Chapter 2 - Linear Circuit Analysis

Chapter 2

Part 1 - Warmups

1. Using the same diameter nichrome wire as in example 2.1, determine the length needed to obtain a resistance of 1 ohm at 100° C. Determine the resistance at 20° C.

The characteristics of #30 AWG wire are:

diameter (mils)	diameter (circular mils)	in^2	Ω/1000ft @20° C	pounds/1000 ft.
10.03	100.5	7.894×10^{-5}	103.2	0.3042

for nichrome wire @20° C

$\rho_{20}=1.08\times10^{-6}\Omega\text{-m}$

$\alpha_{20}=17\times10^{-3}/°\ C$

$\rho_{100}=\rho_{20}[1+\alpha_{20}(T\text{-}20)] = 1.08\times10^{-6}[1+17\times10^{-3}\ (100\text{-}20)]$

$\rho_{100}=1.08\times10^{-6}[1+1.36] = 2.55\times10^{-6}\Omega\text{-m}$

$$A=\pi\left(\frac{d}{2}\right)^2=\frac{\pi d^2}{4}=\frac{\pi\left(10.03\times10^{-3}in\times25.4\frac{mm}{in}\right)^2}{4}=\frac{\pi\left(0.255\times10^{-3}m\right)^2}{4}=5.10\times10^{-8}m^2$$

$$R=\rho\frac{\ell}{A}$$

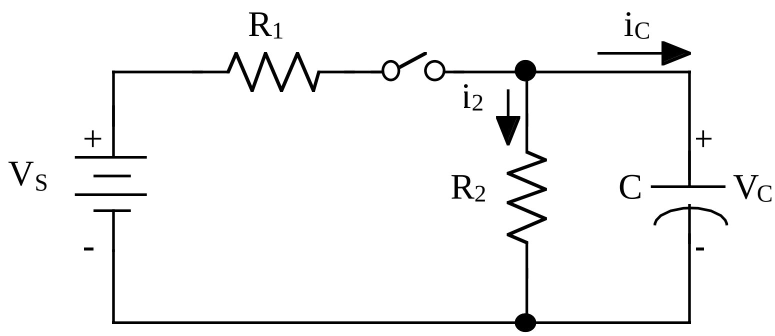
$$1\Omega=\left(2.55\times10^{-6}\Omega-m\right)\frac{\ell}{5.10\times10^{-8}m^2}$$

$$\ell=(1\Omega)\frac{5.10\times10^{-8}m^2}{2.55\times10^{-6}\Omega-m}=0.02m$$

$$R_{20}(\ell=0.02m)=\frac{\left(1.08\times10^{-6}\Omega-m\right)(0.02m)}{5.10\times10^{-8}m^2}=0.42\Omega$$

Note: Circular mils is nothing more than the diameter (expressed in thousandths of an inch) squared.

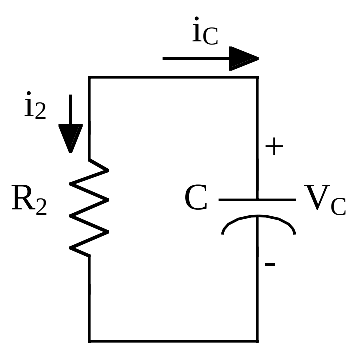
2. For the circuit shown in example 2.2, assume the switch has been closed for a long time and is opened at t=0. Determine (a) the current through the capacitor at the instant that the switch is opened, (b) the time derivative of the capacitor voltage at the instant the switch is opened, and (c) the voltage that will exist across the capacitor after a long time.



The boundary conditions for the capacitor C are
no DC current flow
$v(0^+) = v(0^-)$

By inspection, $v_C(0^-) = \frac{R_2}{R_1 + R_2} V_S$ since there is no DC current through C.

(a) When the switch is open



$$i_2(0^+) = \frac{v_C(0^+)}{R_2} = \frac{v_C(0^-)}{R_2} = \frac{\frac{R_2}{R_1 + R_2} V_S}{R_2} = \frac{V_S}{R_1 + R_2}$$

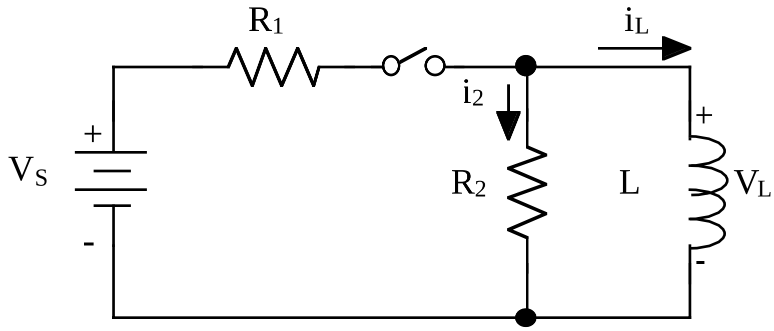
Then, $i_C(0^+) = -i_2(0^+) = \frac{-V_S}{R_1 + R_2}$

(b) Using the derivative relationship $i = C\frac{dv}{dt}$ for C

$$\frac{dv}{dt}(t = 0^+) = \frac{i_C(0^+)}{C} = \frac{-V_S}{(R_1 + R_2)C}$$

(c) $v_C(t = \infty) = 0$

3. For the circuit shown in example 2.3, assume that the switch has been closed for a long time (the final conditions of example 2.3 apply), and the switch is opened at t=0. Determine at the instant the switch opens (a) the current in the inductance, (b) the time rate of change of current in the inductance, and (c) the voltage across the inductance.

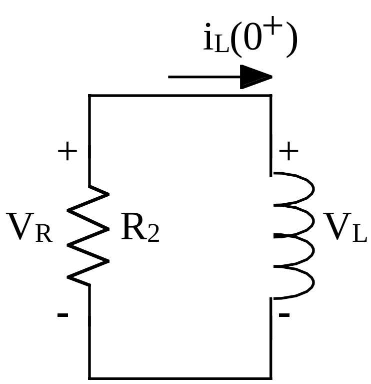


The boundary conditions for an inductor are $i_L\left(0^+\right) = i_L\left(0^-\right)$

(a) $i_L\left(0^-\right) = \dfrac{V_S}{R_1}$ since the inductor is a DC short

Therefore, $i_L\left(0^+\right) = \dfrac{V_S}{R_1}$

(b) For an inductor, $v = L\dfrac{di}{dt}$



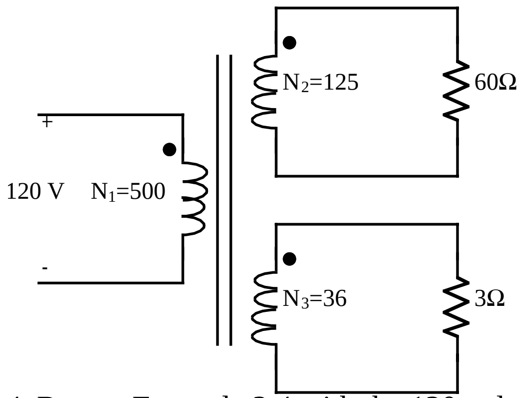
$$V_R = -i_L\left(0^+\right)R_2 = -\frac{V_S R_2}{R_1}$$

$$\frac{di}{dt} = \frac{V_L}{L} = \frac{V_R}{L} = -\frac{V_S R_2}{L R_1}$$

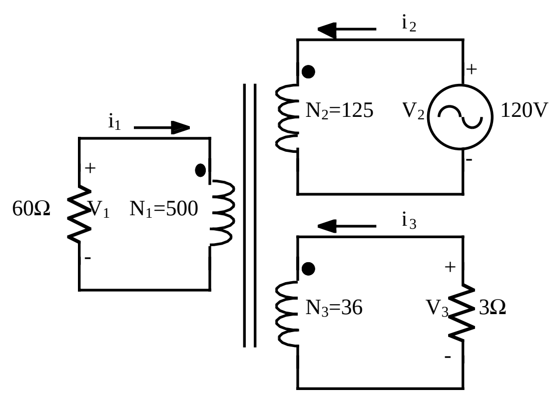
(c)

$$V_L = -\frac{V_S R_2}{R_1}$$

Example 2.4
A transformer consists of a primary winding with 500 turns and two secondary windings of 125 turns and 30 turns. The 125 turn winding has 60 ohms connected to its terminals, and the 30 turn secondary winding has 3 ohms connected to its terminals. If the primary winding is connected to a 120 volt 60 Hz source, determine the current rating for each winding.



4. Repeat Example 2.4 with the 120 volt source connected to winding 2 and the 60 ohm load attached to winding 1.



$$V_1 = \frac{N_1}{N_2}V_2 = \frac{500}{125}(120) = 480 volts$$

$$V_3 = \frac{N_3}{N_2}V_2 = \frac{36}{125}(120) = 34.56 volts$$

$$i_1 = \frac{480V}{60\Omega} = 8 amps$$

$$i_3 = \frac{34.56V}{3\Omega} = 11.52 amps$$

To find i_2 we use conservation of power

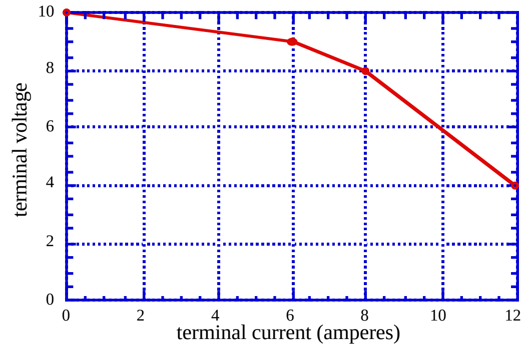
$$v_2 i_2 = v_1 i_1 + v_3 i_3$$

$$120 i_2 = (480)(8) + (34.56)(11.52)$$

$$120 i_2 = 3840 + 398.13$$

$$i_2 = 35.32 amps$$

Example 2.5
Obtain an expression for the characteristic curve shown over the range from 8 to 12 amps. Obtain linear circuit models from the derived expressions.



5. Obtain a linear model of voltage versus current for the characteristic shown in example 2.5 over the current range of 0 to 6 amps. Obtain the current source and voltage source equivalent circuits.

Use the two-point method to fit a straight line to the data

$$\frac{V - V_0}{I - I_0} = \frac{V_1 - V_0}{I_1 - I_0}$$

Read two points from the graph
point 0: 10 volts, 0 amps
point 1: 9 volts, 6 amps

Evaluating the expression,

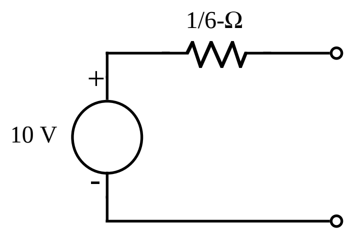
$$\frac{V - 10}{I - 0} = \frac{9 - 10}{6 - 0}$$

$$\frac{V - 10}{I} = -\frac{1}{6}$$

$$6V - 60 = -I$$

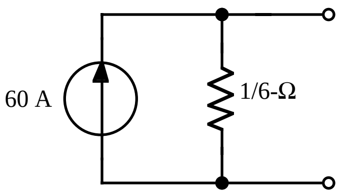
$$6V + I = 60$$

$$V = \frac{-I + 60}{6} = -\frac{1}{6}I + 10$$



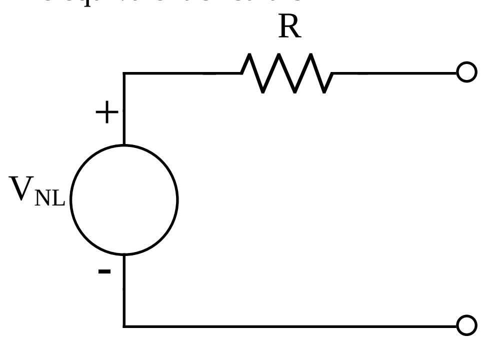
NOTE: Thevenin is usually easier to visualize

$$I = -6V + 60$$



6. A 5 volt, 25 amp d.c. source has voltage regulation of 1%. Obtain an equivalent circuit for this source.

The equivalent circuit is



$$VR = \frac{V_{NoLoad} - V_{FullLoad}}{V_{FullLoad}} \times 100\%$$

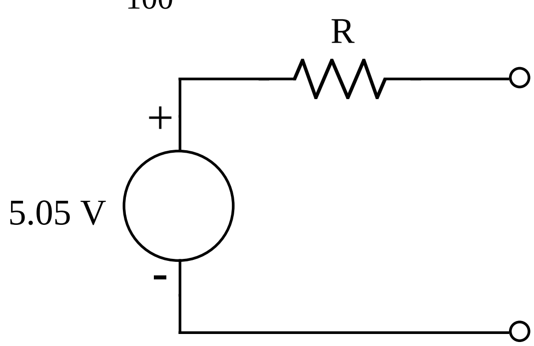
$$1\% = \frac{V_{NoLoad} - 5V}{5V} \times 100\%$$

Solving for V_{NoLoad}

$$5\% = V_{NoLoad} \times 100\% - 500\%$$

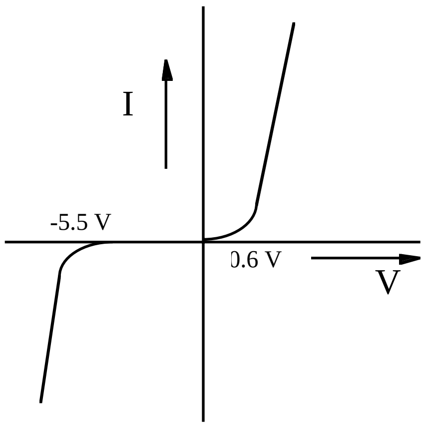
$$5 + 500 = V_{NoLoad}(100)$$

$$V_{NoLoad} = \frac{505}{100} = 5.05$$

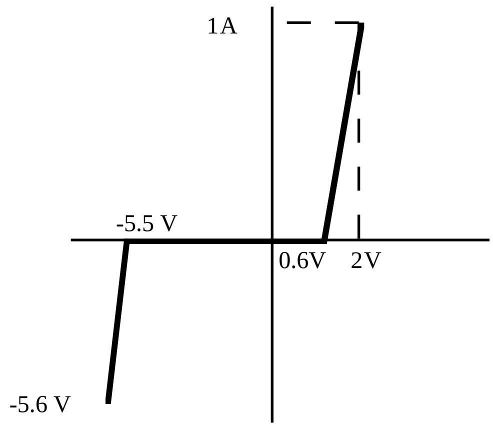


$$R = \frac{5.05 - 5}{25A} = \frac{0.05}{25} = 0.002\Omega$$

7. All silicon diodes have a reverse breakdown voltage essentially the same as the avalanche voltage of a zener diode. This results in three distinct regions of operation: one for the normal forward region, one for the avalanche region, and an intermediate region where the device behaves essentially as an open circuit. Obtain the linear models in these three regions for a 1 amp, 2 watt silicon diode with a reverse breakdown voltage of 5.5 volts and a maximum reverse voltage of 5.6 volts.



