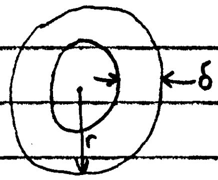
TRANSMISSION LINES

- SKIN EFFECT, MOSTLY AN ISSUE AT RF
  IMPACTS CURRENT CAPACITY OF LINE.

$$\delta = \frac{1}{\sqrt{\frac{\pi f \mu}{\rho}}}$$

$$\delta = 2.53 \times 10^5 \sqrt{\frac{\rho}{f}}$$

EFFECTIVE AREA:



$$A_{eff} = \pi\left(r^2 - (r-\delta)^2\right)$$

- SEE FIGURE 6.1, P. 6-3 FOR HANDY GRAPH OF SKIN EFFECT. (R_o IS D.C. RESISTANCE.)

- INDUCTANCE
  INTERNAL – DUE TO CURRENT IN WIRE ACTING ON ITSELF
  EXTERNAL – INTERACTION OF FIELDS FROM NEARBY WIRES

INTERNAL: $L_i = \frac{\mu}{8\pi}$ (ALWAYS) H/m

EXTERNAL: $L_e = \frac{\mu}{\pi} \ln\left(\frac{D}{r}\right)$ H/m

TOTAL $L = L_i + L_e$ H/m

- CAPACITANCE

$$C = \frac{2\pi\epsilon}{\ln\left[\frac{(D-r_1)(D-r_2)}{r_1 r_2}\right]} \quad F/m$$

POWER TRANSMISSION LINES

- USE TABES TO GET REQURED VALUES.

- CORRECTION FACTOR FOR SPACINGS OTHER THAN 1 FOOT:

$$K_L = 1 + \frac{\ln(D)}{\ln\left(\frac{1}{GMR}\right)}$$

$$K_C = 1 + \frac{\ln(D)}{\ln\left(\frac{1}{r}\right)}$$

EXAMPLE

FIND X_L OF BLUEBIRD LINE, 10' SPACING.

FROM TABEL, GMR = 0.0586,

$$K_L = 1 + \frac{\ln(10)}{\ln\left(\frac{1}{.0586}\right)} = 1.81$$

$$X_L = \underset{\substack{\uparrow \\ \text{FROM TABLE}}}{0.344} \times 1.81 = 0.623 \ \Omega/\text{MILE}$$

THREE PHASE LINES

- USE GEOMETRIC MEAN SPACING

$$d_s = \sqrt[3]{r_{ab} r_{bc} r_{ac}}$$

TO ARRIVE AT SIMPLIFIED EQUATIONS.

$$\left.\begin{aligned} L &= (2\times10^{7}) \ln\left(\frac{d_s}{GMR}\right) \ H/m \\ M_{12} &= (2\times10^{7}) \ln\left(\frac{r_1 r_2}{d_s^2}\right) \ H/m \end{aligned}\right\} \text{AS LONG AS ALL 3 CONDUCTORS ARE IDENTICAL}$$

PLUG & GO SIMPLE!

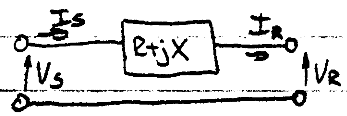
MODELING POWER TRANSMISSION LINES

- UNDER FAIRLY BALANCED CONDX, USE SINGLE LINE ANALYSIS
- MUST USE $\Delta \rightarrow Y$ TRANSFORMATION TO GET EQUIVALENT LINE TO NEUTRAL Z IN CASES OF Δ CONNECTED LOADS:

$$Z_{an} = \frac{Z_{ab} Z_{ac}}{Z_{ab} + Z_{bc} + Z_{ca}}$$

SHORT LINES: (UNDER 50 mi), SHUNT ADMITTANCE NEGLIGIBLE.

$$Z_l = \left[\frac{R}{\text{MILE}} + \frac{jX_L}{\text{MILE}}\right] \times \text{LENGTH}$$

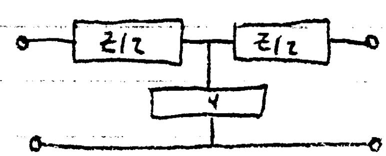


$$\text{REGULATION} = \frac{|V_{R,NL}| - |V_{R,FL}|}{|V_{R,FL}|} \times 100.$$

MEDIUM LINES: (50 - 150 mi)

- SHUNT COMPONENTS SIGNIFICANT, BUT CAN STILL MODEL WITH LUMPED CKT ELEMENTS:

T MODEL



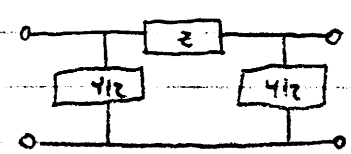
$A = 1 + \frac{YZ}{2}$

$B = Z\left(1 + \frac{YZ}{4}\right)$

$C = Y$

$D = 1 + \frac{YZ}{2}$

π MODEL



$A = 1 + \frac{YZ}{2}$

$B = Z$

$C = Y\left(1 + \frac{YZ}{4}\right)$

$D = 1 + \frac{YZ}{2}$

REMEMBER THAT $\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$

REGULATION AS BEFORE.

LONG LINES:

MUST USE DISTRIBUTED CKT ELEMENTS,

z : SERIES IMPEDANCE/LENGTH

y : SHUNT ADMITTANCE/LENGTH.

$z = R + jX_L$

$y = jB_C = \frac{j}{X_C}$

TO GET PROP CONST $\gamma = \sqrt{yz}$

ATT CONST $\alpha = |\gamma| \cos\left[\frac{1}{2} \tan^{-1}\left(\frac{-R}{X_L}\right)\right]$.

PHASE CONST $\beta = |\gamma| \sin\left[\frac{1}{2} \tan^{-1}\left(\frac{-R}{X_L}\right)\right]$

CHAR IMP $Z_0 = \sqrt{\frac{z}{y}}$

REF. COEFF. $\rho = \frac{Z_L - Z_0}{Z_L + Z_0}$ Z_L = LOAD Z.

MOST THINGS YOU NEED TO KNOW ARE DETAILED IN TEXT EQUATIONS.

RF TRANSMISSION LINES.

AT RF, λ IS SO SHORT EVERY LINE IS A LONG LINE.

BUT $R \to 0$, SO WE MAY MAKE DRASTIC SIMPLIFICATIONS.

$$z = j\omega L$$
$$y = j\omega C$$
$$\gamma = j\omega\sqrt{LC} = j\beta$$
$$Z_0 = \sqrt{\frac{L}{C}}$$

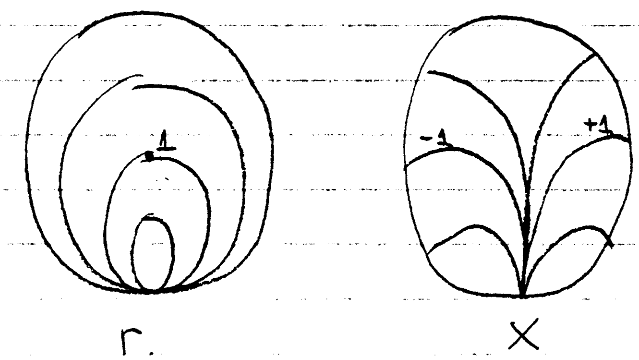
STANDING WAVE RATIO

$$VSWR = \frac{1+|\rho|}{1-|\rho|}$$, INDICATES DEGREE OF MISMATCH ON LINE.

YOU CAN USE EQUATIONS TO SOLVE TRANSMISSION LINE PROBLEMS, BUT SMITH CHART IS MUCH EASIER

→ CAN NOT OVEREMPHASIZE FACT THAT S.C. IS SIMPLER.

SMITH CHART IS A GRAPH OF ALL THE COMPLEX IMPEDANCES (OR ADMITTANCES).

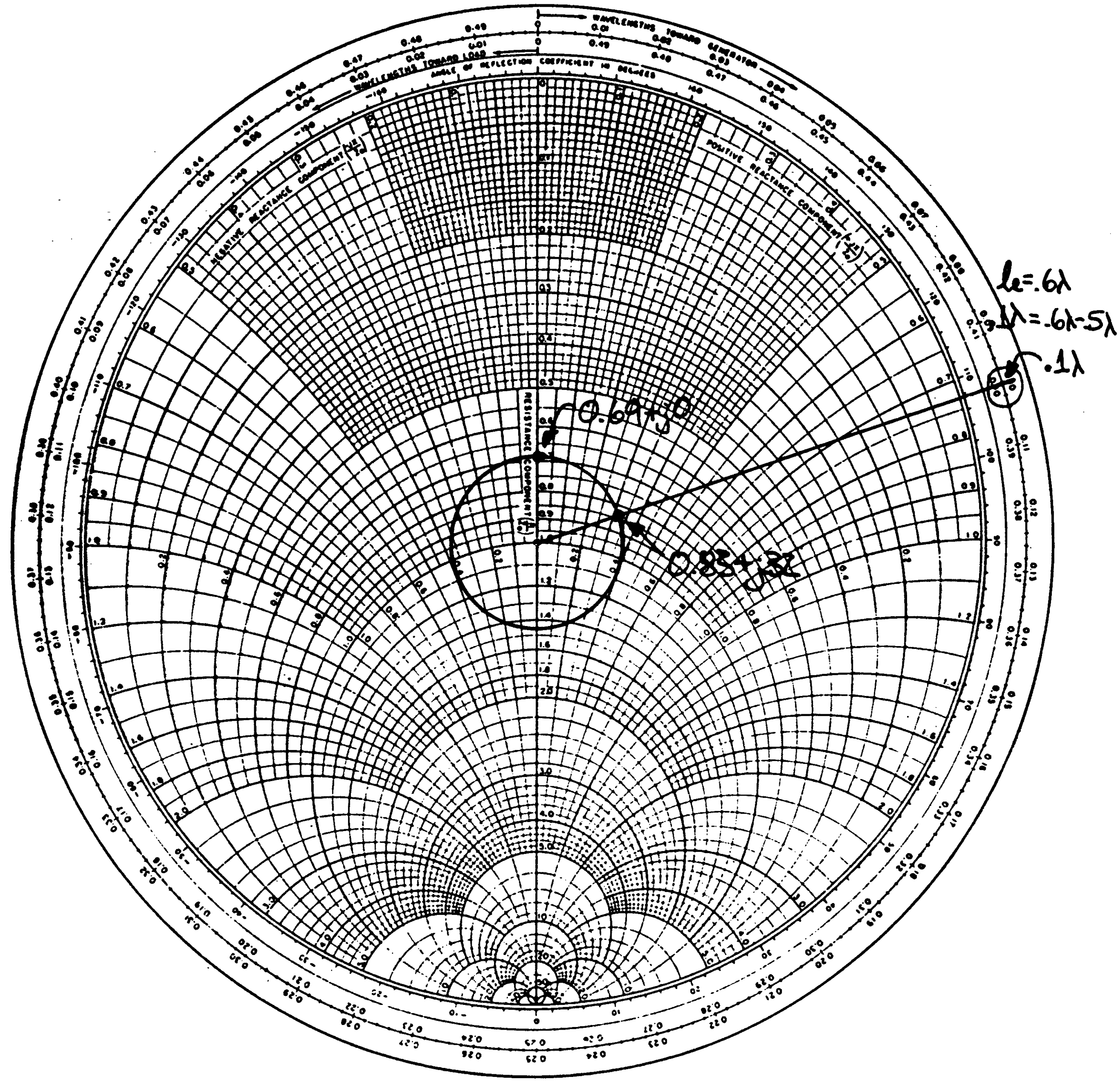


NORMALIZE IMPEDANCES TO BASE 1, SO THAT LINE IMPEDANCE IS AT CENTER OF CHART.

Appendix C

The Smith Chart

EXAMPLE #1.



OPEN & SHORTED LINES

NOTE LOCATION OF OPEN CKT: BOTTOM OF CHART
SHORT CKT: TOP OF CHART.

WE CAN USE OPEN & SHORTED LINES TO GENERATE REACTANCES & SUSEPTENCES WE NEED FOR MATCHING.

EXAMPLE

WE NEEDED SERIES COMPENSATOR $X_c = -160\,\Omega$ IN LAST EXAMPLE. REMEMBER, $Z_0 = 100\,\Omega$, SO $x_c = -1.6\,\Omega$

PLOT $x_c = -1.6\,\Omega$, READ VALUES

IF $Z_0 = 250\,\Omega$, $x_c = \frac{-160}{250} = -.64\,\Omega$, READ VALUES.

EXAMPLE

WE NEEDED SHUNT COMPENSATOR $B_c = 0.016\,S$.
$Y_0 = 0.01\,S$

PLOT $B_c = 1.6\,S$, READ THE LENGTHS.

IF $Z_0 = 250\,\Omega$, $Y_0 = 0.004\,S$,

$$b_c = \frac{B_c}{Y_0} = \frac{0.016}{0.004} = 4\,S,$$ READ VALUES.

#1

EX: TRANSMISSION LINE $Z_0 = 72\,\Omega$, $\beta = .5\ \frac{\text{RAD}}{\text{m}}$, $l = 7.5\text{m}$

~~TBA~~ TERMINATED IN $Z_L = 50\,\Omega$. SOURCE Z?

$$z_L = \frac{Z_L}{Z_0} = \frac{50}{72} = .69$$

PLOT THAT POINT.

DRAW CIRCLE OF CONSTANT $|\rho|$

$$\text{ELECTRICAL LENGTH} = \frac{\beta}{2\pi}\,l$$

$$= \frac{.5}{2\pi}\,7.5 = .6\lambda$$

START FROM LOAD, GO $.6\lambda$ TOWARD GENERATOR

POINT LANDS AT $z_i = 0.83 + j0.32$

SO $Z_i = z_i Z_0 = 60 + j23\,\Omega$.

ANOTHER EXAMPLE (#2)

$Z_0 = 100\,\Omega$ FEEDING $Z_L = 25 + j25\,\Omega$

FIND LUMPED MATCHING ELEMENTS FOR SERIES / SHUNT.

