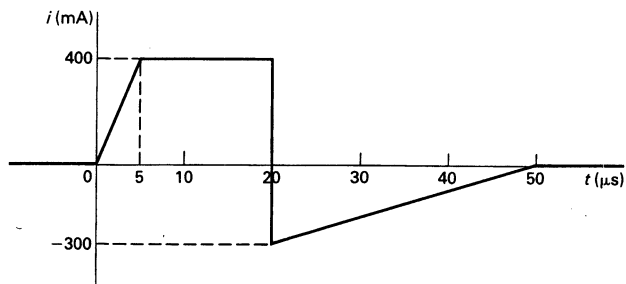
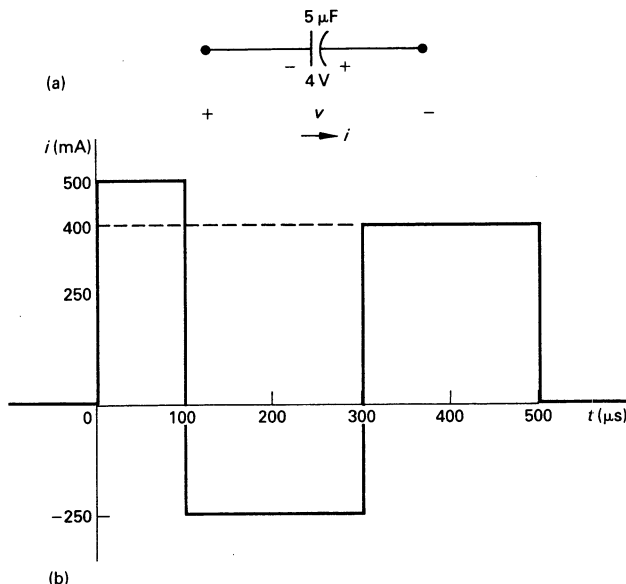


FIGURE P7.16

**7.17** The initial voltage on the  $5\text{-}\mu\text{F}$  capacitor shown in Fig. P7.17(a) is  $4\text{ V}$ . The capacitor current has the waveform shown in Fig. P7.17(b).

- How much energy, in microjoules, is stored in the capacitor at  $t = 400\text{ }\mu\text{s}$ ?
- Repeat part (a) for  $t = 600\text{ }\mu\text{s}$ .



**FIGURE P7.17**

**7.18** A  $20\text{-}\mu\text{F}$  capacitor is subjected to a voltage pulse having a duration of  $1\text{ sec}$ . The pulse is described by the following equations:

$$\begin{aligned} v_c(t) &= 30t^2\text{ V} & 0 \leq t \leq 0.5\text{ s} \\ v_c(t) &= 30(t - 1)^2\text{ V} & 0.5\text{ s} \leq t \leq 1.0\text{ s} \\ v_c(t) &= 0 & \text{elsewhere.} \end{aligned}$$

Sketch the current pulse that exists in the capacitor during the  $1\text{-s}$  interval.

**7.19** The expressions for voltage, power, and energy derived in Example 7.5 involved both integration and manipulation of algebraic expressions. As an engineer, you cannot accept such results on faith alone. That is, you should develop the habit of asking yourself, “Do these results make sense in terms of the known behavior of the circuit they purport to describe?” With these thoughts in mind, test the expressions of Example 7.5 by performing the following checks:

- Check the expressions to see whether the voltage is continuous in passing from one time interval to the next.
- Check the power expression in each interval by selecting a time within the interval and see whether it gives the same result as the corresponding product of  $v$  and  $i$ . For example, test at  $10$  and  $30\text{ }\mu\text{s}$ .
- Check the energy expression within each interval by selecting a time within the interval and see whether the energy equation gives the same result as  $\frac{1}{2}Cv^2$ . Use  $10$  and  $30\text{ }\mu\text{s}$  as test points.

- 7.20** Assume that the initial energy stored in the inductors of Fig. P7.20 is zero. Find the equivalent inductance with respect to the terminals a, b.

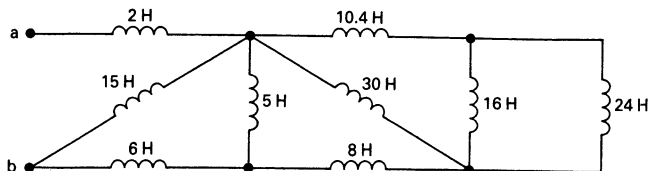


FIGURE P7.20

- 7.21** The two parallel inductors in Fig. P7.21 are connected across the terminals of a black box at  $t = 0$ . The resulting voltage  $v$  for  $t \geq 0$  is known to be  $64e^{-4t}$  V. It is also known that  $i_1(0) = -10$  A and  $i_2(0) = 5$  A.

- Replace the original inductors with an equivalent inductor and find  $i(t)$  for  $t \geq 0$ .
- Find  $i_1(t)$  for  $t \geq 0$ .
- Find  $i_2(t)$  for  $t \geq 0$ .
- How much energy is delivered to the black box in the time interval  $0 \leq t \leq \infty$ ?
- How much energy was initially stored in the parallel inductors?

- How much energy is trapped in the ideal inductors?
- Do your solutions for  $i_1$  and  $i_2$  agree with the answer obtained in part (f)?

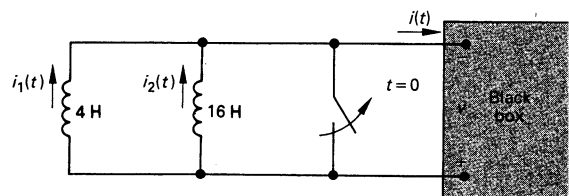


FIGURE P7.21

- 7.22** The three inductors in the circuit in Fig. P7.22 are connected across the terminals of a black box at  $t = 0$ . The resulting voltage for  $t \geq 0$  is known to be

$$v_b = 2000 e^{-100t} \text{ V.}$$

If  $i_1(0) = -6$  A and  $i_2(0) = 1$  A, find

- $i_0(0)$ ;
- $i_0(t)$ ,  $t \geq 0$ ;
- $i_1(t)$ ,  $t \geq 0$ ;
- $i_2(t)$ ,  $t \geq 0$ ;
- the initial energy stored in the three inductors;
- the total energy delivered to the black box; and
- the energy trapped in the ideal inductors.

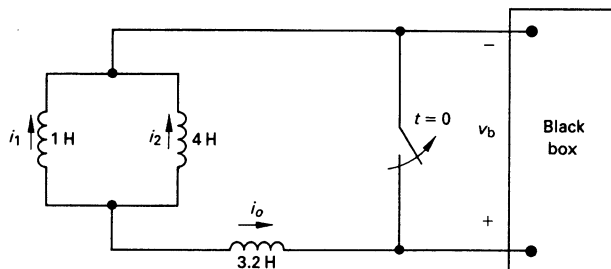


FIGURE P7.22

**7.23** For the circuit shown in Fig. P7.22 how many milliseconds after the switch is opened is the energy delivered to the black box 80% of the total amount delivered?

**7.24** Derive the equivalent circuit for a series connection of ideal capacitors. Assume that each capacitor has its own initial voltage. Denote these initial voltages as  $v_1(t_0)$ ,  $v_2(t_0)$ ,  $\dots$ , etc. (*Hint: Sum the voltages across the string of capacitors, recognizing that the series connection forces the current in each capacitor to be the same.*)

**7.25** Derive the equivalent circuit for a parallel connection of ideal capacitors. Assume that the initial voltage across the paralleled capacitors is  $v(t_0)$ . (*Hint: Sum the currents into the string of*

capacitors, recognizing that the parallel connection forces the voltage across each capacitor to be the same.)

**7.26** Find the equivalent capacitance with respect to the terminals a, b for the circuit shown in Fig. P7.26.

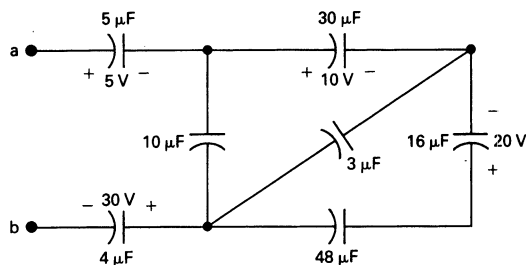


FIGURE P7.26

**7.27** The four capacitors in the circuit in Fig. P7.27 are connected across the terminals of a black box at  $t = 0$ . The resulting current  $i_b$  for  $t \geq 0$  is known to be

$$i_b = -5e^{-50t} \text{ mA.}$$

If  $v_a(0) = -20 \text{ V}$ ,  $v_c(0) = -30 \text{ V}$ , and  $v_d(0) = 250 \text{ V}$ , find for  $t \geq 0$ , (a)  $v_b(t)$ , (b)  $v_a(t)$ , (c)  $v_c(t)$ , (d)  $v_d(t)$ , (e)  $i_1(t)$ , and (f)  $i_2(t)$ .

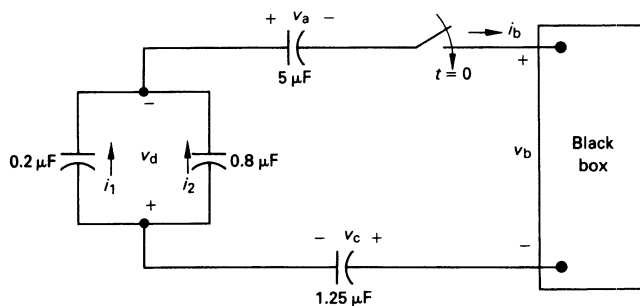


FIGURE P7.27

**7.28** For the circuit in Fig. P7.27 calculate:

- the initial energy stored in the capacitors;
- the final energy stored in the capacitors;
- the total energy delivered to the black box;

- the percentage of the initial energy stored that is delivered to the black box; and
- the time, in milliseconds, it takes to deliver  $7500 \mu\text{J}$  to the black box.

**7.29** The two series-connected capacitors in Fig. P7.29 are connected to the terminals of a black box at  $t = 0$ . The resulting current  $i(t)$  for  $t \geq 0$  is known to be  $800e^{-25t} \mu\text{A}$ .

- Replace the original capacitors with an equivalent capacitor and find  $v_o(t)$  for  $t \geq 0$ .
- Find  $v_1(t)$  for  $t \geq 0$ .
- Find  $v_2(t)$  for  $t \geq 0$ .
- How much energy is delivered to the black box in the time interval  $0 \leq t \leq \infty$ ?
- How much energy was initially stored in the series capacitors?
- How much energy is trapped in the ideal capacitors?
- Do the solutions for  $v_1$  and  $v_2$  agree with the answer obtained in part (f)?

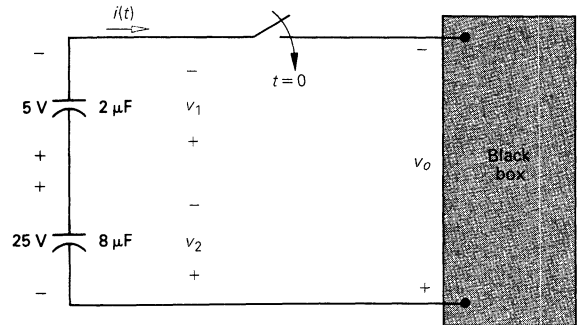


FIGURE P7.29

**7.30** At  $t = 0$  a series-connected capacitor and inductor are placed across the terminals of a black box, as shown in Fig. P7.30. For  $t \geq 0$  it is known that

$$i_o = e^{-80t} \sin 60t \text{ A.}$$

If  $v_c(0) = -300 \text{ V}$  find  $v_o$  for  $t \geq 0$ .

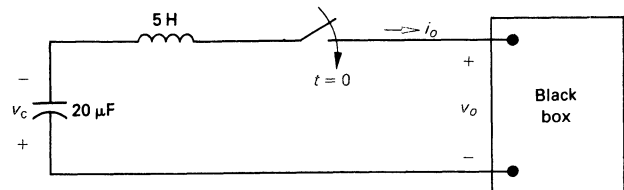


FIGURE P7.30

**7.31** The current in the circuit in Fig. P7.31 is known to be

$$i_o = 5 e^{-2000t} [2 \cos 4000t + \sin 4000t] \text{ A}$$

for  $t \geq 0^+$ .

Find  $v_1(0^+)$  and  $v_2(0^+)$ .

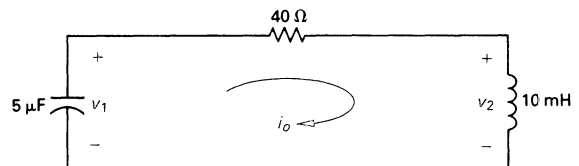


FIGURE P7.31



- The time constant of a capacitive circuit equals the equivalent capacitance times the Thévenin resistance as viewed from the terminals of the equivalent capacitor.
- The natural response corresponds to finding the currents and voltages that exist when stored energy is released to a circuit that contains no independent sources.
- The step response corresponds to finding the currents and voltages that result from abrupt changes in dc sources connected to the circuit. Stored energy may or may not be present at the time the abrupt change takes place.
- The solution for either the natural or step response of both  $RL$  and  $RC$  circuits involves finding the initial and final value of the current or voltage of interest and the time constant of the circuit. Equations (8.56) and (8.57) summarize this approach.
- An unbounded response occurs when the Thévenin resistance is negative, which is possible when the first-order circuit contains dependent sources.
- Sequential switching in first-order circuits is analyzed by dividing the analysis into time intervals corresponding to specific switch positions. Initial values for a particular interval are determined from the solution corresponding to the immediately preceding interval.
- The techniques discussed are used to analyze an integrating amplifier consisting of an ideal op amp, a capacitor in the negative feedback branch, and a resistor in series with the signal source.

## PROBLEMS

- 8.1 In the circuit in Fig. P8.1, the voltage and current expressions are

$$v = 160e^{-10t} \text{ V}, t \geq 0^+;$$

$$i = 6.4e^{-10t} \text{ A}, t \geq 0.$$

Find (a)  $R$ , (b)  $\tau$  (ms), (c)  $L$ , (d) the initial energy stored in the inductor, and (e) the time (ms) it takes to dissipate 60% of the initial stored energy.

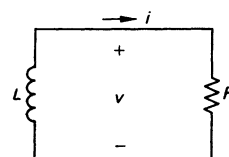


FIGURE P8.1

**8.2** In the circuit shown in Fig. P8.2 the switch makes contact with position b just before breaking contact with position a. This is known as a “make-before-break” switch and is designed so that the switch does not interrupt the current in an inductive circuit. The interval of time between “making” and “breaking” is assumed to be negligible. The switch has been in the a position for a long time. At  $t = 0$  the switch is thrown from position a to position b.

- Determine the initial current in the inductor.
- Determine the time constant of the circuit for  $t > 0$ .

- Find  $i$ ,  $v_1$ , and  $v_2$  for  $t \geq 0$ .
- What percentage of the initial energy stored in the inductor is dissipated in the  $72\text{-}\Omega$  resistor 15 ms after the switch is thrown from position a to position b?

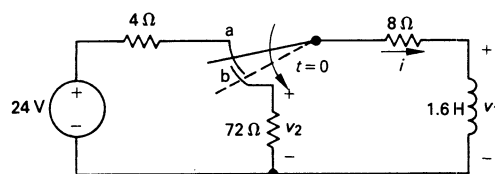


FIGURE P8.2

**8.3** The switch in the circuit in Fig. P8.3 has been open for a long time. At  $t = 0$  the switch is closed.

- Determine  $i_o(0^+)$  and  $i_o(\infty)$ .
- Determine  $i_o(t)$  for  $t \geq 0^+$ .
- How many microseconds after the switch has been closed will the current in the switch equal 3 A?

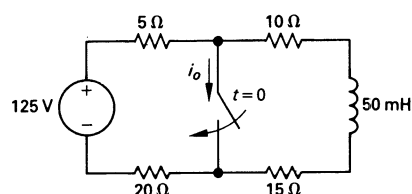


FIGURE P8.3

**8.4** The switch in the circuit in Fig. P8.4 has been closed a long time. At  $t = 0$  it is opened. Find  $v_o(t)$  for  $t \geq 0^+$ .

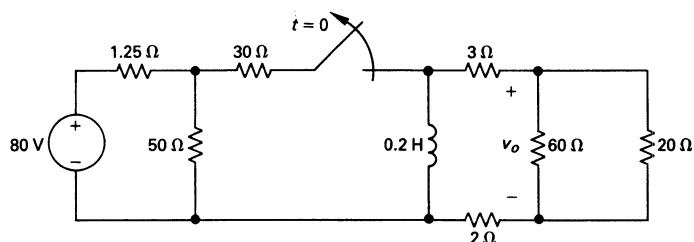


FIGURE P8.4

**8.5** Assume that the switch in the circuit in Fig. P8.4 has been open for two time constants. At this instant, what percentage of the total energy

stored in the  $0.2\text{-H}$  inductor has been dissipated in the  $2\text{-}\Omega$  resistor?

- 8.6 The switch in the circuit in Fig. P8.6 has been in position 1 for a long time. At  $t = 0$ , the switch moves instantaneously to position 2. Find  $v_o(t)$  for  $t \geq 0^+$ .

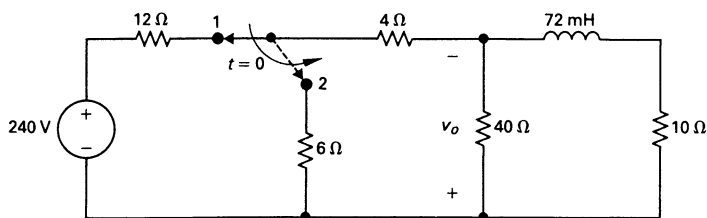


FIGURE P8.6

- 8.7 For the circuit of Fig. P8.6, what percentage of the initial energy stored in the inductor is eventually dissipated in the 40-Ω resistor?

- 8.8 In the circuit in Fig. P8.8 the switch has been closed for a long time before opening at  $t = 0$ .
- Find the value of  $L$  so that  $v_o(t)$  equals  $0.5 v_o(0^+)$  when  $t = 1$  ms.
  - Find the percentage of the stored energy that has been dissipated in the 10-Ω resistor when  $t = 1$  ms.

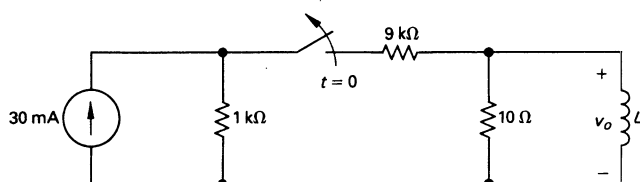


FIGURE P8.8

- 8.9 The switch in the circuit in Fig. P8.9 has been closed for a long time before opening at  $t = 0$ . Find:

- $i_1(0^-)$  and  $i_2(0^-)$ ;
- $i_1(0^+)$  and  $i_2(0^+)$ ;
- $i_1(t)$  for  $t \geq 0$ ;
- $i_2(t)$  for  $t \geq 0^+$ ; and
- Explain why  $i_2(0^-) \neq i_2(0^+)$ .

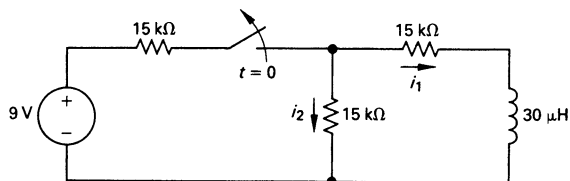


FIGURE P8.9

- 8.10 The switch in the circuit seen in Fig. P8.10 has been in position 1 for a long time. At  $t = 0$  the switch moves instantaneously to position 2. Find the value of  $R$  so that 0.10 of the initial energy stored in the 10-mH inductor is dissipated in  $R$  in 10 μs.

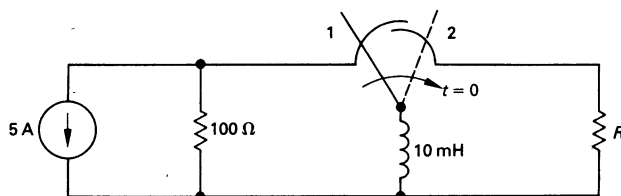


FIGURE P8.10

**8.11** In the circuit shown in Fig. P8.11 the switch has been in position a for a long time. At  $t = 0$ , it moves instantaneously from a to b.

- Find  $v_o(t)$  for  $t \geq 0^+$ .
- Does it take more than or less than two time constants to dissipate 98% of the energy stored in the circuit at  $t = 0^+$ ? Justify your answer.

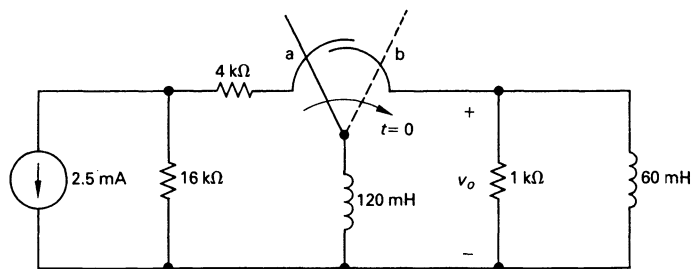


FIGURE P8.11

**8.12** The two switches shown in the circuit in Fig. P8.12 operate simultaneously. Prior to  $t = 0$  each switch has been in its indicated position for a long time. At  $t = 0$  the two switches move instantaneously to their new positions. Find

- $v_o(t)$ ,  $t \geq 0^+$ ,
- $i_o(t)$ ,  $t \geq 0$ .

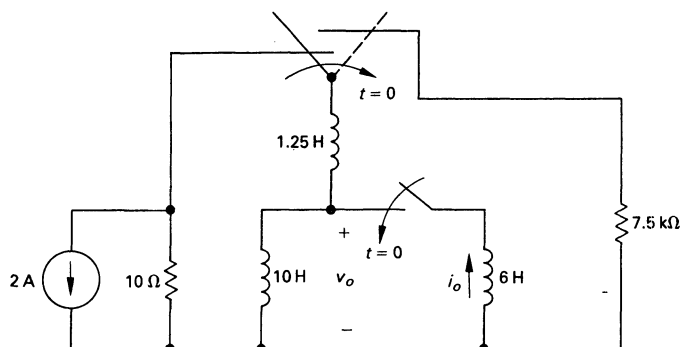


FIGURE P8.12

**8.13** For the circuit seen in Fig. P8.12 find

- the total energy dissipated in the 7.5-kΩ resistor; and
- the energy trapped in the ideal inductors.

**8.14** The switch in the circuit in Fig. P8.14 has been closed for a long time before opening at  $t = 0$ . Find  $v_o(t)$  for  $t \geq 0^+$ .

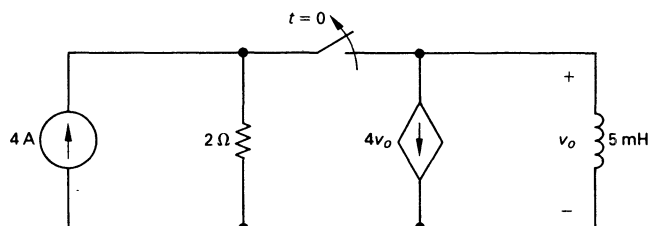


FIGURE P8.14

**8.15** The switch in Fig. P8.15 has been closed for a long time before opening at  $t = 0$ . Find

- $i_L(t)$ ,  $t \geq 0$ ;
- $v_L(t)$ ,  $t \geq 0^+$ ; and
- $i_\Delta(t)$ ,  $t \geq 0^+$ .

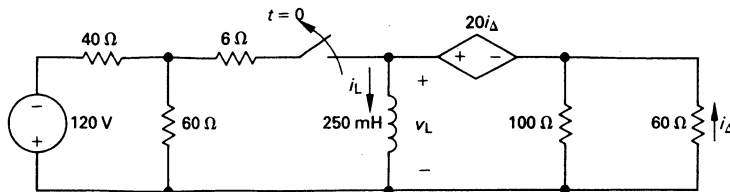


FIGURE P8.15

**8.16** What percentage of the total energy dissipated in the two resistors in the circuit in Fig. P8.15 is supplied by the dependent voltage source?

**8.17** The 240-V, 2-Ω source in the circuit in Fig. P8.17 is inadvertently short-circuited at its terminals a, b. At the time the fault occurs, the circuit has been in operation for a long time.

- What is the initial value of the current  $i_{ab}$  in the short-circuit connection between terminals a, b?
- What is the final value of the current  $i_{ab}$ ?
- How many microseconds after the short-circuit has occurred is the current in the short equal to 114 A?

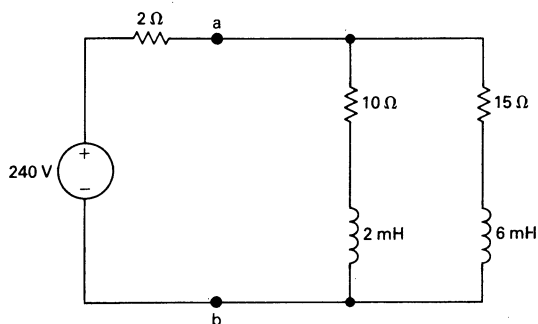


FIGURE P8.17

**8.18** In the circuit in Fig. P8.18 the voltage and current expressions are

$$v = 72e^{-500t} \text{ V}, t \geq 0;$$

$$i = 9e^{-500t} \text{ mA}, t \geq 0^+.$$

Find (a)  $R$ , (b)  $C$ , (c)  $\tau$  (ms), (d) the initial energy stored in the capacitor, and (e) how many milliseconds it takes to dissipate 68% of the initial energy stored in the capacitor.

