P&P Review Notes

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NOTE: Please inform me of your opinion of the relative emphasis of the topics in this review material by any of the above FAX or e-mail number addresses or simply write me your comments and send them by US mail to

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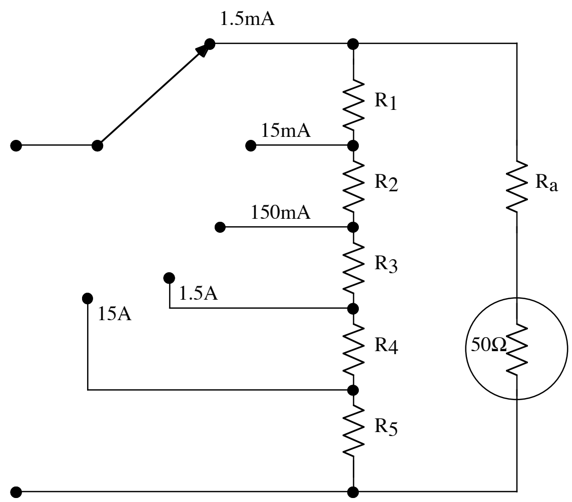
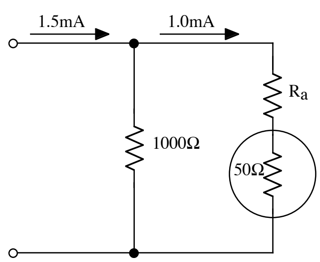
Your input will be used to revise next year's exam review course.

Kirchoff's Law's

3. An ammeter is being designed to measure currents over the five ranges indicated in the accompanying illustration. The indicating meter is a 1.00 milliampere movement with an internal resistance of 50 ohms. The total resistance $(R_1 + R_2 + R_3 + R_4 + R_5)$ is to be 1000 ohms. Specify the resistances R_a and R_1 through R_5.



For 1.5mA



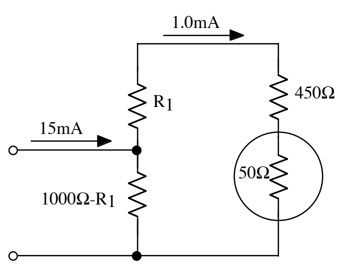
There is 0.5 milliampere through the 1kΩ resistor.
Since the voltages must be equal:

$$1000(0.5mA) = 1mA(R_a + 50\Omega)$$

or

$$R_a = 450\Omega$$

For 15mA



$$1mA(R_1 + 450\Omega + 50\Omega) = (15-1)mA(1000 - R_1)$$

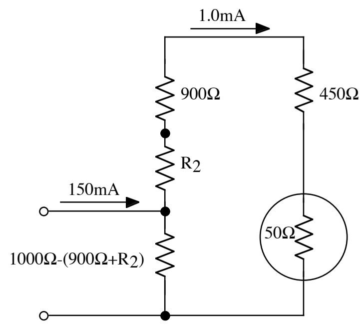
$$R_1 + 500\Omega = 14000 - 14R_1$$

$$15R_1 = 14000 - 500 = 13500$$

$$R_1 = 900$$

For 150mA

$$1mA\left(R_2 + 900\Omega + 450\Omega + 50\Omega\right) = (150-1)mA\left(1000 - \left(900\Omega + R_2\right)\right)$$



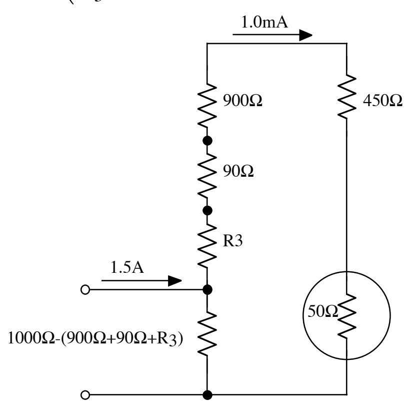
$$R_2 + 1400\Omega = 149\left(100 - R_2\right)$$

$$150R_2 = 14900 - 1400 = 13500$$

$$R_2 = 90$$

For 1.5A

$$1mA\left(R_3 + 900\Omega + 90\Omega + 450\Omega + 50\Omega\right) = (1500-1)mA\left(1000 - \left(900\Omega + 90\Omega + R_3\right)\right)$$



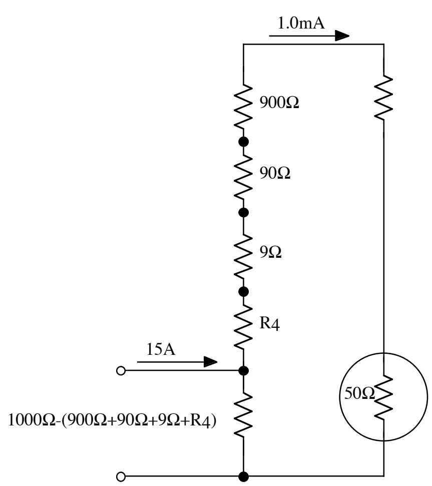
$$R_3 + 1490\Omega = 1499\left(10 - R_3\right)$$

$$1500R_3 = 13500$$

$$R_3 = 9\Omega$$

For 15A

$$1mA\left(R_4 + 9\Omega + 90\Omega + 900\Omega + 450\Omega + 50\Omega\right) = (15000-1)mA\left(1000 - \left(900\Omega + 90\Omega + 9\Omega + R_4\right)\right)$$



$$R_4 + 1499\Omega = 14999\left(1 - R_4\right)$$

$$15000R_4 = 13500$$

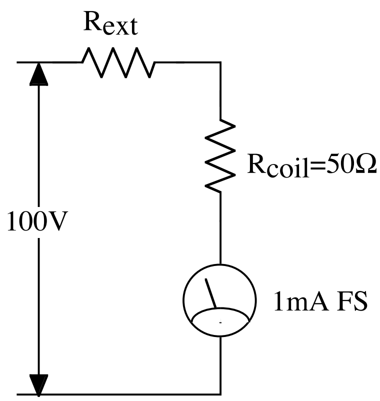
$$R_4 = 0.9\Omega$$

And finally $R_5 = 1000\Omega - \left(900\Omega + 90\Omega + 9\Omega + 0.9\Omega\right) = 0.1\Omega$

1. A voltmeter is being designed to measure voltages in the full-scale ranges of 3, 10, 30 and 100 volts DC. The meter movement to be used has an internal resistance of 50 ohms and a full-scale current of 1 mA. Using a four-pole, single-throw switch, design the voltmeter.

The meter circuit is easily designed using the equivalent circuit of the meter.

@100 volts



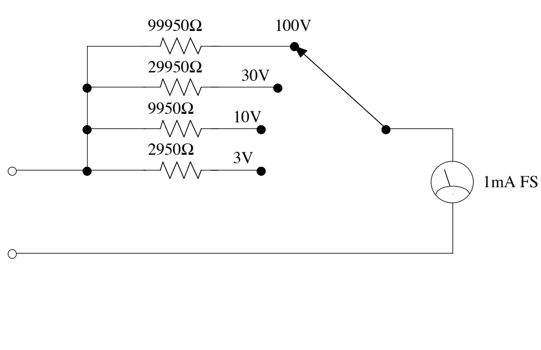
$$R = \frac{V}{I} = \frac{100 volts}{1 mA} = 10^5 \Omega$$
$$R_{ext} = 10^5 \Omega - 50 = 99950 \Omega$$

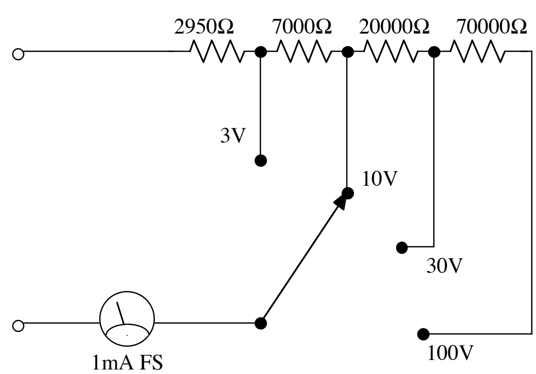
@30V $R = \frac{V}{I} = \frac{30 volts}{1 mA} = 30000 \Omega$ and $R_{ext} = 30000 - 50 = 29950 \Omega$

@10V $R = \frac{V}{I} = \frac{10 volts}{1 mA} = 10000 \Omega$ and $R_{ext} = 10000 - 50 = 9950 \Omega$

@3V $R = \frac{V}{I} = \frac{3 volts}{1 mA} = 3000 \Omega$ and $R_{ext} = 3000 - 50 = 2950 \Omega$

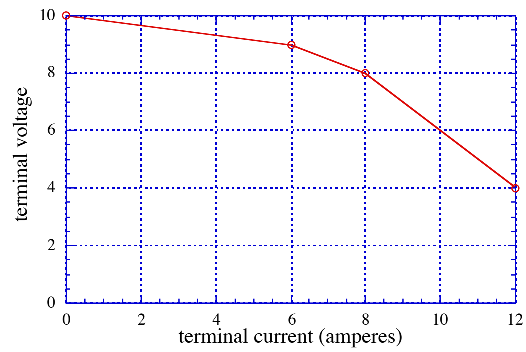
You can design several different types of meter circuits using this data.





Thevenin's Theorem, Norton's Theorem

Example 2.5
Obtain an expression for the characteristic curve shown over the range from 8 to 12 amps. Obtain linear circuit models from the derived expressions.



5. Obtain a linear model of voltage versus current for the characteristic shown in example 2.5 over the current range of 0 to 6 amps. Obtain the current source and voltage source equivalent circuits.

Use the two-point method to fit a straight line to the data

$$\frac{V - V_0}{I - I_0} = \frac{V_1 - V_0}{I_1 - I_0}$$

Read two points from the graph: point 0: 10 volts, 0 amps; point 1: 9 volts, 6 amps

Evaluating the expression,

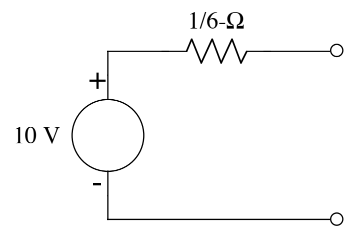
$$\frac{V - 10}{I - 0} = \frac{9 - 10}{6 - 0}$$

$$\frac{V - 10}{I} = -\frac{1}{6}$$

$$6V - 60 = -I$$

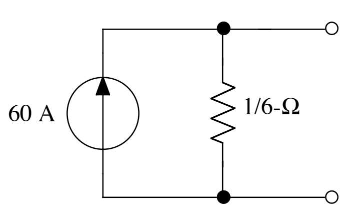
$$6V + I = 60$$

$$V = \frac{-I + 60}{6} = -\frac{1}{6}I + 10$$



NOTE: Thevenin is usually easier to visualize

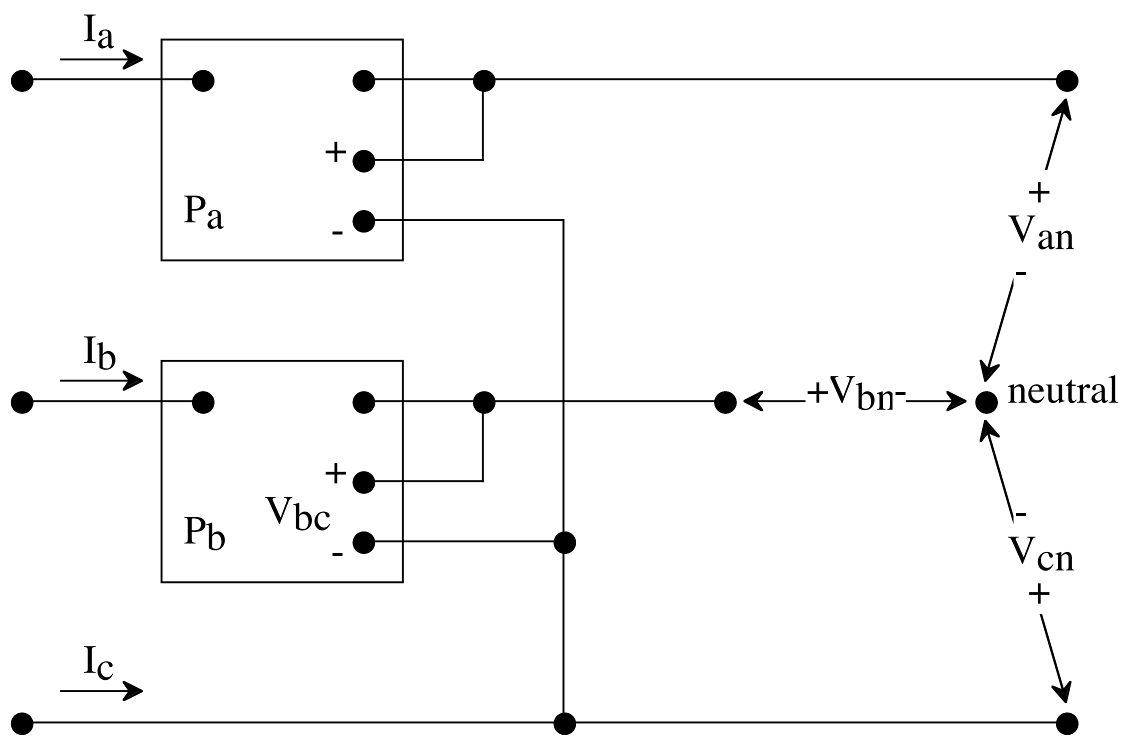
$$I = -6V + 60$$



Power and Energy Calculations

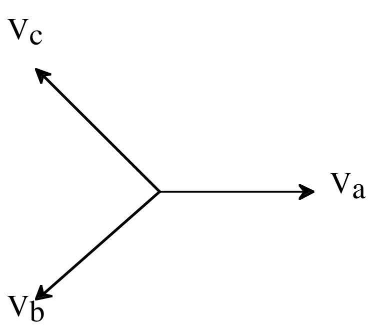
10. The measurement system shown below is used for a balanced load of 1 kW with a lagging power factor of 0.8. Determine the wattmeter readings.

See Section 3-16 for an explanation of the two-wattmeter method of three phase power measurement.



The neutral is an artificial point used to make the two-wattmeter analysis easier.

Since it is not specified assume an ABC phase sequence. For this problem $PF = \cos\theta = 0.8$ lagging so that $\theta = +36.87°$.



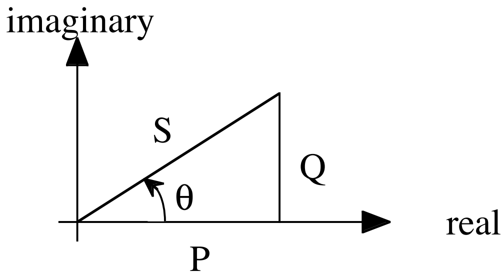
The voltages and currents are then specified by

$V_{an} = V\angle 0°$ $\quad$ $I_a = I\angle -\theta = I\angle -36.87°$

$V_{bn} = V\angle -120°$ $\quad$ $I_b = I\angle -\theta - 120° = I\angle -156.87°$

$V_{cn} = V\angle +120°$ $\qquad$ $I_c = I\angle -\theta + 120° = I\angle +83.13°$

Recall the power triangle and write the expressions for the voltages and currents as seen by the wattmeters.



For wattmeter A:
$V_{ac} = V_{an} - V_{cn} = V\angle 0° - V\angle +120° = V(1.732\angle -30°)$
$I_a = I\angle -36.87°$
$S_{ac} = V_{ac}I_a^* = V(1.732\angle -30°)(I\angle -36.87°)^* = VI(1.732\angle -30°)(1\angle +36.87°) = VI(1.732\angle 6.87°)$
Or, in rectangular form
$S_{ac} = VI(1.72 + j0.207)$

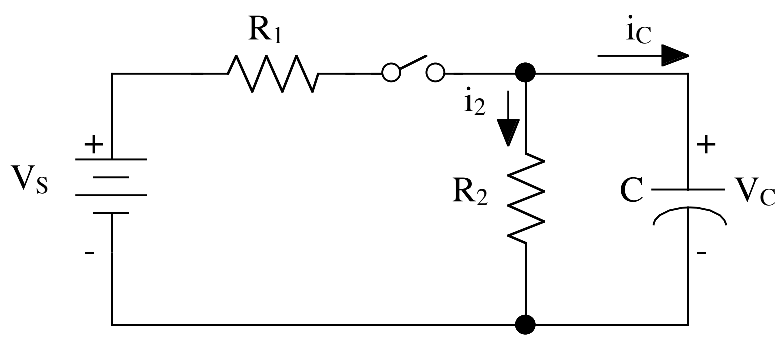
For wattmeter B:
$V_{bc} = V_{bn} - V_{cn} = V\angle -120° - V\angle +120° = V(1.732\angle -90°)$
$I_b = I\angle -156.87°$
$S_{bc} = V_{bc}I_b^* = V(1.732\angle -90°)(I\angle -156.87°)^* = VI(1.732\angle -90°)(1\angle +156.87°) = VI(1.732\angle 66.87°)$
Or, in rectangular form
$S_{bc} = VI(0.68 + j1.59)$

For a balanced load $P = \sqrt{3}V_{line-line}I_{line}\cos\theta$ and $V_{phase} = \frac{V_{line-line}}{\sqrt{3}}$
In this problem V_{an}, V_{bn}, and V_{cn} are phase voltages. V_{ac} and V_{bc} are line-line voltages. Then
$P = \sqrt{3}\sqrt{3}V_{phase}I_{line}\cos\theta = 3V_{phase}I_{line}\cos\theta$
Using the numbers for this problem
$1000 watts = 3V_{phase}I_{line}(0.8)$
or VI=416.67 watts

As a check on our calculations
$P_a = \text{Re}\{S_{ac}\} = 1.72VI = 1.72(416.67) = 716.67\text{watts}$
$P_b = \text{Re}\{S_{bc}\} = 0.68VI = 0.68(416.67) = 283.33$
These powers add up to exactly 1000 watts so the answer looks good.

Transient Analysis

2. For the circuit shown in example 2.2, assume the switch has been closed for a long time and is opened at t=0. Determine (a) the current through the capacitor at the instant that the switch is opened, (b) the time derivative of the capacitor voltage at the instant the switch is opened, and (c) the voltage that will exist across the capacitor after a long time.

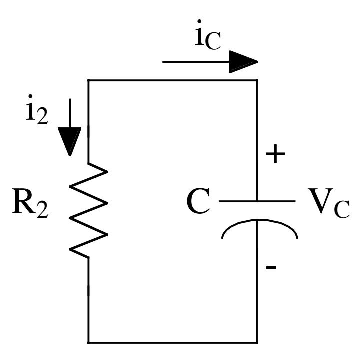


The boundary conditions for the capacitor C are

1. no DC current flow
2. $v(0^+) = v(0^-)$

By inspection, $v_C(0^-) = \frac{R_2}{R_1 + R_2} V_S$ since there is no DC current through C.

(a) When the switch is open



$$i_2(0^+) = \frac{v_C(0^+)}{R_2} = \frac{v_C(0^-)}{R_2} = \frac{\frac{R_2}{R_1 + R_2} V_S}{R_2} = \frac{V_S}{R_1 + R_2}$$

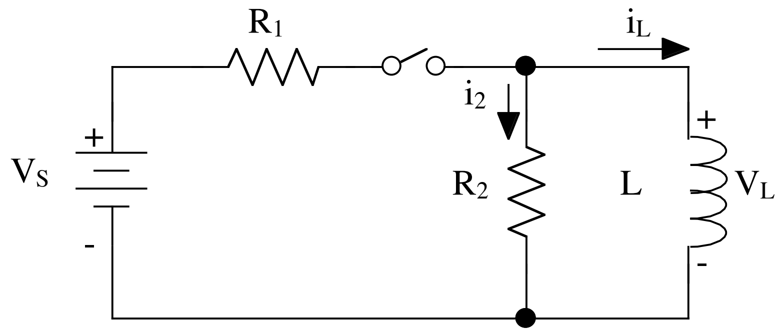
Then, $i_C(0^+) = -i_2(0^+) = \frac{-V_S}{R_1 + R_2}$

(b) Using the derivative relationship $i = C\frac{dv}{dt}$ for C

$$\frac{dv}{dt}(t = 0^+) = \frac{i_C(0^+)}{C} = \frac{-V_S}{(R_1 + R_2)C}$$

(c) $v_C(t = \infty) = 0$

3. For the circuit shown in example 2.3, assume that the switch has been closed for a long time (the final conditions of example 2.3 apply), and the switch is opened at t=0. Determine at the instant the switch opens (a) the current in the inductance, (b) the time rate of change of current in the inductance, and (c) the voltage across the inductance.

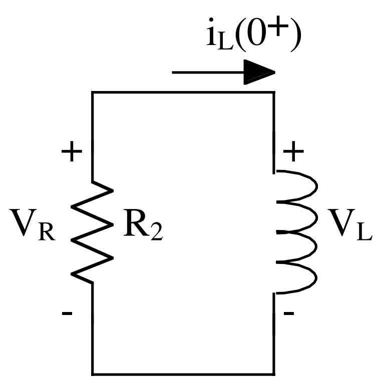


The boundary conditions for an inductor are $i_L(0^+) = i_L(0^-)$

(a) $i_L(0^-) = \frac{V_S}{R_1}$ since the inductor is a DC short

Therefore, $i_L(0^+) = \frac{V_S}{R_1}$

(b) For an inductor, $v = L\frac{di}{dt}$



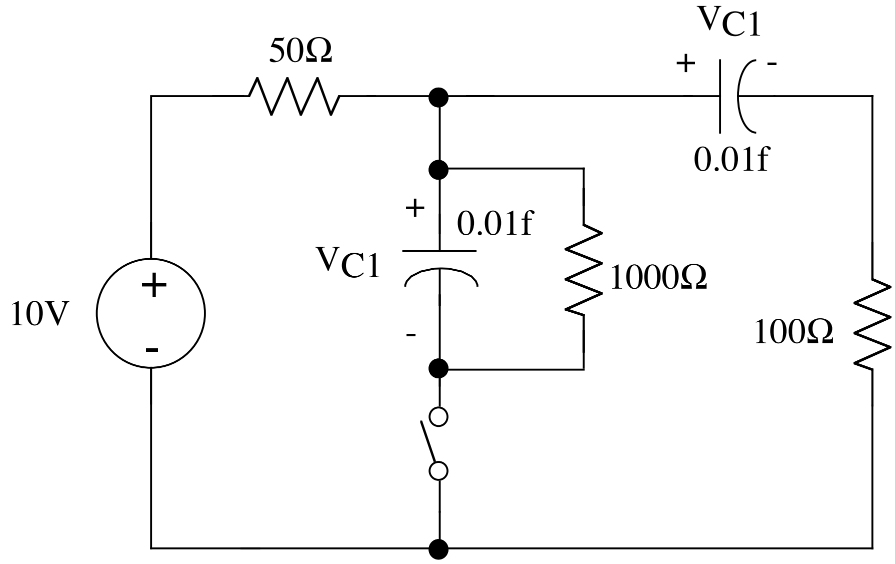
$$V_R = -i_L(0^+)R_2 = -\frac{V_S R_2}{R_1}$$

$$\frac{di}{dt} = \frac{V_L}{L} = \frac{V_R}{L} = -\frac{V_S R_2}{L R_1}$$

(c)

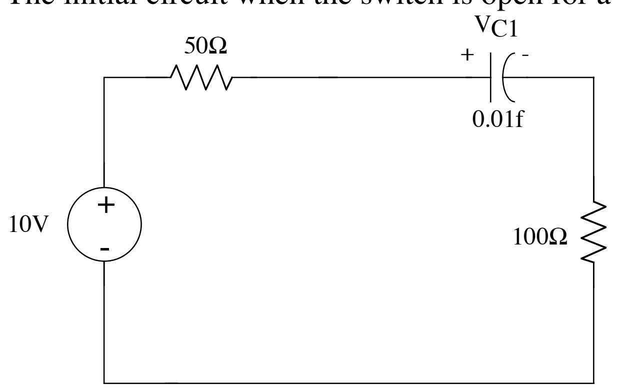
$$V_L = -\frac{V_S R_2}{R_1}$$

10. The switch of the circuit shown has been open for a long time and is closed at t=0.

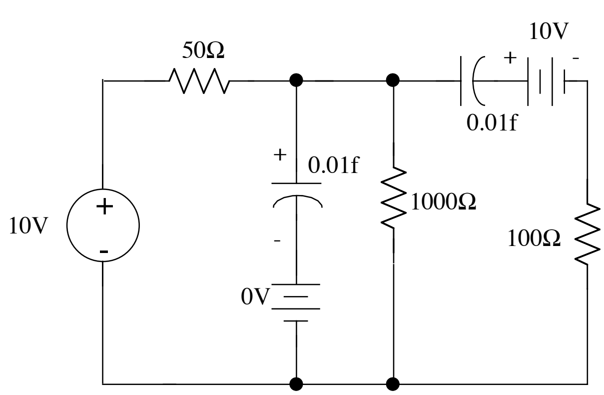


Determine the currents flowing in the two capacitances at the instant the switch closes. Give the magnitude of the capacitances and directions.