SERIES: $z_L = .25 + j.25$ @ $.041\lambda$

SPOT IS $.179\lambda$, $z = 1 + j1.62$

$.179 - .041 = .138\lambda$ IS LOCATION FOR $X_c = 160\,\Omega$

SHUNT: $Y_0 = \frac{1}{Z_0} = .01\,S$

LOCATE y_L ON CHART. $y_L = 2 - j2$ @ $.291\lambda$

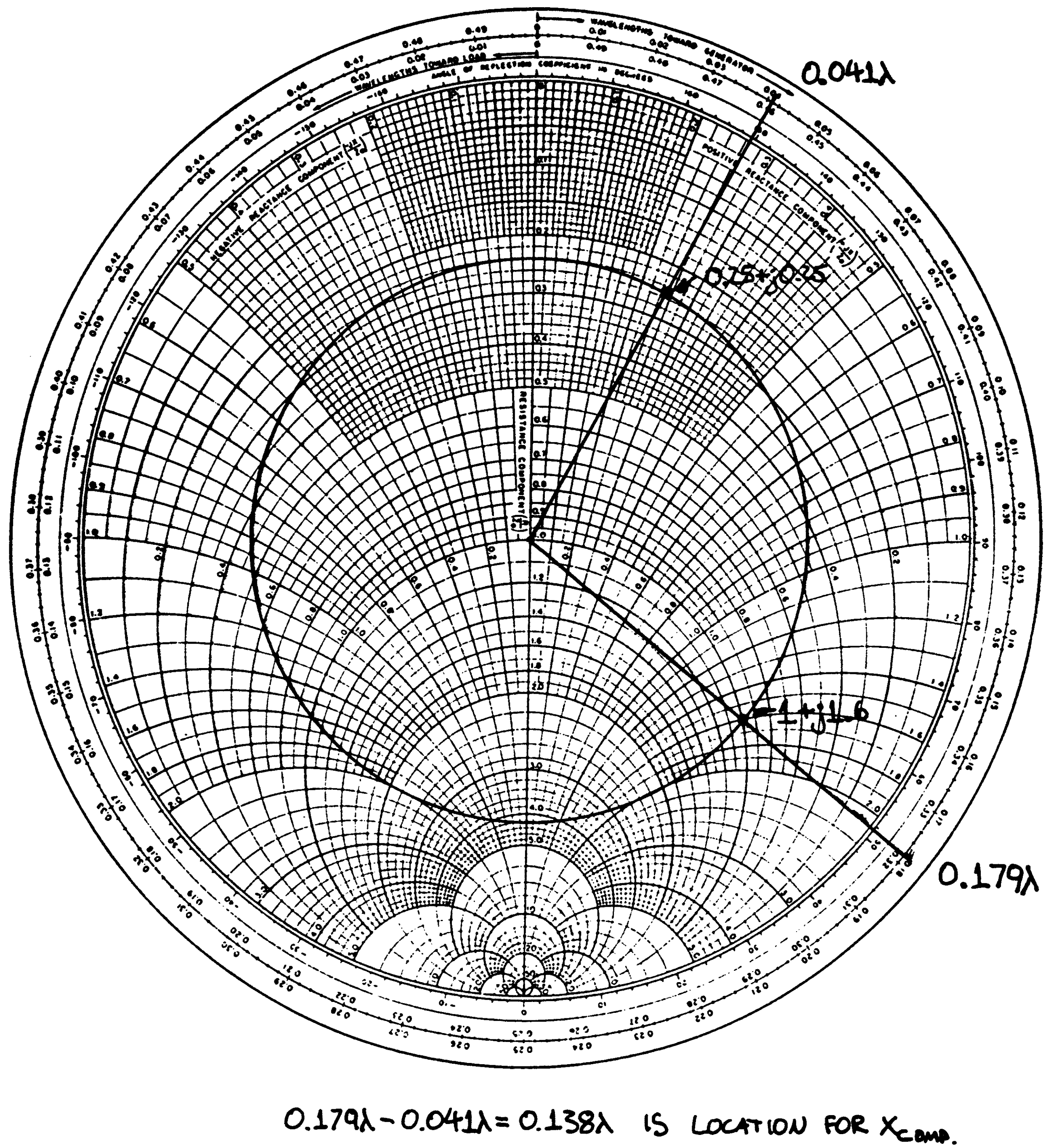
SPOT IS 0.322λ $y = 1 - j1.6\,S$

$0.322 - 0.291 = 0.031\lambda$ IS LOCATION FOR $B_c = 0.016\,S$.

Appendix C

The Smith Chart

EXAMPLE #2: SERIES



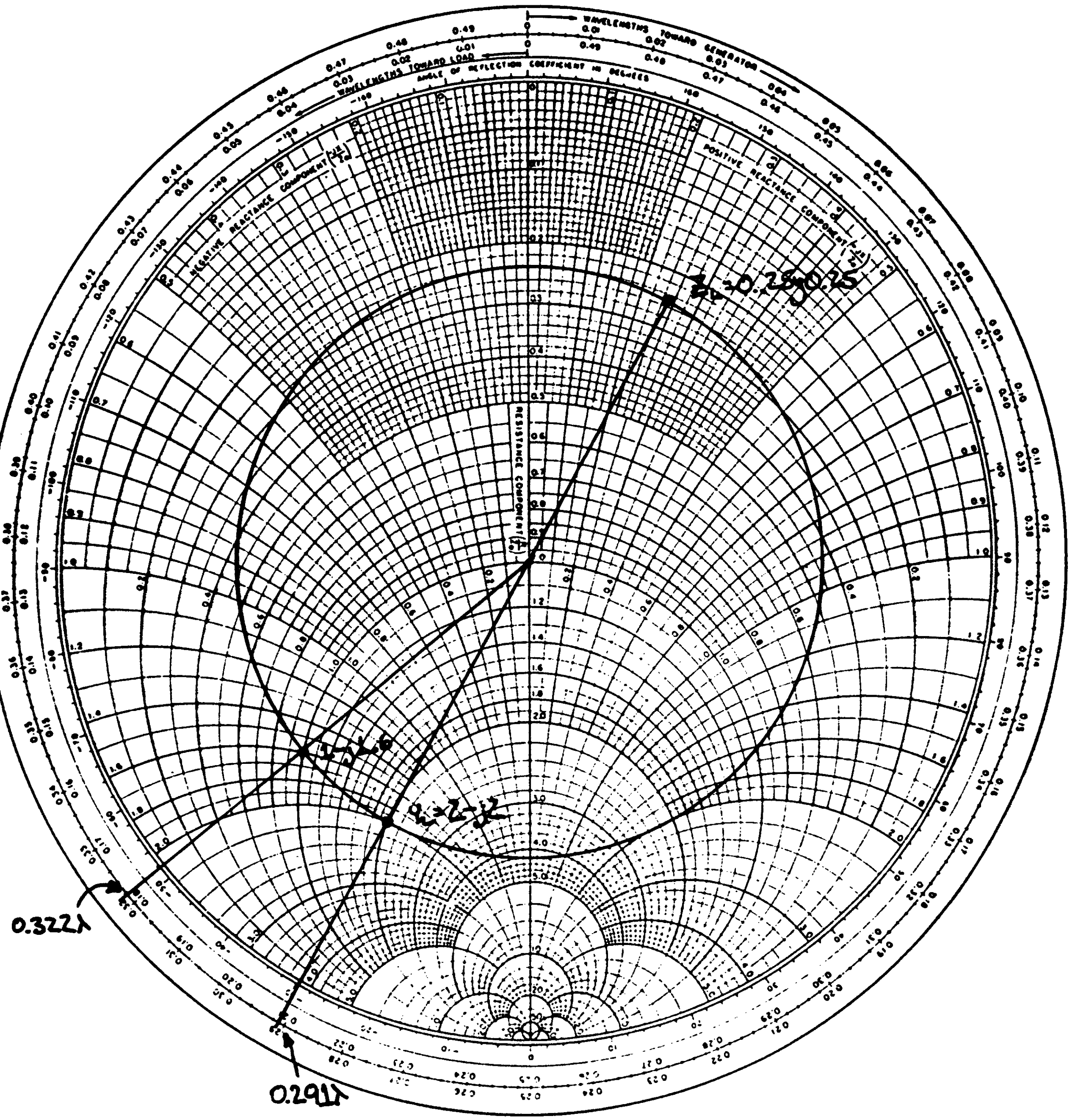
$0.179\lambda - 0.041\lambda = 0.138\lambda$ IS LOCATION FOR X_{COMP}.

$X_{COMP} = -1.6\,\Omega$, SO $X_C = Z_0 X_{COMP} = -160\,\Omega$, CAPACITIVE.

Appendix C

The Smith Chart

EXAMPLE #2: SHUNT



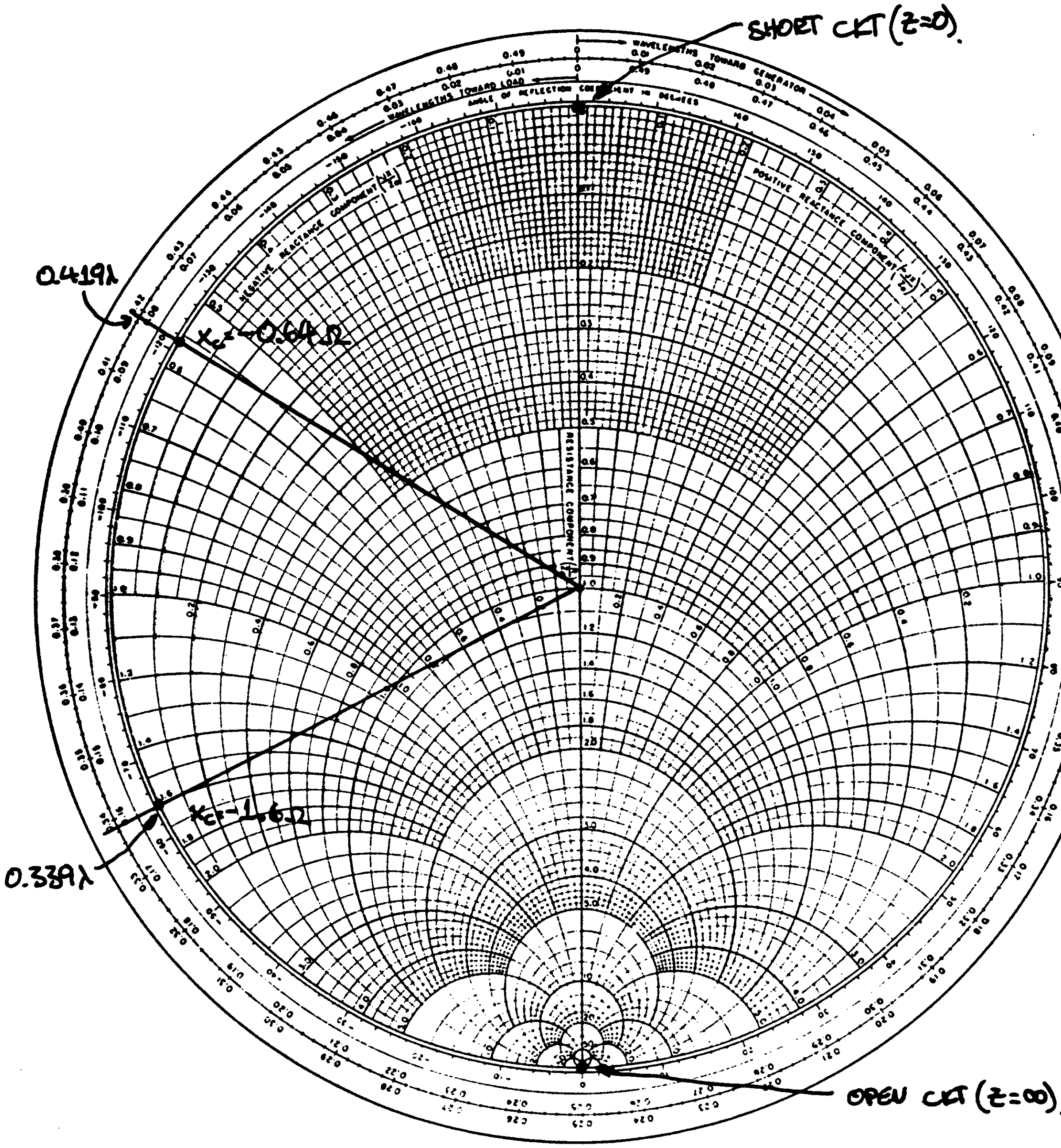
0.322λ−0.291λ=0.031λ IS LOCATION FOR B_{comp}.

B_{comp} = 1.6, SO $B_C = Y_0 B_{comp}$ = .016S, CAPACITIVE.