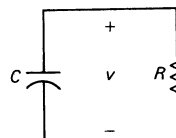


FIGURE P8.18

**8.19** The switch in the circuit in Fig. P8.19 has been in position a for a long time. At  $t = 0$  the switch is thrown to position b.

- Find  $i_o(t)$  for  $t \geq 0^+$ .
- What percentage of the initial energy stored in the capacitor is dissipated in the 3-k $\Omega$  resistor 500  $\mu$ s after the switch has been thrown?

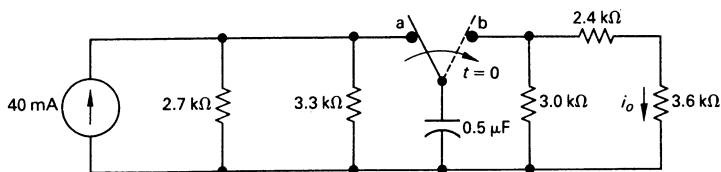


FIGURE P8.19

**8.20** The switch in the circuit in Fig. P8.20 is closed at  $t = 0$  after being open for a long time.

- Find  $i_1(0^-)$  and  $i_2(0^-)$ .
- Find  $i_1(0^+)$  and  $i_2(0^+)$ .
- Explain why  $i_1(0^-) = i_1(0^+)$ .
- Explain why  $i_2(0^-) \neq i_2(0^+)$ .
- Find  $i_1(t)$  for  $t \geq 0$ .
- Find  $i_2(t)$  for  $t \geq 0^+$ .

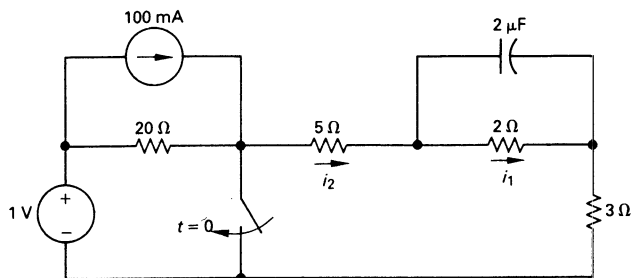


FIGURE P8.20

**8.21** Both switches in the circuit in Fig. P8.21 have been closed for a long time. At  $t = 0$ , both switches open simultaneously.

- Find  $i_o(t)$  for  $t \geq 0^+$ .
- Find  $v_o(t)$  for  $t \geq 0$ .
- Calculate the energy (in microjoules) trapped in the circuit.

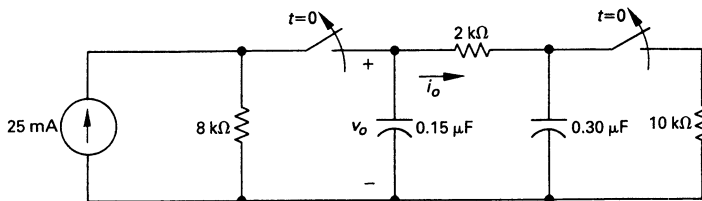


FIGURE P8.21

**8.22** In the circuit shown in Fig. P8.22, switches 1 and 2 operate together—that is, they either open or close at the same time. The switches are closed a long time before opening at  $t = 0$ .

- How many millijoules of energy have been dissipated in the 12-k $\Omega$  resistor 12 ms after the switches open?
- How long does it take to dissipate 75% of the initially stored energy?

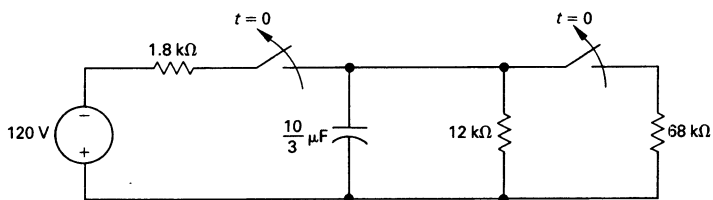


FIGURE P8.22

**8.23** The two switches in the circuit seen in Fig. P8.23 are synchronized. The switches have been closed for a long time before opening at  $t = 0$ .

- How many microseconds after the switches are open is the energy dissipated in the  $10\text{-k}\Omega$  resistor 20% of the initial energy stored in the  $4.4\text{-H}$  inductor?
- At the time calculated in part (a), what percentage of the total energy stored in the inductor has been dissipated?

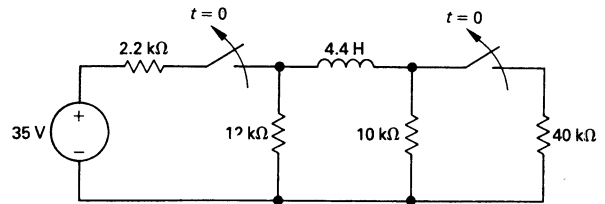


FIGURE P8.23

**8.24** The switch in the circuit in Fig. P8.24 has been in position 1 for a long time before moving to position 2 at  $t = 0$ . Find  $i_o(t)$  for  $t \geq 0^+$ .

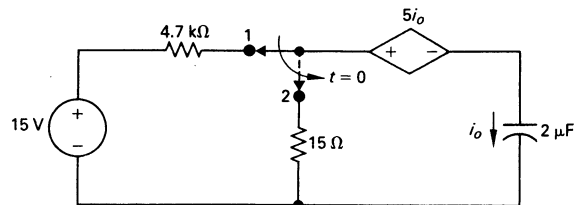


FIGURE P8.24

**8.25** The switch in the circuit seen in Fig. P8.25 has been in position x for a long time. At  $t = 0$  the switch moves instantaneously to position y.

- Find  $\alpha$  so that the time constant for  $t > 0$  is 40 ms.
- For the  $\alpha$  found in part (a), find  $v_\phi$ .

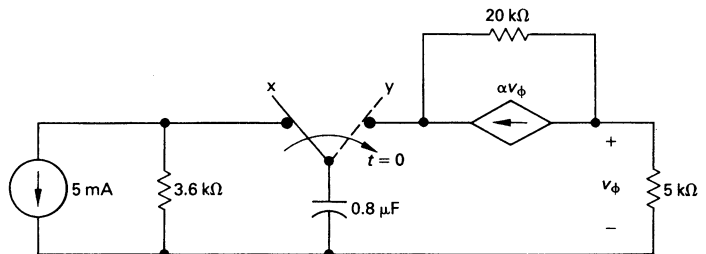


FIGURE P8.25

**8.26** a) In Problem 8.25 how many microjoules of energy are generated by the dependent current source during the time the capacitor discharges to 0 V?

b) Show that for  $t \geq 0$  the total energy stored and generated in the capacitive circuit equals the total energy dissipated.

**8.27** The switch in the circuit in Fig. P8.27 has been in position a for a long time. At  $t = 0$  the switch is thrown to position b.

- Calculate  $i$ ,  $v_1$ , and  $v_2$  for  $t \geq 0^+$ .
- Calculate the energy stored in the capacitor at  $t = 0$ .
- Calculate the energy trapped in the circuit and the total energy dissipated in the  $5\text{-k}\Omega$  resistor if the switch remains in position b indefinitely.

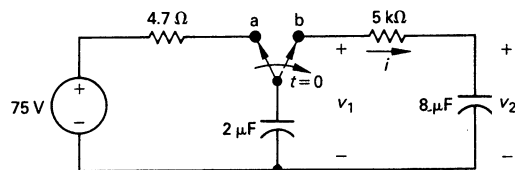


FIGURE P8.27

**8.28** At the time the switch is closed in the circuit shown in Fig. P8.28, the capacitors are charged as shown.

- Find  $v_o(t)$  for  $t \geq 0^+$ .
- What percentage of the total energy initially stored in the three capacitors is dissipated in the  $50\text{-k}\Omega$  resistor?
- Find  $v_1(t)$  for  $t \geq 0$ .
- Find  $v_2(t)$  for  $t \geq 0$ .
- Find the energy (microjoules) trapped in the ideal capacitors.

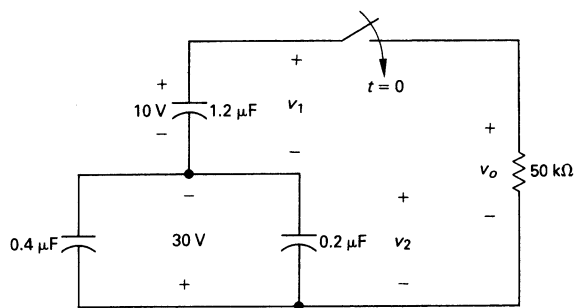


FIGURE P8.28

**8.29** At the time the switch is closed in the circuit in Fig. P8.29 the voltage across the paralleled capacitors is 50 V and the voltage on the  $0.25\text{-}\mu\text{F}$  capacitor is 40 V.

- What percentage of the initial energy stored in the three capacitors is dissipated in the  $24\text{-k}\Omega$  resistor?
- Repeat part (a) for the  $0.4$  and  $16\text{-k}\Omega$  resistors.
- What percentage of the initial energy is trapped in the capacitors?

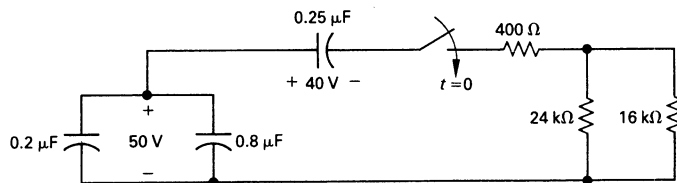


FIGURE P8.29



**8.30** After the circuit in Fig. P8.30 has been in operation for a long time a screwdriver was inadvertently connected across the terminals a, b. Assume the resistance of the screwdriver is negligible.

- Find the current in the screwdriver at  $t = 0^+$  and  $t = \infty$ .
- Derive the expression for the current in the screwdriver for  $t \geq 0^+$ .

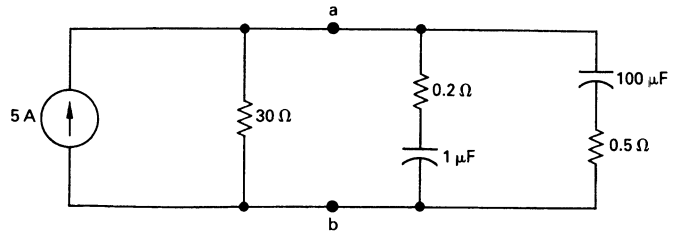


FIGURE P8.30

**8.31** The current and voltage at the terminals of the inductor in the circuit in Fig. 8.16 are

$$i(t) = (25 - 25e^{-400t}) \text{ A}, \quad t \geq 0$$

$$v(t) = 100e^{-400t} \text{ V}, \quad t \geq 0^+.$$

- Specify the numerical values of  $V_s$ ,  $R$ , and  $L$ .
- How many milliseconds after the switch has been closed does the energy stored in the inductor reach 25% of its final value?

**8.32** The switch in the circuit shown in Fig. P8.32 has been in position a for a long time. At  $t = 0$  the switch moves instantaneously to position b.

- Find the numerical expression for  $i_o(t)$  when  $t \geq 0$ .
- Find the numerical expression for  $v_o(t)$  for  $t \geq 0^+$ .

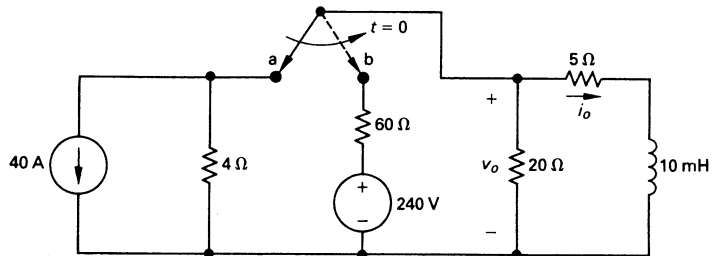


FIGURE P8.32

**8.33** The switch in the circuit shown in Fig. P8.33 has been closed for a long time before opening at  $t = 0$ .

- Find the numerical expressions for  $i_L(t)$  and  $v_o(t)$  for  $t \geq 0$ .
- Find the numerical values of  $v_L(0^+)$  and  $v_o(0^+)$ .

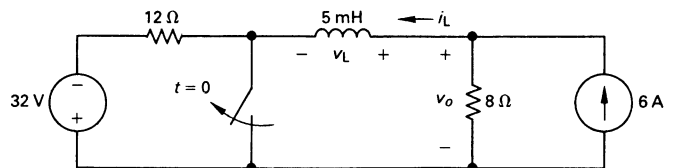


FIGURE P8.33

- 8.34** The switch in the circuit seen in Fig. P8.34 has been closed for a long time. The switch opens at  $t = 0$ . Find the numerical expressions for  $i_o(t)$  and  $v_o(t)$  when  $t \geq 0^+$ .

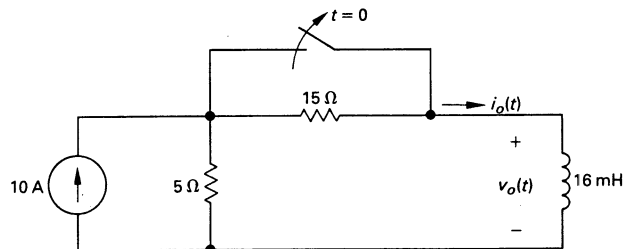


FIGURE P8.34

- 8.35** The switch in the circuit shown in Fig. P8.35 has been closed for a long time. The switch opens at  $t = 0$ . For  $t \geq 0^+$ ,
- find  $v_o(t)$  as a function of  $I_g$ ,  $R_1$ ,  $R_2$ , and  $L$ ;
  - verify your expression by using it to find  $v_o(t)$  in the circuit of Fig. P8.34;
  - explain what happens to  $v_o(t)$  as  $R_2$  gets larger and larger;
  - find  $v_{sw}$  as a function of  $I_g$ ,  $R_1$ ,  $R_2$ , and  $L$ ; and
  - explain what happens to  $v_{sw}$  as  $R_2$  gets larger and larger.

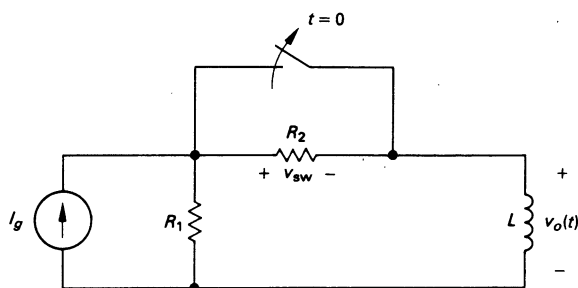


FIGURE P8.35

- 8.36** The switch in the circuit in Fig. P8.36 has been closed for a long time. A student abruptly opens the switch and reports to her instructor that when the switch opened, an electric arc with noticeable persistence was established across the switch and at the same time the voltmeter placed across the coil was damaged. On the basis of your analysis of the circuit in Problem 8.35, can you explain to the student why this happened?

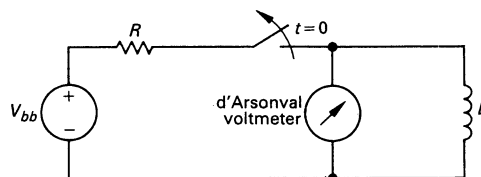


FIGURE P8.36

- 8.37 The switch in the circuit in Fig. P8.37 has been open a long time before closing at  $t = 0$ . Find  $i_o(t)$  for  $t \geq 0$ .

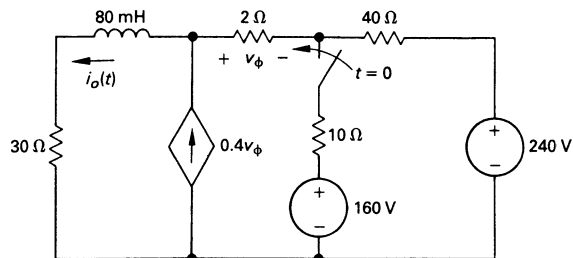


FIGURE P8.37

- 8.38 The “make-before-break” switch in the circuit of Fig. P8.38 has been in position a for a long time. At  $t = 0$  the switch moves instantaneously to position b. Find

- $v_o(t)$ ,  $t \geq 0^+$ ,
- $i_1(t)$ ,  $t \geq 0$ , and
- $i_2(t)$ ,  $t \geq 0$ .

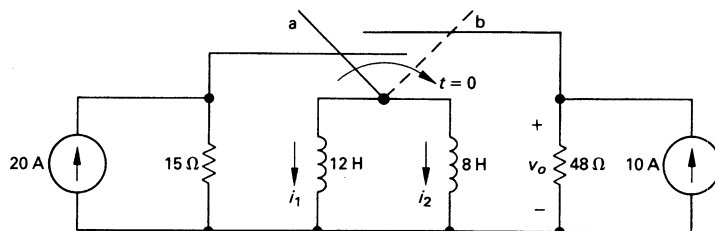


FIGURE P8.38

- 8.39 The switch in the circuit in Fig. P8.39 has been open a long time before closing at  $t = 0$ . Find  $v_o(t)$  for  $t \geq 0^+$ .

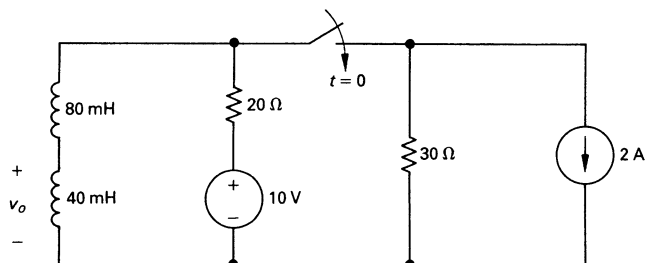


FIGURE P8.39

- 8.40 There is no energy stored in the inductors  $L_1$  and  $L_2$  at the time the switch is opened in the circuit shown in Fig. P8.40.

- Derive the expressions for the currents  $i_1(t)$  and  $i_2(t)$  for  $t \geq 0$ .
- Use the expressions derived in part (a) to find  $i_1(\infty)$  and  $i_2(\infty)$ .

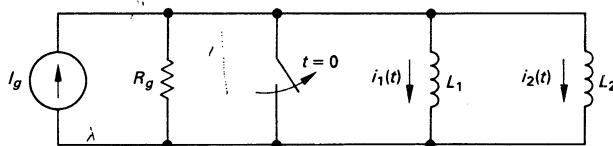


FIGURE P8.40

**8.41** The current and voltage at the terminals of the capacitor in the circuit in Fig. 8.21 are

$$i(t) = 25e^{-500t} \text{ mA} \quad (t \geq 0^+),$$

$$v(t) = (200 - 200e^{-500t}) \text{ V} \quad (t \geq 0).$$

- a) Specify the numerical values of  $I_s$ ,  $R$ ,  $C$ , and  $\tau$ .
- b) How many milliseconds after the switch has been closed does the energy stored in the capacitor reach 36% of its final value?

**8.42** The switch in the circuit shown in Fig. P8.42 has been closed a long time before opening at  $t = 0$ . For  $t \geq 0^+$  find:

- a)  $v_o(t)$ ;  
 b)  $i_o(t)$ ; c)  $i_1(t)$ ;  
 d)  $i_2(t)$ ; and  
 e)  $i_1(0^+)$ .

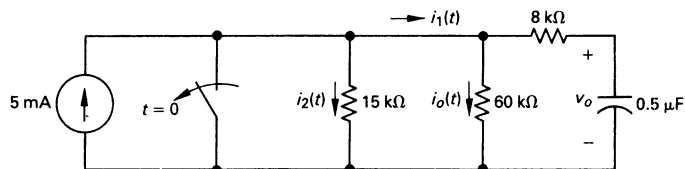


FIGURE P8.42

**8.43** The switch in the circuit seen in Fig. P8.43 has been in position a for a long time. At  $t = 0$  the switch moves instantaneously to position b. For  $t \geq 0^+$  find:

- a)  $v_o(t)$ ;  
 b)  $i_o(t)$ ;  
 c)  $v_g(t)$ ; and  
 d)  $v_g(0^+)$ .

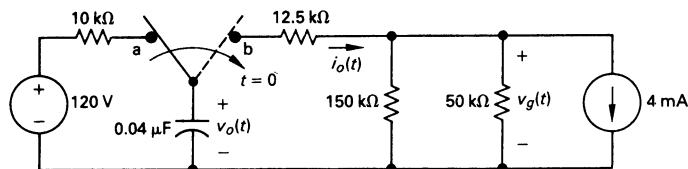


FIGURE P8.43

**8.44** The switch in the circuit shown in Fig. P8.44 has been closed a long time before opening at  $t = 0$ .

- a) What is the initial value of  $i_o(t)$ ?
- b) What is the final value of  $i_o(t)$ ?
- c) What is the time constant of the circuit for  $t \geq 0$ ?
- d) What is the numerical expression for  $i_o(t)$  when  $t \geq 0^+$ ?
- e) What is the numerical expression for  $v_o(t)$  when  $t \geq 0^+$ ?

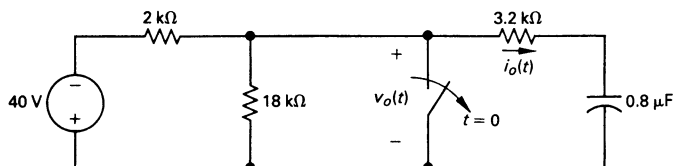


FIGURE P8.44

- 8.45 The switch in the circuit seen in Fig. P8.45 has been in position a for a long time. At  $t = 0$  the switch moves instantaneously to position b. Find  $v_o(t)$  and  $i_o(t)$  for  $t \geq 0^+$ .

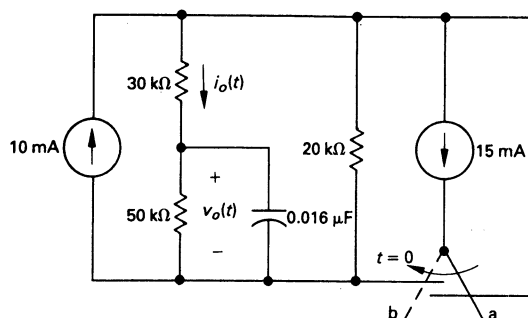


FIGURE P8.45

- 8.46 The switch in the circuit shown in Fig. P8.46 has been in the OFF position for a long time. At  $t = 0$  the switch moves instantaneously to the ON position. Find  $v_o(t)$  for  $t \geq 0$ .

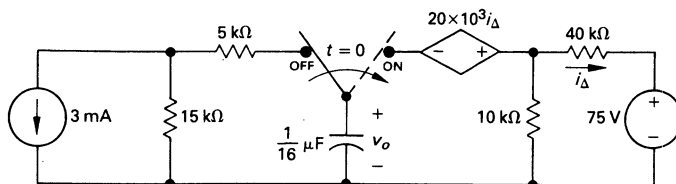


FIGURE P8.46

- 8.47 Assume that the switch in the circuit of Fig. P8.46 has been in the ON position for a long

time before switching instantaneously to the OFF position at  $t = 0$ . Find  $v_o(t)$  for  $t \geq 0$ .

- 8.48 At  $t = 0$  the voltage source in the circuit seen in Fig. P8.48 drops instantaneously from 100 to 25 V. At the same instant, the current source reverses direction. Find  $v_o(t)$  for  $t \geq 0$ .

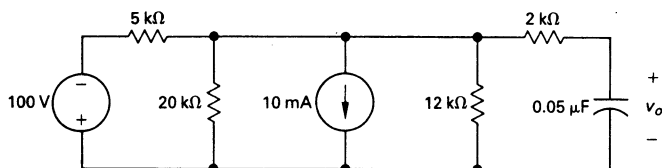


FIGURE P8.48

- 8.49 There is no energy stored in the capacitors  $C_1$  and  $C_2$  at the time the switch is closed in the circuit seen in Fig. P8.49.

- Derive the expressions for  $v_1(t)$  and  $v_2(t)$  for  $t \geq 0$ .
- Use the expressions derived in part (a) to find  $v_1(\infty)$  and  $v_2(\infty)$ .

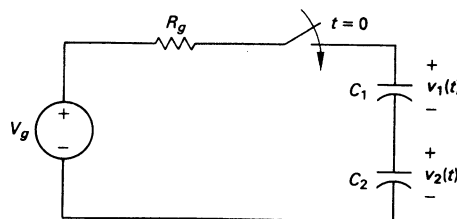


FIGURE P8.49

**8.50** The switch in the circuit of Fig. P8.50 has been in position a for a long time. At  $t = 0$  it moves instantaneously to position b. For  $t \geq 0^+$  find

- $v_o(t)$ ,
- $i_o(t)$ ,
- $v_1(t)$ ,
- $v_2(t)$ ,
- the energy trapped in the capacitors as  $t \rightarrow \infty$ .

