4

ACTUATORS

Actuators are the devices that make robots move. There is a plethora of such devices in the field, and we will restrict our attention in this chapter to four: step motors, DC motors, hydraulic pistons, and pneumatic pistons. The emphasis will be on DC motors and hydraulic systems because of the prevalence of these types of actuators in modern industrial robots. In studying the similarities and differences between these two types of devices, we will learn fundamental principles which we will be able to apply to control of many types of actuators.

4.1 THE DC MOTOR

As do all electromechanical devices, the DC motor makes use of the fact that a wire carrying a current in a magnetic field experiences a force. In a DC motor, the windings wrapped around a rotating armature carry the current. An arrangement of commutator segments and brushes (Figure 4.1) ensures that the DC current is always in the same direction relative to the magnetic field, thus resulting in a constant force direction (or torque).

The principal variation among different types of DC motors lies in the mechanism used to develop the magnetic field. In a permanent magnet DC motor, the field is developed, as the name suggests, by permanent magnets. In such a motor, the torque, T_m, is related to mag-

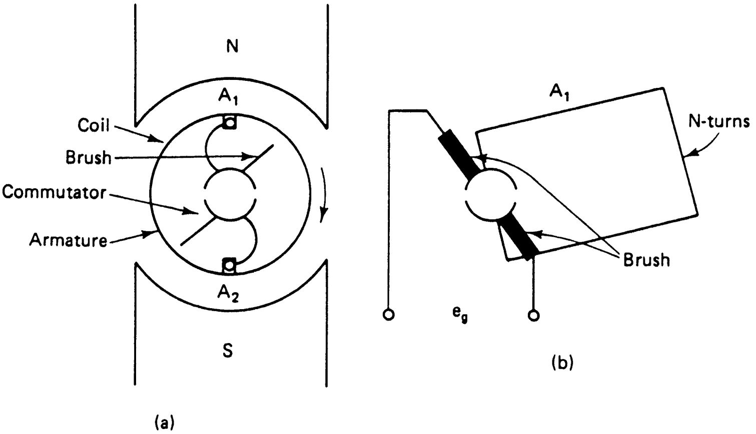


Figure 4.1 An elementary DC machine. (a) A cross-sectional view of poles and armature. (b) A view of the armature coil and the slip rings.

netic flux, Φ, and armature current, I_a by

$$T_m = K_t \Phi I_a \tag{4.1}$$

under steady state conditions, where K_t is a proportionality constant. Since Φ is a constant in a permanent magnet motor, we can say that in the steady state, torque is proportional to armature current.

The magnetic field can also be generated by an electromagnet. If the current for the electromagnet is provided on a pair of wires separate from the armature current, then

$$T_m = K_t K_f I_f I_a. \tag{4.2}$$

where I_f is the current in the field windings and K_f is a constant depending on the number of winding turns and the permeability of the iron around which the windings are turned.

If I_f is supplied by a separate source (Figure 4.2), then the basic

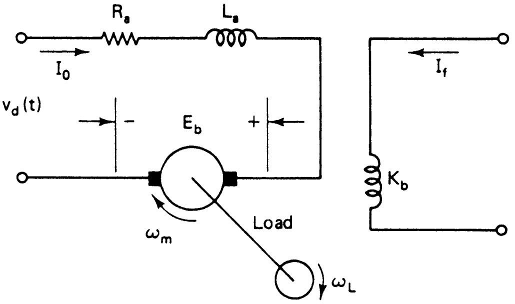


Figure 4.2 (Gourishankar) An armature-controlled DC motor. (*Courtesy Harper & Row*)

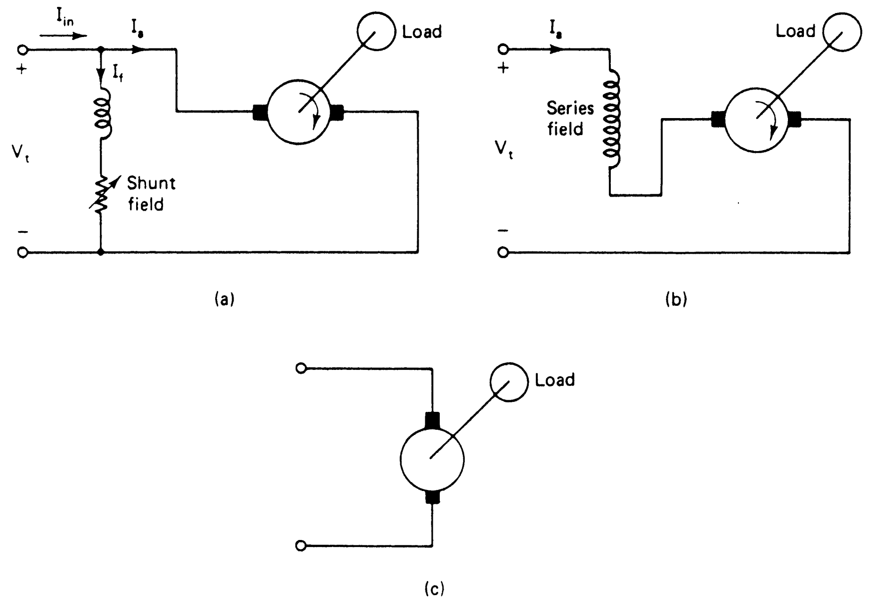


Figure 4.3 (Gourishankar) (a) A DC shunt motor; (b) a DC series motor; (*Courtesy Harper & Row*) (c) Permanent magnet DC motor.

torque-current relation of Eq. 4.1 is still true. In many instances, however, it is desirable to derive the field current from the same pair of wires that provides the armature current. Figure 4.3 shows two of the more popular ways in which to accomplish this, shunt and series motors. The steady-state equivalent circuits are shown in Figure 4.4. In these cases, analysis is complicated by the fact that the motor, when turning, acts as a generator, producing the back EMF (electromotive force), E_b.

