Morphological Image Processing

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Outline

- Introduction
- Binary MORPH
- Gray-scale MORPH
- Miscellaneous MORPH
- Conditional Dilation
- Geodesic Dilation
- Watershed

Introduction

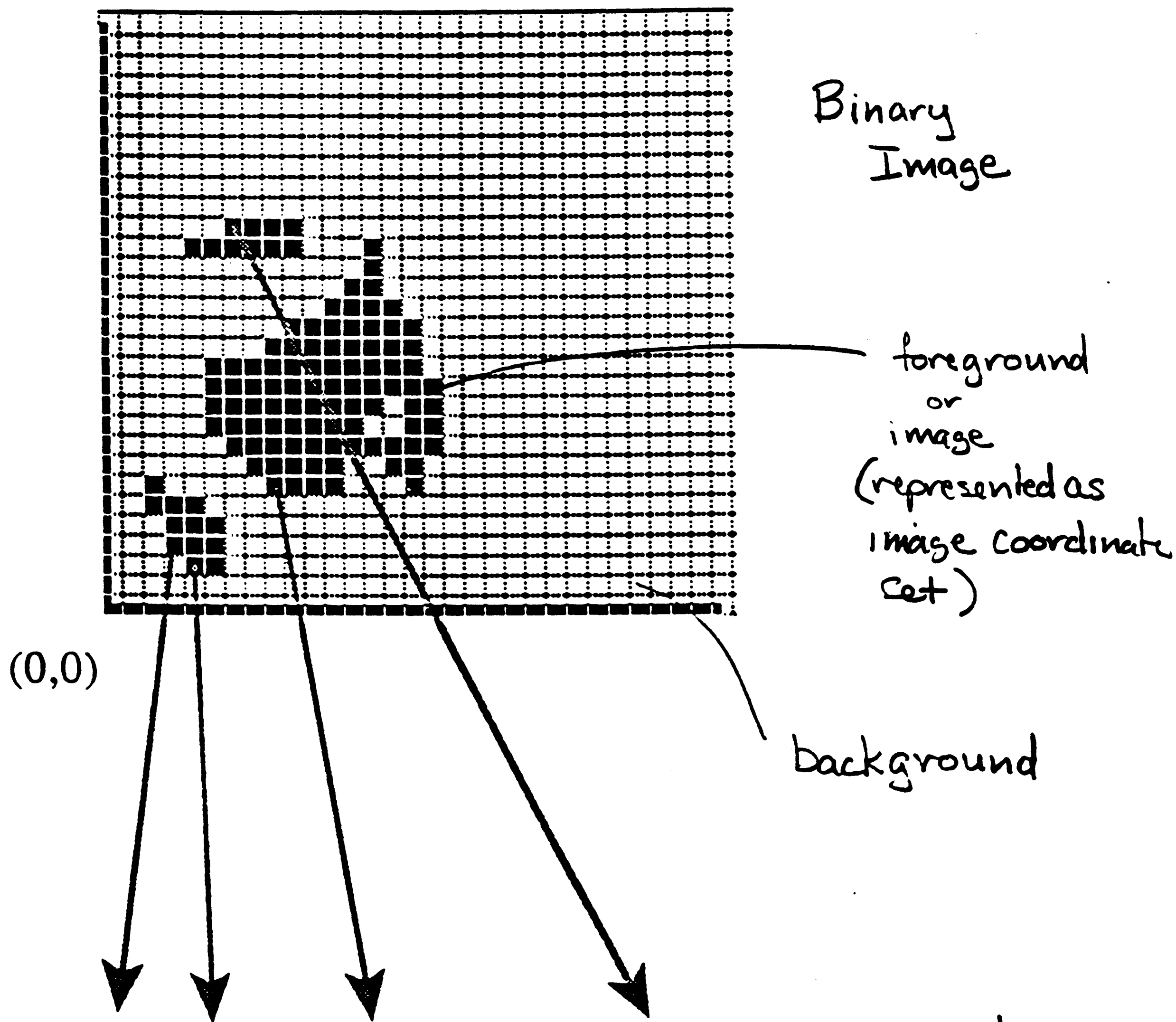
- Images are considered as point sets in E^n or R^n (continuous) or Z^n (discrete).
- Morphology implies shape. Morphological processing is processing with regard to shape and the element of shape is the structuring element, SE.
- Elementary operations include $\ominus$ (erosion) and $\oplus$ (dilation).
- Elementary operations are combined to give $\circ$ (opening) and $\bullet$ (closing).
- MORPH operations are non-linear.
- Erosion, dilation, opening, and closing are filtering operations that use a finite base of support. (An output pixel depends upon a local neighborhood only.)
- Other morphology operations are of "infinite" extent. In such operations, an output pixel can depend upon pixel values throughout the input image.
- MORPH operations are used for noise reduction, edge enhancement, image enhancement, shape recognition, object removal, texture measurement, and segmentation.

Topics in Morphological Image Processing

- binary image filtering
- shape recognition with MORPH
- gray-scale MORPH
- image feature enhancement
- restoration of AFM images
- filtering with multiple SE's
- MORPH edge detection
- granularity
- skeletonization and distance functions
- conditional dilation
- geodesic dilation
- reconstruction
- watershed

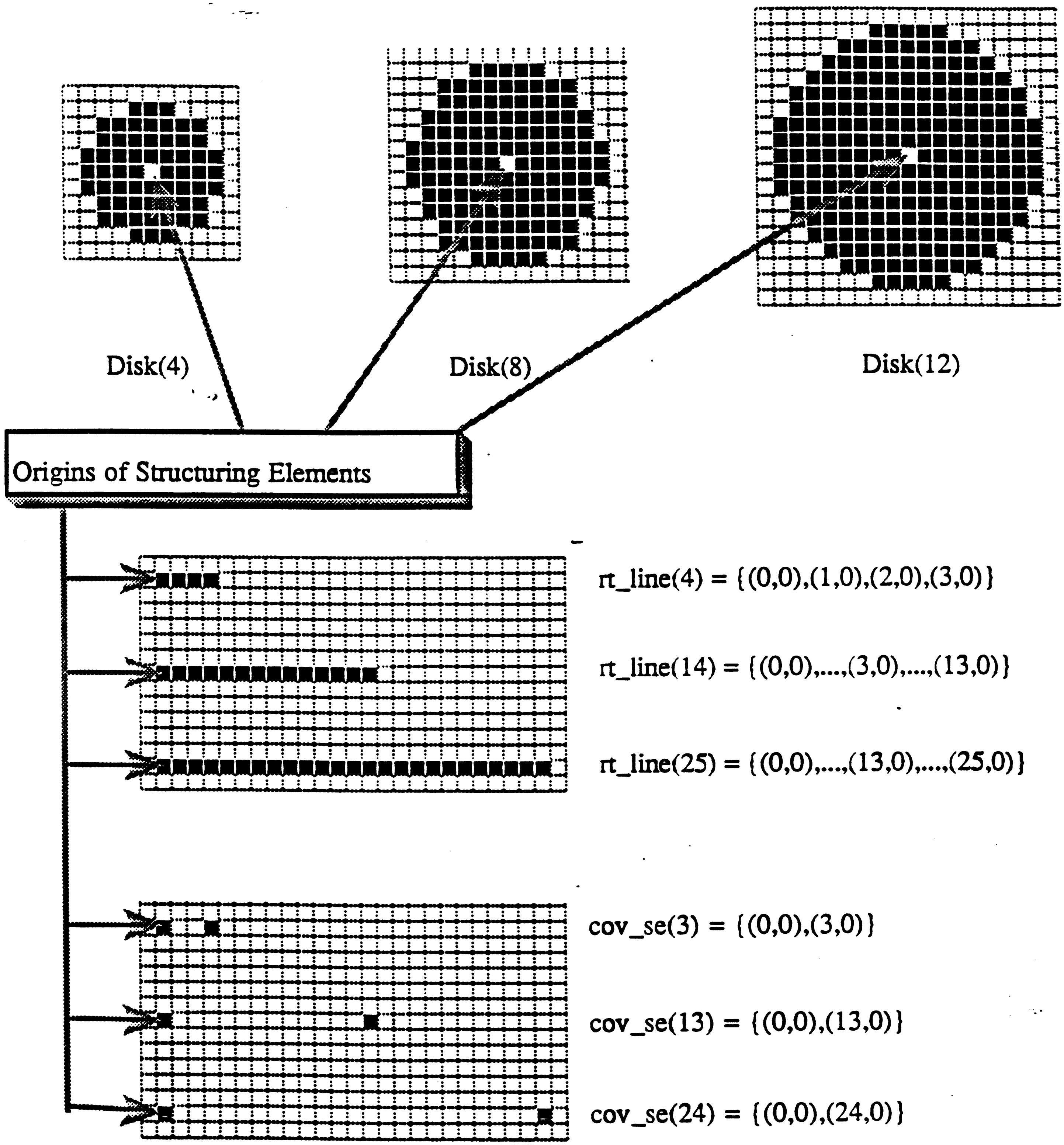
MORPH is useful because it is selective to shape. *A priori* shape information can be included in the processing.

