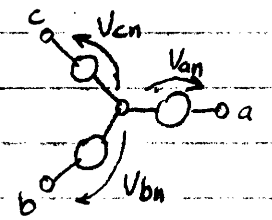
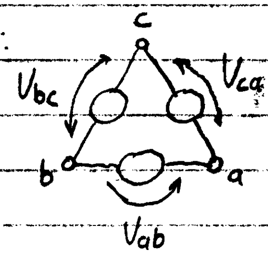
## THREE PHASE POWER:

- 3 VOLTAGES, 120° APART.

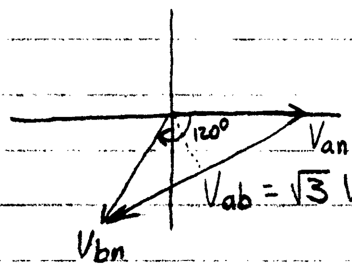
WYE:




DELTA:




- VOLTAGE SEQUENCES IN TEXT
- LOOK AT PHASOR RELATIONSHIP:
  (HOW TO CONVERT LINE-LINE TO LINE-NEUTRAL)




$V_{an}$ IS REFERENCE (RMS)

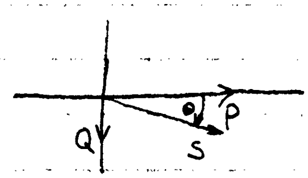
$V_{ab} = \sqrt{3}\, V_{an} = \sqrt{3}\, V_{bn}$

- UNDER CONDITIONS OF BALANCE, (ASSUME BALANCE UNLESS REASON TO BELIEVE OTHERW...
  1) ALL 3 CURRENST ARE EQUAL
  2) " " VOLTAGES " " .
  3) ∴ COMPLETE ANALYSIS CONSISTS OF LOOKING AT 1 LINE.

## TYPES OF POWER:

APPARENT (S) = GRAND TOTAL (VA)
REACTIVE (Q) - QUADRATURE PART OF S. (VA)
TRUE (P) = IN PHASE PART OF S. (W)

QUICK CHECK → ALWAYS $|S|^2 = P^2 + Q^2$, SINCE




$\cos\theta = P.F.$

QUICK PROBLEM:

$3\phi$, 11.5 KV GENERATOR DRIVES 500 KW, .866 LAGGING.

$\boxed{\text{FIND}}$

LINE CURRENT:

$$\frac{11500V}{\sqrt{3}} = 6640 \, V/\text{PHASE}$$

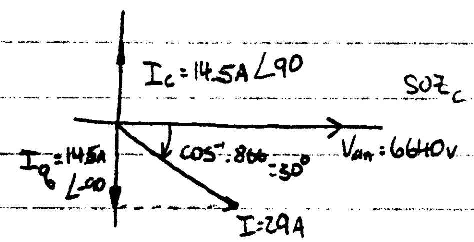
$$P = \frac{500KW}{3} = 167 \, KW/\phi$$

NOW $$|S| = \frac{P}{P.F.} = \frac{167 KW}{.866} = 192 \, KVA/\phi$$

$$I = \frac{192 \, KVA/\phi}{6.640 KV/\phi} = 29A$$

GENERATOR RATING: JUST $3|S| = 576$ KVA

COMPENSATOR:

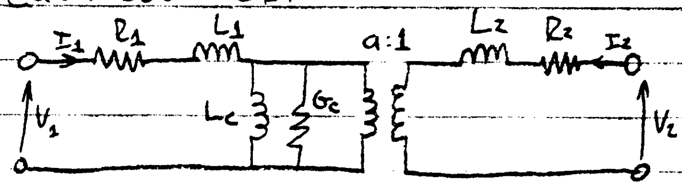



$$\text{SO } Z_c = \frac{V_{an}}{I_c} = \frac{6640\angle 0}{14.5\angle 90} = 458\angle -90$$

IT IS A CAPACITOR, $$\frac{1}{C} = X_c \cdot 2\pi f$$
$$\frac{1}{C} = 458 \cdot 2 \cdot \pi \cdot 60$$
$$C = 5.8\mu F \,/\text{LINE}$$

POWER TRANSFORMERS:

EQUIVALENT CKT:



TRANSFORMER TESTS: (LOW VOLT SIDE – SAFER)

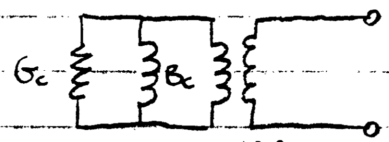
OPEN CKT: $Y_{oc} = G_c - jB_c$, SERIES ELEMENTS USUALLY NEGLIGABLE.
(ASSUME RATED VOLTAGE)

EX: TRANSFORMER RATED 50MVA 115kV : 13.8kV

$P_{2oc} = 250\,kW$ $I_{2oc} = 350\,A$

TESTS ON SECONDARY (MAKE SURE YOU USE RIGHT VOLTAGE! (PRI

$$G_c = \frac{P_{oc}}{V_1^2} = \frac{250\,kW}{(115\,kV)^2} = 18.9\,\mu S.$$



115 : 13.8
$a_{ps} : 8.33$

$$|S_{oc}| = 13.8\,kV \cdot 350\,A$$
$$= 4.83\,MVA$$

$$B_c = \frac{Q_{oc}}{V_1^2} = \frac{\sqrt{|S_{oc}|^2 - P_{oc}^2}}{V_1^2} = 0.36\,mS$$

SHORT CKT: (ASSUME RATED CURRENT)

SHORTED WINDING SHORTS OUT CORE ELEMENTS.

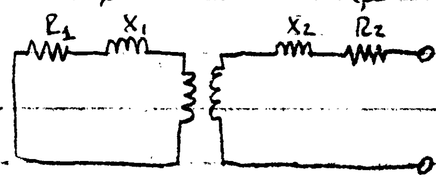
FROM PRIMARY: $Z_{sc} = R_1 + jX_1 + a_{ps}^2(R_2 + jX_2)$

SECONDARY: $Z_{sc} = R_2 + jX_2 + a_{sp}^2(R_1 + jX_1)$.

ASSUME $R_1 = a_{ps}^2 R_2$, $X_1 = a_{ps}^2 X_2$, AND TEST @ RATED CURRENT.

EX: SAME TRANSFORMER.

$P_{2sc} = 300\,kW$ $V_{2sc} = 1.5\,kV$.



$a_{ps} : 8.33$

FIND RATED CURRENT:

$$I_{1sc} = \frac{50\,MVA}{115\,kV} = 435\,A = \text{RATED CURR.}$$

$$(R_1 + a^2 R_2) = \frac{P_{sc}}{(I_{1,sc})^2} = 1.585\,\Omega$$

REAL PWR YIELDS R — IF $R_1 = a^2 R_2$, $R_1 = .793\,\Omega$

$R_2 = 11.4\,m\Omega$

Q POWER YIELDS X:

$$|S_{sc}| = I_{1,sc} \cdot [a_{ps} V_{sc}] = 5.44\,MVA$$

$$X_1 + a^2 X_2 = \frac{Q}{I_{1,sc}} = \frac{\sqrt{|S|^2 - P^2}}{I_{1,sc}^2} = 28.7\,\Omega \quad \text{IF } X_1 = a^2 X_2,$$

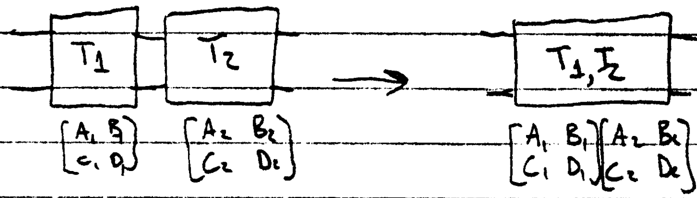
$X_1 : 14.3\,\Omega$

$X_2 : .207\,\Omega$

ABCD PARAMETERS: TRANSFORMER AS 2-PORT NETWORK

ABCD BECOME LIKE 2-PORT PARAMETERS,

MATRICIES MULTIPLY FOR SERIES CONNECTIONS




$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

EX:

- SEE CLASS HANDOUT FOR OUR TRANSFORMER WORKED OUT IN ABCD PARAMS.

NOTICE UNITS!

B IS RESISTANCE

C IS CONDUCTANCE

A, D ARE DIMENSIONLESS CONSTANTS OF PROPORTIONALITY.

PER UNIT SYSTEM.

SIMPLY PERCENT (DECIMAL) OF BASE RATINGS.

EX: 100 KVA TRANS-LINE OPERATING AT 60 KVA CARRIES POWER .6 P.U.

BE CAREFUL NOT TO MIX BASE RATINGS WITH $Z$.

$$\text{BASE } Z = \frac{\text{BASE } V_{LN}}{\text{BASE } I} = \frac{\text{BASE } (V_{LN})^2}{\text{BASE } VA_\phi}$$

SO $$Z(\text{P.U.}) = \frac{\text{ACTUAL } Z}{\text{BASE } Z.}$$

TO CONVERT BASES FOR $Z$:

$$Z_{\text{ACTUAL}} = Z(\text{P.U., OLD}) \times \frac{(\text{BASE}^{\text{OLD}} V_{LN})^2}{\text{BASE} VA_\phi}$$

$$Z(\text{P.U., NEW}) = \frac{Z_{\text{ACTUAL}}}{Z_{\text{NEWBASE}}} = Z(\text{P.U., OLD}) \times \frac{(\text{OLD BASE } V_{LN})^2}{\text{OLD BASE } VA_\phi} \times \frac{\text{NEW BASE } VA_\phi}{(\text{NEW BASE } V_{LN})^2}$$

IN BOOK, BUT CONFUSING.