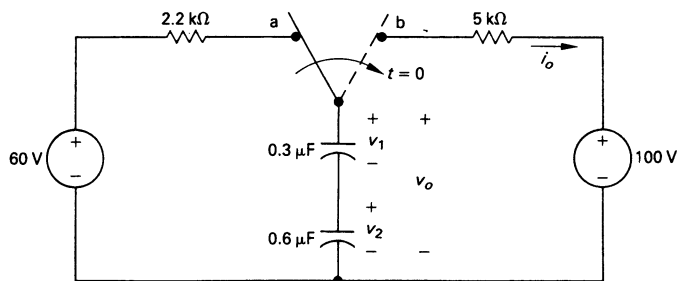


FIGURE P8.50

**8.51** The switch in the circuit shown in Fig. P8.51 opens at  $t = 0$  after being closed for a long time. How many milliseconds after the switch opens is the energy stored in the capacitor 36% of its final value?

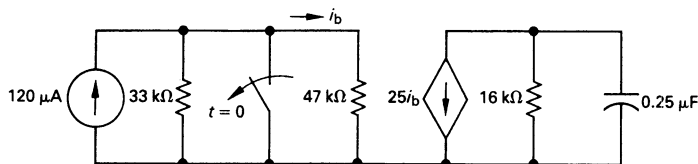


FIGURE P8.51

**8.52** The switch in the circuit in Fig. P8.52 has been open a long time before closing at  $t = 0$ . Find  $v_o(t)$  for  $t \geq 0^+$ .

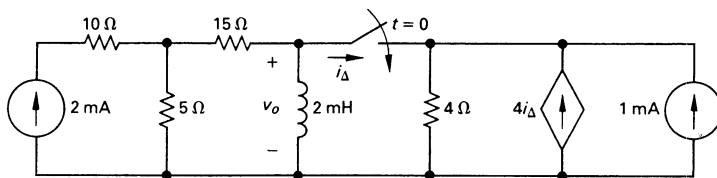


FIGURE P8.52

**8.53** The switch in the circuit shown in Fig. P8.53 has been in position a for a long time. At  $t = 0$  the switch is moved to position b, where it remains for  $800 \mu\text{s}$ . The switch is then moved to position c, where it remains indefinitely.

- Find  $i(0^+)$ .
- Find  $i(300 \mu\text{s})$ .
- Find  $i(1 \text{ ms})$ .
- Find  $v(800^- \mu\text{s})$ .
- Find  $v(800^+ \mu\text{s})$ .

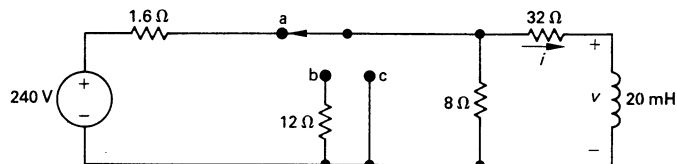


FIGURE P8.53

- 8.54** In the circuit in Fig. P8.54 switch A has been open and switch B has been closed for a long time. At  $t = 0$  switch A closes. One second after switch A closes switch B opens. Find  $i_L(t)$  for  $t \geq 0$ .

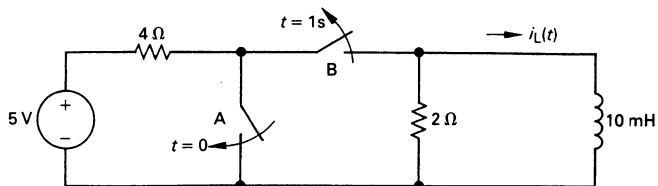


FIGURE P8.54

- 8.55** The action of the two switches in the circuit seen in Fig. P8.55 is as follows. For  $t < 0$ , switch 1 is in position a and switch 2 is open. This state has existed for a long time. At  $t = 0$ , switch 1 moves instantaneously from position a to position b while switch 2 remains open. Ten milliseconds after switch 1 operates switch 2 closes, remains closed for 10 ms, and then opens. Find  $v_o(t)$  25 ms after switch 1 moves to position b.

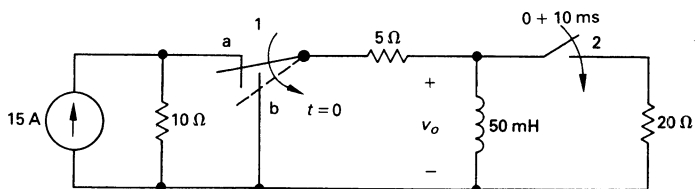


FIGURE P8.55

- 8.56** For the circuit in Fig. P8.55, how many milliseconds after switch 1 moves to position b is

the energy stored in the inductor 4% of its initial value?

- 8.57** The switch in the circuit in Fig. P8.57 has been in position a for a long time. At  $t = 0$  it moves instantaneously to position b where it remains for 5 s before moving instantaneously to position c. Find  $v_o$  for  $t \geq 0$ .

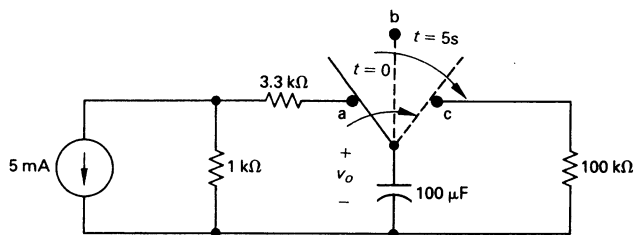


FIGURE P8.57

- 8.58** There is no energy stored in the capacitor in the circuit in Fig. P8.58 when switch 1 closes at  $t = 0$ . Three microseconds later switch 2 closes. Find  $v_o(t)$  for  $t \geq 0$ .

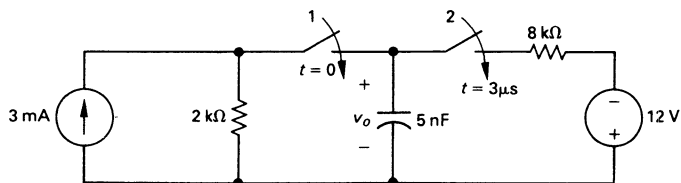


FIGURE P8.58

- 8.59** The capacitor in the circuit seen in Fig. P8.59 has been charged to 300 V. At  $t = 0$ , switch 1 closes, causing the capacitor to discharge into the resistive network. Switch 2 closes  $200 \mu\text{s}$  after switch 1 closes. Find the magnitude and direction of the current in the second switch  $300 \mu\text{s}$  after switch 1 closes.

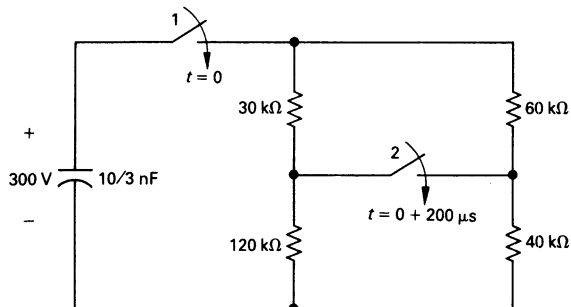


FIGURE P8.59

- 8.60** In the circuit in Fig. P8.60 switch 1 has been in position a, and switch 2 has been closed for a long time. At  $t = 0$ , switch 1 moves instantaneously to position b. Eight hundred microseconds later switch 2 opens, remains open for  $300 \mu\text{s}$  and then recloses. Find  $v_o$  1.5 ms after switch 1 makes contact with terminal b.

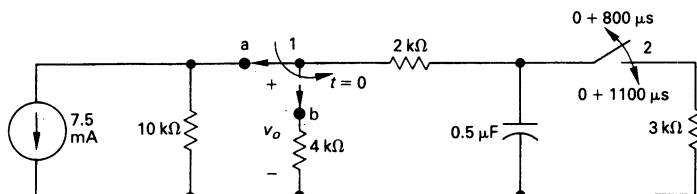


FIGURE P8.60

- 8.61** For the circuit in Fig. P8.60 what percentage of the initial energy stored in the  $0.5\text{-}\mu\text{F}$  capacitor is dissipated in the  $3\text{-k}\Omega$  resistor?

- 8.62** The voltage waveform shown in Fig. P8.62(a) is applied to the circuit of Fig. P8.62(b). The initial voltage on the capacitor is zero.

- Calculate  $v_o(t)$ .
- Make a sketch of  $v_o(t)$  versus  $t$ .

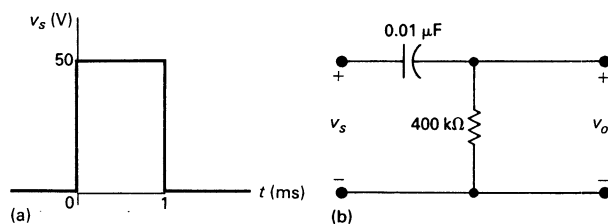


FIGURE P8.62



**8.63** The voltage waveform shown in Fig. P8.63(a) is applied to the circuit of Fig. P8.63(b). The initial current in the inductor is zero.

- Calculate  $v_o(t)$ .
- Make a sketch of  $v_o(t)$  versus  $t$ .

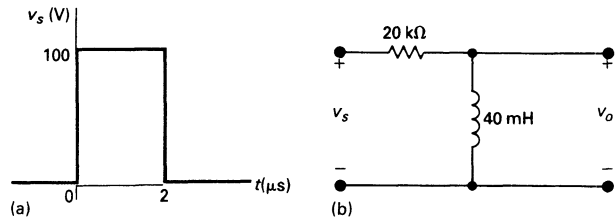


FIGURE P8.63

**8.64** The current source in the circuit in Fig. P8.64(a) generates the current pulse shown in Fig. P8.64(b). There is no energy stored at  $t = 0$ .

- Derive the numerical expressions for  $v_o(t)$  for the time intervals  $t < 0$ ,  $0 < t < 75 \mu\text{s}$ , and  $75 \mu\text{s} < t < \infty$ .
- Calculate  $v_o(75^- \mu\text{s})$  and  $v_o(75^+ \mu\text{s})$ .
- Calculate  $i_o(75^- \mu\text{s})$  and  $i_o(75^+ \mu\text{s})$ .

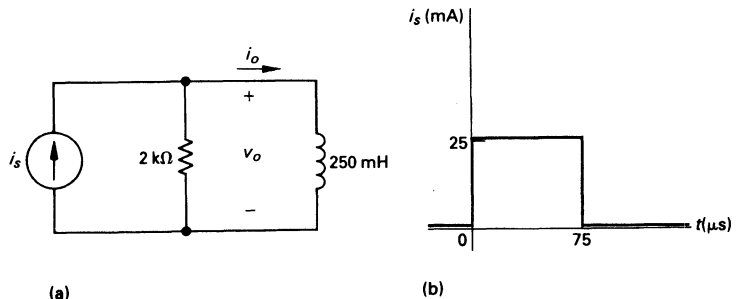


FIGURE P8.64

**8.65** The current source in the circuit in Fig. P8.65(a) generates the current pulse shown in Fig. P8.65(b). There is no energy stored at  $t = 0$ .

- Derive the expressions for  $i_o(t)$  and  $v_o(t)$  for the time intervals  $t < 0$ ;  $0 < t < 0.002 \text{ s}$ ; and  $0.002 \text{ s} < t < \infty$ .
- Calculate  $i_o(0^-)$ ;  $i_o(0^+)$ ;  $i_o(0.002^-)$ ; and  $i_o(0.002^+)$ .
- Calculate  $v_o(0^-)$ ;  $v_o(0^+)$ ;  $v_o(0.002^-)$ ; and  $v_o(0.002^+)$ .
- Sketch  $i_o(t)$  versus  $t$  for the interval  $-1 \text{ ms} < t < 4 \text{ ms}$ .
- Sketch  $v_o(t)$  versus  $t$  for the interval  $-1 \text{ ms} < t < 4 \text{ ms}$ .

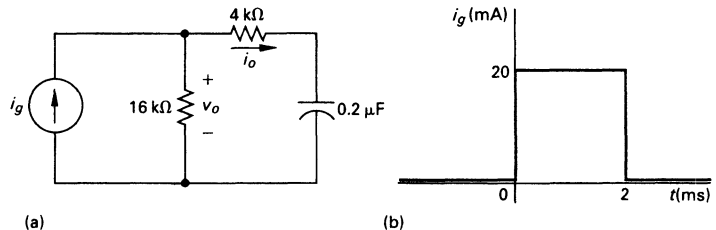


FIGURE P8.65

**8.66** The voltage signal source in the circuit in Fig. P8.66(a) is generating the signal shown in Fig. P8.66(b). There is no stored energy at  $t = 0$ .

- a) Derive the expressions for  $v_o(t)$  that apply in the intervals  $t < 0$ ;  $0 \leq t \leq 4$  ms;  $4 \text{ ms} \leq t \leq 8$  ms; and  $8 \text{ ms} \leq t \leq \infty$ .

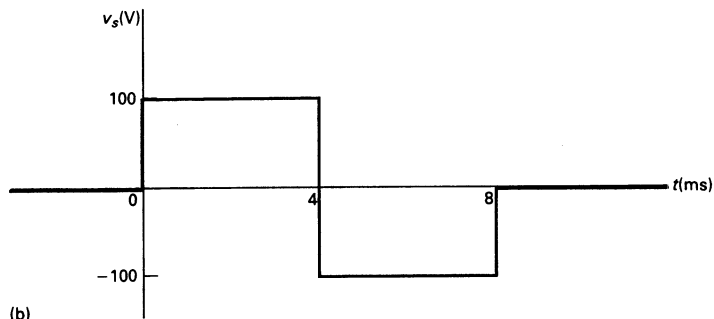
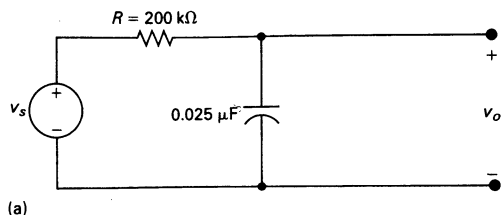


FIGURE P8.66

**8.67** The circuit shown in Fig. P8.67 is used to close the switch between a and b for a predetermined length of time. The electric relay holds its contact arms down so long as the voltage across the relay coil exceeds 5 V. When the coil voltage equals 5 V, the relay contacts return to their initial position by a mechanical spring action. The switch between a and b is initially closed by momentarily pressing the push button. Assume that the capacitor is fully charged when the push button is first pushed down. The resistance of the relay coil is  $25 \text{ k}\Omega$ , and the inductance of the coil is negligible.

- a) How long will the switch between a and b remain closed?
- b) Write the numerical expression for  $i$  from the time when the relay contacts first close to the time when the capacitor is completely charged.

- c) How many milliseconds (after the circuit between a and b is interrupted) does it take the capacitor to reach 85% of its final value?

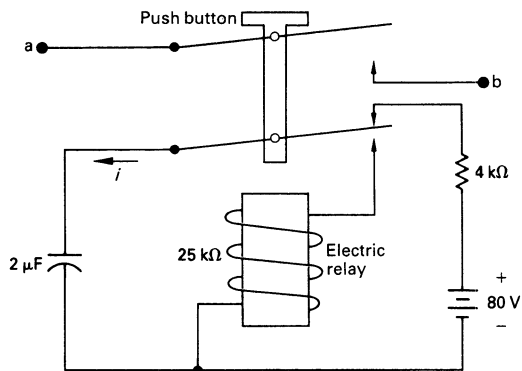


FIGURE P8.67

**8.68** In the circuit of Fig. P8.68, the lamp starts to conduct whenever the lamp voltage reaches 15 V. During the time when the lamp conducts, it can be modeled as a  $10\text{-k}\Omega$  resistor. Once the lamp conducts, it will continue to conduct until the lamp voltage drops to 5 V. When the lamp

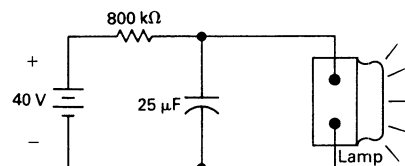


FIGURE P8.68

is not conducting, it appears as an open circuit. Assume that the circuit has been in operation for a long time. Let  $t = 0$  at the instant when the lamp stops conducting.

- a) Derive the expression for the voltage across the lamp for one full cycle of operation.

- b) How many times per minute will the lamp turn on?
- c) The  $800\text{-k}\Omega$  resistor is replaced with a variable resistor  $R$ . The resistance is adjusted until the lamp “flashes” 12 times per minute. What is the value of  $R$ ?

- 8.69 The capacitor in the circuit shown in Fig. P8.69 is charged to  $20\text{ V}$  at the time the switch is closed. If the capacitor ruptures when its terminal voltage equals or exceeds  $20\text{ kV}$ , how long does it take to rupture the capacitor?

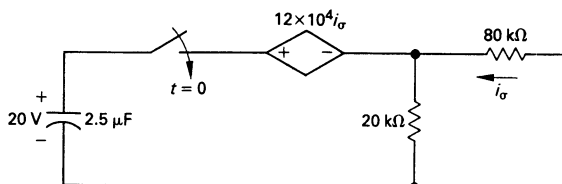


FIGURE P8.69

- 8.70 The inductor current in the circuit in Fig. P8.70 is  $25\text{ mA}$  at the instant the switch is opened. The inductor will malfunction whenever the magnitude of the inductor current equals or exceeds  $5\text{ A}$ . How long after the switch is opened does the inductor malfunction?

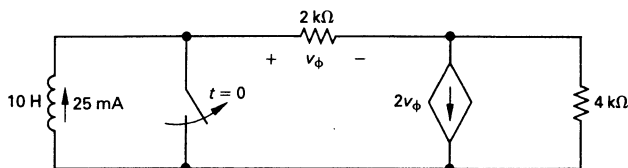


FIGURE P8.70

- 8.71 The gap in the circuit seen in Fig. P8.71 will arc over whenever the voltage across the gap reaches  $36\text{ kV}$ . The initial current in the inductor is zero. The value of  $\beta$  is adjusted so that the Thévenin resistance with respect to the terminals of the inductor is  $-3\text{ k}\Omega$ . How many milliseconds after the switch has been closed will the gap arc over?

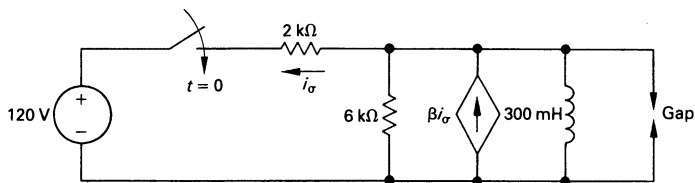


FIGURE P8.71

- 8.72 The switch in the circuit in Fig. P8.72 has been closed for a long time. The maximum voltage rating of the  $1.6\text{ }\mu\text{F}$  capacitor is  $14,400\text{ V}$ . How long after the switch is opened does the voltage across the capacitor reach the maximum voltage rating?

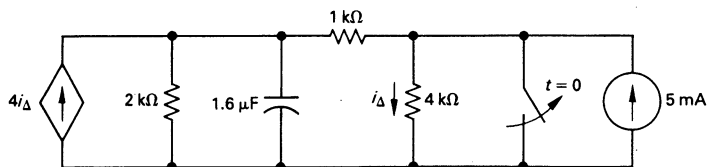


FIGURE P8.72

- 8.73** The energy stored in the capacitor in the circuit shown in Fig. P8.73 is zero at the instant the switch is closed. The ideal operational amplifier reaches saturation in 15 ms. What is the numerical value of  $R$  in kilohms?

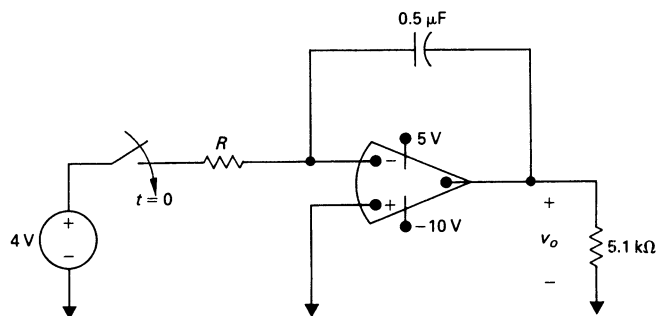


FIGURE P8.73

- 8.74** At the instant the switch is closed in the circuit of Fig. P8.73, the capacitor is charged to 6 V, positive at the left-hand terminal. If the ideal

operational amplifier saturates in 40 ms, what is the value of  $R$ ?

- 8.75** There is no energy stored in the capacitor at the time the switch in the circuit of Fig. P8.75 makes contact with terminal a. The switch remains at position a for 32 ms and then moves instantaneously to position b. How many milliseconds after making contact with terminal a does the op amp saturate?

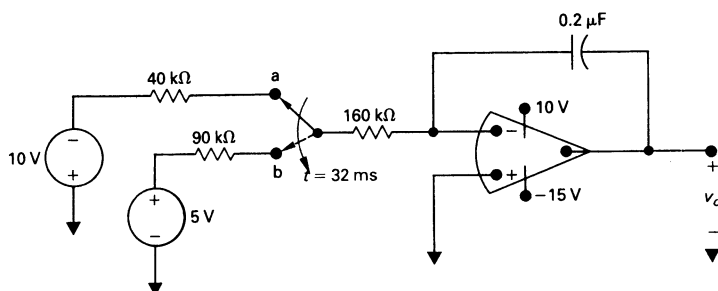


FIGURE P8.75

- 8.76** At the instant the switch of Fig. P8.76 is closed, the voltage on the capacitor is 56 V. Assume an ideal operational amplifier. How many milliseconds after the switch is closed will the output voltage  $v_o$  equal zero?

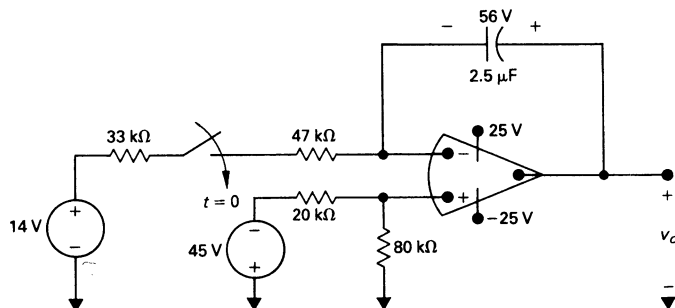


FIGURE P8.76

- 8.77 a) When the switch closes in the circuit seen in Fig. P8.77, there is no energy stored in the capacitor. How long does it take to saturate the op amp?
- b) Repeat part (a) with an initial voltage on the capacitor of 1.0 V, positive at the upper terminal.

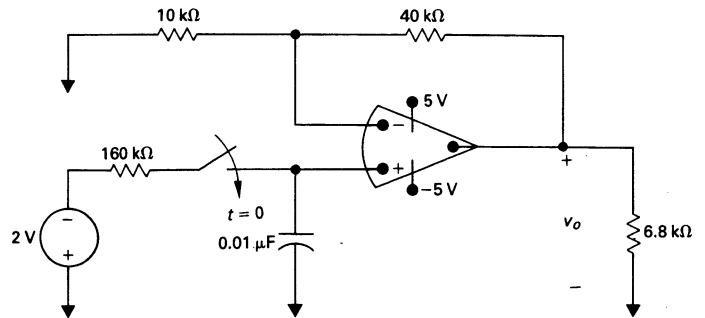


FIGURE P8.77

- 8.78 There is no energy stored in the capacitors in the circuit shown in Fig. P8.78 at the instant the two switches close.

- a) Find  $v_o$  as a function of  $v_a$ ,  $v_b$ ,  $R$ , and  $C$ .
- b) On the basis of the result obtained in part (a), describe the operation of the circuit.
- c) How long will it take to saturate the amplifier if  $v_a = 10$  mV;  $v_b = 60$  mV;  $R = 40$  kΩ;  $C = 25$  nF; and  $V_{CC} = 12$  V?

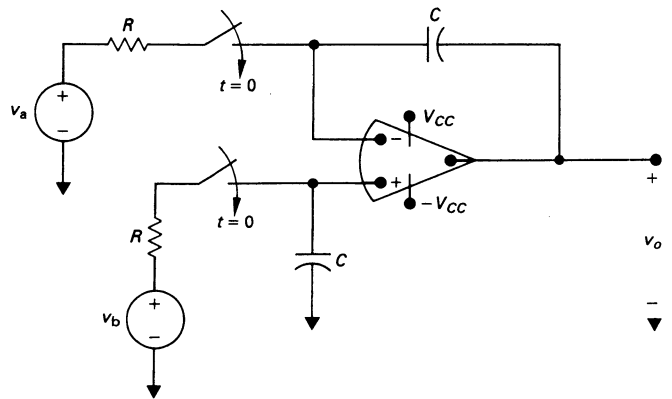


FIGURE P8.78

- 8.79 At the time the double-pole switch in the circuit shown in Fig. P8.79 is closed, the initial voltages on the capacitors are 12 and 4 V, as shown. Find the numerical expressions for  $v_o(t)$ ,  $v_2(t)$ , and  $v_f(t)$  that are applicable as long as the ideal op amp operates in its linear range.