Images: coordinate sets in R^n or Z^n

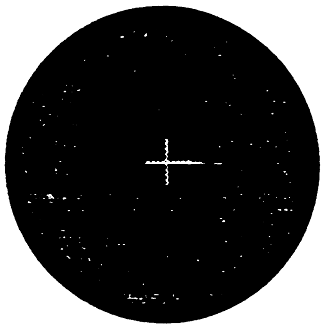


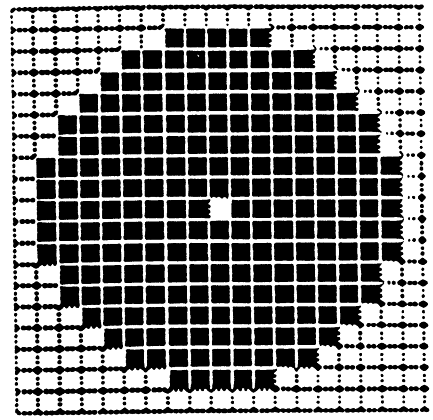
$I = \{ (3,3),(4,2),\ldots\ldots,(8,6),\ldots\ldots\ldots\ldots\ldots\ldots,(6,19)\}$ coordinate set

SE's of increasing size can be used to probe various "scales"



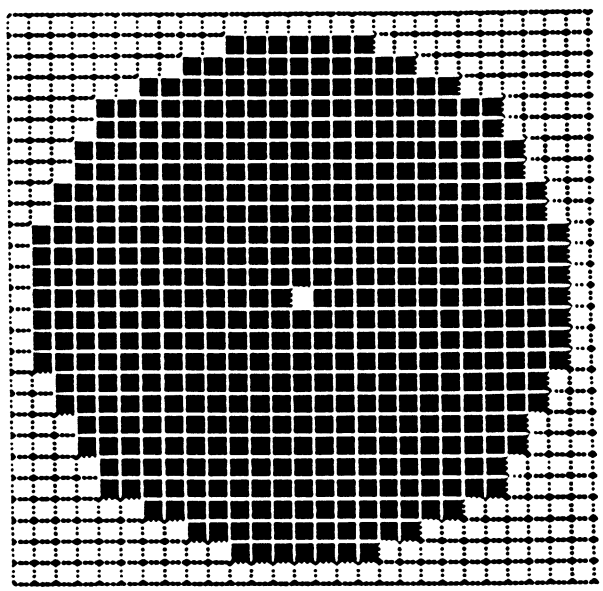
rotationally invariant "disks" are commonly used SE's





$\text{Disk}(r) \subset R^n$ $\qquad$ $\text{Disk}(r) \subset Z^n$

"subset" or "contained within"

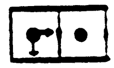


$\text{Duo} - \text{Decagon}(r) \subset Z^n$

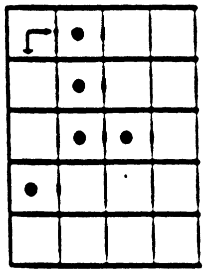
12 sided

$$A = \{(0,1), (1,1), (2,1), (2,2), (3,0)\}$$

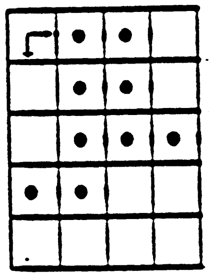
$$B = \{(0,0), (0,1)\}$$



B



A



$A \oplus B$

$$A \oplus B = \{(0,1),\ (1,1),\ (2,1),\ (3,0)\ \ (0,2),\ (1,2),\ (2,2),\ (2,3),\ (3,1)\}$$

1) $A \oplus B = \{ c \in Z^{N} \mid c = a + b$ for some $a \in A$ and $b \in B \}$

2) $A \oplus B = \bigcup_{a \in A} B_a$

3) $A \oplus B = \bigcup_{b \in B} A_b$

$A_t = \{ c \in Z^{N} \mid c = a + t$ for $a \in A \}$

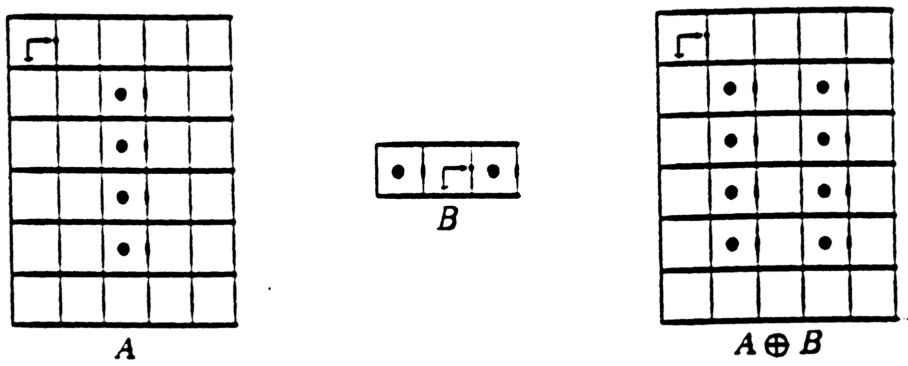
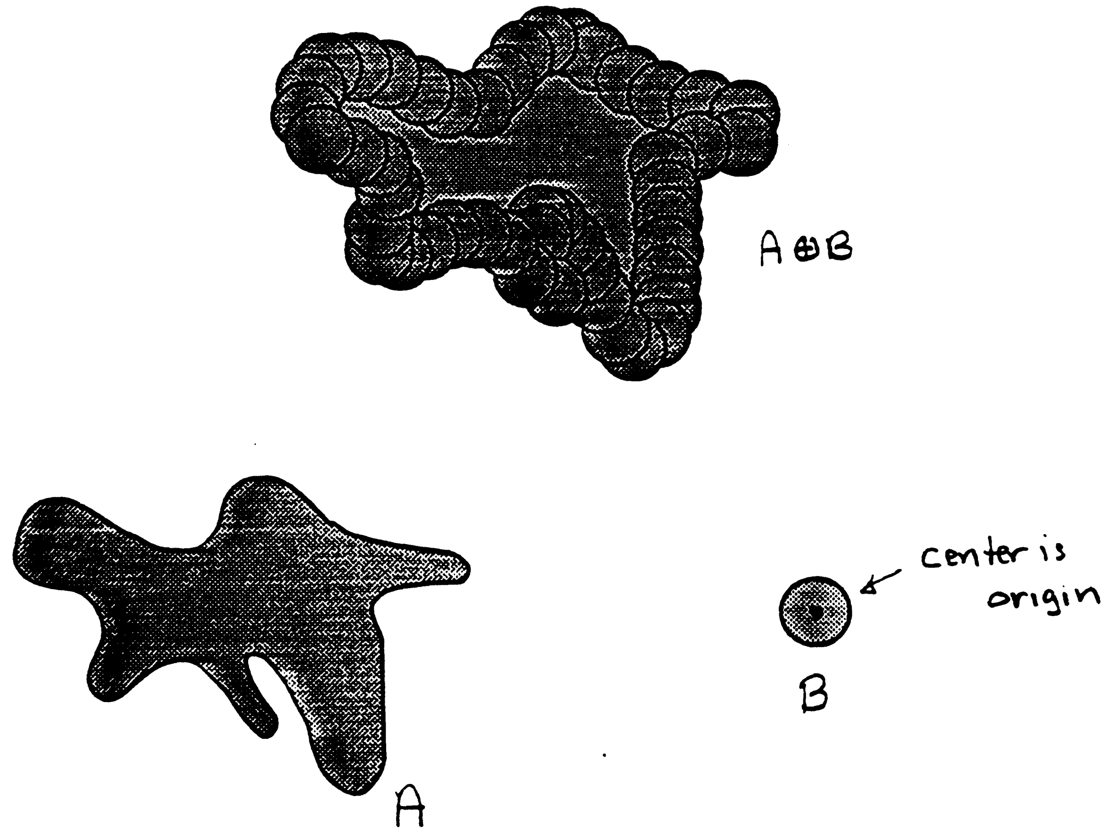


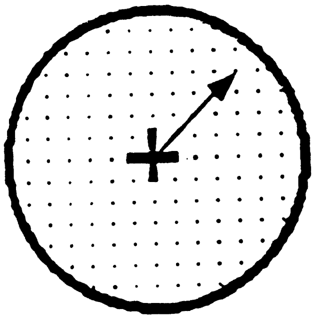
Figure 5.2 illustrates a set A being dilated by structuring element B which does not contain the origin. As a result, the dilated set is not even guaranteed to have a single point in common with A. However, there are always translations of $A \oplus B$ which can contain A.

$$A \oplus B = \bigcup_{a \in A} B_a$$

definition of dilation: $A \oplus B$

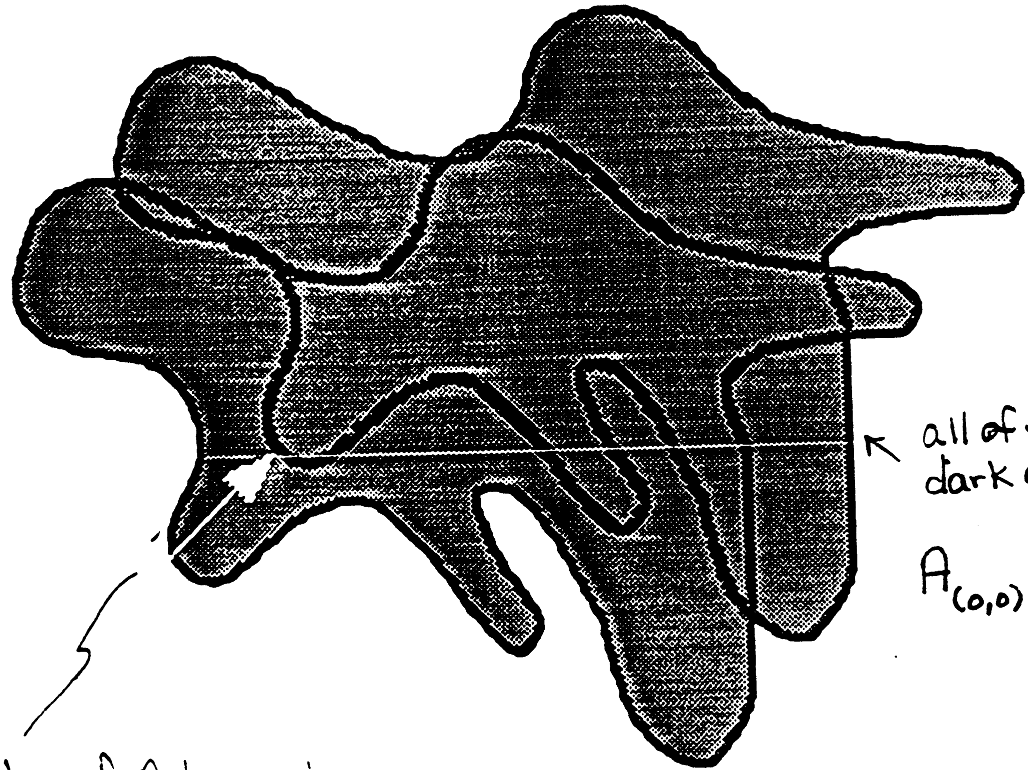


$$A \oplus B = \bigcup_{a \in A} B_a$$



$t_0 = (x_0, y_0) \in B$

a vector



all of the dark area is

$A_{(0,0)} \cup A_{(x_0,y_0)}$

translation of A by vector (x_0, y_0).