Appendix C

The Smith Chart

EXAMPLE #3: SERIES



SHORTED LINE: $l = 0.339\lambda$

OPEN LINE: $l = 0.339\lambda - 0.25\lambda = 0.089\lambda$

$\Big\}\ Z_0 = 100\Omega$

SHORTED LINE: $l = 0.419\lambda$

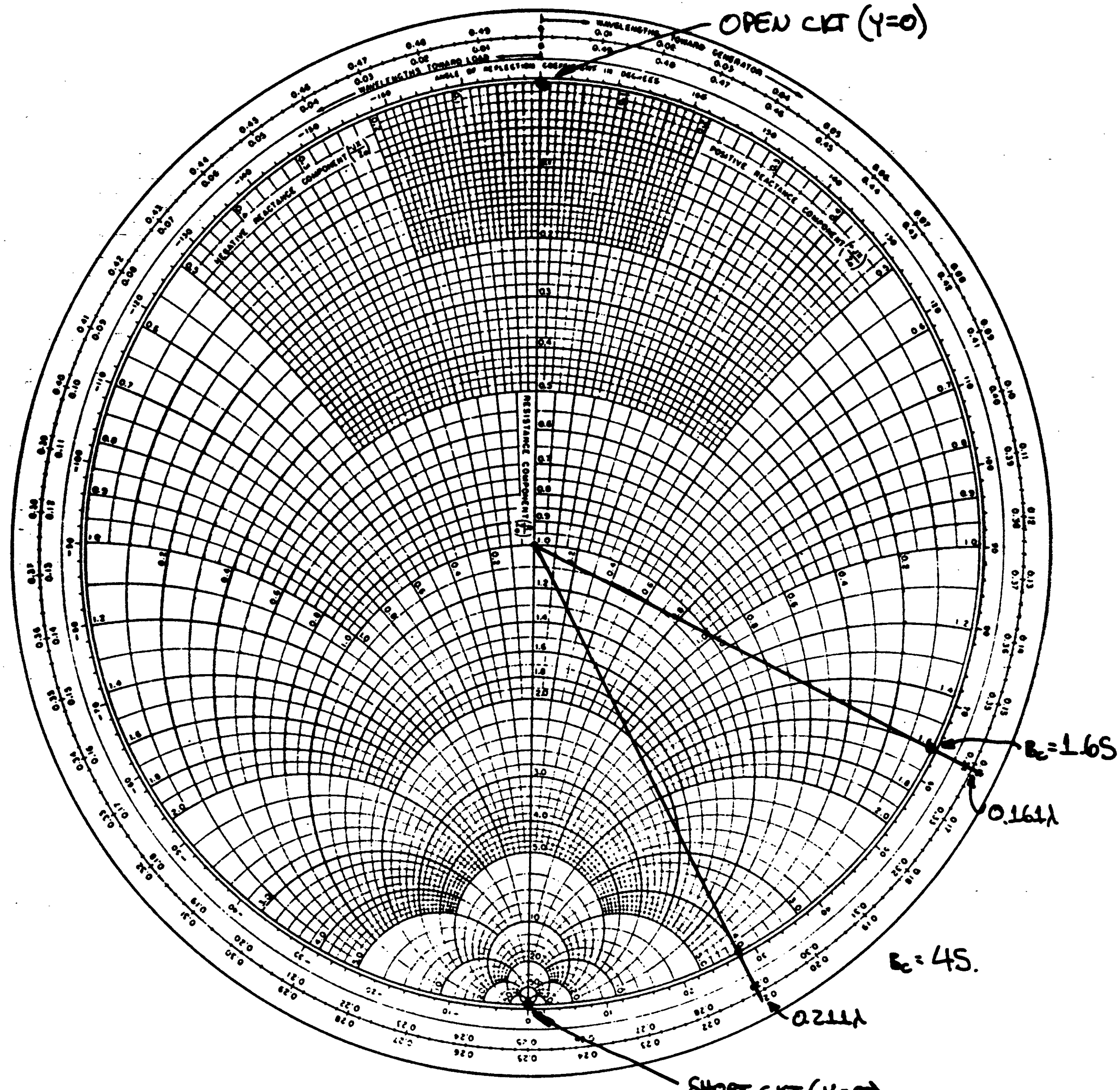
OPEN LINE: $l = 0.419\lambda - 0.25\lambda = 0.169\lambda$

$\Big\}\ Z_0 = 250\Omega$

Appendix C

The Smith Chart

EXAMPLE #3: SHUNT

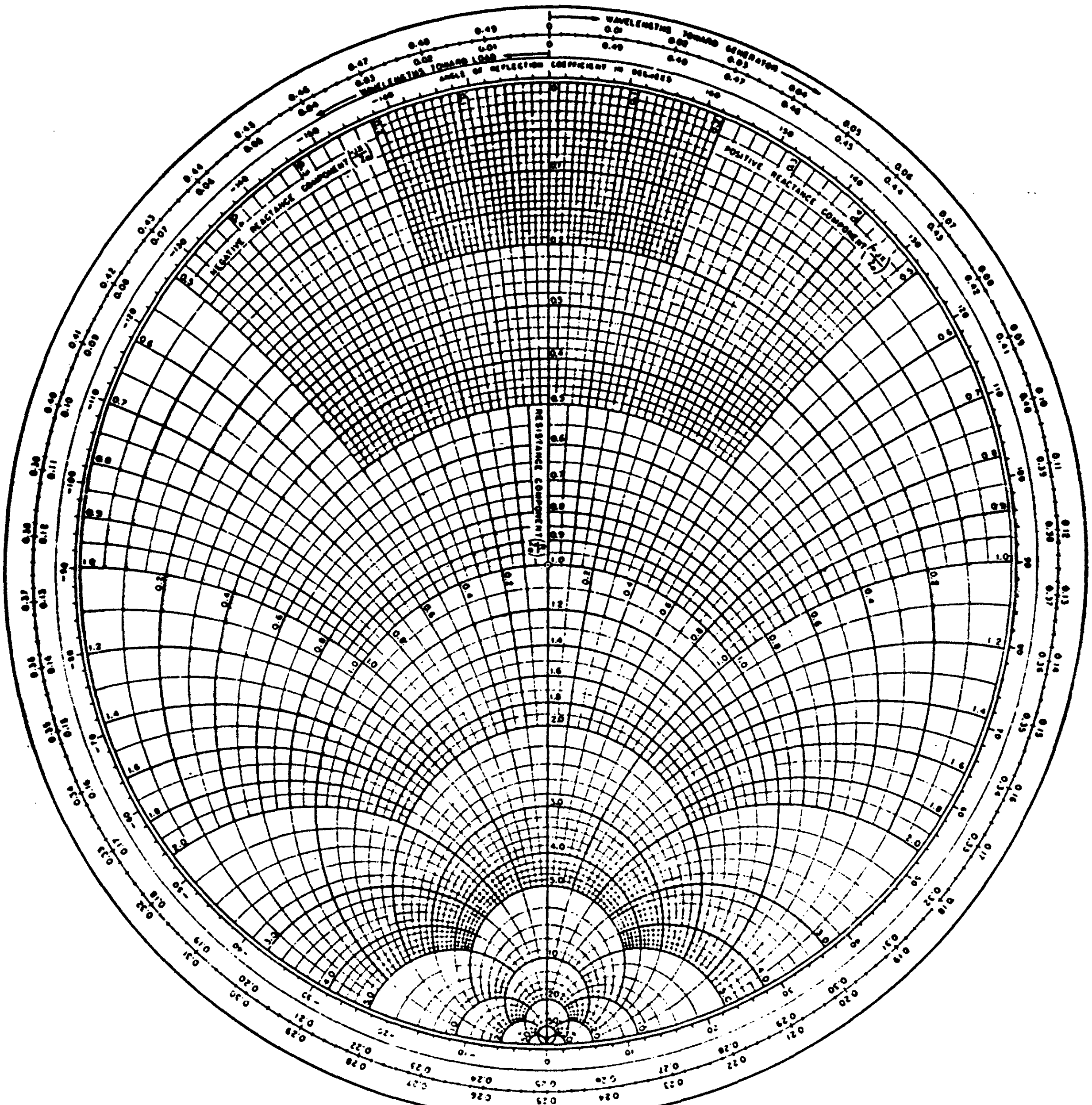


OPEN LINE: $l = 0.161\lambda$
SHORTED LINE: $l = 0.161\lambda + 0.25\lambda = 0.411\lambda$
} $Y_0 = 0.01S.$

OPEN LINE: $l = 0.211\lambda$
SHORTED LINE: $l = 0.211\lambda + 0.25\lambda = 0.461\lambda$
} $Y_0 = 0.004S.$ $(Z_0 = 250\Omega)$

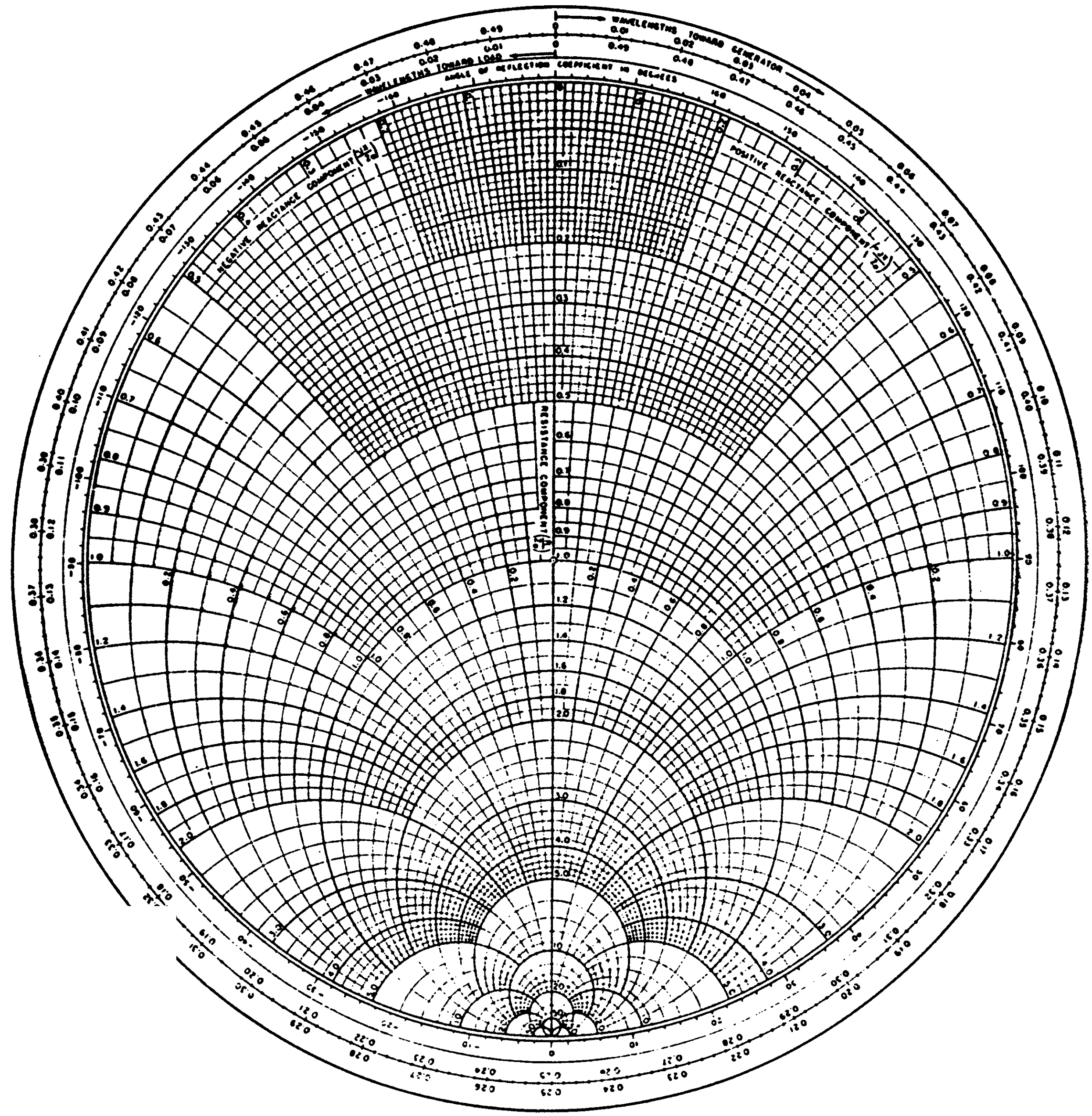
Appendix C

The Smith Chart



Appendix C

The Smith Chart



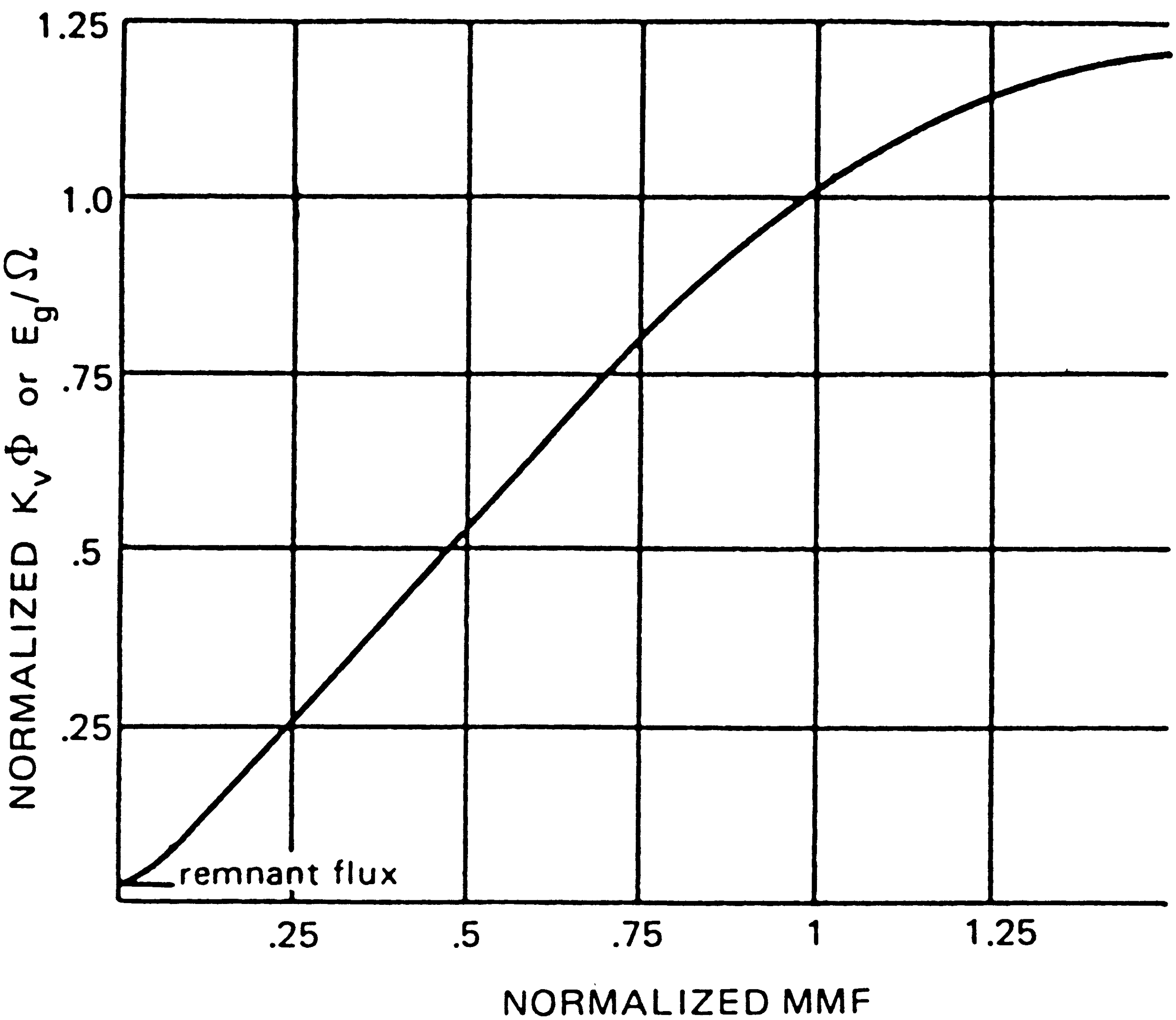
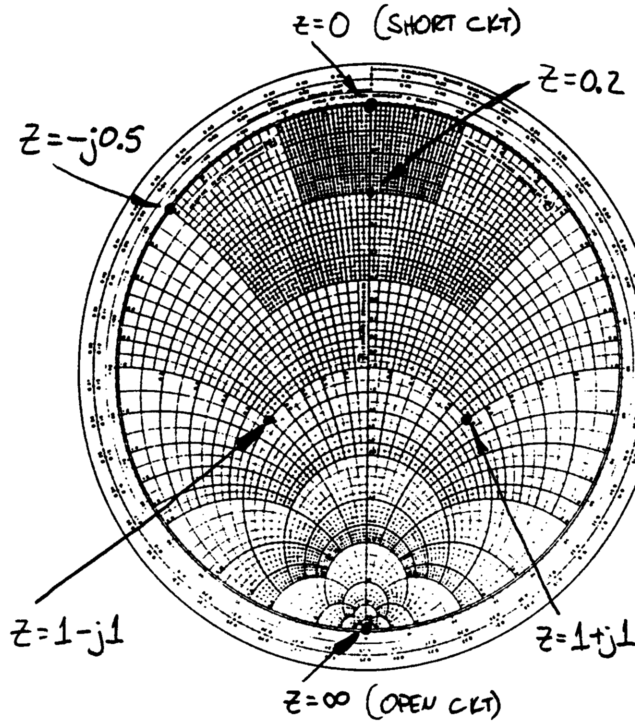