Now, the analysis depends on the driving source. For example, in the shunt motor, if V_t (Figure 4.4) is held constant, then torque varies linearly with armature current.

In the case of the DC series motor, we have $\Phi = K_f I_a$ and

$$T_m = K_t K_f I_a^2 \tag{4.3}$$

and the torque varies as the square of the current.

Most "servo" motors are permanent magnet motors and so obey

$$T_m = K_{pm} I_a \tag{4.4}$$

where K_{pm} results from the product of K_t and Φ in Eq. 4.1, with Φ a constant. This is the model that we will use to analyze the behavior of servo systems in Chapter 5. The other types of DC motors, which drive their field magnetization electrically, may likewise be analyzed, but the

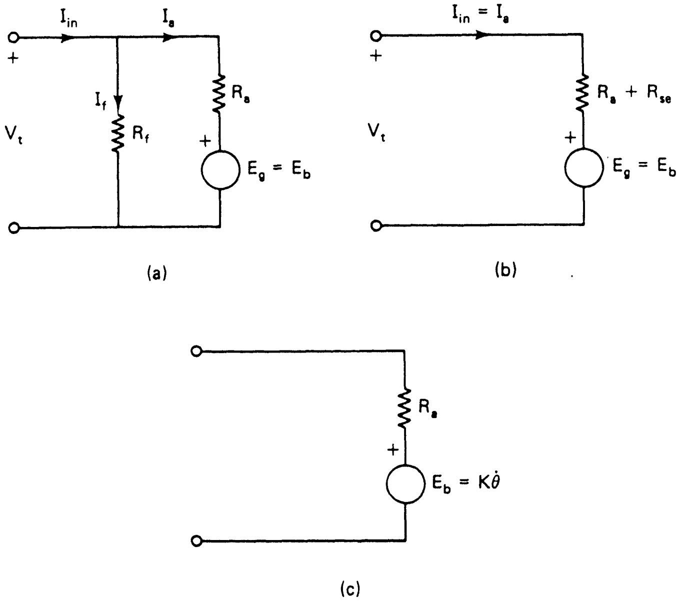


Figure 4.4 (Gourishankar) Equivalent circuits of the motors in Figure 4.3. (*Courtesy Harper & Row*)

analysis is more complex than that required for presentation of fundamental robotics concepts.

4.1.1 Loading DC Motors

The primary loads on motors are friction, inertia, and constant or varying torque loads. At this point, we will consider only friction and inertia, designated F and J, respectively. A rotating physical system, in the absence of outside forces, obeys

$$T = J\ddot{\theta} + F\dot{\theta} \tag{4.5}$$

where T is the torque exerted, θ is the angular position measured in radians, $\dot{\theta}$ is angular velocity in radians per second, and $\ddot{\theta}$ is angular acceleration measured in radians per second squared. The frictionless version of this relation,

$$T = J\ddot{\theta} \tag{4.6}$$

may be seen to be the rotational dual of Newton's law,

$$F = ma \tag{4.7}$$

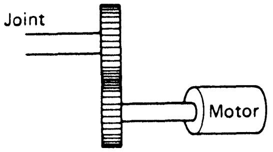


Figure 4.5 N is the number of teeth on the larger gear divided by the number of teeth on the smaller gear.

DC motors typically are capable of high rotational velocities and relatively low torque. Therefore, gearing is used to trade decreased speed for increased torque (Figure 4.5.). If N represents the gear ratio, we find

$$T_{(\text{applied to load})} = N \cdot T_{(\text{applied by motor})} \tag{4.8}$$

$$\dot{\theta}_{\text{load}} = \frac{1}{N}\,\dot{\theta}_{\text{motor}} \tag{4.9}$$

The load is divided by the square of the gear ratio, resulting in equations for the equivalent inertia and friction seen by the motor, J_{eq}, and F_{eq}, respectively.

$$J_{eq} = J_a + \frac{1}{N^2}\,J_1 \tag{4.10}$$

and

$$F_{eq} = F_a + \frac{1}{N^2}\,F_1 \tag{4.11}$$

where J_a and F_a are the inertia and friction of the motor itself and J_1 and F_1 are the inertia and friction of the load. Gear ratios of 100 : 1 are not uncommon, and Eq. 4.10 says that in this case, the inertia of the load is divided by 10^4 before being perceived by the motor. Thus, while the armature inertia of the motor itself may seem at first to be trivially small compared with the load inertia, the gearing may make the exact opposite true.

Equation 4.11 should be taken with a grain of salt, since it assumes frictionless gears. In robot systems, most of the friction is likely to be in the gears themselves, and Eq. 4.11 does not accurately model this effect. Furthermore, the gear friction varies considerably with the temperature of the lubricant. We will see in Chapters 5 and 11 some techniques for dealing with such nonlinear loads.

Example 4.1 Loading of a DC Motor

A permanent magnet DC motor with armature inertia of

$$5 \times 10^{-3} \text{ kg-m}^2$$

is used to drive a load of mass equal to 10 kg. The load is located at an (effective) radius of 0.5 m. A 100:1 gearing is used. Find the equation relating torque to angular rotation. Ignore gear inertia and friction.

Solution: The load inertia is divided by the gear ratio squared, producing

$$J_{leff} = \frac{J}{N^2} = \frac{M_1 r^2}{N^2} = \frac{(10)(0.5)(0.5)}{(100)^2} = \frac{2.5}{10^4}$$

$$J_{leff} = 2.5 \times 10^{-4} \text{ kg m}^2$$

$$J_{eq} = J_a + J_{leff} = 50 \times 10^{-4} + 2.5 \times 10^{-4} = 5.25 \times 10^{-3} \text{ kg m}^2$$

Then

$$T = J_{eq}\,\ddot{\theta}$$

4.1.2 Driving the DC Motor

The DC motor provides a torque which, by Eq. 4.1, is directly proportional to the armature current. (We will henceforth discuss only the permanent magnet DC motor.) Since it will provide this torque, even when stalled, it is desirable to drive the motor with a controllable DC current source. Furthermore, since it will be necessary to drive the motor in either direction, such a DC source must be capable of supplying both positive and negative currents. The most straightforward mechanism for accomplishing this is the DC-coupled push-pull amplifier.

