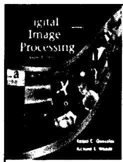

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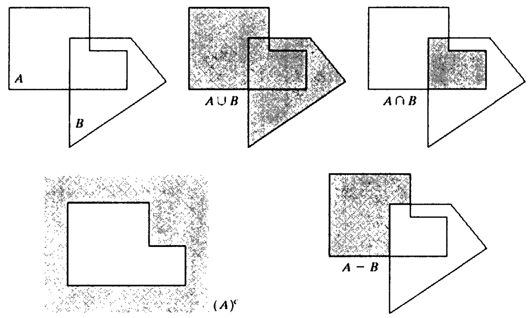

a b c
d e

FIGURE 9.1 (a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

$A^c = \{w \mid w \notin A\}$ $\qquad$ $A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$

morphology — mathematical morphology uses set theory to extract and process image components such as boundaries, skeletons, etc. The outputs are now attributes rather than a conventional image.

Z^2 set of binary images specified by (x,y) locations

$C = \{w \mid w = -d, \text{ for } d \in D\}$

C is the set of elements w, such that w is formed by multiplying each of the two coordinates of all the elements of set D by -1

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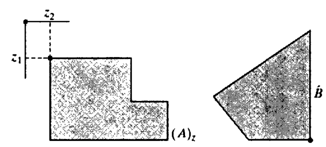

a b
FIGURE 9.2 (a) Translation of A by z. (b) Reflection of B. The sets A and B are from Fig. 9.1.

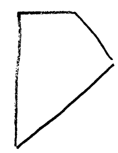

Image morphology uses two definitions not normally used in set theory.

$$\hat{B} = \{ w \mid w = -b, \text{ for } b \in B \}$$

This is the reflection of B about the origin

$$(A)_z = \{ c \mid c = a + z, \text{ for } a \in A \}$$

translate coordinates by $z = (z_1, z_2)$

This is the translation of B.

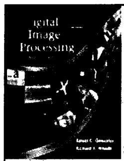

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TABLE 9.1
The three basic logical operations.