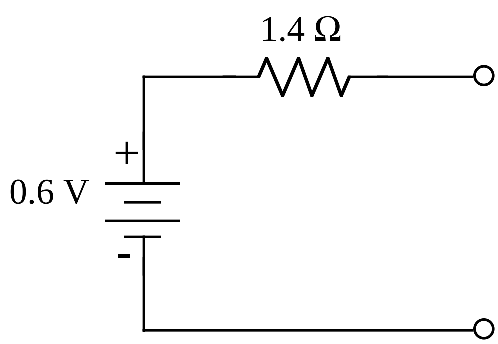
Theoretical



Linear model

$$Power_{rated} = v_{rated} \times i_{rated}$$

$$v_{rated} = \frac{Power_{rated}}{i_{rated}} = \frac{2watts}{1Amp} = 2Volts$$



In region I: $\frac{V-2}{I-1} = \frac{0.6-2}{0-1} = 1.4$

$V - 2 = 1.4I - 1.4$

$V = 1.4I + 0.6$

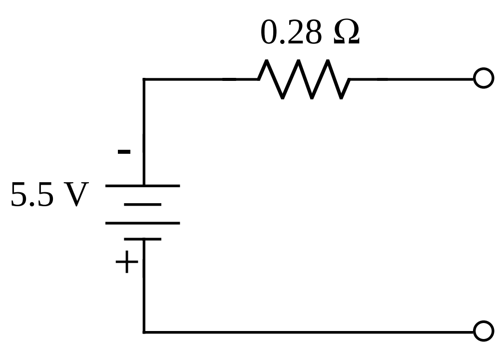
In region II: nothing, diode is an open circuit

In region III: find the rated current using breakdown voltage

$V_{\max} I_{\max} = 2watts$

$(5.6Volts) I_{\max} = 2watts$

$I_{\max} = 0.36Amps$



$$\frac{V-(-5.5)}{I-0} = \frac{-5.6-(-5.5)}{-0.36-0}$$

$V + 5.5 = 0.28I$

$V = 0.28I - 5.5$

8. A three-terminal device is described by the following z-parameter equations.

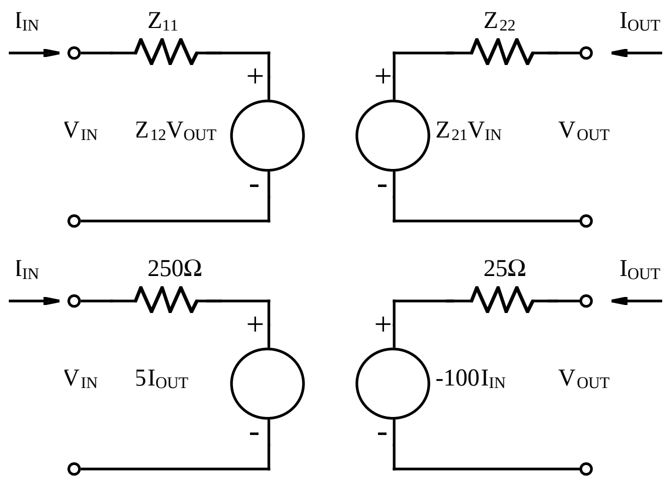
$V_{IN} = 250 i_{IN} + 5 i_{OUT}$

$V_{OUT} = -100 i_{IN} + 25 i_{OUT}$

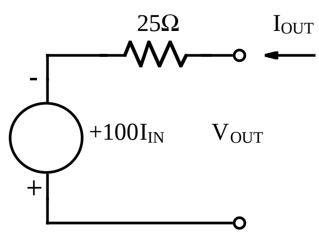
Obtain an equivalent circuit for this device.

Re-writing in matrix form. This is the z-parameter model.

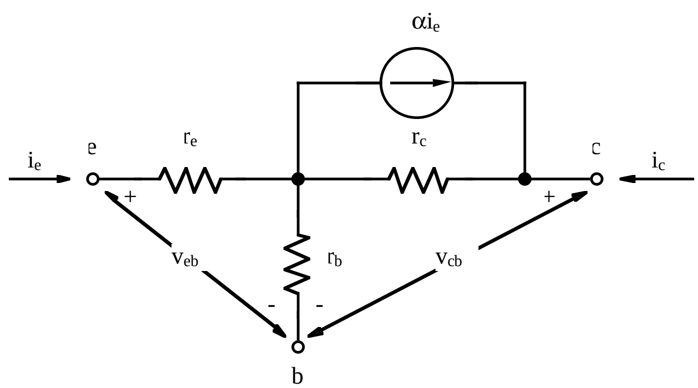
$$\begin{bmatrix} V_{IN} \\ V_{OUT} \end{bmatrix} = \begin{bmatrix} 250 & 5 \\ -100 & 25 \end{bmatrix} \begin{bmatrix} i_{IN} \\ i_{OUT} \end{bmatrix}$$



or draw reversed as



9. Common base transistor configurations are often described in terms of common base y-parameters: y_{ib}, y_{rb}, y_{fb} and y_{ob}. (The b's in the subscripts indicate that the parameters were obtained from a common base configuration with the emitter as the input terminal and the collector as the output terminal.) A common circuit model for the transistor used in this configuration is shown. Obtain the y-parameters for this common base circuit.



This is algebraically a very complex problem. Finding two-port parameters is usually algebraically complex.

The y-parameters are defined by

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

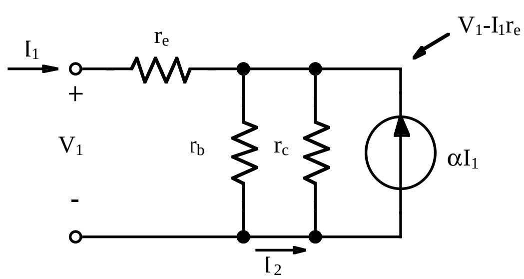
to find y_{11} and y_{21} short V_2:

$$I_1 = y_{11} V$$
$$I_2 = y_{21} V_1$$

to find y_{12} and y_{22} short V_1:

$$I_1 = y_{12} V_2$$
$$I_2 = y_{22} V_2$$

Let's start by shorting V_2 to get:



Using KCL at the common node:

$$I_1 = \frac{V_1 - I_1 r_e}{r_b} + \frac{V_1 - I_1 r_e}{r_c} + \alpha I_1$$

$$I_1(1-\alpha) = V_1\left(\frac{1}{r_b} + \frac{1}{r_c}\right) - I_1 r_e\left(\frac{1}{r_b} + \frac{1}{r_c}\right)$$

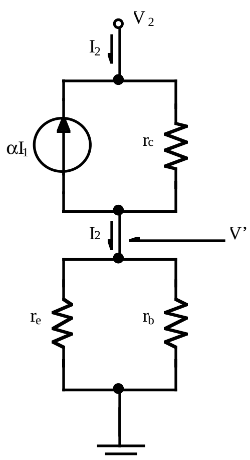
$$I_1(1-\alpha) = V_1\left(\frac{r_b + r_c}{r_b r_c}\right) - I_1 r_e\left(\frac{r_b + r_c}{r_b r_c}\right)$$

$$I_1(1-\alpha)(r_b r_c) = V_1(r_b + r_c) - I_1 r_e(r_b + r_c)$$
$$I_1(1-\alpha)(r_b r_c) + I_1 r_e(r_b + r_c) = V_1(r_b + r_c)$$
$$y_{11} = \frac{I_1}{V_1} = \frac{V_1(r_b + r_c)}{V_1\left[(1-\alpha)(r_b r_c) + r_e(r_b + r_c)\right]}$$
$$\boxed{y_{11} = \frac{(r_b + r_c)}{(1-\alpha)(r_b r_c) + r_e(r_b + r_c)}}$$

$$y_{21} = \frac{I_2}{V_1} = -\frac{V_1 - I_1 r_e + \alpha I_1}{V_1} = -1 + \frac{I_1}{V_1}(r_e - \alpha)$$

$$y_{21} = -1 + y_{11}(r_e - \alpha) = -1 + \frac{(r_b + r_c)}{(1-\alpha)(r_b r_c) + r_e(r_b + r_c)}(r_e - \alpha)$$
$$y_{21} = \frac{(\alpha - 1)(r_b r_c) - r_e(r_b + r_c) + (r_b + r_c)(r_e - \alpha)}{(1-\alpha)(r_b r_c) + r_e(r_b + r_c)}$$
$$y_{21} = \frac{\alpha r_b r_c - r_b r_c - r_e r_b - r_e r_c + r_b r_e + r_c r_e - \alpha r_c - \alpha r_b}{(1-\alpha)(r_b r_c) + r_e(r_b + r_c)}$$
$$y_{21} = \frac{\alpha r_b r_c - r_b r_c - \alpha r_c - \alpha r_b}{(1-\alpha)(r_b r_c) + r_e(r_b + r_c)}$$
$$\boxed{y_{21} = -\frac{(1-\alpha) r_b r_c + \alpha(r_b + r_c)}{(1-\alpha)(r_b r_c) + r_e(r_b + r_c)}}$$

Now short V_1:



Using definitions

$$I_1 = y_{12} V_2$$
$$y_{12} = \frac{I_1}{V_2}$$
$$y_{22} = \frac{I_2}{V_2} \qquad (3)$$

$$I_2 = \frac{V'}{r_e \parallel r_b} = \frac{V_2 - V'}{r_c} - \alpha I_1 \qquad (1)$$

where the first expression is the lower resistances, and the second term is the upper loop

$$I_1 = -\frac{V'}{r_e} \qquad (2)$$

Substituting (2) into (1) gives:

$$I_2 = \frac{V_2 - V'}{r_c} + \alpha\frac{V'}{r_e} = \frac{V_2}{r_c} - V'\left(\frac{1}{r_c} - \frac{\alpha}{r_e}\right)$$

Using (3)

$$I_2 = \frac{V_2}{r_c} - I_2\left(r_e \parallel r_b\right)\left(\frac{1}{r_c} - \frac{\alpha}{r_e}\right)$$

$$I_2\left(1 + \frac{r_e r_b}{r_e + r_b}\frac{r_e - \alpha r_c}{r_c r_e}\right) = \frac{V_2}{r_c}$$

$$I_2\left(\frac{\left(r_e + r_b\right)r_c + r_e r_b - \alpha r_b r_c}{\left(r_e + r_b\right)r_c}\right) = \frac{V_2}{r_c}$$

$$y_{22} = \frac{I_2}{V_2} = \frac{r_e + r_b}{\left(r_e + r_b\right)r_c + r_e r_b - \alpha r_b r_c} = \frac{r_e + r_b}{(1-\alpha)r_b r_c + r_e\left(r_b + r_c\right)}$$

Using (2),

$$y_{12} = \frac{I_1}{V_2} = \frac{-\frac{V'}{r_e}}{V_2}$$

Using (1)

$$I_2 = \frac{V_2}{r_c} - V'\left(\frac{1}{r_c} - \frac{\alpha}{r_e}\right)$$

$$I_2 - \frac{V_2}{r_c} = V'\left(\frac{r_e - \alpha r_c}{r_c r_e}\right)$$

Solving for $-\frac{V'}{r_e}$:

$$-\frac{V'}{r_e} = \left(I_2 - \frac{V_2}{r_c}\right)\left(\frac{r_c}{r_e - \alpha r_c}\right)$$

and substituting into our expression for y_{12}

$$y_{12} = \frac{\left(I_2 - \frac{V_2}{r_c}\right)\left(\frac{r_c}{r_e - \alpha r_c}\right)}{V_2} = \frac{\frac{I_2 r_c}{r_e - \alpha r_c} - \frac{V_2}{r_e - \alpha r_c}}{V_2}$$

$$y_{12} = \frac{I_2}{V_2}\frac{r_c}{r_e - \alpha r_c} - \frac{1}{r_e - \alpha r_c} = \frac{1}{\left(r_e - \alpha r_c\right)}\left[y_{22} r_c - 1\right]$$

Substituting for y_{22}

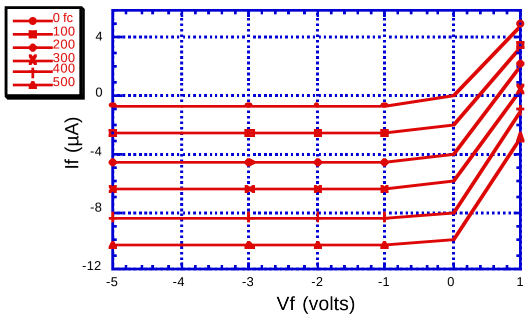
$$y_{12} = \frac{1}{\left(r_e - \alpha r_c\right)}\left[\frac{\left(r_e + r_b\right)r_c}{(1-\alpha)r_b r_c + r_e\left(r_b + r_c\right)} - 1\right]$$

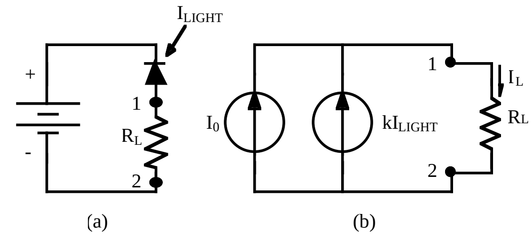
$$y_{12} = \frac{\left(r_e + r_b\right)r_c - (1-\alpha)r_b r_c - r_e\left(r_b + r_c\right)}{\left(r_e - \alpha r_c\right)\left[(1-\alpha)r_b r_c + r_e\left(r_b + r_c\right)\right]}$$

$$y_{12} = \frac{r_e r_c + r_b r_c - r_b r_c + \alpha r_b r_c - r_e r_b - r_e r_c}{\left(r_e - \alpha r_c\right)\left[(1-\alpha)r_b r_c + r_e\left(r_b + r_c\right)\right]}$$

$$y_{12} = \frac{r_b\left[\alpha r_c - r_e\right]}{\left(r_e - \alpha r_c\right)\left[(1-\alpha)r_b r_c + r_e\left(r_b + r_c\right)\right]} = \frac{-r_b}{(1-\alpha)r_b r_c + r_e\left(r_b + r_c\right)}$$

10. For a photodiode with characteristics shown in figure 2.10 and used in the circuit for example 2.11 with a 5 volt source and a 10,000 ohm load, determine the load voltage as a function of illumination level. Over what range of illumination levels will this result be valid? Assume the curves extend to -5 volts on the third quadrant of the characteristic curves.