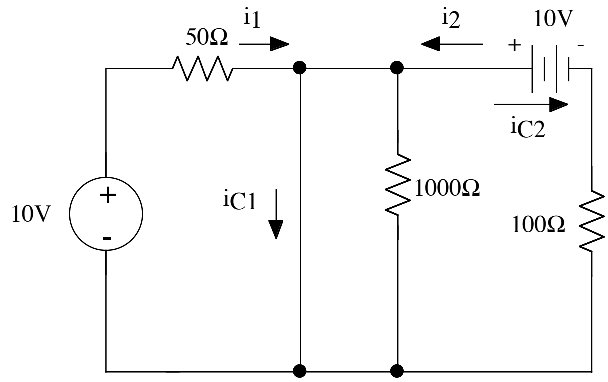
The initial circuit when the switch is open for a long time is very simple.



The initial conditions are $V_{C1}(0^-) = 0$ and $V_{C2}(0^-) = +10V$
When switching capacitors look like ideal voltages sources which retain their initial voltages, i.e., the circuit at t=0+ looks like



which can be reduced to



By inspection (and being careful to follow the passive sign convention)

$$i_1 = \frac{10V}{50\Omega} = \frac{1}{5} Amp$$

$$i_2 = \frac{10V}{100\Omega} = \frac{1}{10} Amp$$

Then,

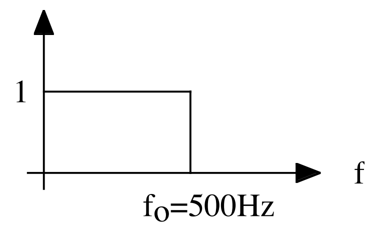
$$i_{C1} = \frac{1}{5} + \frac{1}{10} = 0.3 Amp$$

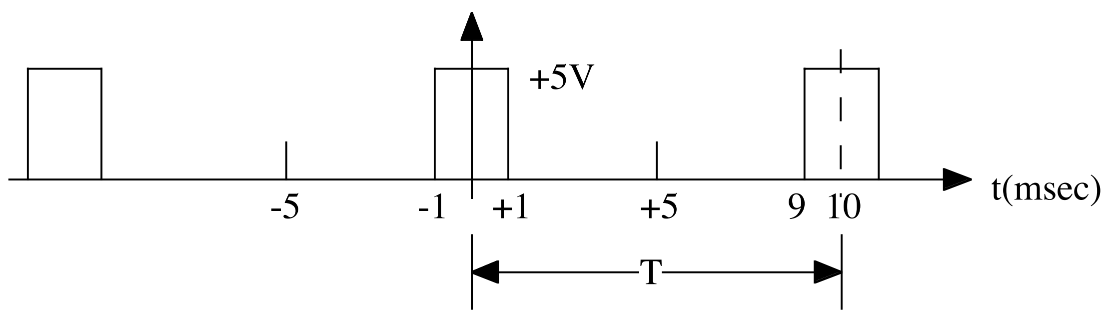
$$i_{C2} = -i_2 = -\frac{1}{10} Amp$$

Fourier Analysis

3. A train of rectangular pulses is applied to an ideal low-pass filter circuit. The pulse height is 5 volts, and the duration of each pulse is 2ms. The repetition period is 10ms. The low-pass filter has a cut-off frequency of 500 Hz. What percentage of the signal power is available at the output of the filter?

The ideal filter





As drawn this pulse train is an even function. That was my option since I prefer even function series.

$$f(t)=\frac{1}{2}a_o+\sum_{n=1}^{\infty}a_n\cos\left(\frac{2\pi n}{T}t\right)+\sum_{n=1}^{\infty}b_n\sin\left(\frac{2\pi n}{T}t\right)$$

The last term of this expression for $f(t)$ can be ignored. All $b_n=0$ since $f(t)$ is an even function and sine is odd.

$$v(t)=\frac{a_o}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{2\pi n}{T}t\right)$$

$$a_o=2f_{avg}=2\frac{1}{10m\sec}\left[(5V)(2m\sec)\right]=2V$$

$$a_n=\frac{2}{T}\int_{t_1}^{t_1+T}f(t)\cos\left(\frac{2\pi n}{T}t\right)dt$$

Do the integral from t=-0.005 to t=+0.005 seconds. For that case, the integral reduces to

$$a_n = 2\frac{2}{0.01}\int_0^{0.001} 5\cos\left(\frac{2\pi n}{0.01}t\right)dt = \frac{4\times 5}{0.01}\left(\frac{0.01}{2\pi n}\right)\sin\left(\frac{2\pi}{0.01}nt\right)\Bigg|_0^{0.001} = \frac{10}{\pi n}\sin\left(\frac{\pi}{5}n\right)$$

$$\omega_n = \frac{2\pi n}{T} = \frac{2\pi}{0.01}n$$

$$f_n = \frac{n}{T} = \frac{n}{0.01} = 100n \text{ Hz}$$

The ideal filter will pass a_o, a_1, a_2, a_3, a_4 and a_5

Power is rms voltage squared.

$$P_{out} = \frac{V_{rms}^2}{R} = \frac{1}{R}\left[\left(\frac{a_o}{2}\right)^2 + \frac{a_1^2}{2} + \frac{a_2^2}{2} + \frac{a_3^2}{2} + \frac{a_4^2}{2} + \frac{a_5^2}{2}\right]$$

The first term is different since the rms value of the dc term is the dc term $\frac{a_o}{2}$. For all other terms the rms voltage is given by $V_{rms} = \frac{V_{peak}}{\sqrt{2}}$. Computing the terms we get

$$a_0 = 2$$

$$a_1 = \frac{10}{\pi}\sin\left(\frac{\pi}{5}\right) = 1.87$$

$$a_2 = \frac{10}{2\pi}\sin\left(\frac{2\pi}{5}\right) = 1.51$$

$$a_3 = \frac{10}{3\pi}\sin\left(\frac{3\pi}{5}\right) = 1.01$$

$$a_4 = \frac{10}{4\pi}\sin\left(\frac{4\pi}{5}\right) = 0.47$$

$$a_5 = \frac{10}{5\pi}\sin\left(\frac{5\pi}{5}\right) = 0$$

The output power is then

$$P_{out} = \frac{V_{rms}^2}{R} = \frac{1}{R}\left[\left(\frac{2}{2}\right)^2 + \frac{(1.87)^2}{2} + \frac{(1.51)^2}{2} + \frac{(1.01)^2}{2} + \frac{(0.47)^2}{2} + 0\right] = \frac{4.51}{R}$$

Using the definition of rms to determine the power for the input pulse waveform

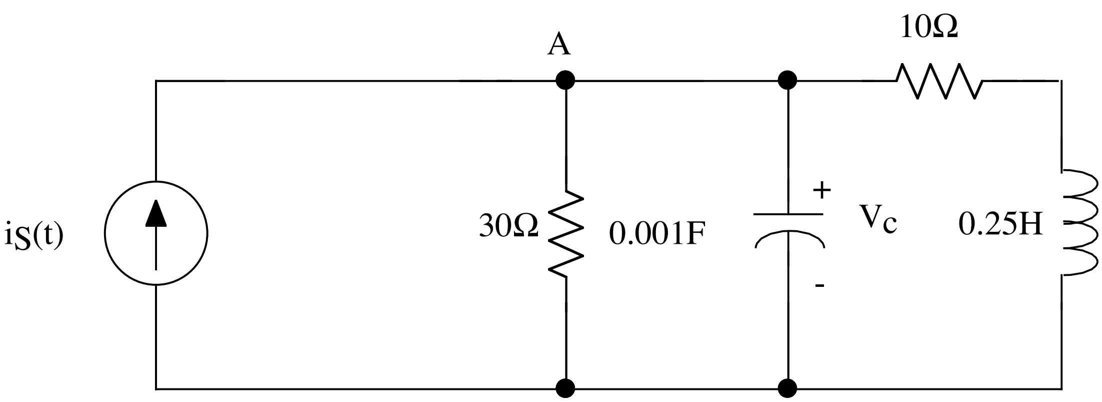
$$P_{in} = \frac{V_{rms}^2}{R} = \frac{1}{R}\left[\sqrt{\frac{1}{T}\int_{-0.001}^{+0.001} 25dt}\right]^2 = \frac{1}{R}\frac{1}{0.01}25(0.002) = \frac{5}{R}$$

The percentage power passed by the filter is then

$$\frac{P_{out}}{P_{in}} = \frac{\frac{4.51}{R}}{\frac{5}{R}} = \frac{4.51}{5} \cong 90\%$$

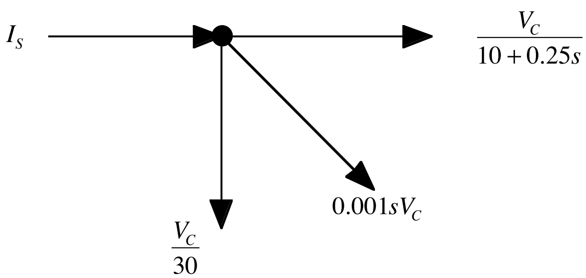
Transfer Functions

7. For the circuit shown, the current source is $i_S(t) = 0.01\cos 500t$



Find the transfer function $G(s) = \dfrac{V_C(s)}{I_S(s)}$

Using KCL at node A (+ out), and that the voltage at A is V_C:



$$-I_S + \frac{V_C}{30} + 0.001sV_C + \frac{V_C}{10+0.25s} = 0$$

$$I_S = V_C\left(\frac{1}{30} + 0.001s + \frac{1}{10+0.25s}\right)$$

Then

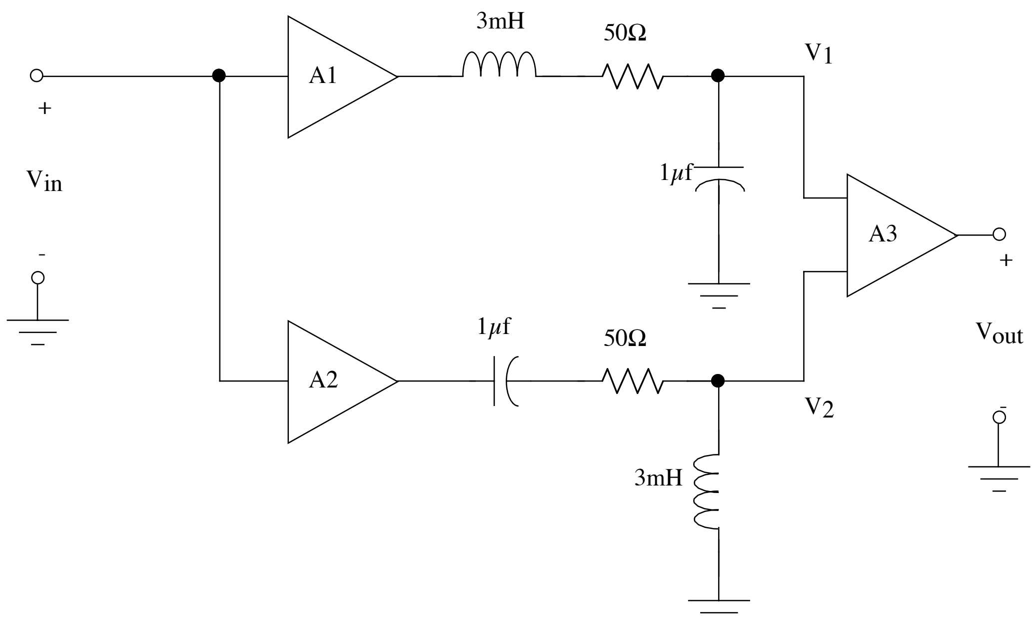
$$G(s) = \frac{V_C(s)}{I_S(s)} = \frac{V_C}{V_C\left(\frac{1}{30} + 0.001s + \frac{1}{10+0.25s}\right)} = \frac{1}{\frac{10+0.25s+0.03s(10+0.25s)+30}{30(10+0.25s)}}$$

$$G(s) = \frac{30(10+0.25s)}{40+0.25s+0.03s(10+0.25s)} = \frac{300+7.5s}{40+0.55s+0.0075s^2}$$

$$G(s) = \frac{7.5(s+40)}{0.0075\left(s^2 + \frac{0.55}{0.0075}s + \frac{40}{0.0075}\right)}$$

$$G(s) = 1000\frac{s+40}{s^2+73.33s+5333}$$

5. Given the circuit shown where the amplifiers are ideal unity-gain voltage amplifiers (infinite input resistance and zero output resistance). Amplifier A3 is a summing amplifier, so its output is the sum of its two inputs. Determine the transfer function of the resulting filter, identify it and its principal parameters.



$$\frac{V_1}{V_{in}} = \frac{\frac{1}{s10^{-6}}}{\frac{1}{s10^{-6}} + 50 + s3\times10^{-3}} = \frac{1}{1+50\times10^{-6}s + 3\times10^{-3}\times10^{-6}s^2}$$

$$\frac{V_1}{V_{in}} = \frac{1}{1+5\times10^{-5}s + 3\times10^{-9}s^2}$$

$$\frac{V_2}{V_{in}} = \frac{s3\times10^{-3}}{\frac{1}{s10^{-6}} + 50 + s3\times10^{-3}} = \frac{3\times10^{-9}s^2}{1+5\times10^{-5}s + 3\times10^{-9}s^2}$$

$$\frac{V_o}{V_{in}} = \frac{V_1}{V_{in}} + \frac{V_2}{V_{in}} = \frac{1+3\times10^{-9}s^2}{1+5\times10^{-5}s + 3\times10^{-9}s^2}$$

Next, we need to put it into a standard form which we can then look up in a table of filters

It is that of a band-reject filter $\dfrac{1+\dfrac{s^2}{\omega_o^2}}{1+\dfrac{s}{Q\omega_o}+\dfrac{s^2}{\omega_o^2}}$

Identify the equivalences,

$\frac{1}{\omega_o^2} = 3\times10^{-9}$ gives $\omega_o^2 = 3.33\times10^{8}$, or $\omega_o = 18257\,{}^{rad}/_{\sec}$ (about 2905 Hz)

$\frac{1}{Q\omega_o} = 5\times10^{-5}$ gives $Q = \frac{1}{5\times10^{-5}\omega_o} = \frac{1}{5\times10^{-5}(18257)} = 1.09$

2. Plot the gain and phase response of the system given by the transfer function:

$$H(s) = \frac{(s + 6500)(s + 8500)}{(s + 350)(s + 100{,}000)}$$

SOLUTION:

Putting the expression into standard form

$$H(s) = \frac{(6500)(8500)}{(350)(100{,}000)} \frac{\left(1 + \frac{s}{6500}\right)\left(1 + \frac{s}{8500}\right)}{\left(1 + \frac{s}{350}\right)\left(1 + \frac{s}{100{,}000}\right)}$$

The gain is given by

$$20\log\left|H(j\omega)\right| = 20\log\left|1.57\right| + 20\log\left|1 + j\frac{\omega}{6500}\right| + 20\log\left|1 + j\frac{\omega}{8500}\right| - 20\log\left|1 + j\frac{\omega}{350}\right| - 20\log\left|1 + j\frac{\omega}{100{,}000}\right|$$

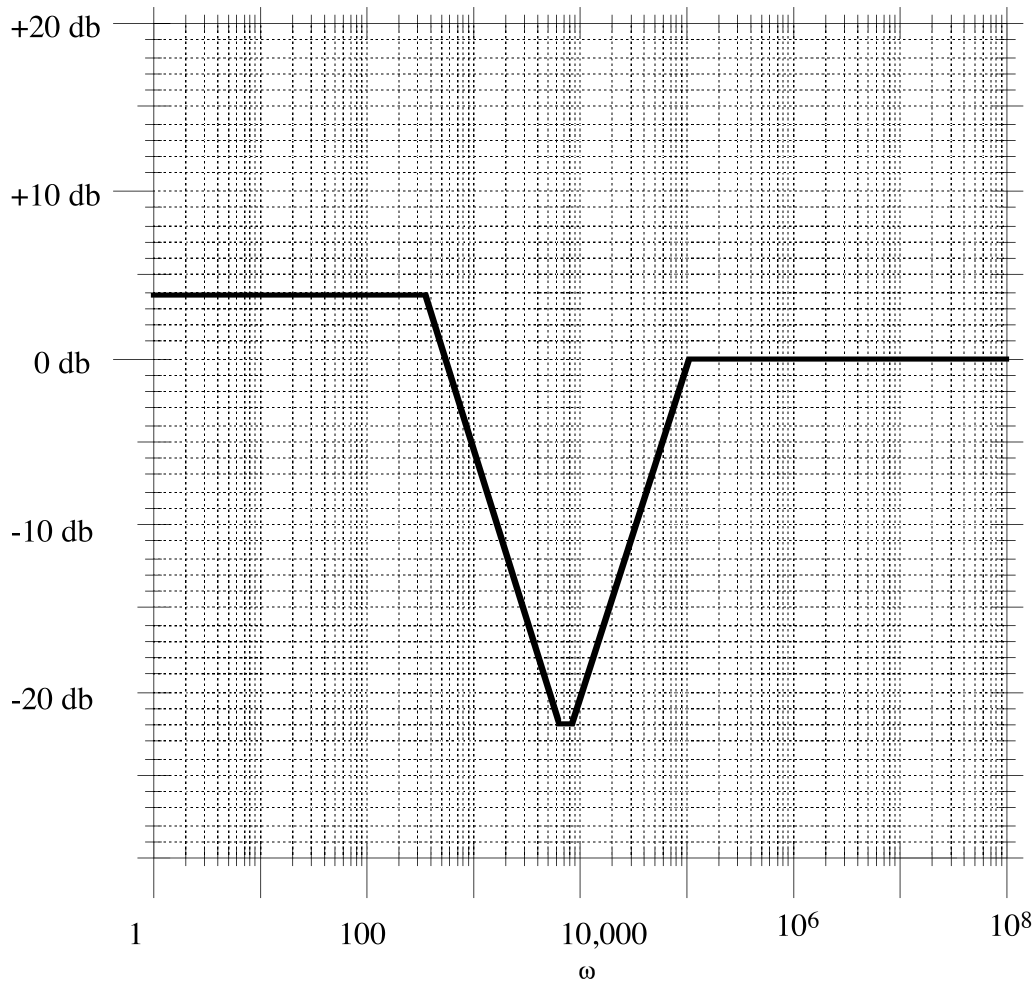
The first term is +4 db. The other terms are evaluated graphically.

The phase angle is given by

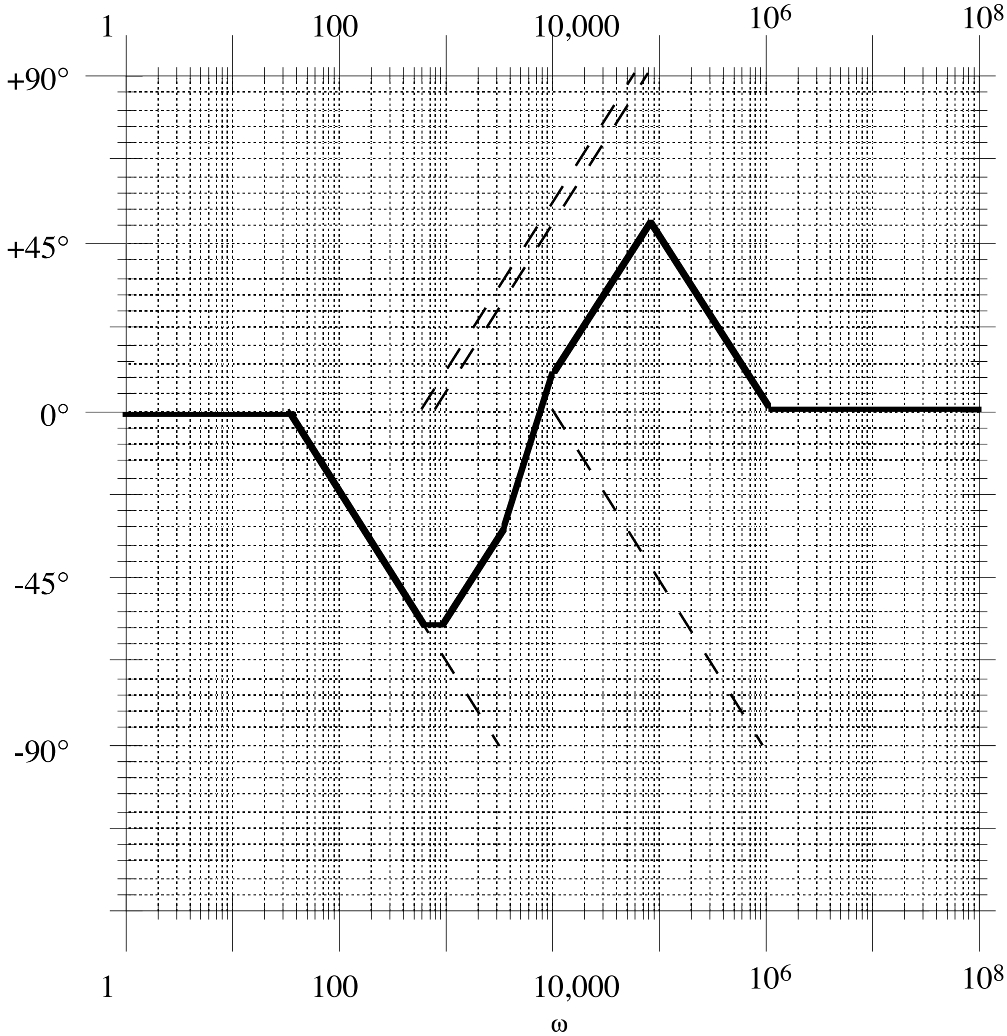
$$\angle H(j\omega) = \angle 1.57 + Tan^{-1}\left(\frac{\omega}{6500}\right) + Tan^{-1}\left(\frac{\omega}{8500}\right) - Tan^{-1}\left(\frac{\omega}{350}\right) - Tan^{-1}\left(\frac{\omega}{100{,}000}\right)$$

The first term evaluates to zero degrees. The other terms are evaluated graphically.

