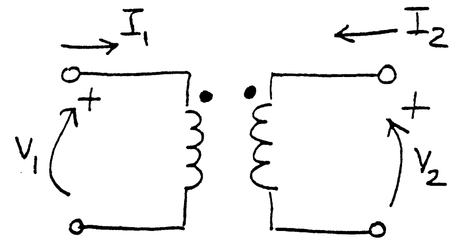
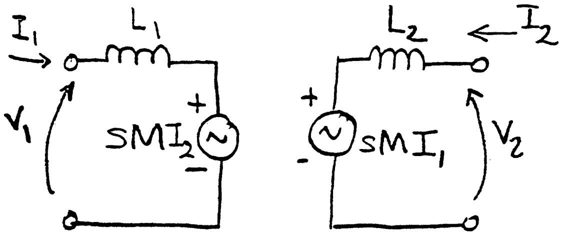
transformer "dot" convention

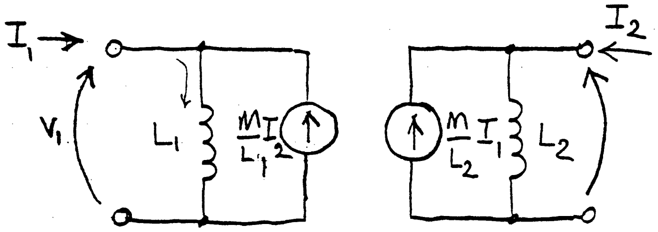
the actual definition is That a current flowing into The dot will cause a plus (at the dotted end) to negative voltage across the inductor and induces a voltage across all other windings such that the dotted end is positive



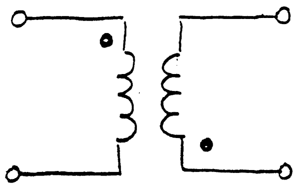


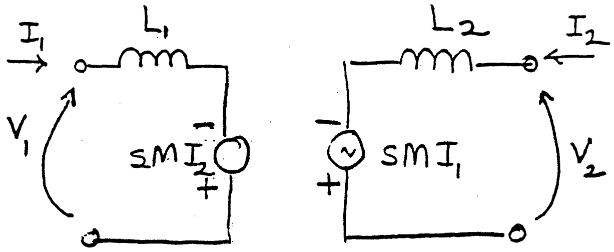
$$V_1 = sL_1 I_1 + sM I_2$$

$$V_2 = sM I_1 + sL_2 I_2$$



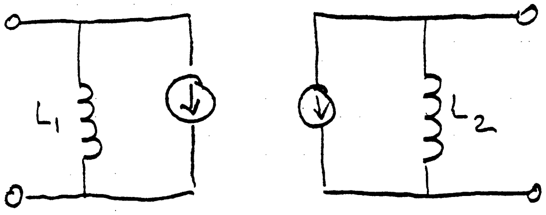
$$I_1 = \frac{V_1}{sL_1} - \frac{M}{L_1} I_2 \quad I_2 = \frac{V_2}{sL_2} - \frac{M}{L_2} I_1$$





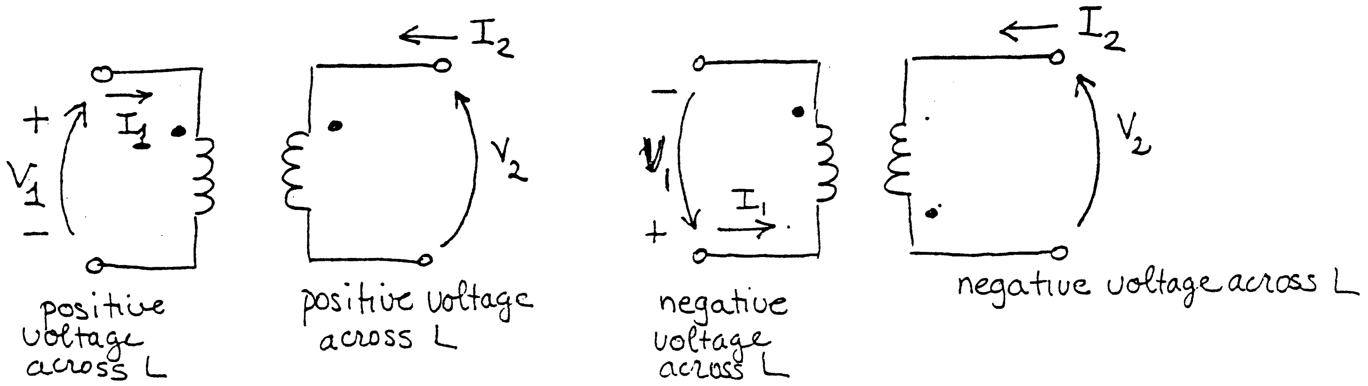
$$V_1 = sL_1 I_1 - sM I_2$$

$$V_2 = -sM I_1 + sL_2 I_2$$



$$I_1 = \frac{V_1}{sL_1} + \frac{M}{L_1} I_2 \quad I_2 = \frac{V_2}{sL_2} - \frac{M}{L_2} I_1$$

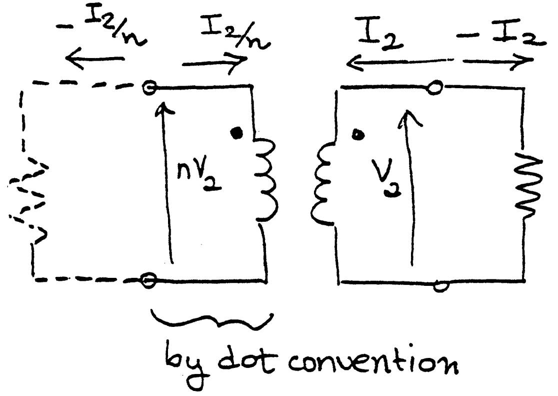
In terms of the "transformer" equations This means:



V_2, I_2 are source

V_1, I_1 are consequences

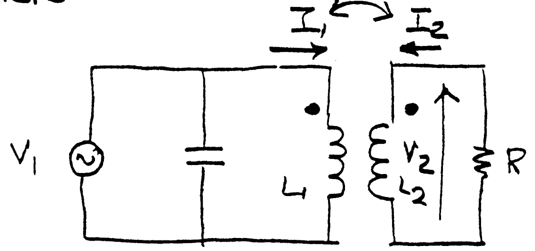
In terms of smith, the arguement is more subtle.



and I_1

transformers

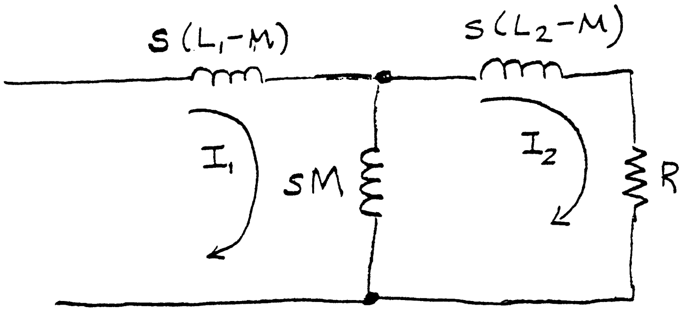
dot convention (with this convention M is always positive)