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

$p \cdot q$ $p + q$ $\sim \bar{p}$

also
called
complement

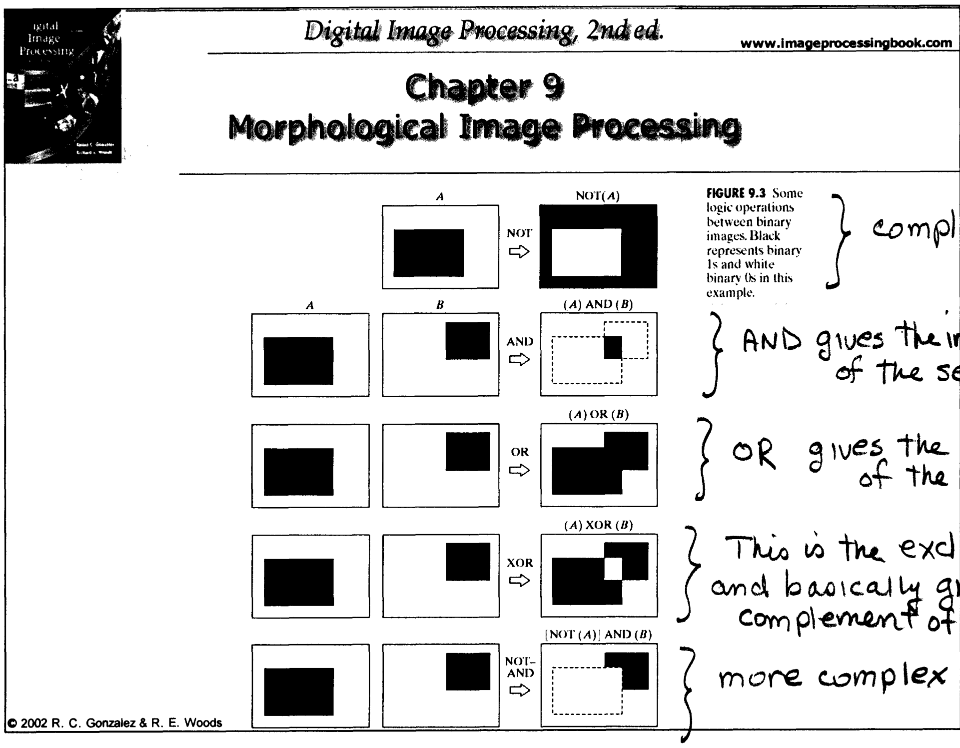


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

complement of A

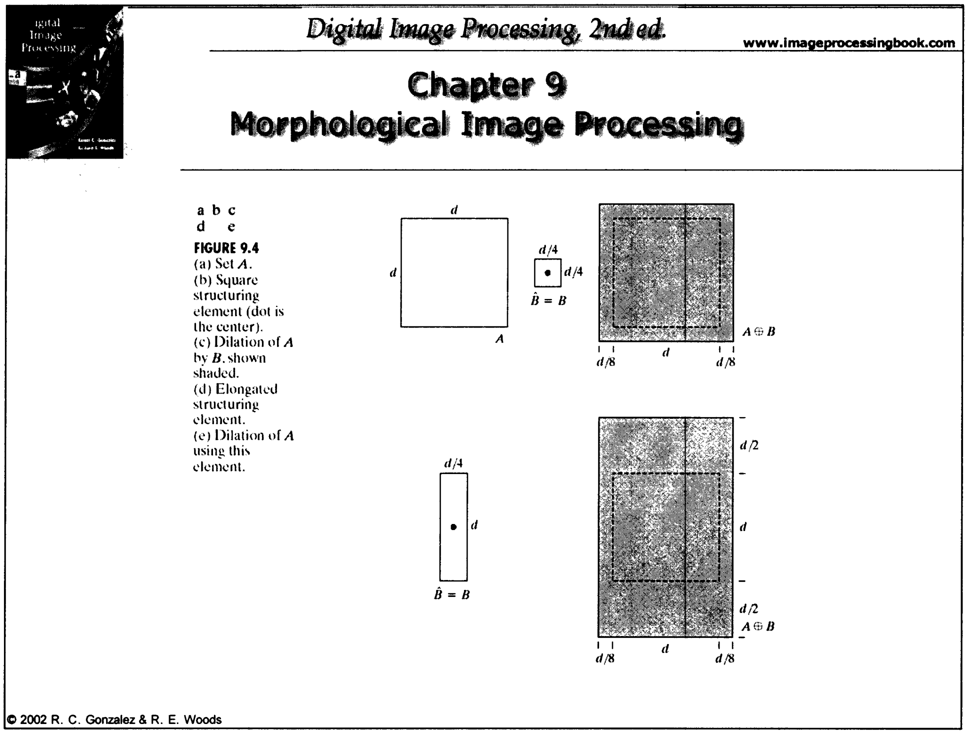
AND gives the intersection of the sets

OR gives the union of the sets

This is the exclusive or and basically gives the complement of the OR

more complex

We can perform logic operations between images on a pixel by pixel basis.



dilation of A by B

$$A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \emptyset \}$$

B is called the structuring element.

Explanation

- Reflect B about the origin $\hat{B}$
- Shift it by z, $(\hat{B})_z$
- $A \oplus B$ is the set of all displacements z such that $\hat{B}$ and A overlap.

This is very similar to a convolution mask.

Other definitions are possible.

