ENGR 210
F03 HW 11
One more to go...

SOLUTIONS

1) 7-15

$$\frac{d}{dt}v(t) + 200v(t) = v_s(t) = 25\cos(100t)\ V$$
$$v(0) = 0\ V$$

$$v(t) = v_N(t) + v_F(t)$$
$$v_N(t) = Ke^{-\frac{t}{T_c}}$$

rewrite original equation

$$\frac{1}{200}\frac{d}{dt}v(t) + v(t) = \frac{1}{8}\cos(100t)\ V$$

($\frac{1}{200} \rightarrow T_c$, $\frac{1}{8} \rightarrow V_A$, $100 \rightarrow \omega$)

$$v_N = Ke^{-200t}$$

$$v_F = \frac{V_A}{1+(\omega T_c)^2}\left[\cos(\omega t) + \omega T_c \sin(\omega t)\right]$$

$$= \frac{\frac{1}{8}}{1+(\frac{1}{2})^2}\left[\cos(100t) + \frac{1}{2}\sin(\omega t)\right]$$

$$= \frac{1}{8+8\cdot\frac{1}{4}}\left[\cos(100t) + \frac{1}{2}\sin(100t)\right]$$

$$= \frac{1}{10}\left[\cos(100t) + \frac{1}{2}\sin(100t)\right]$$

$$v(t) = Ke^{-200t} + \frac{1}{10}\left[\cos(100t) + \frac{1}{2}\sin(100t)\right]$$

$$v(0) = 0 = K + \frac{1}{10}$$
$$K = -\frac{1}{10}$$

$$\boxed{v(t) = -\frac{1}{10}e^{-200t} + \frac{1}{10}\left[\cos(100t) + \frac{1}{2}\sin(100t)\right]}$$

7-18) before $t=0$, inductor behaves as short:

$$i_L(t) = \frac{V}{R} = \frac{5V}{1k\Omega}$$

$$i_L(0) = 5mA$$

after time $t=0$:

as $t \to \infty$, inductor will again be short:

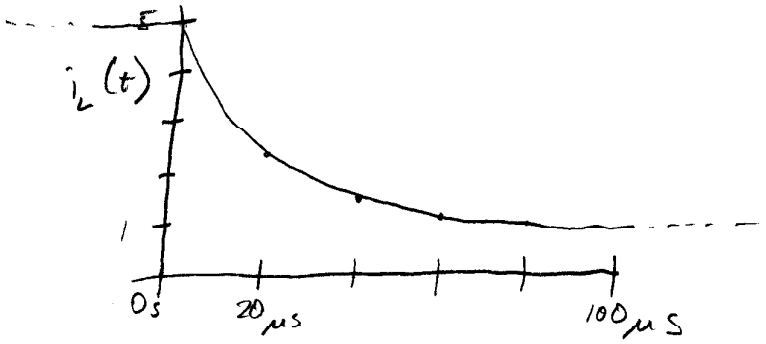
$$i_L(\infty) = \frac{5V}{5k\Omega} = 1mA$$

$$i_L(t) = FC + (IC - FC)e^{-t/\tau_L}$$

$$= 1mA + (4mA)e^{-\frac{t}{20\mu s}}$$

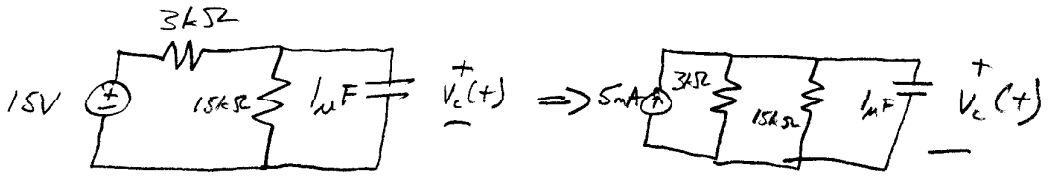
$$\tau_L = \frac{L}{R} = \frac{100mH}{5k\Omega} = 20\mu s$$