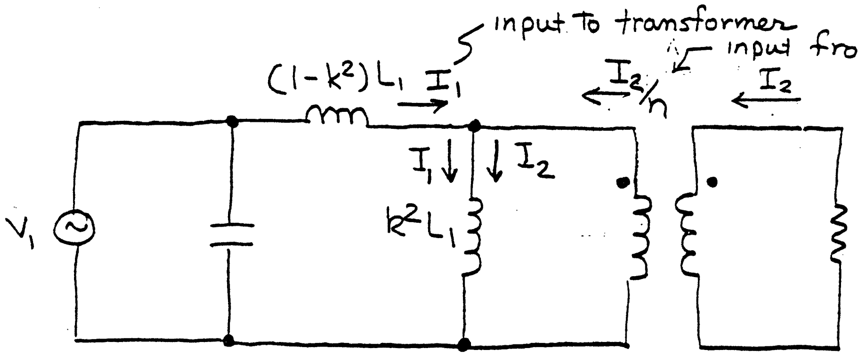
$$V_1 = sL_1 I_1 + sM I_2$$

$$V_2 = sM I_1 + sL_2 I_2$$

equivalent model



These I_2's are not the same



$$V_1 = s\cancel{(1-k^2)L_1} I_1 + sk^2\cancel{L_1} I_1 + sk^2L_1 \frac{I_2}{n}$$

$$V_1 = sL_1 I_1 + \frac{sk^2L_1 I_2}{n}$$

$$nV_2 = sk^2L_1 I_1 + sk^2L_1 \frac{I_2}{n}$$

} use transformer relationship for voltage

identical to previous equations if

$$M = \frac{k^2L_1}{n} \quad \text{and} \quad L_2 = \frac{k^2L_1}{n^2}$$

but for a high k transformer $n = k\sqrt{\frac{L_1}{L_2}}$ or $n^2 = k^2 \frac{L_1}{L_2}$

$$j\omega \frac{k^2 L_1}{n} = j\omega \frac{n^2 L_2}{n} = j\omega n L_2$$

$$\omega^2 C \frac{k^2 L_1}{n^2} = \omega^2 C \frac{n^2 L_2}{n^2} = \omega^2 C L_2$$

$$j\omega \frac{k^2 L_1}{R n^2} = j\omega \frac{n^2 L_2}{R n^2} = j\omega \frac{L_2}{R}$$

$$\therefore \quad Z_{12}(j\omega) = \frac{j\omega n L_2}{1 - \omega^2 C L_2 + j\omega \frac{L_2}{R}}$$

ASIDE: [This looks exactly like a parallel resonant circuit

$$Z_{12}(j\omega) = \frac{V_o(s)}{I(s)} = \frac{sL/R_p}{s^2 LC + s\,L/R_p + 1}$$

$$= \frac{s/R_p C}{s^2 + s/R_p C + \frac{1}{LC}}$$

]

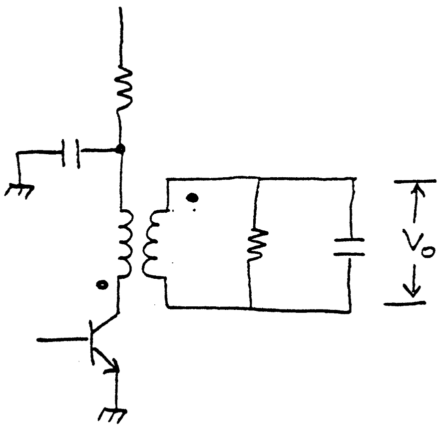
input (collector impedance)

$$Z_c = \frac{V_i(s)}{I_i(s)} = j\omega(1-k^2)L_1 + \underbrace{\frac{nV_o}{I_1}}_{\text{this is } n Z_{12}(j\omega)}$$

$$= j\omega(1-k^2)L_1 + \frac{j\omega n^2 L_2}{1 - \omega^2 L_2 C + j\omega \frac{L_2}{R}}$$

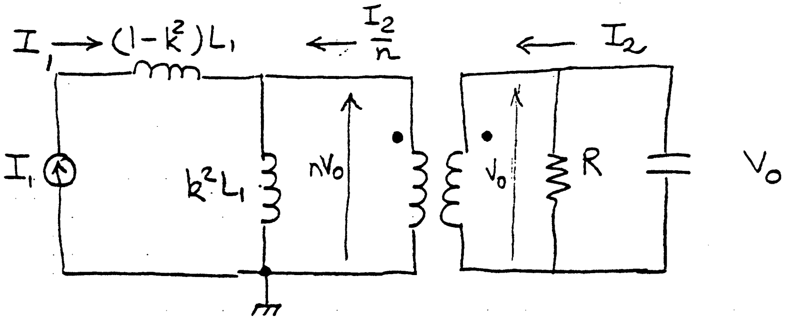
at resonance $\left(1 - \omega^2 L_2 C = 0\right)$

Example:



What is the collector load impedance Z_l? and the output voltage V_o?

small signal circuit



writing the loop equations:

$$nV_o = j\omega k^2 L_1 \left(I_1 + \frac{I_2}{n}\right)$$

$$V_o = - I_2 (R \parallel \tfrac{1}{j\omega C}) = - I_2 \frac{R \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = - I_2 \frac{R}{1+j\omega RC} \;;\; I_2 = -\frac{(1+j\omega}{R}$$

eliminating I_2

$$nV_o = j\omega k^2 L_1 \left(I_1 - \frac{1}{n} \frac{1+j\omega RC}{R} V_o\right)$$

$$nV_o + \frac{(1+j\omega RC)\, j\omega k^2 L_1}{nR} V_o = j\omega k^2 L_1 I_1$$

$$Z_{12} = \frac{V_o}{I_1} = \frac{j\omega k^2 L_1}{1 + \frac{(1+j\omega RC)\, j\omega k^2 L_1}{nR}} = \frac{j\omega k^2 L_1/n}{1 - \omega^2 C \frac{k^2 L_1}{n^2} + j\omega \frac{k^2 L_1}{Rn^2}}$$

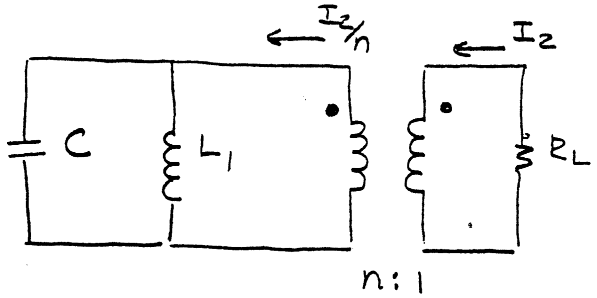
this is a very important result:

$$n^2 = k^2 \frac{L_1}{L_2}$$

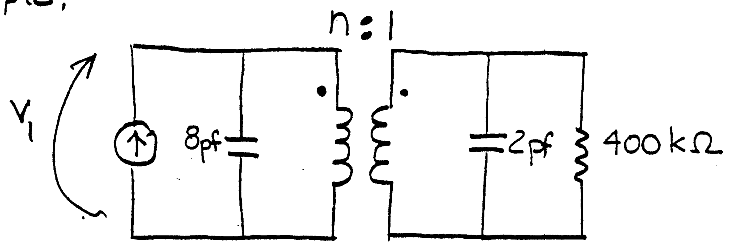
or $n = k\sqrt{\frac{L_1}{L_2}}$ This defines the turns ratio of the transformer

