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FIGURE 10.1 A general 3×3 mask.

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

We can regard a 3x3 mask as either a filter mask or a correlation mask.

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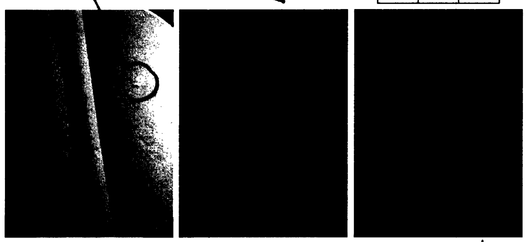
−1	−1	−1
−1	8	−1
−1	−1	−1

single black point we want to find

point in response R



result of thresholding

a
b c d

FIGURE 10.2 (a) Point detection mask. (b) X-ray image of a turbine blade with a porosity. (c) Result of point detection. (d) Result of using Eq. (10.1-2). (Original image courtesy of X-TEK Systems Ltd.)

Consider (a) as a mask for a point, i.e., correlation.

$$R = \sum_{i=1}^{9} w_i z_i$$

If $|R| \geq T$ we have found a point.

The mask here is also that of a Laplacian operator but we can also consider it as a point correlator.

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FIGURE 10.3 Line masks.

Horizontal

−1	−1	−1
2	2	2
−1	−1	−1

+45°

−1	−1	2
−1	2	−1
2	−1	−1

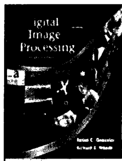
Vertical

−1	2	−1
−1	2	−1
−1	2	−1

−45°

2	−1	−1
−1	2	−1
−1	−1	2

These are examples of correlation (also derivative) masks for single pixel width lines



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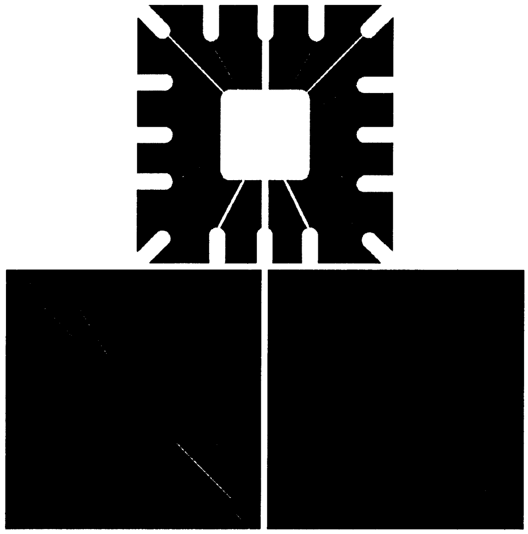

a
b c

FIGURE 10.4 Illustration of line detection. (a) Binary wire-bond mask. (b) Absolute value of result after processing with −45° line detector. (c) Result of thresholding image (b).

(b) shows the result of using a -45° line detector mask on the wire bond mask for an integrated circuit.

2	-1	-1
-1	2	-1
-1	-1	2

(c) shows the result of thresholding (b).
A good rule of thumb for these masks is to use
T = maximum pixel value in original image (a)
The isolated points in (c) could be eliminated using morphological processing.