Figure 7.2 Saturation curve

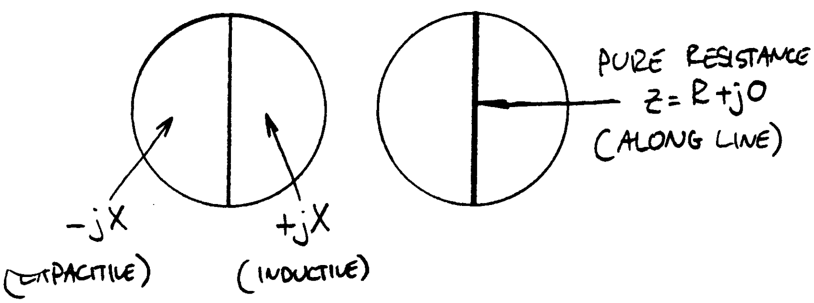
PE REVIEW- TRANSMISSION LINES SECTION
SUPPLEMENTARY NOTES: THE SMITH CHART

THE SMITH CHART IS A GRAPH OF COMPLEX IMPEDANCES OR ADMITTANCES, ARRANGED TO PERMIT RAPID SOLUTION OF WAVE DEPENDENT CIRCUIT PROBLEMS.

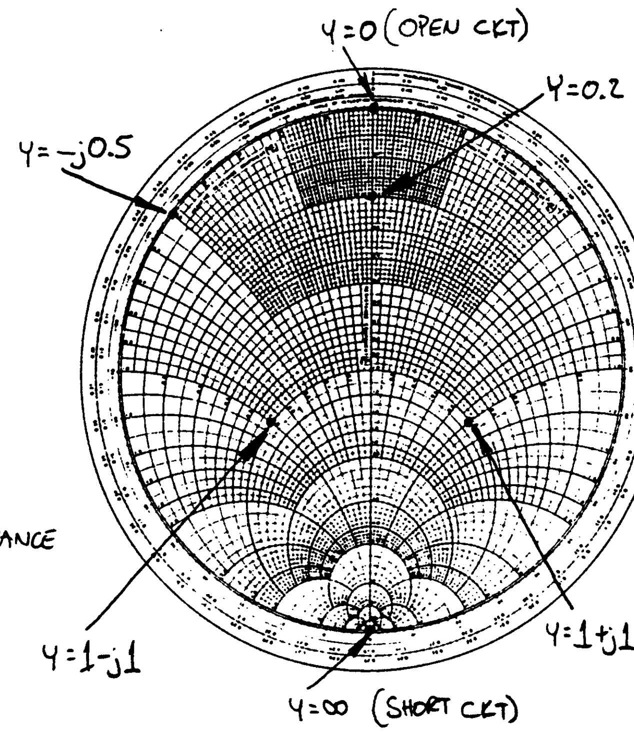
IMPEDANCE CHART



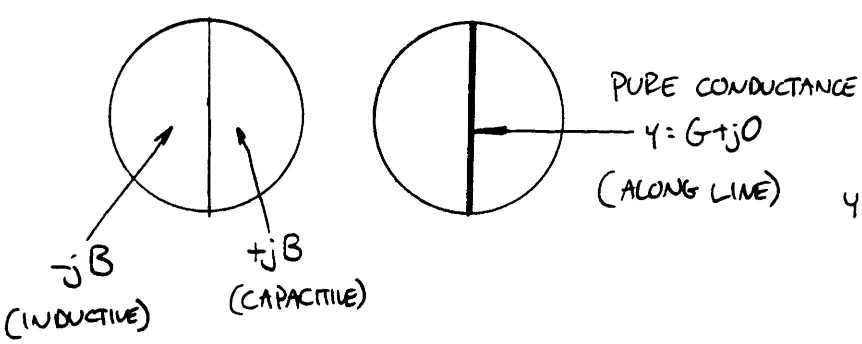
THE CHART IS NORMALIZED WITH RESPECT TO THE LINE IMPEDANCE, Z_0, SO THAT THE CENTER $z=1$.



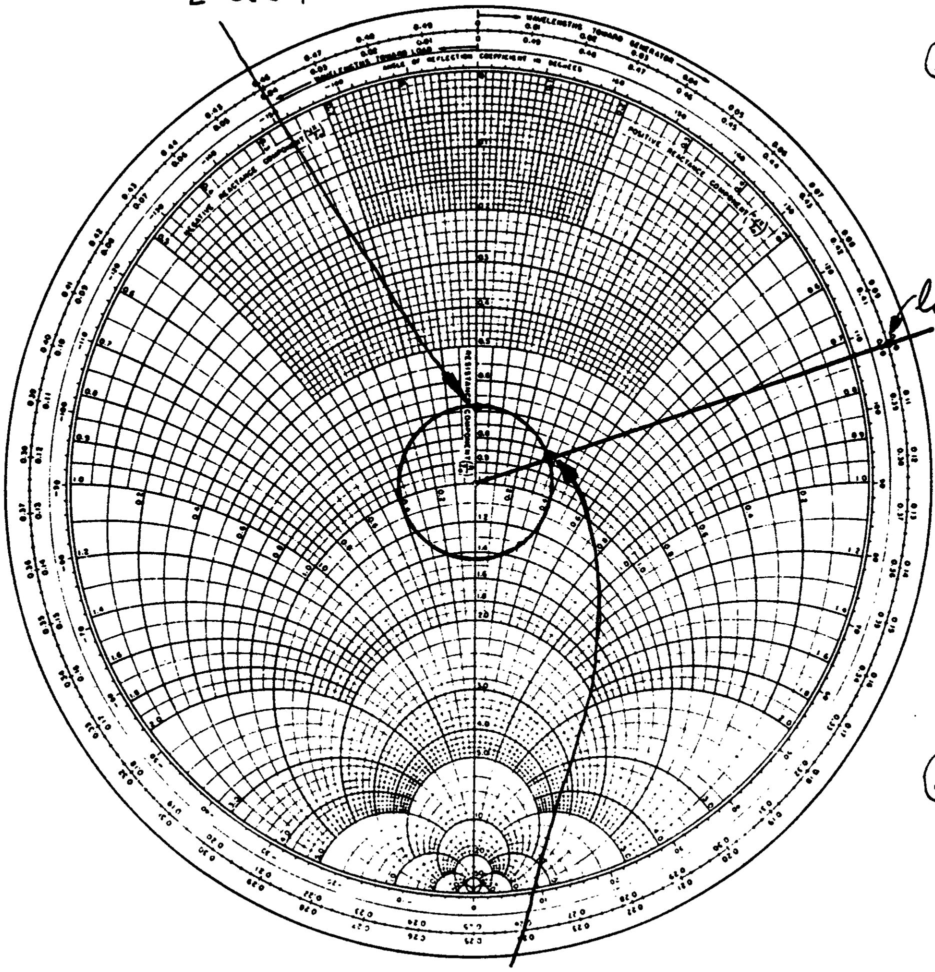
ADMITTANCE CHART



THE CHART IS NORMALIZED WITH RESPECT TO THE LINE ADMITTANCE, Y_0, SO THAT THE CENTER $y=1$.



EXAMPLE #1: TRANSMISSION LINE $Z_o = 72\,\Omega$, $\beta = .5\ \frac{RAD}{m}$, $l = 7.5\,m$.
THE LINE IS TERMINATED IN LOAD $Z_L = 50\,\Omega$.
WHAT IS THE INPUT IMPEDANCE OF THE LINE?



① NORMALIZE Z_L,

$$z_L = \frac{Z_L}{Z_o} = \frac{50}{72} = .69$$

PLOT z_L.

② DRAW CIRCLE OF CONSTANT $|\rho|$ THROUGH z_L.

③ ELECTRICAL LENGTH OF LINE

$$l_e = \frac{\beta}{2\pi} l = \frac{.5}{2\pi} 7.5$$

$$l_e = 0.6\lambda.$$

④ START AT TOP OF CHART, TRAVEL 0.6λ TOWARD GENERATOR. EACH $360°$ TRIP IS $.5\lambda$.

⑤ DRAW A LINE FROM THE CENTER THROUGH $l_e = .6\lambda$.

⑥ READ THE NORMALIZED INPUT z_i FROM THE INTERSECTION.

$z_i = 0.83 + j0.32.$

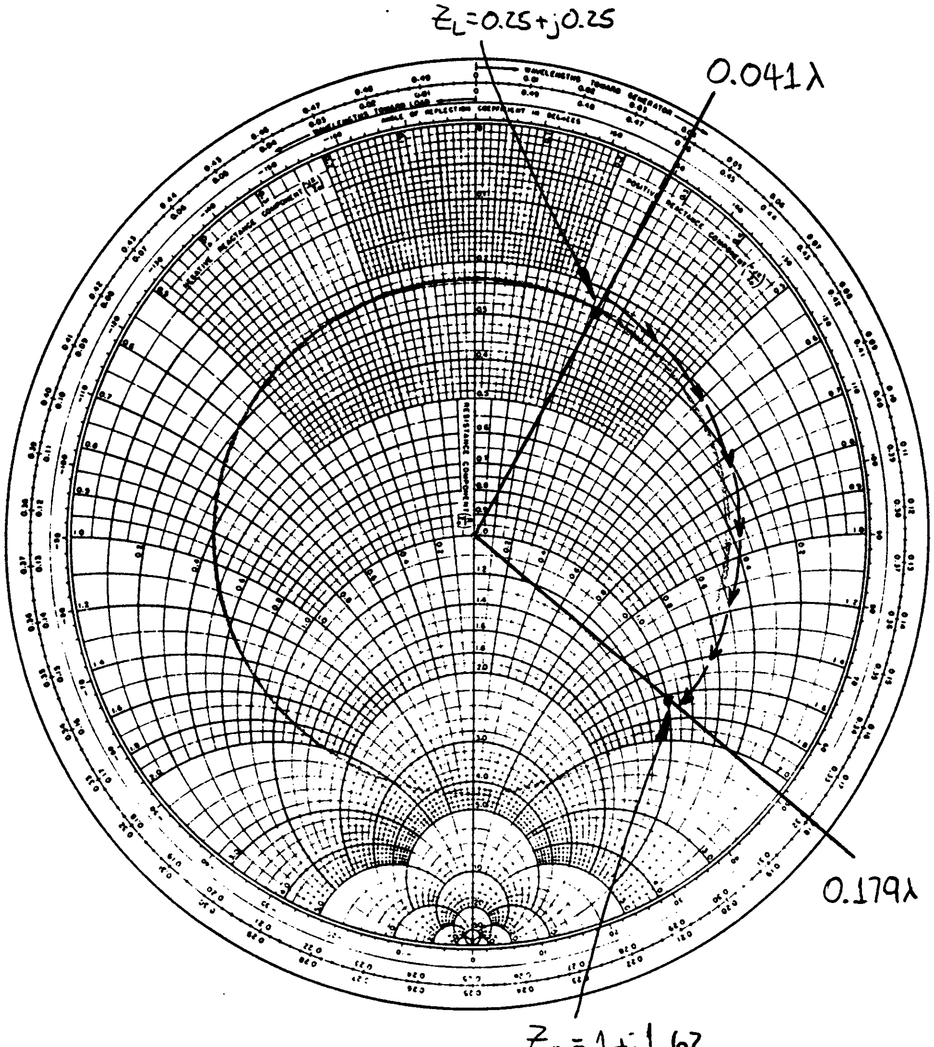
⑦ DIMENSION THE ANSWER

$$Z_i = z_i Z_o$$

$$Z_i = 0.83 + j0.32\,(72\,\Omega)$$

$$Z_i = 60 + j23\,\Omega.$$

EXAMPLE #2: TRANSMISSION LINE $Z_o = 100\Omega$ FEEDS LOAD $Z_L = 25+j25\Omega$. FIND THE COMPONENT VALUES AND LOCATIONS FOR SERIES CONJUGATE MATCHING.



① NORMALIZE

$$z_L = \frac{Z_L}{Z_o} = 0.25+j0.25,$$

AND PLOT.

② PLOT CIRCLE OF CONSTANT $|\rho|$, DRAW LINE FROM CENTER THROUGH z_L, AND READ WAVELENGTH, 0.041λ.

③ FOLLOW ALONG $|\rho|$ CIRCLE, IN DIRECTION TO GENERATOR UNTIL $z_m = 1+jX$. READ $z_m = 1+j1.62$.

FOR CONJUGATE MATCH

$$1 = z_m + z_c$$

$$z_c = 1 - z_m = -j1.62.$$

④ DIMENSION IT

$$Z_c = z_c Z_o = -j\,162\,\Omega,$$

OR SINCE $X_c = \frac{1}{2\pi f C}$, THE CAPACITOR

$$C = \frac{1}{2\pi f(162)}.$$

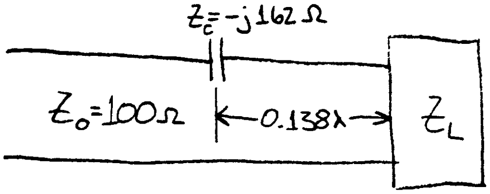
⑤ DRAW A LINE FROM CENTER THROUGH z_m. READ WAVELENGTH, 0.179λ.