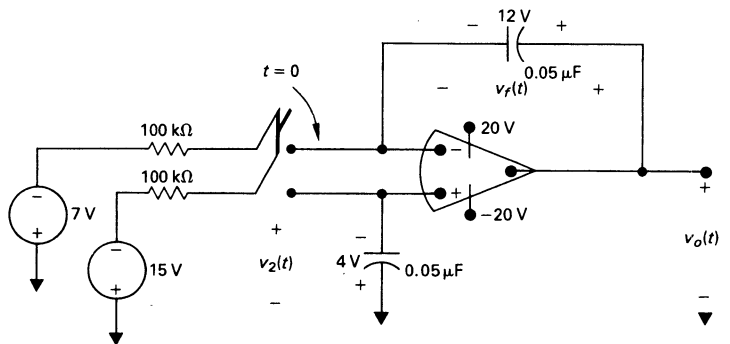


FIGURE P8.79

- 8.80** There is no charge on the capacitor in the circuit of Fig. P8.80 when the switch makes contact with terminal a. The switch remains at terminal a for 20 ms and then moves instantaneously to terminal b. The switch then remains at terminal b. Derive the equations that describe  $v_o(t)$  when the op amp operates in its linear range.

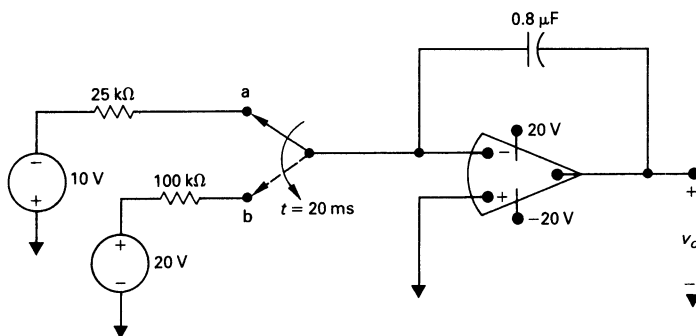


FIGURE P8.80

- 8.81** The capacitor in the circuit seen in Fig. P8.81 has an initial voltage of 5 V at the instant the switch is closed. How many milliseconds after the switch is closed does the op amp saturate?

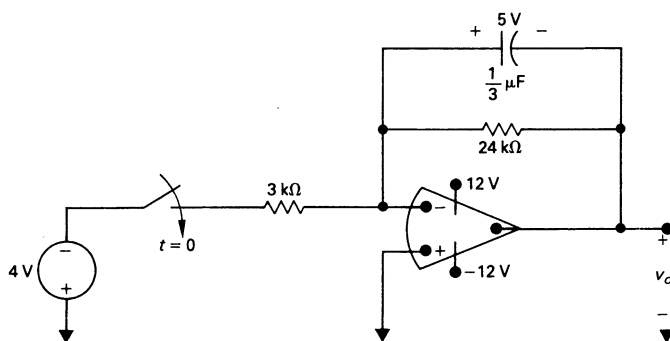


FIGURE P8.81

- 8.82** The voltage pulse shown in Fig. P8.82(a) is applied to the ideal integrating amplifier shown in Fig. P8.82(b). Derive the numerical expressions

for  $v_o(t)$  for the time intervals (a)  $t < 0$ ; (b)  $0 \leq t \leq 250$  ms; (c)  $250 \text{ ms} \leq t \leq 500$  ms; and (d)  $500 \text{ ms} \leq t \leq \infty$  when  $v_o(0) = 0$ .

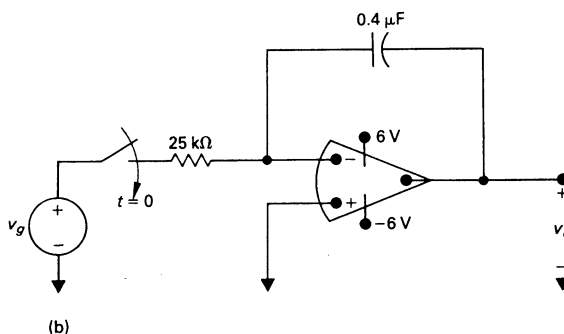
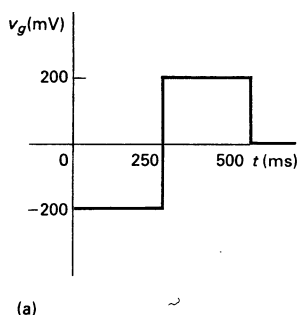


FIGURE P8.82

**8.83** Repeat Problem 8.82 with a 5-M $\Omega$  resistor placed across the 0.4- $\mu$ F feedback capacitor.

- 8.84** a) At the time the switch makes contact with terminal a in the circuit seen in Fig. P8.84 the charge on the capacitor is zero. The switch remains at terminal a for 2 ms and then moves instantaneously to terminal b. How long after the switch makes contact with terminal a is the output voltage zero?
- b) If the switch remains in contact with terminal b, how many milliseconds after making contact with terminal b does the op amp saturate?

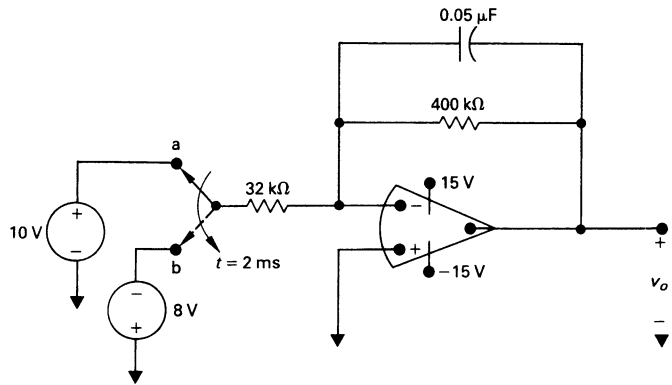
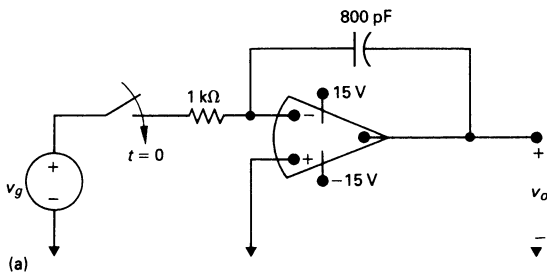


FIGURE P8.84

- 8.85** The voltage source in the circuit in Fig. P8.85(a) is generating the triangular waveform shown in Fig. P8.85(b). Assume the energy stored in the capacitor is zero at  $t = 0$ .
- a) Derive the numerical expressions for  $v_o(t)$  for the following time intervals:  $0 \leq t \leq 1 \mu\text{s}$ ;  $1 \mu\text{s} \leq t \leq 3 \mu\text{s}$ ; and  $3 \mu\text{s} \leq t \leq 4 \mu\text{s}$ .



- b) Sketch the output waveform between 0 and 4  $\mu\text{s}$ .
- c) If the triangular input voltage continues to repeat itself for  $t > 4 \mu\text{s}$  what would you expect the output voltage to be? Explain.

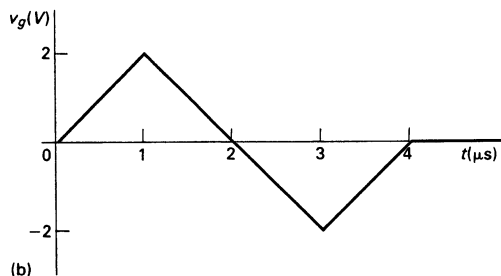


FIGURE P8.85

**8.86** The circuit shown in Fig. P8.86 is known as an *astable multivibrator* and finds wide application in pulse circuits. The purpose of this problem is to relate the charging and discharging of the capacitors to the operation of the circuit. The key to analyzing the circuit is to understand the be-

havior of the ideal transistor switches  $T_1$  and  $T_2$ . The circuit is designed so that the switches automatically alternate between ON and OFF. When  $T_1$  is OFF,  $T_2$  is ON and vice versa. Thus in the analysis of this circuit we assume a switch is either ON or OFF. We also assume that the ideal  
(continued)

transistor switch can change its state instantaneously. In other words, it can snap from OFF to ON and vice versa.

When a transistor switch is ON, (1) the base current  $i_b$  is greater than zero, (2) the terminal voltage  $v_{be}$  is zero, and (3) the terminal voltage  $v_{ce}$  is zero. Thus when a transistor switch is ON, it presents a short circuit between the terminals b, e and c, e.

When a transistor switch is OFF, (1) the terminal voltage  $v_{be}$  is negative, (2) the base current is zero, and (3) there is an open circuit between the terminals c, e. Thus when a transistor switch is OFF, it presents an open circuit between the terminal sets b, e and c, e.

Assume that  $T_2$  has been ON and has just snapped OFF, while  $T_1$  has been OFF and has just snapped ON. You may assume that at this instance  $C_2$  is charged to the supply voltage  $V_{CC}$  and the charge on  $C_1$  is zero. Also assume  $C_1 = C_2$  and  $R_1 = R_2 = 10R_L$ .

- Derive the expression for  $v_{be2}$  during the interval  $T_2$  is OFF.
- Derive the expression for  $v_{ce2}$  during the interval  $T_2$  is OFF.
- Find the length of time  $T_2$  is OFF.

- Find the value of  $v_{ce2}$  at the end of the interval that  $T_2$  is OFF.
- Derive the expression for  $i_{b1}$  during the interval  $T_2$  is OFF.
- Find the value of  $i_{b1}$  at the end of the interval that  $T_2$  is OFF.
- Sketch  $v_{ce2}$  versus  $t$  during the interval  $T_2$  is OFF.
- Sketch  $i_{b1}$  versus  $t$  during the interval  $T_2$  is OFF.

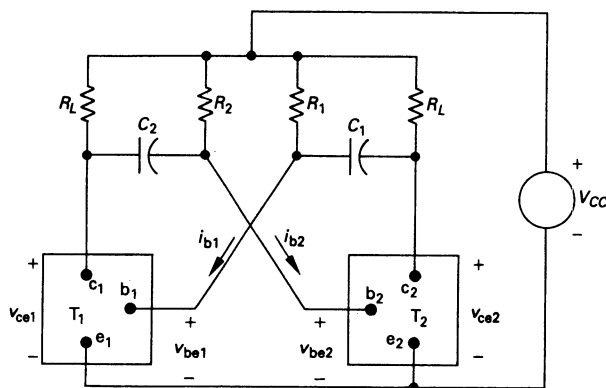


FIGURE P8.86

- 8.87** The component values in the circuit of Fig. P8.86 are  $V_{CC} = 10$  V;  $R_L = 1$  k $\Omega$ ;  $C_1 = C_2 = 1$  nF; and  $R_1 = R_2 = 14.43$  k $\Omega$ .
- How long is  $T_2$  in the OFF state during one cycle of operation?
  - How long is  $T_2$  in the ON state during one cycle of operation?
  - Repeat part (a) for  $T_1$ .

- Repeat part (b) for  $T_1$ .
- At the first instant after  $T_1$  turns ON, what is the value of  $i_{b1}$ ?
- At the instant just before  $T_1$  turns OFF, what is the value of  $i_{b1}$ ?
- What is the value of  $v_{ce2}$  at the instant just before  $T_2$  turns ON?

- 8.88** Repeat Problem 8.87 with  $C_1 = 1$  nF and  $C_2 = 0.8$  nF. All other component values are unchanged.

- 8.89** An astable multivibrator circuit is to satisfy the following criteria: (1) One transistor switch is to be ON for 48  $\mu$ s and OFF for 36  $\mu$ s for each cycle; (2)  $R_L = 2$  k $\Omega$ ; (3)  $V_{CC} = 5$  V; (4)  $R_1 = R_2$ ; and (5)  $6R_L \leq R_1 \leq 50 R_L$ . What are the limiting values for the capacitors  $C_1$  and  $C_2$ ?



**8.90** The circuit shown in Fig. P8.90 is known as a *monostable multivibrator*. The adjective “monostable” is used to describe the fact that the circuit has one stable state. That is, if left alone the electronic switch  $T_2$  will be ON and  $T_1$  will be OFF. (The operation of the ideal transistor switch is described in Problem 8.86.)  $T_2$  can be turned OFF by momentarily closing the switch  $S$ . After  $S$  returns to its open position,  $T_2$  will return to its ON state.

- Show that if  $T_2$  is ON,  $T_1$  is OFF and will stay OFF.
- Explain why  $T_2$  is turned OFF when  $S$  is momentarily closed.
- Show that  $T_2$  will stay OFF for  $RC \ln 2$  s.

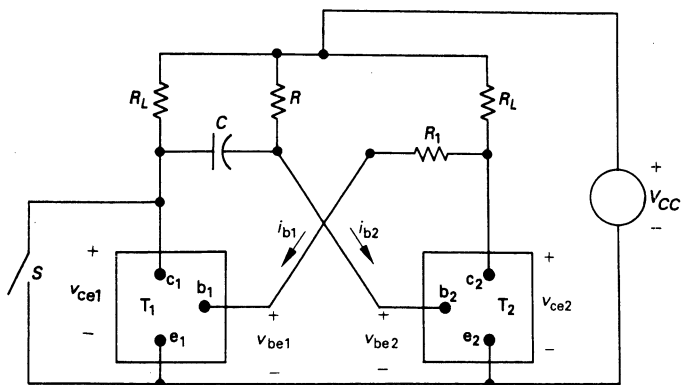


FIGURE P8.90

**8.91** The parameter values in the circuit in Fig. P8.90 are  $V_{CC} = 6$  V;  $R_1 = 5.0$  k $\Omega$ ;  $R_L = 20$  k $\Omega$ ;  $C = 250$  pF; and  $R = 23,083$   $\Omega$ .

- Sketch  $v_{ce2}$  versus  $t$  assuming that after  $S$  is momentarily closed it remains open until the

circuit has reached its stable state. Assume  $S$  is closed at  $t = 0$ . Sketch  $v_{ce2}$  versus  $t$  for the interval  $-5 \leq t \leq 10$   $\mu$ s.

- Repeat part (a) for  $i_{b2}$  versus  $t$ .

and

$$x(t) = x_F + (D_1' t + D_2') e^{-\alpha t} \quad (\text{cd}),$$

where  $x_F$  is the final value of the desired current or voltage response.

- The unknown coefficients (i.e., the  $A$ 's,  $B$ 's, and  $D$ 's) are obtained by evaluating the initial value of the desired response  $[x(0)]$  and the initial value of the first derivative of the desired response  $[dx(0)/dt]$ .

The terms overdamped, underdamped, and critically damped describe the impact of the dissipative element ( $R$ ) on the response. The neper frequency or damping factor  $\alpha$  reflects the effect of  $R$ . The following characteristics define the three types of damping.

- Overdamped: When  $\alpha$  is large compared to the resonant frequency  $\omega_0$ , the voltage or current approaches its final value without oscillation.
- Underdamped: When  $\alpha$  is small compared to  $\omega_0$ , the response oscillates about its final value. The smaller  $\alpha$ 's value, the longer the oscillation persists. If the dissipative element is removed from the circuit,  $\alpha = 0$ , and the response becomes a sustained oscillation.
- Critically damped. When  $\alpha = \omega_0$  the response is on the verge of oscillating.

When two integrating amplifiers with ideal op amps are connected in cascade the output voltage of the second integrator is related to the input voltage of the first integrator by an ordinary, second-order differential equation. Therefore the techniques developed may be used to analyze the behavior of the cascaded integrators.

## PROBLEMS

- 9.1 The resistance, inductance, and capacitance in a parallel  $RLC$  circuit are  $1000 \, \Omega$ ,  $12.5 \, \text{H}$ , and  $2 \, \mu\text{F}$ , respectively.
- Calculate the roots of the characteristic equation that describe the voltage response of the circuit.
  - Will the response be over-, under-, or critically damped?
  - What value of  $R$  will yield a damped frequency of  $120 \, \text{rad/s}$ ?
  - What are the roots of the characteristic equation for the value of  $R$  found in part (c)?
  - What value of  $R$  will result in a critically damped response?

- 9.2** The initial voltage on the  $0.1\ \mu\text{F}$  capacitor in the circuit shown in Fig. 9.1 is  $24\ \text{V}$ . The initial current in the inductor is zero. The voltage response for  $t \geq 0$  is

$$v(t) = -8e^{-250t} + 32e^{-1000t}\ \text{V}.$$

- Determine the numerical values of  $R$ ,  $L$ ,  $\alpha$ , and  $\omega_0$ .
- Calculate  $i_R(t)$ ,  $i_L(t)$ , and  $i_C(t)$  for  $t \geq 0^+$ .

- 9.3** The circuit elements in the circuit in Fig. 9.1 are  $R = 20\ \text{k}\Omega$ ,  $C = 0.02\ \mu\text{F}$ , and  $L = 50\ \text{H}$ . The initial inductor current is  $1.2\ \text{mA}$  and the initial capacitor voltage is zero.

- Calculate the initial current in each branch of the circuit.
- Find  $v(t)$  for  $t \geq 0$ .
- Find  $i_L(t)$  for  $t \geq 0$ .

- 9.4** The natural response for the circuit shown in Fig. 9.1 is known to be

$$v = 3(e^{-100t} + e^{-900t})\ \text{V}, \quad t \geq 0.$$

If  $L = 40/9\ \text{H}$  and  $C = 2.5\ \mu\text{F}$ , find  $i_L(0^+)$  in milliamperes.

- 9.5** The natural voltage response of the circuit of Fig. 9.1 is

$$v = 100e^{-20,000t}(\cos 15,000t - 2 \sin 15,000t)\ \text{V}, \quad t \geq 0,$$

when the capacitor is  $0.04\ \mu\text{F}$ . Find (a)  $R$ ; (b)  $L$ ; (c)  $V_0$ ; (d)  $I_0$ ; and (e)  $i_L(t)$ .

- 9.6** The initial value of the voltage  $v$  in the circuit in Fig. 9.1 is zero and the initial value of the capacitor current  $[i_C(0^+)]$  is  $15\ \text{mA}$ . The expression for the capacitor current is known to be

$$i_C(t) = A_1e^{-160t} + A_2e^{-40t}, \quad t \geq 0^+$$

when  $R$  is  $200\ \Omega$ . Find the numerical:

- value of  $\alpha$ ,  $\omega_0$ ,  $L$ ,  $C$ ,  $A_1$ , and  $A_2$
- expression for  $v(t)$ ,  $t \geq 0$
- expression for  $i_R(t) \geq 0$
- expression for  $i_L(t) \geq 0$ .

- 9.7** The voltage response for the circuit in Fig. 9.1 is known to be

$$v(t) = D_1t e^{-500t} + D_2e^{-500t}, \quad t \geq 0.$$

The initial current in the inductor  $[I_0]$  is  $-10\ \text{mA}$  and the initial voltage on the capacitor  $[V_0]$  is  $8\ \text{V}$ . The inductor has an inductance of  $4\ \text{H}$ .

- Find the value of  $R$ ,  $C$ ,  $D_1$ , and  $D_2$ .
- Find  $i_C(t)$  for  $t \geq 0^+$ .

- 9.8** In the circuit in Fig. 9.1,  $R = 12.5\ \Omega$ ,  $L = 50/101\ \text{H}$ ,  $C = 0.08\ \text{F}$ ,  $V_0 = 0\ \text{V}$ , and  $I_0 = -4\ \text{A}$ .

- Find  $v(t)$  for  $t \geq 0$ .
- Find the first three values of  $t$  for which  $dv/dt$  is zero. Let these values of  $t$  be denoted  $t_1$ ,  $t_2$ , and  $t_3$ .
- Show that  $t_3 - t_1 = T_d$ .
- Show that  $t_2 - t_1 = T_d/2$ .
- Calculate  $v(t_1)$ ,  $v(t_2)$ , and  $v(t_3)$ .
- Sketch  $v(t)$  versus  $t$  for  $0 \leq t \leq t_2$ .

- 9.9** a) Find  $v(t)$  for  $t \geq 0$  in the circuit in Problem 9.8 if the  $12.5\text{-}\Omega$  resistor is removed from the circuit.
- Calculate the frequency of  $v(t)$  in hertz.
  - Calculate the maximum amplitude of  $v(t)$  in volts.

**9.10** In the circuit shown in Fig. 9.1 a 12.5-H inductor is shunted by 3.2-nF capacitor, the resistor  $R$  is adjusted for critical damping,  $V_0 = 100$  V and  $I_0 = 6.4$  mA.

- Calculate the numerical value of  $R$ .
- Calculate  $v(t)$  for  $t \geq 0$ .
- Find  $v(t)$  when  $i_c(t) = 0$ .
- What percentage of the initially stored energy remains stored in the circuit at the instant  $i_c(t)$  is 0?

**9.11** The resistor in the circuit in Example 9.4 is changed to  $3200 \Omega$ .

- Find the numerical expression for  $v(t)$  when  $t \geq 0$ .
- Plot  $v(t)$  versus  $t$  for the time interval  $0 \leq t \leq 7$  ms. Compare this response with that of Example 9.4 ( $R = 20$  k $\Omega$ ) and Example 9.5 ( $R = 4$  k $\Omega$ ). In particular, compare peak values of  $v(t)$  and the times when these peak values occur.

**9.12** The two switches in the circuit seen in Fig. P9.12 operate synchronously. When switch 1 is in position a, switch 2 is in position d. When switch 1 moves to position b, switch 2 moves to position c, and vice versa. Switch 1 has been in position a for a long time. At  $t = 0$  the switches move to their alternate positions. Find  $v_o(t)$  for  $t \geq 0$ .

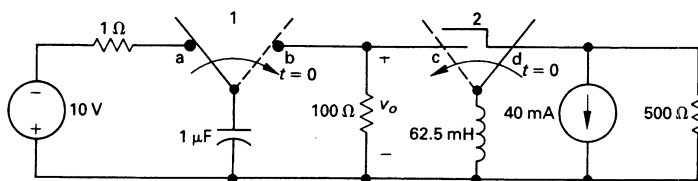


FIGURE P9.12

**9.13** The resistor in the circuit of Fig. P9.12 is increased from 100 to 250  $\Omega$ . Find  $v_o(t)$  for  $t > 0$ .

**9.14** The resistor in the circuit of Fig. P9.12 is increased from 100 to 125  $\Omega$ . Find  $v_o(t)$  for  $t \geq 0$ .

**9.15** The switch in the circuit of Fig. P9.15 has been in position a for a long time. At  $t = 0$  the switch moves instantaneously to position b. Find  $v_o(t)$  for  $t \geq 0$ .

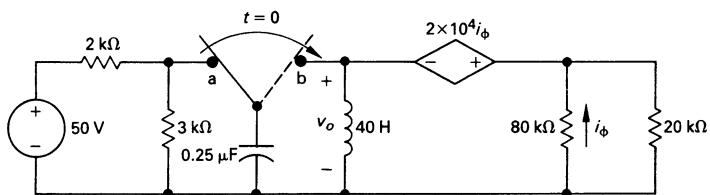


FIGURE P9.15

**9.16** For the circuit in Example 9.6, find for  $t \geq 0$  (a)  $v(t)$ ; (b)  $i_R(t)$ ; and (c)  $i_C(t)$ .

**9.17** For the circuit in Example 9.7, find for  $t \geq 0$  (a)  $v(t)$  and (b)  $i_C(t)$ .

**9.18** For the circuit in Example 9.8, find  $v(t)$  for  $t \geq 0$ .

- 9.19 The switch in the circuit in Fig. P9.19 has been open a long time before closing at  $t = 0$ . Find  $i_L(t)$  for  $t \geq 0$ .

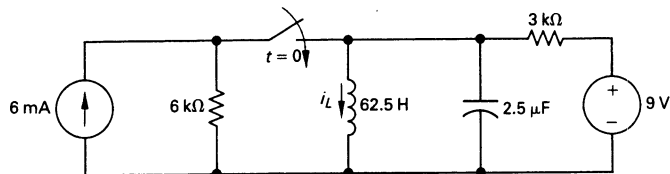


FIGURE P9.19

- 9.20 The two switches in the circuit in Fig. P9.20 are synchronized. For  $t < 0$  switch 1 is in position a and switch 2 is open. At  $t = 0$  switch 1 moves to position b and switch 2 closes. Find  $v_o(t)$  for  $t \geq 0^+$ .

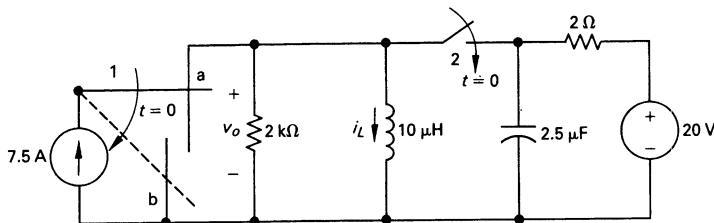


FIGURE P9.20

- 9.21 For the circuit in Fig. P9.20 find  $i_L(t)$  for  $t \geq 0$ .

- 9.22 Switch 2 in the circuit shown in Fig. P9.22 has been ON for a long time. At  $t = 0$ , switch 1 opens and switch 2 moves instantaneously to its OFF position. The initial inductor current is zero. Find  $i_L(t)$  for  $t \geq 0$ .