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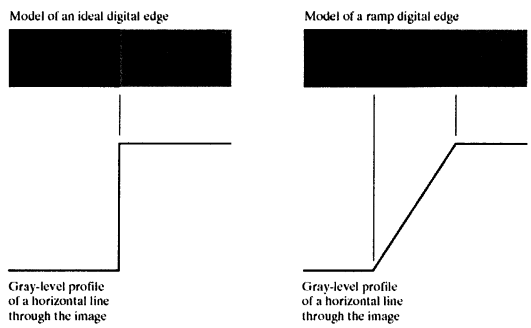
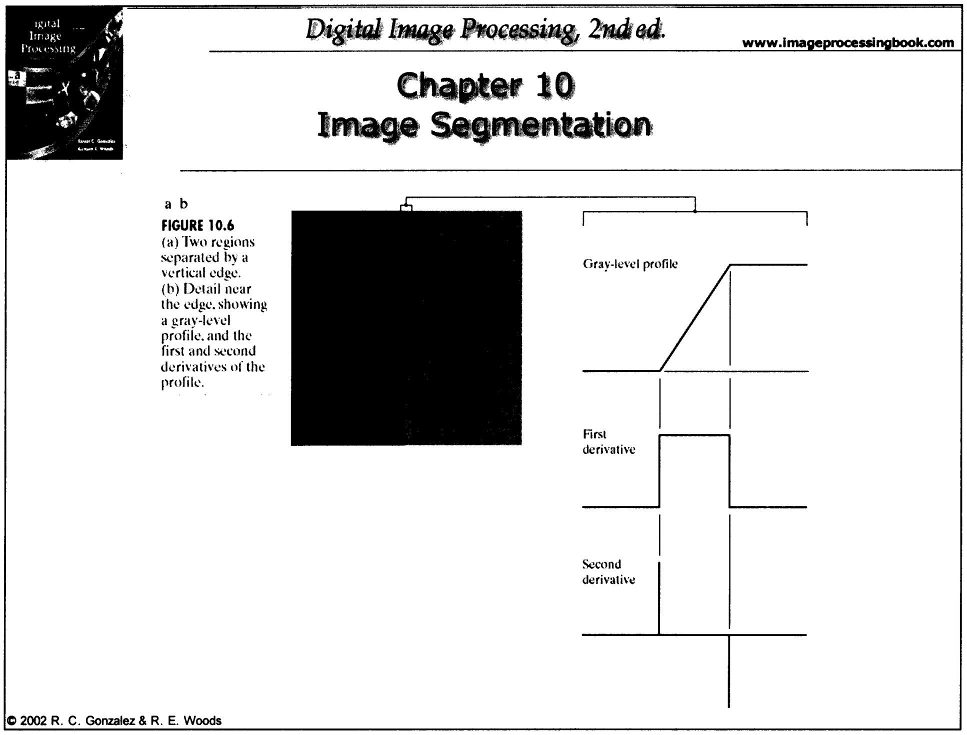


a b

FIGURE 10.5 (a) Model of an ideal digital edge. (b) Model of a ramp edge. The slope of the ramp is proportional to the degree of blurring in the edge.

sharp edge   blurred edge

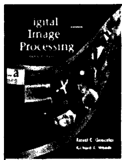
models of gray scale images.
The slope is proportional to the amount of blurring.



"blurred" gray scale edge

the magnitude of the first derivative can be used to detect the presence of an edge

the second derivative's zero crossing property is also appropriate for detecting the presence of an edge, i.e., a line drawn between the two extrema must cross through zero near the center of the edge



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$\sigma = 0$

$\sigma = 0.1$

$\sigma = 1$

$\sigma = 10$

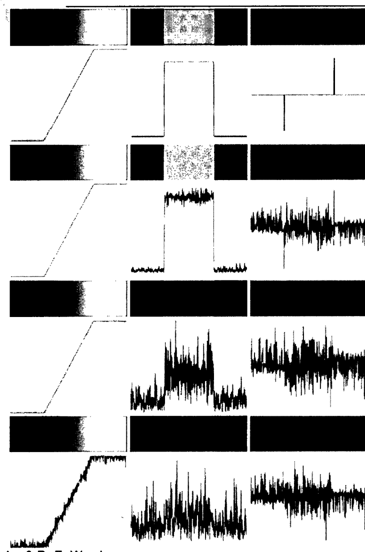

FIGURE 10.7 First column: images and gray-level profiles of a ramp edge corrupted by random Gaussian noise of mean 0 and σ = 0.0, 0.1, 1.0, and 10.0, respectively. Second column: first-derivative images and gray-level profiles. Third column: second-derivative images and gray-level profiles.

a b c d

gray level edges

first derivative

second. derivative

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a
b c
d e
f g

FIGURE 10.8 A 3 × 3 region of an image (the z's are gray-level values) and various masks used to compute the gradient at point labeled z_5.

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

Roberts

−1	−1	−1
0	0	0
1	1	1

−1	0	1
−1	0	1
−1	0	1

Prewitt

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Sobel

most commonly used gradient masks

superior noise characteristics to Prewitt

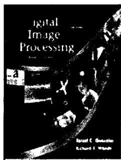
Gradient

$$\underline{\nabla f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$\nabla f = |\underline{\nabla f}| = \sqrt{G_x^2 + G_y^2}$$

$$\alpha(x,y) = \tan^{-1}\left(\frac{G_y}{G_x}\right)$$

The above operators are attempts to implement first-order partial derivatives



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0	1	1	−1	−1	0
−1	0	1	−1	0	1
−1	−1	0	0	1	1

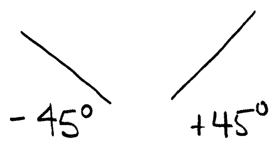
Prewitt

0	1	2	−2	−1	0
−1	0	1	−1	0	1
−2	−1	0	0	1	2

Sobel

a b
c d

FIGURE 10.9 Prewitt and Sobel masks for detecting diagonal edges.



modified Prewitt & Sobel masks for detecting diagonal edges.