EX: $3\phi$ SYNCHRONOUS MOTOR 10 KVA, 440V, $X = .8$ (P.U.) IN SYSTEM 100 KVA SYSTEM.

ACTUAL REACTANCE: FIND $Z$ BASE FIRST

$$Z \text{ BASE} = \frac{(V_{\text{BASE } LN})^2}{\text{BASE } VA_\phi} = \frac{(440/\sqrt{3})^2}{\frac{10000}{3}} =$$

$$= 19.36\,\Omega$$

$$\text{ACTUAL } X = X(\text{P.U.}) \cdot X(\text{BASE})$$
$$= 15.49\,\Omega$$

NEW SYSTEM:

$$X(\text{P.U., NEW}) = \frac{\text{ACTUAL } X}{\left[\frac{(440/\sqrt{3})^2}{\frac{100\,000}{3}}\right]} = 8 \text{ (P.U.)}$$

OMIT.

## ONE LINE DIAGRAMS

- SYMBOLS IN THE BOOK - SHOULD KNOW THEM. (P 5-11)
- JUST LIKE SCHEMATICS (EX P5-12)

## DISTRIBUTION TRANSFORMERS

- ASSUME INFINITE BUSS (5.92, 5.93; P 5-11)
- USE FORMULAS IN TEXT TO GET 25% P.U. TYP IMPEDANCE.

## VOLTAGE REGULATION

- REFER INPUT V TO SECONDARY, CALL IT CONSTANT (INFINITE BUS)
- DIFFERENCE IN P.U. OF $V_{IN}$ AND $V_{OUT}$ IS VOLTAGE REGULATION, WHEN $V_{OUT} = 1$ P.U.

(EXAMPLE TO FOLLOW)

## P.F. CORRECTION (TOUCHED ON BEFORE)

- USE VECTOR (PHASOR DIAGRAMS TO HELP)
- REMEMBER: CURRENT MAGNITUDE (NOT PHASE) DETERMINES CAPACITY OF MACHINES

EX: $3\phi$, 100 KVA TRANSFORMER, $Z_\phi = .03 + j.04$ P.U. $= .05\angle 53.1°$ P.U.
FULL LOAD AT P.F. = .8 LAGGING.
ASSUME RATED CURRENT.
WHAT IS CHANGE IN REGULATION IF P.F. IS CORRECTED TO UNITY?

$|I| = 1$ P.U., $\angle I = -\cos^{-1} .8 = -36.9$

$I = 1\angle -36.9$ P.U.

$V_2 = 1$ P.U.

$V_1 = V_2 + I Z_\phi$

$= 1 + (.048 + j.014)$

$V_1 = 1.048\angle .77$

$V_{REG} = V_1 - V_2 = 1.048 - 1 = 4.8\%$

IF PF = 1,

$I = 1$ P.U.

$V_2 = 1$ P.U.

$V_1 = V_2 + (.03 + j.04) = 1.03 + j.04$

$V_1 = 1.031\angle 2.2°$

$V_{REG} = 3.1\%$

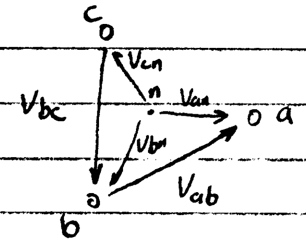
$\Delta$ REGULATION $= 4.8 - 3.1 = 1.7\%$ IMPROVED.

TRANSFORMER CONNECTIONS

ALREADY KNOW Y & Δ,

-LOOK AT OPEN DELTA

EX OPEN DELTA, 3φ 440V CONSISTS OF 2 1φ MACHINES, ca LEG MISSING.
SERVE 50 KVA BAL, 3φ LOAD, .8 LAGGING P.F.
DETERMINE REQUIRED RATINGS IF
A 10KW RESISTIVE LOAD ADDED TO ab LEG.




$V_{ab} = 440\angle 30$ V.

$V_{an}$ IS PHASE REF.

DO BALANCED 3φ LOAD FIRST:

$$I_a = \frac{50\text{KVA}/3 \angle -\cos^{-1} .8}{440/\sqrt{3}\ \angle 0} = 65.6\angle -36.8^\circ \text{ A.}$$

LOAD IS BALANCED, SO $|I_a| = |I_b| = |I_c|$

$$I_b = 65.6\ \frac{\angle -36.8}{\angle +120} = 65.6\angle -156.8^\circ \text{ A}$$

$$I_c = 65.6\ \frac{\angle -36.8}{\angle -120} = 65.6\angle 83.2^\circ \text{ A}$$

ADD 10KW R TO ab LEG:

$$|I_a'| = \frac{10\text{ KW}}{440\text{ V}} = 22.7\text{ A}$$

$\angle I_a' = \angle V_{ab} = 30^\circ$, SINCE IN PHASE.

$$S_{ab} = (I_a + I_a')^* V_{ab}$$
$$= 34.1\angle 51^\circ \text{ KVA}$$

$$S_{bc} = I_b^* V_{bc} = 28.9\ \angle 7^\circ \text{KVA}$$

## FAULT CALCULATIONS

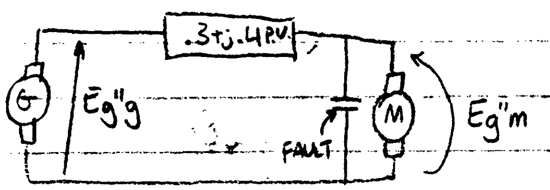
- 3 VOLTAGES PRESENT ON MACHINES, DUE TO EFFECTS OF MECHANICAL INERTIA AND MAGNETIC SATURATION:

  $E$ – STANDARD OPERATING V (PRE FAULT)

  $E''$ – SUBTRANSIENT (FAULT HAS OCCURED, BUT SYS NOT REACTING)

  $E'$ – TRANSIENT (FULL BLOWN FAULT CONDX).

EX: SYNC GEN & SYNC MOTOR BOTH 15 MVA, 13.9 KV, 25% SUBTRANSIENT REACTANCE.
CONNECTED VIA .3+j.4 P.U. LINE, 15MVA, 13.9KV BASE.
MOTOR RUNS AT 12MVA 13KV, .8 LAG P.F.




SUBTRANSIENT V FOR MOTORS GOES DOWN. FOR GENERATORS GOES UP, DUE TO CONTINUITY OF ENERGY.

EQUIV. GENERATED VOLTAGES: (GO TO P.U. QUICKLY)

MOTOR: $E_m = \frac{13}{13.9} = .935$ P.U.

$$I_L = \frac{\frac{12MVA\angle -\cos^{-1}.8}{13KV}}{\frac{15MVA}{13.9KV}} = .855\angle -36.8^\circ \text{ P.U.}$$

$$E_g''m = E_m - I_L(jx_d'') = .935 - (.855\angle -.368)(.25\angle 90^\circ)$$
$$= .807 - j.171 \text{ P.U.}$$

$Z = Z_{LINE} + jX_d'' = .3 + j.65$ P.U.

SO $E_g''g = E_m + I_L Z = 1.47 + j.29$ P.U.

SO MOTOR CURRENT $I_m$ WILL BE $\frac{E_g''m}{jX_d''} = \frac{.807-j.171}{j.25} = \frac{.824\angle -12^\circ}{.25\angle 90^\circ} = 3.29\angle -102^\circ$ P.U.

SIMILARLY, GENERATOR $I_g$ WILL BE $\frac{E_g''g}{Z} = \frac{1.47+j.29}{.3+j.65} = \frac{1.5\angle 11^\circ}{.72\angle 65^\circ} = 2.08\angle -54^\circ$ P.U.

TOTAL FAULT CURRENT $I_g + I_m = .589 - j4.9 = 4.94\angle -83^\circ$ P.U.
THUS COMPONENTS MUST BE ABLE TO TAKE 5X RATED I UNDER FAULT.

## ASYMMETRICAL FAULTS

- CAN BE DONE BY PILE-DRIVER METHOD, BUT VERY HARD.
- CONSIDER ASYMMETRICAL FAULT CURRENT AS THE SUM OF SYMMETRICAL COMPONENTS.

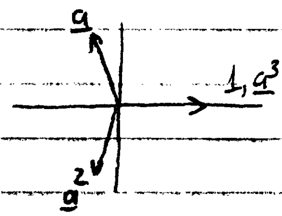
SYMMETRICAL COMPONENT ANALYSIS

- UNIT PHASORS

$a = 1\angle 120°$

$a^2 = 1\angle 240°$

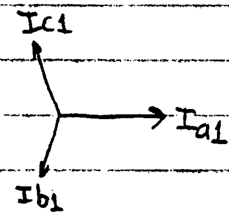
$a^3 = 1$




LET POSITIVE SEQUENCE

$I_{b1} = a^2 I_{a1}$

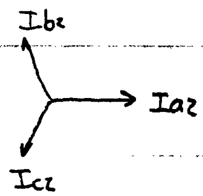
$I_{c1} = a\, I_{a1}$




AND NEGATIVE SEQUENCE

$I_{b2} = a\, I_{a2}$

$I_{c2} = a^2 I_{a2}$




ZERO SEQUENCE: COMMON MODE CURRENT. $I_{x0}$.

SO NOW, BY SUPERPOSITION,

$I_a = I_{a1} + I_{a2} + I_{a0}$

$I_b = I_{b1} + I_{b2} + I_{b0}$

$I_c = I_{c1} + I_{c2} + I_{c0}$

$I_{a0} = I_{b0} = I_{c0} = \frac{1}{3}(I_a + I_b + I_c)$, SINCE $I_0$ IS COMMON.