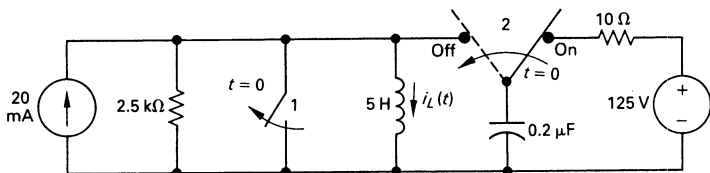


FIGURE P9.22

- 9.23 Switches 1 and 2 in the circuit in Fig. P9.23 operate synchronously. When switch 1 opens, switch 2 closes and vice versa. Switch 1 has been closed for a long time. At  $t = 0$ , switch 1 opens.

- Calculate  $i_L(0^+)$ .
- Calculate  $di_L(0^+)/dt$ .
- Find  $i_L(t)$  for  $t \geq 0$ .

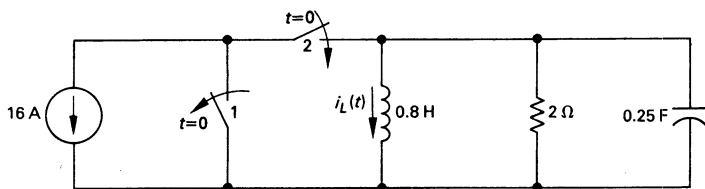


FIGURE P9.23

**9.24** Switches 1 and 2 in the circuit shown in Fig. P9.24 operate synchronously. When switch 1 opens, switch 2 closes and vice versa. Switch 1 has been closed for a long time. At  $t = 0$ , switch 1 opens. Find

- $v_o(0^+)$ ;
- $dv_o(0^+)/dt$ ;
- $v_o(t)$  for  $t \geq 0$ .

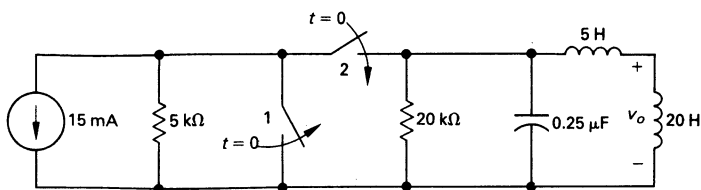


FIGURE P9.24

**9.25** Switches 1 and 2 in the circuit in Fig. P9.25 operate synchronously. When switch 1 opens, switch 2 closes and vice versa. Switch 1 has been closed for a long time. At  $t = 0$ , switch 1 opens. Find

- $v_o(0^+)$ ;
- $dv_o(0^+)/dt$ ;
- $v_o(t)$  for  $t \geq 0$ .

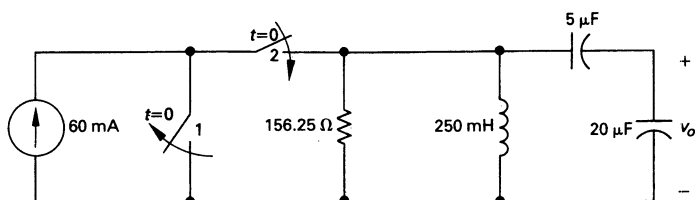


FIGURE P9.25

**9.26** In the circuit in Fig. 9.8 the initial energy stored is 11.76 mJ. The initial voltage on the capacitor is 56 V. The dc current source is delivering 7 mA. The circuit elements are  $R = 10 \text{ k}\Omega$ ;  $L = 20 \text{ H}$ ; and  $C = 2.5 \text{ }\mu\text{F}$ .

- Find the solution for  $i_L(t)$  for  $t \geq 0$ .
- Find the solution for  $v(t)$  for  $t \geq 0$ .
- Find the maximum value of  $v(t)$ .

**9.27** The initial energy stored in the 31.25-nF capacitor in the circuit in Fig. P9.27 is 9  $\mu\text{J}$ . The initial energy stored in the inductor is zero. The roots of the characteristic equation that describes the natural behavior of the current  $i$  are  $-4000 \text{ s}^{-1}$  and  $-16,000 \text{ s}^{-1}$ .

- Find the numerical values of  $R$  and  $L$ .
- Find the numerical values of  $i(0)$  and  $di(0)/dt$  immediately after the switch has been closed.
- Find  $i(t)$  for  $t \geq 0$ .
- How many microseconds after the switch

closes does the current reach its maximum value?

- What is the maximum value of  $i$  in milliamperes?
- Find  $v_L(t)$  for  $t \geq 0$ .

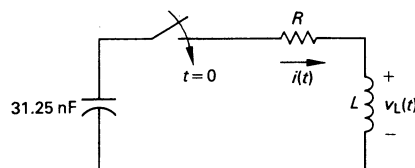


FIGURE P9.27

- 9.28** The current in the circuit in Fig. 9.3 is known to be

$$i = B_1 e^{-800t} \cos 600t + B_2 e^{-800t} \sin 600t, \quad t \geq 0.$$

The capacitor has a value of  $500 \mu\text{F}$ ; the initial value of the current is zero; and the initial voltage on the capacitor is  $12 \text{ V}$ , positive at the upper terminal. Find the values of  $R$ ,  $L$ ,  $B_1$ , and  $B_2$ .

- 9.29** Find the voltage across the  $500\text{-}\mu\text{F}$  capacitor for the circuit described in Problem 9.28. Assume the reference polarity for the capacitor voltage is positive at the upper terminal.

- 9.30** In the circuit in Fig. P9.30, the resistor is adjusted for critical damping. The initial capacitor voltage is  $20 \text{ V}$ , and the initial inductor current is  $30 \text{ mA}$ .

- Find the numerical value of  $R$ .
- Find the numerical values of  $i$  and  $di/dt$  immediately after the switch is closed.
- Find  $v_C(t)$  for  $t \geq 0$ .

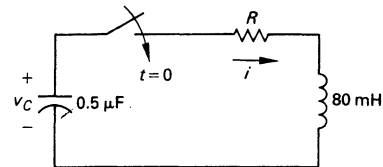


FIGURE P9.30

- 9.31** The switch in the circuit in Fig. P9.31 has been in position a for a long time. At  $t = 0$ , the switch moves instantaneously to position b.

- What is the initial value of  $v_a$ ?
- What is the initial value of  $dv_a/dt$ ?
- What is the numerical expression for  $v_a(t)$  for  $t \geq 0$ ?

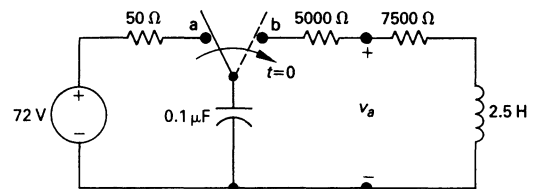


FIGURE P9.31

- 9.32** The “make-before-break” switch in the circuit shown in Fig. P9.32 has been in position a for a long time. At  $t = 0$  the switch is moved instantaneously to position b. Find  $i(t)$  for  $t \geq 0$ .

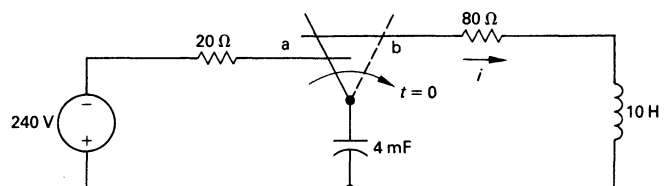


FIGURE P9.32

- 9.33** The switch in the circuit shown in Fig. P9.33 has been closed for a long time. The switch opens at  $t = 0$ . Find  $v_o(t)$  for  $t \geq 0$ .

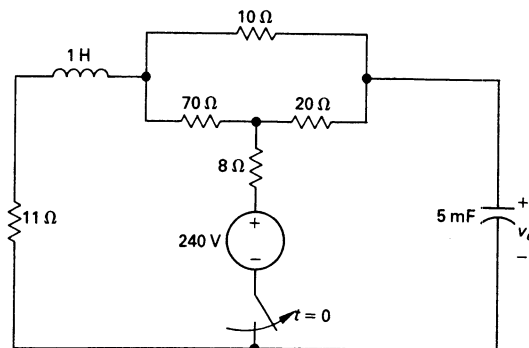


FIGURE P9.33

- 9.34** The switch in the circuit shown in Fig. P9.34 has been closed for a long time. The switch opens at  $t = 0$ .

- a) Find  $i_o(t)$  for  $t \geq 0$ .  
b) Find  $v_o(t)$  for  $t \geq 0$ .

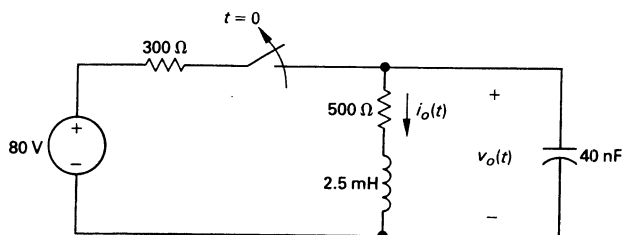


FIGURE P9.34

- 9.35** The initial energy stored in the circuit in Fig. P9.35 is zero. Find  $v_o(t)$  for  $t \geq 0$ .

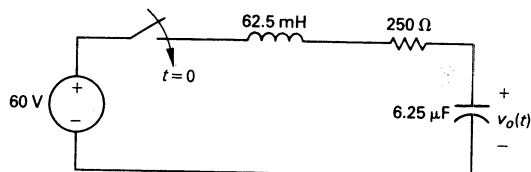


FIGURE P9.35

- 9.36** The two switches in the circuit seen in Fig. P9.36 operate synchronously. When switch 1 is in position a, switch 2 is closed. When switch 1 is in position b, switch 2 is open. Switch 1 has been in position a for a long time. At  $t = 0$  it moves instantaneously to position b. Find  $v_c(t)$  for  $t \geq 0$ .

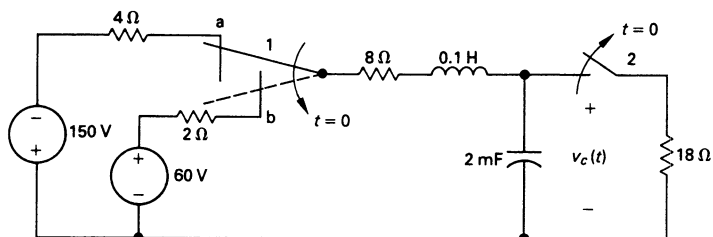


FIGURE P9.36



- 9.37** The circuit shown in Fig. P9.37 has been in operation for a long time. At  $t = 0$  the voltage suddenly jumps to 400 V. Find  $v_c(t)$  for  $t \geq 0$ .

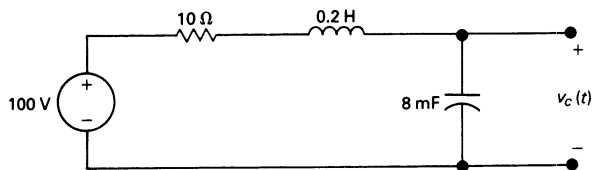


FIGURE P9.37

- 9.38** The switch in the circuit of Fig. P9.38 has been in position a for a long time. At  $t = 0$  the switch moves instantaneously to position b.
- Find  $v_o(0^+)$ .
  - Find  $dv_o(0^+)/dt$ .
  - Find  $v_o(t)$  for  $t \geq 0$ .

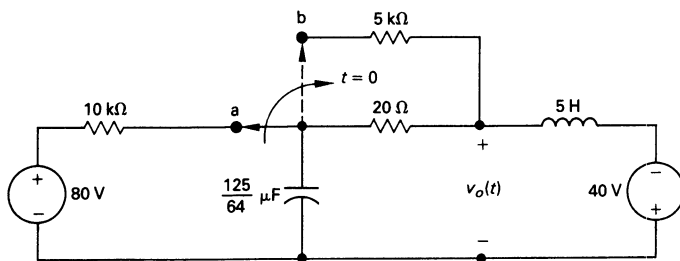


FIGURE P9.38

- 9.39** The switch in the circuit in Fig. P9.39 has been open for a long time before closing at  $t = 0$ . Find  $v_o(t)$  for  $t \geq 0$ .

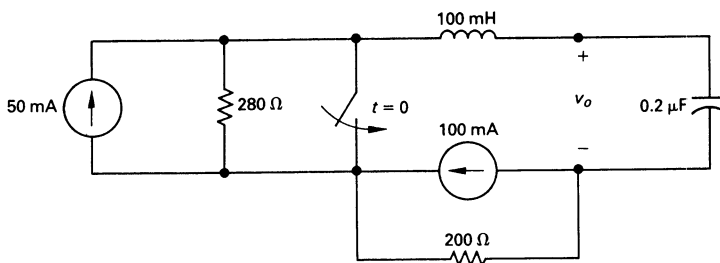


FIGURE P9.39

- 9.40** The switch in the circuit in Fig. P9.40 has been closed for a long time before opening at  $t = 0$ .
- Find  $i_o(t)$  for  $t \geq 0$ .
  - Find  $v_o(t)$  for  $t \geq 0$ .

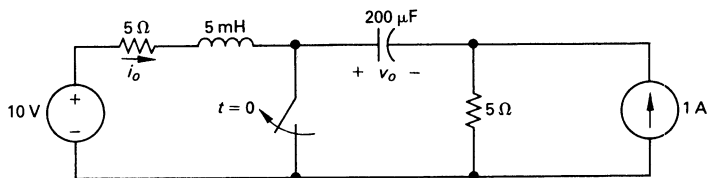


FIGURE P9.40

- 9.41** Assume that the capacitor voltage in the circuit of Fig. 9.10 is underdamped. Also assume that no energy is stored in the circuit elements when the switch is closed.
- Show that  $dv_c/dt = (\omega_0^2/\omega_d)Ve^{-\alpha t} \sin \omega_d t$ .
  - Show that  $dv_c/dt = 0$  when  $t = n\pi/\omega_d$ , where  $n = 0, 1, 2, \dots$ .

- Let  $t_n = n\pi/\omega_d$  and show that  $v_c(t_n) = V - V(-1)^n e^{-\alpha n\pi/\omega_d}$ .
- Show that

$$\alpha = \frac{1}{T_d} \ln \frac{v_c(t_1) - V}{v_c(t_3) - V},$$

where  $T_d = t_3 - t_1$ .

**9.42** The voltage across a  $0.1\text{-}\mu\text{F}$  capacitor in the circuit of Fig. 9.10 is described as follows. After the switch has been closed for several seconds, the voltage is constant at  $100\text{ V}$ . The first time the voltage exceeds  $100\text{ V}$ , it reaches a peak of  $163.84\text{ V}$ . This occurs  $(\pi/7)\text{ ms}$  after the switch has been closed. The second

time the voltage exceeds  $100\text{ V}$ , it reaches a peak of  $126.02\text{ V}$ . This second peak occurs  $(3\pi/7)\text{ ms}$  after the switch has been closed. At the time when the switch is closed, there is no energy stored in either the capacitor or the inductor. Find the numerical values of  $R$  and  $L$ . (Hint: Work Problem 9.41 first.)

**9.43** The switch in the circuit shown in Fig. P9.43 has been closed for a long time before it is opened at  $t = 0$ . Assume that the circuit parameters are such that the response is underdamped.

- Derive the expression for  $v_o(t)$  as a function of  $v_g$ ,  $\alpha$ ,  $\omega_d$ ,  $C$ , and  $R$  for  $t \geq 0$ .
- Derive the expression for the value of  $t$  when the magnitude of  $v_o$  is maximum.

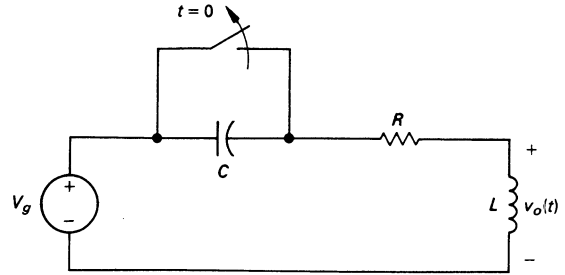


FIGURE P9.43

**9.44** The circuit parameters in the circuit of Fig. P9.43 are  $R = 600\ \Omega$ ,  $L = 20\text{ mH}$ ,  $C = 0.08\ \mu\text{F}$ , and  $v_g = -480\text{ V}$ .

- Express  $v_o(t)$  numerically for  $t \geq 0$ .
- How many microseconds after the switch opens is the inductor voltage maximum?

- What is the maximum value of the inductor voltage?
- Repeat parts (a), (b), and (c), with  $R$  reduced to  $60\ \Omega$ .

**9.45** The voltage signal of Fig. P9.45(a) is applied to the cascaded integrating amplifiers shown in Fig. P9.45(b). There is no energy stored in the capacitors at the instant the signal is applied.

- Derive the numerical expressions for  $v_o(t)$  and  $v_{o1}(t)$  for the time intervals  $0 \leq t \leq 0.5\text{ s}$  and  $0.5\text{ s} \leq t \leq t_{\text{sat}}$ .
- Compute the value of  $t_{\text{sat}}$ .

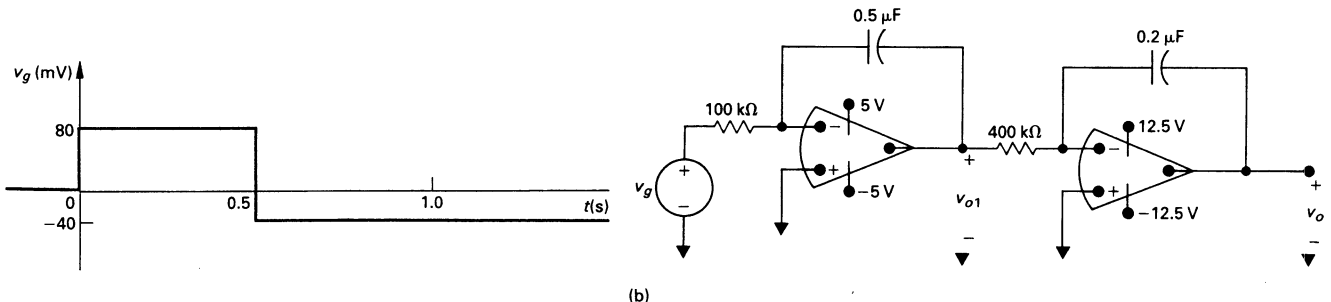


FIGURE P9.45

**9.46** The circuit in Fig. P9.45(b) is modified by adding a  $1\text{-M}\Omega$  resistor in parallel with the  $0.5\text{-}\mu\text{F}$  capacitor and a  $5\text{-M}\Omega$  resistor in parallel with the  $0.2\text{-}\mu\text{F}$  capacitor. As in Problem 9.45, there is no energy stored in the ca-

pacitors at the time the signal is applied. Derive the numerical expressions for  $v_o(t)$  and  $v_{o1}(t)$  for the time intervals  $0 \leq t \leq 0.5\text{ s}$  and  $0.5\text{ s} \leq t \leq \infty$ .

- 9.47** a) Derive the differential equation that relates the output voltage to the input voltage for the circuit shown in Fig. P9.47.  
 b) Compare the result with Eq. (9.86) when  $R_1 C_1 = R_2 C_2 = RC$  in Fig. 9.13.  
 c) What is the advantage of the circuit shown in Fig. P9.47?

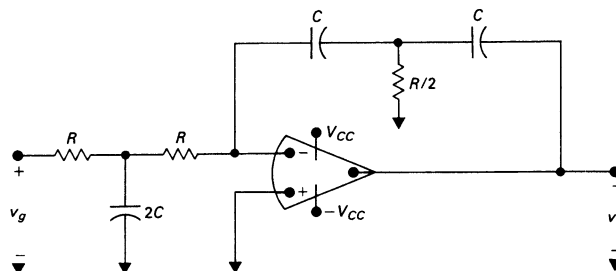


FIGURE P9.47

**9.48** We now wish to illustrate how several op-amp circuits can be interconnected to solve a differential equation.

- a) Derive the differential equation for the spring-mass system shown in Fig. P9.48(a). Assume that the force exerted by the spring is directly proportional to the spring displacement, that the mass is constant, and that the frictional force is directly proportional to the velocity of the moving mass.  
 b) Rewrite the differential equation derived in part (a) so that the highest-order derivative is expressed as a function of all the other terms in the equation. Now assume that a voltage equal to  $d^2x/dt^2$  is available and by successive integrations generate  $dx/dt$  and

$x$ . We can synthesize the coefficients in the equations by scaling amplifiers and combine the terms required to generate  $d^2x/dt^2$  by a summing amplifier. With these ideas in mind, analyze the interconnection shown in Fig. P9.48(b). In particular, describe the purpose of each shaded area in the circuit and describe the signal at the points labeled B, C, D, E, and F, assuming the signal at A represents  $d^2x/dt^2$ . Also discuss the parameters  $R$ ;  $R_1$ ,  $C_1$ ;  $R_2$ ,  $C_2$ ;  $R_3$ ,  $R_4$ ;  $R_5$ ,  $R_6$ ; and  $R_7$ ,  $R_8$  in terms of the coefficients in the differential equation.

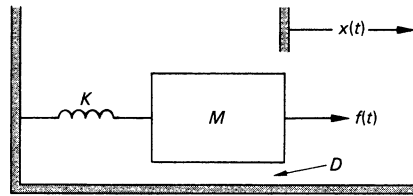
**9.49** Assume the underdamped voltage response of the circuit in Fig. 9.1 is written as

$$v(t) = (A_1 + A_2)e^{-\alpha t} \cos \omega_d t + j(A_1 + A_2)e^{-\alpha t} \sin \omega_d t.$$

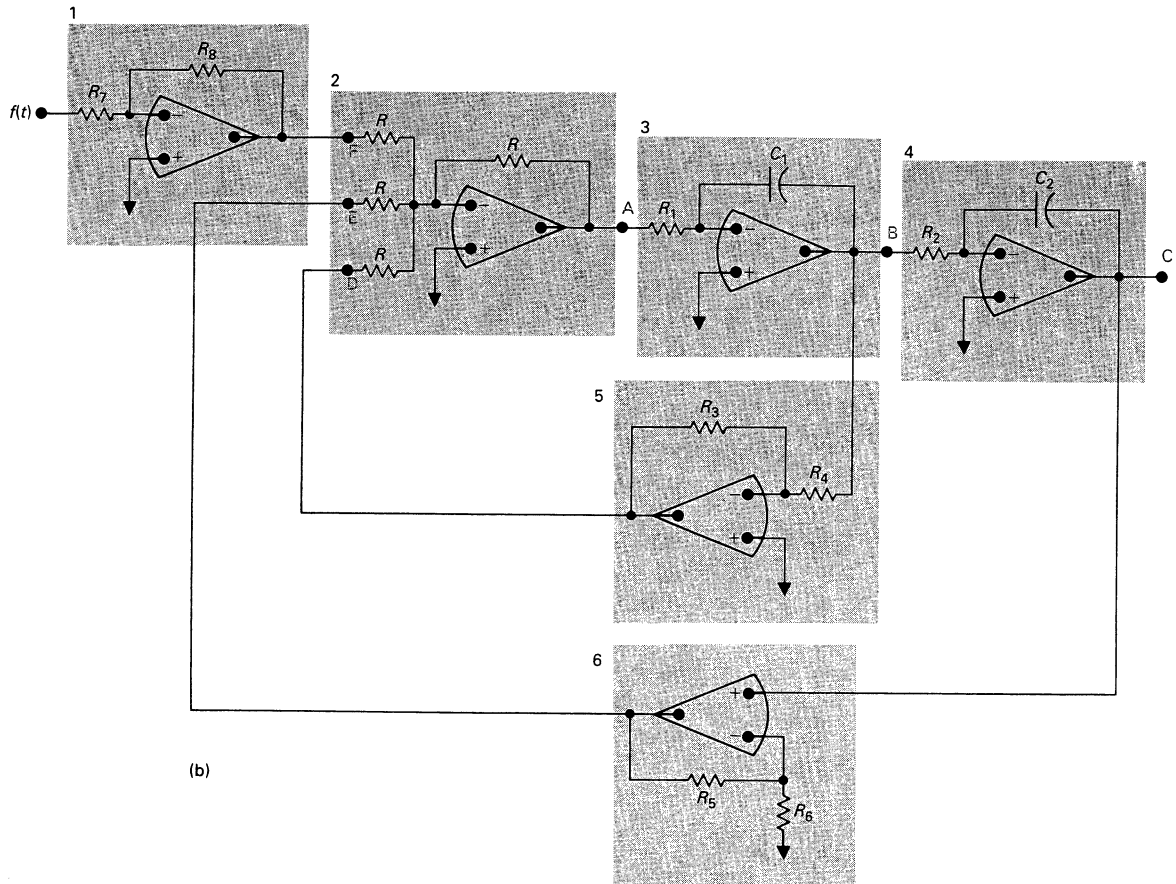
The initial value of the inductor current is  $I_0$  and the initial value of the capacitor voltage is  $V_0$ . Show that  $A_2$  is the conjugate of  $A_1$ .

(Hint: Use the same thought process as outlined in the text to find  $A_1$  and  $A_2$ .)