Push-pull Amplifiers

The push-pull amplifier of Figure 4.6 provides a voltage out at high current levels in response to a signal voltage in. Such amplifiers are the standard circuit configuration used in *servo amplifiers*, which may be obtained commercially.

Since torque in a DC motor is proportional to current, and we really wish to control torque, it would be very desirable to have an amplifier in which the current out (as opposed to the voltage out) is proportional to the voltage in. Since a DC motor in motion acts as a generator,* the current through the armature windings is not simply related to the applied voltage. (This will be discussed in more detail in Section 5.5). Thus, controlling the voltage, as in the case of the push-pull amplifier, does not directly control the torque.

*That is, a voltage, referred to as the "back EMF," is generated that is proportional to motor velocity and is opposing the applied current flow. See Figure 4.8.

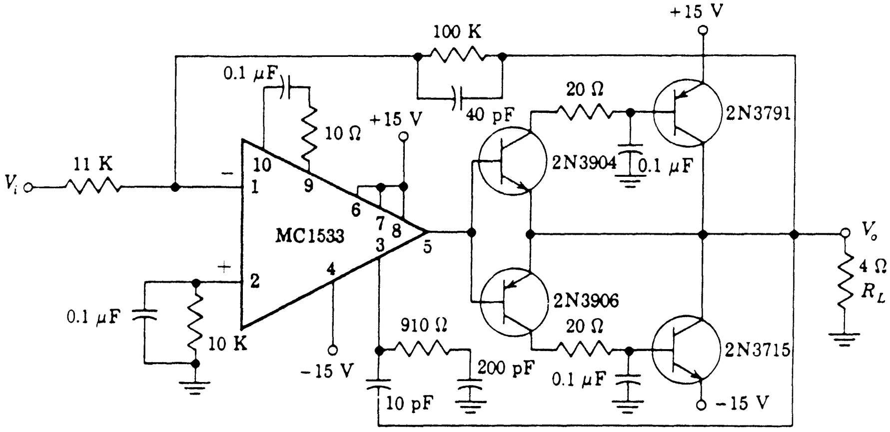


Figure 4.6 Push-pull amplifier. (*Courtesy Motorola*)

Adding a sense resistor and an operational amplifier to a push-pull circuit can provide a current out in response to a voltage in. With this simple addition (Figure 4.7), direct control of torque is possible. In this circuit, the motor current is sensed by a resistor that is small with respect to motor resistance, and the resulting signal is fed back to the operational amplifier. Increasing the voltage out of the D/A will cause the current out of the amplifier to increase proportionally.

It should be noted that a control requiring both high current and high speed simultaneously may place unreasonable demands on a power amplifier. To see this, consider the equivalent circuit of a DC motor as shown in Figure 4.8. As the motor turns rapidly ($\dot{\theta}$ is large), a signif-

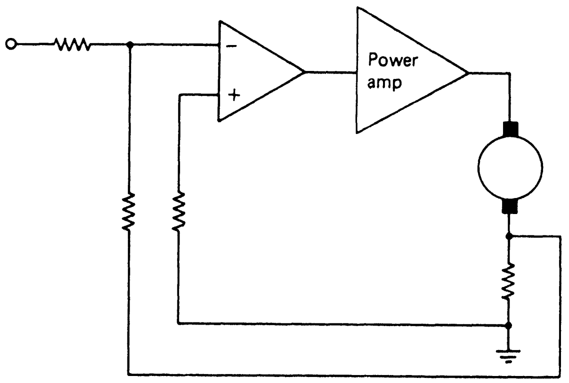


Figure 4.7 Addition of a sense resistor and another operational amplifier to a power amplifier provides a controllable current source.

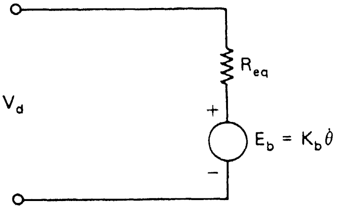


Figure 4.8 Equivalent circuit of a DC motor.

icant back EMF, E_b, is generated which opposes the applied V_d. To maintain a constant current, V_d must increase, which the feedback current source of Figure 4.7 will do. However, V_d is limited by the power supply values, which may be quickly reached at high motor speeds. If the velocity is so high that V_d cannot exceed E_b sufficiently to maintain the current flow through the resistance of the armature, the current will drop and current/torque control will be lost.

A second consideration in designing a motor driver is the fact that excessive armature current will demagnetize the PM (permanent magnet) field magnets. This circumstance generally leads to a requirement for current control.

These observations should lead the reader to realize that simple linear models of power supplies and actuators are in general not adequate to predict accurately the performance of robot systems, and we must strive to develop control schemes that are *robust*, that is, schemes that will give good performance in spite of our inability to develop a simple mathematical model of the system.

Example 4.2 Electrical Load Presented by DC Motor

The motor described in Example 4.1 is subjected to some tests: When stalled, it draws 10 A from a 24-V supply. When allowed to run unloaded, it accelerates and eventually reaches a maximum rotational speed of 3000 rpm (revolutions per minute). (This is the rotary speed of the *motor* shaft, not the output of the gear, which turns 100 times more slowly.) Determine the electrical characteristics of the motor.

Solution: At stall, we determine the resistance of the motor windings in 24/10 = 2.4 Ω (ohms).