THUS

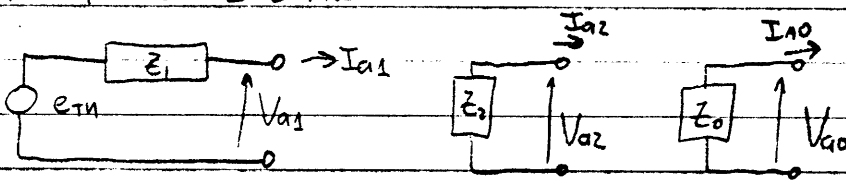
$$I_{a0} = \tfrac{1}{3}\left(I_a + I_b + I_c\right)$$
$$I_{a1} = \tfrac{1}{3}\left(I_a + \underline{a} I_b + \underline{a}^2 I_c\right)$$
$$I_{a2} = \tfrac{1}{3}\left(I_a + \underline{a}^2 I_b + \underline{a} I_c\right)$$

AND
$$V_{a0} = \tfrac{1}{3}\left(V_{an} + V_{bn} + V_{cn}\right)$$
$$V_{a1} = \tfrac{1}{3}\left(V_{an} + \underline{a} V_{bn} + \underline{a}^2 V_{cn}\right)$$
$$V_{a2} = \tfrac{1}{3}\left(V_{an} + \underline{a}^2 V_{bn} + \underline{a} V_{cn}\right)$$

PROCEDURE FOR SOLVING PROBLEMS

1) FOLLOW RULES IN TEXT FOR DRAWING
POSITIVE S.M. (NORMAL)
NEGATIVE S.M. (NO GENERATORS, $X_m = \tfrac{1}{2}(X_d'' + X_q'')$)
ZERO S.M. (USE FIG 5.15, P 5-23)

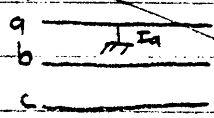
2) LOCATE FAULT ON MODELS. DETERMINE THE THEVENIN EQUIV CKT OF SYSTEM AT FAULT LOCATION



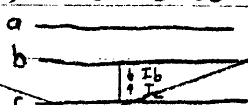

3) DETERMINE HOW THEY COMBINE FROM TYPE OF FAULT.

EXAMPLE

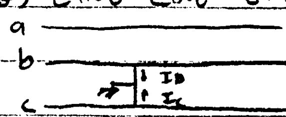
1) LINE TO GND




2) LINE- LINE



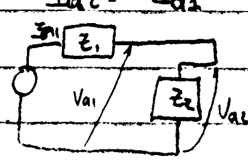

3) LINE-LINE-GND

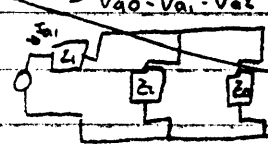



$I_b = I_c = 0,\ V_a = 0$
$\rightarrow I_{a0} = I_{a1} = I_{a2},$

$I_a = 0,\ -I_b = I_c\ \ V_b = V_c$
$\rightarrow I_{a2} = -I_{a1}$

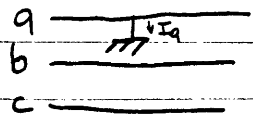
$I_a = 0,\ V_b = V_c = 0$
$\rightarrow V_{a0} = V_{a1} = V_{a2} = \dfrac{V_{a}}{3}$

EXAMPLES:

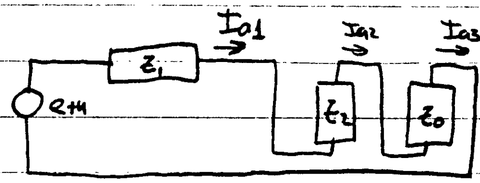
1) LINE – GND




$I_b = I_c = 0$

SO $I_{a0} = I_{a1} = I_{a2} = \frac{I_a}{3}$

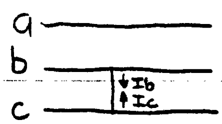
CONNECT:




KNOW $I_{a1}, I_{a2}, I_{a0}$, SO

$I_a = I_{a1} + I_{a2} + I_{a0}$

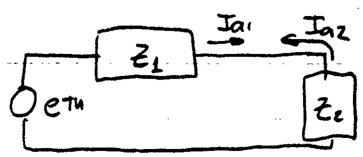
2) LINE-LINE




$I_a = 0,\ I_b = -I_c$

SO $I_{a1} = -I_{a2}$

CONNECT:



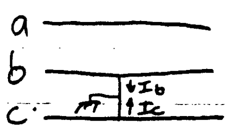

WE KNOW $I_{a1}, I_{a2}, I_{a0}$, SO

$I_b = I_{b0} + I_{b1} + I_{b2}$

$I_b = I_{a0} + \underline{a}^2 I_{a1} + \underline{a}\, I_{a2}$

$I_c = -I_b$

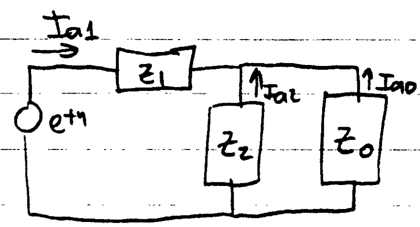
3) LINE-LINE-GND




$I_a = 0,\ V_{bn} = V_{cn} = 0$

SO $V_{a0} = V_{b0} = V_{c0} = \frac{V_{an}}{3}$

CONNECT




SOLVE TO KNOW $I_{a1}, I_{a2}, I_{a0}$

$I_b = I_{a0} + \underline{a}^2 I_{a1} + \underline{a}\, I_{a2}$ (FROM ABOVE)

$I_c = I_{c0} + I_{c1} + I_{c2}$

$I_c = I_{a0} + \underline{a}\, I_{a1} + \underline{a}^2 I_{a2}$

CONCLUSIONS

1) IF FAULT IS SYMMETRICAL, ONLY P.S.M. IS NEEDED.

2) IF NO FAULT I IS GOING TO GROUND, DO NOT NEED Z.S.M.

SUPPLEMENTARY NOTES: DERIVATION OF PHASE-SEQUENCE CONVERSIONS.

GIVEN $I_a = I_{a0} + I_{a1} + I_{a2}$,
$I_b = I_{b0} + I_{b1} + I_{b2}$,
$I_c = I_{c0} + I_{c1} + I_{c2}$, ARE THE PHASE CURRENTS,

AND $I_{b1} = \underline{a}^2 I_{a1}$, $I_{b2} = \underline{a} I_{a2}$,
$I_{c1} = \underline{a} I_{a1}$, $I_{c2} = \underline{a}^2 I_{a2}$, ARE THE SEQUENCE PHASE RELATIONSHIPS, DERIVE THE SEQUENCE CURRENTS IN TERMS OF THE PHASE CURRENTS.

ZERO SEQUENCE CURRENT

NOTE THAT $\boxed{\underline{a} + \underline{a}^2 + 1 = 0}$. BY DEFINITION, $I_{a0} = I_{b0} = I_{c0}$,

SO
$$3I_{a0} = I_a + I_b + I_c - (I_{a1} + I_{a2} + I_{b1} + I_{b2} + I_{c1} + I_{c2}),$$
$$3I_{a0} = I_a + I_b + I_c - (I_{a1} + I_{a2} + \underline{a}^2 I_{a1} + \underline{a} I_{a2} + \underline{a} I_{a1} + \underline{a}^2 I_{a2}),$$
$$3I_{a0} = I_a + I_b + I_c - (I_{a1}(1 + \underline{a} + \underline{a}^2) + I_{a2}(1 + \underline{a} + \underline{a}^2)),$$
$$3I_{a0} = I_a + I_b + I_c - (I_{a1}(0) + I_{a2}(0)),$$
$$I_{a0} = \frac{1}{3}(I_a + I_b + I_c).$$

POSITIVE SEQUENCE CURRENT

$$0 = (1 + \underline{a} + \underline{a}^2) + (1 + \underline{a} + \underline{a}^2),$$
$$0 = I_{a2}(1 + \underline{a} + \underline{a}^2) + I_{a0}(1 + \underline{a} + \underline{a}^2),$$
$$0 = I_{a2} + \underline{a} I_{a2} + \underline{a}^2 I_{a2} + I_{a0} + \underline{a} I_{a0} + \underline{a}^2 I_{a0},$$

SINCE $I_{a0} = I_{b0} = I_{c0}$,

$$0 = I_{a2} + \underline{a} I_{a2} + \underline{a}^2 I_{a2} + I_{a0} + \underline{a} I_{b0} + \underline{a}^2 I_{c0},$$

USING SEQUENCE PHASE RELATIONSHIPS,

$$0 = I_{a2} + \underline{a}^2 I_{c2} + \underline{a} I_{b2} + I_{a0} + \underline{a} I_{b0} + \underline{a}^2 I_{c0},$$
$$0 = (I_{a0} + I_{a2}) + \underline{a}(I_{b0} + I_{b2}) + \underline{a}^2 (I_{c0} + I_{c2}),$$

<CONTINUED>

SINCE $I_{a1} = \underline{a} I_{b1} = \underline{a}^2 I_{c1}$,

$$3 I_{a1} = (I_{a0} + I_{a1} + I_{a2}) + \underline{a}(I_{b0} + I_{b1} + I_{b2}) + \underline{a}^2(I_{c0} + I_{c1} + I_{c2}),$$

$$3 I_{a1} = (I_a) + \underline{a}(I_b) + \underline{a}^2(I_c)$$

$$I_{a1} = \frac{1}{3}(I_a + \underline{a} I_b + \underline{a}^2 I_c).$$

## NEGATIVE SEQUENCE CURRENT

$$0 = (1 + \underline{a} + \underline{a}^2) + (1 + \underline{a} + \underline{a}^2),$$

$$0 = I_{a1}(1 + \underline{a} + \underline{a}^2) + I_{a0}(1 + \underline{a} + \underline{a}^2),$$

$$0 = I_{a1} + \underline{a} I_{a1} + \underline{a}^2 I_{a1} + I_{a0} + \underline{a} I_{a0} + \underline{a}^2 I_{a0},$$