**9.50** Show that the results obtained from Problem 9.49, that is the expressions for  $A_1$  and  $A_2$  are consistent with Eqs. (9.27) and (9.28) in the text.



(a)



(b)

FIGURE P9.43

- Each capacitor appears in the phasor domain as an impedance  $j(-1/\omega C)$  where  $(-1/\omega C)$  is the capacitive reactance.
- Kirchhoff's laws hold for phasor currents and voltages.
- The relationship between the phasor current and voltage at the terminals of an impedance  $Z$  is

$$\mathbf{V} = Z\mathbf{I},$$

where the reference direction for  $\mathbf{I}$  is from the positive to the negative polarity reference for  $\mathbf{V}$  (passive sign convention).

- The reciprocal of the impedance ( $Z$ ) is the admittance ( $Y$ ):

$$Y = \frac{1}{Z}.$$

- All the techniques used in dc circuit analysis (series–parallel and delta–wye simplifications, node voltages, mesh currents, source transformations, Thévenin/Norton equivalents, and superposition) may be used to analyze a phasor-domain circuit.
- Phasor-domain currents and voltages may be transferred to the time domain by means of the inverse phasor transform:

$$\mathcal{P}^{-1}\{Ae^{j\theta}\} = \Re\{Ae^{j\theta}e^{j\omega t}\}.$$

## PROBLEMS

### 10.1 Consider the sinusoidal voltage

$$v = 100 \cos(400\pi t + 60^\circ) \text{ V}.$$

- What is the maximum amplitude of the voltage?
- What is the frequency in hertz?
- What is the frequency in radians per second?
- What is the phase angle in radians?
- What is the phase angle in degrees?
- What is the period in milliseconds?
- What is the first time after  $t = 0$  that  $v = 100$  V?
- The sinusoidal function is shifted  $5/12$  ms to the left along the time axis. What is the expression for  $v(t)$ ?
- What is the minimum number of milliseconds that the function must be shifted to the right if the expression for  $v(t)$  is  $100 \cos 400\pi t$  V?
- What is the minimum number of milliseconds that the function must be shifted to the left if the expression for  $v(t)$  is  $100 \sin 400\pi t$  V?

**10.2** In a single graph, sketch  $v = 60 \cos(\omega t + \phi)$  versus  $\omega t$  for  $\phi = -60^\circ, -30^\circ, 0^\circ, +30^\circ$ , and  $60^\circ$ .

- State whether the voltage function is shifting to the right or left as  $\phi$  becomes more positive.
- What is the direction of shift if  $\phi$  changes from 0 to  $-30^\circ$ ?

**10.3** At  $t = -250 \mu\text{s}$  a sinusoidal voltage is known to be zero and going positive. The voltage is next zero at  $t = 750 \mu\text{s}$ . It is also known that the voltage is 100 V at  $t = 0$ .

- What is the frequency of  $v$  in hertz?
- What is the expression for  $v$ ?

**10.4** A sinusoidal current is zero at  $t = 15 \mu\text{s}$  and increasing at a rate of  $2 \times 10^5 \pi \text{ A/s}$ . The maximum amplitude of the current is 10 A.

- What is the frequency of  $i$  in radians per second?
- What is the expression for  $i$ ?

**10.5 a)** Verify that Eq. (10.8) is the solution of Eq. (10.7). This can be done by substituting Eq. (10.8) into the left-hand side of Eq. (10.7) and then noting that it equals the right-hand side for all values to  $t > 0$ . At  $t = 0$ , Eq. (10.8) should reduce to the initial value of the current.

- Since the transient component vanishes as time elapses and since our solution must satisfy the differential equation for all values of  $t$ , the steady-state component, by itself, must also satisfy the differential equation. Verify this observation by showing that the steady-state component of Eq. (10.8) satisfies Eq. (10.7).

**10.6** Use the concept of the phasor to combine the following sinusoidal functions into a single trigonometric expression:

- $y = 100 \cos(500t + 30^\circ) + 50 \cos(500t - 45^\circ)$ ;
- $y = 100 \sin(377t + 40^\circ) - 50 \cos(377t + 200^\circ)$ ;
- $y = 40 \cos(\omega t + 60^\circ) + 80 \sin(\omega t + 135^\circ) - 100 \cos(\omega t + 270^\circ)$ .

**10.7** Show that

$$\int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \phi) dt = \frac{V_m^2 T}{2}.$$

**10.8** The rms value of the sinusoidal voltage supplied to the convenience outlet of a US home

is 120 V. What is the maximum value of the voltage at the outlet?

- 10.9** A 1000-Hz sinusoidal voltage with a maximum amplitude of 200 V at  $t = 0$  is applied across the terminals of an inductor. The maximum amplitude of the steady-state current in the inductor is 25 A.
- What is the frequency of the inductor current?
  - What is the phase angle of the voltage?
  - What is the phase angle of the current?
  - What is the inductive reactance of the inductor?
  - What is the inductance of the inductor in millihenrys?
  - What is the impedance of the inductor?
- 10.10** A 50-kHz sinusoidal voltage has zero phase angle and a maximum amplitude of 10 mV. When this voltage is applied across the terminals of a capacitor, the resulting steady-state current has a maximum amplitude of  $628.32 \mu\text{A}$ .
- What is the frequency of the current in radians per second?
  - What is the phase angle of the current?
  - What is the capacitive reactance of the capacitor?
  - What is the capacitance of the capacitor in microfarads?
  - What is the impedance of the capacitor?
- 10.11** A  $10\text{-}\Omega$  resistor, a  $8\text{-mH}$  inductor, and a  $2.5\text{-}\mu\text{F}$  capacitor are connected in series. The series-connected elements are energized by a sinusoidal voltage source whose voltage is  $240 \cos(5000t - 40^\circ) \text{ V}$ .
- Draw the phasor-domain equivalent circuit.
  - Reference the current in the direction of the voltage rise across the source and find the phasor current.
  - Find the steady-state expression for  $i(t)$ .
- 10.12** A  $20\text{-}\Omega$  resistor and a  $1\text{-}\mu\text{F}$  capacitor are connected in parallel. This parallel combination is also in parallel with the series combination of a  $1\text{-}\Omega$  resistor and a  $40\text{-}\mu\text{H}$  inductor. These three parallel branches are driven by a sinusoidal current source whose current is  $20 \cos(50,000t - 20^\circ) \text{ A}$ .
- Draw the phasor-domain equivalent circuit.
  - Reference the voltage across the current source as a rise in the direction of the source current and find the phasor voltage.
  - Find the steady-state expression for  $v(t)$ .
- 10.13** Find the steady-state expression for  $i_o(t)$  in the circuit in Fig. P10.13 if  $v_s = 100 \sin 50t \text{ mV}$ .

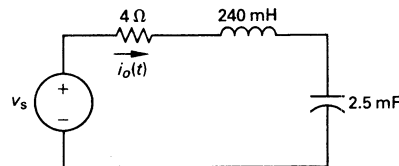


FIGURE P10.13

- 10.14** The circuit in Fig. P10.14 is operating in the sinusoidal steady state. Find the steady-state expression for  $v_o(t)$  if  $v_g = 10 \cos 100t \text{ V}$ .

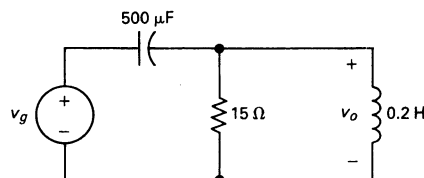


FIGURE P10.14

- 10.15 Find the impedance  $Z_{ab}$  in the circuit seen in Fig. P10.15. Express  $Z_{ab}$  in both polar and rectangular form.

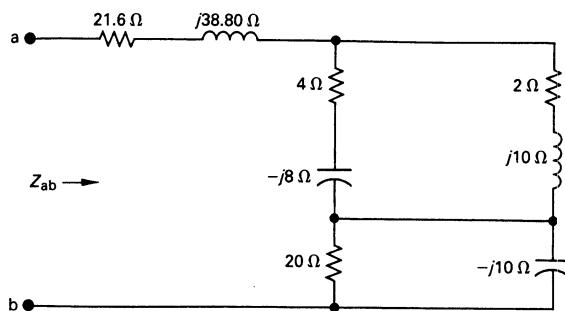


FIGURE P10.15

- 10.16 Find the admittance  $Y_{ab}$  in the circuit seen in Fig. P10.16. Express  $Y_{ab}$  in both polar and rectangular form. Give the value of  $Y_{ab}$  in millimhos.

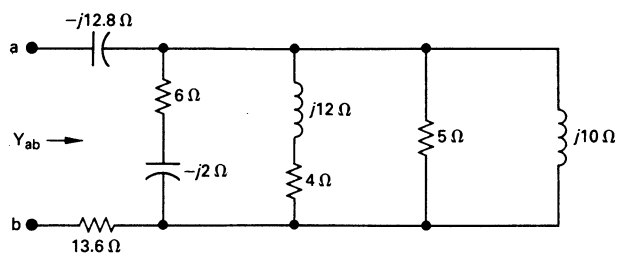


FIGURE P10.16

- 10.17 Find  $Z_{ab}$  in the circuit shown in Fig. P10.17 when the circuit is operating at a frequency of 20 krad/s.

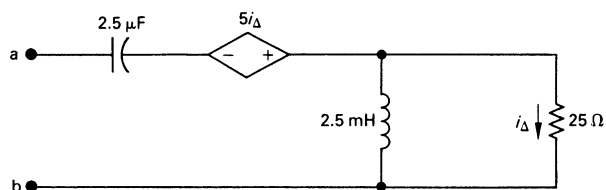


FIGURE P10.17

- 10.18 a) For the circuit shown in Fig. P10.18, find the frequency (in radians per second) at which the impedance  $Z_{ab}$  is purely resistive.  
b) Find the value of  $Z_{ab}$  at the frequency of part (a).

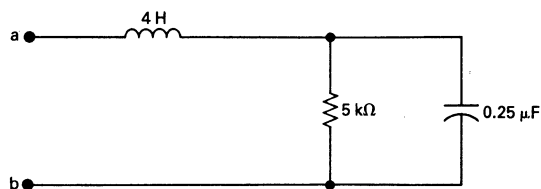


FIGURE P10.18



- 10.19** a) The source voltage in the circuit in Fig. P10.19 is  $v_g = 200 \cos 500t$  V. Find the values of  $L$  such that  $i_g$  is in phase with  $v_g$  when the circuit is operating in the steady state.
- b) For the values of  $L$  found in part (a) find the steady-state expressions for  $i_g$ .

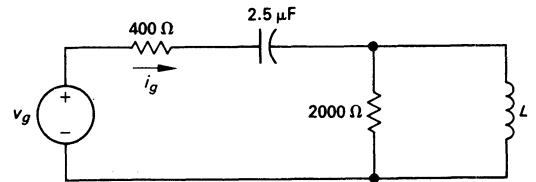


FIGURE P10.19

- 10.20** The circuit shown in Fig. P10.20 is operating in the sinusoidal steady state. The capacitor is adjusted until the current  $i_g$  is in phase with the sinusoidal voltage  $v_g$ .
- a) Specify the values of capacitance in microfarads if  $v_g = 250 \cos 1000t$  V.
- b) Give the steady-state expressions for  $i_g$  when  $C$  has the values found in part (a).

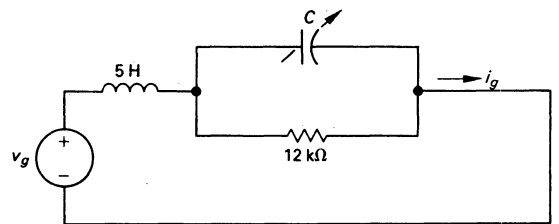


FIGURE P10.20

- 10.21** The frequency of the sinusoidal current source in the circuit in Fig. P10.21 is adjusted until  $v_o$  is in phase with  $i_g$ .
- a) What is the value of  $\omega$  in radians per second?
- b) If  $i_g = 0.25 \cos \omega t$  mA [where  $\omega$  is the frequency found in part (a)] what is the steady-state expression for  $v_o$ ?

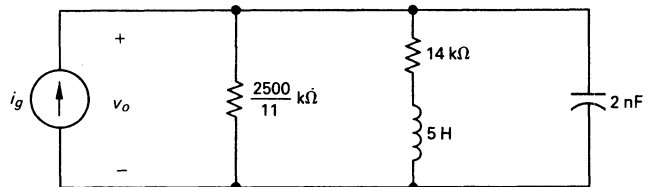


FIGURE P10.21

- 10.22** The frequency of the sinusoidal voltage source in the circuit in Fig. P10.22 is adjusted until the current  $i_o$  is in phase with  $v_g$ .
- a) Find the frequency in hertz.
- b) Find the steady-state expression for  $i_o$  [at the frequency found in part (a)] if  $v_g = 80 \cos \omega t$  V.

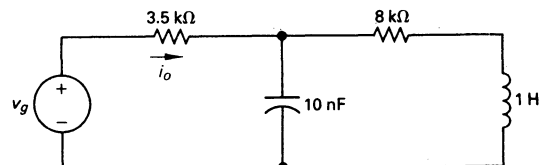


FIGURE P10.22

- 10.23** The circuit shown in Fig. P10.23 is operating in the sinusoidal steady state. Find the value of  $\omega$  if

$$i_o = 0.10 \sin(\omega t + 173.13^\circ) \text{ A}$$

and

$$v_g = 500 \cos(\omega t + 30^\circ) \text{ V.}$$

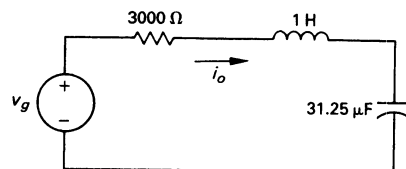


FIGURE P10.23

- 10.24** Find the steady-state expression for  $v_o(t)$  in the circuit shown in Fig. P10.24 if  $i_g = 10 \cos 8t$  A.

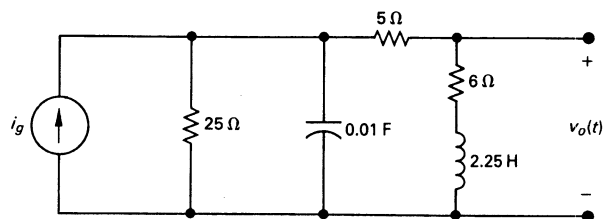


FIGURE P10.24

- 10.25** The current source in the circuit shown in Fig. P10.25 is generating a sinusoidal waveform such that  $i_g = 20 \cos(40,000t - 73.74^\circ)$  A. Find the steady-state expression for  $v_o(t)$ .

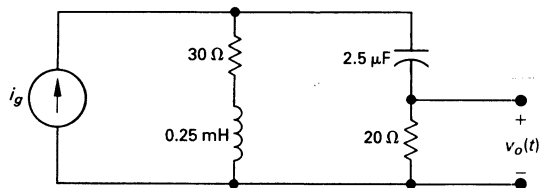


FIGURE P10.25

- 10.26** The expressions for the steady-state voltage and current at the terminals of the circuit seen in Fig. P10.26 are

$$v_g = 240 \sin(2000\pi t + 210^\circ) \text{ V}$$

and

$$i_g = 12 \cos(2000\pi t + 84^\circ) \text{ A.}$$

- What is the impedance seen by the source?
- By how many microseconds is the current out of phase with the voltage?

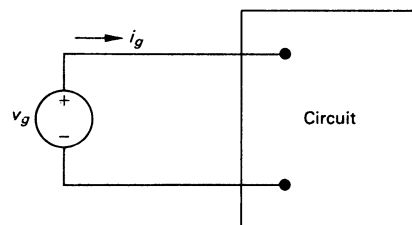


FIGURE P10.26

- 10.27** a) Show that at a given frequency  $\omega$ , the circuits in Fig. P10.27(a) and (b) will have the same impedance between the terminals a, b if

$$R_1 = \frac{R_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{and} \quad C_1 = \frac{1 + \omega^2 R_2^2 C_2^2}{\omega^2 R_2^2 C_2}.$$

- b) Find the values of resistance and capacitance that when connected in series will have the same impedance at 40 krad/s as that of a 1-k $\Omega$  resistor connected in parallel with a 0.05- $\mu$ F capacitor.

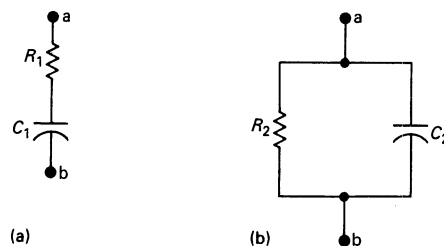


FIGURE P10.27

- 10.28** a) Show that at a given frequency  $\omega$ , the circuits in Fig. P10.27(a) and (b) will have the same impedance between the terminals a, b if

$$R_2 = \frac{1 + \omega^2 R_1^2 C_1^2}{\omega^2 R_1 C_1^2} \quad \text{and} \quad C_2 = \frac{C_1}{1 + \omega^2 R_1^2 C_1^2}.$$

(Hint: The two circuits will have the same impedance if they have the same admittance.)

- b) Find the values of resistance and capacitance that when connected in parallel will give the same impedance at 50 krad/s as that of a 1-k $\Omega$  resistor connected in series with a capacitance of 40 nF.

- 10.29** a) Show that at a given frequency  $\omega$ , the circuits in Fig. P10.29(a) and (b) will have the same impedance between the terminals a, b if

$$R_1 = \frac{\omega^2 L_2^2 R_2}{R_2^2 + \omega^2 L_2^2} \quad \text{and} \quad L_1 = \frac{R_2^2 L_2}{R_2^2 + \omega^2 L_2^2}.$$

- b) Find the values of resistance and inductance that when connected in series will have the same impedance at 8 krad/s as that of a 10-k $\Omega$  resistor connected in parallel with a 2.5-H inductor.

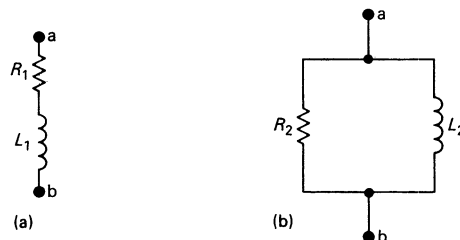


FIGURE P10.29

- 10.30** a) Show that at a given frequency  $\omega$ , the circuits in Fig. P10.29(a) and (b) will have the same impedance between the terminals a, b if

$$R_2 = \frac{R_1^2 + \omega^2 L_1^2}{R_1} \quad \text{and} \quad L_2 = \frac{R_1^2 + \omega^2 L_1^2}{\omega^2 L_1}.$$

(Hint: The two circuits will have the same impedance if they have the same admittance.)

- b) Find the values of resistance and inductance that when connected in parallel will have the same impedance at 4000 rad/s as a 5-k $\Omega$  resistor connected in series with a 0.25-H inductor.

**10.31** The phasor current  $\mathbf{I}_a$  in the circuit shown in Fig. P10.31 is  $5/0^\circ$  A.

- a) Find  $\mathbf{I}_b$ ,  $\mathbf{I}_c$ , and  $\mathbf{V}_g$ .  
 b) If  $\omega = 5$  krad/s, write the expressions for  $i_b(t)$ ,  $i_c(t)$ , and  $v_g(t)$ .

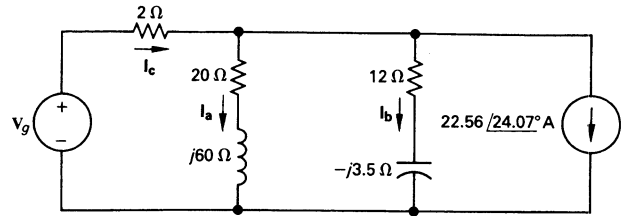


FIGURE P10.31

**10.32** Find  $\mathbf{I}_b$  and  $\mathbf{Z}$  in the circuit shown in Fig. P10.32 if  $\mathbf{V}_g = 25/0^\circ$  V and  $\mathbf{I}_a = 5/90^\circ$  A.

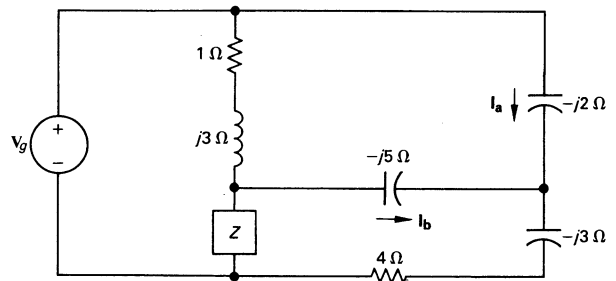


FIGURE P10.32

**10.33** Find the steady-state expression for  $v_o(t)$  in the circuit seen in Fig. P10.33 by using the technique of source transformations. The sinusoidal voltage sources are

$$v_1 = 400 \cos(5000t + 36.87^\circ) \text{ V}$$

and

$$v_2 = 128 \sin 5000t \text{ V.}$$

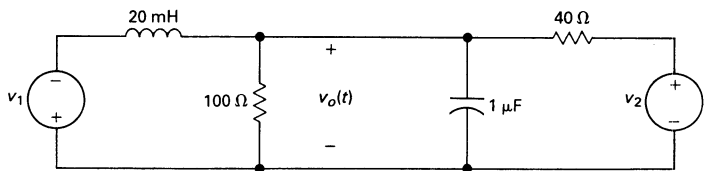


FIGURE P10.33

**10.34** The circuit in Fig. P10.34 is operating in the sinusoidal steady state. Find  $v_o(t)$  if  $i_s(t) = 3 \cos 200t$  mA.

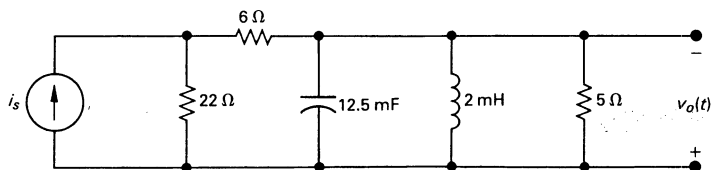


FIGURE P10.34

- 10.35** Use the node-voltage method to find the steady-state expression for  $v_o(t)$  in the circuit seen in Fig. P10.35 if  $L = 0.5$  mH,  $C = 5$   $\mu$ F,  $v_{g1} = 200 \cos 40,000t$  V, and  $v_{g2} = 100 \cos (40,000t + 36.87^\circ)$  V.

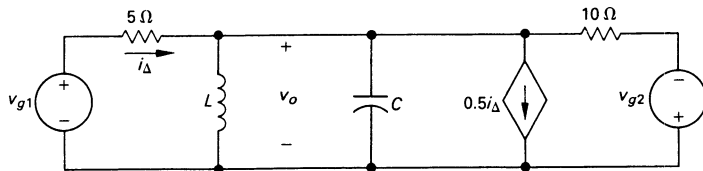


FIGURE P10.35

- 10.36** Use the node-voltage method to find the steady-state expression for  $v_o$  in the circuit seen in Fig. P10.36 if  $v_g = 160 \cos 50,000t$  mV.

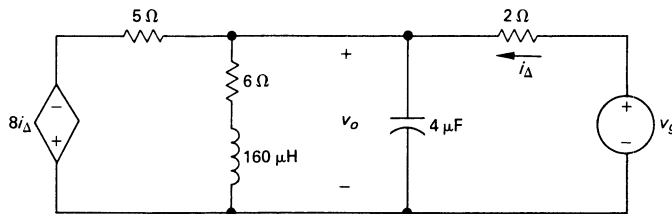


FIGURE P10.36

- 10.37** Use the node-voltage method to find the phasor voltage  $V_o$  in the circuit shown in Fig. P10.37. Express the voltage in both polar and rectangular form.

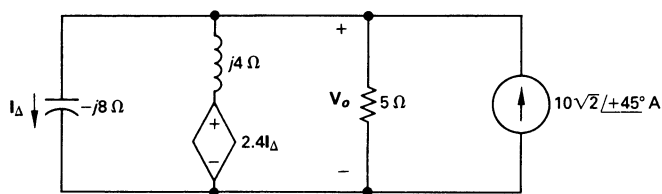


FIGURE P10.37

- 10.38** Use the node-voltage method to find  $V_o$  and  $I_o$  in the circuit seen in Fig. P10.38.

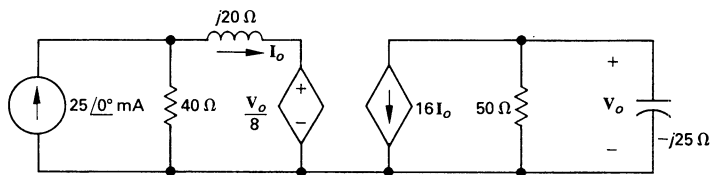


FIGURE P10.38

- 10.39** In the circuit shown in Fig. P10.39  $v_g = 10 \sin 10,000t$  V and  $i_g = 4 \cos 10,000t$  A. Use the node voltage method to determine the steady-state expressions for  $v_1$  and  $v_2$ .