Using Figure 4.8, we write a loop equation and determine

$$24 \text{ V} = I(2.4\ \Omega) + K_b\ (3000 \text{ rpm})$$

We appear to have an insoluble problem, since both back EMF gain and current are unknown. However, we were given one more piece of information. 3000 rpm is the *maximum* speed attained. Since the speed has "topped out," there is no acceleration, but since

$$T = J\ddot{\theta},$$

no acceleration means no torque. Since

$$T = K_m I$$

no torque means no current, and the loop equation simplifies to

$$K_b = \frac{24}{3000} = 8 \times 10^{-3} \text{ V/rpm}$$

Pulse-width Modulation

Rather than drive the DC motor with a constant voltage or current, as was described in the previous section, one could drive the motor with a rapidly changing current. If the rate of change of that current is far higher than the response speed of the motor and attached physical system, the net effect will be a response to the DC component of the drive signal.

One popular way in which to achieve this type of control is to use pulses, as shown in Figure 4.9. The repetition rate, T_c, is a constant and the pulse "on" voltage, indicated positive in Figure 4.8, will be negative for reverse drive. The ratio T_+/T_c defines the *duty cycle*. The average (DC) value of the drive signal is

$$V_{AV} = V_+ \left(\frac{T_+}{T_c}\right) \tag{4.12}$$

The principal advantage of PWM (pulse-width modulation) over linear control is the simplicity of the drive electronics and the ease of computer interfacing. Figure 4.10 shows a simple circuit adequate for driving a small DC motor using PWM. The computer may be used to control the state of the A and B control lines. Timing may be accomplished via software timing loops or by interrupts from an internal timer. Many microprocessors contain software-settable interval timers that make such functions readily available. Since the transistors operate in an on-off mode, the biasing is not critical, and there is no need for temperature compensation. Some care must be taken to ensure that both transistors are never on at the same time and that provisions exist for stopping the motor when the computer is not operational.

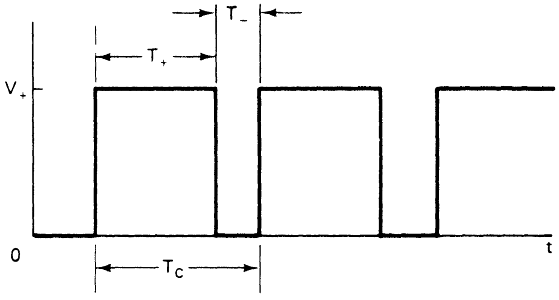


Figure 4.9 Pulse-width modulation.

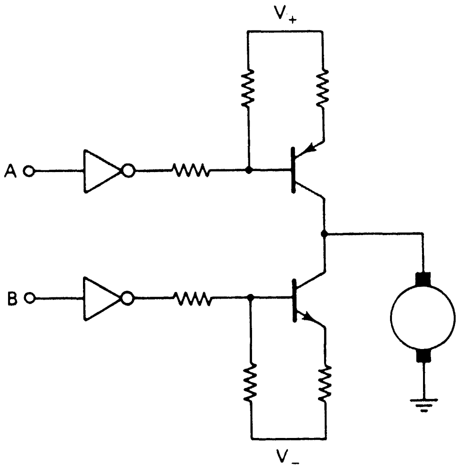


Figure 4.10 A two-transistor circuit designed for driving a DC motor using pulse-width modulation (note that the circuits indicated here as inverters may in fact be other circuits such as operational amplifiers).

An additional advantage of using PWM is the simplicity of computer interface, since only two output bits are required.

There are two disadvantages to the use of PWM. First, current control is not possible without significant additional hardware. If direct control of torque is required, as would be the case, for example, if force control were needed, then linear operation with an amplifier is preferable to PWM.

A second disadvantage associated with PWM is the electrical noise which is created by the rapid switching of current through an inductor (the armature). Proper use of the shielding techniques of Chapter 3 can eliminate this as a problem.

In summary, PWM provides the quickest and easiest mechanism for attaining proportional control of a DC motor. However, it lacks the flexibility required for more sophisticated control strategies such as control of applied force.

4.2 STEP MOTORS

With the continuing *computerization* of electronics, it has become desirable to have a simple digital positioning device. Such a device could be controlled by digital pulses on two inputs: move clockwise and move counterclockwise. The step motor provides one solution to this problem. In this section we will discuss the step motor and its potential applications. We will use a model for the step motor which is slightly simpler than realistic motor systems but which presents the basic concepts.

4.2.1 Organization of the Step Motor

Figure 4.11 shows the electrical organization of a three-phase step motor. The armature is wound about an iron rod and is energized with a DC current. A system of slip rings maintains the DC nature of the armature current as the armature rotates. The armature can thus be considered as a simple electromagnet.

At any instant in the operation of a motor, exactly one pair of pole piece windings is energized. Furthermore, each pair (e.g., windings 1a and 1b of Figure 4.11) is wound in such a way that their magnetic fields are collinear. The armature will then rotate until the armature's magnetic field is aligned with the fields of the single pair of pole pieces. This condition is shown in Figure 4.11 with pole pieces 1a and 1b energized. This is a stable condition, and given sufficient time, the motor will come to rest in this position.

If the motor is at rest as shown in Figure 4.11, coil pair 1 is de-energized and, simultaneously, coil pair 2 is energized, then the armature will rotate to bring the armature electromagnet into alignment with coil pair 2. A single "step" of this three-phase motor thus results in a motion of 360/6 = 60 degrees.

By using more pole pieces and by using more complex armature windings, motors may be built with many more steps per revolution; 24 is common.