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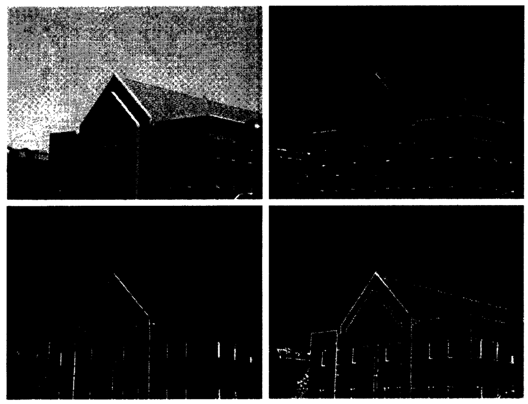
a b
c d

FIGURE 10.10 (a) Original image. (b) $|G_x|$, component of the gradient in the x-direction. (c) $|G_y|$, component in the y-direction. (d) Gradient image, $|G_x| + |G_y|$.

1200×1600 pixel
original image



$|G_x|$

$|G_y|$

gradient approximation
$|G_x| + |G_y|$

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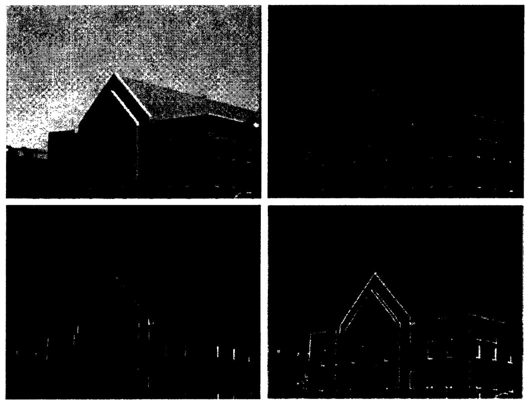
1200x1600

5x5 avg. filter



a b
c d

FIGURE 10.11 Same sequence as in Fig. 10.10, but with the original image smoothed with a 5 × 5 averaging filter.

$|G_x|$

$|G_y|$

$|G_x| + |G_y|$ *

* Averaging causes all the edges to be weaker.
Removed edges of bricks.

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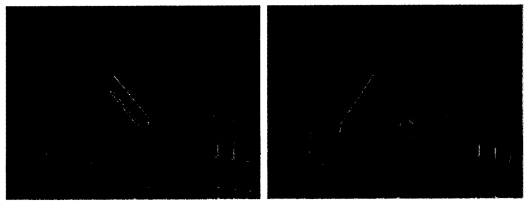

a b

FIGURE 10.12 Diagonal edge detection. (a) Result of using the mask in Fig. 10.9(c). (b) Result of using the mask in Fig. 10.9(d). The input in both cases was Fig. 10.11(a).

$-45°$ sobel $+45°$ sobel

Both can detect horizontal & vertical edges but with a weaker response than a horizontal or vertical operator.

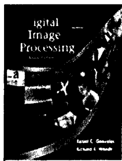

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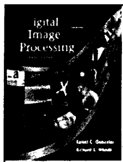
FIGURE 10.13 Laplacian masks used to implement Eqs. (10.1-14) and (10.1-15), respectively.

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

The Laplacian is a second-order derivative which is usually approximated by the above masks.



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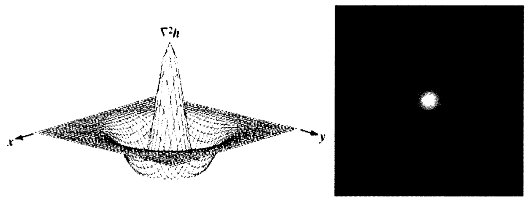
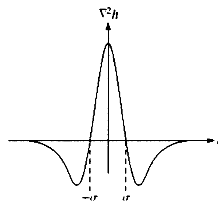
LoG sometimes called the mexican hat function





0	0	−1	0	0
0	−1	−2	−1	0
−1	−2	16	−2	−1
0	−1	−2	−1	0
0	0	−1	0	0

a b
c d

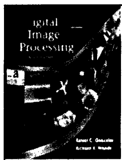
FIGURE 10.14 Laplacian of a Gaussian (LoG). (a) 3-D plot. (b) Image (black is negative, gray is the zero plane, and white is positive). (c) Cross section showing zero crossings. (d) 5×5 mask approximation to the shape of (a).