$k = \frac{M}{\sqrt{L_1 L_2}}$ coefficent of coupling

this was a simple model where $k \approx 1$



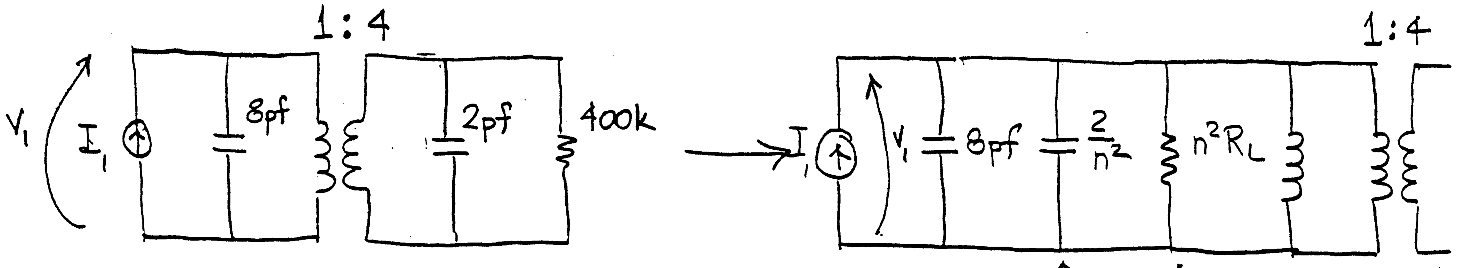
Example:



where $L_1 = 25\mu H$
$L_2 = 400\mu H$

assume $k \approx 1$

$$n = k\sqrt{\frac{L_1}{L_2}} = (1)\sqrt{\frac{25}{400}} = \frac{1}{4}$$



$\frac{2}{\left(\frac{1}{16}\right)} = 32pf$ $\frac{400}{16} = 25k$

outpu
shows
voltage
gain nV_1

for $k \approx 1$

$$Z_c(j\omega_0) \cong \frac{j\omega n^2 L_2}{+j\omega L_2/R} = n^2 R$$

since $j\omega_0(1-k^2)L_1 \approx 0$ for $k \approx 1$

condition (approximate for resonance

$$\omega_0^2 \approx \frac{1}{L_1 C_p} \quad \text{if } C_p \text{ in primary}$$

if I put C_s in secondary

$$\omega_0^2 \approx \frac{1}{L_2 C_s}$$

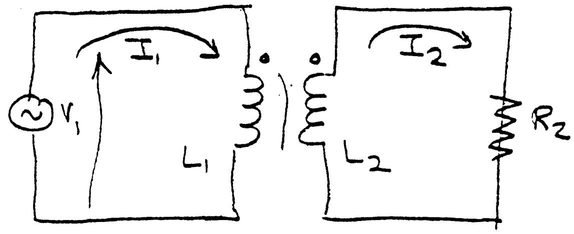
and, for frequency of resonance to remain the same,

$$\frac{1}{L_1 C_p} = \frac{1}{L_2 C_s}$$

$$\frac{C_p}{C_s} = \frac{L_2}{L_1} \text{ which equals } \frac{1}{n^2}$$

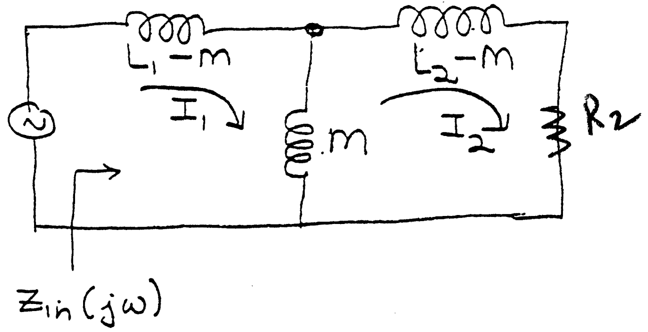
tuned transformers

set up mesh equations



$$V_1 = j\omega L_1 I_1 \underbrace{- j\omega M I_2}_{\text{due to direction of } I_2}$$

redraw as equivalent



$$V_1 = j\omega(L_1 - M) I_1 + j\omega M (I_1 - I_2)$$

$$= j\omega L_1 I_1 - j\omega M I_2$$ (exactly the same as for the transformer)

write the loop equations

$$Z_{in}(j\omega) = j\omega L_1 + \boxed{\frac{\omega^2 M^2}{j\omega L_2 + R_2}} \cdot \frac{\alpha^2}{\alpha^2}$$ parallel combination

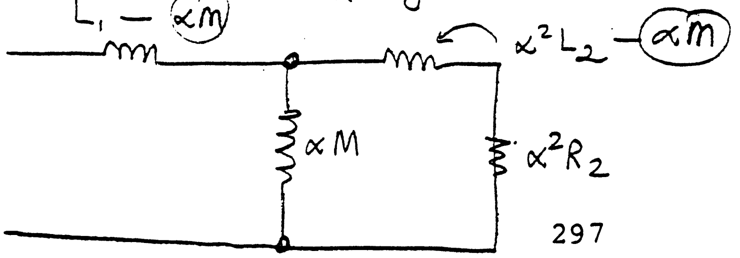
for convenience multiply top & bottom by α^2

$$= j\omega L_1 + \frac{\alpha^2 \omega^2 M^2}{\alpha^2 j\omega L_2 + \alpha^2 R_2}$$

replace M by αM

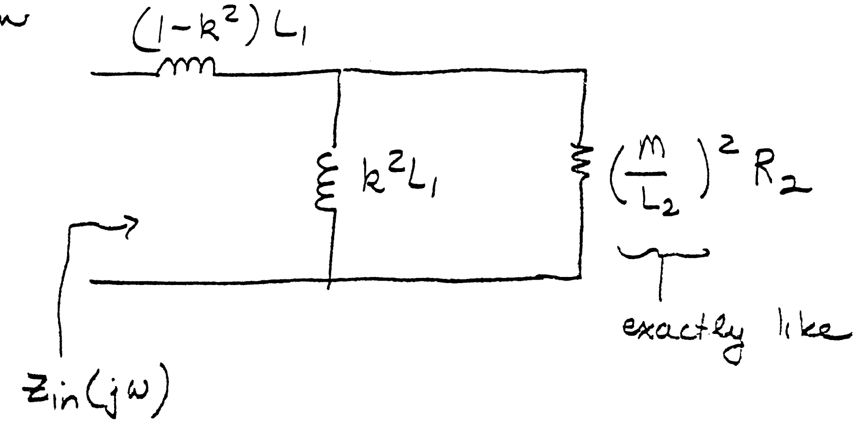
L_2 by $\alpha^2 L_2$

R_2 by $\alpha^2 R_2$ in circuit



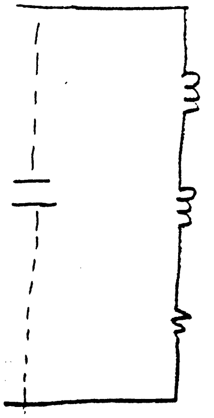
pick $\alpha = \frac{m}{L_2}$ to eliminate the inductance $\alpha^2 L_2 - \alpha m$,

Then



exactly like ideal transformer,

$$Q_p = \frac{R_p}{X_p} = \underbrace{\frac{R_2}{\omega L_2}}_{Q \text{ of coil secondary}}$$

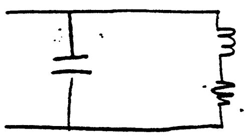


$(1-k^2) L_1$

$$L_p \left(\frac{Q_p^2}{Q_p^2+1}\right) = k^2 L_1 \left(\frac{Q_p^2}{Q_p^2+1}\right)$$

$$\frac{R_p}{1+Q_p^2} = \frac{\left[\frac{m}{L_2}\right]^2 R_2}{1+Q_p^2}$$

problem now looks like LR‖C circuit



$$L = L_1 \left[1 - k^2 + \frac{k^2 Q_p^2}{1+Q_p^2}\right]$$

$$R = \frac{\left(\frac{m}{L_2}\right)^2 R_2}{1+Q_p^2}$$

What do you do in a design problem?