(a) Biasing circuit for photodiode of figure 2.10
(b) equivalent circuit

Fig. 2.10 Typical photodiode characteristics

From example, $\frac{i-0.7}{I_L-0} = \frac{10.3-0.7}{500-0} = 0.192$ where the so-called "dark" current is estimated to be about 0.7µA for 0 fc (footcandles). The other point comes from the estimated line for the 500 fc illumination level.

$i = 0.7 + 0.0192 I_L$ (i is in microamperes) (1)
Considering the load electrical circuit using KVL
$5 = V_f + i(10k\Omega)$ (2)
Substituting (1) into (2) gives
$5 = V_f + \left(0.7 + 0.0192 I_L\right) \times 10^{-6} (10k\Omega)$
$5 - V_f = 0.007 + 0.000192 I_L$
But $5 - V_f = V_{LOAD}$ is simply the voltage across the load resistor.

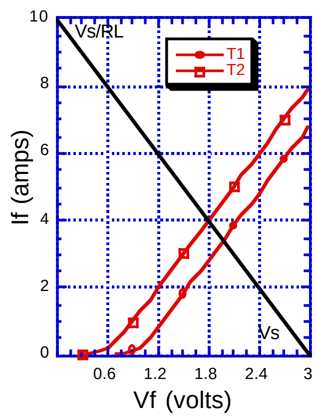
The equation $V_{LOAD} = 0.007 + 0.000192 I_L$ is valid when the illumination I_L>0 and breaks down when the diode becomes forward biased, i.e., the voltage across the load resistor becomes greater than the voltage across the photodiode. At this point the voltage across the diode drops to zero and the entire 5 volts appears across the load resistor.

At this breakdown point,

$$V_{LOAD} = 0.007 + 0.000192 I_L = 5 Volts$$

and the corresponding light level is
$I_L = \frac{5-0.007}{0.000192} \approx 26{,}000$ footcandles.

Example 2.12
The characteristics shown in figure 2.11 have voltage divisions of 0.2 volts and current divisions of 1 amp. Obtain an equivalent circuit for a diode which accounts for the temperature dependence with T_1=20° C and T_2=80° C.



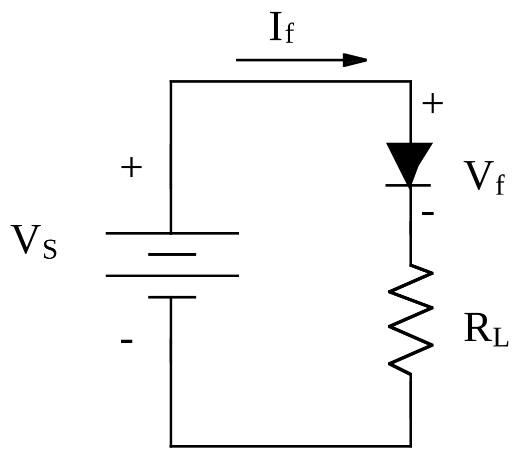


Figure 2.11 Temperature characteristics for a diode

11. Repeat example 2.12 for T_1=0° C and T_2=100°C.

Using T_1=0° C and T_2=100° C.

From the graph for T1 we can estimate the two points on graph to be (0.95V,0Amps) and (1.4Volts,1.5Amps). Using the two point method

$$\frac{V_f - 0.95}{i_f - 0} = \frac{1.4 - 0.95}{1.5 - 0}$$
$$V_f - 0.95 = 0.3 i_f$$
$$V_f = 0.3 i_f + 0.95$$

There are many ways to find the second curve for T_2=100° C. The slope remains the same but the y-intercept changes.

Note that for T_2=100° C at $i_f = 0$, $V_f = 0.6$. The T_2 curve is then

$$V_f = 0.3 i_f + 0.60$$

Assume that the y-intercept is a linear function of temperature and again use the two point method. Using the points (0.95,0) and (0.60,100) for the curve we get an equation for the y-intercept of

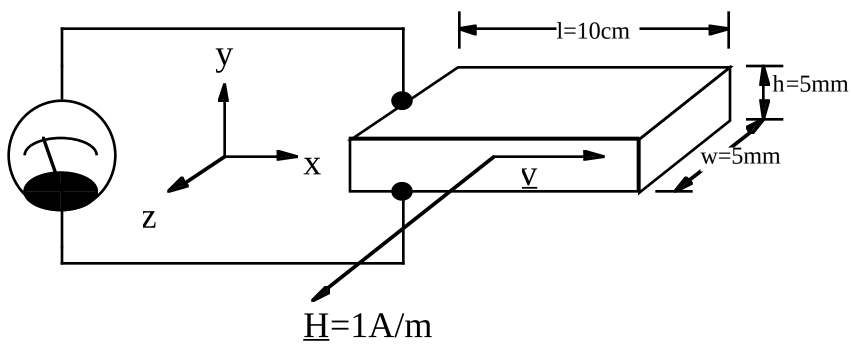
$$\frac{C_o - 0.95}{T - 0} = \frac{0.60 - 0.95}{100 - 0} = -0.0035$$
$$C_o = -0.0035T + 0.95$$

The overall equivalent circuit is

$$V_f(i_f, T) = 0.3 i_f + C_o = 0.3 i_f - 0.0035T + 0.95$$

Example 2.13
Copper has an electron density of 10^{28} electrons per cubic meter. The charge on each electron is 1.6×10^{-19} coulombs. For a copper conductor with width 5 mm, height 5 mm, and length 10 cm, determine the Hall voltage for a current of 1 amp when the magnetic field intensity is 1 amp/m, $B=\mu_o H$ and $\mu_o=4\pi\times10^{-7}$.

12. Use the dimensions given in example 2.13 but assume the conductor is an n-type semiconductor with 10^{19} electrons per cubic meter. The current is 0.1 amp. Obtain an equivalent circuit for a transducer using the Hall voltage to measure magnetic field intensity. Assume that the voltage pickups are capacitative so there is no connection of the output to the current circuit.



The fundamental equation for the Hall effect is $\vec{F}=q\left(\vec{E}+\vec{v}\times\vec{B}\right)=0$
Since there is only a y-component of the E field we can rewrite this as
$E_y+\left(\vec{v}\times\vec{B}\right)_y=0$ (1)
The current I_x in the x-direction is given by $I_x=nqv_xA$ where
n=number of carriers per square meter
q is the charge on the carriers
v_x is the x-component of the carrier velocity and
A=wh is the conductor cross sectional area.

Solving,
$$v_x=\frac{I_x}{nqA}=\frac{I_x}{nqwh} \qquad (2)$$

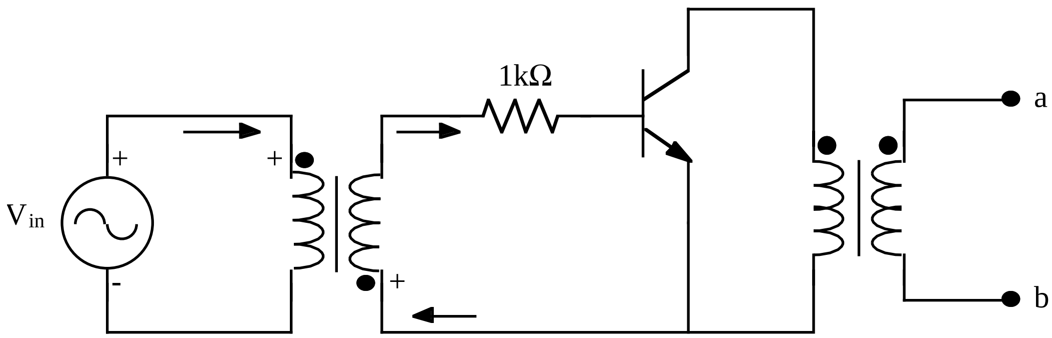
Rewriting (1) and substituting (2) into it

$$E_y-v_xB_z=0$$

$$\frac{V_h}{h}-\left(\frac{I_x}{nqwh}\right)\mu_o H_z=0$$
Solving for V_h

$$V_h=(0.005m)\left(\frac{0.1Amp}{\left(10^{19}\right)\left(1.6\times10^{-19}\right)(0.005m)(0.005m)}\right)\left(4\pi\times10^{-7}\right)H_z=1.57\times10^{-5}H_z$$

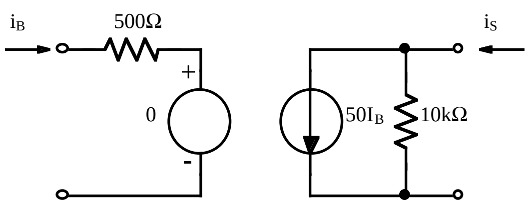
Example 2.14
A simplified small signal amplifier circuit is shown. The voltage source Vin is a variable signal source and can be considered to be an ideal voltage source. The transistor has parameters h_{ie}=500, h_{re}=0, h_{fe}=50, and h_{oe}=0.0001. Determine the Thevenin equivalent circuit (as seen from terminals a and b) in terms of the unknown turns N_1 and N_2 on the right-hand transformer.



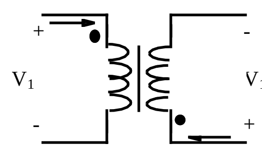
Ideal 1:1 Ideal N_1:N_2

13. For the circuit of example 2.14, obtain the Norton equivalent circuit.

The transistor model is

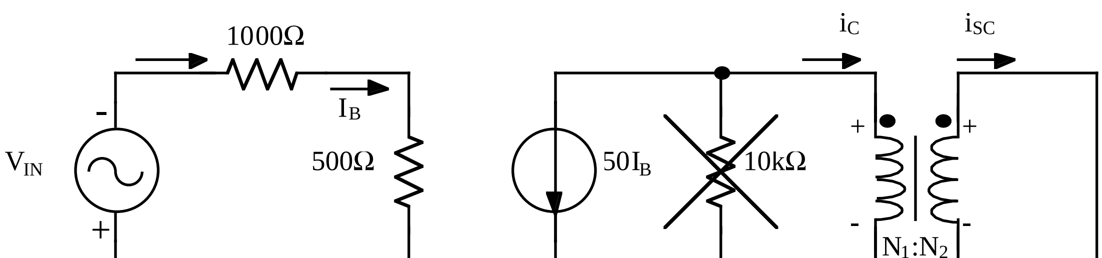


For the transformer, if the +v terminal is at the dot then the current is into the dot.



See p. 2-6 of your reference book

The resulting overall circuit is then



Note the results of the dot convention. The current flowing into the dot results in current flowing out of the top of the transformer secondary. The voltage source's polarity is reversed by the transformer as shown in the first figure. The output is a short circuit because we are determining the short circuit current.

This gives

$$i_B = +\frac{-v_{in}}{1000+500} = \frac{-v_{in}}{1500}$$

Continuing through the transistor,

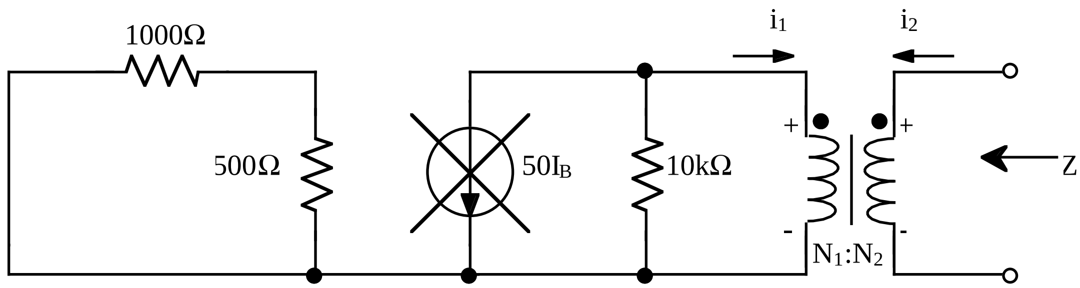
$$i_C = -50 i_B = +\frac{50}{1500} v_{in}$$

By the dot conventions for the output transformer, the output current is going into the dot. The current magnitude is transformed as $\frac{N_1}{N_2}$ and the voltage magnitude as $\frac{N_2}{N_1}$ (Power is conserved).