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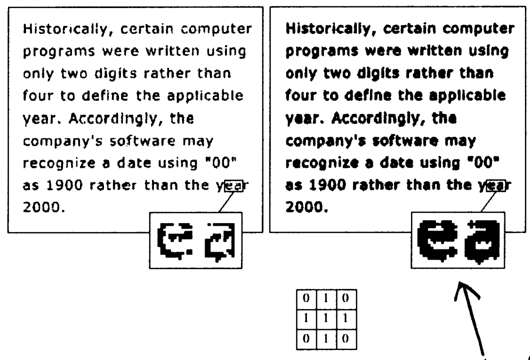
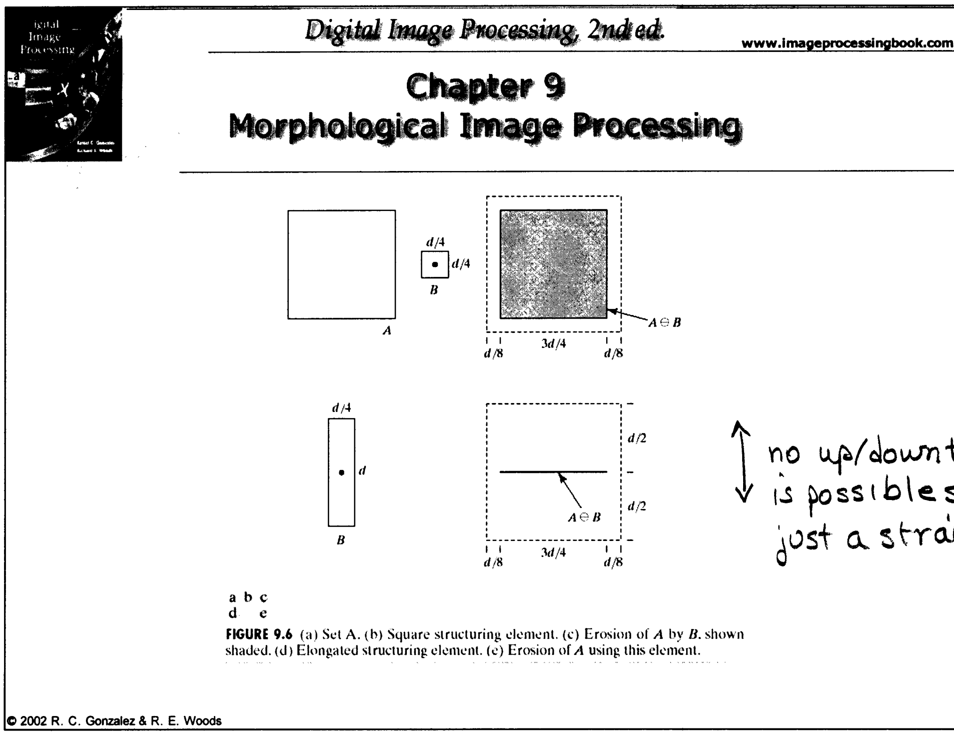


a c
b

FIGURE 9.5 (a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

joined gaps

structuring element $\hat{B} = B$



no up/down translation is possible so this becomes just a straight line.

Erosion

$$A \ominus B = \{ z \mid (B)_z \subseteq A \}$$

subset of

$A \ominus B =$ set of all points z such that the translation of B by z is contained in A

Note that dilation and erosion are dual processes.

$$(A \ominus B)^c = \{ z \mid (B)_z \subseteq A \}^c$$

$$= \{ z \mid (B)_z \cap A^c = \phi \}^c$$

if $(B)_z$ is in A then $(B)_z \cap A^c = \phi$

$$= \{ z \mid (B)_z \cap A^c \neq \phi \}$$

The complement is just the set for which $(B_z) \cap A^c \neq \phi$

$$= A^c \oplus \hat{B}$$

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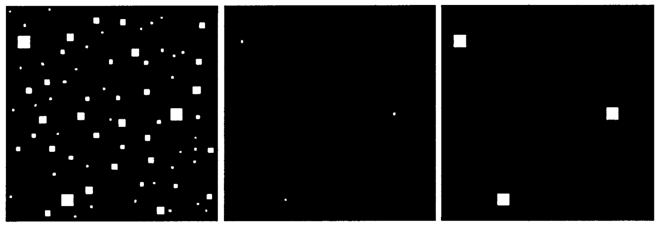


a b c

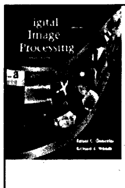
FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

result of erosion by a 13x13 image of 1's

result of dilating the eroded image with a 13x13 all 1's structuring element

Use erosion to eliminate irrelevant (small) detail.

Select a structuring element slightly smaller than the details you want to keep.



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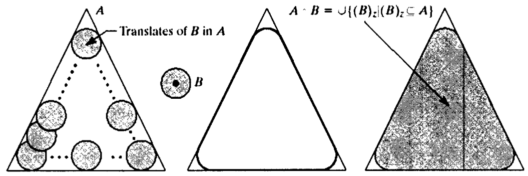

a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

geometric interpretation of opening

Opening

$$A \circ B = (A \ominus B) \oplus B$$

$$= \cup\{(B)_z \mid (B)_z \subseteq A\}$$

this is the set of <u>all</u> translates of B that fit inside A

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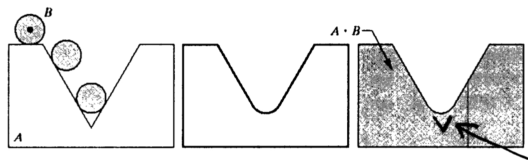


a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

there is no translate of B which can reach inside this region and not intersect A