COMPENSATOR LOCATION

$$\ell_c = 0.179\lambda - 0.041\lambda$$

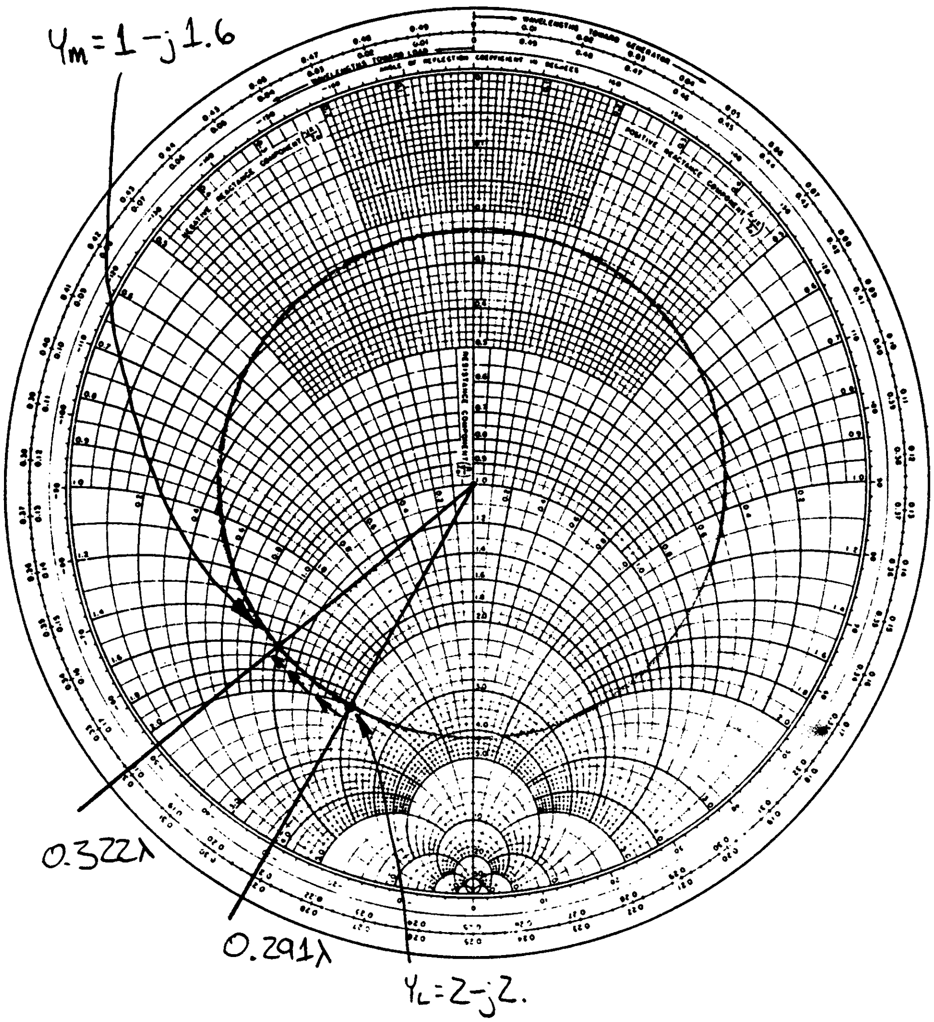
$$\ell_c = 0.138\lambda.$$

FINAL CKT:



EXAMPLE #3: SAME CKT VALUES AS EXAMPLE #2, BUT USE SHUNT CONJUGATE MATCHING.

NOW $Y_o = \frac{1}{Z_o} = .01S$, $Y_L = \frac{1}{Z_L} = .02 - j.02\,S$.



① NORMALIZE

$y_L = \frac{Y_L}{Y_o} = 2 - j2$,

PLOT, AND DRAW |ρ| CIRCLE.

② DRAW LINE, READ WAVELENGTH 0.291 λ.

③ MOVE ALONG |ρ| UNTIL

$y_m = 1 + jB$.

READ

$y_m = 1 - j1.6$

FOR CONJUGATE MATCH,

$1 = y_m + y_c$,

$y_c = j1.6$.

④ DIMENSION IT

$Y_c = Y_o y_c = j0.016S$.

SINCE $B_c = 2\pi f C$,

$C = \frac{.016}{2\pi f}$

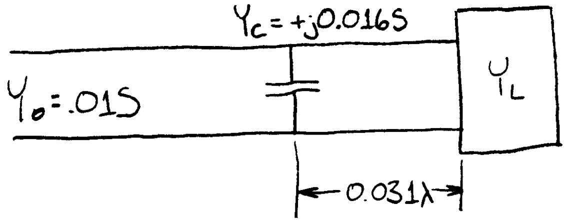
⑤ DRAW LINE FROM CENTER THROUGH y_m, READ WAVELENGTH 0.322λ.

COMPENSATOR LOCATION

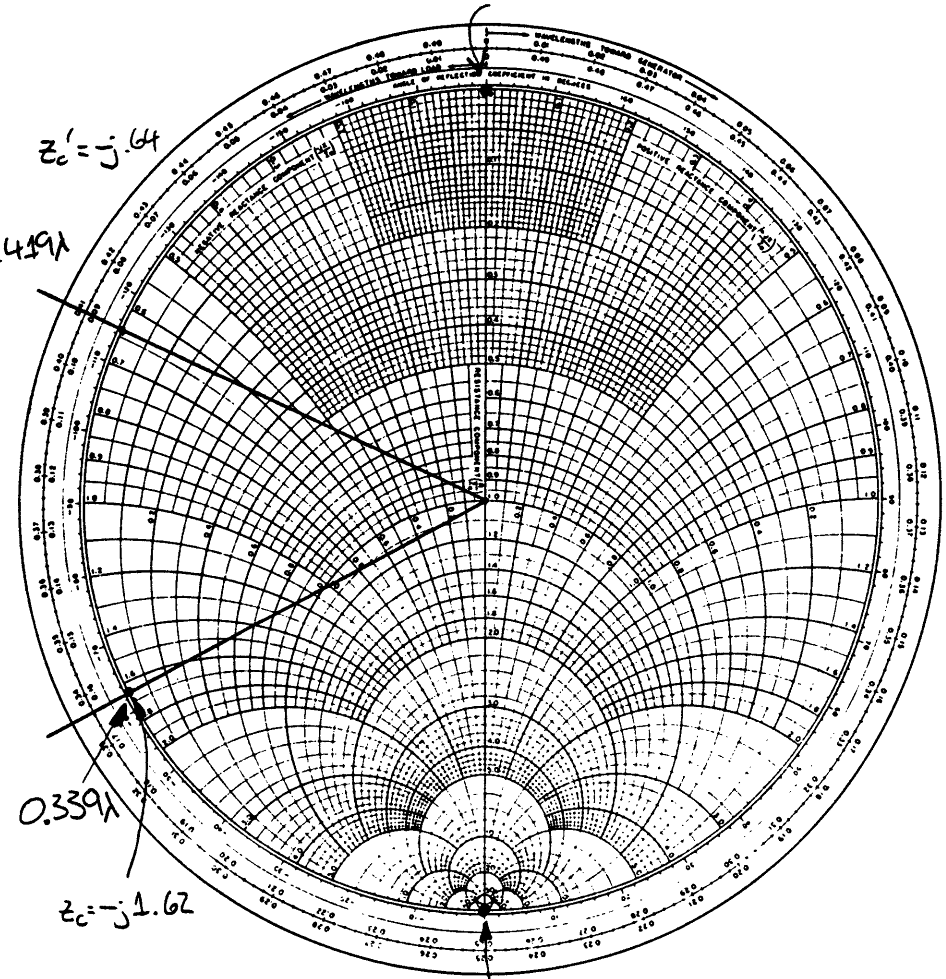
$\ell_c = 0.322\lambda - 0.291\lambda$.

$\ell_c = 0.031\lambda$.

FINAL CKT:



EXAMPLE #4: IN EXAMPLE #2, WE NEEDED A SERIES COMPENSATON ELEMENT $Z_c = -j162\,\Omega$. DESIGN A SUITABLE COMPENSATOR FROM TRANSMISSION LINE STUBS. USE BOTH OPEN & SHORTED STUBS, WITH $Z_0 = 100\,\Omega$ AND $Z_0 = 250\,\Omega$ FOR THE STUB LINE.



① $Z_0 = 100\,\Omega$.

NORMALIZE

$$z_c = \frac{Z_c}{Z_0} = -j1.62.$$

PLOT z_c, DRAW LINE & READ WAVELENGTH 0.339λ.

② SHORTED STUB LENGTH JUST 0.339λ.

OPEN STUB LENGTH IS DISTANCE FROM $z=\infty$ (BOTTOM) TO 0.339λ, OR 0.339λ − 0.25λ = 0.089λ.

③ $Z_0 = 250\,\Omega$.

NORMALIZE,

$$z_c' = \frac{Z_c}{250\,\Omega} = -j.64,$$

PLOT, DRAW, READ 0.419λ.

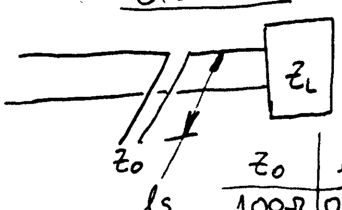
④ AS ABOVE, JUST READ LENGTHS CLOCKWISE

OPEN: 0.419λ − .25λ = 0.169λ

SHORT: 0.419λ.

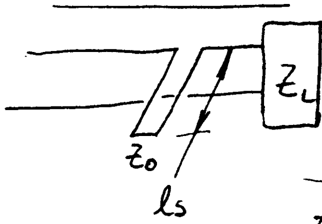
FINAL CKT:

OPEN STUB



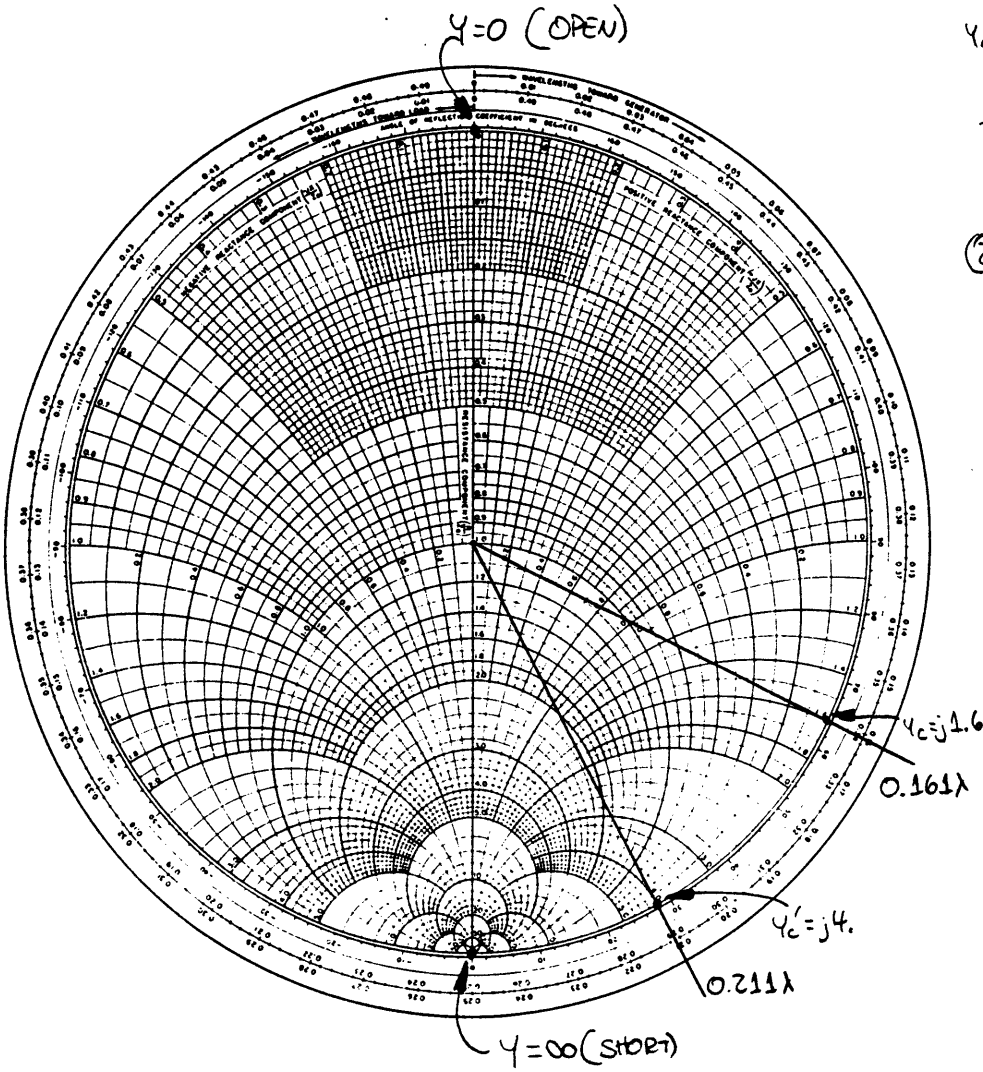
Z_0	l_s
100Ω	0.089λ
250Ω	0.169λ

SHORTED STUB



Z_0	l_s
100Ω	0.339λ
250Ω	0.419λ

EXAMPLE #5: IN EXAMPLE #3 WE NEEDED A SHUNT COMPENSATOR $Y_c = j0.016S$. DESIGN A SUITABLE SHUNT COMPENSATOR FROM OPEN & SHORTED STUBS, $Z_0 = 100\Omega$ AND $Z_0 = 250\Omega$.



① $Z_0 = 100\Omega$, $Y_0 = .01S$.

NORMALIZE,

$$y_c = \frac{Y_c}{Y_0} = j1.6.$$

PLOT, DRAW, READ 0.161λ.

② OPEN STUB = 0.161λ.
SHORTED STUB = 0.161λ + .25λ
= 0.411λ.

③ $Z_0 = 250\Omega$, $Y_0 = 0.004S$.

NORMALIZE

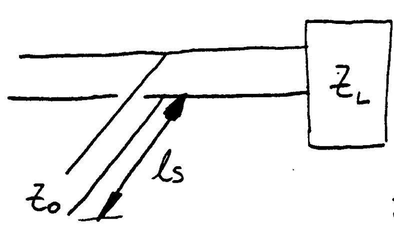
$$y_c' = \frac{Y_c}{.004S} = j4.$$

PLOT, DRAW, READ 0.211λ.