Since we are looking for a Norton equivalent circuit, the output is short circuited and the load voltage is zero. Consequently, we are only looking at the short-circuit current. In this context the transformer also appears as an ac short and we can ignore the 10kΩ output impedance of the transistor.

$$i_{SC} = -i_C \frac{N_1}{N_2} = -\frac{50}{1500} v_{in} \frac{N_1}{N_2} = -\frac{1}{30} \frac{N_1}{N_2} v_{in}$$

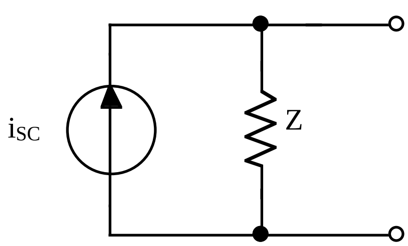
This gives us the short circuit current in the direction shown. To get the Norton equivalent circuit we need to also compute the Thevenin resistance. To get this resistance we short v_{in}, NOT the transistor current source $50 i_B$ and look at the resulting circuit.



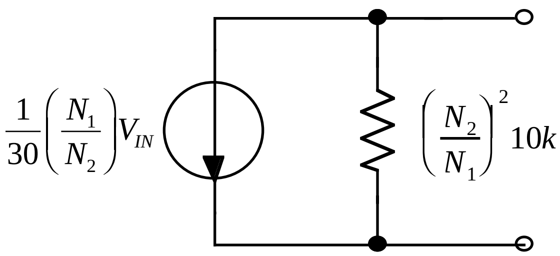
This leaves only the 10kΩ resistor which gets transformed by the transformer as

$$R_N = \left(\frac{N_2}{N_1}\right)^2 10k\Omega$$

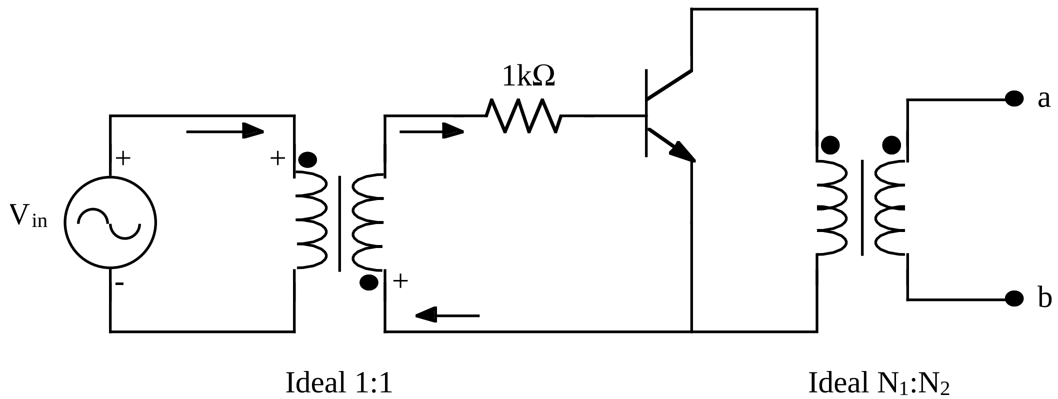
The final Norton equivalent circuit is then



or

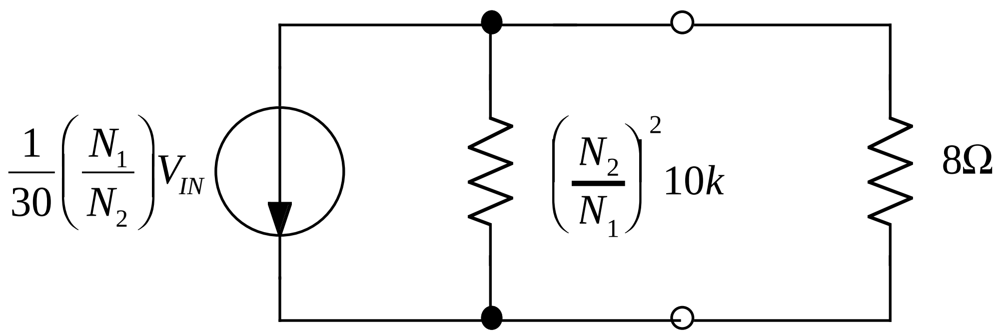


Example 2.14
A simplified small signal amplifier circuit is shown. The voltage source Vin is a variable signal source and can be considered to be an ideal voltage source. The transistor has parameters h_{ie}=500, h_{re}=0, h_{fe}=50, and h_{oe}=0.0001. Determine the Thevenin equivalent circuit (as seen from terminals a and b) in terms of the unknown turns N_1 and N_2 on the right-hand transformer.



14. Consider the circuit of example 2.14 with an 8 ohm speaker attached to terminals a-b. Show that the necessary turns ratio $\left(\frac{N_1}{N_2}\right)$ to provide maximum power transfer is 35.36.

Using the Norton equivalent circuit from the previous problem

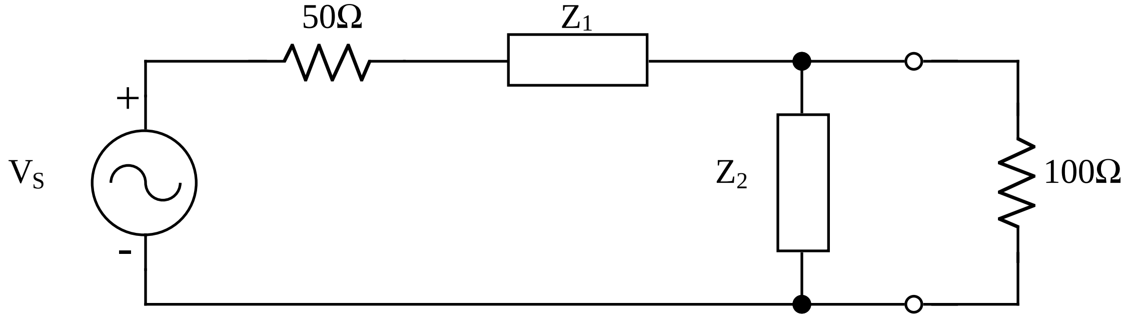


The maximum power transfer occurs when $R_N = \left(\frac{N_2}{N_1}\right)^2 10k\Omega = 8\Omega$

Solving for the turns ratio, $\left(\frac{N_2}{N_1}\right) = \sqrt{\frac{8\Omega}{10k\Omega}} = 0.0283$

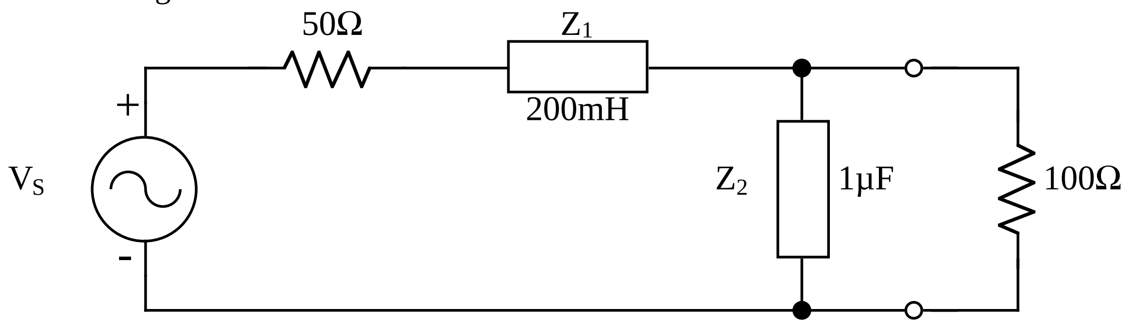
$$\left(\frac{N_1}{N_2}\right) = \frac{1}{0.0283} = 35.36$$

Example 2.16
The circuit shown has Z_1=1μf and Z_2=200mH. The voltage source is represented by the partial Fourier series
$v_S = 20\cos 1000t - 5\cos 4000t$
Determine the voltage across the 100 ohm resistor.

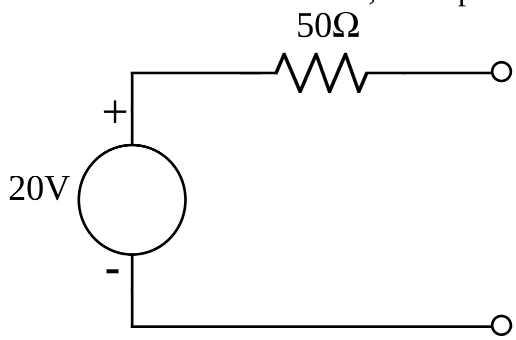


15. The elements Z_1 and Z_2 are interchanged in example 2.16, and the voltage v_S takes on the value of
$v_S = 20 + 5\cos 2236t$
Find the voltage across the 100 ohm resistor.

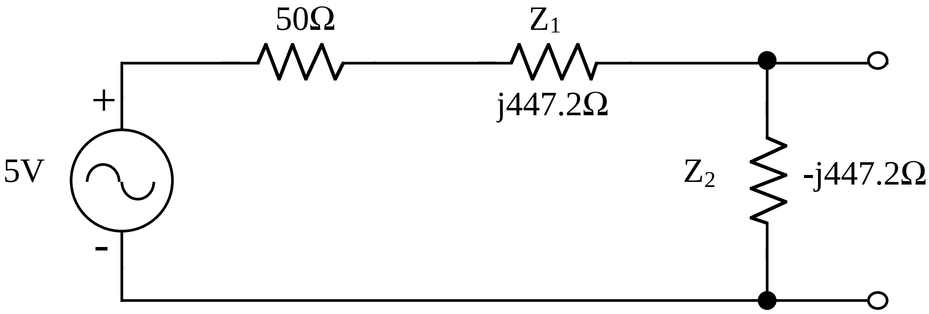
The resulting circuit is then



At DC the inductor is a short, the capacitor is an open.



$V_{T1} = 20Volts$
$R_{T1} = 50\Omega$
At ω=2236 we compute the equivalent impedances to get



$$Z_1 = j\omega L = j(2236)(0.2H) = j447.2\Omega$$
$$Z_2 = \frac{1}{j\omega C} = \frac{1}{j(2236)\left(1\times10^{-6}\right)} = -j447.23\Omega$$

The Thevenin voltage for this AC circuit is found using a voltage divider

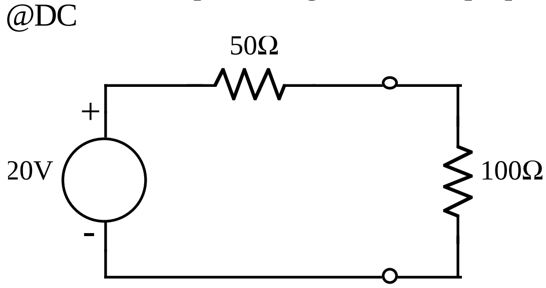
$$V_{T2} = \frac{-j447.2}{50 + j447.2 - j447.2}(5) = -j44.7$$

The Thevenin resistance is that of the two impedances in parallel

$$Z_{T2} = (50 + j447.2) \,||\, (-j447.2) = \frac{(50 + j447.2)(-j447.2)}{50 + j447.2 + (-j447.2)} = 3999. - j447.2$$

To find the output voltage we use superposition

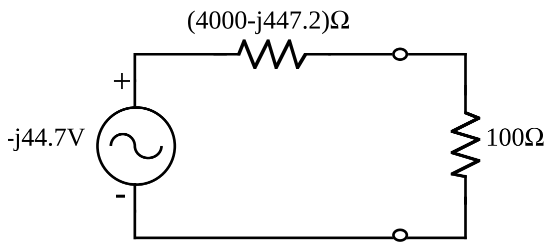
@DC



Using a voltage divider we get

$$V_{OUT1} = \frac{100}{100 + 50}(20) = 13.33Volts$$

@ω=2236 rad/sec



Using a voltage divider we get

$$V_{OUT2} = \frac{100}{100 + 4000 - j447.2}(-j44.7)$$
$$V_{OUT2} = 0.12 - j1.08 = 1.08\angle -83.78°$$

The measured voltage across the 100Ω resistor is then

$$V_{100\Omega} = V_{DC} + V_{AC} = V_{TH1} + V_{TH2} = 13.33 + 1.08\cos(2236t - 83.78°)$$

16. The circuit of example 2.17 has r_b=200 ohms, β=50, r_c=2500 ohms, and r_e=10 ohms. Find the voltage gain $\frac{V_{OUT}}{V_{IN}}$ when a 1000 ohm load is placed across the V_{OUT} terminals.
Note that the output current I_2 is
$$-\frac{V_{OUT}}{1000}$$
for this load.