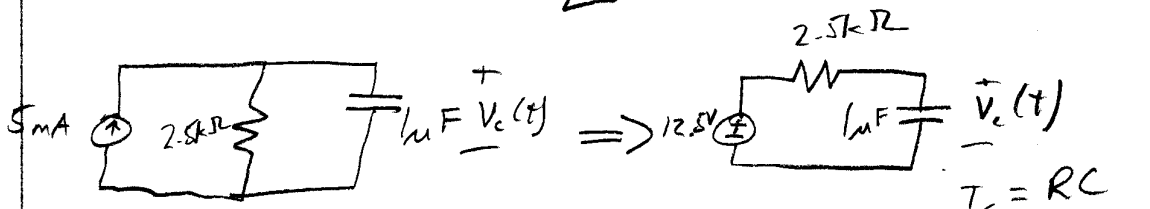
$$\boxed{i_L(t) = 1mA + 4mA\,e^{-50{,}000\,t/s}}$$



3/7-21) by $t=0$, capacitor is at $v_c(0)=0V$ (can discharge through 15kΩ resistor)

for $0 \le t \le 2ms$:





$T_c = RC = 2.5ms$

$$v_c(t) = 12.5V + 12.5V e^{-\frac{t}{2.5ms}}$$

at $t=2ms$:

$$v_c(0.002) = 12.5V - 12.5V e^{-\frac{2}{2.5}}$$
$$\approx 6.88\ V$$

$t > 2ms$:

$T_c = RC = 15ms$

$$v_c(t) = \left(12.5V - 12.5V e^{-\frac{2}{2.5}}\right) e^{-\frac{(t-.002s)}{15ms}}$$

$$\boxed{v_c(t) = \left(12.5V - 12.5V e^{-.8}\right) e^{\frac{0.002s - t}{15ms}}}$$
$$\boxed{\approx 6.88\,V e^{0.133 - 66.7\,t/s}}$$

7-23) a) $T_c = \frac{1}{1000} = 0.001\,s = \boxed{1ms = T_c}$

b) state variable is $v_c(t)$

$\boxed{v_c(0) = 10V}$

c) $i_c(t) = C \frac{d}{dt} v_c(t)$

$-20e^{-1000t}\ mA = C \cdot -1000/s \cdot 10e^{-1000t}\ V$

$\frac{2}{1000}\ \frac{mA \cdot s}{V} = C$

$\boxed{2\mu F = C}$

$R = \frac{T_c}{C} = \frac{1ms}{2\mu F} = 500\,\Omega$

d) $w_c(t) = \frac{1}{2} C v_c^2(t)$

$v_c(t) = 10e^{-1000t}$

$v_c(1ms) = 10e^{-1}$

$= 3.68\,V$

$= \frac{1}{2} \cdot 2\mu F \cdot (3.68\,V)^2$

$\boxed{w_c(1ms) = 13.5\,\mu J}$

8-4) $2V_1 + V_2 = 20e^{-j30^\circ} + 60e^{-j220^\circ}$

$= 20\cos(30^\circ) - j20\sin(30^\circ) + 60\cos(220^\circ) + j60\sin(220^\circ)$

$\approx -28.6 + j28.6$

$= 40.5\,e^{j135^\circ}$

$\boxed{2v_1(t) + v_2(t) = 40.5\cos(10^4 t + 135^\circ)}$

b) $I_1 + 3I_2 = 5e^{j90^\circ} + 3.2e^{j270^\circ}$

$= 5e^{j90^\circ} + 6e^{j270^\circ}$

$= 5\cos 90^\circ + j5\sin 90^\circ + 6\cos 270^\circ + j6\sin 270^\circ$

$= -j$

$= e^{-j90^\circ}$

$\boxed{i_1(t) + 3i_2(t) = \cos(200t - 90^\circ)}$

8-6) a) $\tilde{V}_1 = 10 + j40$

$= 10\sqrt{17}\, e^{+j76°}$

$\boxed{v_1(t) \approx 41.2 \cos(10t + 76°)}$

b) $\tilde{V}_2 = (8 - j3)\, 5 e^{-60°}$

$= \sqrt{73}\, e^{-j20.6°} \cdot 5 e^{-60°}$

$= 5\sqrt{73}\, e^{-j80.6°}$

$\boxed{v_2(t) = 42.7 \cos(20t - 80.6°)}$

c) $\tilde{I}_1 = 8 - j3 + \frac{3}{j}$

$= 8 - j3 - j3$

$= 8 - 6j$

$= 10 e^{-j36.9°}$

$\boxed{i_1(t) = 10 \cos(300t - 36.9°)}$

d) $\tilde{I}_2 = \frac{3 + j}{1 - j3}$

$= \frac{\sqrt{10}\, e^{j18.4°}}{\sqrt{10}\, e^{-j71.6°}}$

$= e^{j90°}$

$\boxed{i_2(t) = \cos(50t + 90°)}$