④ OPEN STUB = 0.211λ
SHORT STUB = 0.211λ + .25λ
= 0.461λ.

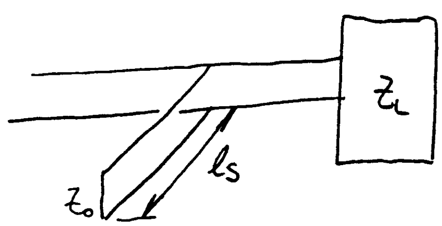
FINAL CKT

OPEN STUB



Z_0	l_s
100Ω	0.161λ
250Ω	0.211λ

SHORTED STUB



Z_0	l_s
100Ω	0.411λ
250Ω	0.461λ

WARMUP 1.

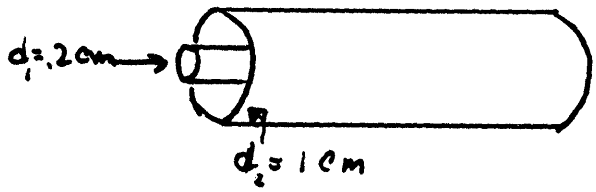
Eq. 6.4 $\delta = \sqrt{\rho/(\pi f \mu)}$

$\rho = 1.8 \times 10^{-8}$ m

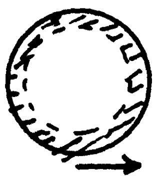
$f = 10^{7}$ Hz

$\mu = \mu_0 = 4\pi \times 10^{-7}$

~~so~~ $\delta = 21.35 \times 10^{-6}$ m



INNER CONDUCTOR



Conducting area $\sim 2\pi r \times \delta$ in meters^2

Approximate width by $2\pi r$

$r = 0.1$ cm

$\delta = 0.0021$ cm

OUTER CONDUCTOR

Conducting area $2\pi R \times \delta$

From eq. 6.5 the impedance per unit length per unit width.

$Z = Z_i + Z_o$

$$= \frac{\rho}{\delta 2\pi r}(1+j) + \frac{\rho}{\delta\, 2\pi R}(1+j)$$

$$= \frac{1.8 \times 10^{-7} \times 100}{2\pi \times 21.35 \times 10^{-6}}\left[\frac{1}{0.1} + \frac{1}{0.5}\right][1+j]$$

$Z = 0.161(1+j)$

WARMUP 2.

From appendix A, AWG #1 wire has a diameter of 289 mils (1 mil = 0.001 inch).

From example 6.1 it was shown that the skin-depth for copper at 10kHz is 26 mils. then the ratio: $\frac{R}{\delta} = \frac{289/2}{26} = 5.56$

Using equation 6.8:

$$\frac{R}{R_o} = 0.25 + 0.5\frac{r}{\delta} = 3.03$$

Using equation 6.9;

$$\frac{X}{R_o} = 0.5025\frac{r}{\delta} = 2.79$$

From exercise 6.3:

$R_o = 0.124\ \Omega/1000$ ft

$Z_L = 0.124[3.03 + j2.79]$

$= 0.376 + j0.346$ ohms/1000 ft.

WARMUP 3.

From Appendix A $r = 16$ mils

From example 6.2 $\delta = 2.6$ mils

From Appendix A $R_o = 10.15 \frac{\Omega}{1000\text{ ft}}$

$\frac{r}{\delta} = 6.15$ and from eq. 6.9:

$$\frac{X_i}{R_o} = 0.5025 \frac{r}{\delta} = 3.09$$

Then, for each conductor

$$X_i = 3.09 \times 10.15 = 31.36 \frac{\Omega}{1000\text{ ft}}$$

so

$$X_{i\,total} = 2X_i = 62.73 \frac{\Omega}{1000\text{ ft.}}$$

$$D/r = \frac{1\text{ cm}}{16\text{ mils}} \times \frac{10^3\text{ mils}}{1\text{ inch}} \times \frac{1\text{ inch}}{2.54\text{ cm}} = 24.61$$

From eq. 6.10

$$L_{ext} = 4 \times 10^{-7} \ln \frac{D}{r} = 1.281 \times 10^{-6} \, H/m$$

$$X_{ext} = 2\pi f L_{ext} = 8.05 \frac{\Omega}{m}$$

$$= 8.05 \frac{\Omega}{m} \times \frac{1\text{ m}}{100\text{ cm}} \times \frac{2.54\text{ cm}}{\text{inch}} \times \frac{12\text{ inch}}{\text{Foot}}$$

$$= 2.454 \frac{\Omega}{ft} = 2{,}454 \frac{\Omega}{1000\text{ ft}}$$

then:

$$X_{TOT} = X_i + X_{ext} = 2{,}517 \frac{\Omega}{1000\text{ ft}}$$

WARMUP 4.

From eq. 6.16

$$C_\ell = \frac{2\pi\epsilon}{\ln D/d}$$, where D is the ID of the outer conductor and d is the diameter of the inner conductor.

$$\epsilon = \epsilon_o = \frac{1}{36\pi \times 10^9}$$

so

$$C_\ell = \frac{\frac{2\pi}{36\pi} \times 10^{-9}}{\ln(2/0.2)} = 24.1 \, pF/m$$

WARMUP 5.

From table 6.2:

$$GMR = 0.0586, \; X_L(1\text{ ft}) = 0.344$$

From eq. 6.19:

$$K_L = 1 + \frac{\ln(D)}{\ln \frac{1}{GMR}} = 1 + \frac{\ln 10}{\ln \frac{1}{GMR}} = 1.812$$

then

$$X_L = 1.812 \times 0.344 = 0.632 \frac{\Omega}{\text{mile}}$$

WARMUP 6.

From table 6.2: $X_C = 0.0776$ MΩ-mi

the correction factor K_C is:

$$K_C = 1 + \frac{\ln(\text{separation in feet})}{\ln \frac{1}{\text{wire radius in feet}}}$$

so

$$K_C = 1 + \frac{\ln 20}{\ln \frac{24}{1.762}} = 2.147$$

$$X_C = 2.147 \times 0.0776 \text{ MΩ-mi/conductor}$$

for 2 miles and 2 conductors:

$$X_C \Rightarrow \frac{2.147 \times 0.0776}{2\text{ cond} \times 2\text{ mi}} = 0.0417\text{ MΩ} = 41.7\text{ KΩ}$$

WARMUP 7.

As Outside Diameter (O.D.) is in inches, and the spacing is 1.0 foot:

$$\ln \frac{d}{r} = \ln \frac{d(\text{ft})}{\frac{\text{O.D.}}{2} \times \frac{1}{12}} = \ln \frac{24\,d(\text{ft})}{\text{O.D.}(\text{in})}$$

Eq. 6.43

$$X_C = 29{,}668 \ln\left(\frac{d}{r}\right) \ \Omega\text{-mi}$$

$$= 29{,}668 \ln \frac{24\,d[\text{ft}]}{\text{O.D.}[\text{in}]}$$

conductor	CALCULATED X_C (MΩ-mi)	TABLE 6.2 X_C (MΩ-mi)
WAXWING	0.1090	0.1090
PARTRIDGE	0.1074	0.1074
BLUEBIRD	0.0775	0.0776

WARMUP 8.

From Chapter 4

let $a \to 1$, $b \to 2$, $c \to 3$

$$Z_1 = \frac{Z_{12} Z_{13}}{Z_{12} + Z_{23} + Z_{13}}$$

$$Z_2 = \frac{Z_{12} Z_{23}}{Z_{12} + Z_{23} + Z_{13}}$$

and

$$Z_3 = \frac{Z_{13} Z_{23}}{Z_{12} + Z_{13} + Z_{23}}$$

so:

$$Z_{an} = \frac{2 \times 1.8}{2 + 2.5 + 1.8} = \frac{3.6}{6.3} = 0.571\ \Omega$$

$$Z_{bn} = \frac{2 \times 2.5}{6.3} = 0.794\ \Omega$$

$$Z_{cn} = \frac{2.5 \times 1.8}{6.3} = 0.714\ \Omega$$

WARMUP 9.

The VOLT-AMPERE ratings are the same.

The voltage rating for the delta connection is $\sqrt{3}$ times that for the WYE CONNECTION.

The current rating for the WYE CONNECTION must be $\sqrt{3}$ times that for the delta connection.

WARMUP 10.

The radiation resistance is obtained from eq 6.142.

$$R_r = \frac{\beta^2 dz^2}{6\pi} \sqrt{\frac{\mu_0}{\epsilon_0}}$$

where $\beta = \frac{\omega}{c} = \frac{\omega_0}{3 \times 10^8}$

WARMUP 10 CONTINUED

with $\sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi$, and

$\omega = 2\pi \times 60 = 120\pi$

$R_e = 3.2 \times 10^{-11}$ ohms

From Appendix A

$R_o = 10.15\Omega/1000$ ft

$= \frac{10.15\Omega}{1000ft} \times \frac{1ft}{12in.} \times \frac{1in}{2.54cm} \times \frac{100cm}{m}$

$= 0.0333$ ohms/meter

$e = \frac{3.2 \times 10^{-11}}{.0333} \times 100$

$= 9.6 \times 10^{-8}$ %

i.e. not very efficient.

CONCENTRATE 1.

From table 6.2: GMR = 0.0217

$d_s = \sqrt[3]{10 \times 10 \times 10\sqrt{2}} = 11.22$ ft.

Eq. 6.33

$L = 2\times10^{-7} \ln \frac{d_s}{GMR} = 1.25\,\mu H/m$

$X_L = 120\pi L = 4.71\times10^{-4}\,\Omega/m$

Eq 6.34

$M_{bc} = 2\times10^{-7} \ln \frac{r_{ab} r_{ac}}{d_s^2}$

then

$M_{ab} = M_{ac} = 2\times10^{-7} \ln \frac{10\times10\sqrt{2}}{d_s^2}$

$= 2.33\times10^{-8}$ H/m

$M_{bc} = 2\times10^{-7} \ln \frac{10\times10}{d_s^2}$

$= -4.60\times10^{-8}$ H/m

as there are 1609 meters per mile:

$X_L = 1609 \times 4.71\times10^{-4} = 0.758\,\Omega/mi$

$X_{ab} = X_{ac} = 1609 \times 120\pi \times M_{ab}$

$= 0.014\,\Omega/mi$

$X_{bc} = 1609 \times 120\pi \times X_{bc}$

$= -0.028\,\Omega/mi$

CONCENTRATE 2.

For a balanced 3-phase load, the WYE-DELTA TRANSFORMATION is a factor of 3. i.e.

$$Z_{l-n} = \frac{Z_{l-l} \times Z_{l-l}}{Z_{l-l} + Z_{l-l} + Z_{l-l}}$$

$$= \frac{Z_{l-l}}{3}$$

Converting to an equivalent Wye configuration:

CONCENTRATE 2. CONTINUED

$$Z_{phase} = 3+j4+\frac{20+j15}{3} = \frac{29+j27}{3}$$

$$P_{Load} = \frac{30KW}{3\text{ phases}} = I_{Line}^2 \times \frac{20}{3}$$

so

$$I_{Line} = \sqrt{1500}$$

Selecting the a-phase current as a reference:

$$V_s = \sqrt{1500}\left[\frac{29+j23}{3}\right]$$

$$= 511.5 \angle 42.95°$$

$$S_{phase} = V_s I_{LINE} = (14.5+j13.5) KVA$$

(a) $V_S = 511.5$ V. l-line
$P = 14.5 KW/phase$
$Q = 13.5 KVAR/phase$

or $V_s = 511.5 \times \sqrt{3} = 886\ V\ l\text{-}l$
$P = 43.5 KW$
$Q = 40.5 KVAR$

(b) Compensation is by means of a parallel capacitance, so the load impedance is converted to admittance

$$Y_L = \frac{3}{20+j15} = \frac{60-j45}{625}$$

Compensation requires a parallel capacitor with

$$B_C = \frac{45}{625} \text{ or } X_C = -13.89\ \Omega$$

the compensated load impedance is then:

$$Z_{L\text{-}comp} = \frac{625}{60} = 10.42\ \Omega$$

Assuming the load voltage remains the same, the previous value is calculated

$$V_L\Big|_{uncomp} = \sqrt{1500}\left(\frac{20+j15}{3}\right)$$

$$= 322.7\angle 36.87°$$

The current is then

$$i_{LINE} = \frac{322.7}{10.42} = 30.97\ A$$

$$V_s = 30.97\left[13.42+j4\right]$$

$$= 433.7\angle 16.60°\ l\text{-}n$$

$P_{phase} = 12.87 KW$
$Q_{phase} = 3.84 KVAR$

or

$$V_{l\text{-}l} = \sqrt{3} \times 433.7 = 751\ V\ l\text{-}l$$

$P = 38.6 KW$
$Q = 11.5 KVAR$

CONCENTRATE 3.

From table 6.2 the resistance per mile is 0.1943 ohms at 20°C and 0.2134 ohms at 50°C. As example 6.9 found a current of 225 A, which is a current density of about 370 A/in², so the temperature should rise, and so the 50°C value of 0.2134 ohms is used.

From table 6.2 the inductive reactance at 1-ft spacing is 0.432 ohms/mile. With GMR = 0.0284, the correction factor is:

$$K_L = 1 + \frac{\ln 7}{\ln \frac{1}{0.0284}} = 1.5464$$

then

$$X_L = 1.5464 \times 0.432 \times 25 \text{ mi}$$
$$= 16.70 \ \Omega/\text{mile}$$

the line impedance is then

$$Z_l = 5.36 + j16.70 \ \Omega$$

The unknown voltage at the receiving end is taken as the reference. The current then leads this voltage by 25.8° ($\cos^{-1} 0.9$). Then:

$$V_S = V_L + 225\angle 25.84 [Z_l]$$
$$= V_L + 3.946\angle 98.01° \text{ [KV]}$$
$$V_S = V_L - 0.550 + j3.908 \text{ [KV]}$$

but

$$V_S = 11\cos\theta + j11\sin\theta \text{ [KV]}$$

and $V_L = V_L + j0$

then $11\sin\theta = 3.908$

$$\sin\theta = 0.355$$
$$\theta = 20.81°$$

$$V_L = 11\cos 20.81° + 0.550$$
$$= 10.83 \text{ KV}$$
$$V_S = 11\cos 20.81° + j11\sin 20.81°$$
$$= 11 \text{ KV}$$

So regulation:

$$\frac{V_{NL} - V_{FL}}{V_{FL}} \times 100 = \frac{11 - 10.83}{10.83} \times 100$$

Regulation = 1.57%

CONCENTRATE 4.