Gain (magnitude) response:



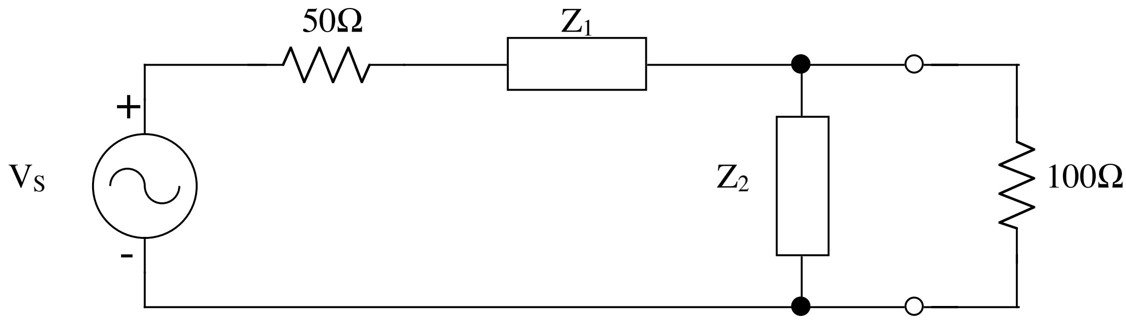
Phase response:



Complex Impedance

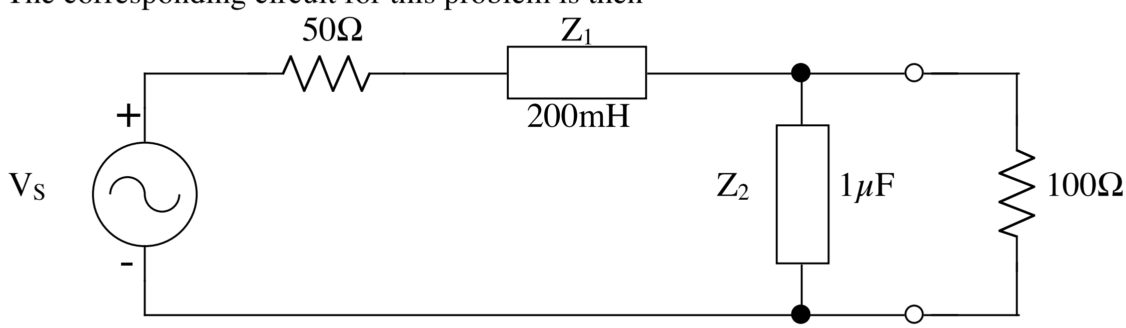
Example 2.16

The circuit shown has Z_1=1μf and Z_2=200mH. The voltage source is represented by the partial Fourier series $v_S = 20\cos 1000t - 5\cos 4000t$. Determine the voltage across the 100 ohm resistor.

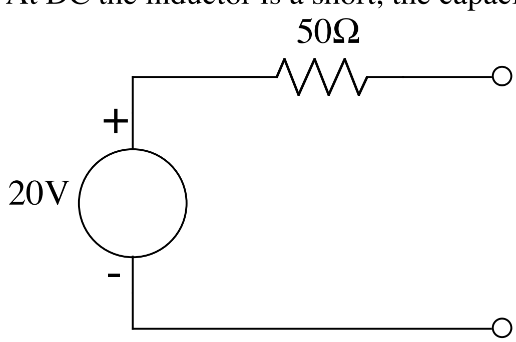


15. The elements Z_1 and Z_2 are interchanged in example 2.16, and the voltage v_S takes on the value of $v_S = 20 + 5\cos 2236t$. Find the voltage across the 100 ohm resistor.

The corresponding circuit for this problem is then



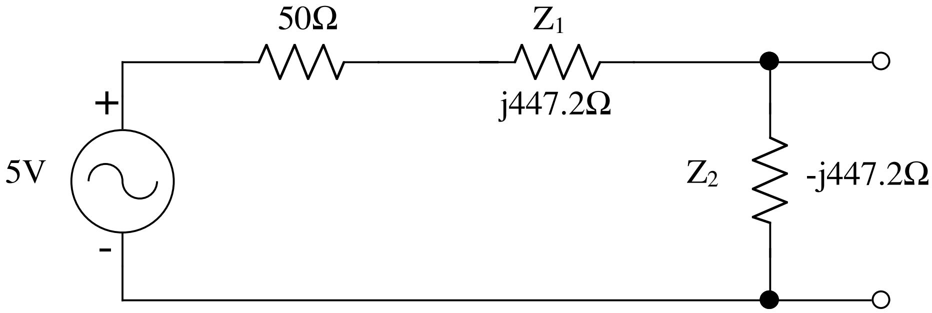
At DC the inductor is a short, the capacitor is an open.



$V_{T1} = 20 Volts$

$R_{T1} = 50\Omega$

At ω=2236 we compute the equivalent impedances to get



$Z_1 = j\omega L = j(2236)(0.2H) = j447.2\Omega$

$Z_2 = \frac{1}{j\omega C} = \frac{1}{j(2236)\left(1\times 10^{-6}\right)} = -j447.23\Omega$

The Thevenin voltage for this AC circuit is found using a voltage divider

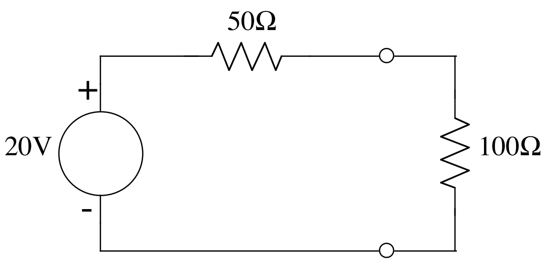
$V_{T2} = \frac{-j447.2}{50 + j447.2 - j447.2}(5) = -j44.7$

The Thevenin resistance is that of the two impedances in parallel

$Z_{T2} = (50 + j447.2) \,||\, (-j447.2) = \frac{(50 + j447.2)(-j447.2)}{50 + j447.2 + (-j447.2)} = 3999. - j447.2$

To find the output voltage we use superposition

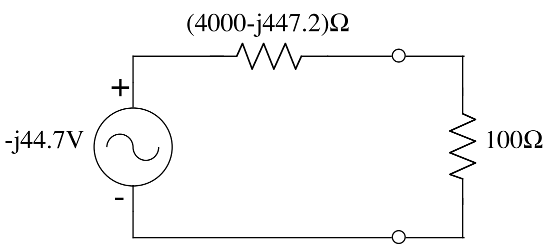
@DC



Using a voltage divider we get

$V_{OUT1} = \frac{100}{100 + 50}(20) = 13.33 Volts$

@ω=2236 rad/sec



Using a voltage divider we get

$V_{OUT2} = \frac{100}{100 + 4000 - j447.2}(-j44.7)$

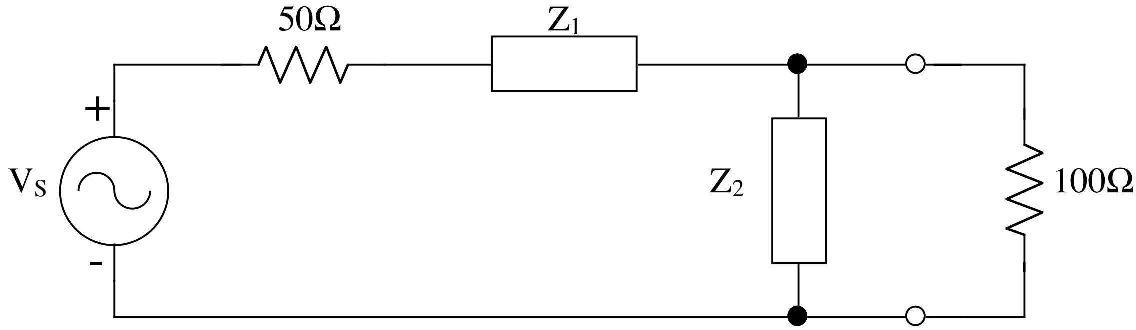
$V_{OUT2} = 0.12 - j1.08 = 1.08\angle -83.78°$

The measured voltage across the 100Ω resistor is then

$V_{100\Omega} = V_{DC} + V_{AC} = V_{TH1} + V_{TH2} = 13.33 + 1.08\cos(2236t - 83.78°)$

Example 2.16

The circuit shown has Z_1=1μf and Z_2=200mH. The voltage source is represented by the partial Fourier series $v_S = 20\cos 1000t - 5\cos 4000t$. Determine the voltage across the 100 ohm resistor.



1. In the circuit shown in example 2.16, the voltage source is operating at a frequency of 2000 rad/sec. Specify the impedances Z_1 and Z_2 so that maximum power is transferred to the 100 ohm load resistance. (Hint: Z_1 and Z_2 must be reactive or else they will absorb some of the power.) In specifying Z_1 and Z_2, give their values in microfarads and/or millihenries. There are two possible sets of answers.

When faced with a source-load problem always Thevenize the source circuit. From the above circuit

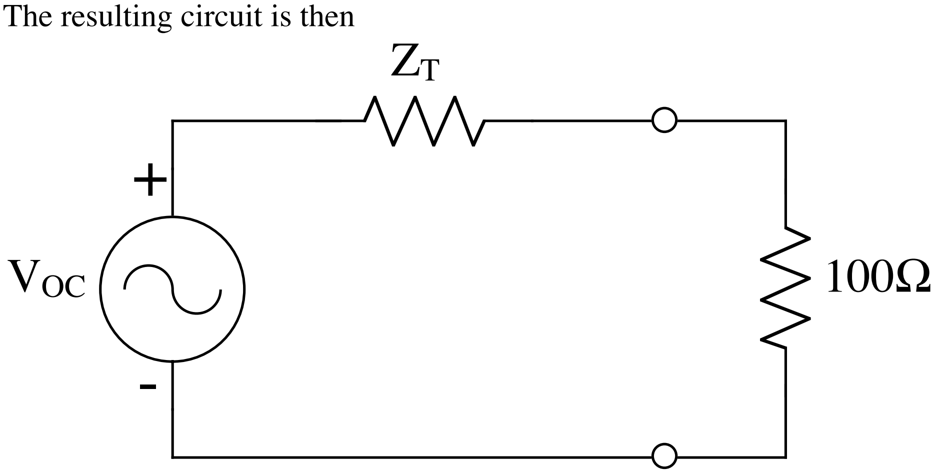
$$V_{OC} = \frac{jX_2}{jX_1 + jX_2 + 50} V_S$$

where we have assumed that Z_1 and Z_2 are purely reactive, i.e., $Z_1 = jX$ and $Z_2 = jX_2$

Similarly,

$$Z_T = (50 + jX_1) \parallel jX_2 = \frac{(50 + jX_1)jX_2}{50 + jX_1 + jX_2}$$

The resulting circuit is then



For maximum power transfer we require that $Z_T = (100)^* = 100\Omega$ or

$$\frac{(50 + jX_1)jX_2}{50 + jX_1 + jX_2} = 100$$

Multiplying out

$j50X_2 - X_1X_2 = 5000 + j100X_1 + j100X_2$

Equating the real and imaginary parts

$X_1X_2 = -5000$ (1)

$50X_2 = 100X_1 + 100X_2$ (2)

Solving (2) for X_2 and substituting into (1)

$-50X_2 = 100X_1$

$X_2 = \frac{100}{-50}X_1 = -2X_1$

$X_1(-2X_1) = -5000$

$X_1^2 = 2500$

Therefore,

$X_1 = \pm 50\Omega$

The corresponding solutions for X_2 are found by substituting this result into (1):

$X_1X_2 = -5000$

$X_2 = -\frac{5000}{X_1} = -\frac{5000}{\pm 50} = -/+\ 50\Omega$

Case 1: $X_1 = +50\Omega,\ X_2 = -100\Omega$

X_1 is inductive so $X_1 = \omega L_1 = 50$

$L_1 = \frac{50}{2000} = 0.025H$

X_2 is capacitive so $X_2 = -\frac{1}{\omega C_2} = -100\Omega$

Solving for the capacitance gives

$C_2 = \frac{1}{(2000)(100)} = 5\mu f$

Case 2: $X_1 = -50\Omega,\ X_2 = +100\Omega$

X_1 is capacitive so $X_1 = -\frac{1}{\omega C_1} = -50\Omega$

Solving for the capacitance gives

$C_1 = \frac{1}{(2000)(50)} = 10\mu f$

X_2 is inductive so $X_2 = \omega L_2 = 100$

$L_2 = \frac{100}{2000} = 0.05H$

4. A 60 Hz source has its voltage measured under various loads, with the results shown.

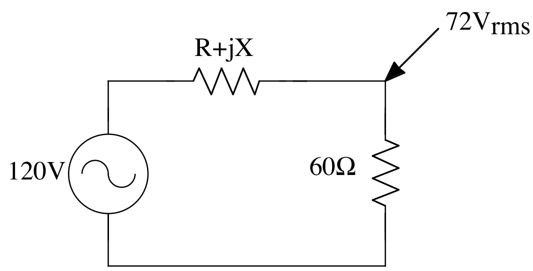
Case	Load	Voltmeter reading
1	open circuit	120 volts rms
2	60Ω resistance	72 volts rms
3	60Ω pure capacitance	360 volts rms
4	60Ω pure inductance	51.43 volts rms

Determine the voltage for a load of 120 ohms pure capacitance.

We will assume that the internal impedance of the source is complex and in series with the source.

Case 1: under no load the open circuit voltage is 120 volts rms

Case 2:



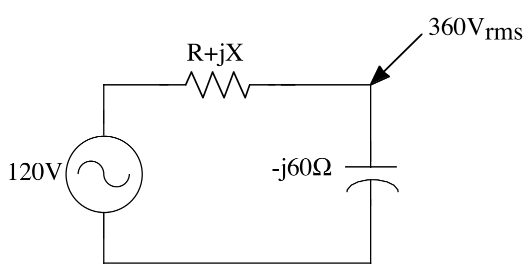
$$V_{OUT} = \left(\frac{60}{60 + R + jX} \right) 120$$

$$\left| \frac{(60)(120)}{(60 + R) + jX} \right| = 72$$

$$\frac{5.184 \times 10^7}{(60 + R)^2 + X^2} = 5184$$

$$(60 + R)^2 + X^2 = 10000 \qquad (1)$$

Case 3:



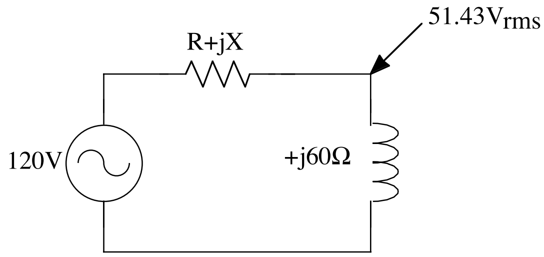
$$V_{OUT} = \left(\frac{-j60}{-j60 + R + jX} \right) 120$$

$$\left| \frac{(-j60)(120)}{R + j(X - 60)} \right| = 360$$

$$\frac{5.184 \times 10^7}{R^2 + (X - 60)^2} = 129600$$

$$R^2 + (X - 60)^2 = 400 \quad (2)$$

Case 4:



$$V_{OUT} = \left(\frac{+j60}{+j60 + R + jX}\right)120$$

$$\left|\frac{(+j60)(120)}{R + j(X + 60)}\right| = 51.4$$

$$\frac{5.184 \times 10^7}{R^2 + (X + 60)^2} = 2645.045$$

$$R^2 + (X + 60)^2 = 19598.911 \quad (3)$$

Solving equations (2) and (3)

$$R^2 + X^2 - 120X + 3600 = 400$$
$$R^2 + X^2 + 120X + 3600 = 19598.911$$

$$R^2 + X^2 - 120X = -3200$$
$$R^2 + X^2 + 120X = 15998.911$$

Subtracting
$-240X = -19198.911$
X=79.995Ω
Substituting this result back into equation (2) gives
$R^2 + (79.995 - 60)^2 = 400$
$R^2 + (19.995)^2 = 400$
$R^2 = 0.200$
$R = 0.447\Omega$

The output voltage for a capacitive -j120Ω load can then be calculated using these results.

$$V_{OUT} = \left|\left(\frac{-j120}{R + jX - j120}\right)120\right| = \left|\frac{(-j120)120}{0.447 + j79.995 - j120}\right| = \left|\frac{-j14400}{0.447 - j40.005}\right| = 359.91 - j4.021$$

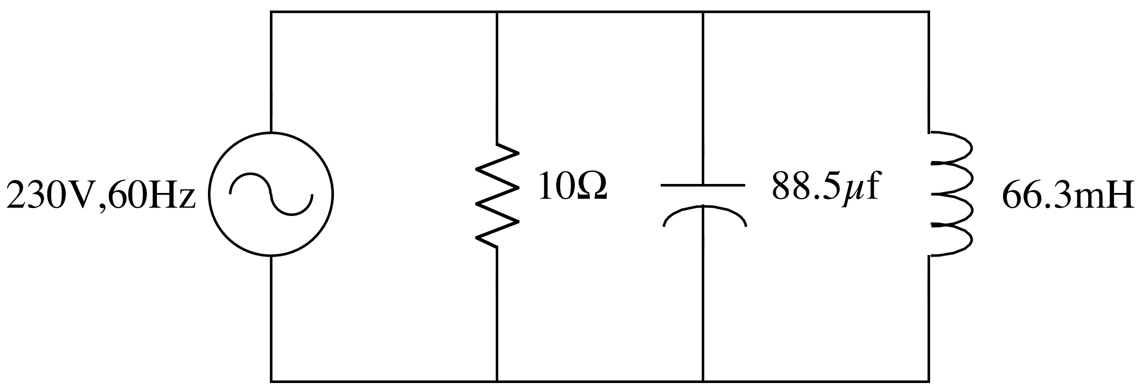
$$V_{OUT} = 359.93\angle -0.64°$$

$$V_{OUT} \approx 360 \text{ volts rms}$$

6. A parallel combination of a resistance (10Ω), capacitance (88.5μf), and inductance (66.3 mH) has 60Hz, 230 volt (RMS) applied. Obtain the:

(a) reactances of C and L.
(b) admittance of each circuit element.
(c) phasor diagram for the currents, using the applied voltage as the reference.
(d) admittance diagram for the circuits, including the total admittance.
(e) input current as a sine function, taking the applied voltage as a reference. (Is the circuit inductive or capacitive?)
(f) power factor.
(g) power triangle.

The circuit is



The angular frequency is $\omega = 2\pi(60) = 376.99\,{rad}/{\sec}$

(a)

$$X_L = j\omega L = j(377)\left(66.3\times10^{-3}\right) = j25$$

$$X_C = \frac{1}{j\omega C} = -j\frac{1}{(377)\left(88.5\times10^{-6}\right)} = -j30$$

(b)

$$B_L = \frac{1}{j25} = -j0.04\,mhos$$

$$B_C = \frac{1}{-j30} = +j0.0334\,mhos$$

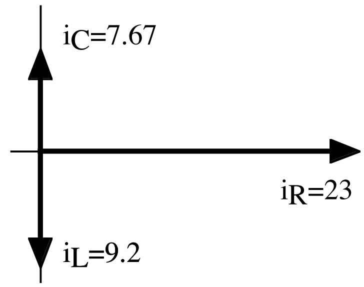
$$G = \frac{1}{R} = \frac{1}{10} = 0.1\,mhos$$

(c)

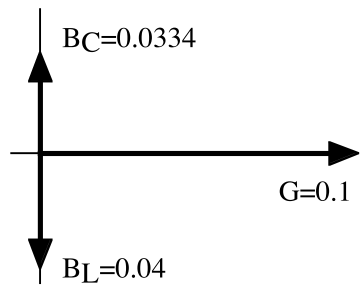
$$i_R = \frac{230\angle 0^\circ}{10} = 23\angle 0^\circ\,amps$$

$$i_C = \frac{230\angle 0^\circ}{-j30} = 7.67\angle +90^\circ\,amps$$

$$i_L = \frac{230\angle 0^\circ}{j25} = 9.2\angle -90^\circ\,amps$$



(d)

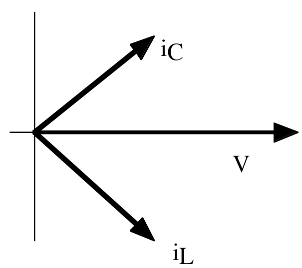


$Y_t = 0.1 + j0.0334 - j0.04 = 0.1 - j0.0066 = 0.1002\angle -3.776°$

(e)

$i = YV = (0.1002\angle -3.776°)(230\angle 0°) = 23.00 - j1.518 = 23.05\angle -3.776°$

The circuit is inductive since the angle is negative. Recall the phasor diagram



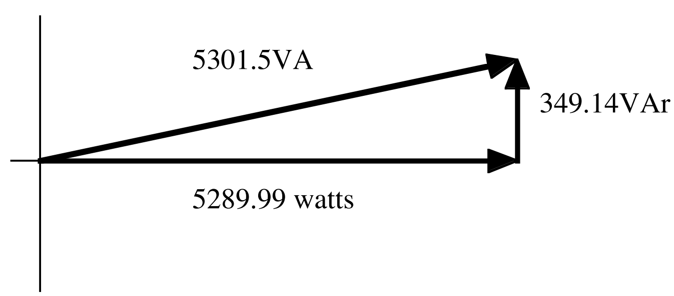
(f)

$PF = \cos(-3.776°) = 0.9978$

(g)

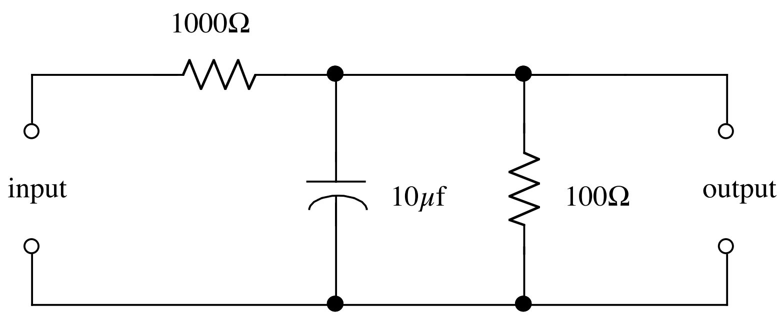
$p = vi^* = (230\angle 0°)(23.05\angle -3.776°)^* = 5301.5\angle +3.776° = 5289.99 + j349.14$

The power triangle will look like this



Laplace Transforms

2. For the circuit shown, find (a) the system transfer function, (b) the response to a unit step at the input, and (c) the response to a unit pulse with a duration of millisecond.