$$A \oplus B = \bigcup_{b \in B} A_b$$

— Erosion —

$$A = \{(1,0),\ (1,1),\ (1,2),\ (1,3),\ (1,4),\ (1,5),$$
$$(2,1),\ (3,1),\ (4,1),\ (5,1),\}$$
$$B = \{(0,0),\ (0,1)\}$$
$$A \ominus B = \{(1,0),\ (1,1),\ (1,2),\ (1,3),\ (1,4)\}$$

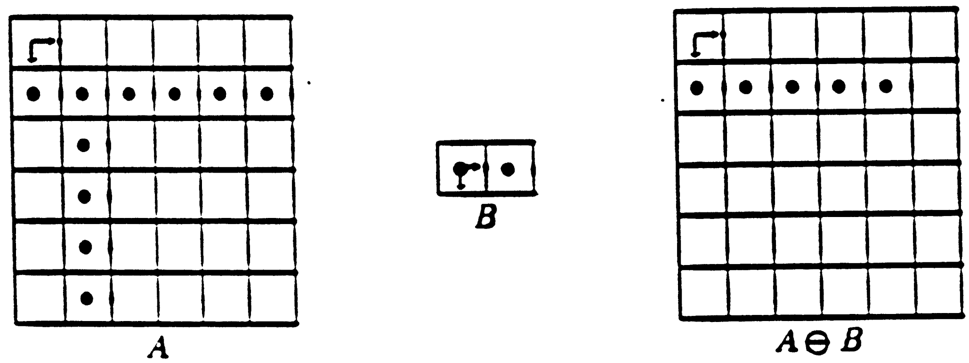


Figure 5.3 illustrates the erosion of a set and its corresponding binary image.

1) $A \ominus B = \{ x \in Z^N \mid x + b \in A \text{ for every } b \in B \}$

2) $A \ominus B = \{ x \in Z^N \mid B_x \subseteq A \}$

3) $A \ominus B = \bigcap_{b \in B} A_{-b}$

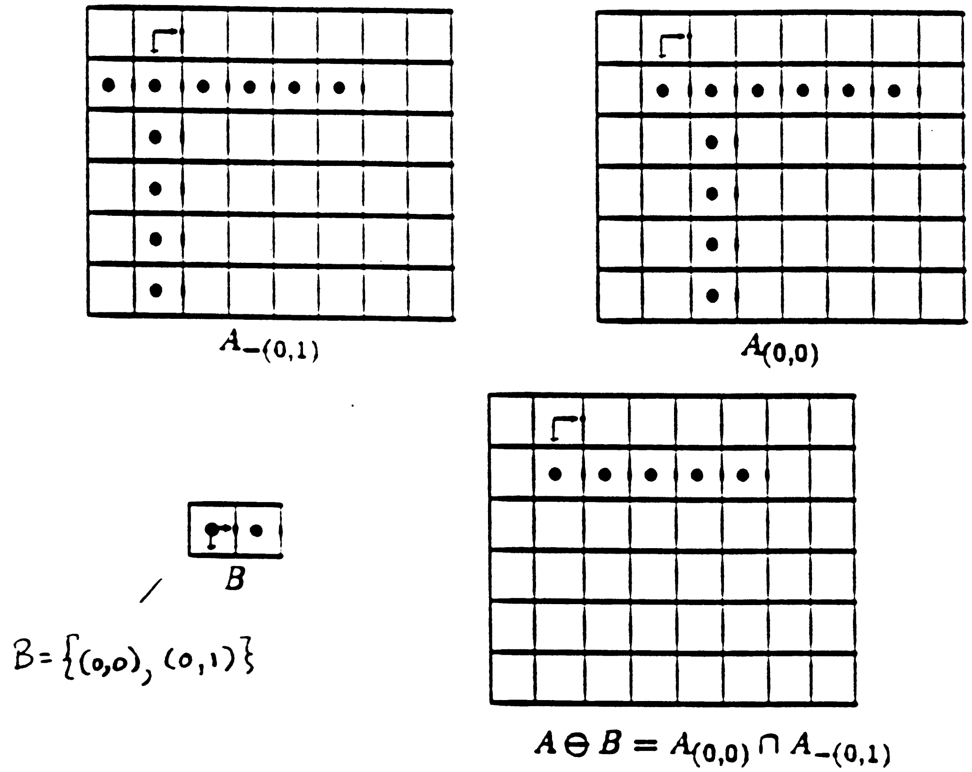


Figure 5.4 illustrates pictorially how the erosion operation can be represented as the intersection of translated sets.

$$A \ominus B = \bigcap_{b \in B} A_{-b}$$

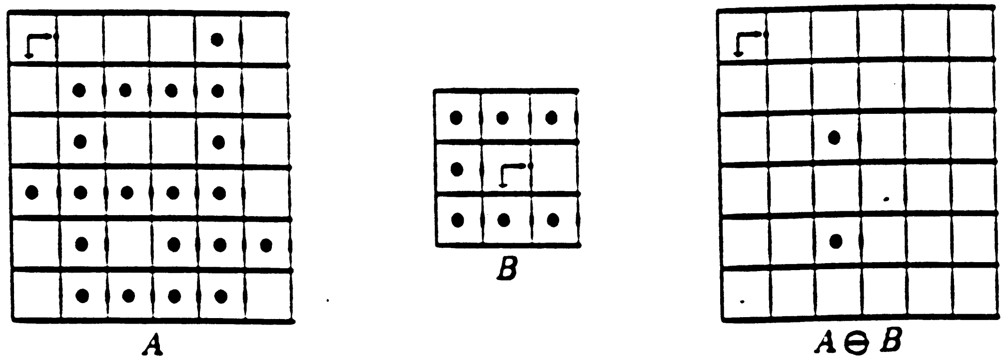
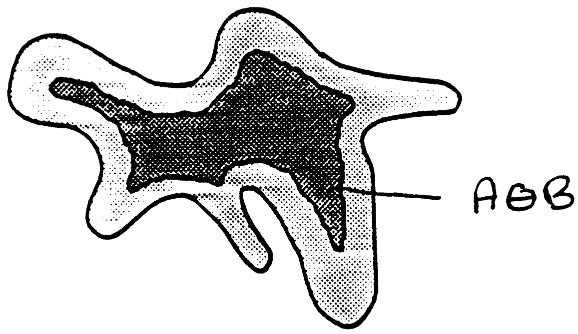
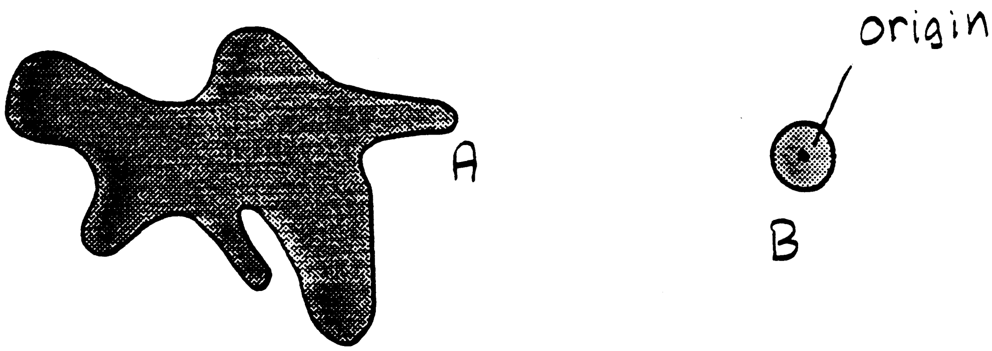


Figure 5.5 illustrates an erosion of a set A with a structuring element B which does not contain the origin. As a result there is no point of the erosion guaranteed to be in common with A. However, there are always translations of $A \ominus B$ which are contained in A.

$$A \ominus B = \{ x \in Z^N \mid B_x \subseteq A \}$$

definton of erosion : $A \ominus B$





$$A \ominus B = \{ x \in Z^2 \mid B_x \subseteq A \}$$

Move B about within the image. Using the origin of B as a marker, mark those pixels where B is totally contained within A.