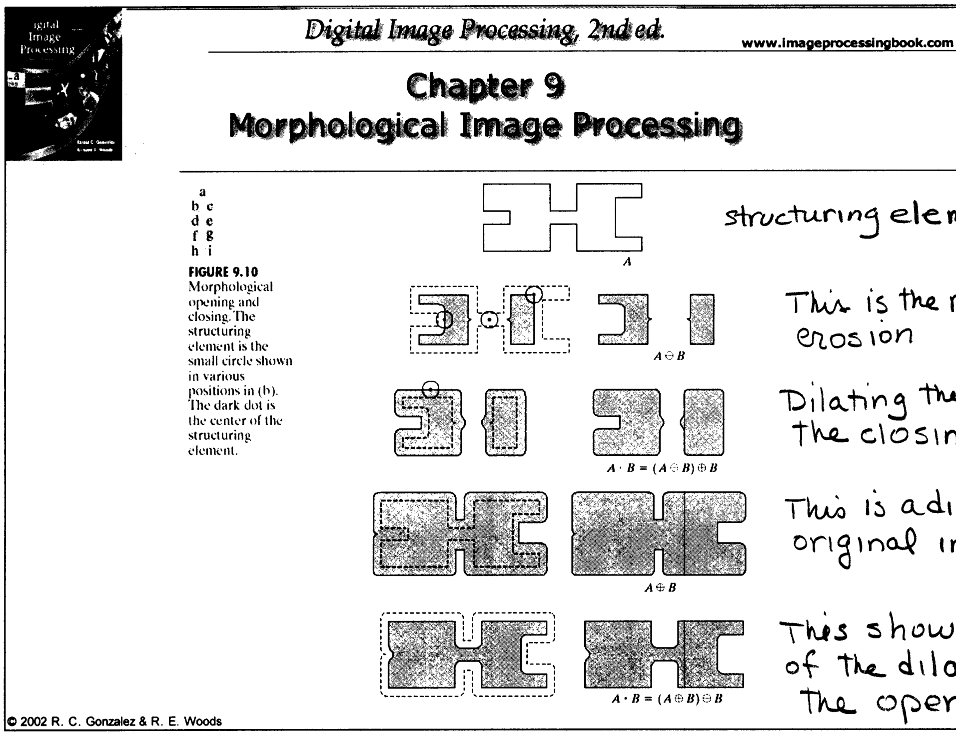
geometric interpretation of closing

Roll B on the outside

$$A \bullet B = (A \oplus B) \ominus B$$

$$A \bullet B = \{ w \mid (B)_z \cap A \neq 0 \}$$

set of all w such that $(B)_z \cap A \neq$ for any translate of $(B)_z$ that contains w.