Given: R_t, R_2, f_0, B

can calculate
$$\begin{cases} Q_t = f_0/B \\ C = \frac{1}{2\pi B R_t} \\ L_t = \frac{1}{\omega_0^2 C} \end{cases}$$

☆

+ 3 equations in 5 unknowns

m, L_1, L_2, Q_p, k

to solve we need either Q_p or k.

$$m = k\sqrt{L_1 L_2}$$

$$L_2 = \frac{R_2}{\omega_0 Q_p}$$

$$\underbrace{L_1 = L_t\left(\frac{Q_p^2+1}{Q_p^2+1-k^2}\right)}$$

Q_p is the Q of the secondary

$$Q_p = \frac{R_2}{\omega_0 L_2}$$

Q_t is the Q of the parallel LR‖C tank

always $Q_p < Q_t$

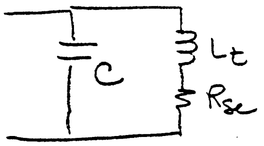
write these in terms of Q_p or k

combine all to get $$\frac{k^{\textcircled{2}4}}{4} + \frac{k^2-1}{Q_t^2} \le 0$$

final hairy results are

$$R_t = R_{se}\,(Q_t^2+1) \quad \text{where} \quad Q_t = \frac{1-k^2+Q_p^2}{Q_p^2 k^2}$$

for iron cores as $k \to 1$



in general :
$$Q_t = \frac{Q_p^2+1-k^2}{Q_p k^2}$$

$$k^2 = \frac{Q_p^2+1}{Q_p Q_t + 1}$$

$$Q_p = Q_t\left[\frac{k^2}{2} + \sqrt{\frac{k^4}{4} + \frac{k^2-1}{Q_t^2}}\right]$$

last equation says that if k is too small the transformer will <u>not</u> work.

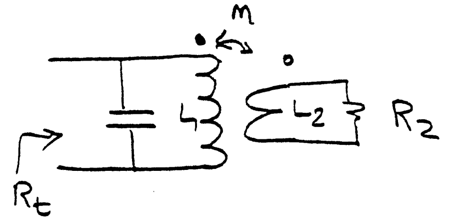
$$k_{min} = \frac{\sqrt{2}}{Q_t}\sqrt{\sqrt{Q_t^2+1} - 1}$$

Approach : ① get Q_t for desired circuit

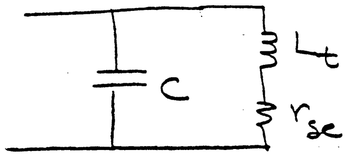
② solve for minimum k_{min}.

Q_p will also be a minimum here so pick $Q_p > Q_{p\,min}(k,$

Example:



we know this model



want $R_t = 2000\,\Omega$

$R_2 = 50\,\Omega$

$f_o = 3.18$ MHz

$B = 159$ kHz

$N^2 = \frac{2000}{50} = 40$ $N = 6.325$ transformer turns ratio

$$Q_t = \frac{\omega_o}{2\pi B} = \frac{3.18\times10^6}{159\times10^3} = 20$$

$$C = \frac{1}{2\pi B R_t} = \frac{1}{2\pi(159\times10^3)(2\times10^3)} = 500.5 \text{ pf.}$$

$$L = \frac{1}{\omega_o^2 C} = \frac{1}{\underbrace{(2\pi\cdot 3.18\times10^6)^2}_{399.2\times10^{12}}(500\times10^{-12})} = 5.005\,\mu H$$

Now, how do you build a transformer to do this.

find $k_{min} = \frac{\sqrt{2}}{Q_t}\sqrt{\sqrt{Q_t^2+1}-1} = \frac{\sqrt{2}}{20}\,4.362 = 0.308$

$$Q_{p,min} = \frac{1}{Q_t}\left\{\sqrt{Q_t^2+1}-1\right\} \cong 0.9512$$

∴ transformer **MUST** have $k > 0.308$

$Q_p > .9512$

we can get an L to give $Q_p = 3$

then $k^2 = \frac{Q_p^2+1}{Q_pQ_t+1} = \frac{10}{61} = .164$ $k = 0.4$

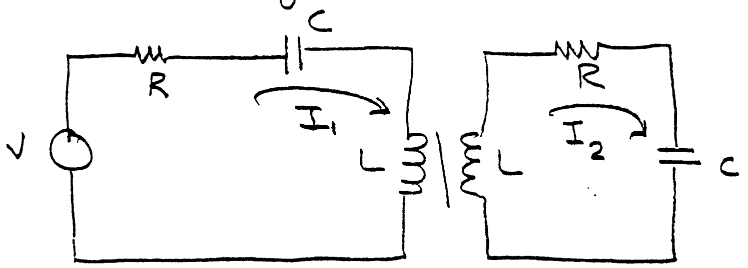
secondary Q $L_2 = \frac{R_2}{\omega_o Q_p} = \frac{50}{(2\pi\,3.18\times10^6)\,3} = 0.83\,\mu H$

$$L_1 = L_t\frac{Q_p^2+1}{Q_p^2+1-k^2} = (5.005\,\mu H)\frac{10}{10-.16} \approx 5\,\mu H.$$

$$m = k\sqrt{L_1L_2} = 0.4\sqrt{(.83)(5)} \cong 0.81\,\mu H.$$

double tuned transformers:

set up mesh equations



$$V = I_1 R - I_1 jX_c + I_1 jX_L \pm j\omega M I_2$$

$$0 = I_2 R - I_2 jX_C + I_2 jX_L \pm j\omega M I_1$$

$$\begin{bmatrix} V \\ 0 \end{bmatrix} = \begin{bmatrix} R + j(X_L - X_c) & \pm j\omega M \\ \pm j\omega M & R + j(X_L - X_c) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

solve for input impedance.

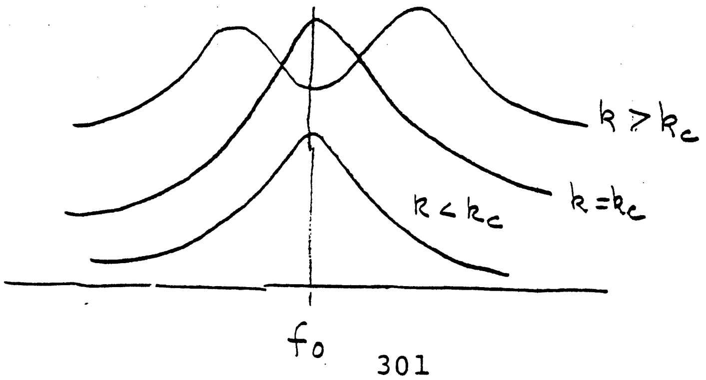
$$Z_{in} = \frac{V}{I_1} = R + j(X_L - X_c) + \frac{(\omega M)^2 R}{R^2 + (X_L - X_c)^2} - j\frac{(X_L - X_c)(\omega M)^2}{R^2 + (X_L - X_c)^2}$$

at resonance $Z_{in} = R + \frac{(\omega_0 M)^2}{R}$

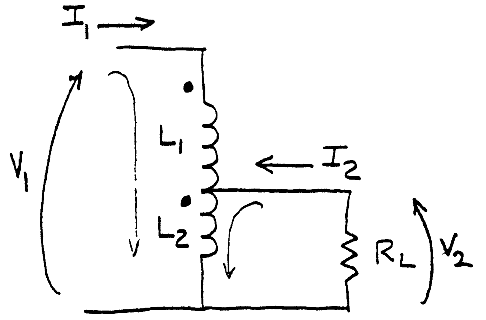
for maximum power transfer $\omega_0 M = R$

this gives rise to a critical coupling for max. power transfer

$$k_c = \frac{1}{Q}$$



Auto transformer



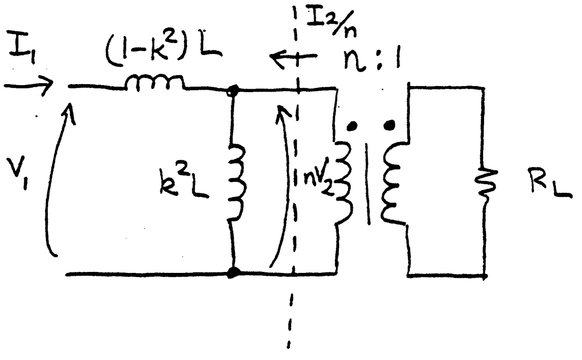
in a parallel circuit $Q \approx R_L$

if R_L is too small we need to increase R_L. A transformer is acceptable but an autotransformer is easier to build

what are the equations for the above autotransformer?

$$V_1 = s(L_1+M)I_1 + s(L_2+M)I_1 + s(L_2)I_2 + sMI_2$$
$$= s(L_1+L_2+M)I_1 + s(L_2+m)I_2$$
$$V_2 = sL_2I_1 + sMI_1 + sL_2I_2$$
$$= s(L_2+M)I_1 + sL_2I_2$$

(The signs of I_1 and I_2 allow all terms to be positive.)



$$V_1 = s(1-k^2)L\,I_1 + sk^2L\left(I_1 + \frac{I_2}{n}\right) = sLI_1 + s\frac{k^2L}{n}I_2$$

$$nV_2 = sk^2L\left(I_1 + \frac{I_2}{n}\right) \text{ or } V_2 = s\frac{k^2L}{n}I_1 + \frac{sk^2L}{n^2}I_2$$

How can these sets of equations be equivalent?