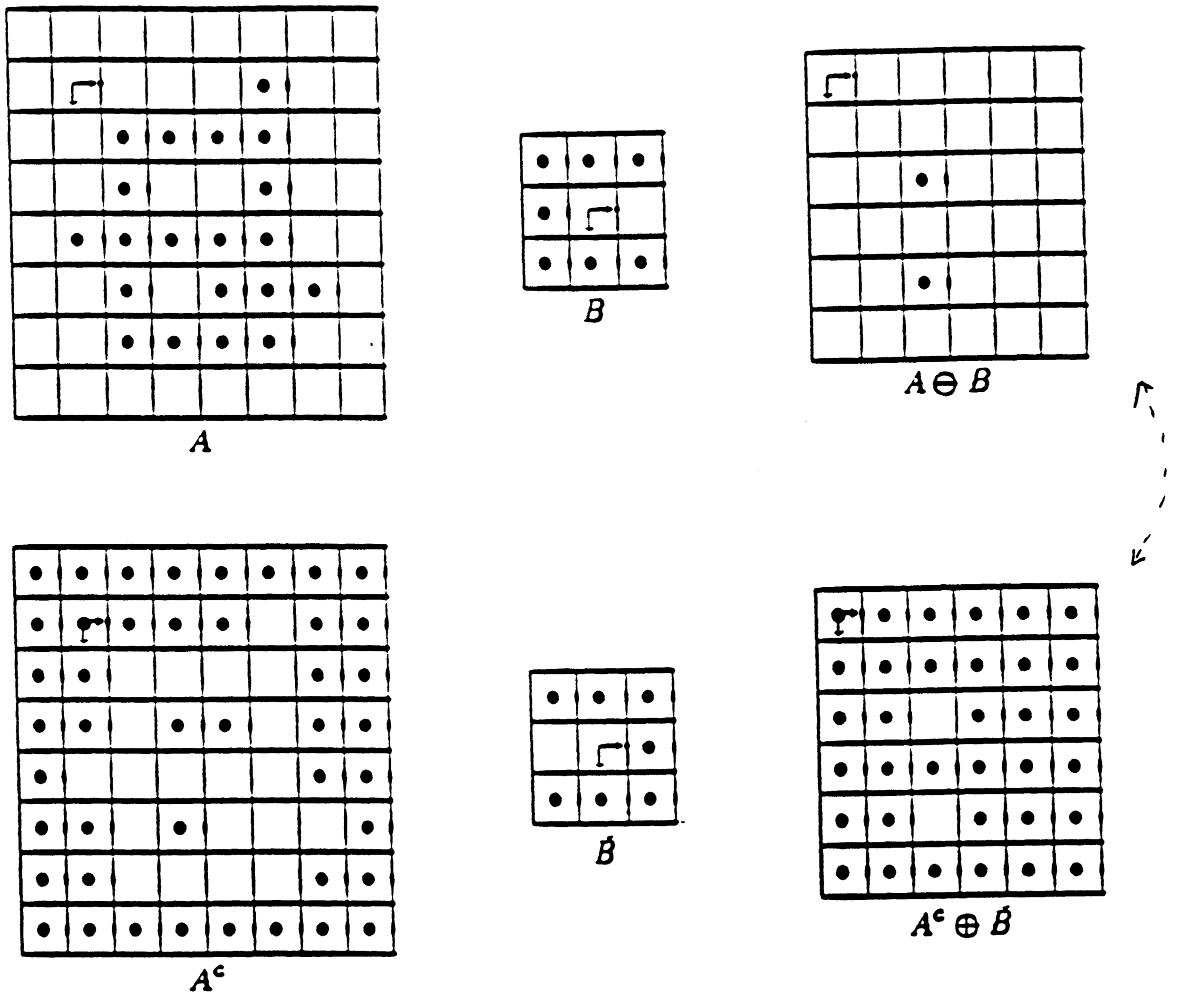


Figure 5.6 illustrates the duality relation between erosion and dilation. The set A eroded by B is the complement of the set A^c dilated by $\check{B}$. By convention, we understand that for the complemented set A^c or $A^c \oplus \check{B}$ all pixels outside the area illustrated are all binary one pixels.

Duality between erosion and dilation

$$(A \ominus B)^c = A^c \oplus \check{B}$$

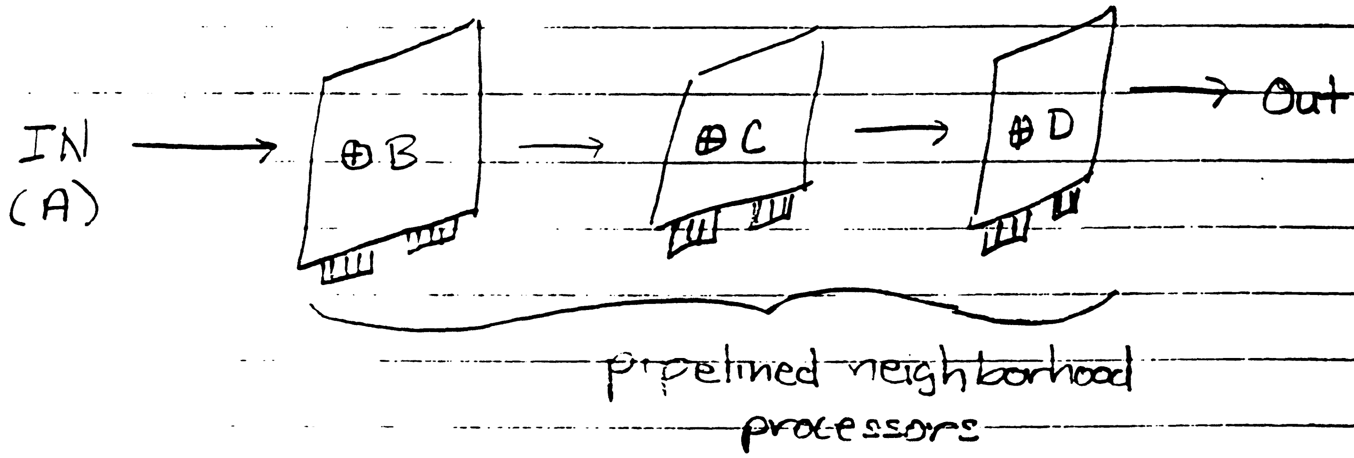
where $\check{B}$ indicates a reflection of B

$$\check{B} = \{ x \mid \text{for some } b \in B,\ x = -b \}$$

Useful properties for computations with a neighborhood processor.

if $Z = B \oplus C \oplus D$ then
$$\left.\begin{aligned} A \oplus Z &= A \oplus (B \oplus C \oplus D) \\ &= [(A \oplus B) \oplus C] \oplus D \end{aligned}\right\} \text{chain rule for } \oplus$$

if $Z = B \oplus C \oplus D$ then
$$\left.\begin{aligned} A \ominus Z &= A \ominus (B \oplus C \oplus D) \\ &= [(A \ominus B) \ominus C] \ominus D \end{aligned}\right\} \text{chain rule for } \ominus$$

