Project #3 Fourier Transforms
Due Tuesday September 28th.

In this programming assignment, you will perform a number of experiments using the Fourier Transform (FT). The role of these experiments is to help you get a better understanding of the FT and its applications.

Part1
Take the 2-D FFT of an image of your choice followed by its 2-D inverse FFT. What do you get and why?

An approximation: To simplify this and the following, you may ignore image padding (Section 4.6.3). Although your results will not be strictly correct, significant simplifications will be gained not only in image sizes, but also in the need for cropping the final result. The principles should not be affected by this approximation.

Part 2
For this part you need to generate a 512 x 512 image which contains a 32 x 32 square placed at the center of the image (i.e., at (256,256)). Set the background to black (i.e., 0) and the interior of the square (and its boundary) to white (i.e. 255). Your images should roughly look like Fig. 4.3 from your textbook (page 156). Take the FT of the image and display its magnitude without shifting it to the center of the frequency domain. Then, shift the magnitude to the center of the frequency domain and display it again. (Note: for all of the following parts, the magnitude should be displayed centered in the frequency domain). Determine the average value of this image from your Fourier transform.

Part 3
In this part, you are going to examine the importance of magnitude and phase. For this, take the FT of the lena image (from the course web site). First, set the phase equal to zero, and take the inverse FT (hint: to set the phase to zero, set the imaginary part to zero). The resulting image should look nothing like the original. Can you explain why? Then, let the phase be the original one and set the magnitude equal to one and take the inverse FT (hint: to set the magnitude equal to one, set the real part to cos(theta) and the imaginary part to sin(theta) where theta=arctan(imag/real) - show why this should work).

Since the magnitude is set to such a small value in the Fourier domain, all the values in the spatial domain will be very small when you take the inverse FT. To alleviate this problem, rescale the pixel values after the inverse FFT has been taken (i.e., values should be in [0, 255]).

Part 4

In this experiment, you will consider the effects of additive noise and the use of FT to remove this kind of noise. The noisy image shown below (it will be put on the course's web page) has been generated by adding some noise in the form of a cosine function.

If we denote the original image as f(x,y), then the noisy image can be denoted as f(x,y)+n(x,y) where n(x,y) is a cosine function. Using the additive property of the FT, the FT of the noisy image will be F(u,v)+N(u,v) where F(u,v) is the FT of f(x,y) and N(u,v) is the FT of n(x,y). Your goal is to remove the cosine interference. This can be done as follows:

(i) Compute the DFT of the noisy image

(ii) Compute the magnitude and find the frequencies ((u,v) values) corresponding to the four largest values of the magnitude (why ??) (do not consider values of the magnitude at its center - very large values)

(iii) Replace each one of these values by the average of its 8 neighbors (do this averaging both for the real and imaginary parts).

(vi) Take the inverse DFT transform and display the resulted image.

If all of the above steps have been carried out correctly, you should get an image without the cosine interference.