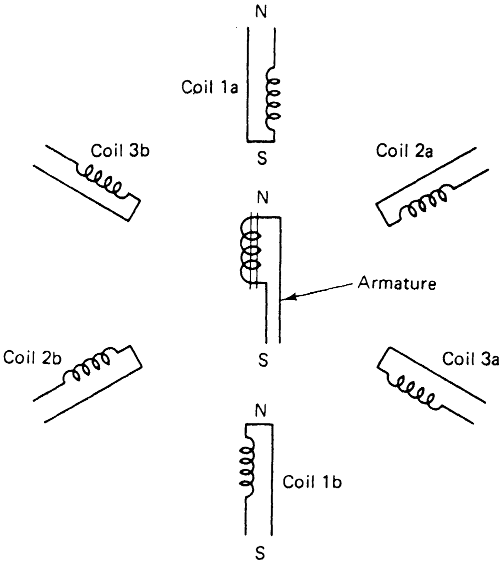


Figure 4.11 Schematic organization of a step motor.

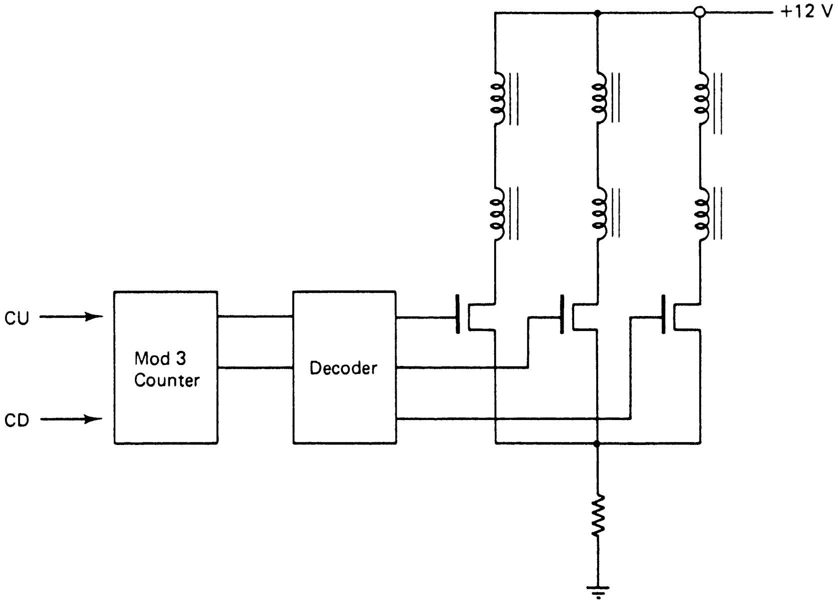


Figure 4.12 Representation of a step motor driver.

4.2.2 The Step Motor Driver

Figure 4.12 shows a schematic representation for a step motor driver. A modulo-3 counter followed by a decoder selects one of three possible FET's (field-effect transistors). The selected FET, when energized, allows current to flow through one coil pair.* CU (count up) pulses increment the counter and energize the coils in sequence, resulting in clockwise rotation of the motor. CD (count down) pulses decrement the counter and result in counterclockwise rotation.

4.2.3 Performance of Step Motors

To cause a specific clockwise rotation, the computer need only output the appropriate number of CU pulses, resulting in a very simple control scheme. Since no feedback is required, this scheme has found a great number of applications, particularly in printers and plotters.

*Figure 4.12 shows field-effect transistors being used to switch the current through fairly high inductance coils. Such switching would result in unacceptably high transient voltages. Hence, commercially available driver circuits are somewhat more complex than shown in this figure. This figure also omits the current reversing logic required to keep the pole alignment correct.

In robotics applications, step motors have found less application, as we shall see.

Figure 4.13 shows the velocity response of a typical step motor running in *single-step mode.* The negative excursions of $\dot{\theta}$ indicate a momentary direction reversal of the motor. This behavior typically results in a high degree of vibration (ringing) in the object being moved. (CU pulse rates of 100 hertz are typical.) The frequency of the ringing is a function of the frictional and inertial loading of the motor. Ringing in single step mode can be reduced by passive *inertial dampers*, which are available from step motor manufacturers.

As the input clock rate is increased, the point will be reached where the next clock pulse occurs before the direction reversal. When running in this mode, the step motor is said to be *slewing*. By slewing, high motor speeds can be accomplished with much less vibration. When a motor is slewing, however, positioning errors can occur. If a motor is moving rapidly, in slewing mode, and the control pulses suddenly stop, the motor may easily have enough forward momentum to carry it all the way past the appropriate stopping point around to the next stable point. In the motor of Figure 4.11, this is an entire revolution. Since the motor has no feedback sensor, it is impossible to detect that this error has occurred.

If the inertia of the load is a constant, then acceleration and deceleration algorithms can be developed that allow step motors to be used at high speed without loss of positional accuracy. Such algorithms are commonly implemented in printers and plotters. Unfortunately, when inertia changes, as it does in robots, the physical performance of the motor system may be faster or slower than that used in the design, and the controller may "lose" steps.

This lack of feedback is both the greatest attribute and the greatest

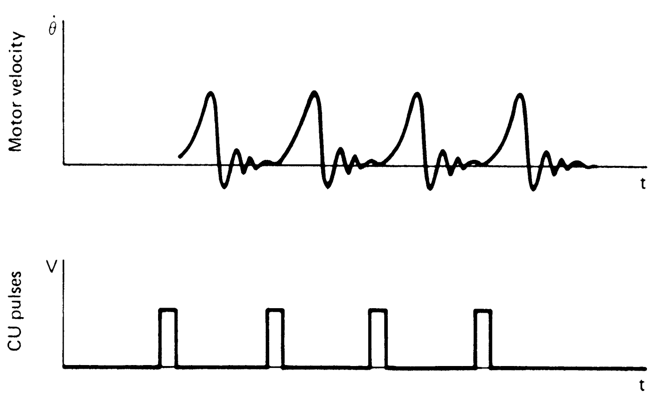


Figure 4.13 Response of a step motor (angular velocity versus time).

drawback of the step motor. On the one hand, it provides exceptional simplicity of interface and control, but the range of applications where this control is suitable is quite limited.