The Laplacian is never used directly because of its strong noise sensitivity.

We usually use it with a Gaussian smoothing (low-pass) filter to minimize noise. $h(r) = -e^{-\frac{r^2}{2\sigma^2}}$

Combining these operators leads to the Laplacian of a Gaussian (LoG)

$$\nabla^2 h(r) = -\left[\frac{r^2-\sigma^2}{\sigma^4}\right] e^{-\frac{r^2}{2\sigma^2}}$$



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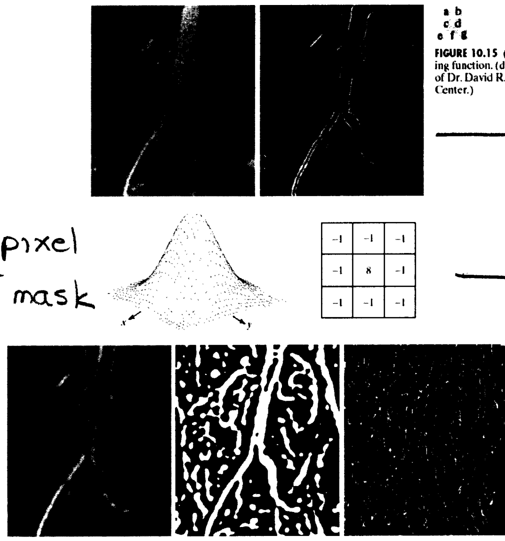

FIGURE 10.15 (a) Original image. (b) Sobel gradient (shown for comparison). (c) Spatial Gaussian smoothing function. (d) Laplacian mask. (e) LoG. (f) Thresholded LoG. (g) Zero crossings. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

— Sobel image

27 x 27 pixel Gaussian smoothing mask

— gradient mask $\nabla^2 f$

grayscale LoG result

thresholded LoG image gradient output is + and −

zero-crossings obtained from thresholded image.

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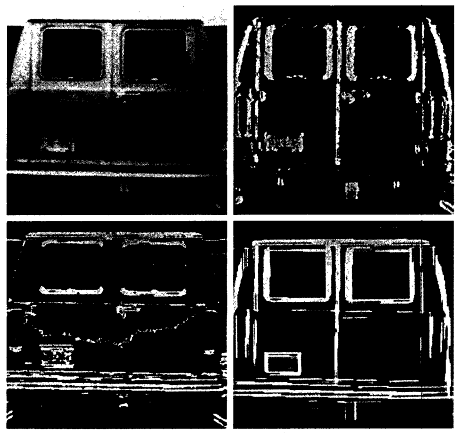
a b
c d

FIGURE 10.16
(a) Input image.
(b) G_y component of the gradient.
(c) G_x component of the gradient.
(d) Result of edge linking. (Courtesy of Perceptics Corporation.)



$|G_x|$

$|G_y|$

simply
$\alpha = 15^\circ$
$|\nabla f| > 25$

detect license plate using 2:1 rectangle ratio

edge linking – uses similarity of edge pixels to produce meaningful edges

in a neighborhood (usually 3x3 or 5x5)

Edge pixels are similar if

$$|\nabla f(x,y) - \nabla f(x_0,y_0)| \leq E$$

$$|\alpha(x,y) - \alpha(x_0,y_0)| < A$$

Remember the edge direction is perpendicular to ∇f.

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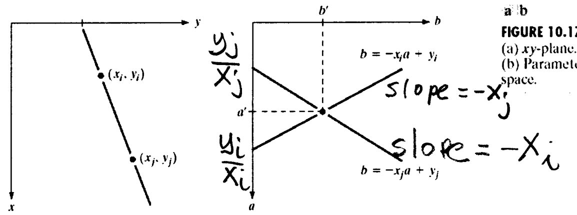

a b

FIGURE 10.17
(a) *xy*-plane.
(b) Parameter space.

equation of line
$y_i = a x_i + b$

rewrite line as $b = -x_i a + y_i$

these come from image edge point

This is a line in a-b parameter space.
Intersections of lines from all image edge points locates lines,

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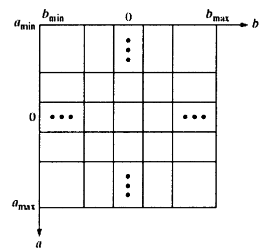

FIGURE 10.18 Subdivision of the parameter plane for use in the Hough transform.