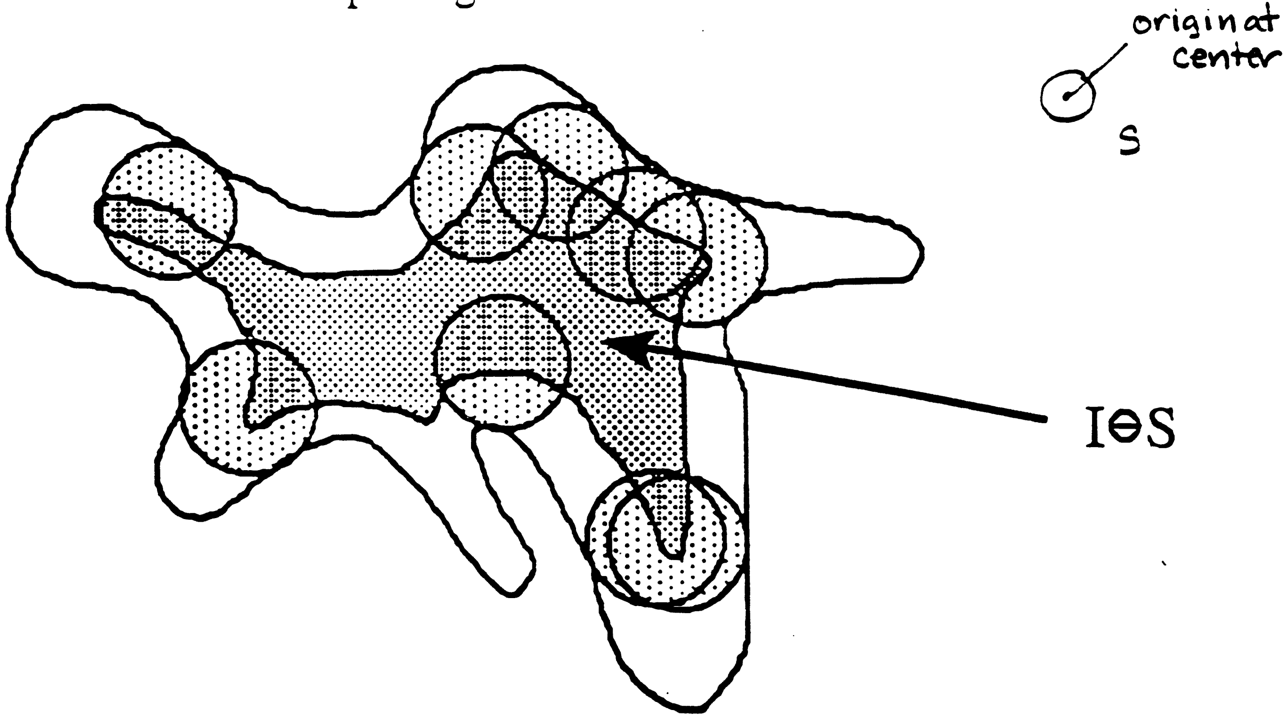


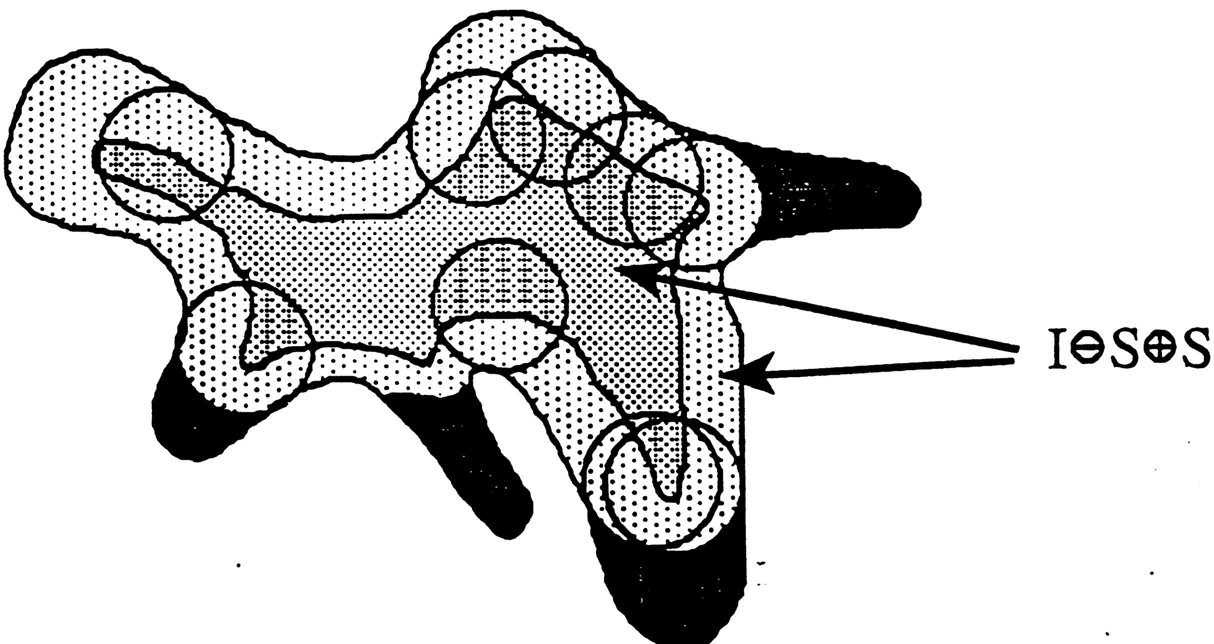
Other potentially useful properties –

$$A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$$

$$A \ominus (B \cup C) = (A \ominus B) \cap (A \ominus C)$$

Opening: $I \circ S = I \ominus S \oplus S$ ← perform operations left-to-right





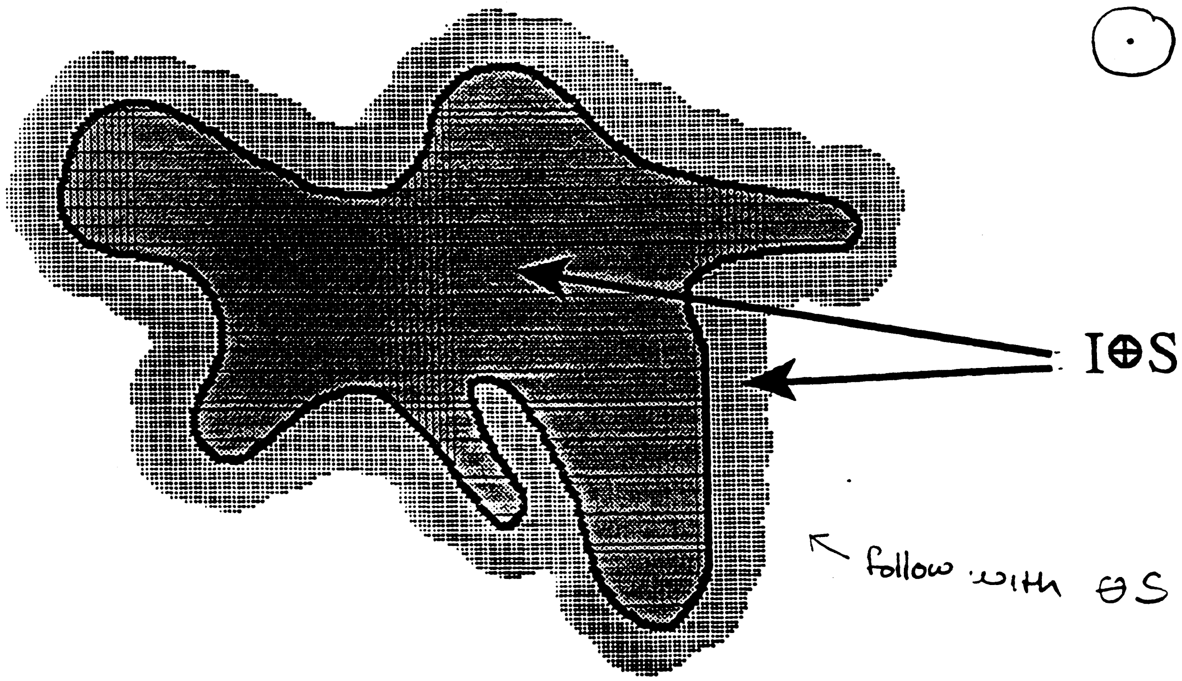
Intuitions:

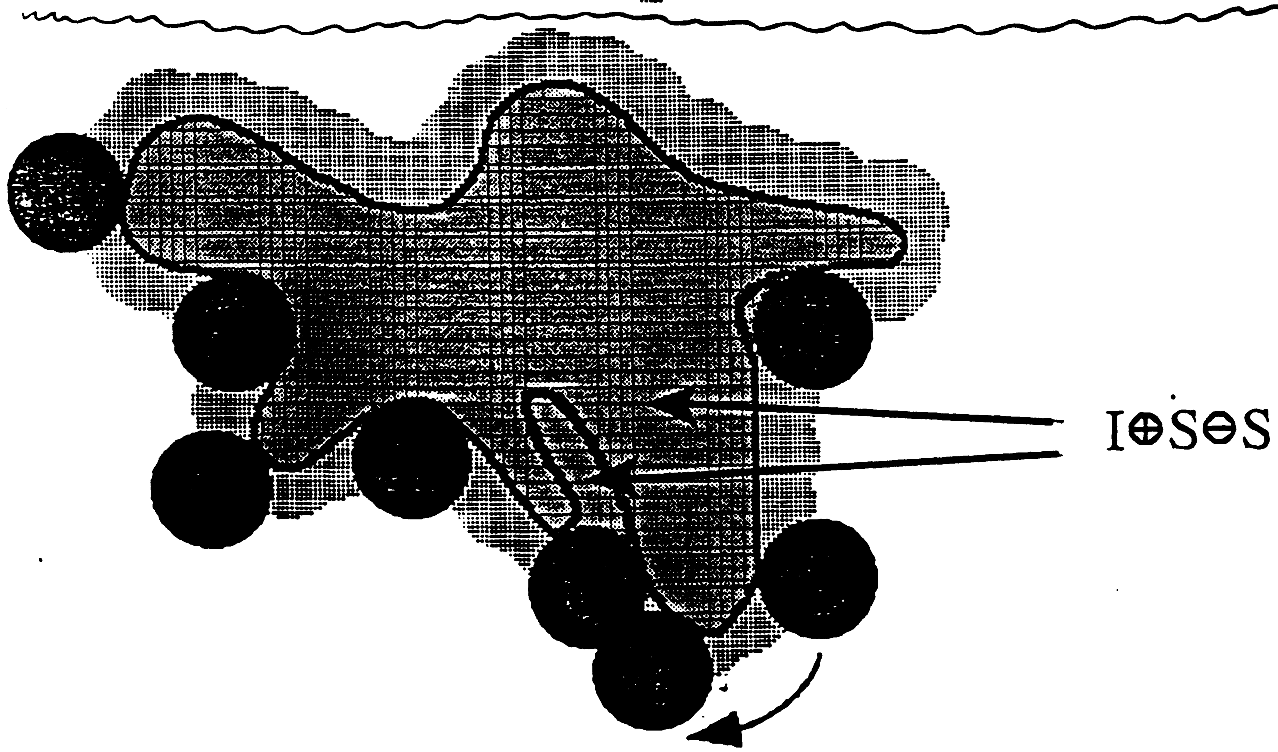
$$\boxed{I \circ S = \bigcup_{S_y \subseteq I} S_y}$$