1-ft spacing

O.D. inches	R/mi 60 Hz 50°C	GMR feet	X_L Ω/mi	X_C MΩ-mi
1.545	0.0667	0.0523	0.358	0.0814

Table parameters for FALCON (table 6.2)

Correction factors for 20 ft spacing:

$$K_L = 1 + \frac{\ln 20}{\ln \frac{1}{0.0523}} = 2.015$$

$$K_C = 1 + \frac{\ln 20}{\ln \frac{24}{1.545}} = 2.092$$

$$X_L = 0.358 \times 2.015 = 0.7214 \ \Omega/\text{mi}$$

$$X_C = 0.0814 \times 2.092 = 0.1703 \times 10^6 \ \Omega\text{-mi}$$

then

$$Z = 0.0667 + j0.7214 \ \Omega/\text{mi}$$

$$Y = j\frac{1}{0.1703 \times 10^6} = j5.8724 \times 10^{-6} \ \text{mho/mi}$$

These parameters are converted to a per meter basis:

There are 1609.3 meters/mile

$$Z = 41.45 \times 10^{-6} + j448.3 \times 10^{-6}$$

$$= 450.2 \times 10^{-6} \angle 84.72° \ \Omega/\text{m}$$

$$Y = j3.649 \times 10^{-9} \ \text{mhos/m}$$

Then:

$$YZ = 1.643 \times 10^{-12} \angle 174.7° \ /\text{m}^2$$

$$\sqrt{YZ} = 1.282 \angle 87.36 = \gamma$$

(a) $\gamma = 59.06 \times 10^{-9} + j1.280 \times 10^{-6}$

(b) $\gamma = \alpha + j\beta$

$$\alpha = 59.06 \times 10^{-9} \ [\text{meter}]^{-1}$$

(c) $\beta = 1.280 \times 10^{-6} \ [\text{meter}]^{-1}$

(d) $$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{450.2 \times 10^{-6} \angle 84.72°}{3.649 \times 10^{-9} \angle 90°}}$$

$$= 351.2 \angle \tfrac{1}{2}(84.72 - 90)$$

$$= 351.2 \angle -5.28°$$

$$= 349.7 - j32.3$$

(e) $$\rho = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-249.7 + j32.3}{449.7 - j32.3}$$

$$= -0.558 \angle -3.26°$$

$$= 0.558 \angle 176.7°$$

CONCENTRATE 5.

$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{70-50}{70+50} = \frac{1}{6} \quad \text{(a)}$$

(b) $$Z_i = Z_0 \frac{Z_L \cos\beta l + j Z_0 \sin\beta l}{Z_0 \cos\beta l + j Z_L \sin\beta l}$$

$$= 50 \frac{70 \cos\frac{\pi}{2} + j 50 \sin\frac{\pi}{2}}{50 \cos\frac{\pi}{2} + j 70 \sin\frac{\pi}{2}} = 35.71\ \Omega$$

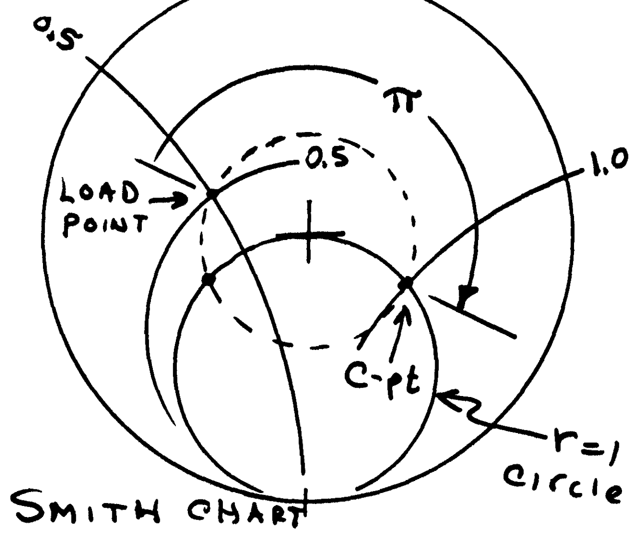
(c)
$$\text{VSWR} = \frac{1+|\rho|}{1-|\rho|} = \frac{1+\frac{1}{6}}{1-\frac{1}{6}} = 1.40$$

CONCENTRATE 6.

$$\frac{Z_L}{Z_0} = \frac{25 - j25}{50} = \frac{1}{2} - j\frac{1}{2}$$

The load point is shown on the smith chart below.

Impedance loci is shown as a dashed circle.



The compensation point for series capacitance is where the impedance loci intersects the r=1 circle on the right-hand side of the chart.

For this particular load the compensation point is 180° (on the chart) from the load point, at

$z_i = 1 + j1$

180° on the smith chart is $\lambda/4$ or $\pi/2$ radians for βl.

$$\frac{Z_i}{Z_0} = \frac{\frac{Z_L}{Z_0}\cos\beta l + j \sin\beta l}{\cos\beta l + j\frac{Z_L}{Z_0}\sin\beta l}$$

$$= \frac{\frac{1}{2}\cos\beta l + j(\sin\beta l - \frac{1}{2}\cos\beta l)}{\cos\beta l + \frac{1}{2}\sin\beta l + j\frac{1}{2}\sin\beta l}$$

at $\beta l = \pi/2$ $\sin\beta l = 1$, $\cos\beta l = 0$

$$\frac{Z_i}{Z_0} = \frac{j}{\frac{1}{2} + j\frac{1}{2}} = \frac{2}{\frac{1}{j}+1} = 1 + j$$

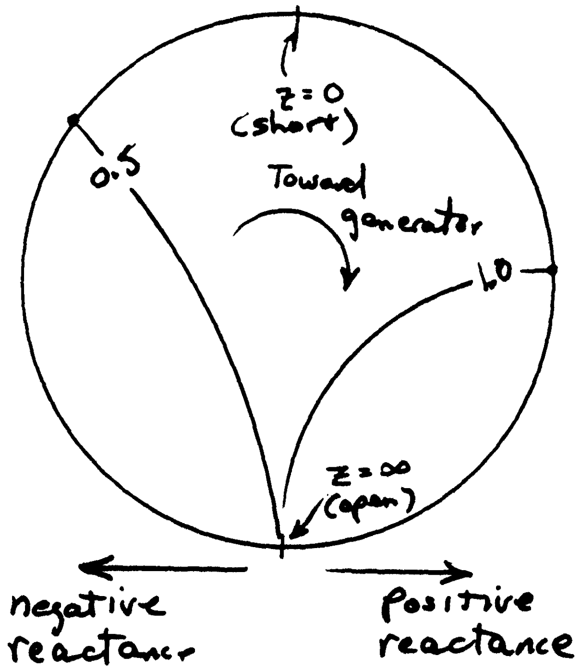
So the compensating reactance must be $-j1$ or

$$X_C = -Z_0 = -50\ \Omega$$

Concentrate 7.

a) $\frac{Z}{Z_0} = \frac{-j25}{50} = -0.5$

The bottom of the chart is open.



The shortest length to obtain $-j.5\times 50$ is to start with an open line. Going clockwise 126.5° to the intersection with $x = 0.5$ on left half of smith chart. This corresponds to $\beta l = \frac{126}{2}\frac{\pi}{180}$ or $\beta l = 1.1$ rad $= 0.175\lambda$.

(b) $\frac{Z}{Z_0} = \frac{j50}{50} = 1$

The shortest line is found for a shorted line (short point at top of the smith chart) going clockwise to intersect the $x = 1$ circle (on the right), which is 90 degrees on the chart, corresponding to $\beta l = 45\times\frac{\pi}{180}$

$\beta l = 0.785$ radians or $\lambda/8$

Concentrate 8.

Assume $c = 3\times 10^8$ m/s

$$\beta = \frac{2\pi f}{c} = \frac{2\pi\times 10^8}{3\times 10^8} = \frac{2\pi}{3} \text{ rad/m}$$

$\beta l = \frac{2\pi}{3}$ rad as $l = 1$ m.

$\tan\beta l = -1.732 = -\sqrt{3}$

Eq. 6.131

$$\frac{Z_L}{Z_0} = \frac{1 - j\,VSWR\tan\beta l}{VSWR - j\tan\beta l}$$

$VSWR = 2$

$$\frac{Z_L}{Z_0} = \frac{1+2\sqrt{3}\,j}{2+j\sqrt{3}} = 1.143 + j0.742$$

$Z_0 = 50\,\Omega$

$Z_L = 57.14 + j\,37.12$

$R = 57.14\,\Omega$

$X = 37.12\,\Omega$

CONCENTRATE 9.

Following eq. 6.157, the radiated power from a half-wave dipole antenna is:

$$P_r = 73.08\, I_m^2$$

The radiation resistance corresponds:

$$P_r = I_m^2 R_r$$

$$R_r = 73.08\,\Omega$$

At a frequency of 100 MHz, the wavelength is 3 m, and $h = \lambda/4 = 0.75$ m

The skin depth for copper at 100 MHz is (see example 6.1) 0.26 mils.

AWG #20 wire has a diameter of 32 mils, and a d.c. resistance of 10.15 Ω/1000 feet, so

$$R_o = \frac{10.15}{1000}\frac{\Omega}{\text{ft}} \times \frac{1\text{ ft}}{12\text{ in}} \times \frac{1\text{ in}}{2.54\text{ cm}}\ \frac{100\text{ cm}}{\text{m}} \times .75\text{ m}$$

$$= 0.025\,\Omega$$

at high frequencies (see eq. 6.8)

$$R = .025\left[.25 + \frac{r}{\delta}\right]$$

$$= 0.025\left[.25 + \frac{16}{0.26}\right] = 15.93\,\Omega$$

The radiation efficiency is

$$e = \frac{R_r}{R_r + R} \times 100 = \frac{73.08 \times 100}{73.08 + 15.93}\ \%$$

$$e = 82.1\%$$

CONCENTRATE 10.

From eq. 6.145:

$$g = e \times g_D$$

e = efficiency = 0.90

g_D = directivity = 20

$$g = 20 \times 0.9 = 18$$

or $g|_{db} = 10 \log_{10} 18 = 12.5$ db

From equation 6.176:

$$\Omega_A = \frac{4\pi}{g_D} = \frac{\pi}{5}$$

TIMED 1.

OD (in)	R_{AC} 60 Hz 50°C	GMR ft	1-ft spacing reactance/conductance X_L Ω/mi	X_C MΩ-mi
0.846	0.2134	0.284	0.432	0.0992

correction factors:

$$K_L = 1 + \frac{\ln 7}{\ln \frac{1}{0.284}} = 1.5464$$

$$K_C = 1 + \frac{\ln 7}{\ln \frac{24}{0.846}} = 1.5817$$

$$X_L = 0.432\, K_L = 0.6680\ \Omega/\text{mi}$$

$$X_C = -0.0992\, K_C = -156.90 \times 10^3\ \Omega\text{-mi}$$

TIMED 1 CONTINUED

$Z=(0.2134+j0.6680)\times 100$

$= 21.34+j66.8 = 70.126\angle 72.28^\circ$

$X_C = -\dfrac{157\times 10^3\ \Omega\text{-mi}}{100\ \text{mi}} = -1.57\ \text{k}\Omega$

$Y = -\dfrac{j}{X_C} = 637\times 10^{-6}\angle 90^\circ$

$YZ = 44.67\times 10^{-3}\angle 162.28^\circ$

EQ 6.59

$V_S = \left(1+\dfrac{YZ}{2}\right)V_R + Z I_R$

EQ 6.60

$I_S = Y\left(1+\dfrac{YZ}{4}\right)V_R + \left(1+\dfrac{YZ}{2}\right)I_R$

$1+\dfrac{YZ}{2} = 0.9787+j0.0068$

$1+\dfrac{YZ}{4} = 0.9894+j0.0034$

$V_{\ell\text{-}n} = V_R = \dfrac{86.6}{\sqrt{3}}\ \text{kV} = 50\ \text{kV}$

$|I_R| = \dfrac{(8500/3)\ \text{kVA}}{50\ \text{kV}} = 56.67\ \text{A}$

$\angle I_R = -\cos^{-1}\dfrac{75}{85} = -28.07^\circ$

$I_R = 56.67\angle -28.07^\circ$

EQ 6.59

$V_S = [.9787\angle 0.4^\circ]50\text{K}$

$+ [70.126\angle 72.28]\,56.67\angle -28.07$

Then: $V_S = 51.88\ \text{kV}\angle 3.44^\circ$

WHEN $I_R = 0$, from eq 6.59:

$51.88\text{K}\angle 3.44^\circ = .9788\angle .4^\circ\ V_R$

or $V_R = 53.00\angle 3.04^\circ$

then:

$\text{Regulation} = \dfrac{53-50}{50}\times 100 = 6.0\%$

TIMED 2.