structuring element ⊙

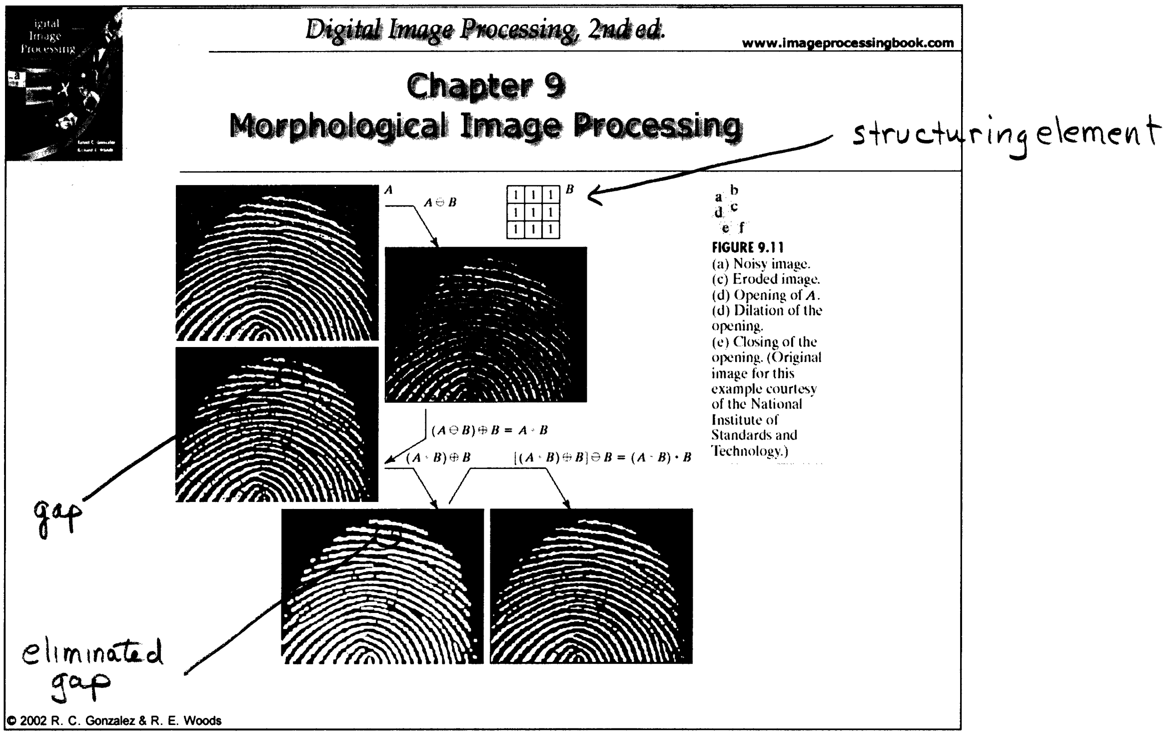
This is the resuet of erosion

Dilating the above gives the closing.

This is a dilation of the original image.

This shows the erosion of the dilation to complete the opening.

(d), (e) shows the dilation. The topology (connection) was not preserved because B can not fit in A at the connection and the two right fingers.
B must fit inside A to preserve topology.



Noise is white on black and black on white.

(c) erode A by B

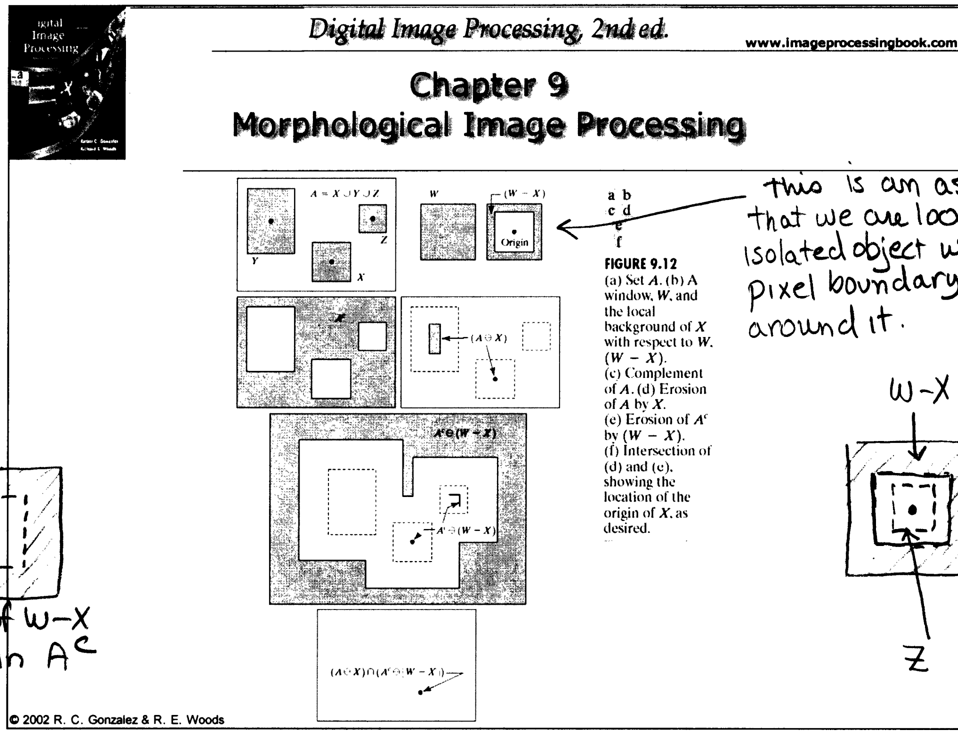
eliminates white on black background noise since the white spots (1's) are smaller than the structuring element but the dark noise in the whorls increased since erosion decreased the size of the white objects

(d) dilate (c) to restore the whorls, i.e. increase the size of the white

(e) dilate (d) to eliminate gaps in the ridges

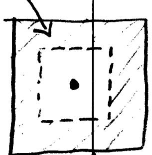
(f) erode (e) to thin the thickened ridges

Still some gaps because we did not consider how to maintain connectivity.



this is an assertation that we are looking for an isolated object with at least a pixel boundary completely around it.