To compute the hough transform divide the parameter space into accumulator cells which span the expected ranges of a and b.

Set all values to zero.

Enter each edge point incrementing all appropriate accumulator cells. Round-off a and b as appropriate.

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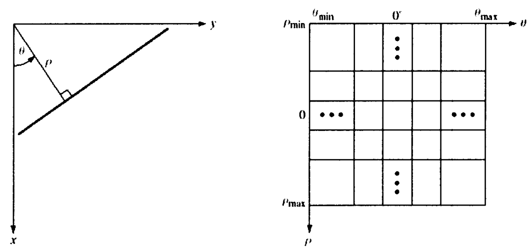


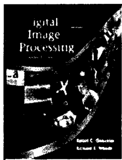
a b
FIGURE 10.19 (a) Normal representation of a line. (b) Subdivision of the $\rho\theta$-plane into cells.

accumulator array for $\rho - \theta$
note zero at center of each axis.

A major problem with using lines of the form $y = ax + b$ is that the slope $\rightarrow \infty$ which exceeds the accumulator array.

Solution is to use the polar form of the line

$$\rho = x\cos\theta + y\sin\theta$$



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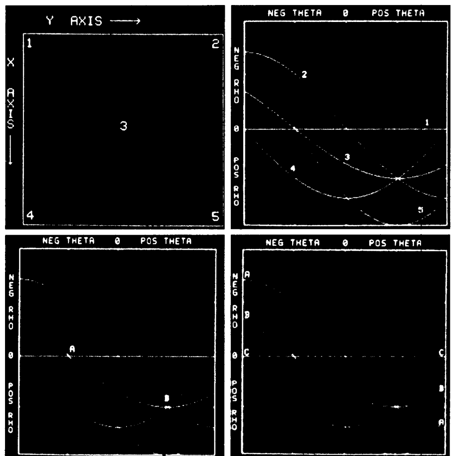
a b
c d

FIGURE 10.20 Illustration of the Hough transform. (Courtesy of Mr. D. R. Cate, Texas Instruments, Inc.)

original image



each edge point gives
$x_i \cos\theta + y_i \sin\theta = \rho$
which a sinusoidal curve
in $\rho\theta$ space
Note: point 1 is $\rho = 0$

Note how ρ and θ
change signs across
ρ-θ space

points 1, 3, 5
intersect at A
in ρ-θ space

points 2, 3, 4
intersect at B
in ρ-θ space

Note: you can do Hough transforms based upon
more complex and generalized functions

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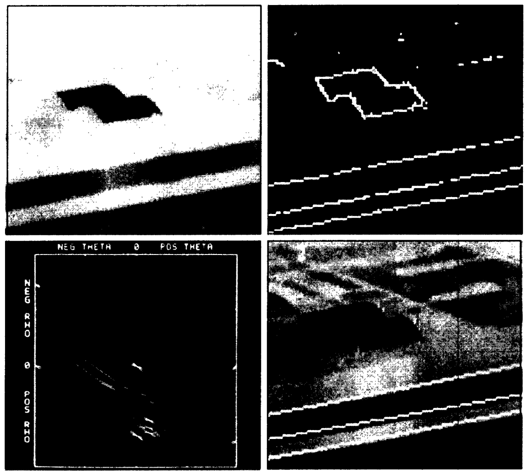

a b
c d

FIGURE 10.21 (a) Infrared image. (b) Thresholded gradient image. (c) Hough transform. (d) Linked pixels. (Courtesy of Mr. D. R. Cate, Texas Instruments, Inc.)

(a) IR image of a runway and two hangers

(b) thresholded gradient image

(c) Hough transform of (b) using $\rho = x\cos\theta + y\sin\theta$

(d) Linked pixels from strongest points in (c)
No gaps in linked image.

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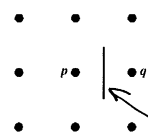

FIGURE 10.22 Edge element between pixels p and q.

each edge element has a cost

$$c(p,q) = H - [\underbrace{f(p) - f(q)}_{\text{gray scale image values}}]$$

H: highest gray scale value in image

let $f(p) = 7$

$f(q) = 0$

$H = 7$

Then $c(p,q) = 7 - [7-0] = 0$

cost is low traveling along edge from to to bottom

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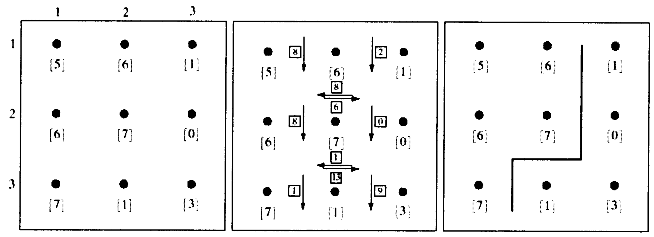

a b c

FIGURE 10.23 (a) A 3 × 3 image region. (b) Edge segments and their costs. (c) Edge corresponding to the lowest-cost path in the graph shown in Fig. 10.24.