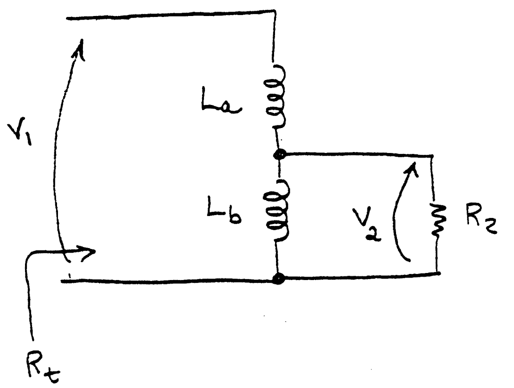
if $L = L_1 + L_2 + 2M$

and $L_2 + M = \frac{k^2L}{n}$ or $n = \frac{k^2L}{L_2+M}$

and $\frac{k^2L}{n^2} = L_2$ or $n^2 = k^2\frac{L}{L_2}$

inductor / transformer circuits

tapped coil <u>with</u> mutual inductance



M = mutual inductance

k = coupling coefficient

L looking into coil is

$L_a + L_b + 2M$

when k=1 (100% coupling) we have an ideal transformer

$$\frac{R_t}{R_2} = \left(\frac{L}{L_b+M}\right)^2 = \frac{V_1^2}{V_{2\,o.c.}^2}$$

where does this come from

① $V_1 = j\omega L$; $V_2 = j\omega(L_b+M)$

② $$\frac{V_1^2}{V_2^2} = \frac{L^2}{(L_b+M)^2}$$

③ from power considerations we get V_1/V_2

$$\frac{V_1^2}{R_t} = \frac{V_2^2}{R_2} \Rightarrow \frac{V_1^2}{V_2^2} = \frac{R_t}{R_2}$$

what does Y_i look like? $Y_i = G_i + jB_i$

transformed R_2 (for G_i); no capacitor in circuit NOT resonant (for B_i)

If we add a parallel C we can resonant

$$Q_t = \frac{B_i}{G_t} = \frac{R_t}{\omega_0 L}$$

and $B = \dfrac{f_0}{Q_t}$

suppose $k \neq 1$

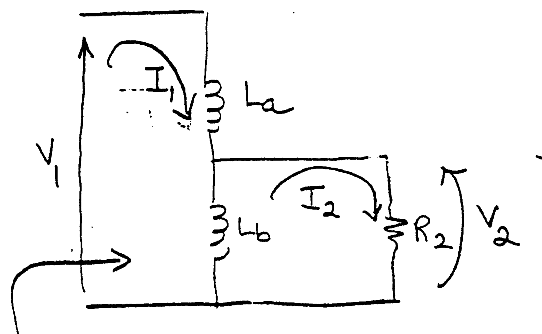
solve by mesh equations

$$V_1 = j\omega\,(L_1+L_2)\,I_1 + \boxed{2}\, j\omega m I_1 + j\omega\,\underbrace{(L_2+m)}_{}\,I_2$$

current flowing in lower section + its inductance

$$V_2 = j\omega(L_2+m)I_1 + j\omega L_2 I_2$$

current I_1 flows through L_1 causing m



$Y_i = G_i + jB_i$

✓

$$V_2 = I_2R_2 = -\underbrace{j\omega\,(L_b+m)}_{\text{total voltage across } L_b \text{ due to } I_2}I_2 + \underbrace{j\omega\,\overset{L_b}{\text{[illegible]}}\,I_1}_{}$$

~~mutual inductance from La~~

I_1 in

Some errors here

voltage around I_1 loop.

$$V_1 = j\omega\,(L_a+m)\,I_1 + j\omega\,(L_b+m)\,I_1 - j\omega\,(L_b+M)\,I_2$$

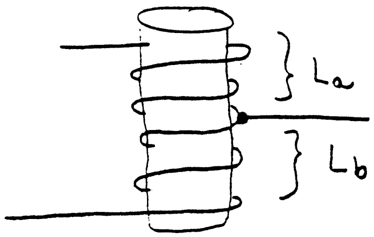
$$0 = j\omega m I_1 \quad -j\omega(L_b+m)I_2 \quad - I_2R_2$$

$$\begin{bmatrix} V_1 \\ 0 \end{bmatrix} = \begin{bmatrix} j\omega(L_a+m)+j\omega\,(L_b+m) & -j\omega(L_b+m) \\ j\omega m & -j\omega\,(L_b+m)-R_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

solve $R_t = \dfrac{L^2}{(L_b+m)^2}R_2 + \dfrac{\omega^2L^2}{R_2}\left[\dfrac{L_b}{L_b+m} - \dfrac{L_b+m}{L}\right]^2$

$$B_i = -\frac{1}{\omega L}\left\{\frac{\omega^2 L_b\left[L_b - \frac{(L_b+m)^2}{L}\right] + R_2^2}{\omega^2\left[L_b - \frac{(L_b+m)^2}{L}\right]^2 + R_2^2}\right\}$$

for certain types of coils this looks like a transformer with turns ratio N



L_a & L_b share a common form and, except for length are identical.

For solenoidal fields

$$L_a = \underset{\nwarrow \text{some constant}}{a} L_b$$

object — reduce our complex expressions for Y_i

$$k \triangleq \frac{M}{\sqrt{L_a L_b}} \quad \text{or} \quad M = k\sqrt{L_a L_b}$$

$$= k\sqrt{aL_bL_b} = k\sqrt{a}\,L_b$$

$$L = L_a + L_b + 2M = aL_b + L_b + 2k\sqrt{a} \quad \nwarrow L_b$$

$$L_b = \frac{L}{1+a+2k\sqrt{a}}$$

$$\frac{L_b}{L_b+M} = \frac{1}{1+k\sqrt{a}} \qquad \text{after some algebra}$$

for the opencircuited V_2 we had $\frac{L_b+M}{L} = \frac{V_{2\,o.c.}}{V_1} \triangleq \frac{1}{N}$

define this way so $N > 1$

$$V_1 = \left(\frac{L}{L_b+M}\right) V_{2oc} = NV_{2oc}$$

combine results to get

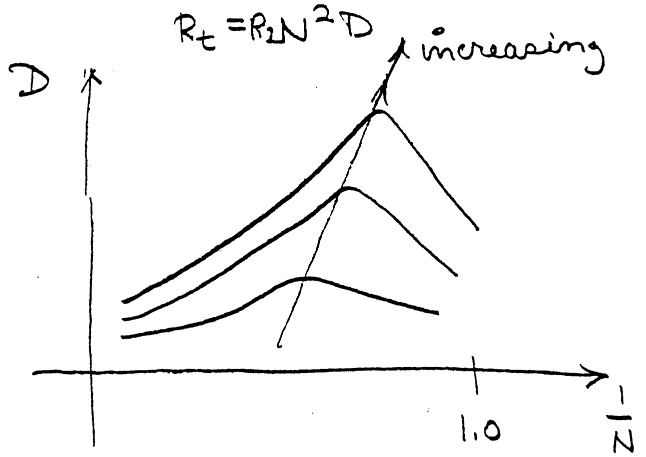
$$R_t = R_2\left[N^2 + \left(\frac{\omega L}{R_2}\right)^2 K^2\right] = R_2\left[N^2 \times D\right]$$