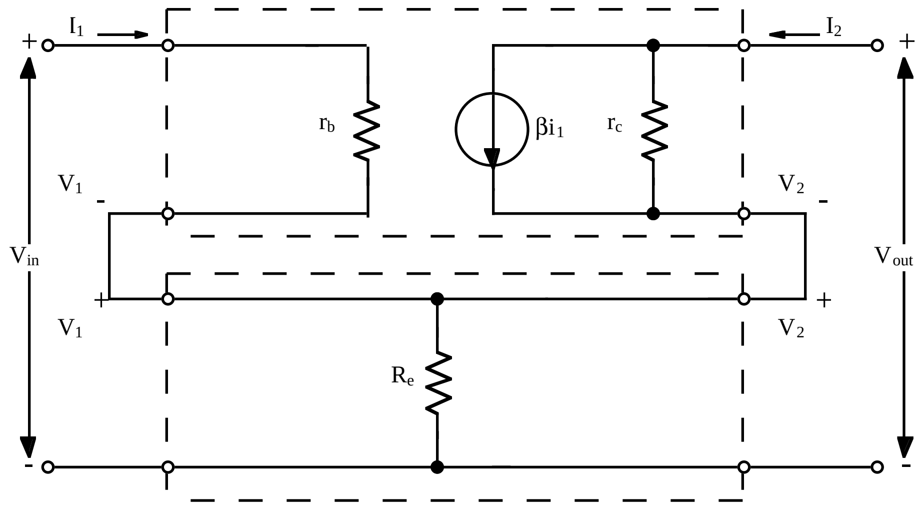
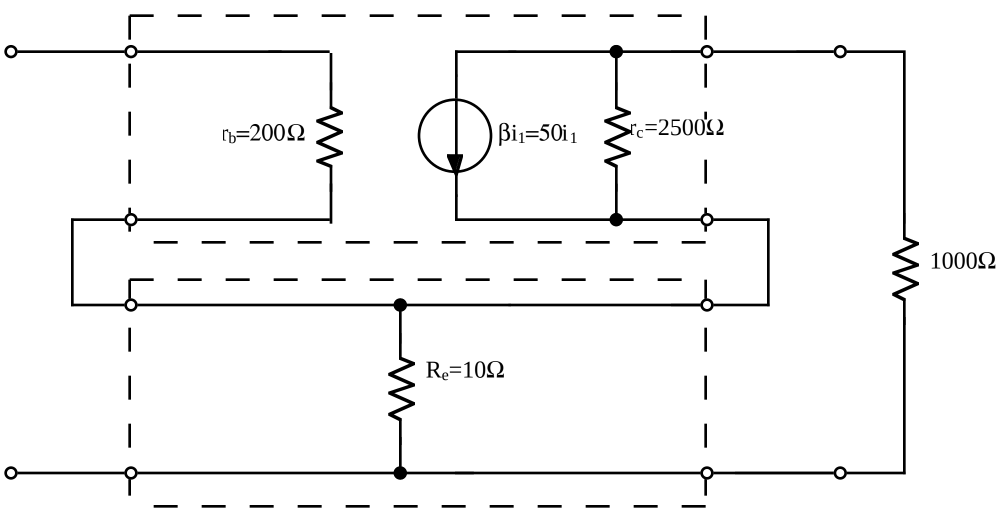


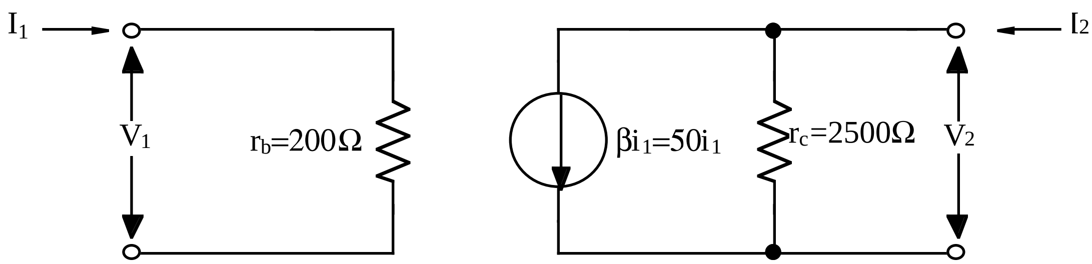
Figure 2.16. Combination of 2-port networks with common currents.

The circuit is then



We will convert both the upper and lower circuits to z-parameters and then combine them.
for z-parameters
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$
For an open output $I_2 = 0$ and
$$V_1 = z_{11} I_1$$

$V_2 = z_{21} I_1$



By inspection

$$z_{11} = \frac{V_1}{I_1} = 200\Omega$$

$$z_{21} = \frac{V_2}{I_1} = \frac{-(50I_1)(2500)}{I_1} = -1.25\times10^5$$

if $I_1 = 0$

$V_1 = z_{12} I_2$

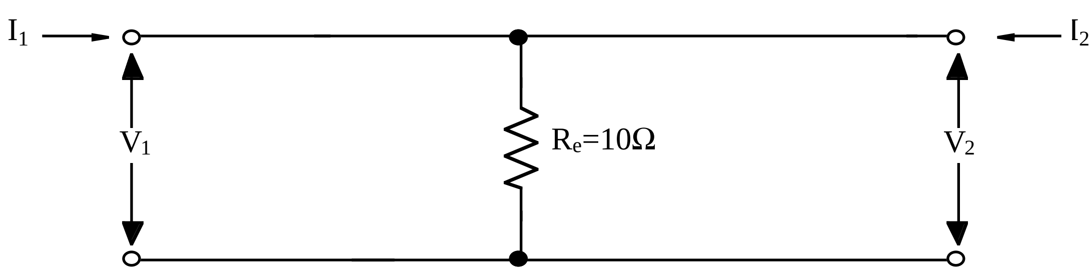
$V_2 = z_{22} I_2$

and

$$z_{12} = \frac{V_1}{I_2} = 0$$

$$z_{22} = \frac{V_2}{I_2} = 2500\Omega$$

For the 10Ω resistor



$$Z = \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix}$$

The total admittance matrix is then

$$Z_T = \begin{bmatrix} 200 & 0 \\ -1.25\times10^5 & 2500 \end{bmatrix} + \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix} = \begin{bmatrix} 210 & 10 \\ -124990 & 2510 \end{bmatrix}$$

To find the voltage gain we write the network equations resulting from Z_T

$V_1 = 210I_1 + 10I_2 \qquad (1)$

$V_2 = -124990I_1 + 2510I_2 \qquad (2)$

For the output,

$V_2 = -I_2 R_L$

Using (2) and (3)

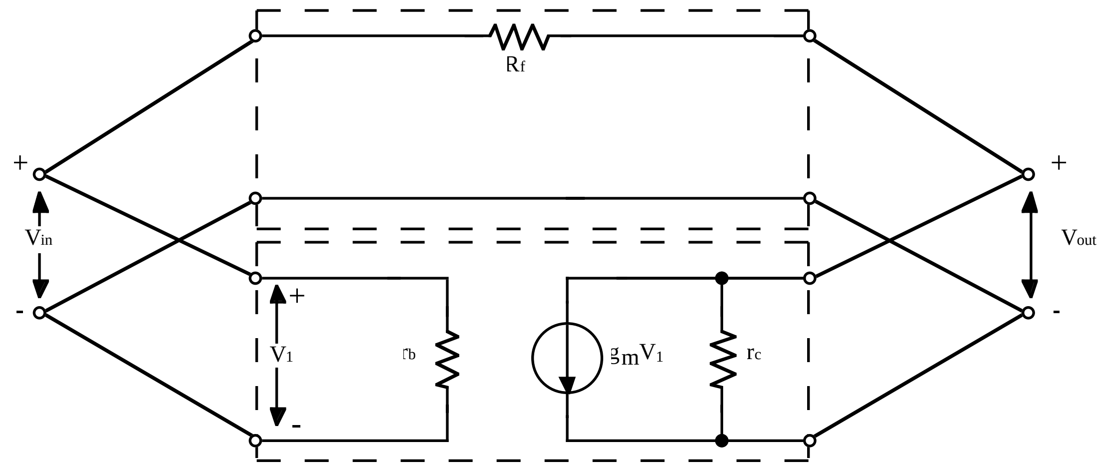
$$V_2 = -124990I_1 + 2510\left(-\frac{V_2}{R_L}\right)$$

$$V_2 + \frac{2510}{1000} V_2 = -124990 I_1$$
$$I_1 = -\frac{1 + 2.51}{124990} V_2 = -2.81 \times 10^{-5} V_2$$
Substituting this result into (1) and using (3)
$$V_1 = 210\left(-2.81 \times 10^{-5} V_2\right) + 10\left(\frac{-V_2}{1000}\right)$$

$$V_1 = -5.90 \times 10^{-3} V_2 - 0.01 V_2 = -1.59 \times 10^{-2} V_2$$

$$\frac{V_2}{V_1} = -62.9$$

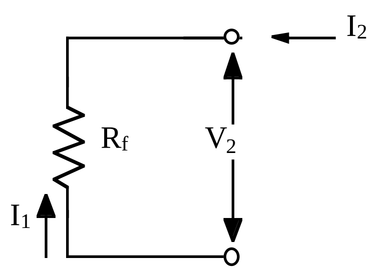
17. Using y-parameters, obtain the total circuit y-parameters for the circuits indicated by the dashed lines. Hint: first find the y-parameters of the two indicated two-port networks, then combine them to obtain the total network y-parameters.



The solution of this problem is similar to that of problem 16 except that y-parameters will be used.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

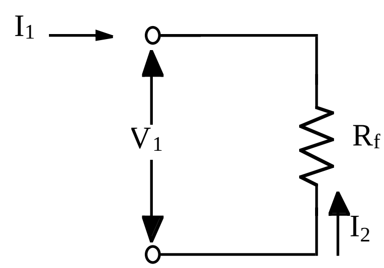
For $V_1 = 0$:
$I_1 = y_{12} V_2$
$I_2 = y_{22} V_2$



$$y_{12} = \frac{I_1}{V_2} = \frac{-I_2}{V_2} = -\frac{1}{R_f}$$

$$y_{22} = \frac{I_2}{V_2} = \frac{1}{R_f}$$

For $V_2 = 0$:
$I_1 = y_{11} V_1$
$I_2 = y_{21} V_1$

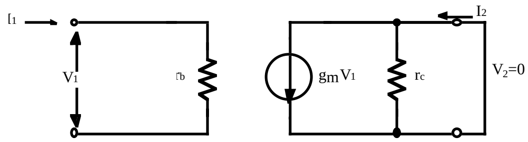


$$y_{11} = \frac{I_1}{V_1} = \frac{1}{R_f}$$

$$y_{21} = \frac{I_2}{V_1} = -\frac{I_1}{V_1} = -\frac{1}{R_f}$$

$$\text{Therefore, } Y_R = \begin{bmatrix} \frac{1}{R_f} & -\frac{1}{R_f} \\ -\frac{1}{R_f} & \frac{1}{R_f} \end{bmatrix}$$

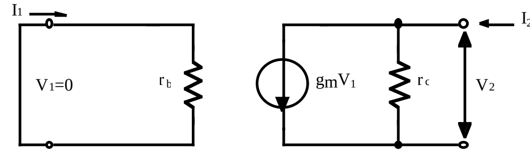
For $V_2 = 0$



$$y_{11} = \frac{I_1}{V_1} = \frac{1}{r_b}$$

$$y_{21} = \frac{I_2}{V_1} = \frac{g_m V_1}{V_1} = +g_m$$

For $V_1 = 0$



$$y_{12} = \frac{I_1}{V_2} = 0$$

$$y_{22} = \frac{I_2}{V_2} = \frac{1}{r_c}$$

And, therefore, $Y_{transistor} = \begin{bmatrix} \frac{1}{r_b} & 0 \\ g_m & \frac{1}{r_c} \end{bmatrix}$

$$Y_{total} = Y_{transistor} + Y_R = \begin{bmatrix} \frac{1}{r_b} & 0 \\ g_m & \frac{1}{r_c} \end{bmatrix} + \begin{bmatrix} \frac{1}{R_f} & -\frac{1}{R_f} \\ -\frac{1}{R_f} & \frac{1}{R_f} \end{bmatrix}$$

$$Y_{total} = \begin{bmatrix} \frac{1}{R_f} + \frac{1}{r_b} & -\frac{1}{R_f} \\ g_m - \frac{1}{R_f} & \frac{1}{R_f} + \frac{1}{r_c} \end{bmatrix}$$

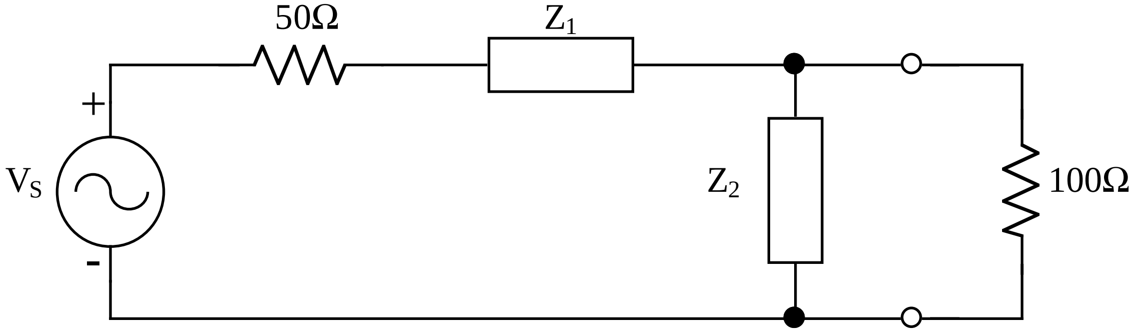
CONCENTRATES

Example 2.16

The circuit shown has Z_1=1μf and Z_2=200mH. The voltage source is represented by the partial Fourier series

$v_S = 20\cos 1000t - 5\cos 4000t$

Determine the voltage across the 100 ohm resistor.



1. In the circuit shown in example 2.16, the voltage source is operating at a frequency of 2000 rad/sec. Specify the impedances Z_1 and Z_2 so that maximum power is transferred to the 100 ohm load resistance. (Hint: Z_1 and Z_2 must be reactive or else they will absorb some of the power.) In specifying Z_1 and Z_2, give their values in microfarads and/or millihenries. There are two possible sets of answers.