An error of one or two steps over a large motion may not be significant, so if the actuator returns to a standard "reset" position fairly often, errors will not accumulate to a measurable extent. The reset position is generally determined by the closing of a switch, introducing a simple degree (1 bit) of feedback.

More sophisticated feedback strategies may be used with step motors, including even force control (Kuo, 1979). However, if feedback needs to be introduced, one is forced to question the use of a step motor at all, since it provides a lower power to weight ratio than a comparable DC motor and may require a controller of comparable complexity.

4.3 HYDRAULIC ACTUATORS

Hydraulic systems make use of an incompressible fluid, an oil, which is forced under pressure into a cylinder. The cylinder contains a piston which moves in response to the pressure on the fluid. Both rotary and telescoping (prismatic) actuators are available, and either may be the actuator of choice in robot applications requiring high power. In this section, we will discuss the principles of operation of hydraulic actuators and compare them with electric actuators, in preparation for the discussion of control of such systems in Chapter 5.

4.3.1 Principles of Operation of Hydraulic Actuators

The double-acting hydraulic piston is the principal moving part in a hydraulic system. In such a piston (Figure 4.14), fluid can flow into side A and out of side B or vice versa, resulting in movement of the piston to the right or left respectively.

Control over the direction of fluid flow is accomplished by the hydraulic servo valve, detailed in Figure 4.15. A small, high-precision electric motor displaces the valve piston slightly, allowing fluid to flow from the source to the actuator over one hose, returning to the valve over another hose. If displaced in the opposite direction, the valve directs flow in the opposite direction. It should be noted that Figure 4.15 is a simplification of the true construction of a servo valve.

The ideal hydraulic rotary actuator provides shaft torque, T_m, proportional to differential pressure, ΔP_1, across the servo valve,

$$T_m = K_P D_m \Delta P_1 \qquad (4.13)$$

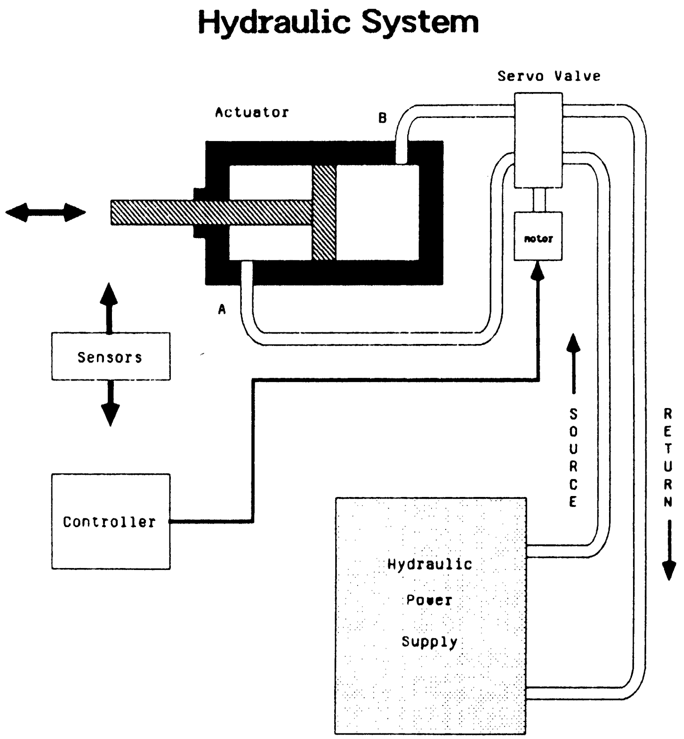


Figure 4.14 Hydraulic systems.

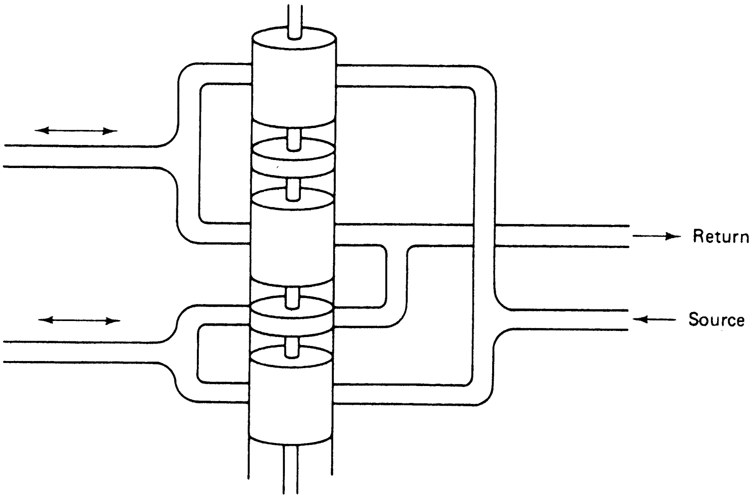
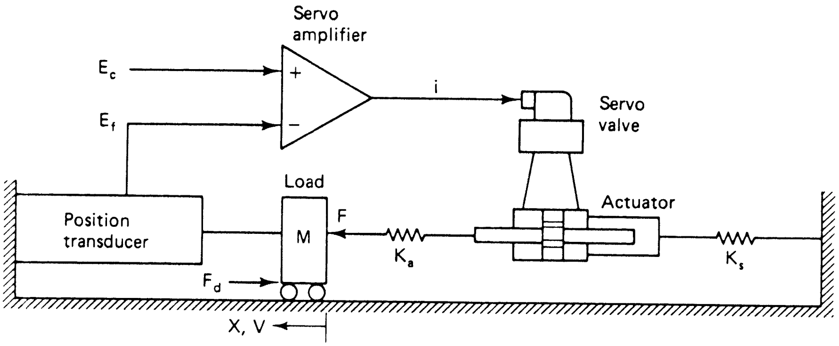


Figure 4.15 Hydraulic servo valve.