"Roll" SE about interior of set to define new set

removes "peninsula" of images and small islands

Closing: $I \bullet S = I \oplus S \ominus S$

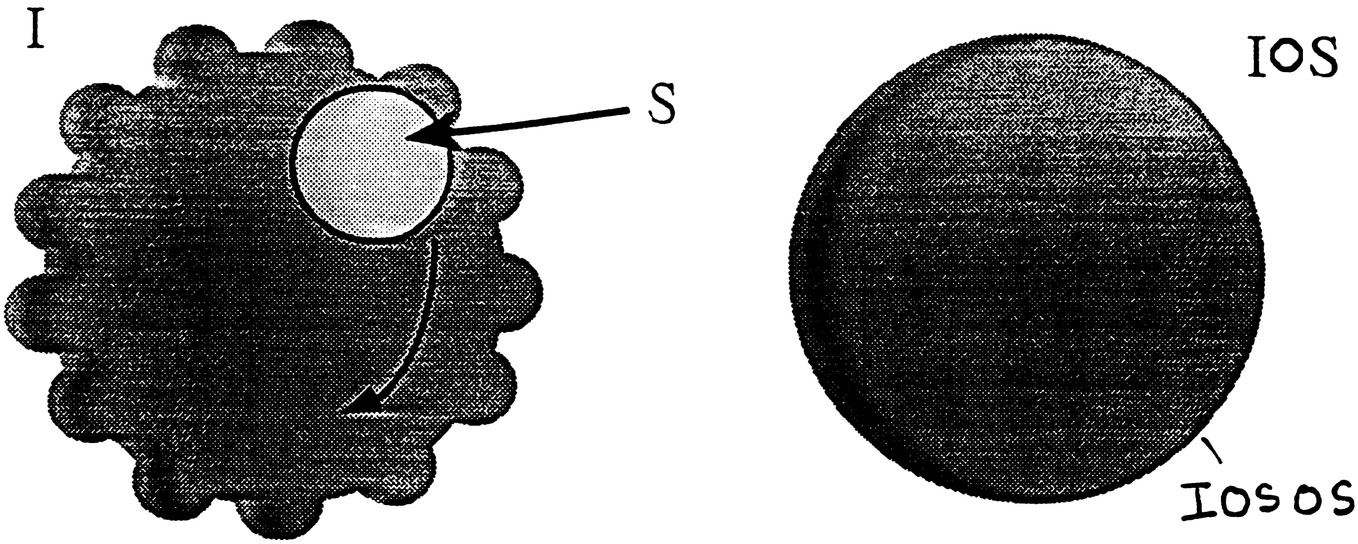


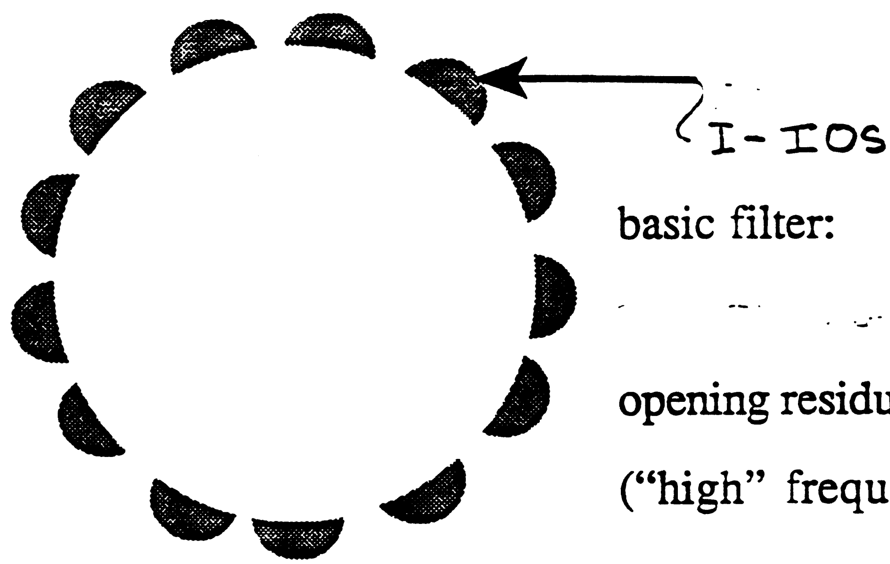


Intuitions:

"Roll" SE about exterior of set to define new set

fills in "fiords" and small "holes" of images idempotence and extensivity





basic filter:

opening residues

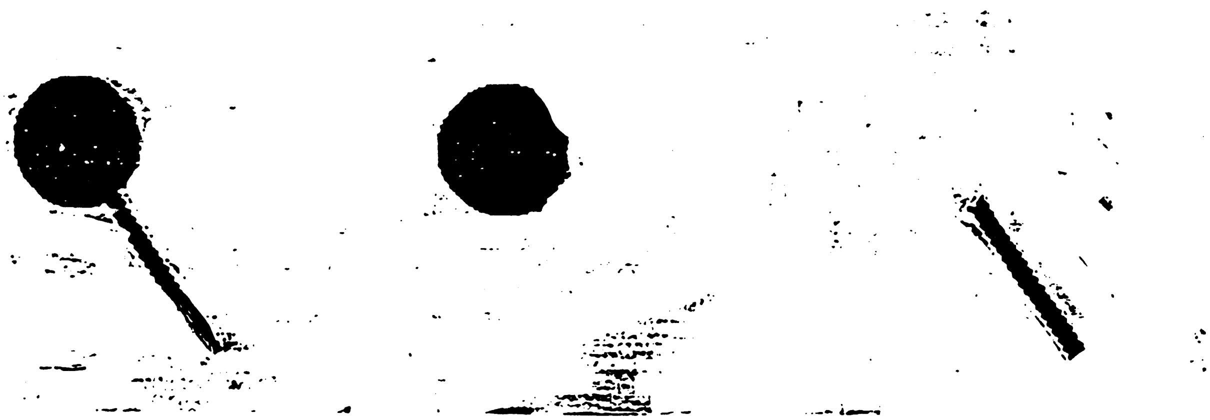
("high" frequency)

→ <u>idempotence</u>: transform of transform equals transform: IOS = IOSOS

→ <u>anti-extensivity</u>: transform set will be contained within original set: IOS ⊆ I

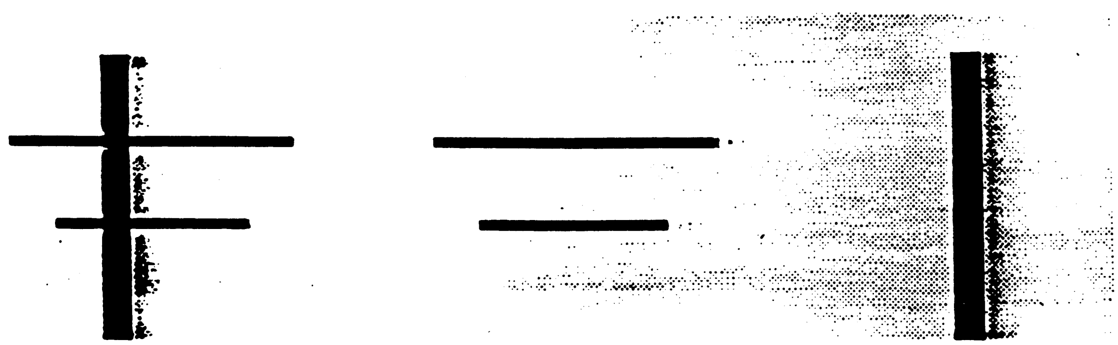
→ if opening (or closing) equals original set,

then I is said to be open (or closed) with respect to S



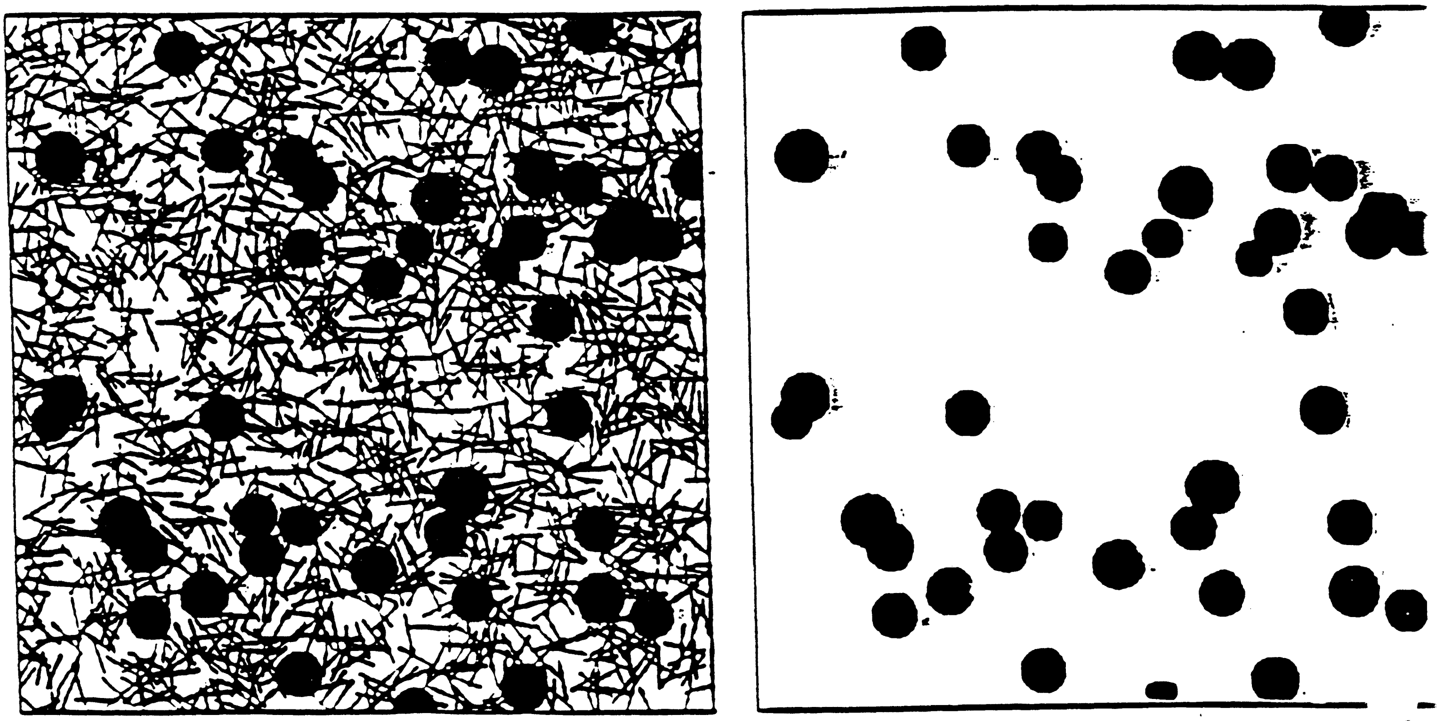
(a) (b) (c)

Figure 5.16 illustrates the extraction of the body and handle of a shape F by opening with L for the body and taking the residue of the opening for the handle; (a) F, (b) $F \circ L$, and (c) $F - F \circ L$.



(a) F (b) $F \circ$ horizontal segment (c) $F \circ$ Vertical segment

Figure 5.17 illustrates the extraction of the trunk and arms of a shape F by opening with vertically and horizontally oriented structuring elements.



(a) (b)

Figure 5.19 (a) shows a binary image. (b) shows the opening of (a) with a disk structuring element.