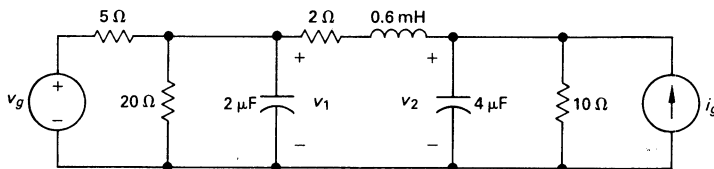


FIGURE P10.39

- 10.40** Use the mesh-current method to find the phasor branch current  $I_1$  in the circuit shown in Fig. P10.40.

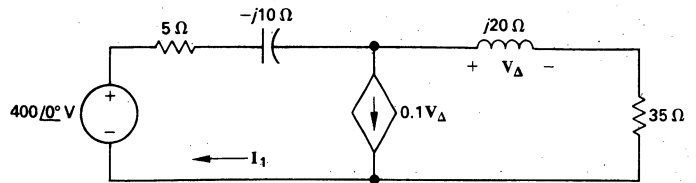


FIGURE P10.40

- 10.41** Use the mesh-current method to find the phasor current  $I_g$  in the circuit shown in Fig. P10.41.

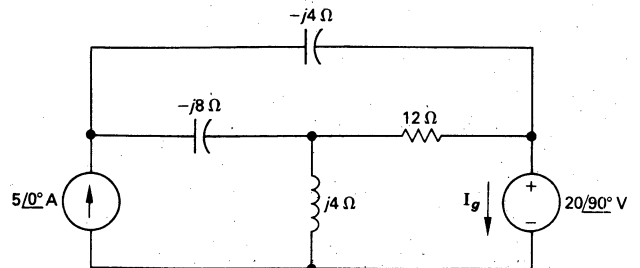


FIGURE P10.41

- 10.42** Use the mesh-current method to find the phasor branch currents  $I_a$ ,  $I_b$ , and  $I_c$  in the circuit shown in Fig. P10.42.

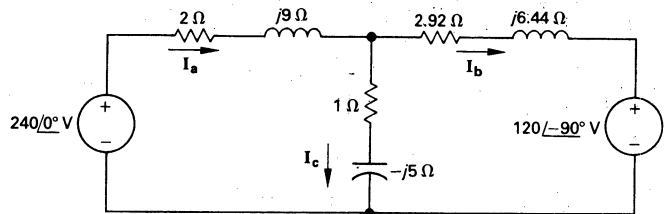


FIGURE P10.42

- 10.43** Use the mesh-current method to find the branch currents  $I_a$ ,  $I_b$ ,  $I_c$ , and  $I_d$  in the circuit shown in Fig. P10.43.

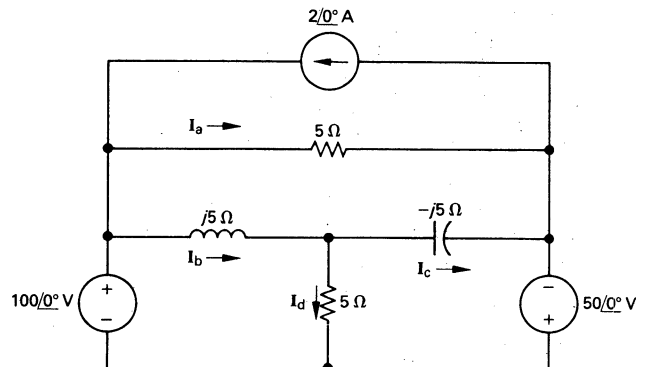


FIGURE P10.43

- 10.44** Use the mesh-current method to find the steady-state expression for  $v_o$  in the circuit seen in Fig. P10.44 if  $v_s$  equals  $800 \cos 8000t$  V.

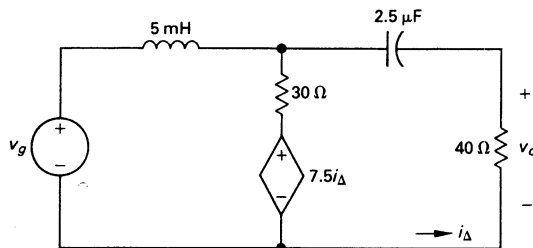


FIGURE P10.44

- 10.45** In order to introduce you to a circuit configuration that is widely used in residential wiring, we have shown a representation circuit in Fig. P10.45. In this simplified model, the resistor  $R_3$  is used to model a 250-V appliance (such as an electric range), and the resistors  $R_1$  and  $R_2$  to model 125-V appliances (such as a lamp, toaster, and iron). The branches carrying  $I_1$  and  $I_2$  are modeling what electricians refer to as the “hot” conductors in the circuit, and the branch carrying  $I_n$  is modeling the neutral conductor. Our purpose in analyzing the circuit is to show the importance of the neutral conductor in the satisfactory operation of the circuit. You are to choose the method for analyzing the circuit.

- Show that  $I_n$  is zero if  $R_1 = R_2$ .
- Show that  $V_1 = V_2$  if  $R_1 = R_2$ .
- Open the neutral branch and calculate  $V_1$

and  $V_2$  if  $R_1 = 40 \Omega$ ,  $R_2 = 400 \Omega$ , and  $R_3 = 8 \Omega$ .

- Close the neutral branch and repeat part (c).
- On the basis of your calculations, explain why the neutral conductor is never fused in such a manner that it could open while the “hot” conductors are energized.

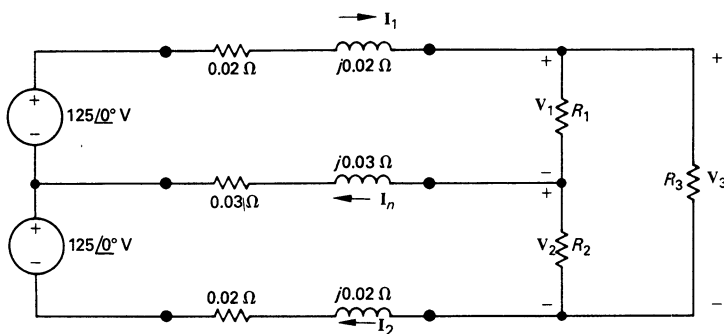


FIGURE P10.45

- 10.46** Find the steady-state expressions for the branch currents  $i_a$ ,  $i_b$ , and  $i_c$  in the circuit seen in Fig. P10.46 if  $v_a = 240 \sin 10^5 t$  V and  $v_b = 120 \cos 10^5 t$  V.

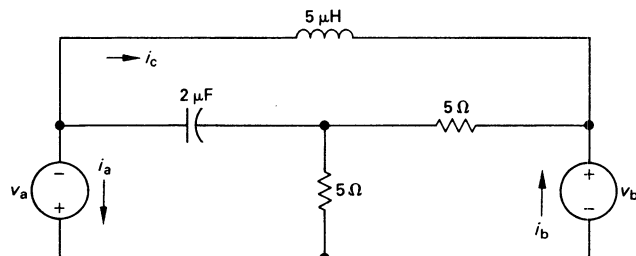


FIGURE P10.46

- 10.47 a) For the circuit shown in Fig. P10.47, find the steady-state expression for  $v_o$  if  $i_g = 5 \cos(8 \times 10^5 t)$  A.
- b) By how many microseconds does  $v_o$  lag  $i_g$ ?

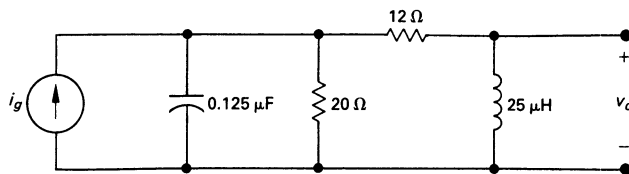


FIGURE P10.47

- 10.48 Find the value of  $Z$  in the circuit seen in Fig. P10.48 if  $\mathbf{V}_g = 100 - j50$  V,  $\mathbf{I}_g = 30 + j20$  A, and  $\mathbf{V}_b = 140 + j30$  V.

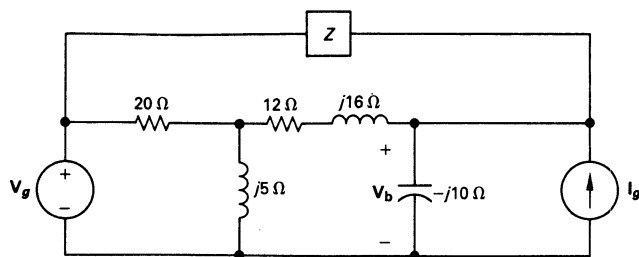


FIGURE P10.48

- 10.49 Find the Thévenin equivalent circuit with respect to the terminals a, b for the circuit shown in Fig. P10.49.

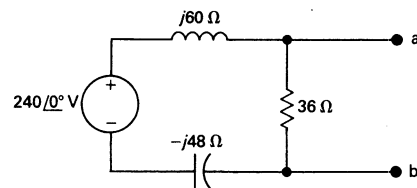


FIGURE P10.49

- 10.50 Find the Norton equivalent circuit with respect to the terminals a, b for the circuit shown in Fig. P10.50.

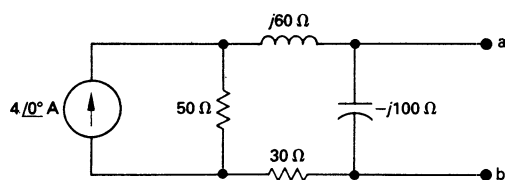


FIGURE P10.50



- 10.51** Find the Thévenin equivalent circuit with respect to the terminals a, b for the circuit shown in Fig. P10.51.

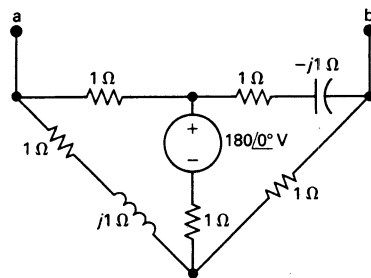


FIGURE P10.51

- 10.52** Find the Thévenin equivalent circuit with respect to the terminals a, b of the circuit shown in Fig. P10.52.

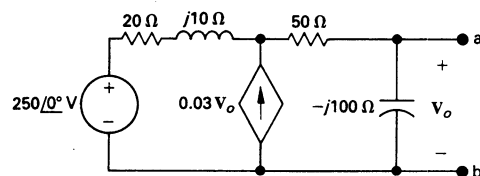


FIGURE P10.52

- 10.53** Find the Norton equivalent circuit with respect to the terminals a, b for the circuit shown in Fig. P10.53 when  $V_s = 5/0^\circ$  V.

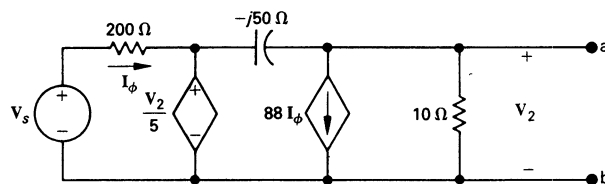


FIGURE P10.53

- 10.54** The circuit in Fig. P10.54 is operating in the sinusoidal steady state at a frequency of  $(500/\pi)$  Hz.
- Find the Thévenin impedance seen looking into the terminals a, b.
  - Repeat part (a) for the terminals c, d.

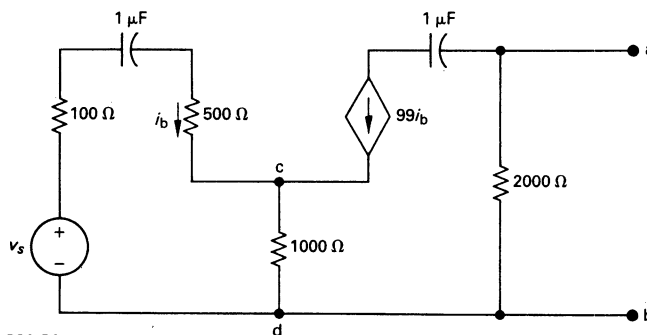


FIGURE P10.54

- 10.55** Find the Thévenin impedance seen looking into the terminals a, b of the circuit in Fig. P10.55 if the frequency of operation is  $(200/\pi)$  Hz.

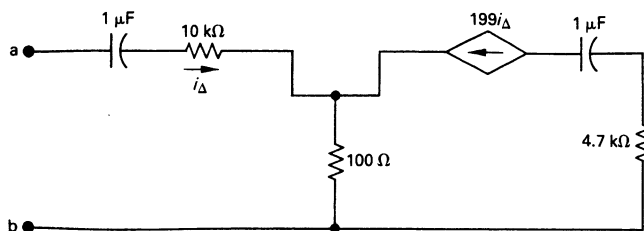


FIGURE P10.55

- 10.56** The device in Fig. P10.56 is represented in the phasor domain by a Norton equivalent. When an inductor having an impedance of  $j100 \Omega$  is connected across the device the value of  $V_o$  is  $100/120^\circ$  mV. When a capacitor having an impedance of  $-j100 \Omega$  is connected across the device the value of  $I_o$  is  $-3/210^\circ$  mA. Find the Norton current  $I_n$  and the Norton impedance  $Z_n$ .

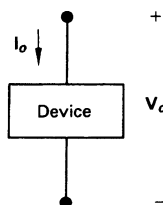


FIGURE P10.56

- 10.57** The circuit shown in Fig. P10.57 is operating at a frequency of 10 rad/s. Assume  $\sigma$  is real and lies between  $-10$  and  $+10$ , i.e.,  $-10 \leq \sigma \leq 10$ .
- Find the value of  $\sigma$  so that the Thévenin impedance looking into the terminals a, b is purely resistive.
  - What is the value of the Thévenin impedance for the  $\sigma$  found in part (a)?
  - Can  $\sigma$  be adjusted so that the Thévenin

impedance equals  $500 - j500 \Omega$ ? If so, what is the value of  $\sigma$ ?

- For what values of  $\sigma$  will the Thévenin impedance be inductive?

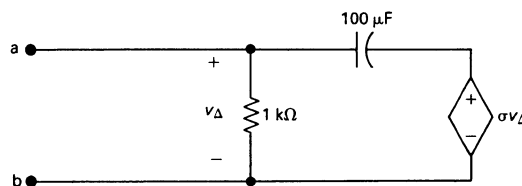


FIGURE P10.57

- 10.58** The ideal sinusoidal current sources in the circuit in Fig. P10.58 are generating the currents

$$i_{g1} = 6 \cos 25t \text{ A}$$

and

$$i_{g2} = 5 \cos (50t + 30^\circ) \text{ A}.$$

Find the steady-state expression for  $v_o(t)$ .

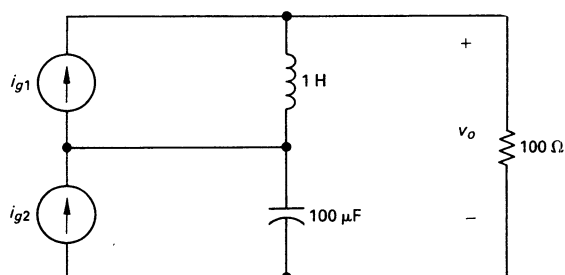


FIGURE P10.58

- 10.59** a) For the circuit shown in Fig. P10.59, compute  $V_s$  and  $V_L$ .
- b) Construct a phasor diagram showing the relationship between  $V_s$ ,  $V_L$ , and the load voltage of  $240\angle 0^\circ$  V.
- c) Repeat parts (a) and (b), given that the load voltage remains at  $240\angle 0^\circ$  V when a capacitive reactance of  $-j5\ \Omega$  is connected across the load terminals.

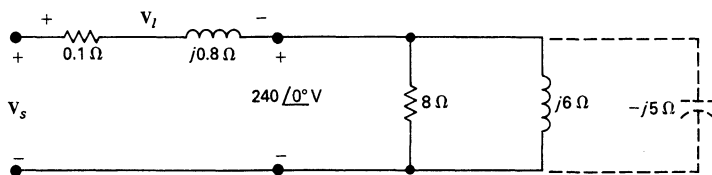


FIGURE P10.59

- 10.60** Show by using a phasor diagram what happens to the magnitude and phase angle of the voltage  $v_o$  in the circuit in Fig. P10.60 as  $R_x$  is varied from zero to infinity. The amplitude and phase angle of the source voltage are held constant as  $R_x$  varies.

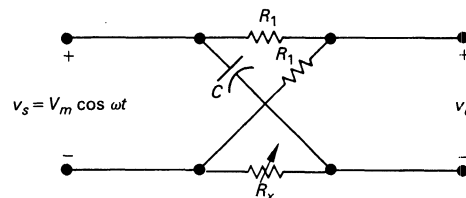


FIGURE P10.60

- 10.61** The operational amplifier in the circuit in Fig. P10.61 is ideal.

- a) Find the steady-state expression for  $v_o(t)$ .
- b) How large can the amplitude of  $v_g$  be before the amplifier saturates?

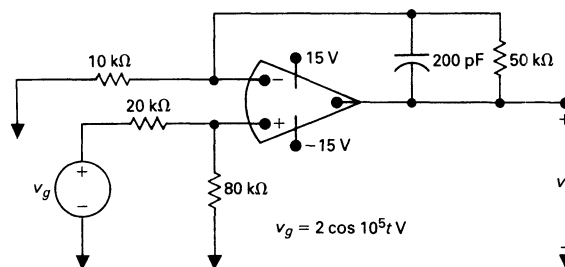


FIGURE P10.61

- 10.62** The operational amplifier in the circuit seen in Fig. P10.62 is ideal. Find the steady-state expression for  $v_o(t)$  when  $v_g = 2 \cos 10^6 t$  V.

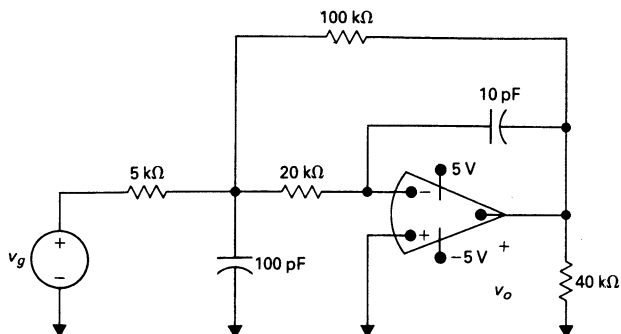


FIGURE P10.62

- 10.63** The operational amplifier in the circuit shown in Fig. P10.63 is ideal. The voltage of the ideal sinusoidal source is  $v_g = 6 \cos 10^5 t$  V.

- How small can  $C_o$  be before the steady-state output voltage no longer has a pure sinusoidal waveform?
- For the value of  $C_o$  found in part (a), write the steady-state expression for  $v_o$ .

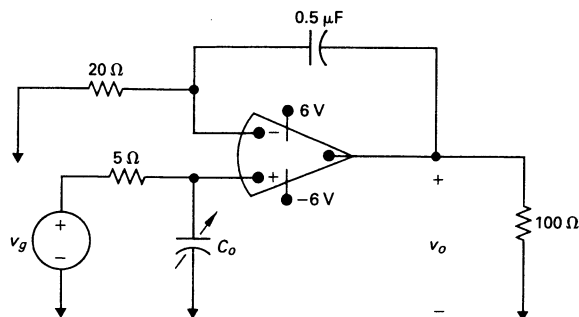


FIGURE P10.63

- 10.64** The sinusoidal voltage source in the circuit shown in Fig. P10.64 is generating the voltage  $v_g = 4 \cos 200t$  V. If the op amp is ideal, what is the steady-state expression for  $v_o(t)$ ?

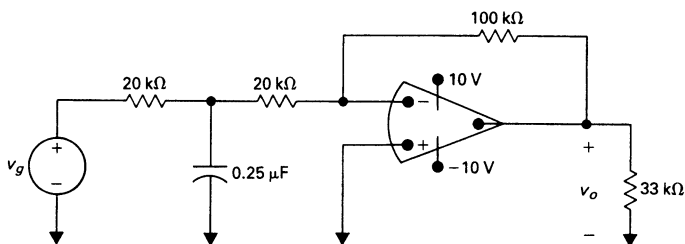


FIGURE P10.64

**10.65** The  $0.25\text{-}\mu\text{f}$  capacitor in the circuit seen in Fig. P10.64 is replaced with a variable capacitor. The capacitor is adjusted until the output voltage leads the input voltage by  $135^\circ$ .

- Find the value of  $C$  in microfarads.
- Write the steady-state expression for  $v_o(t)$  when  $C$  has the value found in part (a).

- 10.66** a) Find the input impedance  $Z_{ab}$  for the circuit in Fig. P10.66. Express  $Z_{ab}$  as a function of  $Z$  and  $K$  where  $K = (R_2/R_1)$ .
- b) If  $Z$  is a pure capacitive element what is the capacitance seen looking into the terminals a, b?

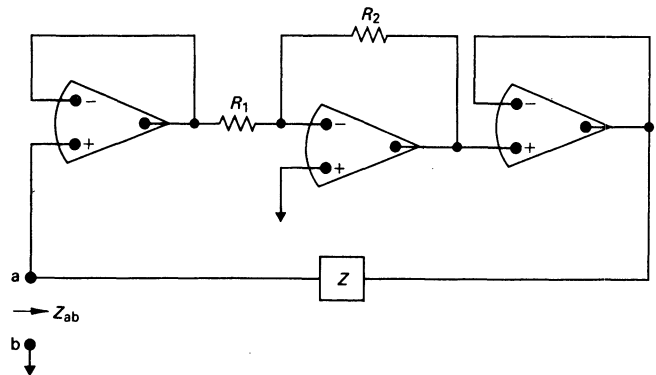


FIGURE P10.66

**10.67** You may have the opportunity as an engineering graduate to serve as an expert witness in lawsuits involving either personal injury or property damage. As an illustrative example of the type of problem on which you may be asked to give an opinion, consider the following event.

At the end of a day of fieldwork, a farmer returns to his farmstead, checks his hog-confinement building, and finds to his dismay that the hogs are dead. The problem is traced to a blown fuse that caused a 240-V fan motor to stop. The loss of ventilation led to the suffocation of the livestock. The interrupted fuse is located in the main switch that connects the farmstead to the electrical service.

Before the insurance company settles the claim, they want to know if the electric circuit supplying the farmstead functioned properly. The lawyers for the insurance company are puzzled because the farmer's wife, who was in the house on the day of the accident convalescing from minor surgery, was able to watch TV during the afternoon. Furthermore, when she went to the kitchen to start preparing the

evening meal, the electric clock indicated the correct time.

The lawyers have hired you to explain (1) why the electric clock in the kitchen and the television set in the living room continued to operate after the fuse in the main switch blew and (2) why the second fuse in the main switch didn't blow after the fan motor stalled.

After ascertaining the loads on the three-wire distribution circuit prior to the interruption of fuse A, you are able to construct the circuit model shown in Fig. P10.67. The

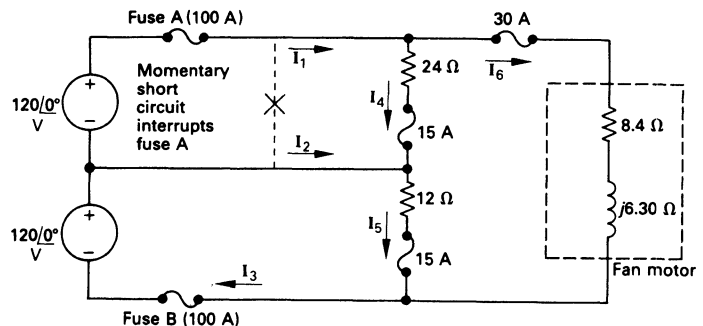


FIGURE P10.67

---

impedances of the line conductors and the neutral conductor are assumed negligible.

- a) Calculate the branch currents  $I_1$ ,  $I_2$ ,  $I_3$ ,  $I_4$ ,  $I_5$ , and  $I_6$  prior to the interruption of fuse A.
  - b) Calculate the branch currents after the interruption of fuse A. Assume the stalled fan motor behaves as a short circuit.
  - c) Explain why the clock and television set were not affected by the momentary short circuit that interrupted fuse A.
  - d) Assume the fan motor is equipped with a thermal cutout that is designed to interrupt the motor circuit if the motor current becomes excessive. Would you expect the thermal cutout to operate? Explain.
  - e) Explain why fuse B is not interrupted when the fan motor stalls.
-

- The transfer function  $H(j\omega)$  can be used to analyze the frequency response of the circuit because the amplitude and phase angle of  $H(j\omega)$  determine the amplitude and phase angle of the output signal. (If the poles and zeros of  $H(s)$  are spread out, Bode plots can be used to give a graphic picture of the amplitude and phase angle characteristics.)

The amplitude of  $H(j\omega)$  is expressed in decibels, a unit first introduced by engineers concerned with the power loss in telephone transmission circuits. Decibels relative to 1 milliwatt (dBm) was also discussed.

## PROBLEMS

- 17.1** There is no energy stored in the circuit seen in Fig. P17.1 at the time the two sources are energized.

- Use the principle of superposition to find  $V_o$ .
- Find  $v_o$  for  $t > 0$ .

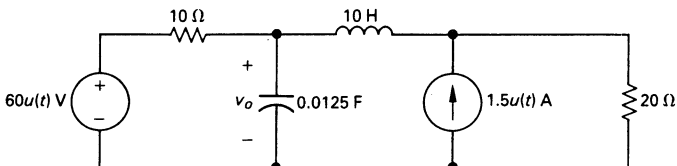


FIGURE P17.1

- 17.2** Verify that the solution of Eqs. (17.19) and (17.20) for  $V_2$  yields the same expression as that given by Eq. (17.18).