$Z_0 = 72\,\Omega \quad Z_L = 50\,\Omega \quad \beta = 0.5$

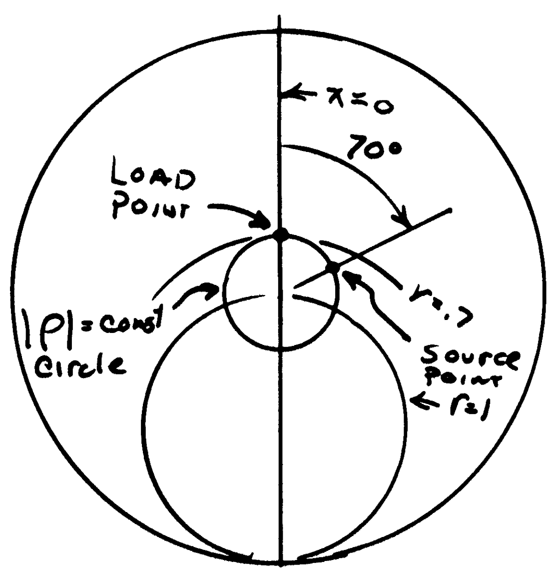
$\ell = 7.5\ \text{m}$

$\dfrac{Z_L}{Z_0} = z_L = \dfrac{50}{72} = 0.6944$

$\beta\ell = 3.75\ \text{rad} = 214.86^\circ$

$2\beta\ell = 429.72^\circ = 360 + 69.72^\circ$

the load point as shown
the source point as shown



TIMED 2 CONTINUED

READING from the enlarged chart of appendix C, the value is $\frac{Z_i}{Z_0} \approx 0.84 + j0.29$

or $Z_i \approx 60 + j21$

Using eq. 6.117

$$\frac{Z_i}{Z_0} = \frac{50 + j\tan 3.75\,(\text{rad})}{1 + j50 \tan 3.75}$$

$$= 0.8358 + j0.2923$$

So

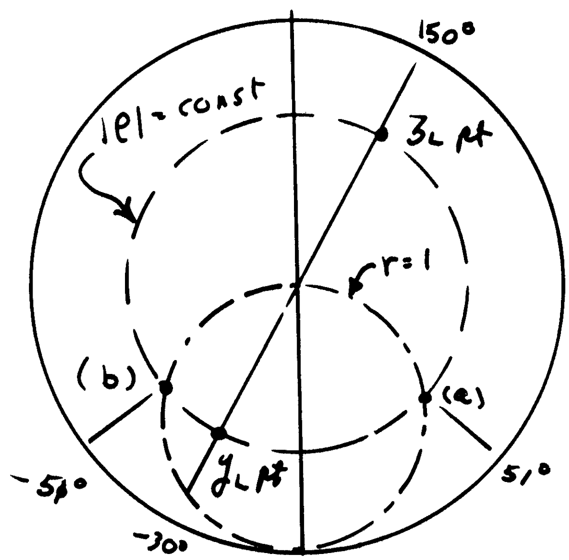
$$Z_L = 60.18 + j21.05$$

TIMED 3.

$Z_0 = 100 \quad Z_L = 25 + j25$

$\therefore z_L = 0.25 + j0.25$

$y_L = 2 - j2$



(a) The z_L pt, $\frac{1}{4} + j\frac{1}{4}$ is at 150°. Following constant $|\rho|$ circle toward generator, the compensation point is at 51° where $z_i = 1 + j1.6$, where series compensation of $-j1.6$ can be added. The line length is $\frac{150° - 51°}{720°}\lambda = 0.1375\lambda$ from the load.

$$Z_C = -j100 \times 1.6 = -j160\,\Omega$$

Using a 72 Ω line for the compensator, its reactance is

$$x_c = -\frac{160}{72} = -2.22$$

STARTING from the open position (bottom of the smith chart), the location of -2.22 is at about $-48°$, for a compensator length of

$$l_{comp} = \frac{48°}{720°}\lambda = \frac{\lambda}{15}$$

Using eq 6.117 with $Z_L = \infty$

$$\frac{Z_i}{Z_0} = \frac{\cos\beta l}{j\sin\beta l} = \frac{-j}{\tan\beta l}$$

So $\tan\beta l = \frac{1}{2.22}$

$$\beta l = \frac{2\pi}{\lambda}l = 0.423$$

$$l = \frac{.423}{2\pi}\lambda = \frac{\lambda}{14.85}$$

TIMED 3. CONTINUED

(b) The y_L point is 180° from the z_L point on the same $|\rho|$ = constant circle.

The parallel compensation point is at $-51°$, so the distance from the load for compensation is at $\frac{51-30}{270}\lambda = 0.0704\lambda$.

at that point

$$y_L = 1 - j1.6$$

$$y = \frac{z_0}{Z} = z_0 Y \quad \text{so}$$

$$Y_{comp} = \frac{y}{z_0} = j\frac{1.6}{100}$$

For the 72Ω line

$$y_{comp} = j\frac{1.6}{100}\times 72 = j1.152$$

1.152 is on the right side of the chart at 82°. The shortest line is from the top of the chart ($y = 0$: open) at 180°

so $l = \frac{180-82}{720}\lambda = 0.136\lambda$

more exactly, using eq 6.117

with $Z_L = \infty$

$$y_C = j\tan\beta l = j1.152$$

so $\beta l = \tan^{-1}1.152 = .8559$

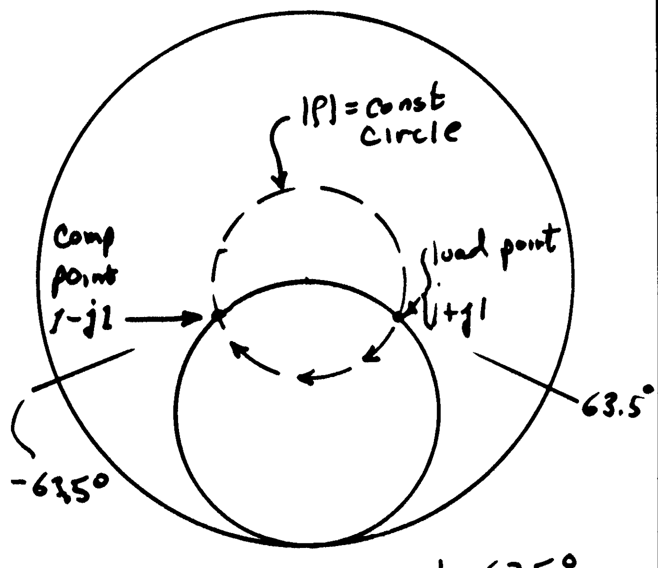
$$l = \frac{.8559\lambda}{2\pi} = 0.1362\lambda$$

TIMED 4.

$$y_i = Yz_0 = \frac{50}{25-j25} = 1+j1$$

This is to be compensated with a shunt capacitance:

$$y_{comp} = jb_c \qquad B_c z_0 = b_c$$



the load point is at 63.5°.
traverse the $|\rho|$ = const circle clockwise to $1-j1$ at $-63.5°$

Distance from load is

$$d = \frac{63.5-(-63.5)}{720}\lambda = 0.1764\lambda$$

$$B_c = b_c/z_0 = \omega C$$

$$\therefore C = 1/50\omega = 0.02/\omega \text{ F}$$

TIMED 5.

The line-neutral voltage is $220/\sqrt{3}$ KV; Load current is

$$i_R = \frac{500\,\text{MW}/(3\times.9)}{220/\sqrt{3}\,\text{KV}} \angle -\cos^{-1} 0.9$$

Take $V_R = V_{BASE}$, $|I_R| = I_{base}$

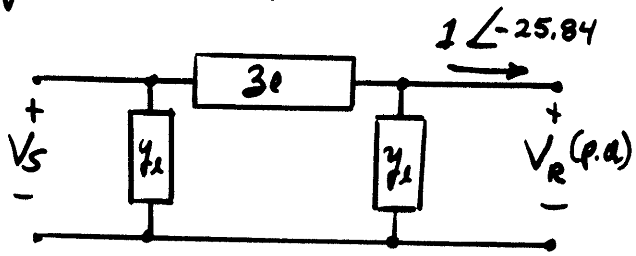
then $$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{(220/\sqrt{3})^2\,K^2}{500/(3\times.9)\,M}$$

$$Z_{base} = 87.12\ \Omega$$

then

$$z_{line} = z_l = \frac{40+j160}{87.12} = 0.4591+j1.8365$$

$$y_l = Y Z_{base} = j0.09261$$



$$V_S = (I_R + y_l V_R) z_l + V_R$$

$$= (1 + y_l z_l) V_R + I_R z_l$$

$$V_S = 1 + y_l z_l + z_l \angle -25.84^\circ$$

because $|V_R| = 1$, $|I_R| = 1$

$$V_S = 0.8299 + j0.04252 + 1.2137 + j1.4528$$

$$V_S = 2.0436 + j1.4953 = 2.53\angle 36.19^\circ \text{ p.u.}$$

Then the line-to-line sending end voltage is:

$$V_S = 220 \times 2.53 = 557 \text{ volts}$$

TIMED 6.

Taking V_R and $|I_R|$ as 1 p.u.,

$$Z_{base} = \frac{(132K)^2}{75M/.92} = 213.73\ \Omega$$

$$Z_{line} = (0.1 + j0.6)100$$

$$z = \frac{Z_{line}}{Z_{base}} = 0.2846\angle 80.54^\circ$$

$$-X_c = \frac{0.15\times 10^6}{100\text{ mi}} = 1.5\times 10^3\ \Omega$$

$$y = j\frac{Z_{base}}{-X_c} = j0.1425$$

$$1 + yz = 0.960 + j0.0067$$

as in timed 5:

$$V_S = 1 + yz + z\angle -\cos^{-1} 0.92$$

$$= 1 + yz + 0.1531 + j0.2399$$

$$= 1.113 + j0.247 \text{ p.u.}$$

$$= 1.14\angle 12.5 \text{ p.u.}$$

the base voltage is 132 KV, so

$$V_S = 132\text{ KV} \times 1.14 = 150.5\text{ KV}$$

TIMED 7.

The power density from an isotropic antenna at a distance r will be:

$$\rho_r = \frac{e_t P_t}{4\pi r^2} \quad \text{(isotropic)}$$

where e_t is the transmitting antenna efficiency.

For a directional antenna with a beam solid angle of Ω_T, the maximum power density will be increased over the isotropic case by the directivity:

$$g_D = \frac{4\pi}{\Omega_T}$$

so:

$$\rho_r = \frac{e_t P_t}{\Omega_T r^2} \quad \text{(anisotropic)}$$

The receiving antenna will have a power P_r:

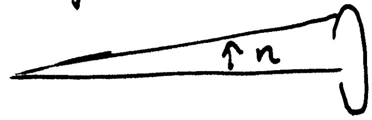
$$P_r = e_r \rho_r A_{eff}$$

where $A_{eff} = \frac{\lambda^2}{4\pi} g_D = \frac{\lambda^2}{\Omega_r}$

thus

$$P_r = \frac{e_r e_t \lambda^2 P_t}{\Omega_T \Omega_r r^2}$$

For a conical beam with half-angle n:



$$\Omega = 2\pi(1 - \cos n)$$

Both antennas have $n = 1.5°$

$$\Omega_T = \Omega_r = 2\pi(1 - \cos 1.5°)$$
$$= 2.153 \times 10^{-3} \text{ steradians}$$

For $f = 10$ gHz $\lambda = \frac{3 \times 10^8}{10 \times 10^9} = 0.030$ m

so for $P_r = 10^{-9}$ mW

$$P_t = \frac{(2.153 \times 10^{-3})^2 (4 \times 10^5)^2 \times 10^{-9}}{e_r e_t (0.03)^2}$$

$$P_t = \frac{0.8241}{e_r e_t} \text{ mW} \approx 1 \text{ mW}$$

TIMED 8.

For a half-wave dipole:

$$P_r = 73.08 I_m^2$$

For an isotropic antenna

$$P = 120\pi I_m^2$$

so

$$g_D = \frac{120}{73.08}$$

For $f = 50$ mHz $\lambda = 6$ m, so the ohmic resistance will be small, and is therefore neglected, which is to assume $e = 1$.

TIMED 8 CONTINUED

then:

$$\rho_{max} = \frac{e g_D P_t}{4\pi r^2} = \frac{E^2}{120\pi}$$

and:

$$r^2 = \frac{120\pi e g_D P_t}{4\pi E^2}$$

$$= 30 \times \frac{120}{73.08} \times 1 \times \frac{100}{(10^{-4})^2}$$

and

$$r = 948.7 \text{ Km}$$

TIMED 9.

$$P_{amp} = \left(\frac{\lambda}{4\pi r}\right)^2 g_{Dr} g_{Dr} P_T$$

$$g_D = \frac{4\pi}{\lambda^2} A_{eff}$$

so

$$P_{amp} = \frac{\lambda^2}{4^2\pi^2 r^2}\left(\frac{4\pi}{\lambda^2}\right)^2 A_{eff}^2 P_T$$

$$= \frac{A_{eff}^2}{r^2\lambda^2} P_T$$

or

$$r = \frac{A_{eff}}{\lambda}\sqrt{\frac{P_T}{P_{amp}}}$$

Assume A_{eff} = 50% of physical aperture of 0.2 m^2

$$\lambda = \frac{3\times 10^8}{3\times 10^9} = 0.1 \quad \text{so} \quad \frac{A_{eff}}{\lambda} = 1$$

$$r = \sqrt{\frac{1}{10^{-9}}} = 31.62 \text{ Km.}$$

TIMED 10.

At the first null of the shorted line we find the half wavelength: $\beta l = \pi$, so

$$\beta = \pi/3 = 60°$$

With the antenna connected to the line, the first minimum occurs at the VSWR condition which is a purely real impedance.

It was given that $R = 73\,\Omega$, and X is small. then:

$$\frac{Z_i}{Z_0} = \frac{\frac{R+jX}{Z_0} + j\tan\frac{1.6\pi}{3}}{1 + j\frac{R+jX}{Z_0}\tan\frac{1.6\pi}{3}}$$

$$\frac{1.6\pi}{3} = 1.6 \times 60 = 96°$$

$\frac{Z_i}{Z_0}$ is a real number, so the angles of the numerator and denominator must be equal:

TIMED 10 CONTINUED

$$\frac{\tan 96° + (X/Z_0)}{(R/Z_0)} = \frac{(R/Z_0)\tan 96°}{1-(X/Z_0)\tan 96°}$$

From which:

$$\left(\frac{X}{Z_0}\right)^2 + \frac{\tan^2 96 - 1}{\tan 96}\left(\frac{X}{Z_0}\right) - \left[1 - \left(\frac{R}{Z_0}\right)^2\right] = 0$$

with $R/Z_0 = 73/75$

this is solved using the quadradic formula

$$\frac{X}{Z_0} = 4.7046 \pm \sqrt{\left\{4.7046^2 + 1 - (73/75)^2\right\}}$$

as X was made small by tuning, the minus sign is appropriate and

$$\frac{X}{Z_0} = -5.5893 \times 10^{-3}$$

In order that the amplifier appear as the complex conjugate of the antenna impedance:

$$\frac{Z_{ant}}{Z_0} = \frac{73}{75} - j5.5893 \times 10^{-3}$$

The amplifier must have the same impedance as the minimum seen from the transmission line:

$$\frac{Z_{min}}{Z_0} = \frac{R/Z_0}{1 - \frac{X}{Z_0}\tan 96°} = 1.028$$

Then with a transmission line of 1.6 meters the impedance seen from the antenna will be:

$$\frac{Z}{Z_0} = \frac{1.028 + j\tan 96°}{1 + j1.028\tan 96°}$$

$$= 0.9733 + j5.589 \times 10^{-3}$$

$$Z = 73.00 + j0.4192$$

which is the desired complex conjugate.