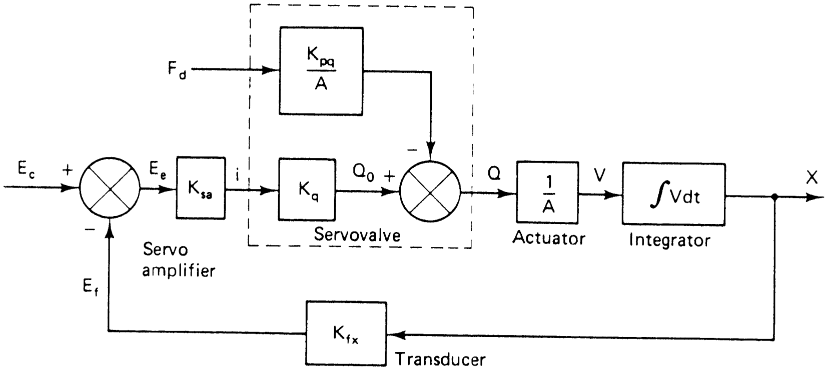
where D_m is the displaced volume measured in cubic inches. The ideal hydraulic actuator also provides angular velocity, $\dot{\theta}_m$ proportional to flow, Q:

$$\dot{\theta}_m = \frac{K_Q}{D_m} Q \tag{4.14}$$

Either $\dot{\theta}_m$ or T_m may be controlled by use of the appropriate type of servo valve (but not both simultaneously, since they are interrelated). Flow control valves are the most common, and Figure 4.16 shows a block diagram of a control system using such a valve. In Chapter 5, we will discuss the control of hydraulic systems in more detail. At this point, only a few additional comments are required regarding implementational details.



(a) Schematic diagram



(b) Idealized block diagram (dynamic effects neglected)

Figure 4.16 (Clark) A hydraulic servo system utilizing position feedback. (*Courtesy MOOG*)

4.3.2 Implementation Considerations

Clark (1969) points out that

> there are several significant advantages to electrohydraulic servo systems over electric motor drives:
>
> 1. Hydraulic drives have substantially higher power to weight ratios, resulting in higher machine frame resonant frequencies for a given power level.
> 2. Hydraulic drives are much stiffer than electric drives, resulting in higher loop gain capability, greater accuracy, and better frequency response.
> 3. Hydraulic drives give smoother performance at low speeds and have a wide speed range without special control circuits. They can usually be direct-coupled to the load without the requirement for intermediate gearing.
> 4. Hydraulic drives are self-cooling and can be operated in a stall condition indefinitely without damage.

While Clark's evaluations are valid, electric motor drives may be more appropriate for certain applications because of the following considerations.

First, it must be emphasized that the hydraulic fluid is incompressible, and when the servo valve is closed, flow cannot occur. This means that when a control system reaches the no-flow condition, it is impossible to backdrive the actuator without incurring physical damage. Thus, compliance with external forces is more difficult to accomplish than with DC motors. It is not impossible; the external forces can be explicitly sensed and the sensory data used to control the actuator. However, this is a fairly complex control technique, as we will learn in Chapter 11.

Second, hydraulic systems are highly nonlinear. These nonlinearities make it more straightforward to implement sophisticated and delicate control using electric drives.

These issues will be discussed in Chapter 5 as they relate to computer control of position and velocity of hydraulic systems and in Chapter 11 as they relate to force control and compliance, important topics for assembly applications.

4.4 PNEUMATIC ACTUATORS*

In a pneumatic control system, a compressible fluid, air, is used to drive a piston. As in the case of hydraulic actuators, an electrical signal controls a valve which, in turn, controls the flow to a cylinder. Some of

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*The author is grateful to Windell Malpass of Schraeder Bellows Corp. for his assistance in preparation of this section.

the variations in complexity of pneumatic control systems will be discussed in the paragraphs that follow.

In the simplest pneumatic control system, a solenoid opens a valve which allows air flow into a cylinder. The force, f, developed by the piston is given by

$$f = pA \tag{4.15}$$

where p is the supply pressure and A is the cross-sectional area of the piston. To return the piston, the supply valve is closed, and an exhaust valve is opened. With supply pressure eliminated, the piston may be returned by a spring, or, if a double-acting cylinder is used, a constant pressure on the other side will cause a return.

Such simple control is ideal for grippers; the pneumatic piston simply closes the gripper until the gripping force equals the piston force. Unlike a hydraulic gripper, a pneumatic gripper can open and close in response to external forces and hence can *comply* to some extent with unpredictability in its environment.

In addition to their use in grippers, pneumatic components are often used in simple robots. Advocates of pneumatic actuators point to the following advantages:

High speed and relatively high power-to-weight ratio.
Very low cost
Simplicity of control
Noncontamination of work space (unlike an oil leak, a leak in a pneumatic system causes no mess)

A totally pneumatic robot can be sequenced through a complex series of operations by a simple controller which opens and closes valves in order. Such robots normally use fixed stops. That is, a joint moves until its travel is halted by a collision with a rigid piece of metal.

More complex control can be accomplished using variable stops. In variable stop control, a number of mechanical stops exist, which may intercept the joint travel at varying points. The stops are typically engaged by a solenoid and are released by a spring.

Proportional control is possible with pneumatic actuators, although more sophisticated (and, therefore, more expensive) valves are required. Several manufacturers provide i to p transducers that accept a current as input and produce an output pressure directly related to that current. With such transducers, as well as the existence of pressure or flow amplifiers, it becomes feasible to use servo control of a pneumatic joint.