$W-X$



one translate of $W-X$ fits X and is in A^c

$W-X$

Z

There are several translates of $W-X$ such that $W-X$ is in A^c

Use the "hit-or-miss" algorithm to find shapes in images.

(a) We want to find the location of X.

(b) Let X be enclosed by a small background W. Define the local background $W-X$ as shown in (b).

(c) This is simply the complement A^c of the set of all shapes, i.e. $A = X \cup Y \cup Z$

(d) Erode A by X. Since Z is smaller than X it disappears. X eroded by X is a single point, and Y eroded by X is a rectangular region of all locations of X inside Y.

(e) The most complicated thing is the erosion of A^c by $W-X$. It's the set of all translates of $W-X$ such that the center of $W-X$ is in A^c. Note that $W-X$ fits around X giving a center point. Since Z is smaller than X there is also a set of points inside Z corresponding to translates around Z.

(f) Intersection of $A \ominus X$ and $A^c \ominus (W-X)$ is the location of X.

$$A \circledast B = (A \ominus X) \cap [A^c \ominus (W-X)]$$

can be written in other forms as well.

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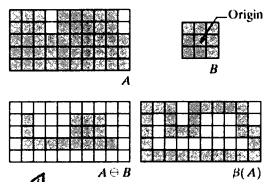


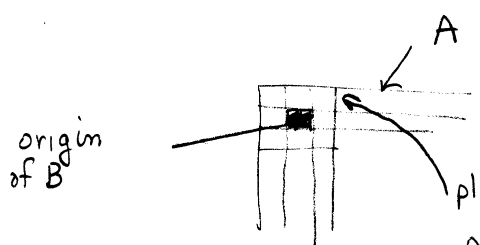
a b
c d

FIGURE 9.13 (a) Set *A*. (b) Structuring element *B*. (c) *A* eroded by *B*. (d) Boundary, given by the set difference between *A* and its erosion.

$A \ominus B$ $\qquad$ $\beta(A) = A - (A \ominus B)$

Boundary extraction:

Erode a set A by an appropriate structuring element B.
Subtract this from A to get the boundary $\beta(A)$.



places B 1-pixel inside A giving 1-pixel boundaries.
A 5x5 would give 2-pixel boundaries but may eliminate detail.

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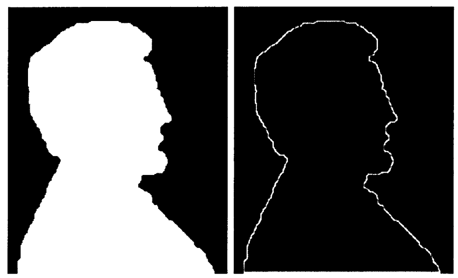
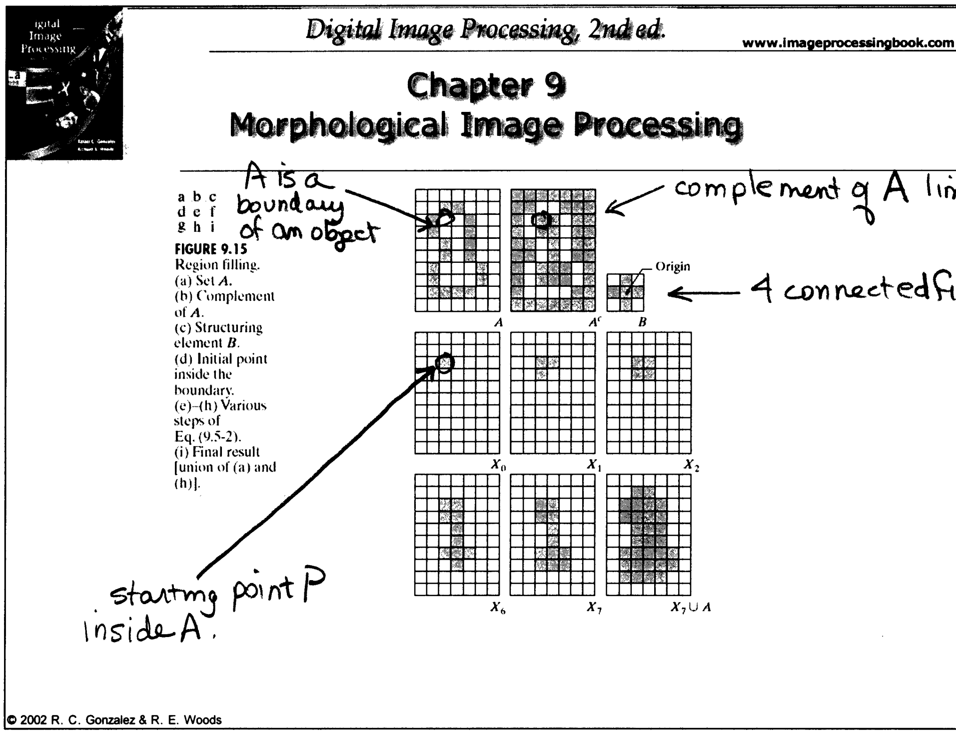