When faced with a source-load problem always Thevenize the source circuit. From the above circuit

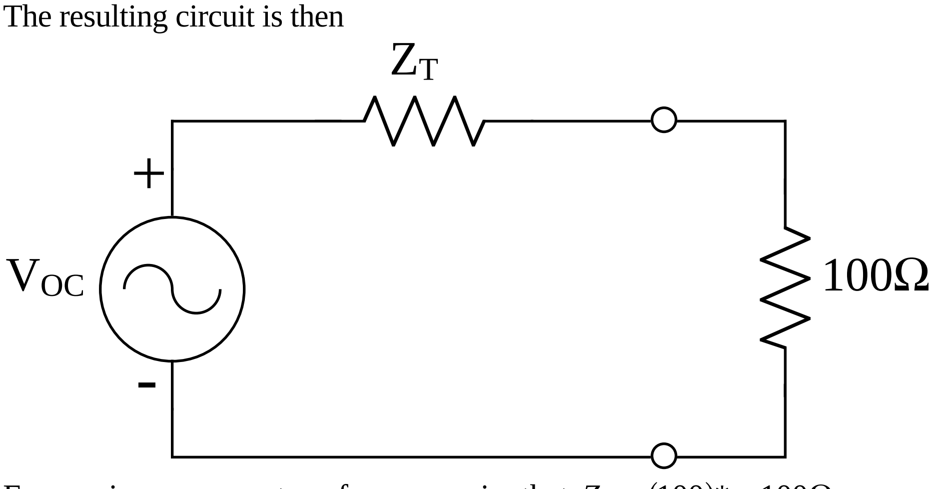
$$V_{OC} = \frac{jX_2}{jX_1 + jX_2 + 50} V_S$$

where we have assumed that Z_1 and Z_2 are purely reactive, i.e., $Z_1 = jX$ and $Z_2 = jX_2$

Similarly,

$$Z_T = (50 + jX_1) \,||\, jX_2 = \frac{(50 + jX_1) jX_2}{50 + jX_1 + jX_2}$$

The resulting circuit is then



For maximum power transfer we require that $Z_T = (100)^* = 100\Omega$ or

$$\frac{(50 + jX_1) jX_2}{50 + jX_1 + jX_2} = 100$$

Multiplying out

$$j50X_2 - X_1X_2 = 5000 + j100X_1 + j100X_2$$

Equating the real and imaginary parts

$$X_1X_2 = -5000 \qquad (1)$$

$$50X_2 = 100X_1 + 100X_2 \qquad (2)$$

Solving (2) for X_2 and substituting into (1)

$$-50X_2 = 100X_1$$

$$X_2 = \frac{100}{-50}X_1 = -2X_1$$

$$X_1(-2X_1) = -5000$$

$$X_1^2 = 2500$$

Therefore,

$$X_1 = \pm 50\Omega$$

The corresponding solutions for X_2 are found by substituting this result into (1):

$$X_1X_2 = -5000$$

$$X_2 = -\frac{5000}{X_1} = -\frac{5000}{\pm 50} = -/+\,50\Omega$$

Case 1: $X_1 = +50\Omega$, $X_2 = -100\Omega$

X_1 is inductive so $X_1 = \omega L_1 = 50$

$$L_1 = \frac{50}{2000} = 0.025H$$

X_2 is capacitive so $X_2 = -\frac{1}{\omega C_2} = -100\Omega$

Solving for the capacitance gives

$$C_2 = \frac{1}{(2000)(100)} = 5\mu f$$

Case 2: $X_1 = -50\Omega$, $X_2 = +100\Omega$

X_1 is capacitive so $X_1 = -\frac{1}{\omega C_1} = -50\Omega$

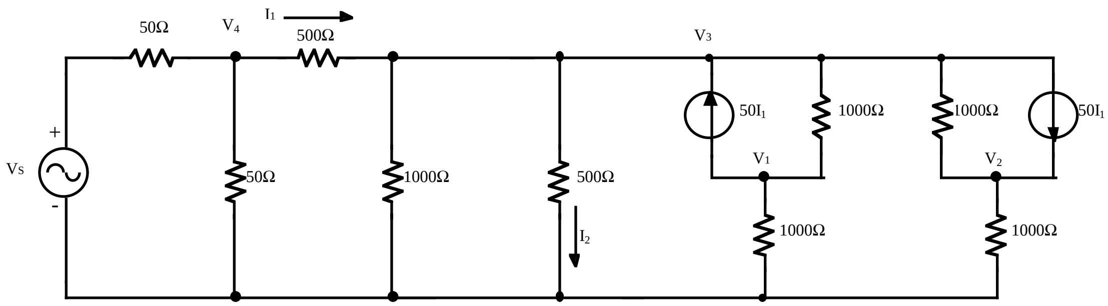
Solving for the capacitance gives

$$C_1 = \frac{1}{(2000)(50)} = 10\mu f$$

X_2 is inductive so $X_2 = \omega L_2 = 100$

$$L_2 = \frac{100}{2000} = 0.05H$$

2. For the circuit shown, obtain the node voltage equations in matrix form. Solve these equations for V_1 and V_2. (The actual output is V_1- V_2.) Obtain the voltage gain $\frac{V_{OUT}}{V_S}$ for this emitter -coupled amplifier.



This is a complex problem which requires that we know how to construct an admittance matrix representation for the circuit.

The standard ("formal") admittance matrix formulation is:

$$\begin{bmatrix} \sum I_1 \\ \sum I_2 \\ \\ \\ \end{bmatrix} = \begin{bmatrix} \sum Y_{11} & \sum Y_{12} & \\ \sum Y_{21} & \sum Y_{22} & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \\ \\ \end{bmatrix}$$

where

$\sum I_i$ =sum of <u>current sources</u> feeding into node i

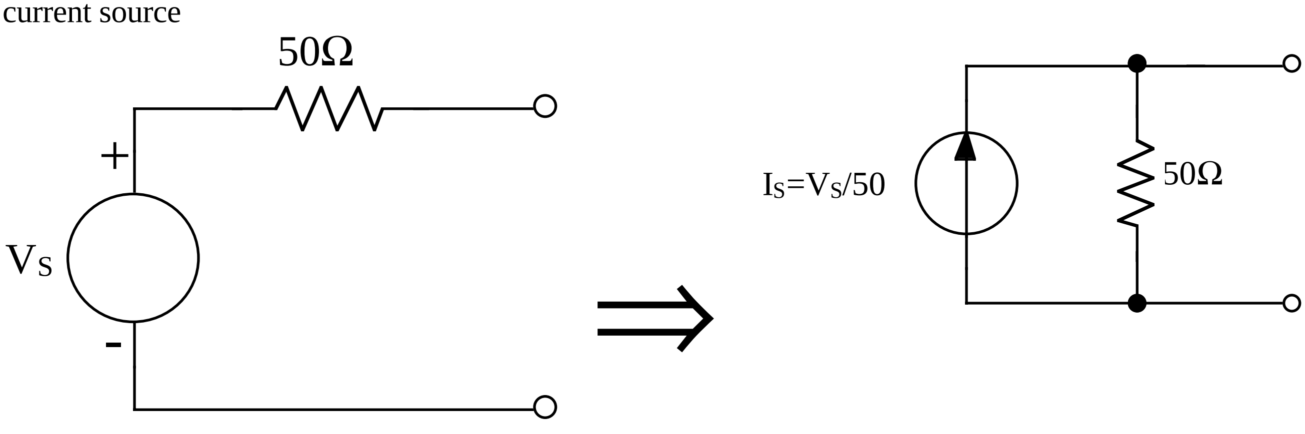
(into=positive, out=negative)

$\sum Y_{ii}$ =sum of all admittances connected to node i

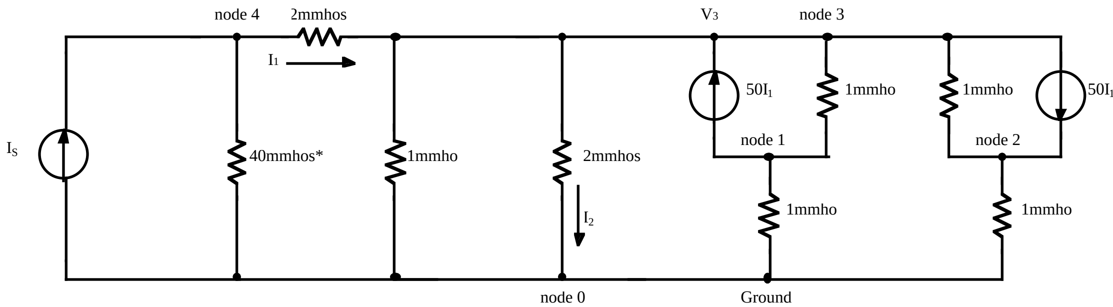
$\sum Y_{ij}$ = -sum of all admittances connected between nodes i and j

V_i=node voltages (including dependent sources)

The first step in solving the problem is to Nortonize the voltage source, converting it into a current source



Now, convert all resistances into admittances (I will use the unit of millimhos for convenience) and label the nodes.



Note that node 0, the reference node, must always be present.
*The 40millimhos is the combination of the two 50Ω resistors in parallel.

Constructing the admittance matrix representation

$$\begin{bmatrix} -50i_1 \\ +50i_2 \\ 50i_1 - 50i_2 \\ I_S \end{bmatrix} = \begin{bmatrix} 1+1 & 0 & -1 & 0 \\ 0 & 1+1 & -1 & 0 \\ -1 & -1 & 2+1+2+1+1 & -2 \\ 0 & 0 & -2 & 40+2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Consider node 1:
A current source of $50i_1$ leaves node 1, therefore the current is $-50i_1$
The sum of all admittances connected to node 1 is 1+1 millimhos
There are no admittances directly connected between nodes 2 and 1 or 4 and 1, therefore $Y_{12} = Y_{14} = 0$
There is 1 millimho connected between nodes 1 and 3 so $Y_{13} = -1$ millimho since all off-diagonal terms are negative.

Actually each row of the matrix represents a KCL equation for that node and this is the way I prefer to generate the matrix. At node 1, assuming that currents into the node are positive,

$$-50i_1 - 1(V_1 - 0) - 1(V_1 - V_3) = 0$$

which can be re-written as

$$-50i_1 = (1+1)V_1 - V_3 = 0$$

which is exactly the first row of the matrix equation.

The rest of the problem is simply solving the matrix:

$$\begin{bmatrix} -50i_1 \\ +50i_2 \\ 50i_1 - 50i_2 \\ I_S \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 2 & -1 & 0 \\ -1 & -1 & 7 & -2 \\ 0 & 0 & -2 & 42 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Expressing i_1 and i_2 in terms of independent variables V_3 and V_4

$$i_1 = \frac{V_4 - V_3}{500} = 2(V_4 - V_3)$$

$$i_2 = \frac{V_3}{500} = 2V_3$$

and converting the source term

$$\frac{V_S}{50} = 20V_S$$

Then,

$$\begin{bmatrix} -100V_4 + 100V_3 \\ 100V_3 \\ 50(2V_4 - 2V_3) - 50(2V_3) \\ 20V_S \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 2 & -1 & 0 \\ -1 & -1 & 7 & -2 \\ 0 & 0 & -2 & 42 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

$$\begin{bmatrix} -100V_4 + 100V_3 \\ 100V_3 \\ 100V_4 - 200V_3 \\ 20V_S \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 2 & -1 & 0 \\ -1 & -1 & 7 & -2 \\ 0 & 0 & -2 & 42 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Rearranging the equations,

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 20V_S \end{bmatrix} = \begin{bmatrix} 2 & 0 & -101 & +100 \\ 0 & 2 & -101 & 0 \\ -1 & -1 & 207 & -102 \\ 0 & 0 & -2 & 42 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

This equation can now be solved for the variables we are interested in (V_1 and V_2) using any method you want.

I highly recommend using a calculator which can solve systems of equations since this is where I usually make math errors. However, I will use expansion by minors to solve for V_1 and V_2 since the matrices are only 4×4.

$$V_1 = \frac{-20V_S \begin{bmatrix} 0 & -101 & 100 \\ 2 & -101 & 0 \\ -1 & 207 & -102 \end{bmatrix}}{2\begin{bmatrix} 2 & -101 & 0 \\ -1 & 207 & -102 \\ 0 & -2 & 42 \end{bmatrix} - 1\begin{bmatrix} 0 & -101 & 100 \\ 2 & -101 & 0 \\ 0 & -2 & 42 \end{bmatrix}}$$

The coefficients of all other minors are zero and not shown. Recall that solving for V_1 requires that the column vector of sources be substituted for column 1. Note also that the signs of the minor coefficients alternate.