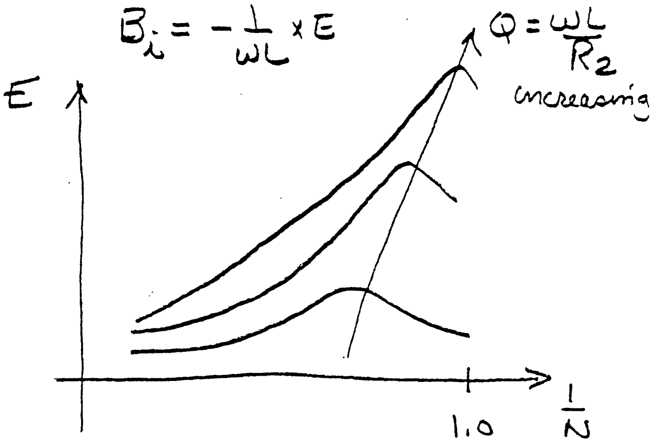
$$\text{where } K \triangleq \frac{1}{1+k\sqrt{a}} - \frac{1}{N}$$

$$B_i = -\frac{1}{\omega L}\left[\frac{\left(\frac{\omega L}{R_2}\right)^2\left[\frac{K}{(1+a+2k\sqrt{a})\,N}\right]+1}{\left(\frac{\omega L}{R_2}\right)\left(\frac{K}{N}\right)^2+1}\right]$$

$$= -\frac{1}{\omega L} \times E$$

D & E represent departures from ideal transformers and are algebraically quite complicated.





transformer action is
<u>least</u> ideal in center
largest mutual inductance

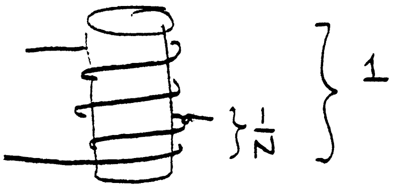
inductance change is
largest near full loop

as Q increases D increases E increases

k increases D decreases E decreases

why plot these as functions of 1/N ?

$\frac{1}{N}$ is the "fraction" of the coil tapped



overall turns ratio is $\frac{1}{1/N} = N$

Analysis of autotransformers:

The basic autotransformer is simply two coils which have a non-zero mutual inductance AND a load resistor across one of the inductances as shown below.

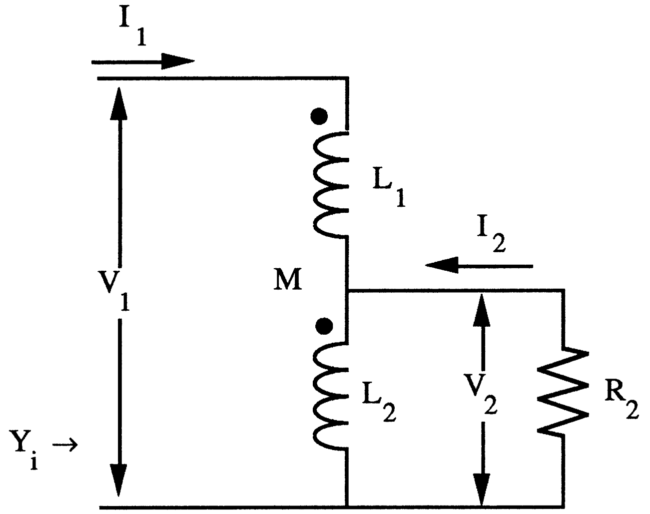


Figure 1 - "Physical " tapped inductor

The loop equations for this network can be exactly written as

$$V_1 = j\omega(L_1+L_2+2M)I_1 + j\omega(L_2+M)I_2 \tag{1a}$$

and

$$V_2 = j\omega(L_2+M)I_1 + j\omega(L_2)I_2 \tag{1b}$$

These equations were formally analyzed in Krauss, Bostian and Raab and can be solved without any approximations to give the input impedance of the network shown in Figure 1 as

$$Y_i = G_i + jB_i = \frac{R_2 + j\omega L_2}{-\omega^2(LL_2 - (L_2+M)^2) + j\omega LR_2} \tag{2}$$

This result is a little awkward to manipulate so we introduced the idea that the network can be modeled as a parallel resistance and susceptance. The input resistance is given by

$$\frac{R_t}{R_2} = \left[\frac{L}{L_2 + M}\right]^2 \times D = N^2 \times D \tag{3}$$

where N is an "effective" turns ratio and D is a deviation parameter which is a complex function of Q_p, M, k, the "tap ratio" and other parameters. The input susceptance is given by

$$B_i = \frac{-1}{\omega L} \times E \tag{4}$$

Notice that this transformation ratio is NOT that of a transformer model because it directly transforms the secondary resistance to the input and gives the equivalent parallel inductance across the input.

An equivalent transformer model is easily to work with and understand. Such a model is shown below

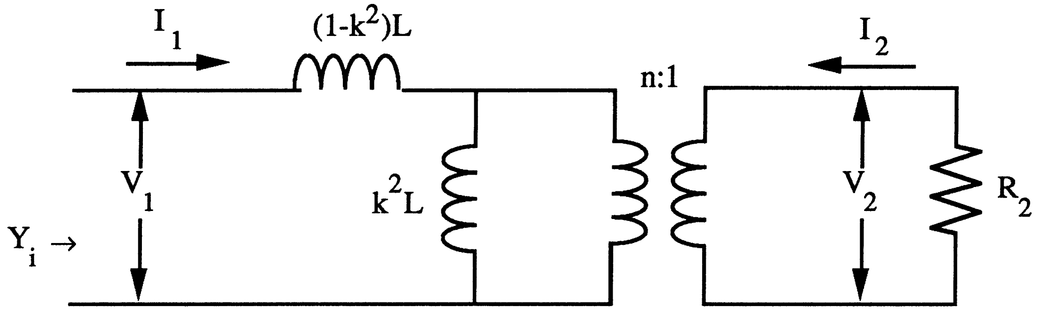


Figure 2 - "Transformer" equivalent model of tapped inductor

Note that the same letter "n" has been used to denote this turns ratio which is the actual turns ratio for an "equivalent" transformer. The k used in this model is confusing because it is not the actual k of the original physically coupled coils. Instead, it is a fictious k which makes the two models equivalent. In the same manner, the L shown in the above figure is the L which makes the two models equivalent.

The loop equations for Figure 2 are, after some simplification,

$$V_1 = j\omega L I_1 + \frac{j\omega k^2 L}{n} I_2 \tag{5a}$$

and

$$V_2 = \frac{j\omega k^2 L}{n} I_1 + \frac{j\omega k^2 L}{n^2} I_2 \tag{5b}$$

which, because it is the same physical network, must match equation (1). For equations (1) and (5) to be identical, it is necessary that the coefficients of I_1 and I_2 match, i.e.

$$L = L_1 + L_2 + 2M \tag{6a}$$

$$L_2 + M = \frac{k^2 L}{n} \tag{6b}$$

and

$$L_2 = \frac{k^2 L}{n^2} \tag{6c}$$

The first equation is already in a usable form. The second and third equations are two equations in two unknowns, k and n, which must be solved simultaneously. Substituting (6c) into (6b) we get

$$L_2 + M = \frac{k^2 L}{n} = \left(\frac{L_2 n^2}{L}\right)\frac{L}{n} = L_2 n$$

which can be solved for n to give

$$n = \frac{L_2 + M}{L_2} = 1 + \frac{M}{L_2} \tag{7}$$

This result can be substituted into equation (6b) to give the expression for k

$$k^2 = \frac{n(L_2 + M)}{L} = \frac{(L_2 + M)^2}{L L_2} \tag{8}$$

Equations (6a), (7) and (8) completely define the transformer equivalent circuit shown in Figure 2. Note that the expression for k is NOT even similar to the expression

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

which applies to the physical transformer.