Compute the parallel impedance of the 10μf capacitor and the 100Ω resistance and then use a voltage divider relationship.

Compute the parallel impedance

$$Z_{parallel} = \frac{\frac{1}{s(10\times 10^{-6})}100}{\frac{1}{s(10\times 10^{-6})}+100} = \frac{\frac{100}{10^{-5}s}}{\frac{1+10^{-3}s}{10^{-5}s}} = \frac{100}{1+10^{-3}s}$$

(a) system transfer function

$$\frac{V_{out}}{V_{in}} = \frac{\frac{100}{1+10^{-3}s}}{1000+\frac{100}{1+10^{-3}s}} = \frac{\frac{100}{1+10^{-3}s}}{\frac{1000+s+100}{1+10^{-3}s}} = \frac{100}{s+1100}$$

(b) response to a unit step $\frac{1}{s}$

$$V_{out} = \frac{100}{s+1100}\frac{1}{s} = \frac{A}{s}+\frac{B}{s+1100}$$

$$As + A1100 + Bs = 100$$

Equating the real parts $A1100 = 100$ or $A = \frac{1}{11}$

Equating the imaginary parts $As + Bs = 0$ or $B = -A = -\frac{1}{11}$

$$V_{out} = \frac{\frac{1}{11}}{s} - \left(\frac{\frac{1}{11}}{s+1100}\right) = \frac{1}{11}\left(\frac{1}{s}\right) - \frac{1}{11}\left(\frac{1}{s+1100}\right)$$

Inverse Laplace transforming

$$V_{out}(t) = \frac{1}{11}u(t) - \frac{1}{11}e^{-1100t}u(t)$$

(c) response to a unit pulse with duration of 1msec.

$$V_{in}(t) = u(t) - u\left(t - 10^{-3}\right)$$

Laplace transforming this gives $V_{in}(s) = \frac{1}{s} - \frac{e^{-0.001s}}{s}$

The output is then given by

$$V_{out}(s) = \frac{1}{11}\left(\frac{1}{s} - \frac{1}{s+1100}\right) - \frac{1}{11}e^{-0.001s}\left(\frac{1}{s} - \frac{1}{s+1100}\right)$$

$$V_{out}(t) = \frac{1}{11}\left(1 - e^{-1100t}\right)u(t) - \frac{1}{11}e^{-1100\left(t-10^{-3}\right)}u\left(t - 10^{-3}\right)$$

6. A Butterworth filter has its transfer function given below. Determine its corner frequency, and its response to a 1 volt step input.

$$G(s) = \frac{100{,}000}{s^4 + 26.13s^3 + 341.4s^2 + 2613s + 10{,}000}$$

From a table of Butterworth filter transfer functions [our book no longer has one], for n=4

$$\theta_1 = 22.5°,\ \left(\frac{s}{\omega_o}\right)^2 + 0.765\frac{s}{\omega_o} + 1 \text{ and } \theta_2 = 67.5°,\ \left(\frac{s}{\omega_o}\right)^2 + 1.848\frac{s}{\omega_o} + 1$$

We need to identify ω_o so divide by 10,000 to get 1 in denominator.

$$G(s) = \frac{100{,}000 / 10{,}000}{\left(\frac{s}{10}\right)^4 + \frac{26.13}{10}\left(\frac{s}{10}\right)^3 + \frac{341.4}{100}\left(\frac{s}{10}\right)^2 + \frac{2613}{1000}\left(\frac{s}{10}\right) + 1}$$

By inspection $\omega_o = 10$

From the table of filter transfer functions, the poles are at ±22.5˚ and ±67.5˚

Putting this into standard form for a transfer function [again from a table of transfer functions] we have

$$G(s) = \frac{10}{\left[\left(\frac{s}{10}\right)^2 + 0.765\left(\frac{s}{10}\right) + 1\right]\left[\left(\frac{s}{10}\right)^2 + 1.848\left(\frac{s}{10}\right) + 1\right]}$$

The step response for this function is truly NASTY to compute. Do by partial fraction expansion.

$$R(s) = G(s)\frac{1}{s} = \frac{A'}{s} + \frac{B's + C'}{\left(\frac{s}{10}\right)^2 + 0.765\left(\frac{s}{10}\right) + 1} + \frac{D's + E'}{\left(\frac{s}{10}\right)^2 + 1.848\left(\frac{s}{10}\right) + 1}$$

Put into more standard form for Laplace analysis.

$$R(s) = \frac{A}{s} + \frac{Bs + C}{s^2 + 7.65s + 100} + \frac{Ds + E}{s^2 + 18.48s + 100}$$

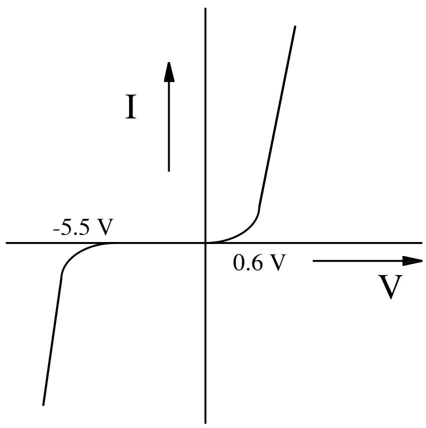
This transforms to an expression of the form

$$r(t) = Au(t) + B''e^{-at}\cos bt + C''e^{-at}\sin bt + D''e^{-ct}\cos dt + E''e^{-ct}\sin dt$$

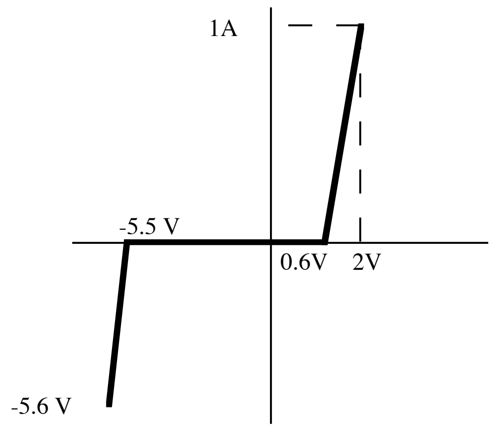
where I did not bother to compute the coefficients.

Solid-state Device Characteristics and Ratings

7. All silicon diodes have a reverse breakdown voltage essentially the same as the avalanche voltage of a zener diode. This results in three distinct regions of operation: one for the normal forward region, one for the avalanche region, and an intermediate region where the device behaves essentially as an open circuit. Obtain the linear models in these three regions for a 1 amp, 2 watt silicon diode with a reverse breakdown voltage of 5.5 volts and a maximum reverse voltage of 5.6 volts.



Theoretical



Linear model

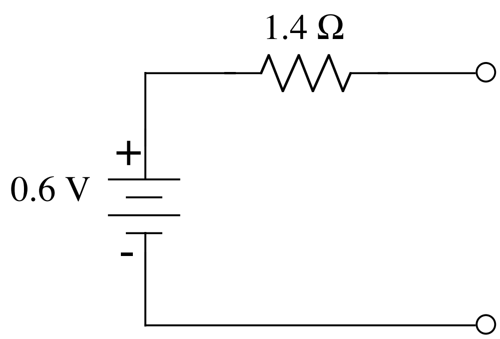
$$Power_{rated} = v_{rated} \times i_{rated}$$

$$v_{rated} = \frac{Power_{rated}}{i_{rated}} = \frac{2watts}{1Amp} = 2Volts$$

In region I: $\frac{V-2}{I-1} = \frac{0.6-2}{0-1} = 1.4$

$V - 2 = 1.4I - 1.4$

$V = 1.4I + 0.6$



In region II: nothing, diode is an open circuit

In region III: find the rated current using breakdown voltage

$V_{max} I_{max} = 2watts$

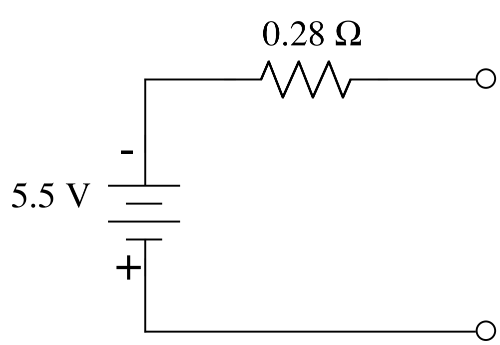
$(5.6Volts) I_{max} = 2watts$

$I_{max} = 0.36Amps$

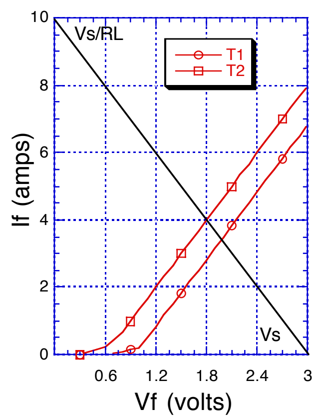
$$\frac{V-(-5.5)}{I-0} = \frac{-5.6-(-5.5)}{-0.36-0}$$

$V + 5.5 = 0.28I$

$V = 0.28I - 5.5$



Example 2.12
The characteristics shown in figure 2.11 have voltage divisions of 0.2 volts and current divisions of 1 amp. Obtain an equivalent circuit for a diode which accounts for the temperature dependence with T_1=20° C and T_2=80° C.



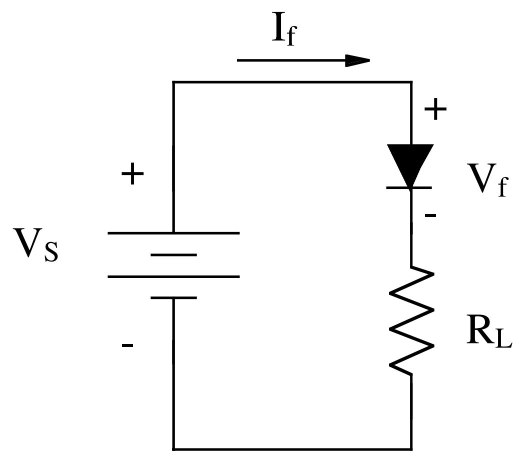


Figure 2.11 Temperature characteristics for a diode

11. Repeat example 2.12 for T_1=0° C and T_2=100°C.

Using T_1=0° C and T_2=100° C.
From the graph for T1 we can estimate the two points on graph to be (0.95V,0Amps) and (1.4Volts,1.5Amps). Using the two point method

$$\frac{V_f - 0.95}{i_f - 0} = \frac{1.4 - 0.95}{1.5 - 0}$$
$$V_f - 0.95 = 0.3 i_f$$
$$V_f = 0.3 i_f + 0.95$$

There are many ways to find the second curve for T_2=100° C. The slope remains the same but the y-intercept changes.

Note that for T_2=100° C at $i_f = 0$, $V_f = 0.6$. The T_2 curve is then

$$V_f = 0.3 i_f + 0.60$$

Assume that the y-intercept is a linear function of temperature and again use the two point method. Using the points (0.95,0) and (0.60,100) for the curve we get an equation for the y-intercept of

$$\frac{C_o - 0.95}{T - 0} = \frac{0.60 - 0.95}{100 - 0} = -0.0035$$
$$C_o = -0.0035T + 0.95$$

The overall equivalent circuit is

$$V_f\left(i_f, T\right) = 0.3 i_f + C_o = 0.3 i_f - 0.0035T + 0.95$$

Transducers/Sensors

10. For a photodiode with characteristics shown in figure 2.10 and used in the circuit for example 2.11 with a 5 volt source and a 10,000 ohm load, determine the load voltage as a function of illumination level. Over what range of illumination levels will this result be valid? Assume the curves extend to -5 volts on the third quadrant of the characteristic curves.

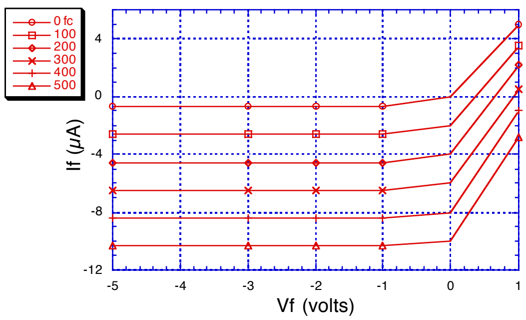
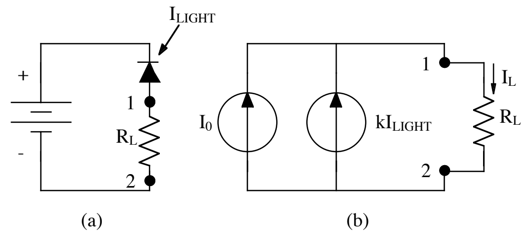


Fig. 2.10 Typical photodiode characteristics



(a) Biasing circuit for photodiode of figure 2.10
(b) equivalent circuit

Fit alinear function to the variation of current with respect to light level gives $\frac{i-0.7}{I_L-0}=\frac{10.3-0.7}{500-0}=0.192$ where the so-called "dark" current is estimated to be about 0.7μA for 0 fc (footcandles). The other point comes from the estimated line for the 500 fc illumination level.

$i=0.7+0.0192I_L$ (i is in microamperes) (1)

Considering the load electrical circuit using KVL

$5=V_f+i(10k\Omega)$ (2)

Substituting (1) into (2) gives

$5=V_f+\left(0.7+0.0192I_L\right)\times10^{-6}(10k\Omega)$

$5-V_f=0.007+0.000192I_L$

But $5-V_f=V_{LOAD}$ is simply the voltage across the load resistor.

The equation $V_{LOAD}=0.007+0.000192I_L$ is valid when the illumination I_L>0 and breaks down when the diode becomes forward biased, i.e., the voltage across the load resistor becomes greater than the voltage across the photodiode. At this point the voltage across the diode drops to zero and the entire 5 volts appears across the load resistor.

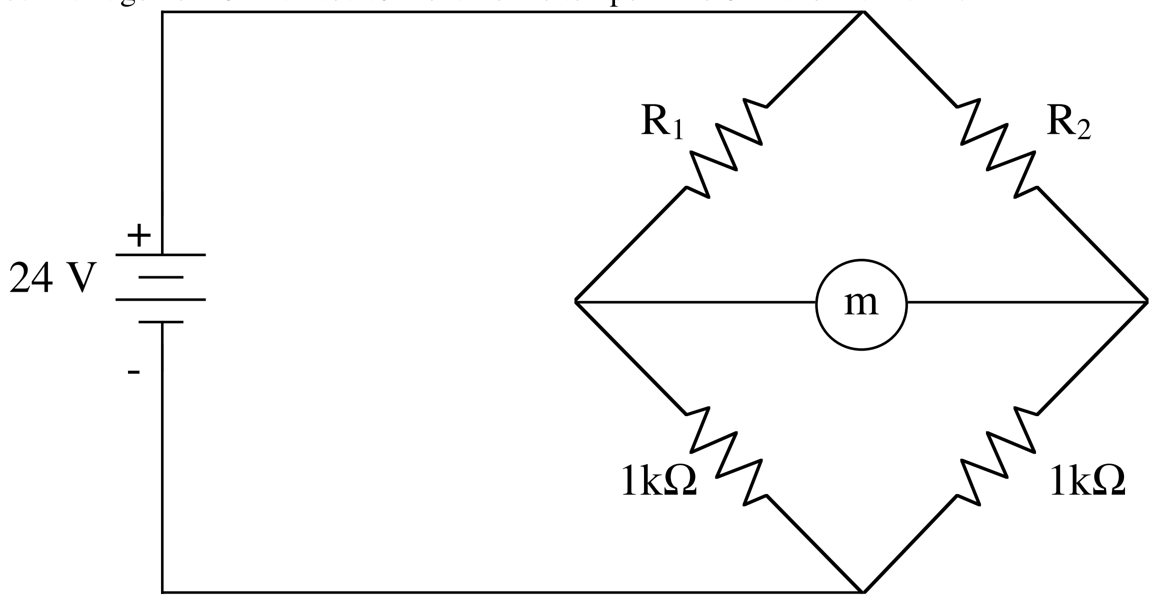
At this breakdown point is given by

$V_{LOAD}=0.007+0.000192I_L=5Volts$

and the corresponding light level is given by

$I_L=\frac{5-0.007}{0.000192}\approx 26{,}000$ footcandles.

5. A bridge network is used to measure the temperature of a chemical bath.



R_1 and R_2 are identical thermistors with negative temperature coefficients of resistance of -4%/° C. At 25° C they are both 1500Ω. The meter (m) has an internal resistance of 800Ω. Resistance R_2 is held at 25° C.

(a) Determine the meter current when R_1 is at 20° C.

(b) Determine the meter current when R_1 is at 30° C.

The first step is to calculate the different values of R_1.

R_1=1500Ω @ 25° C

temperature coefficient -4%/° C

@20° C

$$R_1 = 1500(1 - 0.04(T - 25°C))$$

$$R_1 = 1500(1 - 0.04(20 - 25)) = 1500(1 + 0.04(5))$$

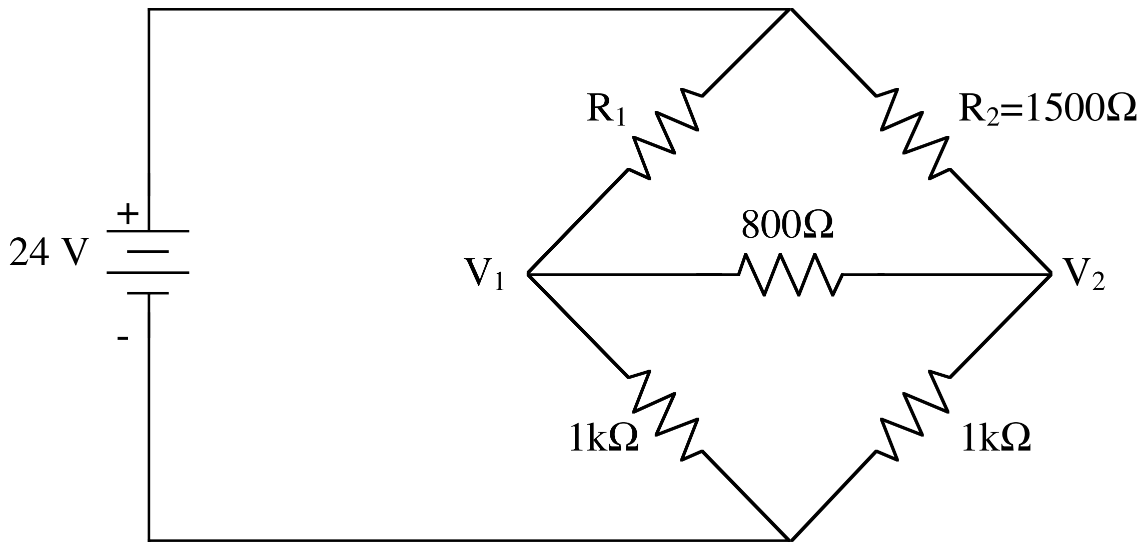
$$R_1 = 1500(1 + 0.2) = 1800\Omega$$

@30° C

$$R_1 = 1500(1 - 0.04(T - 25°C))$$

$$R_1 = 1500(1 - 0.04(30 - 25)) = 1500(1 - 0.04(5))$$

$$R_1 = 1500(1 - 0.2) = 1200\Omega$$



We will write the nodal equations as shown below:

$$\frac{24-V_1}{1800}-\frac{V_1-V_2}{800}-\frac{V_1}{1000}=0$$

$$\frac{24-V_2}{1500}+\frac{V_1-V_2}{800}-\frac{V_2}{1000}=0$$

Re-writing

$$24-V_1-2.25V_1+2.25V_2-1.8V_1=0$$

$$24-V_2+1.875V_1-1.875V_2-1.5V_2=0$$

In matrix form,

$$\begin{bmatrix}24\\24\end{bmatrix}=\begin{bmatrix}1+2.25+1.8 & -2.25\\ -1.875 & 1.5+1.875+1\end{bmatrix}\begin{bmatrix}V_1\\V_2\end{bmatrix}$$

$$\begin{bmatrix}24\\24\end{bmatrix}=\begin{bmatrix}5.05 & -2.25\\ -1.875 & 4.375\end{bmatrix}\begin{bmatrix}V_1\\V_2\end{bmatrix}$$

Solving,

$$V_1=\frac{\begin{bmatrix}24 & -2.25\\ 24 & 4.375\end{bmatrix}}{\begin{bmatrix}5.05 & -2.25\\ -1.875 & 4.375\end{bmatrix}}=\frac{105-(-54)}{(5.05)(4.375)-(-1.875)(-2.25)}=\frac{159}{22.094-4.219}=\frac{159}{17.875}=+8.895$$

$$V_2=\frac{\begin{bmatrix}5.05 & 24\\ -1.875 & 24\end{bmatrix}}{\begin{bmatrix}5.05 & -2.25\\ -1.875 & 4.375\end{bmatrix}}=\frac{(5.05)(24)-(-1.875)(24)}{(5.05)(4.375)-(-1.875)(-2.25)}=\frac{166.2}{17.875}=+9.2979$$

$$I_m(@20°C) = \frac{V_1 - V_2}{800\Omega} = \frac{8.8951 - 9.2979}{800} = -0.504mA$$

At 30° C the node equations are:

@node 1 $\frac{24 - V_1}{R_1} - \frac{V_1 - V_2}{800} + \frac{V_1}{1000} = 0$

@node 2 $\frac{24 - V_2}{R_2} + \frac{V_1 - V_2}{800} - \frac{V_2}{1000} = 0$

$$\frac{24 - V_1}{1200} - \frac{V_1 - V_2}{800} - \frac{V_1}{1000} = 0$$

$$\frac{24 - V_2}{1500} + \frac{V_1 - V_2}{800} - \frac{V_2}{1000} = 0$$

In matrix form,

$$\begin{bmatrix} -\frac{24}{1200} \\ -\frac{24}{1500} \end{bmatrix} = \begin{bmatrix} -\frac{1}{1200} - \frac{1}{800} - \frac{1}{1000} & \frac{1}{800} \\ \frac{1}{800} & -\frac{1}{1000} - \frac{1}{800} - \frac{1}{1500} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Multiplying the first row by 1200 and the second row by 1500 we get:

$$\begin{bmatrix} -24 \\ -24 \end{bmatrix} = \begin{bmatrix} -1 - 1.5 - 1.2 & 1.5 \\ 1.875 & -1.5 - 1.875 - 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} -24 \\ -24 \end{bmatrix} = \begin{bmatrix} -3.7 & 1.5 \\ 1.875 & -4.375 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$V_1 = \frac{\begin{bmatrix} -24 & 1.5 \\ -24 & -4.375 \end{bmatrix}}{\begin{bmatrix} -3.7 & 1.5 \\ 1.875 & -4.375 \end{bmatrix}} = \frac{105 + 36}{13.375} = 10.5421$$

$$V_2 = \frac{\begin{bmatrix} -3.7 & -24 \\ 1.875 & -24 \end{bmatrix}}{\begin{bmatrix} -3.7 & 1.5 \\ 1.875 & -4.375 \end{bmatrix}} = \frac{88.8 + 45}{13.375} = 10.0037$$

$$I_m(@20°C) = \frac{V_1 - V_2}{800\Omega} = \frac{10.5421 - 10.0037}{800} = +0.67mA$$

Conductivity/Resitivity

1. Using the same diameter nichrome wire as in example 2.1, determine the length needed to obtain a resistance of 1 ohm at 100° C. Determine the resistance at 20° C.