Solving:

$$V_1 = \frac{-20V_S\left[(2)(207)(100) - (100)(-101)(-1) - (2)(-101)(-102)\right]}{2\left[(2)(207)(42) - (2)(-2)(-102) - (-1)(-101)(42)\right] - 1\left[(2)(-2)(100) - (2)(42)(-101)\right]}$$

$$V_1 = \frac{-20V_S[41400 - 10100 - 20604]}{2[17388 - 408 - 4242] - 1[-400 + 8484]} = \frac{-20V_S[10696]}{2[12738] - 1[8084]}$$

$$V_1 = \frac{-213920}{17392} V_S = -12.29 V_S$$

Similarly,

$$V_2 = \frac{-20V_S \begin{bmatrix} 2 & -101 & 100 \\ 0 & -101 & 0 \\ -1 & 207 & -102 \end{bmatrix}}{2\begin{bmatrix} 2 & -101 & 0 \\ -1 & 207 & -102 \\ 0 & -2 & 42 \end{bmatrix} - 1\begin{bmatrix} 0 & -101 & 100 \\ 2 & -101 & 0 \\ 0 & -2 & 42 \end{bmatrix}} = \frac{-20V_S\left[(2)(-101)(-102) - (-1)(-101)(100)\right]}{17392}$$

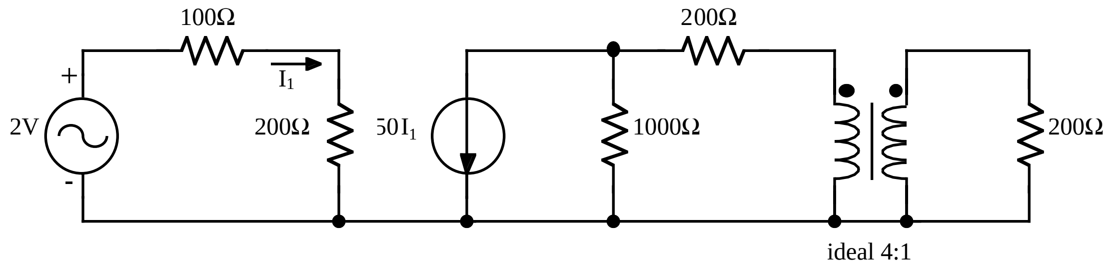
where we saved a little effort by recognizing that the denominator is the same as before.

$$V_2 = \frac{20V_S[20604 - 10100]}{17392} = +12.079V_S$$

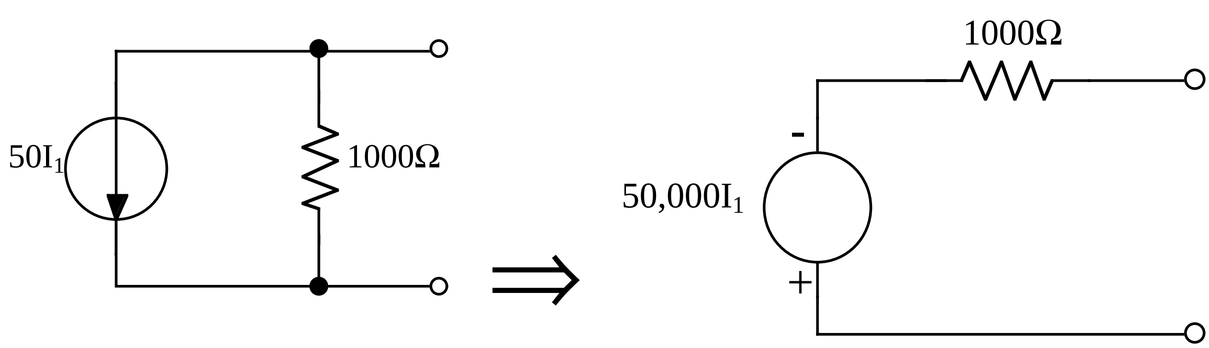
$$A_V = \frac{V_1 - V_2}{V_S} = \frac{-12.29V_S - 12.079V_S}{V_S} = -24.37$$

Expansion by minors:
Check sign:

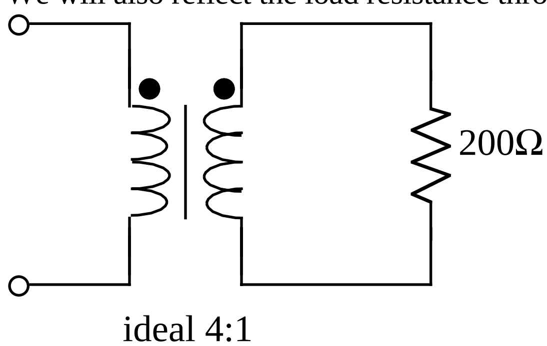
3. For the circuit shown, obtain the loop current equations. Solve for the current in the 200 ohm resistance connected to the secondary of the ideal transformer.

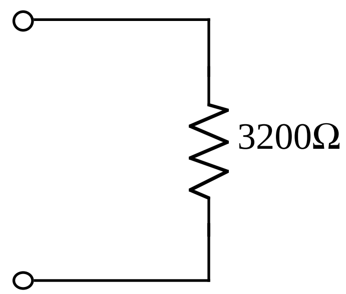


As usual we will Thevenize the dependent current source to make the loop analysis easier.



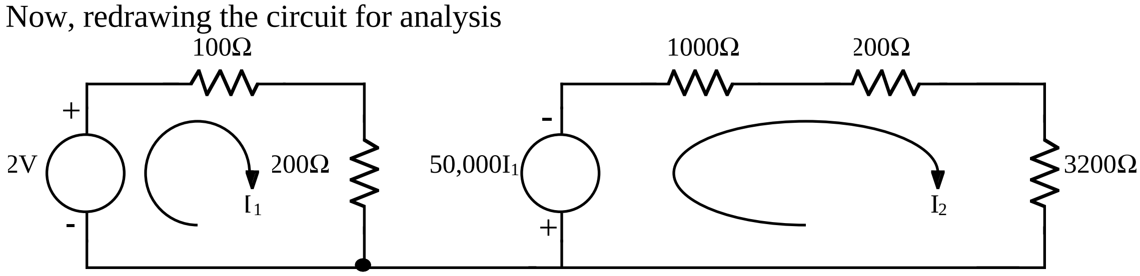
We will also reflect the load resistance through the transformer to make analysis simpler.





where $(4)^2(200) = (16)(200) = 3200\Omega$

Now, redrawing the circuit for analysis



Writing the loop equations

$-2 + 100i_1 + 200i_1 = 0$

$+50000i_1 + 1000i_2 + 200i_2 + 3200i_2 = 0$

$$\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 300 & 0 \\ 50000 & 4400 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

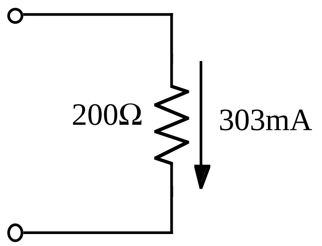
We are interested in the transformer primary current i_2. Solving for i_2:

$$i_2 = \frac{\begin{bmatrix} 300 & 2 \\ 50000 & 0 \end{bmatrix}}{\begin{bmatrix} 300 & 0 \\ 50000 & 4400 \end{bmatrix}} = \frac{0-(50000)(2)}{(300)(4400)-0} = -75.76mA$$

The transformer secondary current is then

-75.76mA×4=-303mA

with the current as shown below



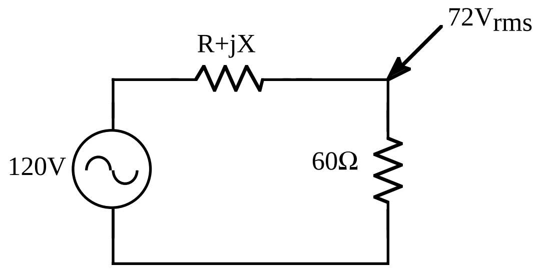
4. A 60 Hz source has its voltage measured under various loads, with the results shown.

Case	Load	Voltmeter reading
1	open circuit	120 volts rms
2	60Ω resistance	72 volts rms
3	60Ω pure capacitance	360 volts rms
4	60Ω pure inductance	51.43 volts rms

Determine the voltage for a load of 120 ohms pure capacitance.

We will assume that the internal impedance of the source is complex and in series with the source.

Case 1: under no load the open circuit voltage is 120 volts rms
Case 2:



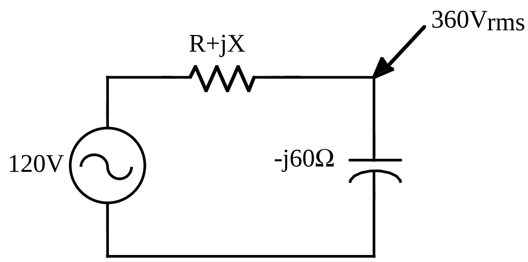
$$V_{OUT} = \left(\frac{60}{60 + R + jX} \right) 120$$

$$\left| \frac{(60)(120)}{(60 + R) + jX} \right| = 72$$

$$\frac{5.184 \times 10^7}{(60 + R)^2 + X^2} = 5184$$

$$(60 + R)^2 + X^2 = 10000 \qquad (1)$$

Case 3:



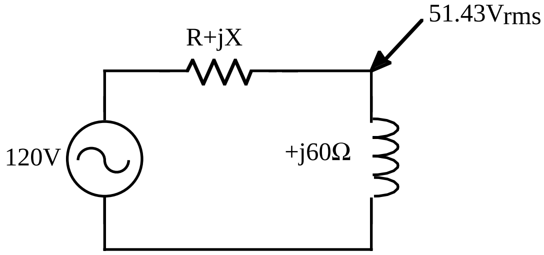
$$V_{OUT} = \left(\frac{-j60}{-j60 + R + jX} \right) 120$$

$$\left| \frac{(-j60)(120)}{R + j(X - 60)} \right| = 360$$

$$\frac{5.184 \times 10^7}{R^2 + (X - 60)^2} = 129600$$

$$R^2 + (X - 60)^2 = 400 \quad (2)$$

Case 4:



$$V_{OUT} = \left(\frac{+j60}{+j60+R+jX}\right)120$$

$$\left|\frac{(+j60)(120)}{R+j(X+60)}\right| = 51.4$$

$$\frac{5.184\times10^{7}}{R^2+(X+60)^2} = 2645.045$$

$$R^2+(X+60)^2 = 19598.911 \quad (3)$$

Solving equations (2) and (3)

$$R^2+X^2-120X+3600 = 400$$
$$R^2+X^2+120X+3600 = 19598.911$$

$$R^2+X^2-120X = -3200$$
$$R^2+X^2+120X = 15998.911$$

Subtracting

$-240X = -19198.911$

X=79.995Ω

Substituting this result back into equation (2) gives

$$R^2+(79.995-60)^2 = 400$$
$$R^2+(19.995)^2 = 400$$
$$R^2 = 0.200$$
$$R = 0.447\Omega$$

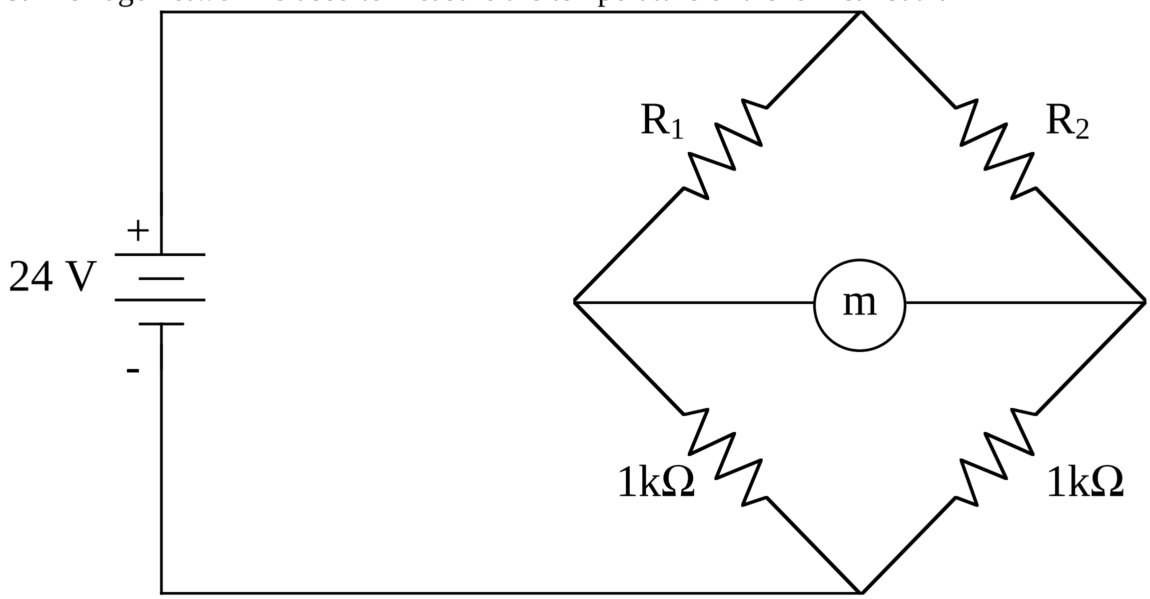
The output voltage for a capacitive -j120Ω load can then be calculated using these results.

$$V_{OUT} = \left|\left(\frac{-j120}{R+jX-j120}\right)120\right| = \left|\frac{(-j120)120}{0.447+j79.995-j120}\right| = \left|\frac{-j14400}{0.447-j40.005}\right| = 359.91-j4.021$$

$$V_{OUT} = 359.93\angle-0.64°$$

$$V_{OUT} \approx 360 \text{ volts rms}$$

5. A bridge network is used to measure the temperature of a chemical bath.



R_1 and R_2 are identical thermistors with negative temperature coefficients of resistance of -4%/° C. At 25° C they are both 1500Ω. The meter (m) has an internal resistance of 800Ω.

Resistance R_2 is held at 25° C.

(a) Determine the meter current when R_1 is at 20° C.

(b) Determine the meter current when R_1 is at 30° C.

The first step is to calculate the different values of R_1.