pick lowest starting cost and traverse lowest cost path to bottom

edge segments and all computed costs

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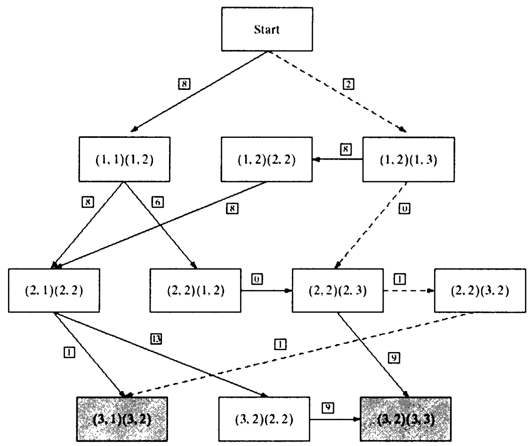

FIGURE 10.24 Graph for the image in Fig. 10.23(a). The lowest-cost path is shown dashed.

graph of problem in previous figure
nodes correspond to edge pixels in figures
arcs are potential linked edges

lowest cost path shown in dashes

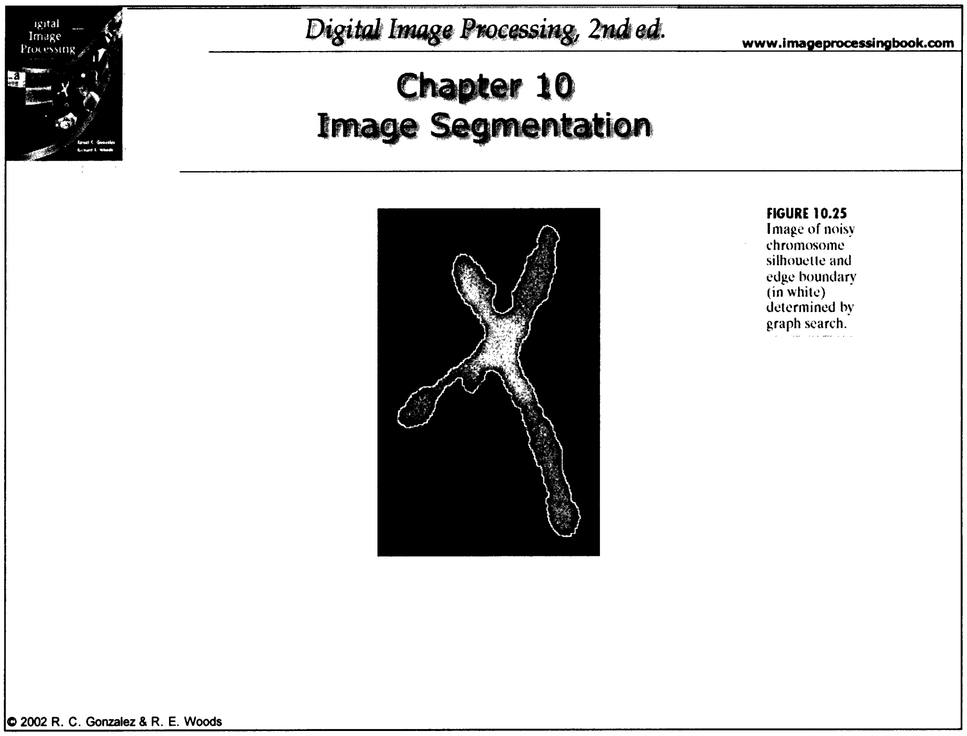

FIGURE 10.25 Image of noisy chromosome silhouette and edge boundary (in white) determined by graph search.

improved algorithm will estimate the cost to the end as well

$$\underset{\substack{\text{estimate of}\\ \text{minimum}\\ \text{cost path}}}{r(n)} = \underbrace{g(n)}_{\substack{\text{lowest cost}\\ \text{path found}\\ \text{to } n}} + \underbrace{h(n)}_{\substack{\text{estimated cost}\\ \text{to goal node}}}$$