SINCE $I_{a0} = I_{b0} = I_{c0}$,

$$0 = I_{a1} + \underline{a} I_{a1} + \underline{a}^2 I_{a1} + I_{a0} + \underline{a} I_{c0} + \underline{a}^2 I_{b0},$$

USING SEQUENCE PHASE RELATIONSHIPS,

$$0 = I_{a1} + \underline{a}^2 I_{b1} + \underline{a} I_{c1} + I_{a0} + \underline{a} I_{c0} + \underline{a}^2 I_{b0},$$

$$0 = (I_{a0} + I_{a1}) + \underline{a}(I_{c0} + I_{c1}) + \underline{a}^2(I_{b0} + I_{b1}),$$

SINCE $I_{a2} = \underline{a}^2 I_{b2} = \underline{a} I_{c2}$,

$$3 I_{a2} = (I_{a0} + I_{a1} + I_{a2}) + \underline{a}(I_{c0} + I_{c1} + I_{c2}) + \underline{a}^2(I_{b0} + I_{b1} + I_{b2}),$$

$$3 I_{a2} = (I_a) + \underline{a}(I_c) + \underline{a}^2(I_b),$$

$$I_{a2} = \frac{1}{3}(I_a + \underline{a}^2 I_b + \underline{a} I_c).$$

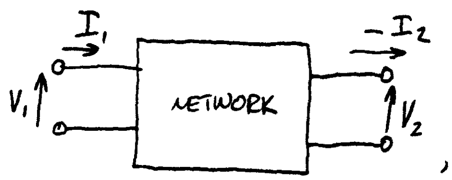
# PE REVIEW - POWER SYSTEMS SECTION
## SUPPLEMENTARY NOTES ON ABCD PARAMETERS

LIKE ALL 2-PORT NETWORK PARAMETERS, ABCD PARAMETERS ARE BASED ON THE LINEAR SEPARATION OF INDEPENDENT VARIABLES AND "SUMMING" THEM TOGETHER AGAIN (SUPERPOSITION IN ACTION).

FROM THE DEFINITION OF THE ABCD MATRIX,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}, \quad \text{OR} \quad \begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2, \end{aligned}$$

WHERE




,

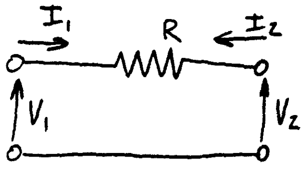
WE MAY WRITE THE FORMULAS FOR THE ABCD PARAMETERS

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad B = -\left.\frac{V_1}{I_2}\right|_{V_2=0}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \qquad D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}.$$

GIVEN ANY 2-PORT NETWORK, WE MAY DERIVE THE ABCD PARAMETERS BY SIMPLY SOLVING THE ABOVE FOUR EQUATIONS.

## EXAMPLE #1 - SERIES RESISTOR

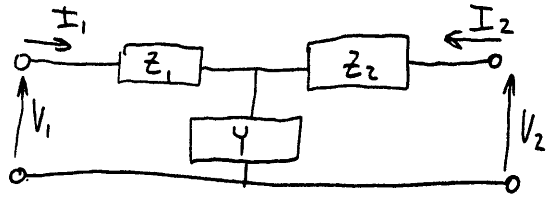



$$V_1 = V_2 - RI_2 \qquad I_1 = -I_2$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1 \qquad B = -\left.\frac{V_1}{I_2}\right|_{V_2=0} = R$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 0 \qquad D = -\left.\frac{I_1}{I_2}\right|_{V_2=0} = 1.$$

## EXAMPLE #2 - T NETWORK




$$V_2\Big|_{I_2=0} = V_1 \frac{1/Y}{1/Y + Z_1}\Big|_{I_2=0},$$

$$V_2\Big|_{I_2=0} = V_1 \frac{1}{1+YZ_1}\Big|_{I_2=0},$$

$$A = \frac{V_1}{V_2}\Big|_{I_2=0} = 1 + YZ_1.$$

$$-I_2\Big|_{V_2=0} = I_1 \frac{1}{1+YZ_2}\Big|_{V_2=0},$$

$$-I_2\Big|_{V_2=0} = \frac{V_1}{Z_1 + \frac{Z_2}{1+YZ_2}} \frac{1}{1+YZ_2}\Big|_{V_2=0},$$

$$-I_2\Big|_{V_2=0} = \frac{V_1}{Z_1+Z_2+YZ_1Z_2}\Big|_{V_2=0},$$

$$B = -\frac{V_1}{I_2}\Big|_{V_2=0} = Z_1 + Z_2 + YZ_1Z_2.$$

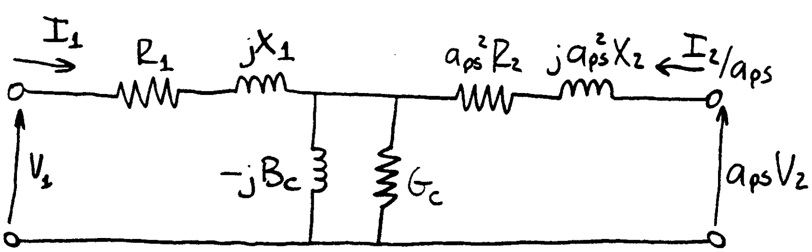
$$V_2\Big|_{I_2=0} = \frac{I_1}{Y}\Big|_{I_2=0},$$

$$C = \frac{I_1}{V_2}\Big|_{I_2=0} = Y.$$

$$-I_2\Big|_{V_2=0} = I_1 \frac{1}{1+YZ_2},$$

$$D = -\frac{I_1}{I_2}\Big|_{V_2=0} = 1 + YZ_2.$$

## EXAMPLE #3 - TRANSFORMER




LET $Z_1 = R_1 + jX_1,$
$Z_2 = R_2 + jX_2,$
$Y = G_c - jB_c.$

NOTE THAT THE TRANSFORMER IS SIMPLY A T-NETWORK, WITH

$$V_2 \longrightarrow a_{ps}V_2,$$
$$I_2 \longrightarrow I_2/a_{ps},$$
$$Z_2 \longrightarrow a_{ps}^2 Z_2.$$

SO NOW

$$\left.\frac{V_1}{a_{ps}V_2}\right|_{I_2=0} = 1+YZ_1, \qquad \left.-\frac{V_1}{I_2/a_{ps}}\right|_{V_2=0} = Z_1 + a_{ps}^2 Z_2 + a_{ps}^2 Z_1 Z_2 Y,$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = a_{ps}(1+YZ_1). \qquad B = \left.-\frac{V_1}{I_2}\right|_{V_2=0} = \frac{1}{a_{ps}}\left(Z_1 + a_{ps}^2 Z_2(1+Z_1Y)\right).$$

$$\left.\frac{I_1}{a_{ps}V_2}\right|_{I_2=0} = Y, \qquad \left.-\frac{I_1}{I_2/a_{ps}}\right|_{V_2=0} = 1 + a_{ps}^2 Z_2 Y,$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = a_{ps}Y. \qquad D = \left.-\frac{I_1}{I_2}\right|_{V_2=0} = \frac{1}{a_{ps}}\left(1 + a_{ps}^2 Z_2 Y\right).$$

IN A PROPERLY DESIGNED TRANSFORMER, WE MAY ASSUME THAT $Z_1 = a_{ps}^2 Z_2$, SO THAT

$$A = a_{ps}(1+YZ_1), \qquad B = \frac{1}{a_{ps}} Z_1(2+Z_1Y),$$

$$C = a_{ps}Y, \qquad D = \frac{1}{a_{ps}}(1+Z_1Y).$$

THE TRANSFORMER TESTED IN CLASS PROVIDED THE PARAMETERS:

$$a_{ps} = 8.33, \quad Z_1 = .793 + j\,14.3\,\Omega, \quad Y = .0189 - j\,.36 \text{ mS},$$

$$YZ_1 = (5.16 - j\,.015)\times 10^{-3}$$

SO $A = 8.33(1+YZ_1)$

$A = 8.37 + j0.$

$B = \frac{1}{8.33} Z_1(2+Z_1Y)$

$B = 0.191 + j\,3.45\ \Omega.$

$C = 8.33Y$

$C = 0.157 - j3.00$ mS

$D = \frac{A}{a_{ps}^2} = \frac{8.37}{8.33^2} = .121 + j0.$

WARMUP 1.

$$I_{phase} = \frac{|S|/3}{V} = \frac{2\times10^8}{3\times230\times10^3}$$

$$= 289.9\text{ A}$$

the line current is $\sqrt{3}\times$ phase current

$$|I_{line}| = \sqrt{3}\cdot289.9 = 502\text{ A}$$

The powerfactor correction is on a phase basis. For unity power factor the lagging part of the current is cancelled:

$$I_{phase} = I_p + jI_q \;:\; I_p = |I_{phase}|\cos\theta$$

$$I_q = |I_{phase}|\sin\theta \qquad \cos\theta = .9$$

$$I_q = 289.9\sin(-25.8^\circ) = -126.4$$

$$I_{Cap} = j(126.4) = \frac{230\times10^3}{-jX_C}$$

$$X_C = \frac{230\times10^3}{126.4} = 1820\,\Omega$$

WARMUP 2.