A servo system uses a position sensor to detect true position and compare it with desired position. If an error exists, the servo control moves the joint in a direction to reduce the error. Using a double-acting cylinder or pneumatic motor, with controlled pressure on both sides, servo control is possible. However, at this time only a very few servo-controlled pneumatic robots are in the field.*

Pneumatic servo systems have been seldom used because their dynamic performance is poor in comparison with electrical or hydraulic servos. This poor performance is usually attributed to the compressibility of the fluid and (therefore) to its sluggish time delay. The time delay, τ, for a signal to pass down a pipe of length l is

$$\frac{\tau}{l} = \sqrt{\frac{\rho}{N}} \tag{4.16}$$

where

ρ = density (kg/cm^3)
N = bulk modulus = χP
$\chi = C_p/C_v = 1.4$
P = absolute pressure

For pneumatic systems at room temperature, this works out to

$$\frac{\tau}{l} = 3 \text{ ms/m}$$

which might be compared with that of a typical hydraulic system:

$$\frac{\tau}{l} \text{ (hyd)} = 0.7 \text{ ms/m}$$

Therefore, propagation speed in a hydraulic system is about four times greater than in a pneumatic system. This difference in speed, while significant, does not explain the inferior performance usually attributed to pneumatic systems when they are compared with hydraulic systems. Mannetje (1981) suggests that the real reason for inferior performance is the use of flow control valves. He recommends that in a pneumatic system, pressure control should be used. He then shows how different strategies in servo design can result in a radical improvement in performance, making pneumatic control viable in many previously unconsidered applications.

4.5 SYNOPSIS

Vocabulary

You should know the definition and application of the terms on the following page.

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*Lord Corp.; Pendar, Inc.; International Robomation/Intelligence.

armature
back EMF
compliance
current control
cylinder
DC motor
field
friction
hydraulic system
i to *p* transducer
incompressible fluid
inertia
permanent magnet DC motor
pneumatic system
pulse-width modulation
push-pull amplifier
series motor
servo valve
shunt motor
slewing
step motors
torque

Notation

Symbol	Meaning
K_t	Proportionality constant, relating torque, magnetic flux, and armature current
Φ	Magnetic flux
I_a	Armature current
T_m	Torque produced by a DC motor
K_f	Proportionality constant, relating field current to flux
I_f	Field current
J_a	Inertia of the armature
F_a	Friction of the motor
J_l	Inertia of the load
F_l	Friction of the load
J_{eq}	Total inertial load seen by the motor
F_{eq}	Total frictional load seen by the motor
T	A torque
θ	Angular position
$\dot{\theta}$	Angular velocity
$\ddot{\theta}$	Angular acceleration

Symbol	Meaning
E_b	Back EMF produced by a motor
V_d	Voltage applied to a DC motor
K_P	Proportionality constant, relating torque and differential pressure
K_b	Proportionality constant, relating back EMF to rotational velocity
K_Q	Proportionality constant, relating velocity to flow
PWM	Abbreviation for pulse-width modulation
T_c	Period of time for one cycle of a PWM driver
T_+	Period of time the output of a PWM driver is high
V_{AV}	Effective voltage produced by a PWM driver
ΔP_1	Differential pressure across a hydraulic servo valve
D_m	Displaced volume
Q	Flow
f	Force
P	Pressure
A	Area
τ	Time delay for a signal to travel down a fluid line
l	Length of a fluid line
ρ	Density of the fluid

4.6 REFERENCES

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4.7 PROBLEMS

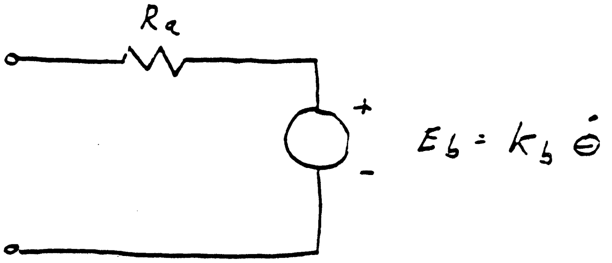
1. A permanent magnet DC motor is tested as follows. With a constant voltage of 15 V applied, and stalled, the motor draws 5 A. When rotating at 50 rad/sec,

the motor draws 1 A. Use the model of Figure 4.8 and find the effective armature resistance and the back EMF constant, K_b.

2. The motor in problem 1, when stalled, draws 5 A and exerts 1 foot-pound of torque. Find K_{pm}, as defined in Eq. 4.4.
3. The motor in problem 1 is presented with a voltage step input of 15 V and, unloaded, is measured to accelerate immediately at a rate of 15 rad/sec^2. Assume a frictionless system and compute the armature inertia.

4.1

To find R_a we use the model



when $\dot{\theta} = 0$, $E_b = 0$, and $R = \frac{15V}{5A} = 3\Omega$

AT 50 rad/sec, we have

$$15V = (3\Omega)(1A) + K_b(50\,rad/sec)$$

$$k_b = \frac{12V}{50\,rad/sec} = 0.24\,\frac{V\text{-}sec}{rad}$$

4.2

$$T_m = k_{pm} I_a$$

$$1\,ft\text{-}lb = (k_{pm})(5A)$$

$$K_{pm} = 0.2\,\frac{ft\text{-}lb}{A}$$

4.3

$$T_m = J\ddot{\theta}$$

$$J = \cancel{\frac{15\,rad/sec^2}{T_m}}$$

$\frac{1}{15}$ lb·ft·sec²/rad

To find T_m, note that the problem stated an <u>instantaneous</u> acceleration, so let $\dot{\theta} = 0$, Then $I_a = 5A$, and $T_m = 1\,ft\text{-}lb$ ∴ $J = \cancel{\frac{15\,rad/sec^2}{ft\text{-}lb}}$ (bad choice of units.)