The characteristics of #30 AWG wire are:

diameter (mils)	diameter (circular mils)	in^2	Ω/1000ft @20° C	pounds/1000 ft.
10.03	100.5	7.894×10^{-5}	103.2	0.3042

for nichrome wire @20° C

$\rho_{20}=1.08\times10^{-6}\Omega\text{-m}$

$\alpha_{20}=17\times10^{-3}/°\ C$

$\rho_{100}=\rho_{20}[1+\alpha_{20}(T-20)] = 1.08\times10^{-6}[1+17\times10^{-3}\ (100-20)]$

$\rho_{100}=1.08\times10^{-6}[1+1.36] = 2.55\times10^{-6}\Omega\text{-m}$

$$A=\pi\left(\frac{d}{2}\right)^2=\frac{\pi d^2}{4}=\frac{\pi\left(10.03\times10^{-3}in\times25.4\frac{mm}{in}\right)^2}{4}=\frac{\pi\left(0.255\times10^{-3}m\right)^2}{4}=5.10\times10^{-8}m^2$$

$$R=\rho\frac{\ell}{A}$$

$$1\Omega=\left(2.55\times10^{-6}\Omega-m\right)\frac{\ell}{5.10\times10^{-8}m^2}$$

$$\ell=(1\Omega)\frac{5.10\times10^{-8}m^2}{2.55\times10^{-6}\Omega-m}=0.02m$$

$$R_{20}(\ell=0.02m)=\frac{\left(1.08\times10^{-6}\Omega-m\right)(0.02m)}{5.10\times10^{-8}m^2}=0.42\Omega$$

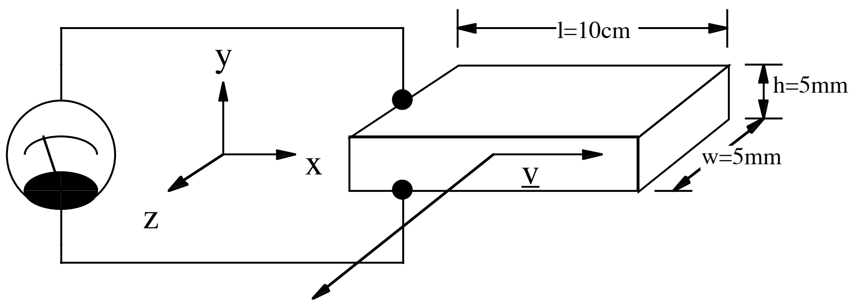
Note: Circular mils is nothing more than the diameter (expressed in thousandths of an inch) squared.

Semiconductors

Example 2.13

Copper has an electron density of 10^{28} electrons per cubic meter. The charge on each electron is 1.6×10^{-19} coulombs. For a copper conductor with width 5 mm, height 5 mm, and length 10 cm, determine the Hall voltage for a current of 1 amp when the magnetic field intensity is 1 amp/m, $B=\mu_o H$ and $\mu_o=4\pi\times10^{-7}$.

12. Use the dimensions given in example 2.13 but assume the conductor is an n-type semiconductor with 10^{19} electrons per cubic meter. The current is 0.1 amp. Obtain an equivalent circuit for a transducer using the Hall voltage to measure magnetic field intensity. Assume that the voltage pickups are capacitative so there is no connection of the output to the current circuit.



The fundamental equation for the Hall effect is $\vec{F}=q\left(\vec{E}+\vec{v}\times\vec{B}\right)=0$

Since there is only a y-component of the E field we can rewrite this as

$$E_y+\left(\vec{v}\times\vec{B}\right)_y=0 \qquad (1)$$

The current I_x in the x-direction is given by $I_x=nqv_xA$ where

n=number of carriers per square meter

q is the charge on the carriers

v_x is the x-component of the carrier velocity and

A=wh is the conductor cross sectional area.

Solving,

$$v_x=\frac{I_x}{nqA}=\frac{I_x}{nqwh} \qquad (2)$$

Rewriting (1) and substituting (2) into it

$$E_y-v_xB_z=0$$

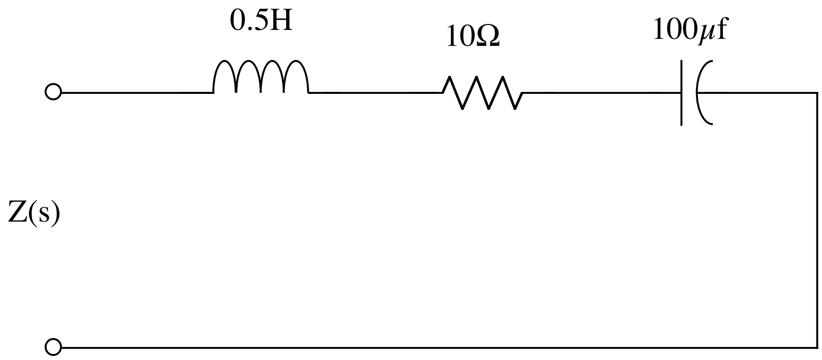
$$\frac{V_h}{h}-\left(\frac{I_x}{nqwh}\right)\mu_o H_z=0$$

Solving for V_h

$$V_h=(0.005m)\left(\frac{0.1Amp}{\left(10^{19}\right)\left(1.6\times10^{-19}\right)(0.005m)(0.005m)}\right)\left(4\pi\times10^{-7}\right)H_z=1.57\times10^{-5}H_z$$

Frequency Response

8. For a series RLC circuit of 10 ohms, 0.5 henrys, and 100μf, determine the resonant frequency, the quality factor, and the bandwidth.



$$Z(s) = sL + R + \frac{1}{sC}$$

$$Z(s) = \frac{1}{2}s + 10 + \frac{1}{s\left(10^2 \times 10^{-6}\right)}$$

$$Z(s) = \frac{1}{2}\left[s + 20 + \frac{2\times 10^4}{s}\right] = \frac{1}{2}\left[\frac{s^2 + 20s + 2\times 10^4}{s}\right]$$

$$Z(s) = \frac{1}{2s}\left[s^2 + 20s + 2\times 10^4\right]$$

The standard form is $s^2 + \frac{\omega_0}{Q}s + \omega_0^2$

$$\omega_0^2 = 2\times 10^4$$
$$\omega_0 = 100\sqrt{2} = 141.2\,{}^{rad}\!/_{\sec}$$

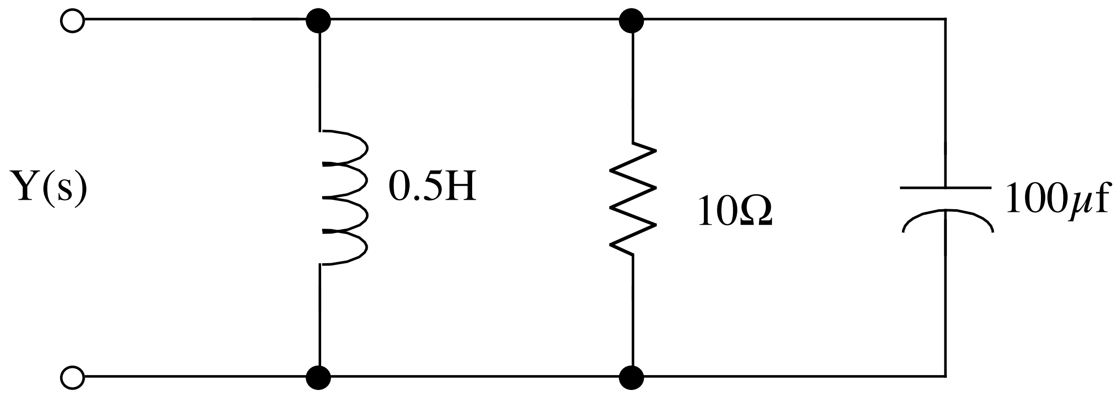
$$\frac{\omega_0}{Q} = \frac{100\sqrt{2}}{Q} = 20$$

$$Q = \frac{100\sqrt{2}}{20} = 5\sqrt{2} \cong 7.07$$

$$Q = \frac{\omega_0}{\Delta\omega}$$

$$\Delta\omega = \frac{\omega_0}{Q} = \frac{100\sqrt{2}}{5\sqrt{2}} = 20$$

9. For a parallel RLC circuit of 10 ohms, 0.5 henrys, and 100μf, determine the resonant frequency, the quality factor, and the bandwidth.



Use the same approach as for a series RLC circuit but use Y(s) instead.

$$Y(s)=\frac{1}{Z(s)}=\frac{1}{sL}+\frac{1}{R}+sC=\frac{1}{s\left(\frac{1}{2}\right)}+\frac{1}{10}+s\left(10^2\times10^{-6}\right)$$

$$Y(s)=\frac{2}{s}+\frac{1}{10}+10^{-4}s=\frac{20+s+10-10^{-3}s^2}{10s}$$

$$Y(s)=\frac{s^2+1000s+20,000}{10^4 s}$$

As before the model is $s^2+\frac{\omega_0}{Q}s+\omega_0^2$

$$\omega_0^2=20,000$$
$$\omega_0=141.42\,{}^{rad}\!/_{\sec}$$

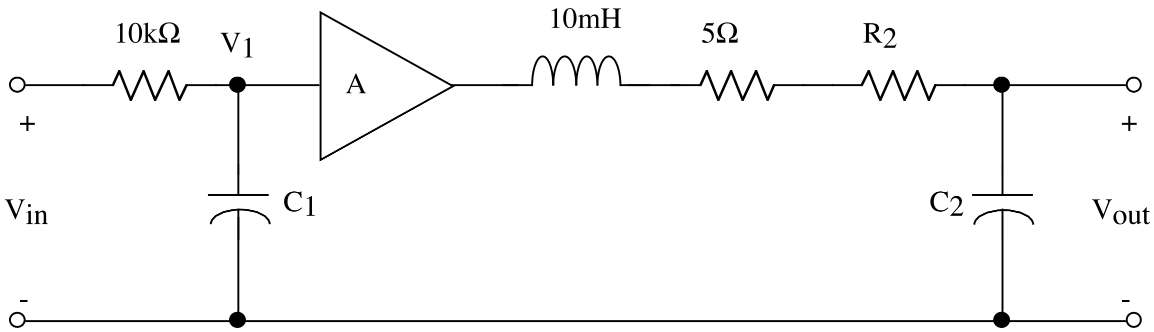
$$\frac{\omega_0}{Q}=1000$$

$$Q=\frac{\omega_0}{1000}=0.141$$

$$Q=\frac{\omega_0}{\Delta\omega}$$
$$\Delta\omega=\frac{\omega_0}{Q}=\frac{141.42}{0.141}=1000$$

Frequency Selective Filters

4. Specify R2 and capacitances C1 and C2 to cause the following circuit to be a Butterworth low-pass filter with a -3dB (corner) frequency of 150 Hz. The amplifier is to be considered an ideal unity gain amplifier (infinite input resistance and zero output resistance).



$$\frac{V_1}{V_{in}} = \frac{\frac{1}{sC_1}}{\frac{1}{sC_1} + 10^4} = \frac{1}{1 + s10^4 C_1}$$

$$\frac{V_o}{V_1} = \frac{\frac{1}{sC_2}}{\frac{1}{sC_2} + (R_2 + 5) + s10^{-2}} = \frac{1}{s^2 10^{-2} C_2 + (R_2 + 5)sC_2 + 1}$$

$$\frac{V_o}{V_{in}} = \frac{V_1}{V_{in}}\frac{V_o}{V_1} = \frac{1}{1 + s10^4 C_1} \times \frac{1}{s^2 10^{-2} C_2 + (R_2 + 5)sC_2 + 1}$$

The standard form for a Butterworth filter from a table of standard transfer functions (3rd order) is

$$\frac{V_o}{V_{in}} = \frac{1}{1 + \frac{s}{\omega_o}} \times \frac{1}{\left(\frac{s}{\omega_o}\right)^2 + \left(\frac{s}{\omega_o}\right) + 1}$$

We want $\omega_o = 2\pi f_o = 2\pi(150) = 300\pi$

Identifying terms

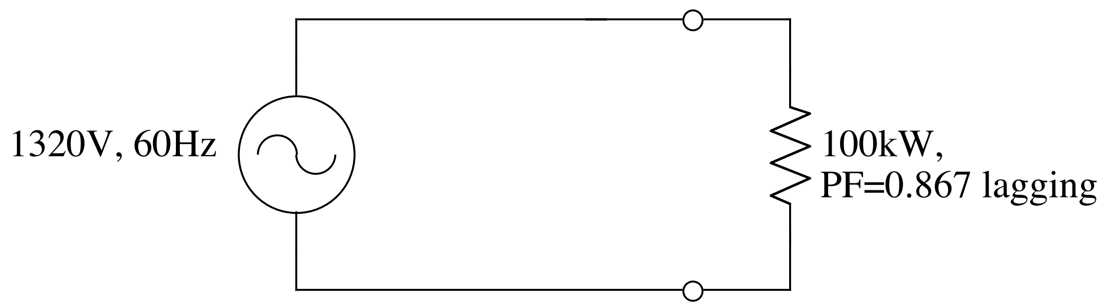
$10^4 C_1 = 300\pi$, or $C_1 = \frac{10^{-4}}{300\pi} = 106nf$

$10^{-2} C_2 = \frac{1}{(300\pi)^2}$, or $C_2 = \frac{10^2}{(300\pi)^2} = 113\mu f$

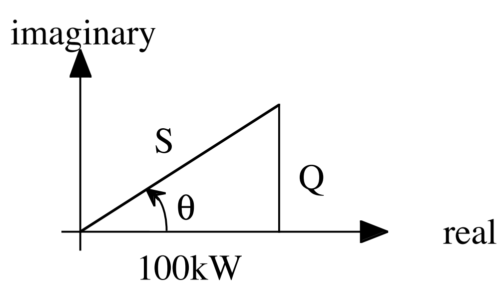
$(R_2 + 5)C_2 = \frac{1}{300\pi}$, or $R_2 = \frac{1}{300\pi}\frac{(300\pi)^2}{10^2} - 5 = 4.4\Omega$

Power Factor Correction

A load is connected to a voltage of 1320 volts at 60Hz. The load dissipates 100kW with a 0.867 lagging power factor. Specify the capacitance needed to correct the power factor to:
(a) 0.895 lagging
(b) 0.95 leading
(Give the voltage and volt-ampere-reactive ratings at 60Hz for the capacitors.)

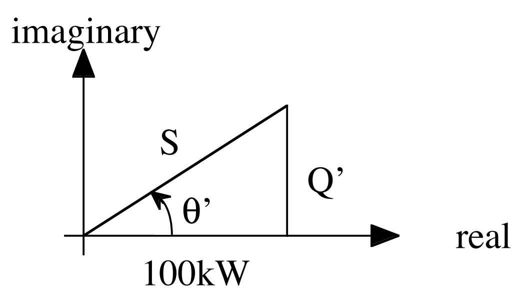


Determine the initial reactive power before correction.



$PF = 0.867 lagging = \cos\theta$ therefore $\theta = 29.89°$
From the power triangle
$Q = 100kW \tan\theta = 100\tan 29.89° = 100(0.5747) = +j57.47kVAR$

This reactive power must be corrected as per the problem specification. The desired power factor is $PF = 0.895 lagging = \cos\theta'$. Therefore, the new angle must be $\theta' = 26.49°$. The new reactive power for this angle comes from the new power triangle.



$Q' = 100kW \tan\theta' = 100\tan 26.49° = 100(0.4984) = +j49.84kVAR$
The difference in reactive powers must be supplied by the correction capacitor.

$$Q + Q_{capacitor} = Q'$$

$$Q_{capacitor} = Q' - Q = +j49.84 - j57.47 = -j7.63kVAR$$

$$P_{capacitor} = VI*$$

$$-j7.63 \times 10^3 = (1320)\left(\frac{1320}{X_C}\right)*$$

Solving for the capacitive reactance:

$$X_C* = \frac{(1320)^2}{-j7.63 \times 10^3} = \frac{1}{-j2\pi(60)C}$$

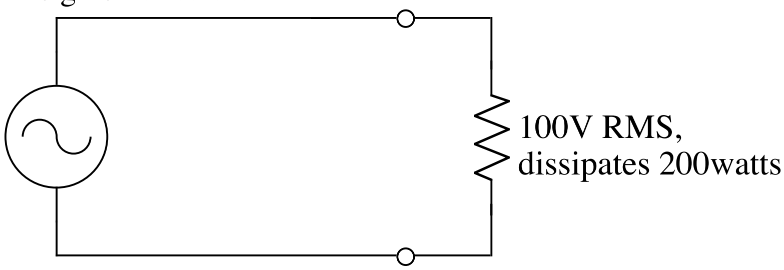
Solving for the required capacitance

$$C = \frac{7630}{(1320)^2 2\pi(60)} = 1.16 \times 10^5 = 11.6\mu f$$

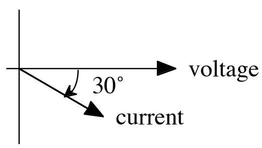
We already know the power rating to be 7.6kVAR and the voltage rating to be 1320 volts.

5. An impedance receives a line current which lags the voltage by 30˚. When the voltage across the impedance is 100 volts (RMS), the impedance dissipates 200 watts. Specify the reactance of a capacitance to be places in parallel with the impedance which would make the line current be in phase with the voltage.

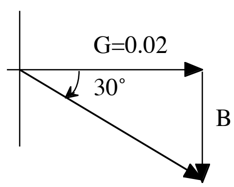
The given circuit is



The current lags the voltage

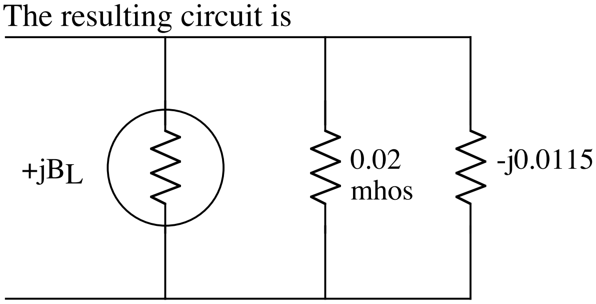


From the power specification $P=\frac{V^2}{R}$ or $R=\frac{V^2}{P}=\frac{(100)^2}{200}=50$. This gives G=0.02mhos. At this point we have $I=YV=(G+jB)V=(0.02+jB)V$. Since the phase angle is known to be 30˚ we can use the relationship between voltage and current to find B.



$\tan 30° = \frac{B}{G}$ or $B = G\tan 30° = 0.02\tan 30° = 0.0115$. Note that B is actually negative.

The resulting circuit is



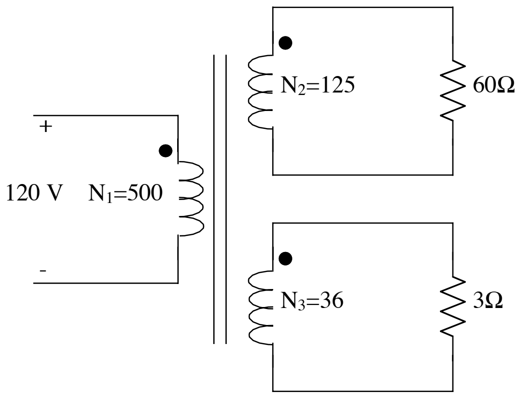
Having the line current being in phase means that the reactance is zero. This requires

$$jB_C + jB_L = 0$$
$$B_C = -B_L = -(-0.0115) = 0.0115$$
$X_C = -86.6\Omega$ for the desired power factor correction

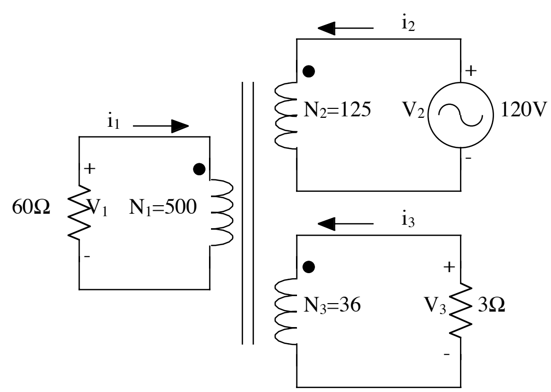
Transformers

Example 2.4

A transformer consists of a primary winding with 500 turns and two secondary windings of 125 turns and 30 turns. The 125 turn winding has 60 ohms connected to its terminals, and the 30 turn secondary winding has 3 ohms connected to its terminals. If the primary winding is connected to a 120 volt 60 Hz source, determine the current rating for each winding.



4. Repeat Example 2.4 with the 120 volt source connected to winding 2 and the 60 ohm load attached to winding 1.



$$V_1 = \frac{N_1}{N_2} V_2 = \frac{500}{125}(120) = 480 volts$$

$$V_3 = \frac{N_3}{N_2} V_2 = \frac{36}{125}(120) = 34.56 volts$$

$$i_1 = \frac{480V}{60\Omega} = 8 amps$$

$$i_3 = \frac{34.56V}{3\Omega} = 11.52 amps$$

To find i_2 we use conservation of power

$$v_2 i_2 = v_1 i_1 + v_3 i_3$$

$$120 i_2 = (480)(8) + (34.56)(11.52)$$

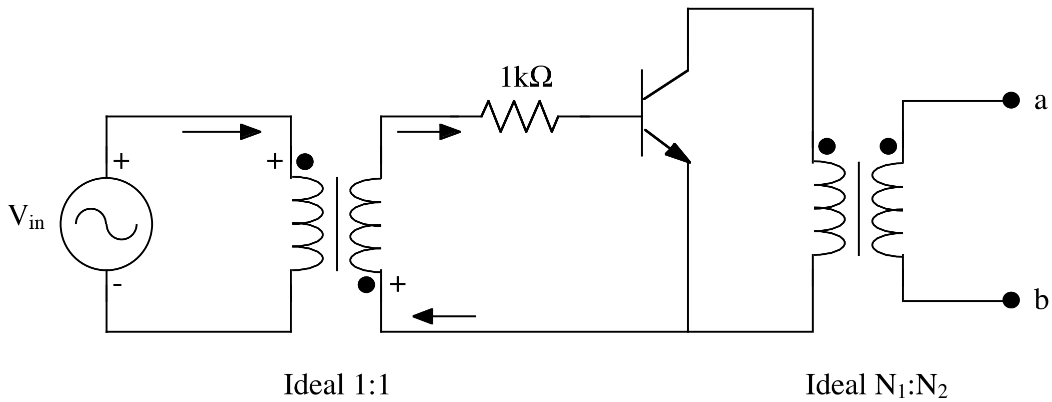
$$120 i_2 = 3840 + 398.13$$

$$i_2 = 35.32 amps$$

Small signal and large signal

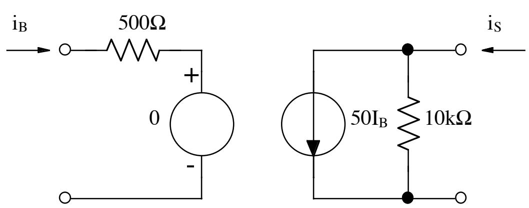
Example 2.14

A simplified small signal amplifier circuit is shown. The voltage source Vin is a variable signal source and can be considered to be an ideal voltage source. The transistor has parameters h_{ie}=500, h_{re}=0, h_{fe}=50, and h_{oe}=0.0001. Determine the Thevenin equivalent circuit (as seen from terminals a and b) in terms of the unknown turns N_1 and N_2 on the right-hand transformer.