The open circuit test obtains the core parameters G and B:

$$G = \frac{P_{o.c.}}{V_{o.c.}^2} = \frac{900}{13800^2} = 4.73\times10^{-6}\text{ mhos}$$

$$Q^2 = S^2 - P^2 =$$

$$(13800\times.2)^2 - 900^2$$

$$Q = 2609$$

$$B = \frac{Q}{V^2} = \frac{2609}{13{,}800^2} = 13.7\times10^{-6}\text{ mhos}$$

the turns ratio:

$$\frac{V_p}{N_p} = \frac{V_s}{N_s} \;:\; \frac{N_p}{N_s} = \frac{V_p}{V_s}$$

$$\frac{N_p}{N_s} = \frac{13800}{460} = 30$$

WARMUP 3.

$$N_pI_p = N_sI_s \qquad a = \frac{N_p}{N_s} = \frac{I_s}{I_p}$$

$$a = \frac{100\text{A}}{20\text{A}} = 5$$

$$P_{SC} = I_p^2R_{wp} + I_s^2R_{ws} = 1000\text{ W}$$

$$I_p^2(R_{wp} + a^2R_{ws}) = 1000$$

$$R_{wp} + a^2R_{ws} = \frac{1000}{20^2} = 2.5\,\Omega$$

Assume $R_{wp} = a^2R_{ws} = 1.25\,\Omega$

$$R_{ws} = \frac{1.25}{5^2} = 0.05\,\Omega$$

$$|S| = V_pI_p = 80\times20 = 1600$$

$$S = P + jQ \;\therefore\; Q^2 = S^2 - P^2 =$$

$$|Q| = 1249\text{ VARs}$$

$$|Q| = I_1^2(X_p + a^2X_s)$$

## WARMUP 3 CONTINUED

$$X_p + a^2 X_s = \frac{1249}{20^2} = 3.12\ \Omega$$

$$X_p = \frac{3.12}{2} = 1.56\ \Omega$$

$$X_s = \frac{1.56}{a^2} = 0.0624\ \Omega$$

## WARMUP 4.

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{V_{l\text{-}l}^2}{VA} = \frac{440^2}{10{,}000}$$

$$Z_{base} = 19.36\ \Omega$$

$$Z = Z_{pu} \times Z_{base} = j.8 \times 19.36$$

$$Z = 15.49\ \Omega$$

SYSTEM BASE:

$$Z_{base\text{-}2} = \frac{440^2}{100\text{ K}} = 1.936\ \Omega$$

$$Z_{pu\text{-}2} = \frac{Z}{Z_{base\text{-}2}} = \frac{15.49}{1.936} = 8\text{ p.u.}$$

## WARMUP 5.

The short circuit test is done at rated current:

$$I_{rated} = \frac{500\text{ KVA}}{115\text{ KV}} = 4.35\text{ A}$$

$$|S_{sc}| = 4.35\text{ A} \times 2.5\text{ KV} = 10.88\text{ KVA}$$

$$= P + jQ = 435 + jQ$$

$$|Q| = \sqrt{10{,}880^2 - 435^2} = 10{,}870$$

$$P = I^2R \qquad R = \frac{435}{(4.35)^2} = 23\ \Omega$$

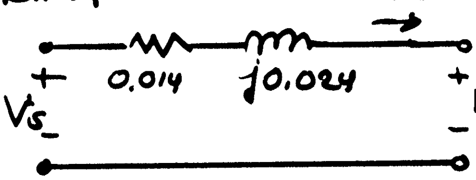
$$|Q| = I^2X \qquad X = \frac{10{,}870}{(4.35)^2} = 574\ \Omega$$

$$Z_{BASE} = \frac{115\text{ KV}}{4.35\text{ A}} = 26{,}440$$

$$Z_{pu} = \frac{23 + j574}{26{,}440} = 0.0009 + j0.022$$

$$Z_{pu} \cong 0.001 + j0.022$$

## WARMUP 6.




$$V_s = 1 + iZ = 1 + 0.014 + j0.024$$

$$= 1.014 + j0.024 = 1.014\angle 1.4°$$

It is assumed that $V_s$ does not change amplitude.

At half load $V_s = V_L + iZ$

$i = 0.5\angle 0°$, $V_L = |V_L|\angle 0° = V_L$

$$V_s = V_L + 0.5[0.014 + j0.024]$$

$$= V_L + 0.007 + j0.012$$

but $|V_s|^2 = 1.014^2$ so

$$1.014^2 = (V_L + 0.007)^2 + 0.012^2$$

solving for $V_L$:

WARMUP 6. CONTINUED:

$V_{half\text{-}load} = 1.007$

$V_{No\text{-}load} = 1.014$

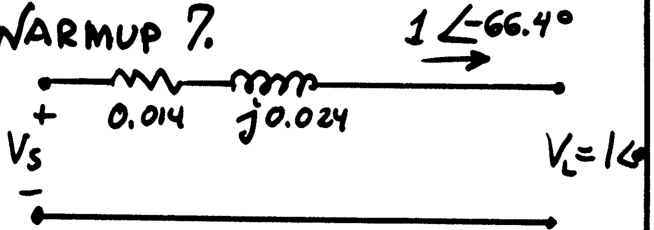
$V_{full\text{-}load} = 1.000$

REGULATION:

half to full load: $1.007 - 1.001 = 0.007$ or $0.7\%$

no load to full load: $1.014 - 1.001 = 0.014$ or $1.4\%$

WARMUP 7.




$0.014 + j0.024 = 0.0278\angle 59.7$

$ZI = .0278\angle 59.7° \times 1\angle -66.4°$

$= 0.0278\angle -6.7° = 0.0276 - j0.0032$

$V_s = 1 + ZI = 1.0276 - j0.0032$

$|V_s| = 1.0276$

At half load

$V_s = V_L + 0.0138 - j0.0016$

$|V_s| = 1.0276,\ V_L = 1.014$

REGULATION:

half to full load $\Rightarrow 1.014 - 1 = .014 \rightarrow 1.4\%$

No load to full load $\Rightarrow 1.028 - 1 = .028 \rightarrow 2.8\%$

WARMUP 8.

$V_L = 1$ p.u. $\quad I_L = 1\angle -36.9$

$Z_{line} = 0.03 + j0.04 = 0.05\angle 53.1°$

$V_S = 1 + 0.05\angle(53.1 - 36.9)$

$= 1.048 + j0.014$

$|V_S| = 1.048$ so the regulation is $0.048$ or $4.8\%$

For unity PF $I_L = 1\angle 0$

$V_s = 1 + 0.03 + j0.04 = 1.03 + j0.04$

$|V_s| = 1.03$ and the regulation is $0.03$ or $3\%$

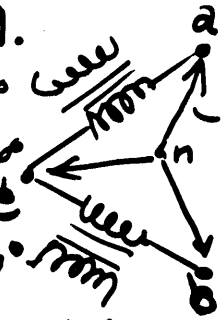
The regulation is improved from $4.8\%$ to $3\%$, a percentage change of $\frac{4.8-3}{4.8} \times 100 = 37.5\%$

WARMUP 9.

$V_{an} = 254\angle 0°$

$V_{bn} = 254\angle -120°$

$V_{cn} = 254\angle 120°$




$V_{a\text{-}n} = \frac{440}{\sqrt{3}} = 254\text{V}$

$i_{an} = 65.6\angle 0°$

$i_{bn} = 65.6\angle -120°$

$V_{ac} = V_{an} - V_{cn} = 440\angle -30°$

## WARMUP 9 CONTINUED

$$S_{ac} = V_{ac} I_{an}^{*} = 440\angle{-30^\circ} \times 65.6$$

$$= 29.9 \text{ KVA} @ -30^\circ$$

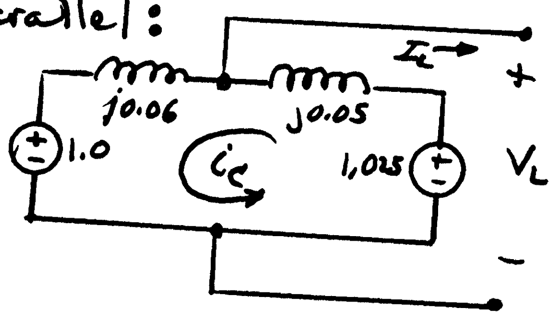
$$V_{bc} = V_{bn} - V_{cn} = 440\angle{-90^\circ}$$

$$S_{bc} = V_{bc} I_{bn}^{*} = 440\angle{-90^\circ} \times 65.6\angle{120^\circ}$$

$$= 29.9 \text{ KVA} @ +30^\circ$$

## WARMUP 10

One transformer has voltage 1 p.u. and the other 1.025 p.u. they are connected in parallel:




IGNORING the effect of the load current (which has a differential effect on the circulatin current)

$$i_c = \frac{1.025 - 1.0}{j.06 + j.05} = \frac{.025}{j0.11} = 0.227$$

So the circulating current is 22.7% of rated current.