- 17.3** Find the numerical expression for the transfer function ( $V_o/V_i$ ) of each circuit in Fig. P17.3 and give the numerical value of the poles and zeros of each transfer function.

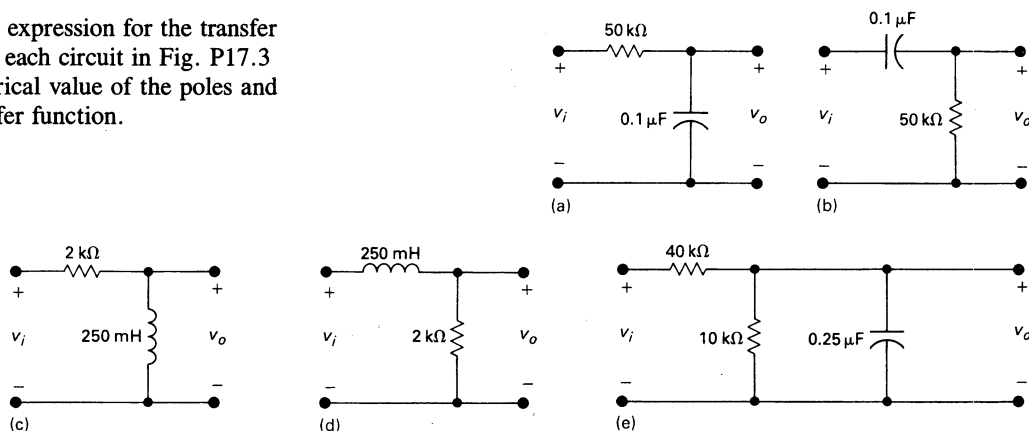


FIGURE P17.3

17.4 The voltage source in the circuit in Example 17.1 is changed to a unit impulse, that is,  $v_g = \delta(t)$ .

- How much energy does the impulsive voltage source store in the capacitor?
- How much energy does it store in the inductor?
- Use the transfer function to find  $v_o(t)$ .
- Show that the response found in part (c) is identical with the response generated by

first charging the capacitor to 1000 V and then releasing the charge to the circuit, as shown in Fig. P17.4.

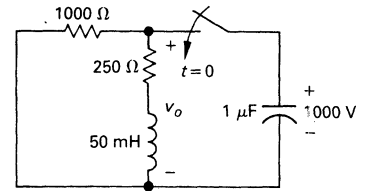


FIGURE P17.4

- 17.5 a) Find the numerical expression for the transfer function  $H(s) = V_o/V_i$  for the circuit in Fig. P17.5.
- b) Give the numerical value of each pole and zero of  $H(s)$ .

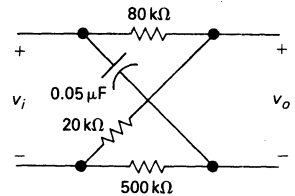


FIGURE P17.5

- 17.6 a) Derive the numerical expression of the transfer function  $H(s) = V_o/V_g$  for the circuit in Fig. P17.6.
- b) Give the numerical value of each pole and zero of  $H(s)$ .

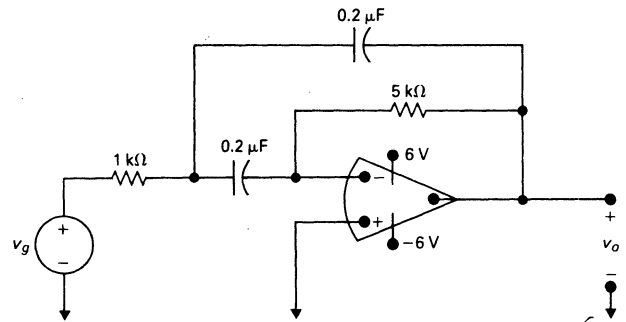


FIGURE P17.6

17.7 There is no energy stored in the circuit in Fig. P17.7 at the time the switch is opened. The sinusoidal current source is generating the signal  $100 \cos 10^4 t$  mA. The response signal is the current  $i_o$ .

- Find the transfer function  $I_o/I_g$ .
- Find  $I_o(s)$ .

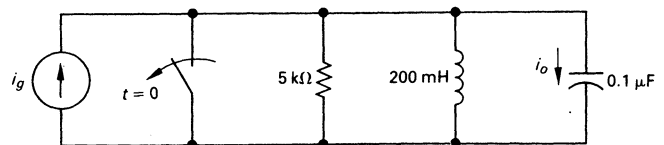


FIGURE P17.7

(continued)



- c) Describe the nature of the transient component of  $i_o(t)$  without solving for  $i_o(t)$ .
- d) Describe the nature of the steady-state component of  $i_o(t)$  without solving for  $i_o(t)$ .
- e) Verify the observations made in parts (c) and (d) by finding  $i_o(t)$ .

- 17.8** a) Find the transfer function  $I_o/I_g$  as a function of  $\mu$  for the circuit seen in Fig. P17.8.
- b) Find the largest value of  $\mu$  that will produce a bounded output signal for a bounded input signal.
- c) Find  $i_o$  for  $\mu = -0.5, 0, 1.0, 1.5$ , and  $2.00$  if  $i_g = 10u(t)$  A.

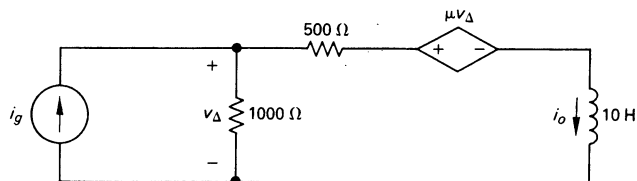


FIGURE P17.8

- 17.9** In the circuit of Fig. P17.9  $v_o$  is the output signal and  $v_g$  is the input signal. Find the poles and zeros of the transfer function.

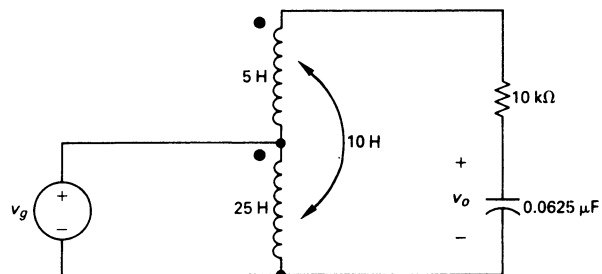


FIGURE P17.9

- 17.10** a) Find  $h(t) * x(t)$  when  $h(t)$  and  $x(t)$  are the rectangular pulses shown in Fig. P17.10(a).
- b) Repeat part (a) when  $x(t)$  changes to the rectangular pulse shown in Fig. P17.10(b).
- c) Repeat part (a) when  $h(t)$  changes to the rectangular pulse shown in Fig. P17.10(c).

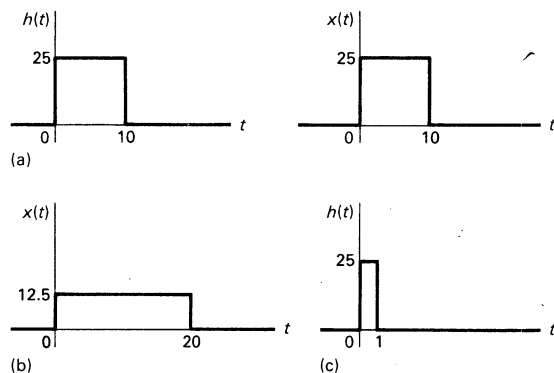


FIGURE P17.10

**17.11** The voltage impulse response of a circuit is shown in Fig. P17.11(a). The input signal to this circuit is the triangular voltage pulse shown in Fig. P17.11(b).

- Use the convolution integral to derive the expressions for the output voltage.
- Sketch the output voltage over the interval 0 to 25 s.
- Repeat parts (a) and (b) if the voltage impulse response of a second circuit is as shown in Fig. P17.11(c).
- Compare the two output voltages relative to their being scaled replicas of the input voltage.

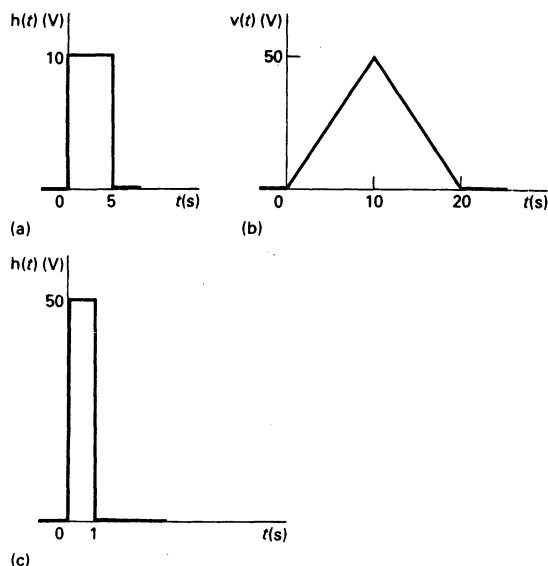


FIGURE P17.11

**17.12** The voltage impulse response of a circuit is shown in Fig. P17.12(a). The input signal to the circuit is the rectangular voltage pulse shown in Fig. P17.12(b).

- Derive the equations for the output voltage. Note the range of time for which each equation is applicable.
- Sketch  $v_o$  for  $-2 \leq t \leq 35$  s.

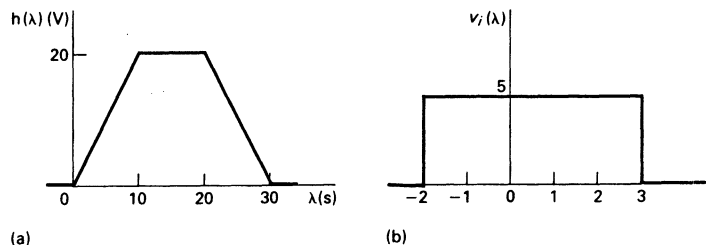


FIGURE P17.12

- 17.13**
- Use the convolution integral to find the output voltage of the circuit in Fig. P17.3(a) if the input voltage is the rectangular pulse shown in Fig. P17.13.
  - Sketch  $v_o(t)$  versus  $t$  for the time interval  $0 \leq t \leq 15$  ms.

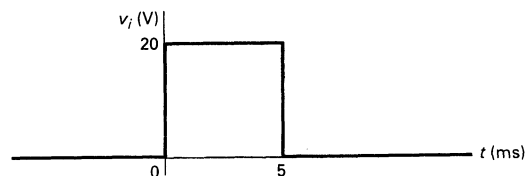


FIGURE P17.13

- 17.14**
- Repeat Problem 17.13, given that the resistor in the circuit in Fig. P17.3(a) is reduced to 5 kΩ.

- Does decreasing the resistor increase or decrease the “memory” of the circuit?
- Which circuit comes closest to transmitting a replica of the input voltage?

- 17.15** The input voltage in the circuit seen in Fig. P17.15 is

$$v_i = 5[u(t) - u(t - 0.5)] \text{ V.}$$

- a) Use the convolution integral to find  $v_o$ .  
b) Sketch  $v_o$  for  $0 \leq t \leq 1$  s.

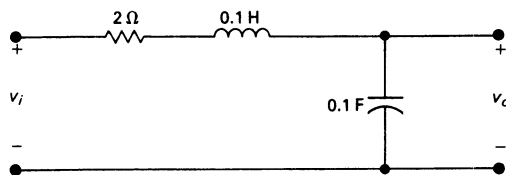


FIGURE P17.15

- 17.16** Use the convolution integral to find  $v_o$  in the circuit seen in Fig. P17.16 if  $v_i = 75u(t)$  V.

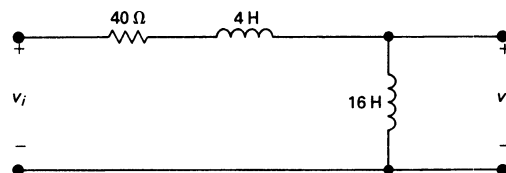
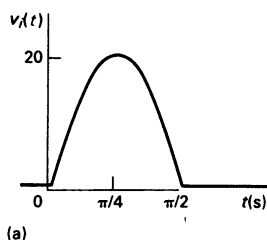
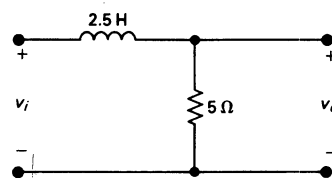


FIGURE P17.16

- 17.17** The sinusoidal voltage pulse shown in Fig. P17.17(a) is applied to the circuit shown in Fig. P17.17(b). Use the convolution integral to find the value of  $v_o$  at  $t = 2.2$  s.



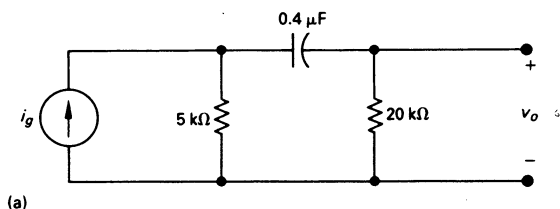
(a)



(b)

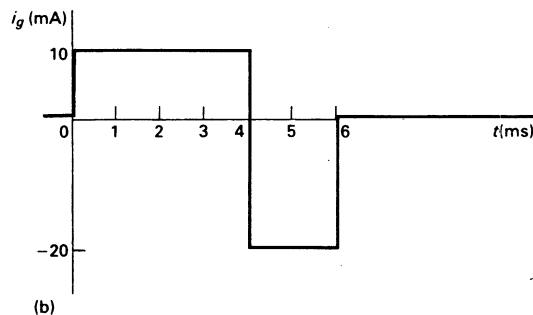
FIGURE P17.17

- 17.18** The current source in the circuit shown in Fig. P17.18(a) is generating the waveform shown



(a)

- in Fig. P17.18(b). Use the convolution integral to find  $v_o$  at  $t = 5$  ms.



(b)

FIGURE P17.18

- 17.19 a) Use the convolution integral to find  $v_o$  in the circuit in Fig. P17.19(a) if  $i_g$  is the pulse shown in Fig. P17.19(b).  
 b) Use the convolution integral to find  $i_o$ .  
 c) Show that your solutions for  $v_o$  and  $i_o$  are consistent by calculating  $i_o$  at  $100^-$  ms,  $100^+$  ms,  $200^-$  ms, and  $200^+$  ms.

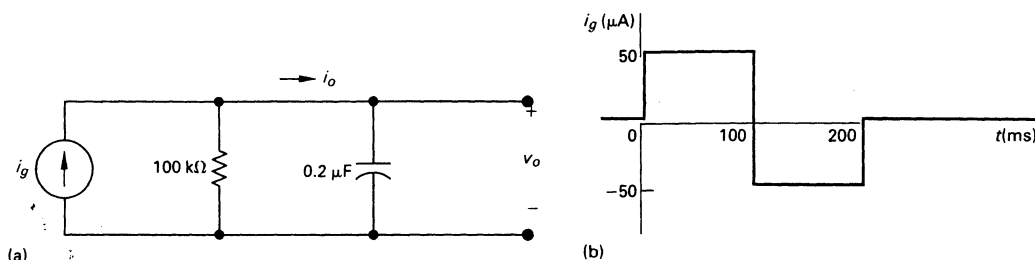


FIGURE P17.19

- 17.20 a) Find the impulse response of the circuit shown in Fig. P17.20(a) if  $v_g$  is the input signal and  $v_o$  is the output signal.  
 b) Given that  $v_g$  has the waveform shown in Fig. P17.20(b), use the convolution integral to find  $v_o$ .  
 c) Does  $v_o$  have the same waveform as  $v_g$ ? Why?

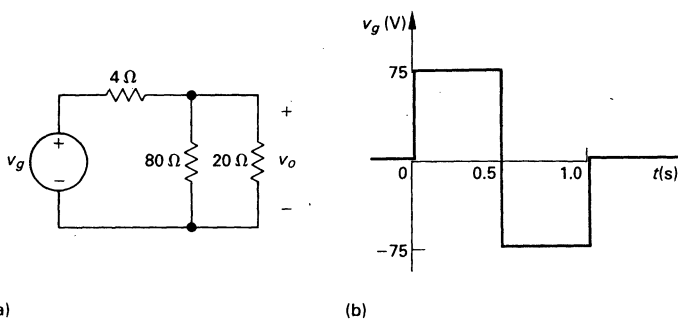


FIGURE P17.20

- 17.21 a) Find the impulse response of the circuit seen in Fig. P17.21 if  $v_g$  is the input signal and  $v_o$  is the output signal.  
 b) Assume that the voltage source has the waveform shown in Fig. P17.20(b). Use the convolution integral to find  $v_o$ .  
 c) Sketch  $v_o$  for  $0 \leq t \leq 2$  s.  
 d) Does  $v_o$  have the same waveform as  $v_g$ ? Why?

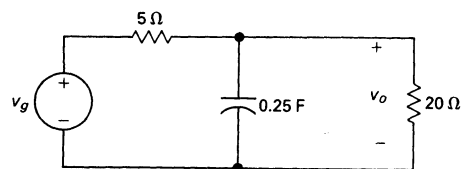


FIGURE P17.21

- 17.22 a) Show that if  $y(t) = h(t) * x(t)$  then  $Y(s) = H(s)X(s)$ .  
 b) Use the result given in part (a) to find  $f(t)$  if

$$F(s) = \frac{a}{s(s+a)^2}.$$

- 17.23 The transfer function for a linear time-invariant circuit is

$$H(s) = \frac{V_o}{V_g} = \frac{4(s+3)}{s^2 + 8s + 41}.$$

If  $v_g = 40 \cos 3t$  V, what is the steady-state expression for  $v_o$ ?

- 17.24 The operational amplifier in the circuit seen in Fig. P17.24 is ideal and is operating within its linear region.

- a) Calculate the transfer function  $V_o/V_g$ .  
 b) If  $v_g = 800 \cos 100t$  mV, what is the steady-state expression for  $v_o$ ?

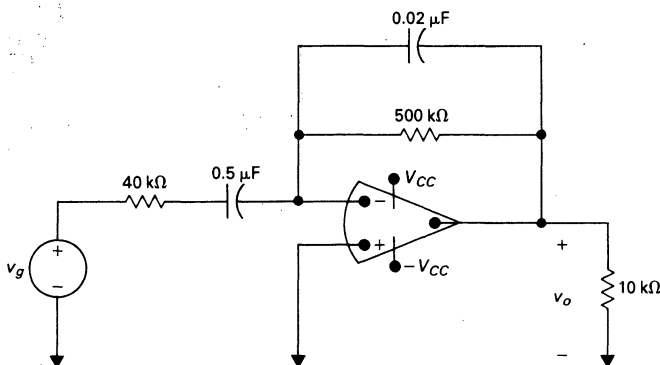


FIGURE P17.24

- 17.25 The operational amplifier in the circuit seen in Fig. P17.25 is ideal.

- a) Find the transfer function  $V_o/V_g$ .  
 b) Find  $v_o$  if  $v_g = 600u(t)$  mV.  
 c) Find the steady-state expression for  $v_o$  if  $v_g = 2 \cos 10,000t$  V.

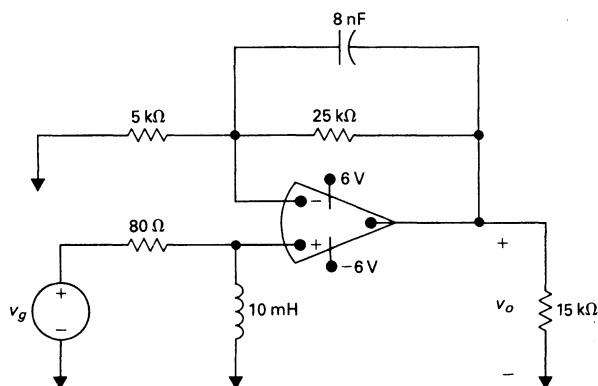


FIGURE P17.25

- 17.26** When an input voltage of  $50u(t)$  V is applied to a circuit, the response is known to be

$$v_o = (20 - 500te^{-25t} - 20e^{-25t})u(t) \text{ V.}$$

What will the steady-state response be if  $v_g = 100 \cos 50t$  V?

- 17.27** Make straight-line (uncorrected) amplitude and phase-angle plots for each of the transfer functions derived in Problem 17.3.

- 17.28** Make straight-line amplitude and phase-angle plots for the voltage transfer function derived in Problem 17.5.

- 17.29** a) Derive the numerical expression of the transfer function  $I_o/I_g$  for the circuit in Fig. P17.29.  
 b) Make a corrected amplitude plot for the transfer function derived in part (a).  
 c) At what frequency is the amplitude maximum?  
 d) What is the maximum amplitude in decibels?  
 e) At what frequencies is the amplitude down 3 dB from the maximum?  
 f) What is the bandwidth of the circuit?  
 g) Check your graphical results by calculating the actual amplitude in decibels at the frequencies read from the plot.

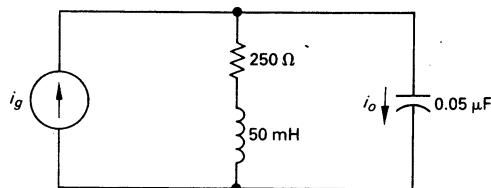


FIGURE P17.29

- 17.30** Use Bode diagrams to describe the behavior of the circuit in Problem 10.60 as  $R_x$  is varied from zero to infinity.

- 17.31** Given the following current transfer function:

$$H(s) = \frac{V_o}{V_i} = \frac{10^8}{s^2 + 3000s + 10^8}.$$

- a) At what frequencies (in radians per second) is the ratio of  $V_o/V_i$  equal to unity?  
 b) At what frequency is the ratio maximum?  
 c) What is the maximum value of the ratio?

**17.32** The circuit shown in Fig. P17.32 resembles the interstage coupling network of an amplifier.

a) Show that

$$H(s) = \frac{V_o}{V_i} = \frac{(1/R_1 C_2)s}{s^2 + [(1/R_1 C_1) + (1/R_2 C_2) + (1/R_1 C_2)]s + (1/R_1 C_1 R_2 C_2)}.$$

- b) Find the numerical expression for  $H(s)$  if  $R_1 = 40 \text{ k}\Omega$ ,  $C_1 = 0.1 \text{ }\mu\text{F}$ ,  $R_2 = 10 \text{ k}\Omega$ , and  $C_2 = 250 \text{ pF}$ .
- c) Give the numerical values of the poles and zeros of  $H(s)$ .
- d) Give an approximate numerical expression for  $H(s)$  for values of  $s$  much less than the highest corner frequency, that is, for values of  $s$  close to the lowest corner frequency.
- e) Give an approximate numerical expression for  $H(s)$  for values of  $s$  much larger than the lowest corner frequency.
- f) Show that the expression derived in part (d) is equivalent to the transfer function of the circuit in Fig. P17.32 when  $C_2$  is neglected at low frequencies.
- g) Show that the expression derived in part (e) is equivalent to the transfer function of the circuit in Fig. P17.32 when  $C_1$  is neglected at high frequencies.

This problem illustrates that when the numerical values of the circuit parameters are known, it is sometimes possible to use different circuit models in different frequency ranges. Quite frequently electronic-amplifier equivalent circuits can be divided into models that apply to low-, mid-, and high-frequency ranges.

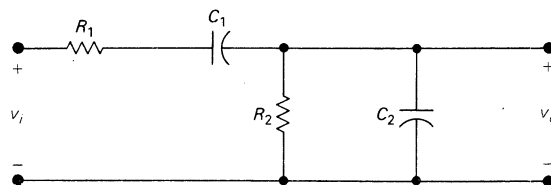


FIGURE P17.32

- 17.33** a) Find the resistance seen looking into the terminals a, b of the circuit in Fig. P17.33.
- b) Find the power loss through the network, in decibels, when the output power is the power delivered to the  $25\text{-}\Omega$  resistor.

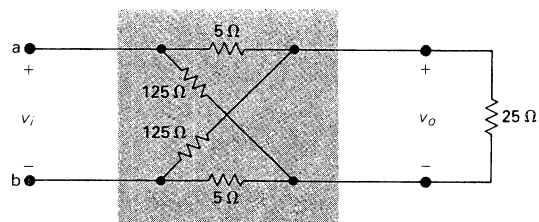


FIGURE P17.33

- 17.34 The amplitude plot of a transfer function is shown in Fig. P17.34. What is the numerical expression for  $H(s)$ ?

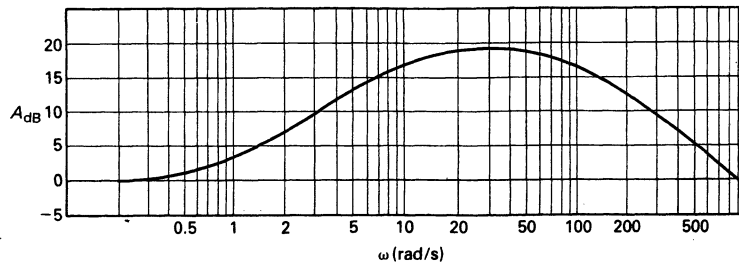


FIGURE P17.34