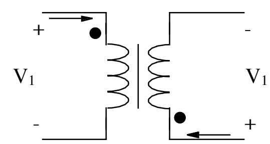


13. For the circuit of example 2.14, obtain the Norton equivalent circuit.

The transistor model is

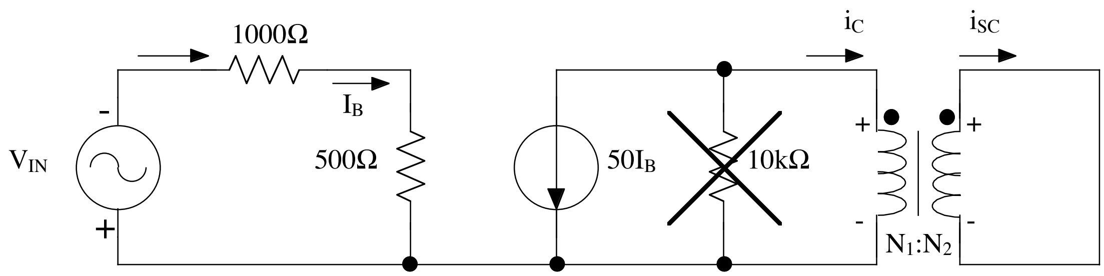


For the transformer, if the +v terminal is at the dot then the current is into the dot.



See p. 28-3 of your reference book

The resulting overall circuit is then



Note the results of the dot convention. The current flowing into the dot results in current flowing out of the top of the transformer secondary. The voltage source's polarity is reversed by the transformer as shown in the first figure. The output is a short circuit because we are determining the short circuit current.

This gives

$$i_B = +\frac{-v_{in}}{1000+500} = \frac{-v_{in}}{1500}$$

Continuing through the transistor,

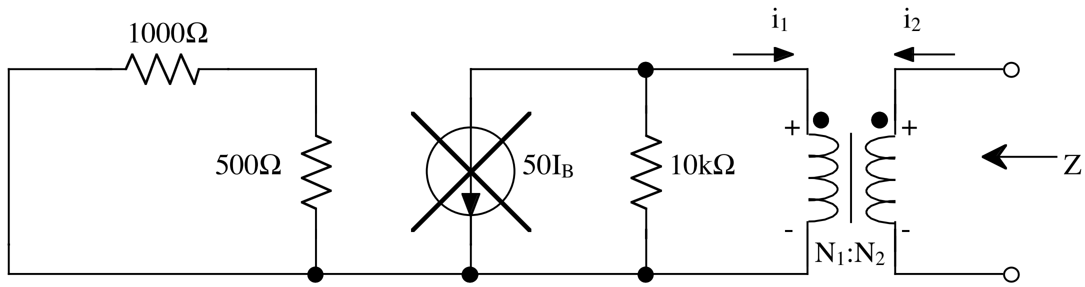
$$i_C = -50 i_B = +\frac{50}{1500} v_{in}$$

By the dot conventions for the output transformer, the output current is going into the dot. The current magnitude is transformed as $\frac{N_1}{N_2}$ and the voltage magnitude as $\frac{N_2}{N_1}$ (Power is conserved).

Since we are looking for a Norton equivalent circuit, the output is short circuited and the load voltage is zero. Consequently, we are only looking at the short-circuit current. In this context the transformer also appears as an ac short and we can ignore the 10kΩ output impedance of the transistor.

$$i_{SC} = -i_C \frac{N_1}{N_2} = -\frac{50}{1500} v_{in} \frac{N_1}{N_2} = -\frac{1}{30}\frac{N_1}{N_2} v_{in}$$

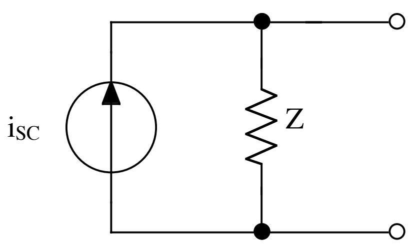
This gives us the short circuit current in the direction shown. To get the Norton equivalent circuit we need to also compute the Thevenin resistance. To get this resistance we short v_{in}, NOT the transistor current source $50 i_B$ and look at the resulting circuit.



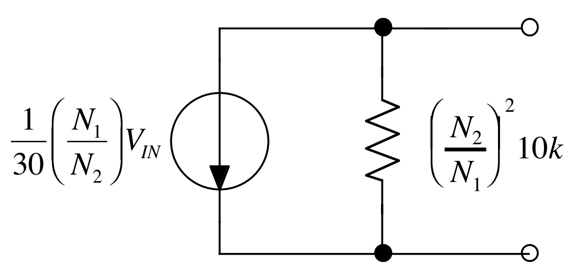
This leaves only the 10kΩ resistor which gets transformed by the transformer as

$$R_N = \left(\frac{N_2}{N_1}\right)^2 10k\Omega$$

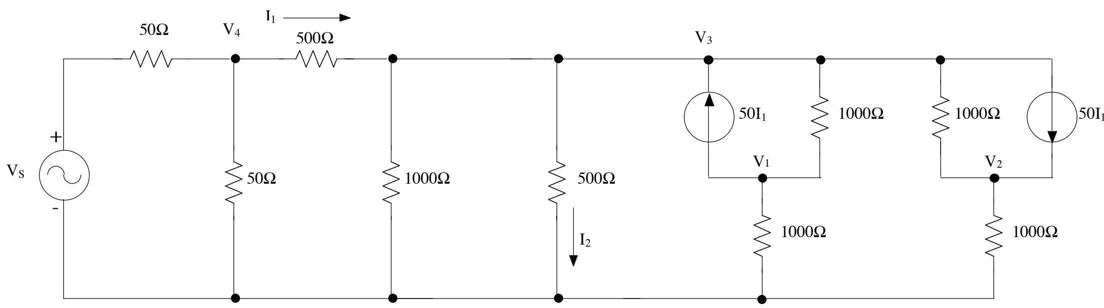
The final Norton equivalent circuit is then



or



2. For the circuit shown, obtain the node voltage equations in matrix form. Solve these equations for V_1 and V_2. (The actual output is V_1- V_2.) Obtain the voltage gain $\frac{V_{OUT}}{V_S}$ for this emitter -coupled amplifier.



This is a complex problem which requires that we know how to construct an admittance matrix representation for the circuit.

The standard ("formal") admittance matrix formulation is:

$$\begin{bmatrix} \sum I_1 \\ \sum I_2 \\ \\ \\ \end{bmatrix} = \begin{bmatrix} \sum Y_{11} & \sum Y_{12} & \\ \sum Y_{21} & \sum Y_{22} & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \\ \\ \end{bmatrix}$$

where

$\sum I_i$ =sum of <u>current sources</u> feeding into node i

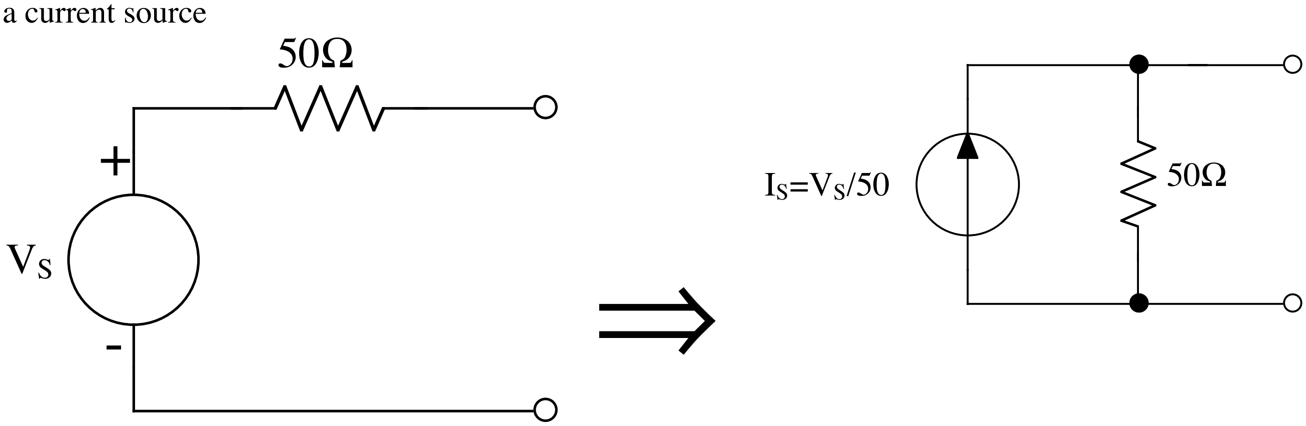
(into=positive, out=negative)

$\sum Y_{ii}$ =sum of all admittances connected to node i

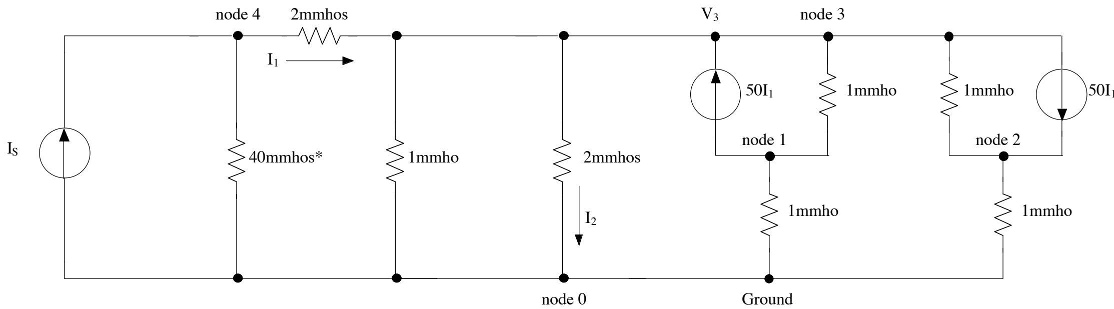
$\sum Y_{ij}$ = -sum of all admittances connected between nodes i and j

V_i=node voltages (including dependent sources)

The first step in solving the problem is to Nortonize the voltage source, converting it into a current source



Now, convert all resistances into admittances (I will use the unit of millimhos for



convenience) and label the nodes.
Note that node 0, the reference node, must always be present.
*The 40millimhos is the combination of the two 50Ω resistors in parallel.

Constructing the admittance matrix representation

$$\begin{bmatrix} -50i_1 \\ +50i_2 \\ 50i_1 - 50i_2 \\ I_S \end{bmatrix} = \begin{bmatrix} 1+1 & 0 & -1 & 0 \\ 0 & 1+1 & -1 & 0 \\ -1 & -1 & 2+1+2+1+1 & -2 \\ 0 & 0 & -2 & 40+2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Consider node 1:
A current source of $50i_1$ leaves node 1, therefore the current is $-50i_1$
The sum of all admittances connected to node 1 is 1+1 millimhos
There are no admittances directly connected between nodes 2 and 1 or 4 and 1, therefore $Y_{12} = Y_{14} = 0$
There is 1 millimho connected between nodes 1 and 3 so $Y_{13} = -1$ millimho since all off-diagonal terms are negative.

Actually each row of the matrix represents a KCL equation for that node and this is the way I prefer to generate the matrix. At node 1, assuming that currents into the node are positive,

$$-50i_1 - 1(V_1 - 0) - 1(V_1 - V_3) = 0$$

which can be re-written as

$$-50i_1 = (1+1)V_1 - V_3 = 0$$

which is exactly the first row of the matrix equation.

The rest of the problem is simply solving the matrix:

$$\begin{bmatrix} -50i_1 \\ +50i_2 \\ 50i_1 - 50i_2 \\ I_S \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 2 & -1 & 0 \\ -1 & -1 & 7 & -2 \\ 0 & 0 & -2 & 42 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Expressing i_1 and i_2 in terms of independent variables V_3 and V_4

$$i_1 = \frac{V_4 - V_3}{500} = 2(V_4 - V_3)$$
$$i_2 = \frac{V_3}{500} = 2V_3$$
and converting the source term
$$\frac{V_S}{50} = 20V_S$$
Then,
$$\begin{bmatrix} -100V_4 + 100V_3 \\ 100V_3 \\ 50(2V_4 - 2V_3) - 50(2V_3) \\ 20V_S \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 2 & -1 & 0 \\ -1 & -1 & 7 & -2 \\ 0 & 0 & -2 & 42 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

$$\begin{bmatrix} -100V_4 + 100V_3 \\ 100V_3 \\ 100V_4 - 200V_3 \\ 20V_S \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 2 & -1 & 0 \\ -1 & -1 & 7 & -2 \\ 0 & 0 & -2 & 42 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Rearranging the equations,
$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 20V_S \end{bmatrix} = \begin{bmatrix} 2 & 0 & -101 & +100 \\ 0 & 2 & -101 & 0 \\ -1 & -1 & 207 & -102 \\ 0 & 0 & -2 & 42 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$
This equation can now be solved for the variables we are interested in (V_1 and V_2) using any method you want.

I highly recommend using a calculator which can solve systems of equations since this is where I usually make math errors. However, I will use expansion by minors to solve for V_1 and V_2 since the matrices are only 4×4.

$$V_1 = \frac{-20V_S \begin{bmatrix} 0 & -101 & 100 \\ 2 & -101 & 0 \\ -1 & 207 & -102 \end{bmatrix}}{2\begin{bmatrix} 2 & -101 & 0 \\ -1 & 207 & -102 \\ 0 & -2 & 42 \end{bmatrix} - 1\begin{bmatrix} 0 & -101 & 100 \\ 2 & -101 & 0 \\ 0 & -2 & 42 \end{bmatrix}}$$
The coefficients of all other minors are zero and not shown. Recall that solving for V_1 requires that the column vector of sources be substituted for column 1. Note also that the signs of the minor coefficients alternate.

Solving:

$$V_1 = \frac{-20V_S\left[(2)(207)(100)-(100)(-101)(-1)-(2)(-101)(-102)\right]}{2\left[(2)(207)(42)-(2)(-2)(-102)-(-1)(-101)(42)\right]-1\left[(2)(-2)(100)-(2)(42)(-101)\right]}$$

$$V_1 = \frac{-20V_S[41400-10100-20604]}{2[17388-408-4242]-1[-400+8484]} = \frac{-20V_S[10696]}{2[12738]-1[8084]}$$

$$V_1 = \frac{-213920}{17392}V_S = -12.29V_S$$

Similarly,

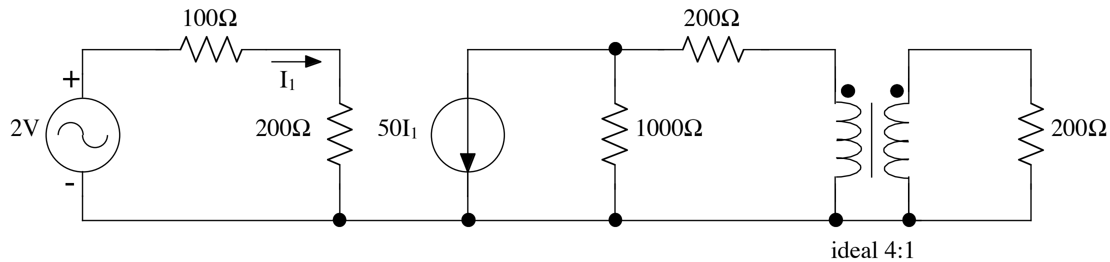
$$V_2 = \frac{-20V_S\begin{bmatrix}2 & -101 & 100\\ 0 & -101 & 0\\ -1 & 207 & -102\end{bmatrix}}{2\begin{bmatrix}2 & -101 & 0\\ -1 & 207 & -102\\ 0 & -2 & 42\end{bmatrix}-1\begin{bmatrix}0 & -101 & 100\\ 2 & -101 & 0\\ 0 & -2 & 42\end{bmatrix}} = \frac{-20V_S[(2)(-101)(-102)-(-1)(-101)(100)]}{17392}$$

where we saved a little effort by recognizing that the denominator is the same as before.

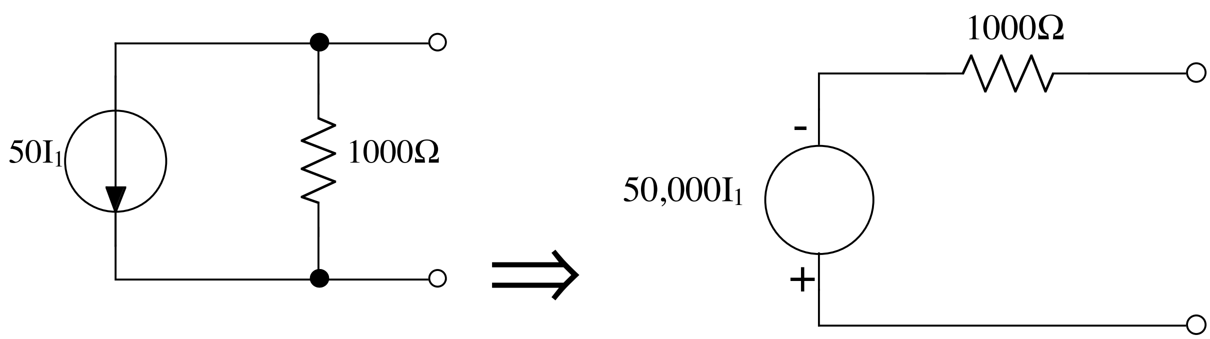
$$V_2 = \frac{20V_S[20604-10100]}{17392} = +12.079V_S$$

$$A_V = \frac{V_1-V_2}{V_S} = \frac{-12.29V_S-12.079V_S}{V_S} = -24.37$$

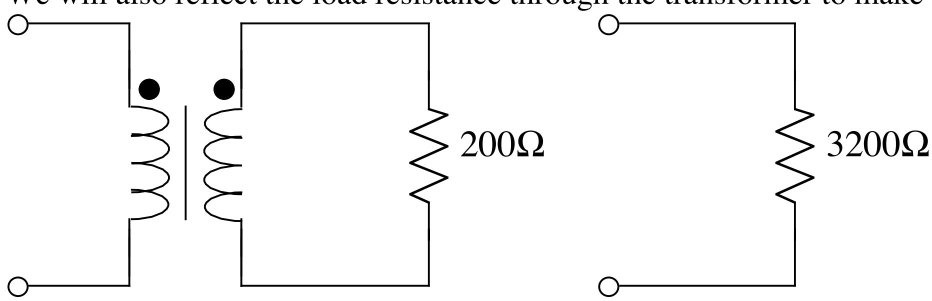
3. For the circuit shown, obtain the loop current equations. Solve for the current in the 200 ohm resistance connected to the secondary of the ideal transformer.



As usual we will Thevenize the dependent current source to make the loop analysis easier.



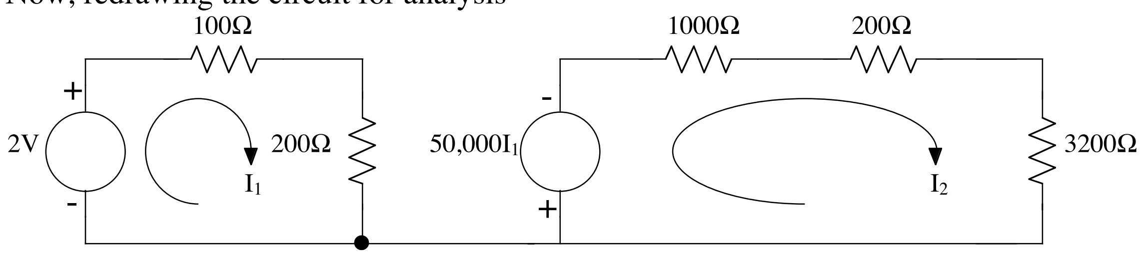
We will also reflect the load resistance through the transformer to make analysis simpler.



ideal 4:1

where $(4)^2(200) = (16)(200) = 3200\Omega$

Now, redrawing the circuit for analysis



Writing the loop equations

$$-2 + 100i_1 + 200i_1 = 0$$

$$+50000i_1 + 1000i_2 + 200i_2 + 3200i_2 = 0$$

$$\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 300 & 0 \\ 50000 & 4400 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

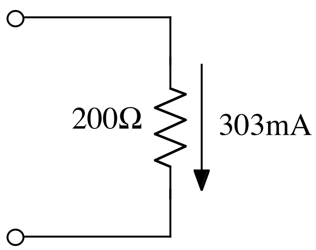
We are interested in the transformer primary current i_2. Solving for i_2:

$$i_2 = \frac{\begin{bmatrix} 300 & 2 \\ 50000 & 0 \end{bmatrix}}{\begin{bmatrix} 300 & 0 \\ 50000 & 4400 \end{bmatrix}} = \frac{0-(50000)(2)}{(300)(4400)-0} = -75.76mA$$

The transformer secondary current is then

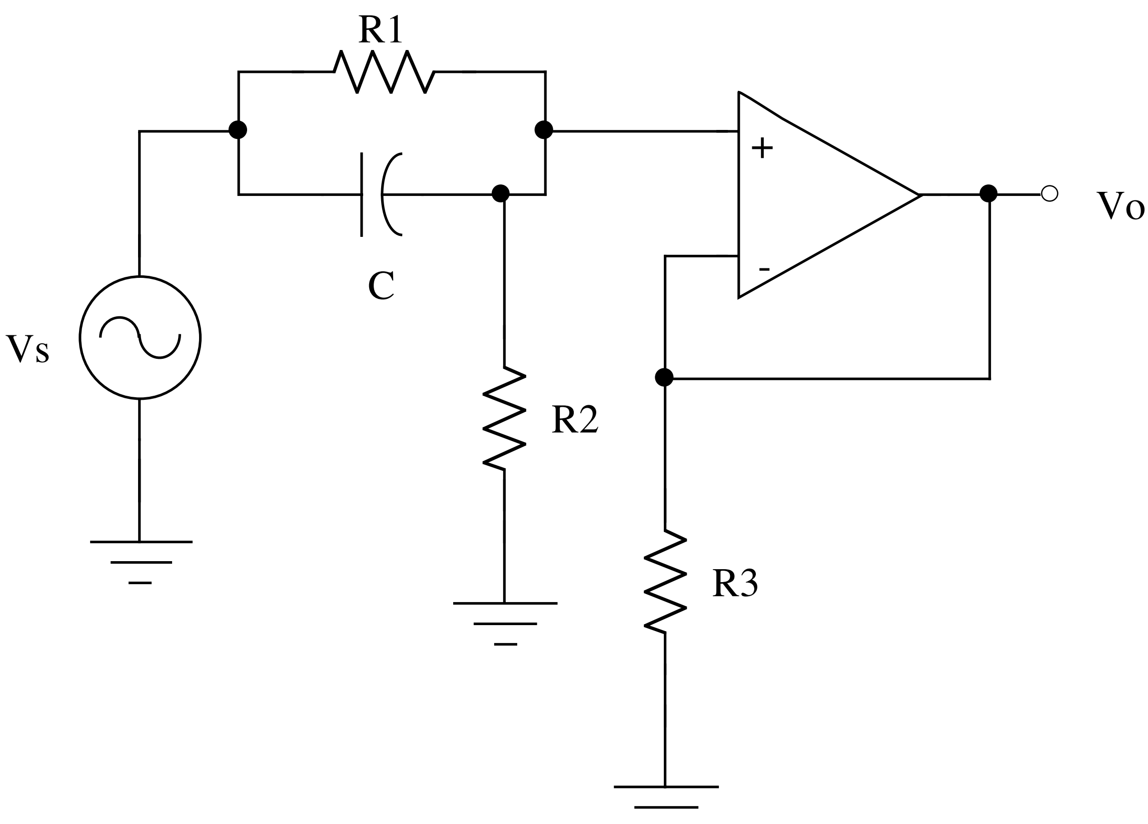
-75.76mA×4=-303mA

with the current as shown below



Active networks and filters

1. Consider the OP-AMP circuit shown below.
   (a) Determine the system transfer function.
   (b) Sketch the gain of the circuit as a function of ω. Assume that R_1=1000Ω, R_2=200Ω, R_2=1000Ω and C=1μF.
   (c) What type of filter is this: low-pass, high-pass, etc.?