R_1=1500Ω @ 25° C

temperature coefficient -4%/° C

@20° C

$$R_1 = 1500(1 - 0.04(T - 25°C))$$

$$R_1 = 1500(1 - 0.04(20 - 25)) = 1500(1 + 0.04(5))$$

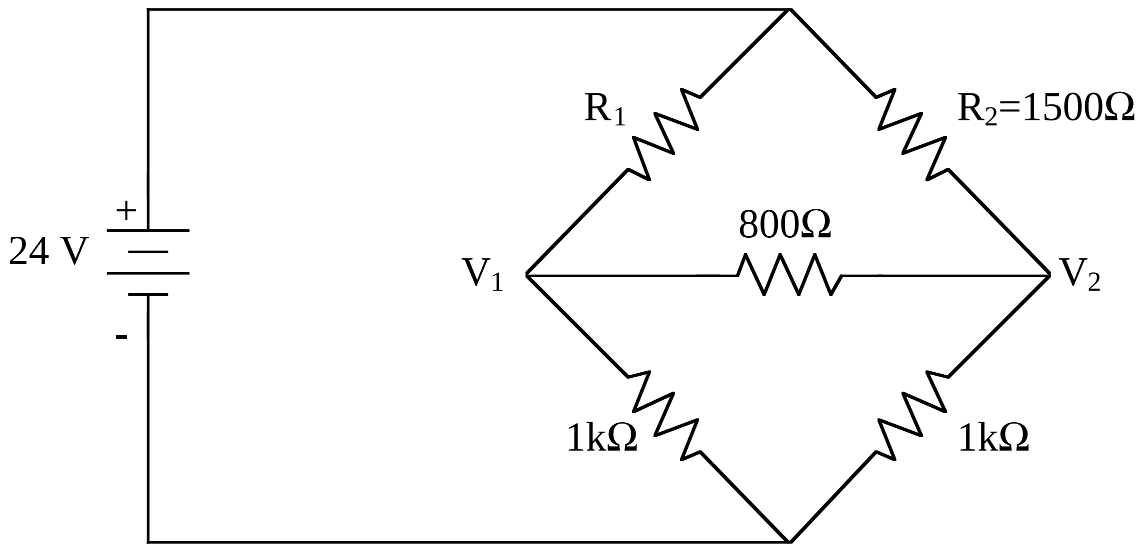
$$R_1 = 1500(1 + 0.2) = 1800\Omega$$

@30° C

$$R_1 = 1500(1 - 0.04(T - 25°C))$$

$$R_1 = 1500(1 - 0.04(30 - 25)) = 1500(1 - 0.04(5))$$

$$R_1 = 1500(1 - 0.2) = 1200\Omega$$



We will write the nodal equations as shown below:

$$\frac{24-V_1}{1800}-\frac{V_1-V_2}{800}-\frac{V_1}{1000}=0$$

$$\frac{24-V_2}{1500}+\frac{V_1-V_2}{800}-\frac{V_2}{1000}=0$$

Re-writing

$$24-V_1-2.25V_1+2.25V_2-1.8V_1=0$$

$$24-V_2+1.875V_1-1.875V_2-1.5V_2=0$$

In matrix form,

$$\begin{bmatrix}24\\24\end{bmatrix}=\begin{bmatrix}1+2.25+1.8 & -2.25\\ -1.875 & 1.5+1.875+1\end{bmatrix}\begin{bmatrix}V_1\\V_2\end{bmatrix}$$

$$\begin{bmatrix}24\\24\end{bmatrix}=\begin{bmatrix}5.05 & -2.25\\ -1.875 & 4.375\end{bmatrix}\begin{bmatrix}V_1\\V_2\end{bmatrix}$$

Solving,

$$V_1=\frac{\begin{bmatrix}24 & -2.25\\ 24 & 4.375\end{bmatrix}}{\begin{bmatrix}5.05 & -2.25\\ -1.875 & 4.375\end{bmatrix}}=\frac{105-(-54)}{(5.05)(4.375)-(-1.875)(-2.25)}=\frac{159}{22.094-4.219}=\frac{159}{17.875}=+8.895$$

$$V_2=\frac{\begin{bmatrix}5.05 & 24\\ -1.875 & 24\end{bmatrix}}{\begin{bmatrix}5.05 & -2.25\\ -1.875 & 4.375\end{bmatrix}}=\frac{(5.05)(24)-(-1.875)(24)}{(5.05)(4.375)-(-1.875)(-2.25)}=\frac{166.2}{17.875}=+9.2979$$

$$I_m(@20°C) = \frac{V_1 - V_2}{800\Omega} = \frac{8.8951 - 9.2979}{800} = -0.504mA$$

At 30° C the node equations are:

@node 1 $\frac{24 - V_1}{R_1} - \frac{V_1 - V_2}{800} + \frac{V_1}{1000} = 0$

@node 2 $\frac{24 - V_2}{R_2} + \frac{V_1 - V_2}{800} - \frac{V_2}{1000} = 0$

$$\frac{24 - V_1}{1200} - \frac{V_1 - V_2}{800} - \frac{V_1}{1000} = 0$$

$$\frac{24 - V_2}{1500} + \frac{V_1 - V_2}{800} - \frac{V_2}{1000} = 0$$

In matrix form,

$$\begin{bmatrix} -\frac{24}{1200} \\ -\frac{24}{1500} \end{bmatrix} = \begin{bmatrix} -\frac{1}{1200} - \frac{1}{800} - \frac{1}{1000} & \frac{1}{800} \\ \frac{1}{800} & -\frac{1}{1000} - \frac{1}{800} - \frac{1}{1500} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Multiplying the first row by 1200 and the second row by 1500 we get:

$$\begin{bmatrix} -24 \\ -24 \end{bmatrix} = \begin{bmatrix} -1 - 1.5 - 1.2 & 1.5 \\ 1.875 & -1.5 - 1.875 - 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

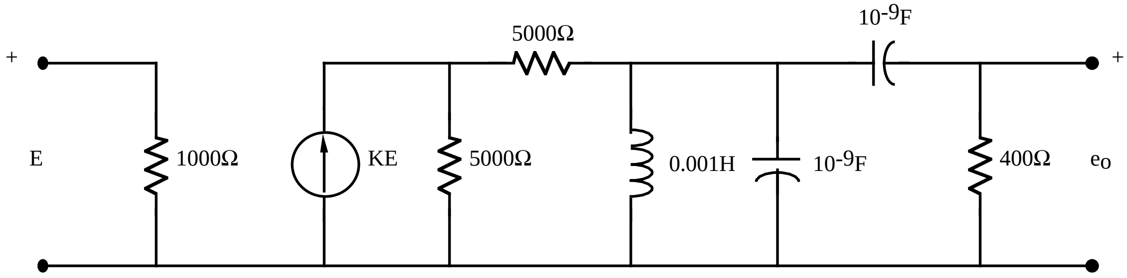
$$\begin{bmatrix} -24 \\ -24 \end{bmatrix} = \begin{bmatrix} -3.7 & 1.5 \\ 1.875 & -4.375 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$V_1 = \frac{\begin{bmatrix} -24 & 1.5 \\ -24 & -4.375 \end{bmatrix}}{\begin{bmatrix} -3.7 & 1.5 \\ 1.875 & -4.375 \end{bmatrix}} = \frac{105 + 36}{13.375} = 10.5421$$

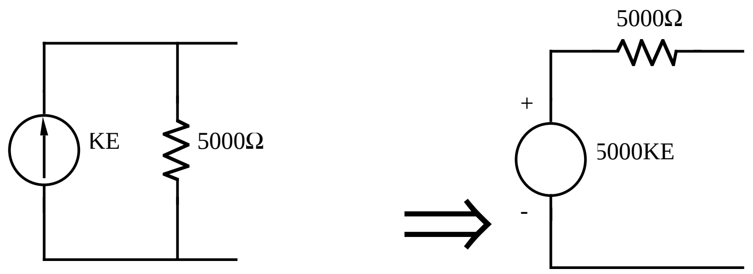
$$V_2 = \frac{\begin{bmatrix} -3.7 & -24 \\ 1.875 & -24 \end{bmatrix}}{\begin{bmatrix} -3.7 & 1.5 \\ 1.875 & -4.375 \end{bmatrix}} = \frac{88.8 + 45}{13.375} = 10.0037$$

$$I_m(@20°C) = \frac{V_1 - V_2}{800\Omega} = \frac{10.5421 - 10.0037}{800} = +0.67mA$$

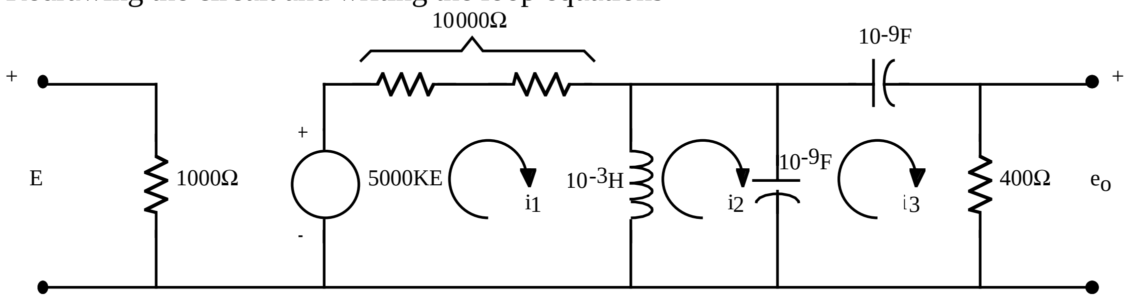
6. An oscillator circuit is shown below. Determine the minimum value of K such that e_o=E. Also, find the frequency at which e_o=E.



As usual we Thevenize the source so that we can combine resistances.



Redrawing the circuit and writing the loop equations



$$-5000KE + 10000 i_1 + (i_1 - i_2) j\omega 10^{-3} = 0$$

$$j\omega 10^{-3}(i_2 - i_1) + (i_2 - i_3)\frac{1}{j\omega 10^{-9}} = 0$$

$$\frac{1}{j\omega 10^{-9}}(i_3 - i_2) + \frac{1}{j\omega 10^{-9}} i_3 + 400 i_3 = 0$$

In matrix form,

$$\begin{bmatrix} +5000KE \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10000 + j\omega 10^{-3} & -j\omega 10^{-3} & 0 \\ -j\omega 10^{-3} & +j\omega 10^{-3} + \dfrac{1}{j\omega 10^{-9}} & -\dfrac{1}{j\omega 10^{-9}} \\ 0 & -\dfrac{1}{j\omega 10^{-9}} & \dfrac{1}{j\omega 10^{-9}} + \dfrac{1}{j\omega 10^{-9}} + 400 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

Since we are only interested in i_3:

$$i_3 = \frac{\begin{bmatrix} 10000 + j\omega 10^{-3} & -j\omega 10^{-3} & +5000KE \\ -j\omega 10^{-3} & +j\omega 10^{-3} + \dfrac{1}{j\omega 10^{-9}} & 0 \\ 0 & -\dfrac{1}{j\omega 10^{-9}} & 0 \end{bmatrix}}{\begin{bmatrix} 10000 + j\omega 10^{-3} & -j\omega 10^{-3} & 0 \\ -j\omega 10^{-3} & +j\omega 10^{-3} + \dfrac{1}{j\omega 10^{-9}} & -\dfrac{1}{j\omega 10^{-9}} \\ 0 & -\dfrac{1}{j\omega 10^{-9}} & \dfrac{1}{j\omega 10^{-9}} + \dfrac{1}{j\omega 10^{-9}} + 400 \end{bmatrix}}$$

$$i_3 = \frac{+5000KE\left(-j\omega 10^{-3}\right)\left(-\frac{1}{j\omega 10^{-9}}\right)}{\left\{\left(10^4 + j\omega 10^{-3}\right)\left(j\omega 10^{-3} + \frac{1}{j\omega 10^{-9}}\right)\left(+\frac{j2}{j\omega 10^{-9}} + 400\right) - \left(-j\omega 10^{-3}\right)\left(-j\omega 10^{-3}\right)\left(+\frac{j2}{j\omega 10^{-9}} + 400\right) - \left(-\frac{1}{j\omega 10^{-9}}\right)\left(-\frac{1}{j\omega 10^{-9}}\right)\left(10000 + j\omega 10^{-3}\right)\right\}}$$

$$i_3 = \frac{+5000KE \times 10^6}{\left\{2\times 10^{10} - \dfrac{2\times 10^{22}}{\omega^2} - \dfrac{2j\times 10^{15}}{\omega} + j\omega 4000 - \dfrac{4j\times 10^{15}}{\omega} + 4\times 10^8 + \dfrac{10^{22}}{\omega^2} + \dfrac{j10^{15}}{\omega}\right\}}$$

$$i_3 = \frac{+5\times 10^6 KE}{\left\{\left(2.04\times 10^{10} - \dfrac{10^{22}}{\omega^2}\right) + j\left(4000\omega - \dfrac{5\times 10^{15}}{\omega}\right)\right\}}$$

For $e_o \cong E$ for oscillation we require i_3 to be real which requires $4000\omega - \dfrac{5\times 10^{15}}{\omega} = 0$ or $4000\omega^2 = 5\times 10^{15}$

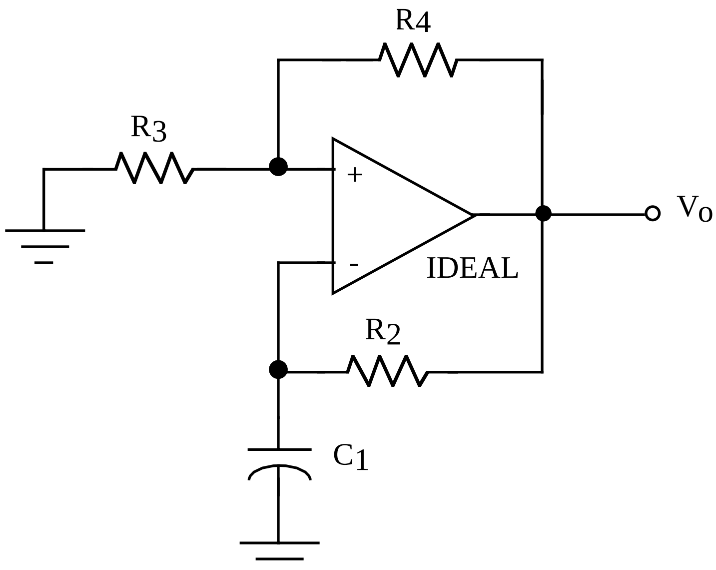
Solving for ω gives $\omega^2 = \dfrac{5\times 10^{15}}{4000} = 1.25\times 10^{12}$ or $\omega = 1.12\times 10^6 rad/\sec$

Using this value, $\dfrac{+5\times 10^6 KE}{2.04\times 10^{10} - \dfrac{10^{22}}{1.25\times 10^{12}}}(400) = E$

Solving for K gives $\dfrac{+5\times 10^6 K(400)}{2.04\times 10^{10} - 8\times 10^9} = 1$ or $K = \dfrac{1.24\times 10^{10}}{+5\times 10^6(400)} = +\dfrac{2480}{400} = +6.2$

The following problem is similar to a problem I encountered on my PE exam (morning session)

Sketch the steady-state output of the following circuit. Be sure to indicate values of voltages and time. Assume the op-amp operates on ±15 power supplies.



The component values are C_1=0.1µf, R_2=5kΩ, R_3=10kΩ and R_4=10kΩ.