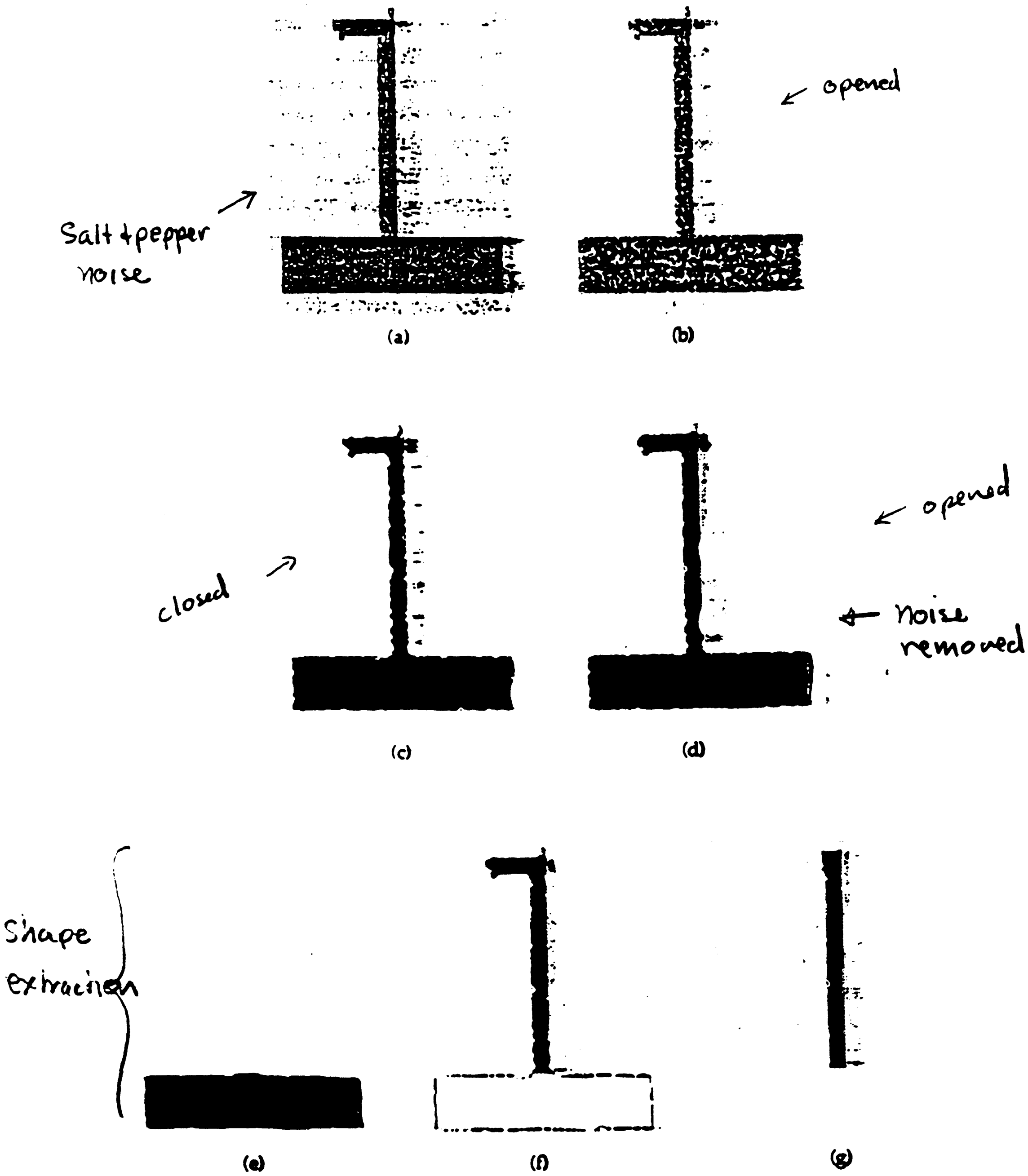
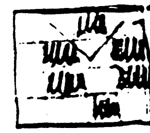


Figure 5.18 illustrates the extraction of the base, trunk, horizontal, and vertical areas of a shape F immersed in salt and pepper background noise by conditioning and then opening: (a) original image, (b) opening with a disk of radius one, (c) closing with a disk of radius 4, (d) opening with a disk of radius 3, (e) opening with a rectangle of size 21×20, (f) residue of the opening, and (g) opening residue of opening with a vertical structuring element.

erosion/dilation on hexagonal grid

background connected

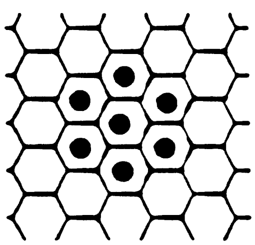


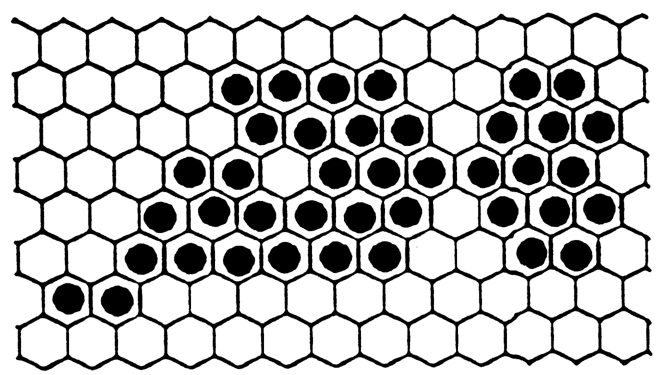
Advantages of hex lattice

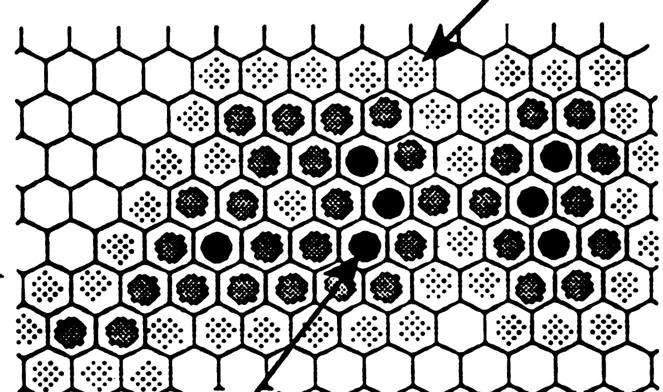
- no connectivity paradox
- simpler SE decomposition
- constant distance to neighbors

Disadvantages of hex lattice

not supported in I/O







elemental window

original figure

after erosion and dilation

Some Basic Properties of Binary Morph Filters

Opening

1) increasing: if $A \subseteq B$, then $A \circ S \subseteq B \circ S$

2) translation invariant: $A_t \circ S = (A \circ S)_t$

3) antiextensive: $A \circ S \subseteq A$

4) idempotent: $A \circ S \circ S = A \circ S$

Closing

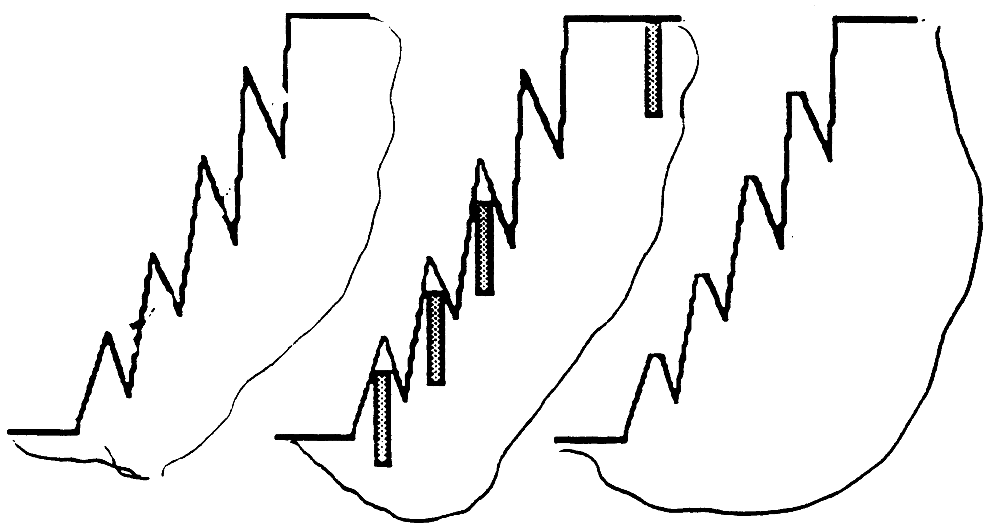
1) increasing: if $A \subseteq B$ then $A \bullet S \subseteq B \bullet S$

2) translation invariant: $A_t \bullet S = (A \bullet S)_t$

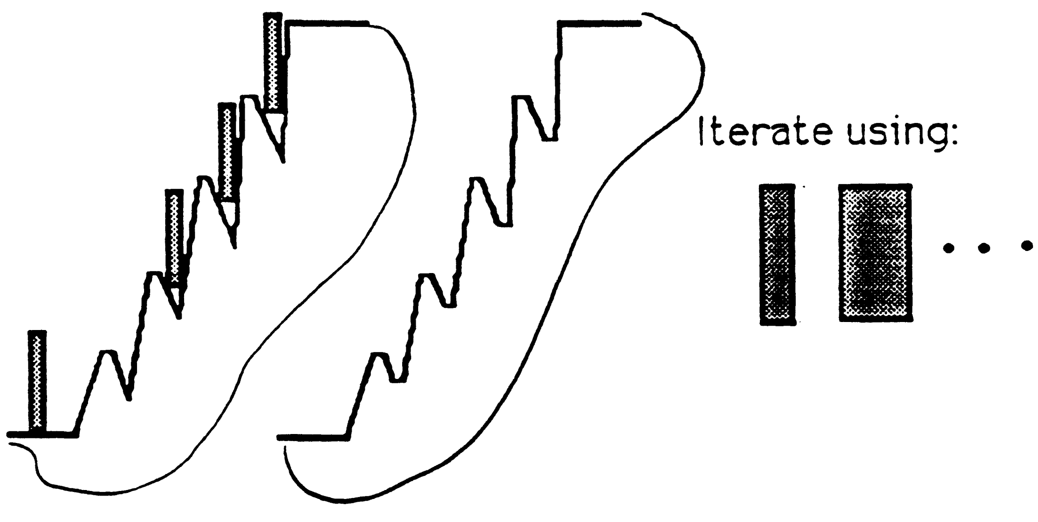
3) extensive: $A \bullet S \supseteq A$

4) idempotent: $A \bullet S \bullet S = A \bullet S$

iterative filters $I \circ S_1 \bullet S_1 \circ S_2 \bullet S_2 \cdots$



Opening with small SE removes small convexities



Closing with small SE fills in small concavities

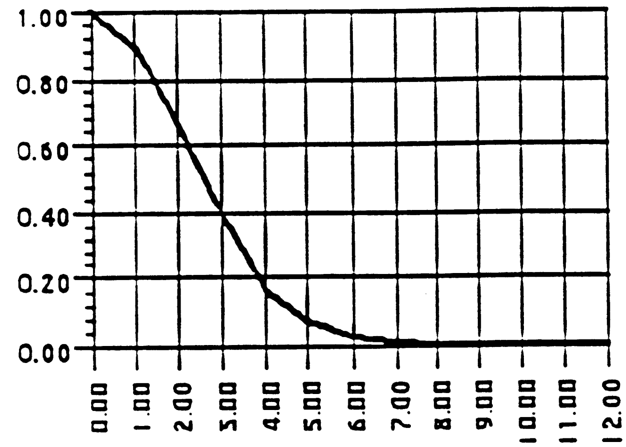
In general, get something different when do a single ~~Contrast with simple~~ opening (or closing) with max SE

$\Rightarrow$ don't get a good approximation to the edge.