SOLUTION:

(a) Apply KCl at the inverting input of the op-amp. Assuming that current into the node is positive we can write

$$\frac{V_s - V_o}{R_1} + \frac{V_s - V_o}{\frac{1}{sC}} - \frac{V_o}{R_2} = 0$$

Solving for the transfer function gives

$$\left(\frac{1}{R_1} + sC\right)V_s = \left(\frac{1}{R_1} + \frac{1}{R_2} + sC\right)V_o$$

$$\frac{V_o}{V_s} = \frac{\frac{1}{R_1} + sC}{\frac{1}{R_1} + \frac{1}{R_2} + sC} = \frac{s + \frac{1}{R_1 C}}{s + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{1}{C}} = \frac{s + \frac{1}{R_1 C}}{s + \frac{1}{(R_1 \| R_2)C}}$$

This transfer function has a single pole and a single zero.

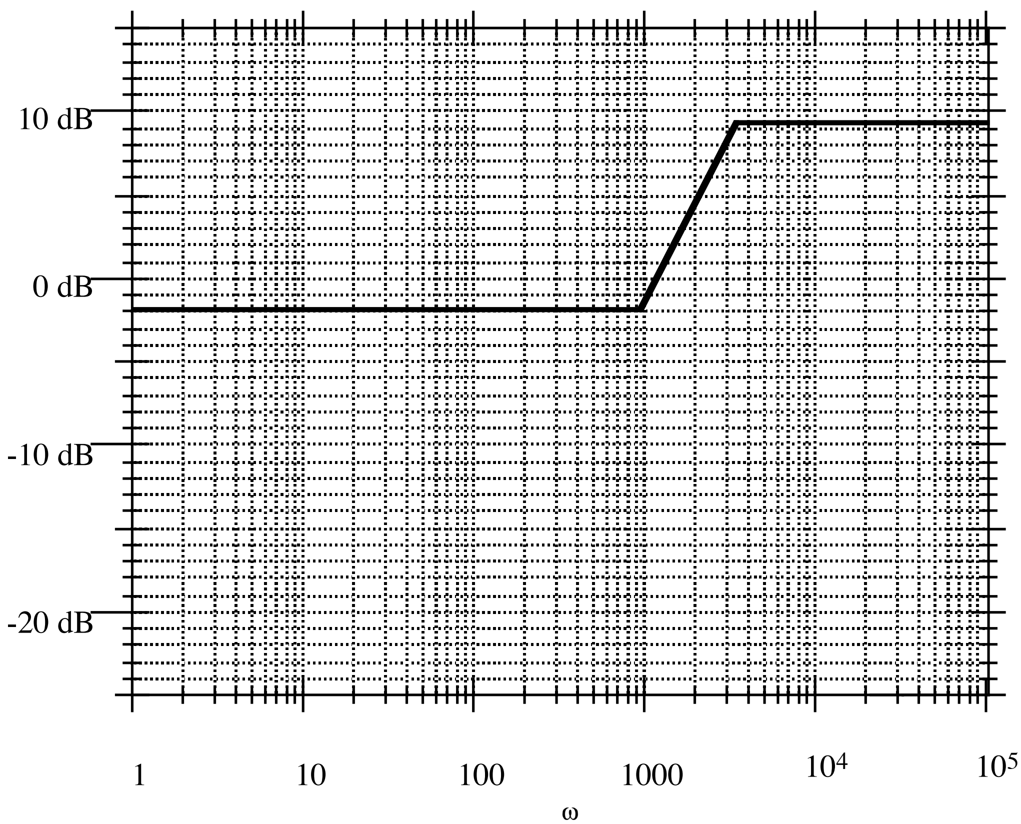
(b) For the given values, the transfer function evaluates to

$$\frac{V_o}{V_s} = \frac{s + \frac{1}{(1000)(1\times 10^{-6})}}{s + \frac{1}{\left(\frac{(1000)(200)}{1000 + 200}\right)(1\times 10^{-6})}} = \frac{s + 1000}{s + 1250} = 0.8\frac{1 + \frac{s}{1000}}{s + \frac{s}{1250}}$$

$$\left|H(j\omega)\right| = 20\log(0.8) + 20\log\left|1 + \frac{s}{1000}\right| - 20\log\left|1 + \frac{s}{1250}\right|$$

The first term is approximately -1.9 dB.

(c) This is a high-pass filter.



Two-port theory (see p. 29-16 for definitions)

8. A three-terminal device is described by the following z-parameter equations.

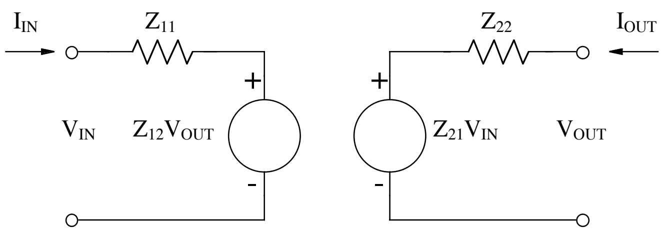
$V_{IN} = 250 i_{IN} + 5 i_{OUT}$

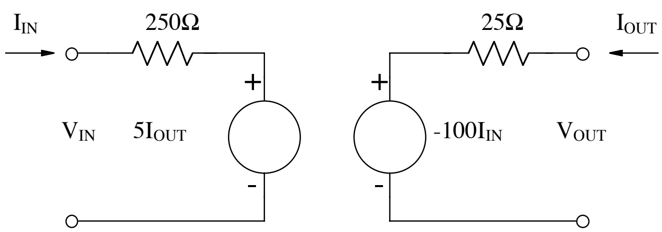
$V_{OUT} = -100 i_{IN} + 25 i_{OUT}$

Obtain an equivalent circuit for this device.

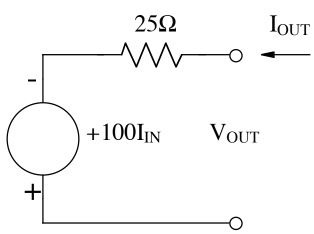
Re-writing in matrix form. This is the z-parameter model.

$$\begin{bmatrix} V_{IN} \\ V_{OUT} \end{bmatrix} = \begin{bmatrix} 250 & 5 \\ -100 & 25 \end{bmatrix} \begin{bmatrix} i_{IN} \\ i_{OUT} \end{bmatrix}$$





or draw reversed as



16. The circuit of example 2.17 has r_b=200 ohms, β=50, r_c=2500 ohms, and r_e=10 ohms. Find the voltage gain $\frac{V_{OUT}}{V_{IN}}$ when a 1000 ohm load is placed across the V_{OUT} terminals. Note that the output current I_2 is

$$-\frac{V_{OUT}}{1000}$$

for this load.

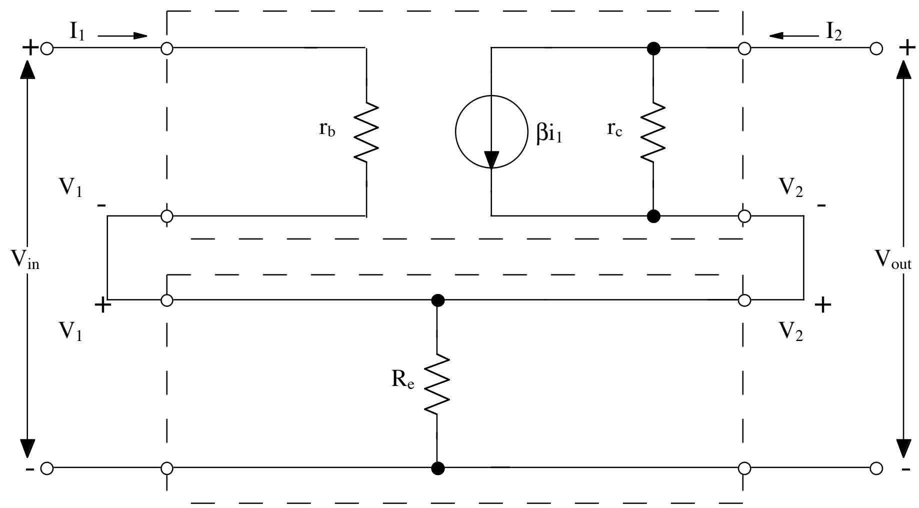
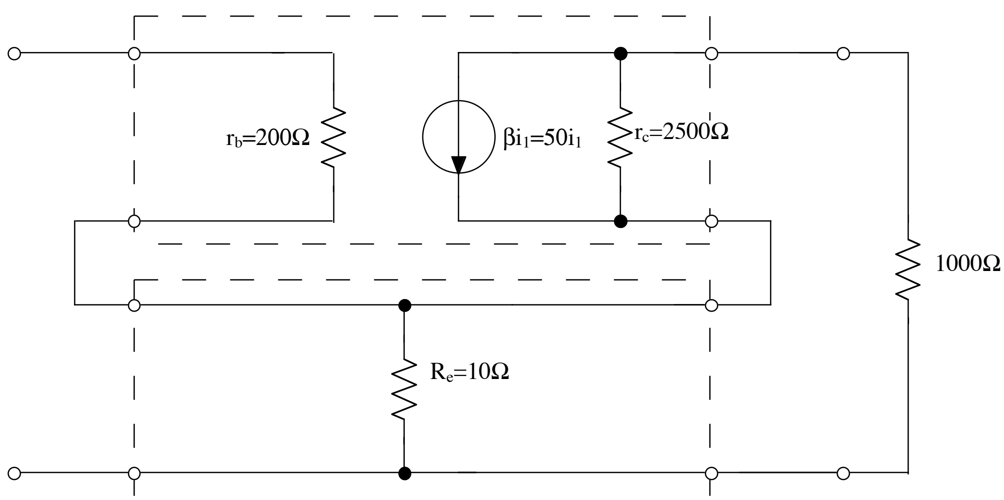


Figure 2.16. Combination of 2-port networks with common currents.

The circuit is then

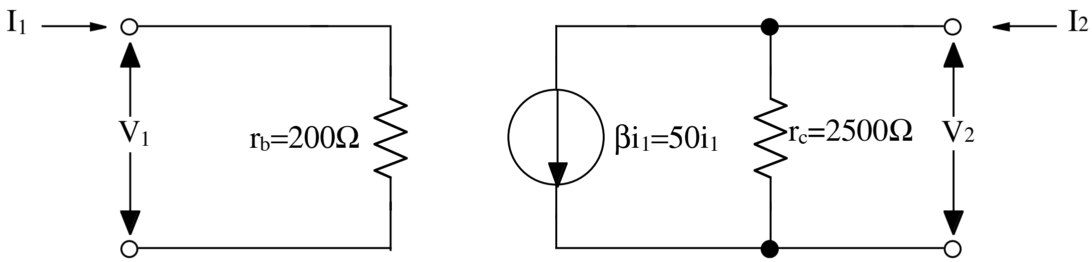


We will convert both the upper and lower circuits to z-parameters and then combine them.

for z-parameters

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

For an open output $I_2 = 0$ and
$V_1 = z_{11} I_1$
$V_2 = z_{21} I_1$



By inspection

$$z_{11} = \frac{V_1}{I_1} = 200\Omega$$

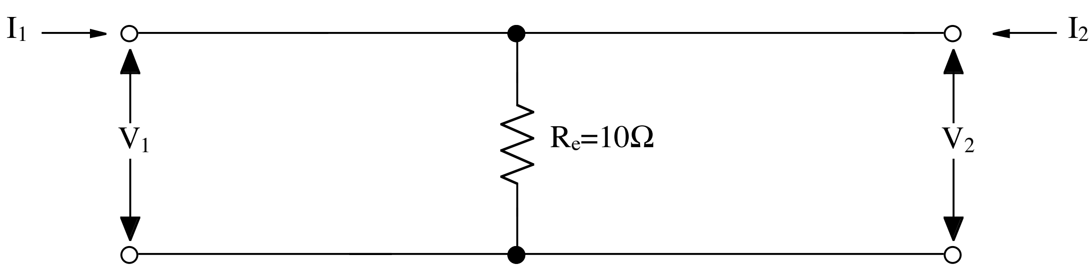
$$z_{21} = \frac{V_2}{I_1} = \frac{-(50I_1)(2500)}{I_1} = -1.25 \times 10^5$$

if $I_1 = 0$
$V_1 = z_{12} I_2$
$V_2 = z_{22} I_2$
and

$$z_{12} = \frac{V_1}{I_2} = 0$$

$$z_{22} = \frac{V_2}{I_2} = 2500\Omega$$

For the 10Ω resistor



$$Z = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix}$$

The total admittance matrix is then

$$Z_T = \begin{bmatrix} 200 & 0 \\ -1.25 \times 10^5 & 2500 \end{bmatrix} + \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix} = \begin{bmatrix} 210 & 10 \\ -124990 & 2510 \end{bmatrix}$$

To find the voltage gain we write the network equations resulting from Z_T

$V_1 = 210 I_1 + 10 I_2$ (1)

$V_2 = -124990 I_1 + 2510 I_2$ (2)

For the output,

$V_2 = -I_2 R_L$

Using (2) and (3)

$$V_2 = -124990 I_1 + 2510\left(-\frac{V_2}{R_L}\right)$$

$$V_2 + \frac{2510}{1000} V_2 = -124990 I_1$$

$$I_1 = -\frac{1+2.51}{124990} V_2 = -2.81\times 10^{-5} V_2$$

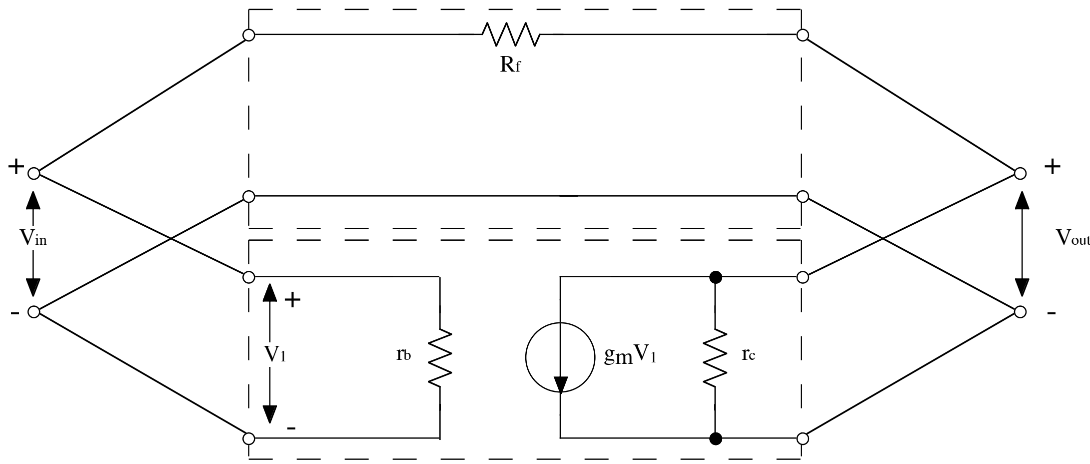
Substituting this result into (1) and using (3)

$$V_1 = 210\left(-2.81\times 10^{-5} V_2\right) + 10\left(\frac{-V_2}{1000}\right)$$

$$V_1 = -5.90\times 10^{-3} V_2 - 0.01 V_2 = -1.59\times 10^{-2} V_2$$

$$\frac{V_2}{V_1} = -62.9$$

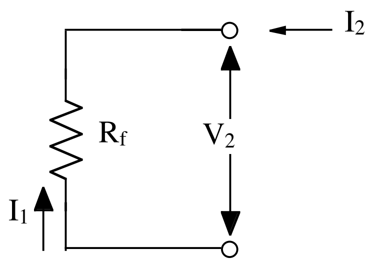
17. Using y-parameters, obtain the total circuit y-parameters for the circuits indicated by the dashed lines. Hint: first find the y-parameters of the two indicated two-port networks, then combine them to obtain the total network y-parameters.



The solution of this problem is similar to that of problem 16 except that y-parameters will be used.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

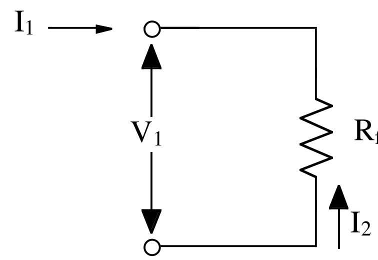
For $V_1 = 0$:
$I_1 = y_{12} V_2$
$I_2 = y_{22} V_2$



$$y_{12} = \frac{I_1}{V_2} = \frac{-I_2}{V_2} = -\frac{1}{R_f}$$

$$y_{22} = \frac{I_2}{V_2} = \frac{1}{R_f}$$

For $V_2 = 0$:
$I_1 = y_{11} V_1$
$I_2 = y_{21} V_1$

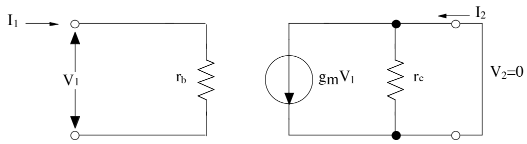


$$y_{11} = \frac{I_1}{V_1} = \frac{1}{R_f}$$

$$y_{21} = \frac{I_2}{V_1} = -\frac{I_1}{V_1} = -\frac{1}{R_f}$$

Therefore, $Y_R = \begin{bmatrix} \frac{1}{R_f} & -\frac{1}{R_f} \\ -\frac{1}{R_f} & \frac{1}{R_f} \end{bmatrix}$

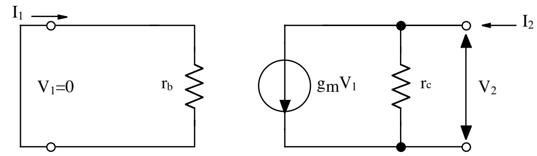
For $V_2 = 0$



$$y_{11} = \frac{I_1}{V_1} = \frac{1}{r_b}$$

$$y_{21} = \frac{I_2}{V_1} = \frac{g_m V_1}{V_1} = +g_m$$

For $V_1 = 0$



$$y_{12} = \frac{I_1}{V_2} = 0$$

$$y_{22} = \frac{I_2}{V_2} = \frac{1}{r_c}$$

And, therefore, $Y_{transistor} = \begin{bmatrix} \frac{1}{r_b} & 0 \\ g_m & \frac{1}{r_c} \end{bmatrix}$

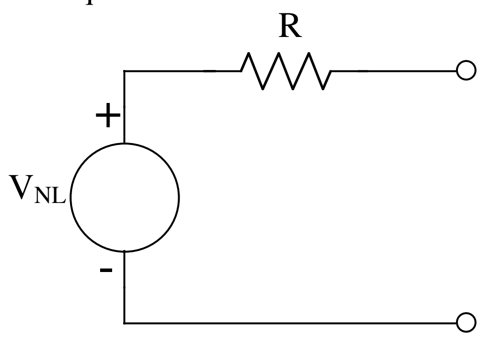
$$Y_{total} = Y_{transistor} + Y_R = \begin{bmatrix} \frac{1}{r_b} & 0 \\ g_m & \frac{1}{r_c} \end{bmatrix} + \begin{bmatrix} \frac{1}{R_f} & -\frac{1}{R_f} \\ -\frac{1}{R_f} & \frac{1}{R_f} \end{bmatrix}$$

$$Y_{total} = \begin{bmatrix} \frac{1}{R_f} + \frac{1}{r_b} & -\frac{1}{R_f} \\ g_m - \frac{1}{R_f} & \frac{1}{R_f} + \frac{1}{r_c} \end{bmatrix}$$

Power supplies

6. A 5 volt, 25 amp d.c. source has voltage regulation of 1%. Obtain an equivalent circuit for this source.

The equivalent circuit is



$$VR = \frac{V_{NoLoad} - V_{FullLoad}}{V_{FullLoad}} \times 100\%$$

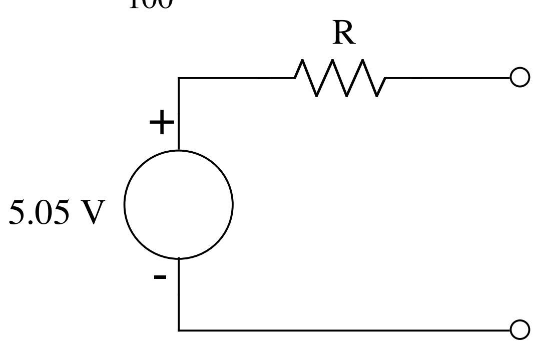
$$1\% = \frac{V_{NoLoad} - 5V}{5V} \times 100\%$$

Solving for V_{NoLoad}

$$5\% = V_{NoLoad} \times 100\% - 500\%$$

$$5 + 500 = V_{NoLoad}(100)$$

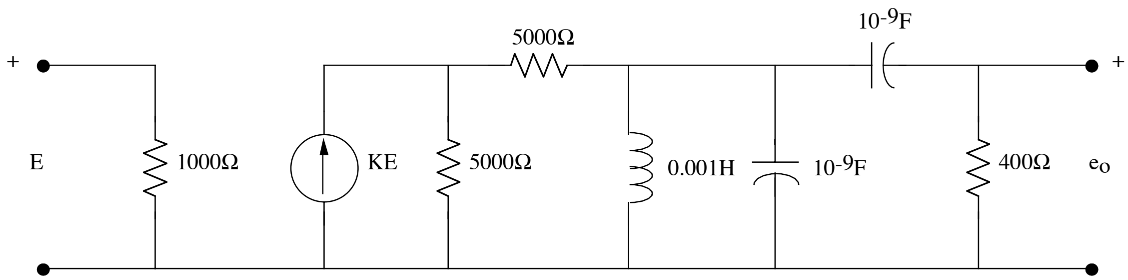
$$V_{NoLoad} = \frac{505}{100} = 5.05$$



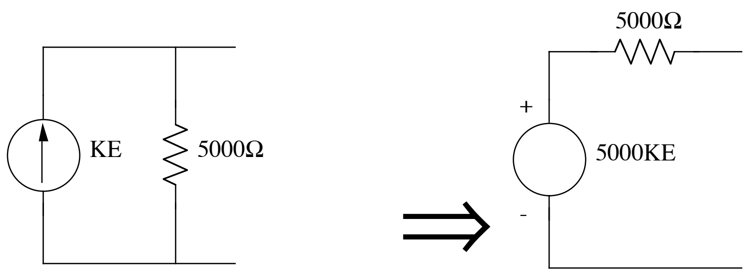
$$R = \frac{5.05 - 5}{25A} = \frac{0.05}{25} = 0.002\Omega$$

Oscillators

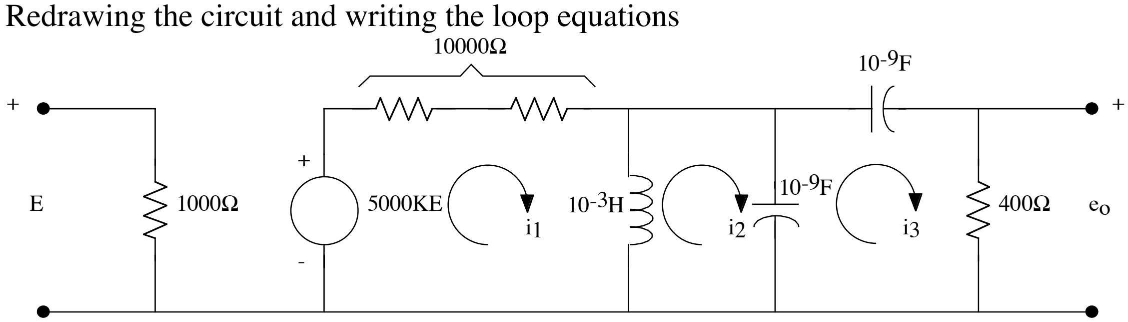
6. An oscillator circuit is shown below. Determine the minimum value of K such that $e_o=E$. Also, find the frequency at which $e_o=E$.