## CONCENTRATE 1.

(a)

$$I_{line} = \frac{500 \text{ KW}/3 \text{ phases}}{\frac{11.5 \text{ KV}}{\sqrt{3}} \times 0.866} = 29 \text{A}$$

(b)

$$|S| = \frac{500 \text{KV}}{0.866} = 577 \text{ KVA}$$

(c)

$$|I_q| = 29\sqrt{1 - 0.866^2} = 14.5 \text{ A}$$

$$X_{comp} = -\frac{11.5 \text{KV}/\sqrt{3}}{14.5 \text{ A}} = -458\,\Omega$$

## CONCENTRATE 2.

The turns ratio $a \left[= \frac{N_p}{N_s}\right]$ is found so the low-voltage secondary measurements can be referred to the primary:

$$a = \frac{115 \text{ KV}}{13.8 \text{ KV}} = 8.33$$

ASSUMPTION: open circuit measurements are made at rated voltage.

$$I_{p\text{-}oc}\Big|_{equv} = \frac{I_{s\text{-}oc}}{a} = \frac{350}{8.33} = 42.0 \text{A}$$

$$|S_{oc}| = |V_s I_s| = 13.8\text{K} \times 350 = 4.83 \text{ MVA}$$

$$|Q_{oc}| = \sqrt{S_{oc}^2 - P_{oc}^2} = 4.82 \text{ MVAR}$$

$$G = P/V^2 = \frac{250 \text{KW}}{(115 \text{KV})^2} = 18.9 \times 10^{-6} \text{ mhos}$$

$$B = Q/V^2 = \frac{4.82 \text{MVAR}}{(115 \text{KV})^2} = 364 \times 10^{-6} \text{ mhos}$$

CONCENTRATE 2 CONTINUED

ASSUMPTION: Short circuit test is at rated current.

$$I_{pri\text{-}rated} = \frac{50\,MVA}{115\,KV} = 435\,A$$

$$V_{p\text{-}oc}\Big|_{equiv} = a \times 1.5\,kV = 12.5\,KV$$

$$|S_{sc}| = 12.5\,KV \times 435\,A = 5.44\,MVA$$

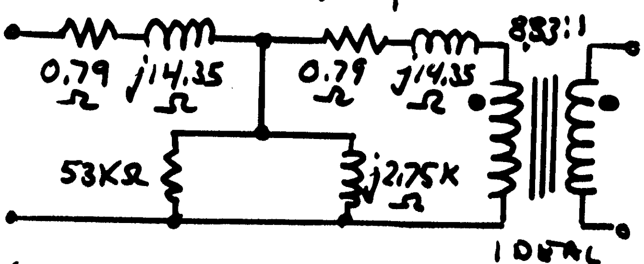
$$R_p + a^2 R_s = \frac{P}{I^2} = \frac{300\,KW}{435^2} = 1.585\,\Omega$$

$$|Q_{sc}| = \sqrt{S_{sc}^2 - P_{sc}^2} = 5.43\,MVAR$$

$$X_p + a^2 X_s = \frac{Q_{oc}}{I^2} = 28.7\,\Omega$$

ASSUMPTIONS: $R_p = a^2 R_s$
$X_p = a^2 X_s$

$R_p = a^2 R_s = 0.79$, $X_p = a^2 X_s = 14.35$




(a) EQUIVALENT CIRCUIT

(b) $Z_1 = a^2 Z_2 = 0.79 + j14.35$

$Y = Y_a = (18.9 - j364) \times 10^{-6}$

$YZ = 5.24 \times 10^{-3} \angle -0.3° = 0.0052$

FROM FIGURE 5.10(c)

$A = D = 1 + YZ \approx 1.0$

$B = 2Z_1 + YZ_1^2 = 28.7 \angle 86.9°$

$C = Y = 364 \times 10^{-6} \angle -87°$

FROM EQ'S 5.61 & 5.62:

(c) $$\frac{V_1}{I_1} = \frac{Aa^2Z_L + B}{Ca^2Z_L + D} \quad \& \quad a^2 Z_L = 0.5$$

$$Z_{IN} = \frac{.5 + 28.7\angle 86.9}{.5 \times 364 \times 10^{-6} \angle -87° + 1} = \frac{28.7\angle 85.9}{1}$$

CONCENTRATE 3.

REFER TO EQ'S 5.65 through 5.68.

$Y_c Z_1 = (0.0002 - j0.001)(0.01 + j0.02)$

$= 22.8 \times 10^{-6} \angle -15.3$

$= (22.0 - j6.0) \times 10^{-6}$

$A = 1 \times 1 + 2 Z_1 Y_c = 1.00$

$B = 2Z_1 + 2Z_2 = 0.04 + j0.08$

$C = Y_c + Y_c = 0.0004 - j0.002$

$D = 2 Z_1 Y_c + 1 \approx 1.00$

$a^4 Z_L = 0.1$

$$\frac{V_1}{I_1} = Z_{IN} = \frac{Aa^4 Z_L + B}{Ca^4 Z_L + D} = 0.14 + j0.08$$

CONCENTRATE 4.

$$a_{ps} = \frac{6.8\,KV}{440\,V} = 15.45$$

$$a_{pt} = \frac{6.8\,KV}{1.38\,KV} = 4.93$$

FOR THE SECONDARY BASE:

$Z_{BASE\text{-}S} = (440\,V)^2 / 30\,KVA = 6.453\,\Omega$

CONCENTRATE 4 continued

REFERRED TO THE SECONDARY:

$$Z_p \rightarrow \frac{49.5 + j110}{(15.45)^2}$$

$$Z_p(\text{p.u.})\Big|_{\text{SEC BASE}} = \frac{49.5+j110}{(15.45)^2 6.453} = 0.032 + j0.071$$

$$Z_s(\text{p.u.})\Big|_{\text{SEC BASE}} = \frac{70.5+j90}{(15.45)^2 6.453} = 0.046 + j0.058$$

$$Z_t(\text{p.u.})\Big|_{\text{SEC BASE}} = \frac{50.5+j70}{(15.45)^2 6.453} = 0.033 + j0.045$$

FOR THE TERCIARY

$$Z_{\substack{\text{base}\\ \text{TERT.}}} = \frac{1380^2}{20{,}000} = 95.22 \text{ ohms}$$

$$Z_p\Big|_{\text{TERT BASE}} = \frac{45+j110}{(4.93)^2 95.22} = 0.021 + j0.048 \text{ p.u.}$$

$$Z_s\Big|_{\text{TERT BASE}} = \frac{70.5+j90}{(4.93)^2 95.22} = 0.030 + j0.039 \text{ p.u.}$$

$$Z_t\Big|_{\text{TERT BASE}} = \frac{50.5+j70}{(4.93)^2 95.22} = 0.022 + j0.030 \text{ p.u.}$$

ON A PERCENTAGE BASIS:

In secondary base:

$$Z_p = 3.2 + j7.1\ \%$$

$$Z_s = 4.6 + j5.8\ \%$$

$$Z_t = 3.3 + j4.5\%$$

In TERTIARY BASE:

$$Z_p = 2.1 + j4.8\%$$

$$Z_s = 3.0 + j3.9\%$$

$$Z_T = 2.2 + j3.0\%$$

CONCENTRATE 5.

Using the transformer base:

$$V_{\text{base}} = \frac{440}{\sqrt{3}} = 254;\quad I_{\text{base}} = \frac{250\text{ kVA}/3}{V_{\text{base}}}$$

$$I_{\text{base}} = 328\text{ A} \quad Z_{\text{base}} = \frac{254}{328} = 0.744\ \Omega$$

wire resistance (p.u.)

$$\frac{0.053\ \Omega}{1000\text{ ft}} \times \frac{500\ \Omega}{0.744} = 0.0342 \text{ p.u.}$$

$$Z_{\text{trans}} + R = 0.0442 + j0.05$$

$V_{in}$ is transformer input,
$V_L$ is assumed 1 p.u. in all cases:

$$V_{in} = V_L + (0.0442 + j0.05) I_L$$

UNCOMPENSATED

p.f. = .6 lagging.

$$|I| = \frac{120\text{ kW}/3}{254\,(0.6)} \times \frac{1}{328} = 0.8 \text{ p.u.}$$

$$V_{in} = V_L + (Z+R)\,0.8\angle{-\cos^{-1}.6}$$

$$= 1 + (.067\angle 48.5^\circ)(0.8\angle{-53.1})$$

$$V_{in} = 1.053\angle{-0.2^\circ}$$

CONCENTRATE 5 CONTINUED

COMPENSATION

rated wire current: $\frac{225}{328} = 0.686$ p.u.

The compensation will be just sufficient to deliver the same load power at 0.686 p.u. amps:

$0.686 \cos\theta = 0.8 \cos(\cos^{-1} .6)$

$\cos\theta = \frac{0.48}{0.686} = 0.7$

$I_{Line} = 0.686\angle -45.6°$

$V_{in} = 1 + (0.067\angle 48.5°) I_{Line}$

$V_{in} = 1.046$

The input voltage decreases from 1.053 to 1.046

KEEPING the primary voltage at 1.053, the load voltage increases by the same percentage:

$V_L = \frac{1.053}{1.046} = 1.007$ volts.

Concentrate 6.

(a) $I_L = 0.855\angle 0°$, $Z = 0.716\angle 65.2°$

$E_{g''m} = 0.935 - 0.855(j.25)$

$= 0.959\angle -12.9°$

$I_m = \frac{E_{g''m}}{j0.25} = 3.84\angle -102.9°$

$E_{g''g} = 0.935 + 0.855\,Z = 1.315\angle 25°$

$I_g = \frac{E_{g''g}}{Z} = 1.84\angle -40.2°$

Breaker RATINGS:

GEN: $1.84 \times 15$ MVA $= 27.6$ MVA @ 13.9 KV

Motor: $3.84 \times 15$ MVA $= 57.5$ MVA @ 13.9 KV

(b)

$I_L = 0.855\angle 36.9°$

$I_m = \frac{0.935 - j.25 I_L}{j0.25} = 4.308\angle -99.1°$

$I_g = \frac{0.935 - Z I_L}{Z} = 1.403\angle -28.6°$

Breaker RATINGS.

GEN: 21 MVA @ 13.9 KV

MOT: 64.6 MVA @ 13.9 KV

(C) $pf = 1$ $\quad I_{fault} = 4.96\angle -83.6°$

$pf = .8$ load $I_{fault} = 4.96\angle -83.6°$

Total fault current not affected by power factor.