The use of the wrong k in the transformer equivalent model is a very common student error.

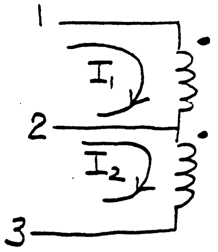
The ac output voltage ΔV_{O2} is then, approximately,

$$\Delta V_{O2} \approx -\frac{\alpha_F I_{EE} R_C}{1 + e^{\frac{-\Delta V_i}{V_T}}} = -\frac{\alpha_F I_{EE} R_C}{1 + \left(1 - \frac{\Delta V_i}{V_T}\right)} = -\frac{\alpha_F I_{EE} R_C}{2\left(1 - \frac{\Delta V_i}{2V_T}\right)} = -\frac{\alpha_F I_{EE} R_C}{2}\left(1 + \frac{\Delta V_i}{2V_T}\right) \qquad (15)$$

Note that the first term contains only I_{EE} whereas the second term contains the product term $I_{EE}\Delta V_i$ which is the desired mixer term. Examination of the second term reveals that the mixer term is proportional to I_{EE}, R_C and ΔV_i. Unfortunately, the first term is also proportional to I_{EE} and R_C and making these terms too large will lead to saturation of the mixer output by the unwanted first term. In practice, optimizing your mixer output will require a careful adjustment of the bias current and load resistance.

The current I_{EE} is actually an ac current source controlled by the local oscillator. A mixer design I tested used a relatively small nominal I_{EE} of about 1mA and collector resistances of 1kΩ. [Actually, one collector load resistance was 1kΩ and the other was a 42IF104 i.f. transformer.] The local oscillator voltage was approximately 500 mV and the rf input voltage was estimated to be about 100μV. The voltage out of the i.f. amplifier was on the order of one volt.

Example: You are given the coil shown below. Measured inductance values are $L_{12} = 4 \mu H$, $L_{23} = 2 \mu H$, $L_{13} = 9 \mu H$.



(a) For I_1 and I_2 as shown above write loop equations for V_{12} and V_{23}.

$$V_{12} = j\omega L_{12} I_1 + j\omega M I_2$$

$$V_{23} = j\omega L_{23} I_2 + j\omega M I_1$$

(b) Find values for the mutual inductance M and coupling coefficient k.

$$L_{13} = L_{12} + L_{23} + 2M$$

$$9 = 4 + 2 + 2M$$

$$m = 1.5 \mu H$$

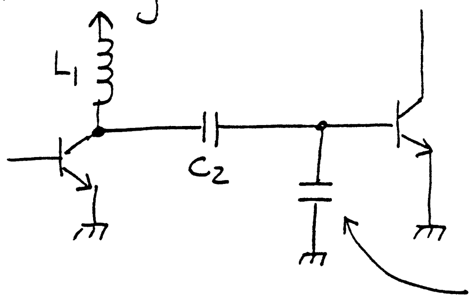
$$k = \frac{M}{\sqrt{L_{12} L_{23}}} = \frac{1.5}{\sqrt{(4)(2)}} = 0.53$$

(c) If $V_{13} = 10$ volts rms, what are the corresponding values for V_{12} and V_{23}?

$$V_{12} = 10 \frac{L_{12} + M}{L_{13}} = 10 \frac{4 + 1.5}{9} = 6.11 \text{ volts}$$

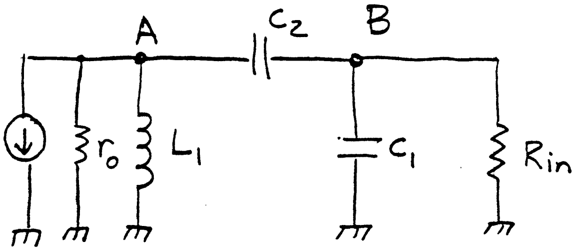
$$V_{23} = 10 \frac{L_{23} + M}{L_{13}} = 10 \frac{2 + 1.5}{9} = 3.89 \text{ volts.}$$

Example : matching network

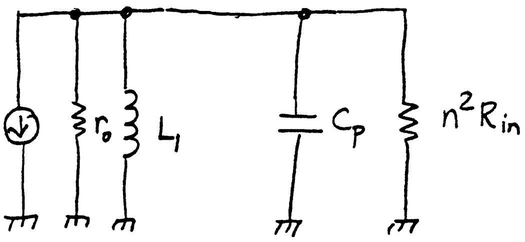


C_1 is added to make this a resonant circuit

equivalent circuit



which becomes



where $C_p = \frac{C_1 C_2}{C_1 + C_2}$

$\underline{\underline{\underline{\text{if}}}}$ $\omega^2 R^2 (C_1 + C_2)^2 \gg 1$

and

$$R = n^2 R_{in} = \left(\frac{C_1 + C_2}{C_2}\right)^2 R_{in}$$

we can pick C_1 & C_2 to give the n we need.

standard parallel RLC circuit

$$\omega_0^2 = \frac{1}{L_1 C_p}$$

$$Q = \frac{R_P}{\omega_0 L} = \frac{r_0 \parallel n^2 R_{in}}{\omega_0 L_1}$$

voltage at point A: (at resonance)

$$V_{\text{collector } Q_1} \doteq -(\beta i_b)\left(r_o \| n^2 R_{in}\right)$$

$$V_{base, Q_2} = V_{collector, Q_1} \frac{\frac{1}{j\omega C_1} \| R_{in}}{\frac{1}{j\omega C_1} \| R_{in} + \frac{1}{j\omega C_2}}$$

$$= V_{c, Q_1} \frac{\frac{R_{in}/j\omega C_1}{R_{in} + 1/j\omega C_1}}{\frac{R_{in}/j\omega C_1}{R_{in} + 1/j\omega C_1} + \frac{1}{j\omega C_2}}$$

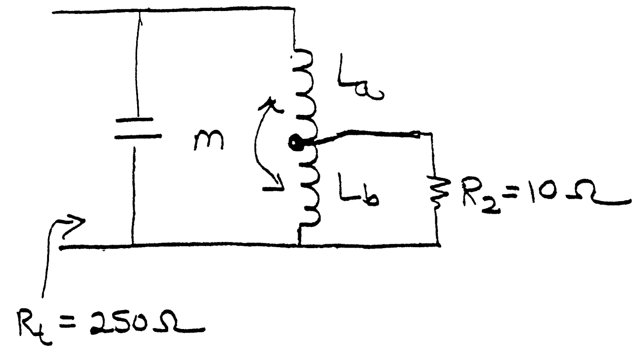
$$= V_{c,Q} \frac{\frac{R_{in}}{1 + j\omega R_{in} C_1}}{\frac{R_{in}}{1 + j\omega R_{in} C_1} + \frac{1}{j\omega C_2}} \qquad \frac{j\omega C_2 (1 + j\omega R_{in} C_1)}{j\omega C_2 (1 + j\omega R_{in} C_1)}$$

$$= V_{c,Q} \frac{j\omega C_2 R_{in}}{j\omega C_2 R_{in} + 1 + j\omega R_{in} C_1}$$