a b

FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Because the structuring element is 3x3 the boundary is 1 pixel thick.



Region filling

$X_0 = P.$

$$X_1 = (\underbrace{P \oplus B}_{\text{expands } P \text{ in four directions}}) \cap A^c$$

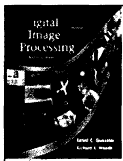
stops all pixels in boundary
this is called conditional dilation

$$X_2 = (\underbrace{X_1 \oplus B}_{\text{continues to expand (dilate) } P}) \cap A^c$$

stops all pixels at boundary.

In general $\quad X_k = (X_{k-1} \oplus B) \cap A^c, \quad k = 1, 2, 3, \ldots$

stop when $X_k = X_{k-1}$



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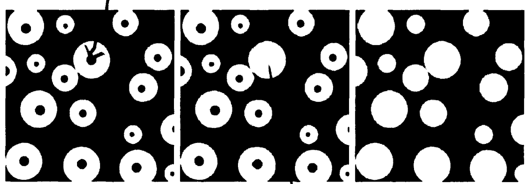
put white dot here



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

filled

Application of filling

(a) possible thresholded image from ball bearings.
The spots might be due to reflections. One dot inside spot

(b) Fill upto boundary of circle.

Need additional algorithm to identify circles for this to become automated.

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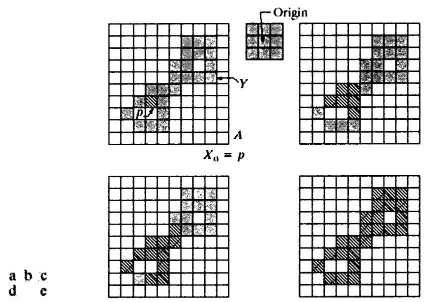


FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

B as given assumes 8-connectivity of Y.

Extraction of connected components.

We want to construct a list of connected components in A.

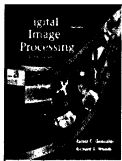
Suppose we are given p which is a part of object Y.

Dilate $X_0 = p$ by B

$X_1 = X_0 \cap A$ gives all the 1's in A connected to p, i.e, elements of Y

$X_2 = (X_1 \oplus B) \cap A$

etc.



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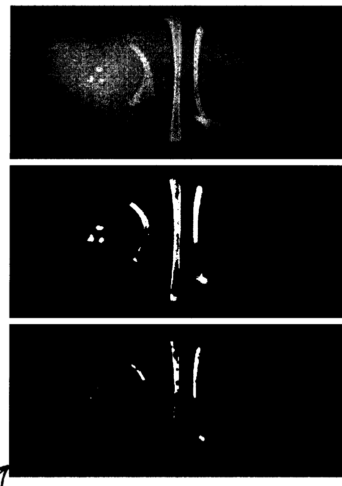
a
b
c d



X-ray of bones in a chicken fillet

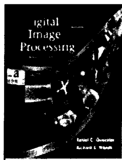
Thresholded

eroded by a 5x5 structuring element of 1's

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

connected components.

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5 × 5 structuring element of 1's. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)



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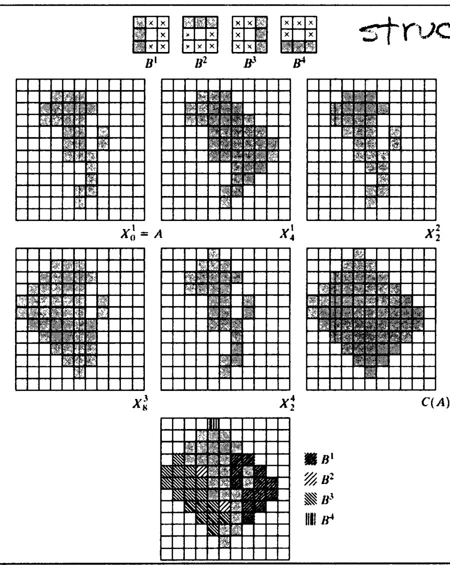
a
b c d
e f g
h

FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.



structuring elements.
X = don't care

This is a repetitive "hit or miss" algorithm.