**CONCENTRATE 7.**

As in example 5.20, the pre-fault current is $0.76\angle 2.3°$.

at point A:

$$V_{th} = 0.9 + (0.76\angle 2.3°)(0.02 + j0.25)$$
$$= 0.927\angle 11.85°$$

The impedance seen from point A is:

$$Z_{th} = (j0.09) \| (0.02 + j0.45) \| (1.6 + j1.2)$$
$$= 0.075\angle 89.2°$$

then:

$$I_{a1} = \frac{V_{th}}{Z_{th}} = 12.4\angle -77.3° \text{ p.u.}$$

For a balanced fault, $I_{a0} = I_{a2} = 0$ so $I_{fault} = I_{a1}$: $|I_{fault}| = 12.4$ p.u.

**CONCENTRATE 8.**

From problem 7:

$$E_{th,a} = 0.927\angle 11.85°$$
$$Z_{th,1} = 0.075\angle 89.2° = Z_1$$
$$Z_2 = j.085 \| [.02 + j.25 + \{j.195 \| (1.6 + j1.2)\}]$$
$$= 0.071\angle 89.3°$$
$$Z_0 = 3(j0.6) + j0.03 = j1.83$$

Then:

$$I_{a1} = \frac{e_{th,a}}{Z_0 + Z_1 + Z_2} = \frac{0.927\angle 11.85°}{1.967\angle 89.94°}$$

$$I_{a1} = 0.469\angle -78.1°$$

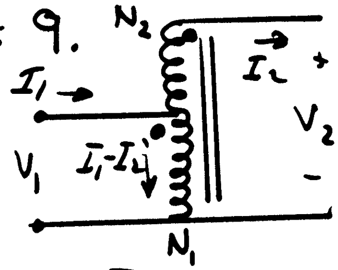
$$I_a = I_{a1} + I_{a2} + I_{a0}$$

but $I_{a1} = I_{a2} = I_{a0}$

then

$$I_a = 1.41\angle -78.1°$$

**CONCENTRATE 9.**




$$\frac{V_1}{N_1} = \frac{V_2}{N_2 + N_1}$$

$$N_1(I_1 - I_2) = N_2 I_2$$

RQD: $V_1 I_1 = V_2 I_2 = \frac{750 \text{ MVA}}{3}$

Common:

$$V_1(I_1 - I_2) = \text{RATING}$$

series: $(V_2 - V_1) I_2 = \text{RATING}$

$$V_1 = 345 \text{ kV} \quad I_1 = \frac{250\text{M}}{345\text{k}}$$
$$V_2 = 500 \text{ kV} \quad I_2 = \frac{250\text{M}}{500\text{K}}$$

Common RATING $= 345\left(\frac{250}{345} - \frac{250}{500}\right)\text{M} = 77.5$ MVA

Series RATING $= (500 - 345)\left(\frac{250}{500}\right)\text{M} = 77.5$ MVA

CONCENTRATE 10.

Using Motor base, assume 90% efficiency & p.f. = 0.85 lag.

$P_{out} = 0.9\,P_{IN}$; $VA_{in} = \frac{P_{in}}{0.85}$

$$VA_{IN} = \frac{500 \times 0.746}{0.9 \times 0.85} = 488 \text{ KVA}$$

CONVERTING SYSTEM Z to motor base:

$$Z_s = (0.05 + j0.2)\frac{488\text{KVA}}{100\text{ MVA}}$$

$$Z_s = 0.0078 + j0.0544 \text{ p.u.}$$

Converting transformer Z to motor base:

$$Z_t = .05\left[\cos 81.9° + j\sin 81.9°\right]\frac{488}{2500}$$

$$= 0.0014 + j\,0.0097 \text{ p.u.}$$

Estimating Motor starting reactance:

$$I_{start} = 6 \times \text{p.f.} = 5.1 \text{ p.u}$$

$$X_{START} = \frac{1}{5.1} = 0.1961$$

Assumption

At full-load steady state the motor voltage is 1 p.u. and the current is 1 p.u. Then: the source voltage is found:

$$V_s = V_L + I Z_t \qquad I = 1\angle{-31.8°}$$

$$V_s = 1 + (1\angle{-31.8°})(0.0092 + j0.0641)$$

$$= 1.043\angle{2.7°}$$

By line starting the motor, initially the voltage is determined by voltage division:

$$V_m = V_s \frac{Z_{START}}{Z_{start} + Z_t}$$

$$= 1.043 \frac{j0.1961}{.0092 + j0.2602}$$

$$= 0.786\angle{2°}$$

so the voltage is 78.6% of the running voltage for a dip of 21.4%

TIMED 1.

The Transformer base is used.

(a) $$HP_{rated} = \frac{0.9 \times 0.85 \times KVA_{rated}}{0.746}$$

$$\approx 1.025 \text{ KVA}_{rated}$$

$$KVA_{rated} = 3 \times \frac{0.44}{\sqrt{3}} I_{RATED}$$

$$= 0.762\, I_{RATED}$$

MAXIMUM VOLTAGE DROP – taking the starting current as $6 \times pf \times I_{rated}$:

## TIMED 1 CONTINUED

$I_{STARTING} = 5.1\, I_{rated}$

for a 3% maximum voltage drop:

$$0.03(p.u.) = 0.04 \times 5.1\, I_{rated}(p.u.)$$

$$I_{rated}(p.u.) = \frac{0.03}{0.04 \times 5.1} = 0.147$$

BASE CURRENT:

$$I_{base} = \frac{250\,KVA/3}{440V/\sqrt{3}} = 328\,A$$

$$I_{rated} = 0.147 \times 328 = 48.2\,A$$

$$KVA_{rated} = 0.762 \times 48.2 = 36.75$$

$$HP_{rated} = 1.025 \times 36.75 = 37.7\,HP$$

(b) Switching from Δ to Y results in 3 times the starting impedance, reducing the current by $\frac{1}{3}$. Thus a motor with 3 times the starting current & 3 times the HP could be used:

$$Hp_{rated} \rightarrow 3 \times 37.7 = 113.1\,HP$$

(c) A 40% tap corresponds to a turns-ratio of 1:0.4

So the impedance seen by the transformer primary would be $\frac{1}{(0.4)^2} Z_L = \frac{Z_L}{0.16}$

The starting current would then be $\frac{I_{rated}}{0.16}$

and

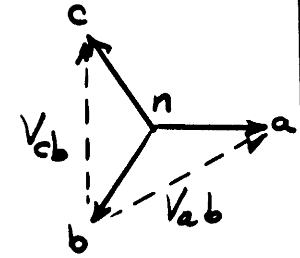
$$Hp_{rated} = \frac{37.7}{0.16} = 235.6\,HP$$

## TIMED 2.

$V_{an} = 254 \angle 0°$

$V_{ab} = 440 \angle 30°$

$V_{cb} = 440 \angle 90°$




BALANCED LOAD:

$$I_{an} = \frac{50\,KVA/3}{254} \angle -\cos^{-1} .8$$

$$= 65.6 \angle -36.9°$$

$$I_{cn} = 65.6 \angle 83.1°$$

(a) $\frac{|V_{ab}|^2}{10\,KW} = 19.36\,\Omega$

$$I_{1\phi} = \frac{440}{19.36} = 22.7\,A$$

in phase with $V_{ab}$

then

$$I_a = I_{an} + 22.7 \angle 30°$$

$$= 77.4 \angle -21.2°$$

$$S_{ab} = (440 \angle 30°)(77.4 \angle +21.2°)$$

## TIMED 2 CONTINUED

$S_{AB} = 34.05 \angle 51.23$

$= 21.3\text{ KW} + j\,26.5\text{ KVAR}$

$S_{CB} = (440 \angle 90°)(65.62 \angle -83.13°)$

$= 28.87 \angle 6.87°$

$= 28.66\text{KW} + j3.45\text{ KVAR}$

(b) $I_C = 65.6 \angle 83.1° + 22.7 \angle 90°$

$= 88.2 \angle 84.9°$

$S_{CB} = (440 \angle 90)(88.2 \angle -84.9°)$

$= 38.81 \angle 5.1°$

$= 38.65\text{KW} + j3.45\text{KVAR}$

$S_{AB} = (440 \angle 30°)(65.6 \angle 36.9°)$

$= 28.87 \angle 66.9$

$= 11.34\text{KW} + j\,26.56\text{KVAR}$

## TIMED 3.

The 25 MVA Base is used.
Converting 15 MVA impedance TO 25 MVA BASE

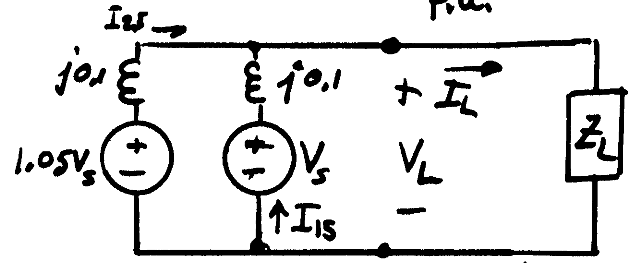
$Z_{15} = j0.06 \times \frac{25}{15} = j0.10$

So both windings have the same impedance.

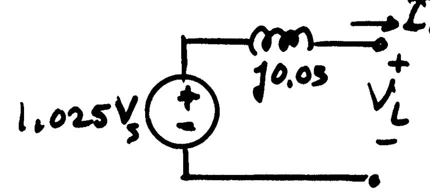
(a) With equal reactances and equal voltages, the two transformers carry equal currents, i.e. 50% of the load current.

(b) The load voltage is taken as 1 p.u., and on the 25 MVA base,

$I_L = \frac{30\text{MVA}}{25\text{MVA}} \angle \cos^{-1} .8 = 1.2 \angle -36.87°$ p.u.