$$= V_{c,Q} \frac{j\omega R_{in} C_2}{1 + j\omega R_{in}(C_1 + C_2)}$$

$$\approx V_{c,Q} \frac{C_2}{C_1 + C_2}$$

Example 3-7.2

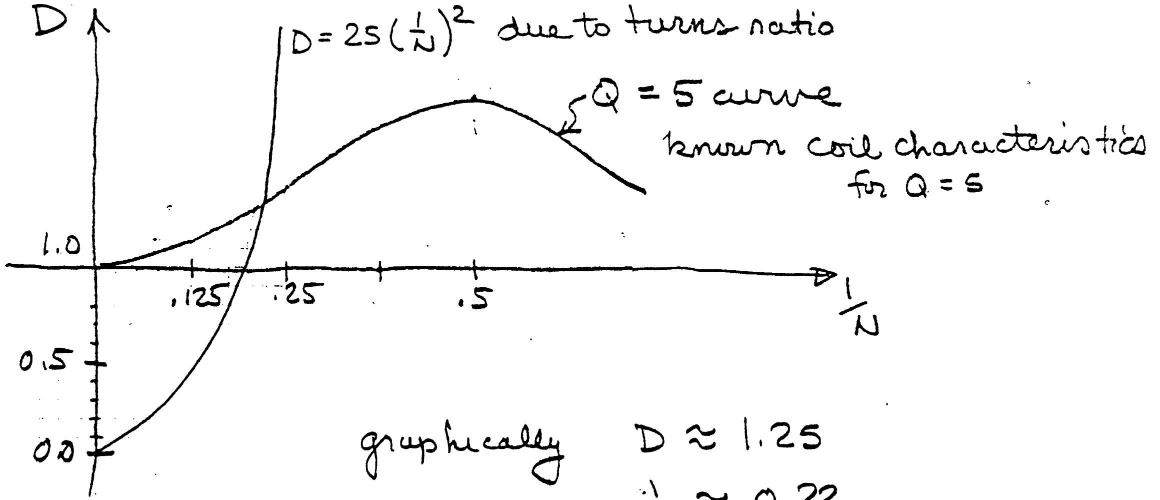


$f_0 = 4\,mHz$
$L = 2\,\mu H$
$k = 0.25$

$$Q = \frac{\omega L}{R_2} = \frac{2\pi\ (4\times10^{6})(2\times10^{-6})}{10\,\Omega} = \frac{16\pi}{10} \cong 5$$

$$\frac{R_t}{R_2} = \frac{250}{10} = 25 = N^2 D$$

$$D = \frac{25}{N^2} = 25\left(\frac{1}{N}\right)^2$$ for $\underline{\underline{\text{this}}}$ coil



graphically $D \approx 1.25$

$\frac{1}{N} \approx 0.22$

now easy to get E — read from graph in Chapter 3

for $Q = 5$, $1/N \approx 0.22$ $\quad E \sim 1.1$

$$\Rightarrow B_i = -\frac{1}{\omega L} \times E = -\frac{1}{2\pi(4\times10^{6})(2\times10^{-6})}\,1.1 = .0438 \times 10^{-6}$$

math errors

$$= -\ 0.0438\,m\mho$$

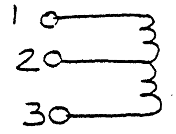
Capacitance needed to resonate

$$\omega C = +\ 43.8 \times 10^{-3}$$

$$C = \frac{43.8\times10^{-3}}{2\pi\times 4\times10^{6}} = 1.74\times10^{-9}$$

$$C = 1740\ pf$$

You are given an unknown r.f. coil which is tapped. You number the pins as shown below. Using an inductance meter you measure 250μH inductance between pins 1 and 2, 50μH between pins 2 and 3, and 516 μH between pins 1 and 3.



(a) What is the mutual inductance M?

(b) What is the coupling coefficient k?

(c) I decide to use this transformer (coil) as part of a tuned circuit and connect a 0.075μf capacitor between pins 2 and 3. What is the resonan frequency as seen at pins 1 and 3. State all assumptions.

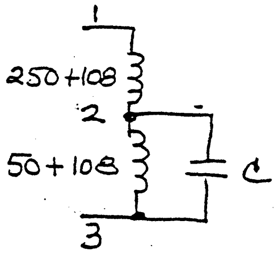
(a) $250 + 50 + 2m = 516$

$300 + 2m = 516$

$m = 108$

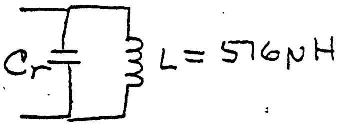
(b) $k = \frac{M}{\sqrt{L_a L_b}} = \frac{108}{\sqrt{50(250)}} = \frac{108}{111.8} \approx 0.965$

(c)



step-up ratio $N = \frac{L}{\underset{L_{2,3}=}{L_b+M}} = \frac{516}{50+108} = \frac{516}{158} \approx 3.27$

circuit looks like



where C_r decreased from C by square of turns ratio, i.e.

$$\frac{1}{j\omega C_r} = N^2 \frac{1}{j\omega C} \Rightarrow C_r = \frac{C}{N^2} = \frac{.075\times10^{-6}}{(3.27)^2}$$

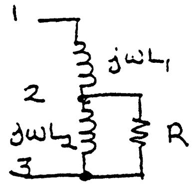
$$C_r \approx 7\times10^{-9} = 7000\,pf$$

The resonant frequency is then given by

$$\omega_0 = \frac{1}{\sqrt{LC_p}} = \frac{1}{\sqrt{(516\times10^{-6})(7.03\times10^{-9})}} = 5.25\times10^5$$

$f_0 = 83.6$ kHz

The above solution can be solved also by direct network analysis.
For algebraic simplicity let R be The impedance connected across pins 2&3



let Z_{in} be the impedance seen across terminals 1 and 3.

$$Z_{in} = j\omega L_1 + j\omega L_2 \| R = j\omega L_1 + \frac{j\omega L_2 R}{j\omega L_2 + R}$$

Note this assumes no mutual inductance, it will be included in L_1 & L_2. $Z_{in} = \frac{-\omega^2 L_1 L_2 + j\omega(L_1+L_2)R}{R + j\omega L_2}$

This can be inverted to give

$$Y_{IN} = \frac{1}{Z_{in}} = \frac{-\omega^2 L_1 L_2 R + \omega^2 L_2(L_1+L_2)R - j\omega^3 L_1 L_2^2 - j\omega(L_1+L_2)}{\omega^4(L_1L_2)^2 + \omega^2(L_1+L_2)^2R^2}$$

Since Y_{IN} has real and imaginary parts Y_{IN} looks like a parallel RL circ where

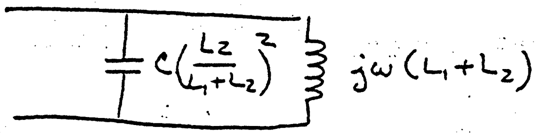
$$R_p = R\left[\left(\frac{L_1+L_2}{L_2}\right)^2 + \frac{\omega^2 L_1^2}{R^2}\right] \qquad L_p = (L_1+L_2)\left[\frac{(L_1+L_2) + \omega^2\frac{L_1^2L_2^2}{(L_1+L_2)R^2}}{(L_1+L_2) + \omega^2\frac{L_1L_2^2}{R^2}}\right]$$

these can be regarded as equivalent to the D factor

$$\text{if } \frac{\omega L_1}{R} << 1$$

$$R_p \approx R\left(\frac{L_1+L_2}{L_2}\right)^2 \quad \text{and} \quad L_p \approx L_1 + L_2$$

in terms of our original circuit where $R = \frac{1}{j\omega C}$ we now have.



$$\omega_o^2 = \frac{1}{LC} = \frac{1}{L_1+L_2}\,\frac{(L_1+L_2)^2}{L_2^2\,C} = \frac{1+\frac{L_1}{L_2}}{L_2C}$$

$$\omega_o^2 = \frac{1+\frac{358}{158}}{(158\times10^{-6})(.075\times10^{-6})} \cong 0.28\times10^2$$

$$\omega_o = 0.525\times10^6$$

$$f_o = 83.5\text{ kHz}.$$