As usual we Thevenize the source so that we can combine resistances.



Redrawing the circuit and writing the loop equations



$$-5000KE + 10000i_1 + (i_1 - i_2)j\omega 10^{-3} = 0$$

$$j\omega 10^{-3}(i_2 - i_1) + (i_2 - i_3)\frac{1}{j\omega 10^{-9}} = 0$$

$$\frac{1}{j\omega 10^{-9}}(i_3 - i_2) + \frac{1}{j\omega 10^{-9}}i_3 + 400i_3 = 0$$

In matrix form,

$$\begin{bmatrix} +5000KE \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10000 + j\omega 10^{-3} & -j\omega 10^{-3} & 0 \\ -j\omega 10^{-3} & +j\omega 10^{-3} + \dfrac{1}{j\omega 10^{-9}} & -\dfrac{1}{j\omega 10^{-9}} \\ 0 & -\dfrac{1}{j\omega 10^{-9}} & \dfrac{1}{j\omega 10^{-9}} + \dfrac{1}{j\omega 10^{-9}} + 400 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

Since we are only interested in i_3:

$$i_3 = \frac{\begin{bmatrix} 10000 + j\omega 10^{-3} & -j\omega 10^{-3} & +5000KE \\ -j\omega 10^{-3} & +j\omega 10^{-3} + \frac{1}{j\omega 10^{-9}} & 0 \\ 0 & -\frac{1}{j\omega 10^{-9}} & 0 \end{bmatrix}}{\begin{bmatrix} 10000 + j\omega 10^{-3} & -j\omega 10^{-3} & 0 \\ -j\omega 10^{-3} & +j\omega 10^{-3} + \frac{1}{j\omega 10^{-9}} & -\frac{1}{j\omega 10^{-9}} \\ 0 & -\frac{1}{j\omega 10^{-9}} & \frac{1}{j\omega 10^{-9}} + \frac{1}{j\omega 10^{-9}} + 400 \end{bmatrix}}$$

$$i_3 = \frac{+5000KE\left(-j\omega 10^{-3}\right)\left(-\frac{1}{j\omega 10^{-9}}\right)}{\left\{\left(10^4 + j\omega 10^{-3}\right)\left(j\omega 10^{-3} + \frac{1}{j\omega 10^{-9}}\right)\left(+\frac{j2}{j\omega 10^{-9}} + 400\right) - \left(-j\omega 10^{-3}\right)\left(-j\omega 10^{-3}\right)\left(+\frac{j2}{j\omega 10^{-9}} + 400\right) - \left(-\frac{1}{j\omega 10^{-9}}\right)\left(-\frac{1}{j\omega 10^{-9}}\right)\left(10000 + j\omega 10^{-3}\right)\right\}}$$

$$i_3 = \frac{+5000KE \times 10^6}{\left\{2\times 10^{10} - \frac{2\times 10^{22}}{\omega^2} - \frac{2j\times 10^{15}}{\omega} + j\omega 4000 - \frac{4j\times 10^{15}}{\omega} + 4\times 10^8 + \frac{10^{22}}{\omega^2} + \frac{j10^{15}}{\omega}\right\}}$$

$$i_3 = \frac{+5\times 10^6 KE}{\left\{\left(2.04\times 10^{10} - \frac{10^{22}}{\omega^2}\right) + j\left(4000\omega - \frac{5\times 10^{15}}{\omega}\right)\right\}}$$

For $e_o \cong E$ for oscillation we require i_3 to be real which requires $4000\omega - \frac{5\times 10^{15}}{\omega} = 0$ or $4000\omega^2 = 5\times 10^{15}$

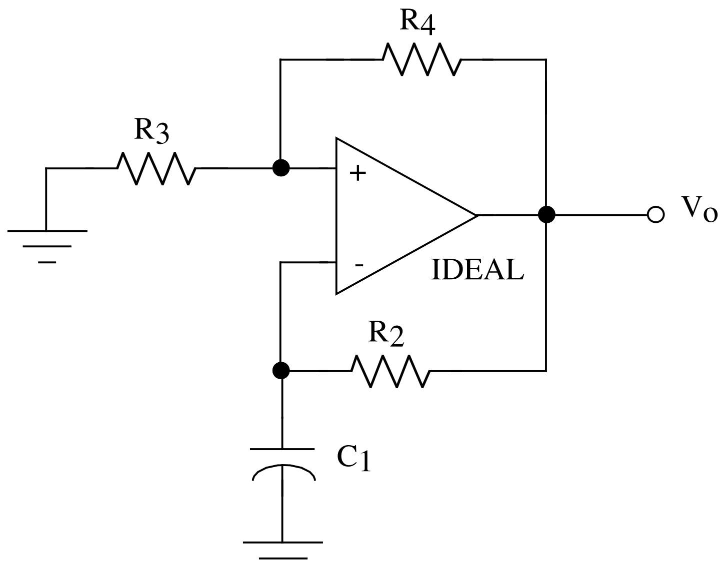
Solving for ω gives $\omega^2 = \frac{5\times 10^{15}}{4000} = 1.25\times 10^{12}$ or $\omega = 1.12\times 10^6 rad/\sec$

Using this value, $\frac{+5\times 10^6 KE}{2.04\times 10^{10} - \frac{10^{22}}{1.25\times 10^{12}}}(400) = E$

Solving for K gives $\frac{+5\times 10^6 K(400)}{2.04\times 10^{10} - 8\times 10^9} = 1$ or $K = \frac{1.24\times 10^{10}}{+5\times 10^6(400)} = +\frac{2480}{400} = +6.2$

Transients

Sketch the steady-state output of the following oscillator circuit. Be sure to indicate values of voltages and time. Assume the op-amp operates on ±15 power supplies.



The component values are C_1=0.1μf, R_2=5kΩ, R_3=10kΩ and R_4=10kΩ.

This problem can only be solved by making some assumptions about the capacitor (initially uncharged) and the output (assume it starts at 15 volts).

Assuming that $V_o = +15$ then $V_+ = \frac{R_3}{R_3 + R_4} V_o = \frac{10k\Omega}{10k\Omega + 10k\Omega}(15) = +7.5$. This will be the threshold at which the output switches state, i.e., if V-<+7.5 then Vo=+15 and, if V->7.5 volts, then Vo will switch to –15 volts.

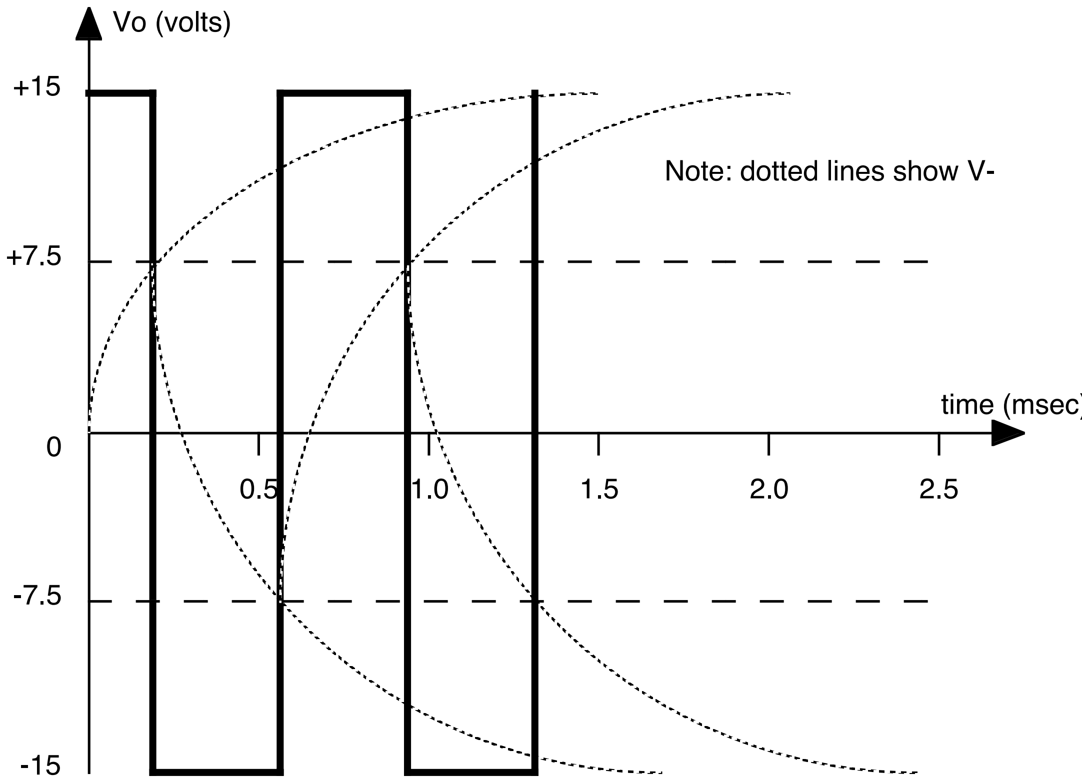
Since the capacitor C1 is initially uncharged the voltage V- varies according to

$$V_- = V_o e^{-\frac{t}{R_2 C_1}} = 15e^{-\frac{t}{(5\times10^3)(0.1\times10^{-6})}} = 15e^{-\frac{t}{0.5m\sec}}$$

This means that V- will initially be at 0 volts but will rise towards +15 with exponential time constant 0.5msec. I can sketch this and see that the switching will occur at approximately t=0.5msec. I can also do this mathematically and find out that

$15e^{-\frac{t}{0.5m\sec}} = 7.5$, or $e^{-\frac{t}{0.5m\sec}} = 0.5$.

Taking the logarithm and solving for t gives $-\frac{t}{0.5m\sec} = -0.69315$, or t=34.66 msec so my graphical drawing of exponentials was not really all that good.



Note that the period changes as the oscillator approaches its steady state values.

Characteristic equations

1. A system is described by the differential equation

$$20\sin 4t = \frac{d^2 i}{dt^2} + 4\frac{di}{dt} + 4i \qquad (1)$$

The initial conditions are $i(0) = 0$ and $\frac{di(0)}{dt} = 4$. Determine $i(t)$ for t>0.

To find the steady state solution convert to phasors

$$20\sin 4t = 20\cos(4t - 90°) \rightarrow -j20e^{j\omega t}$$

Then (1) becomes

$$-j20 = -\omega^2 i + 4j\omega + 4i$$

Since $\omega = 4$

$$i = \frac{-j20}{-\omega^2 + 4j\omega + 4} = \frac{-j20}{-16 + j16 + 4} = \frac{-j20}{-12 + j16} = 1\angle 141.3°$$

$$i_{steady-state} = \cos(4t + 141.3°)$$

To find the transient solution set the forcing function to zero.

$$\frac{d^2 i}{dt^2} + 4\frac{di}{dt} + 4i = 0$$

$$s^2 + 4s + 4 = 0$$

$$s = \frac{-4 \pm \sqrt{16 - 4(4)}}{2} = -2$$

Since this is a second order equation we MUST have two linearly independent solutions. we get the second solution by multiplying by t

$$Ae^{-2t} + Bte^{-2t}$$

$$i(t) = \cos(4t + 143.1°) + Ae^{-2t} + Bte^{-2t}$$
$$i(0) = 0 = \cos(143.1°) + A$$
$$A = -\cos(143.1°) = 0.8$$

$$\frac{di}{dt} = -2Ae^{-2t} + Be^{-2t} - 2Bte^{-2t} - 4\sin(4t + 143.1°)$$

$$\left.\frac{di}{dt}\right|_{t=0} = 4 = -2(0.8) + B - 4\sin(143.1°)$$

$$B = 4\sin(143.1°) + 1.6 + 4 = 8$$

The final solution is then

$$i(t) = \cos(4t + 143.1°) + 0.8e^{-2t} + 8te^{-2t}$$

An Abbreviated List of Laplace Transform Pairs

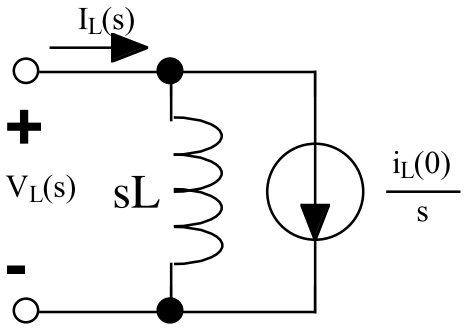
f(t) ($t>0^-$)	TYPE	F(s)
$\delta(t)$	(impulse)	1
u(t)	(step)	$\frac{1}{s}$
t	(ramp)	$\frac{1}{s^2}$
e^{-at}	(exponential)	$\frac{1}{s+a}$
$\sin(\omega t)$	(sine)	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	(cosine)	$\frac{s}{s^2+\omega^2}$
te^{-at}	(damped ramp)	$\frac{1}{(s+a)^2}$
$e^{-at}\sin(\omega t)$	(damped sine)	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	(damped cosine)	$\frac{s+a}{(s+a)^2+\omega^2}$

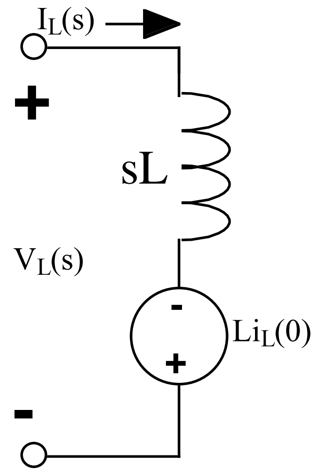
An Abbreviated List of Operational Transforms

f(t)	F(s)
$Kf(t)$	$KF(s)$
$f_1(t)+f_2(t)-f_3(t)+\ldots$	$F_1(s)+F_2(s)-F_3(s)+\ldots$
$\frac{df(t)}{dt}$	$sF(s)-f(0^-)$
$\frac{d^2f(t)}{dt^2}$	$s^2F(s)-sf(0^-)-\frac{df(0^-)}{dt}$
$\frac{d^nf(t)}{dt^n}$	$s^nF(s)-s^{n-1}f(0^-)-s^{n-2}\frac{df(0^-)}{dt}-s^{n-3}\frac{d^2f(0^-)}{dt^2}-\ldots-\frac{d^{n-1}f(0^-)}{dt^{n-1}}$
$\int_0^t f(x)dx$	$\frac{F(s)}{s}$
$f(t-a)u(t-a),\ a>0$	$e^{-as}F(s)$
$e^{-at}f(t)$	$F(s+a)$
$f(at),\ a>0$	$\frac{1}{a}f\left(\frac{s}{a}\right)$
$tf(t)$	$-\frac{dF(s)}{ds}$
$t^nf(t)$	$(-1)^n\frac{d^nF(s)}{ds^n}$
$\frac{f(t)}{t}$	$\int_s^\infty F(u)du$

Summary of initial conditions for Laplace transforms:

Inductor:

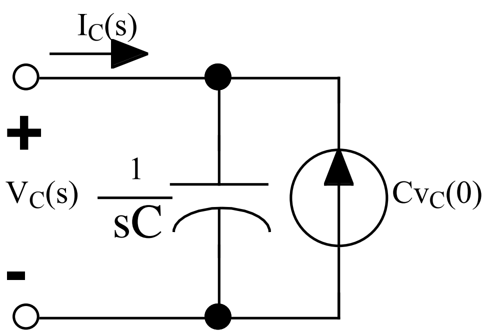


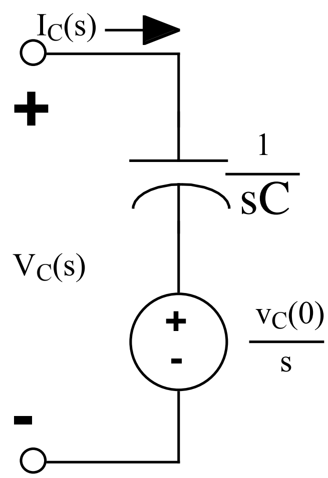


$$I_L(s) = \frac{1}{Ls}V_L(s) + \frac{i_L(0)}{s}$$

$$V_L(s) = LsI_L(s) - Li_L(0)$$

Capacitor:





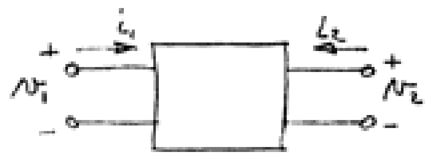
$$I_C(s) = CsV_C(s) - Cv_C(0)$$

$$V_C(s) = \frac{1}{Cs}I_C(s) + \frac{v_C(0)}{s}$$

Two-port Networks

(1)

TWO-PORT NETWORKS



IMPEDANCE PARAMETERS:

$$v_1 = z_{11} i_1 + z_{12} i_2$$
$$v_2 = z_{21} i_1 + z_{22} i_2$$

WHERE
$$z_{11} = \left.\frac{v_1}{i_1}\right|_{i_2=0}$$
$$z_{12} = \left.\frac{v_1}{i_2}\right|_{i_1=0}$$
$$z_{21} = \left.\frac{v_2}{i_1}\right|_{i_2=0}$$
$$z_{22} = \left.\frac{v_2}{i_2}\right|_{i_1=0}$$

ALSO CALLED
OPEN CIRCUIT IMPEDANCES

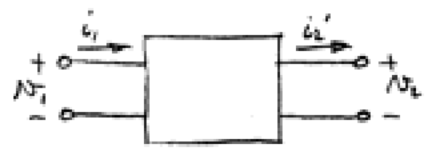
ADMITTANCE PARAMETERS:

$$i_1 = y_{11} v_1 + y_{12} v_2$$
$$i_2 = y_{21} v_1 + y_{22} v_2$$

WHERE
$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0}$$
$$y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0}$$
$$y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0}$$
$$y_{22} = \left.\frac{i_2}{v_2}\right|_{v_1=0}$$

ALSO CALLED
SHORT CIRCUIT ADMITTANCES

ABCD PARAMETERS:



$$v_1 = A v_2 + B i_2'$$
$$i_1 = C v_2 + D i_2'$$

WHERE
$$A = \left.\frac{v_1}{v_2}\right|_{i_2'=0}$$
$$B = \left.\frac{v_1}{i_2'}\right|_{v_2=0}$$
$$C = \left.\frac{i_1}{v_2}\right|_{i_2'=0}$$
$$D = \left.\frac{i_1}{i_2'}\right|_{v_2=0}$$

SOMETIMES NAMED AS FOLLOWS:

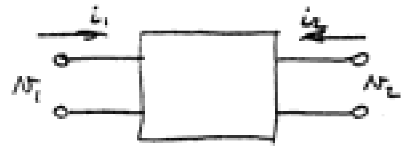
A^{-1} = OPEN-CIRCUIT VOLTAGE GAIN

B^{-1} = SHORT-CIRCUIT TRANSFER ADMITTANCE

C^{-1} = OPEN-CIRCUIT TRANSFER IMPEDANCE

D^{-1} = SHORT-CIRCUIT CURRENT GAIN

TWO-PORT NETWORKS (CONT') ②



HYBRID PARAMETERS:

$$i_1 = g_{11} v_1 + g_{12} i_2$$
$$v_2 = g_{21} v_1 + g_{22} i_2$$

WHERE

$$g_{11} = \frac{i_1}{v_1}\bigg|_{i_2=0} = \frac{1}{z_{11}}$$

$$g_{12} = \frac{i_1}{i_2}\bigg|_{e_1=0}$$

$$g_{21} = \frac{v_2}{v_1}\bigg|_{i_2=0}$$

$$g_{22} = \frac{v_2}{i_2}\bigg|_{v_1=0} = \frac{1}{y_{22}}$$

$$v_1 = h_{11} i_1 + h_{12} v_2$$
$$i_2 = h_{21} i_1 + h_{22} v_2$$

WHERE

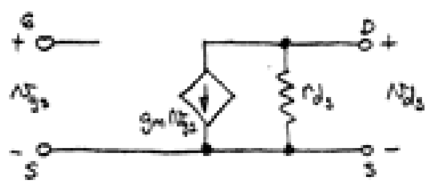
$$h_{11} = \frac{v_1}{i_1}\bigg|_{v_2=0} = \frac{1}{y_{11}}$$

$$h_{12} = \frac{v_1}{v_2}\bigg|_{i_1=0}$$

$$h_{21} = \frac{i_2}{i_1}\bigg|_{v_2=0}$$

$$h_{22} = \frac{i_2}{v_2}\bigg|_{i_1=0} = \frac{1}{z_{22}}$$

FOR FET'S (AND VACUUM TUBES):



$v_1 = v_{gs}$ $\quad v_2 = v_{ds}$

$i_1 = i_g = 0$

$g_{11} = 0$

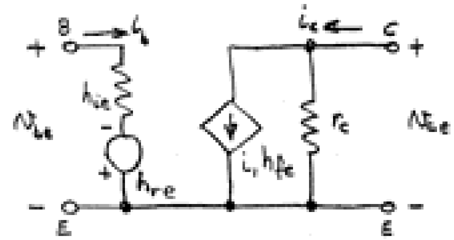
$g_{12} = 0$

$g_{21} = g_m$

$g_{22} = r_{ds}$

NOTE THAT OUTPUT CAN ALSO BE MODELED WITH ITS THEVININ EQUIV.

FOR BI-POLAR TRANSISTORS



$v_1 = v_{be}$ $\quad v_2 = v_{ce}$

$i_1 = i_b$ $\quad i_2 = i_c$

$h_{11} = h_{ie}$ (A RESISTANCE)

$h_{12} = h_{re}$ (FEEDBACK TERM – SMALL)

$h_{21} = h_{fe} = \beta$ (CURRENT GAIN)

$h_{22} = h_{oe} = \frac{1}{r_c}$