The thevenin equivalent of the transformers:




$1.025\,V_S = V_L + j0.05\,I_L$

$= 1 + 0.037 + j0.048$

So $V_S = 1.012 \angle 2.65°$

then

$I_{25} = \frac{1.05V_s - 1}{j0.1} = 0.787 \angle -51.4°$

$= 0.491 - j0.615$

TIMED 3 CONTINUED

and

$$I_{1S} = \frac{V_S - 1}{j0.1} = 0.481\angle{-13.1^\circ}$$

$$= 0.468 - j0.109$$

and

$$I_L = I_{2S} + I_{1S} = 0.959 - j0.724$$

$$= 1.202\angle 37.0^\circ$$

which checks within rounding errors.

$I_{circ}$: $I_{2S} = I_{circ} + \frac{I_L}{2}$

$$I_{1S} = \frac{I_L}{2} - I_{circ}$$

$$\therefore I_{circ} = \frac{I_{2S} - I_{1S}}{2}$$

$$= \frac{0.023 - j0.505}{2}$$

$$= 0.253\angle{-87.4^\circ}$$

TIMED 4.

Using the calculations of CONCENTRATE 8:

$$e_{th,a} = 0.927\angle 11.5^\circ$$

$$Z_1 = 0.075\angle 89.2^\circ$$

$$Z_2 = 0.071\angle 89.3^\circ$$

Then, from figure 5.18:

$$-I_{a2} = I_{a1} = \frac{e_{th,a}}{Z_1 + Z_2} = 6.35\angle{-77.4^\circ} \text{ p.u.}$$

Using eq. 5.150

$$I_{fault} = \sqrt{3}\, j I_{a1} = 11.0\angle 12.7^\circ$$

Using eq. 5.144 with $V_{a0} = 0$

$$V_a = -2V_b$$

eq. 5.145:

$$3V_a = V_a + (a + a^2)V_b = -3V_b$$

figure 5.18:

$$V_{a1} = \frac{Z_2}{Z_1 + Z_2} e_{th,a} = 0.464\angle 11.9^\circ$$

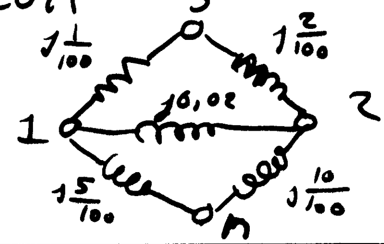
$$V_b = -0.464\angle 11.9^\circ$$

$$V_a = 0.928\angle 11.9^\circ$$

TIMED 5.

As no current flows when bus #3 is open, $e_{th,a} = 1\angle 0^\circ$

The thevenin equivalent impedance seen from bus #3 is found by shorting the sources, to obtain a bridge circuit



TIMED 5 CONTINUED

Notice that the bridge is balanced, so that no current flows in the branch from bus 1 to bus 2. So busses 1 and 2 constitute a single node, and

$$Z_{th} = \frac{j}{100} \Big\| \frac{j2}{100} + \frac{j10}{100} \Big\| \frac{j5}{100}$$

$$= j0.04$$

From figure 5.18:

$$-I_{a2} = I_{a1} = \frac{e_{th}}{Z_1 + Z_2} = 12.5 \angle -90°$$

From eq 5.150:

$$I_{fault} = -j\sqrt{3}\, I_{a1} = -21.65 \angle 0°$$

Because the voltages at Busses 1 and 2 are equal, the current delivered by each is inversely proportional to the impedance To bus 3:

$$I_{13} = \frac{2}{3} I_{fault}, \quad I_{23} = \frac{1}{3} I_{fault}.$$

at bus #1:

$V_{a1} = 1\angle 0$ no fault current, fault between b and c.

$$V_{1b} = 1\angle -120° - j0.05 \times \frac{2}{3} I_{fault}$$

$$= 0.52 \angle -164° \text{ p.u.}$$

$$V_{1c} = 1\angle +120° - j0.05 \times \frac{2}{3} I_{fault}$$

$$= 1.66 \angle 107.5° \text{ p.u.}$$

at bus #2

$$V_{2a} = 1 \angle 0$$

$$V_{2b} = 1\angle -120° - j0.1 \frac{1}{3} I_{fault}$$

$$= 0.52 \angle -164° \text{ p.u.}$$

$$V_{2c} = V_{1c} = 1.66 \angle 107.5° \text{ p.u.}$$

TIMED 6.

THE TRANSFORMER PRIMARY is rated at 60 MVA for the secondary and 21 MVA for the tertiary, so its total rating is 81 MVA.

Assuming all impedances are referred to the primary, and given in the primary base:

$$Z_{base} = \frac{(138\text{kV})^2}{81\text{MVA}} = 235\,\Omega$$

TIMED 6. CONTINUED

$$Z_\ell = \frac{j3.1}{235} = j0.0132 \text{ p.u.}$$

(a) For a 3-phase fault on the X winding, ignoring any T-current (which would be quite small)

$$Z_\ell + Z_{H\text{-}X} = j.0132 + j0.0977$$
$$= 0.111j \text{ p.u.}$$

Then the primary fault current is:

$$I_{H\text{-}fault} = \frac{1}{0.111j} = -j9.02$$

The actual fault current is amplified by the turns ratio $\frac{138\text{KV}}{34.5\text{KV}} = 4.0$

The primary current base is $\frac{81\text{ M}/3}{138\text{ K}/\sqrt{3}} = 339\text{ A}$

FAULT current $= -j339 \times 4 \times 9.02$
$= 12{,}231\angle -90^\circ$ A

(b) For a single-phase fault to ground (see fig. 5.15e) with both H and X in grounded neutral configurations:

$$Z_0 = Z_1 = Z_2 = j0.111$$

Then, from fig 5.17(b)

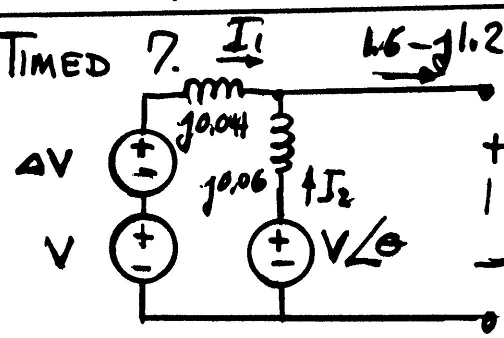
$$\bar{I}_{a1} = \frac{e_{th,a}}{Z_1+Z_2+Z_0} = \frac{1}{3 \times j0.111} = -j3$$

From eq. 5.136:

$$\bar{I}_a = 3\bar{I}_{a1} = -j9 \text{ p.u.}$$

$$I_{FAULT} = 4 \times 339 \times (-j9)$$
$$= -j12{,}200 \text{ A}$$

TIMED 7.




Transformer base is used

$$I_1 = \frac{V(1+\Delta)-1}{j0.04}$$

$$I_2 = \frac{V\cos\theta + jV\sin\theta - 1}{j0.06}$$

Because $I_1 + I_2 = 1.6 - j1.2$ and $|I_1| = |I_2| = 1$ p.u., it is necessary that

$$I_1 = I_2 = 0.8 - j0.6$$

TIMED 7. CONTINUED

then:

$$V(1+\Delta) = j0.04[0.8 - j0.6] + 1$$

$$V(1+\Delta) = 1.0 \angle 1.8^\circ$$

$$V\angle\theta = j0.06[0.8 - j0.6] + 1$$

$$= 1.037 \angle 2.7^\circ$$

Then $|V| = 1.037$

an $\angle\theta = 2.7 - 1.8 = 0.9^\circ$

$$V = 1.037 \angle 1.8^\circ$$

$$1 + \Delta = \frac{1.026}{1.037} = 0.989$$

$$\Delta = -0.011$$

as $\Delta = \frac{.1}{32/2} n \quad n = 1.8 \to 2$

$$\theta = 0.9^\circ$$

TIMED 8.

The load is $\frac{34 MW}{0.85 pf} = 40 MVA$, so the transformer must be on the 50 MVA setting.

Then: $Z_{base} = \frac{(12 KV)^2}{50 K^2 VA} = 2.88 \Omega$

using the transformer secondary base.

On the same base:

$$Z_l = \frac{1.555}{2.88}(0.02 + j0.1)$$

$$= 0.011 + j0.054 \text{ p.u.}$$

$$Z_{tr} = R(1+j15) \therefore R = \frac{.075}{|1+j15|} = 0.005$$

$$Z_{tr} = 0.005 + j0.075 \text{ p.u.}$$

$$V_{LOAD} = \frac{12.16}{12} = 1.013 \text{ p.u.}$$

$$I_{Load} = \frac{40 MVA/50 MVA}{1.013} \angle \cos^{-1} 0.85$$

$$= 0.789 \angle -31.8^\circ$$

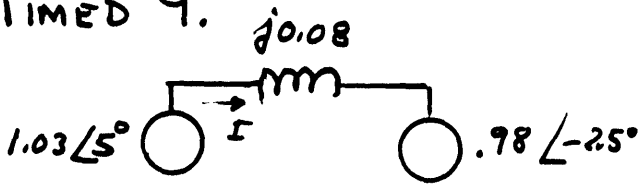
$$Z = Z_{tr} + Z_l = 0.13 \angle 82.9^\circ$$

$$V_S = IZ + 1.013 = 1.08 \angle 4.2^\circ$$

REGULATION: $1.08 - 1.013 = 0.067$

or 6.7%

TIMED 9.




$$I = \frac{1.03\angle 5^\circ - 0.98\angle -2.5^\circ}{j0.08} = 1.758 \angle -19.5^\circ$$

$$S = VI^* = 1.03\angle 5^\circ \times 1.758 \angle +19.5^\circ$$

$$= 1.810 \angle 24.5^\circ$$

$$= 1.648 + j0.751$$