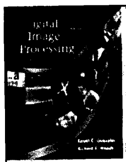
You apply B^1 to A, i.e., $X_0^1 \circledast B^1$
which finds the locations of B^1 and you add them to A.

$$X_1^1 = (X_0^1 \circledast B^1) \cup A$$

Continue this process until nothing changes.
Do this for each shape B^i starting with A
Combine the results to get the convex hull.

$$X_k^i = (X_{k-1}^i \circledast B^i) \cup A \qquad i = 1, 2, 3, \ldots \quad k = 1, 2, 3, 4$$

$$C(A) = \bigcup_{i=1}^{4} D^i \quad \text{where } D^i = X^i_{\text{converged}}$$



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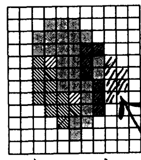


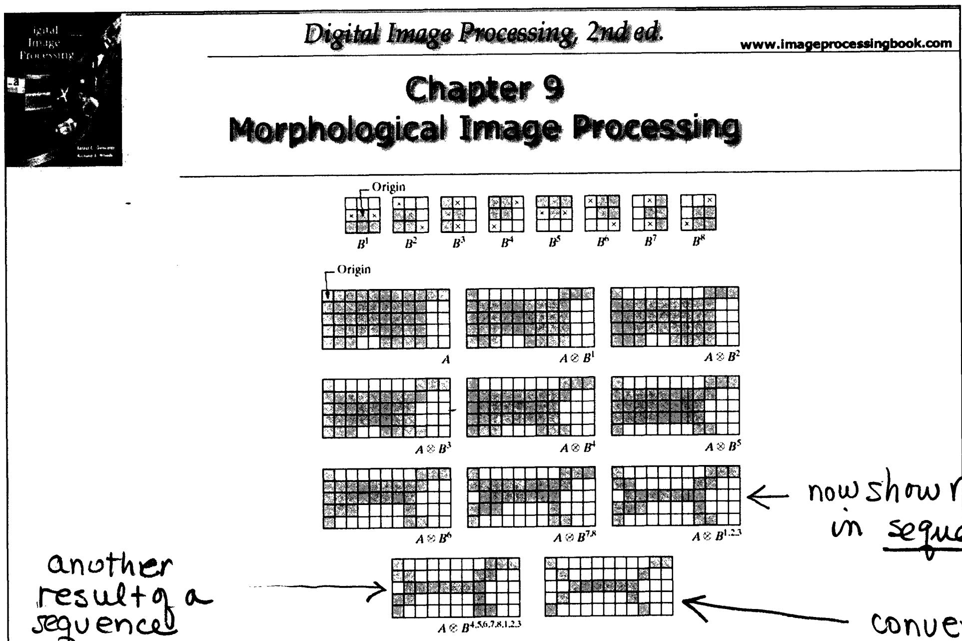
original boundaries

FIGURE 9.20 Result of limiting growth of convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

original boundaries.

bounding prevents this growth past the original boundaries.

LIMITING GROWTH



a
b c d
e f g
h i j
k l

FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first element again (there were no changes for the next two elements). (k) Result after convergence. (l) Conversion to m-connectivity.

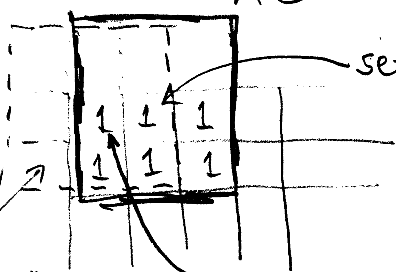
now show result of doing in sequence.

another result of a sequence

converted to m-connectivity

Thinning is often used to reduce thick objects to "skeletons"
In this case if the structuring elements match we remove that pixel.

$$A \otimes B = A - (A \circledast B) = A \cap (A \circledast B)^c$$



set matches B^1 so remove this pixel.

treat as a "0"
doesn't match B^1 so don't remove