Granulometry Measurement

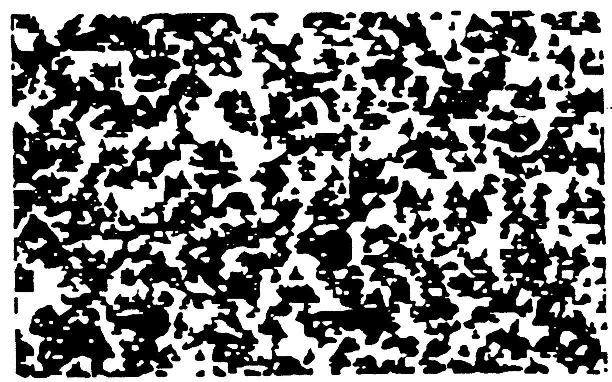
opening distributions

$$\frac{\mathrm{Area}(I \circ \mathrm{Disk}(r))}{\mathrm{Area}(I)}$$

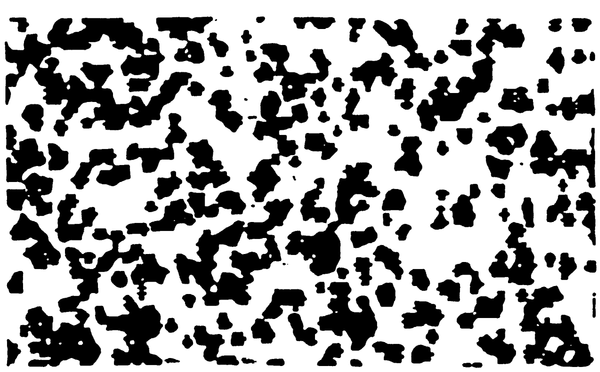


r

$I \circ \mathrm{Disk}(0)$



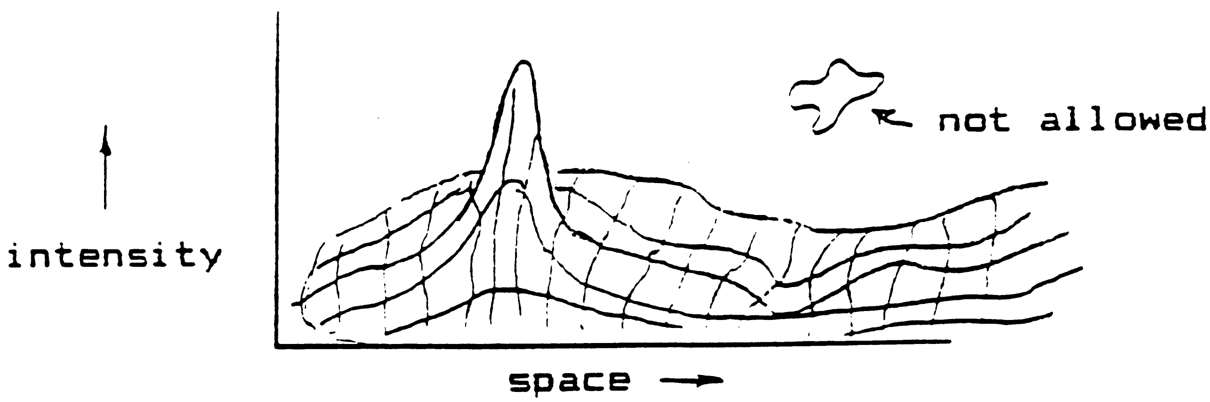
$I \circ \mathrm{Disk}(1)$



$I \circ \mathrm{Disk}(2)$

$I \circ \mathrm{Disk}(4)$

Gray-Scale Morphology



The image is a continuous landscape with hills and valleys. (No birds.)

- Gray-scale image $\Rightarrow$ surface
- 2 1/2 dimensions $\longleftrightarrow$ 3D has voxels
- SE's have 2 1/2 dimensions as well

$$\begin{tikzcd} \text{binary morph} \arrow[r, bend left, "\text{top}"] & \text{gray-scal morph} \arrow[l, bend left, "\text{umbra}"] \end{tikzcd}$$

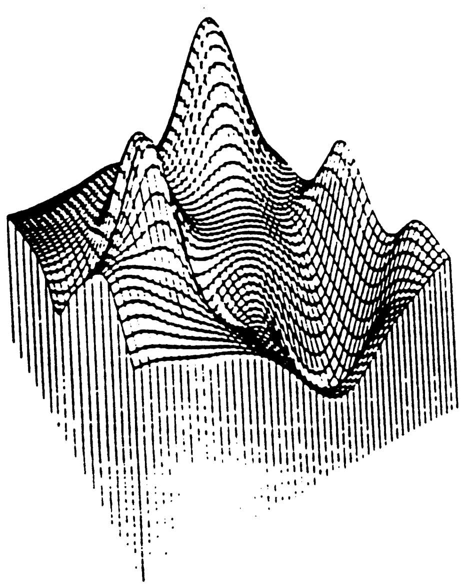
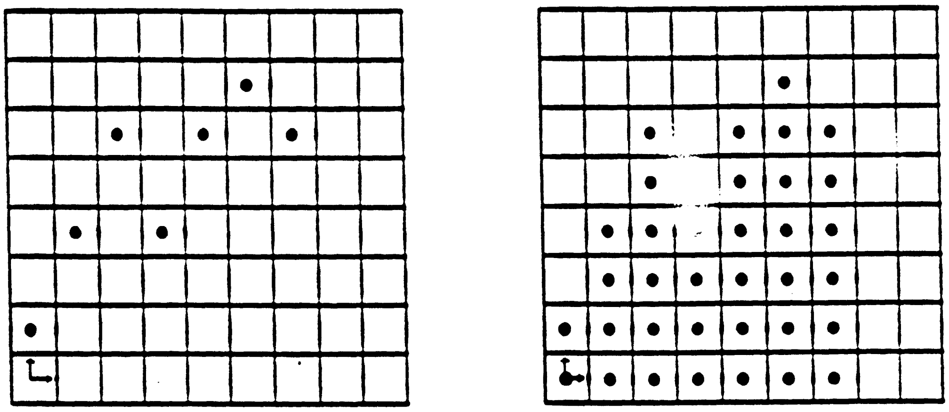


FIG. 1. Illustration of an umbra.

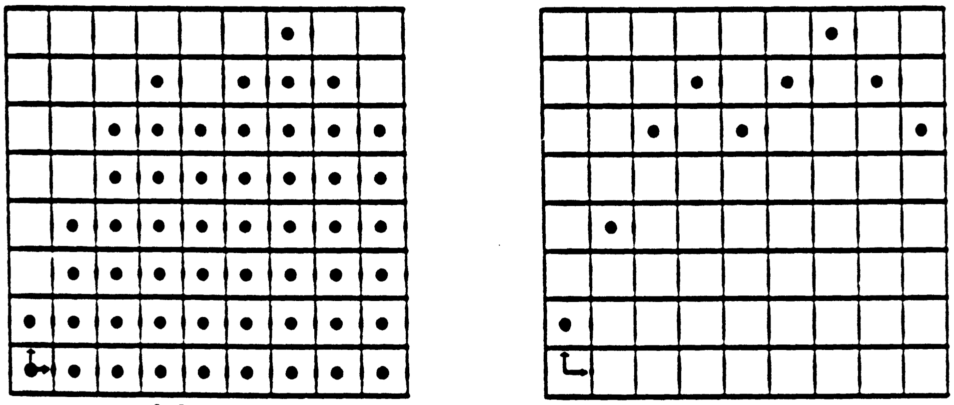


f $\qquad$ $U[f]$

Figure 5.28 illustrates a function and its umbra.

k $\qquad$ $U[k]$

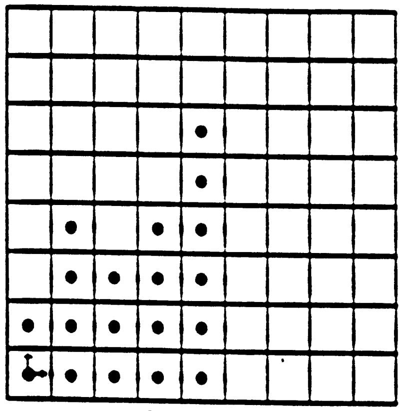
Figure 5.29 illustrates a small structuring element k and its umbra $U[k]$.



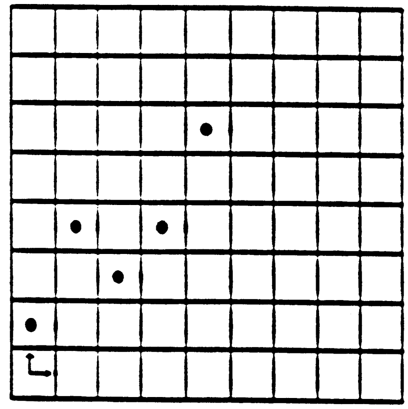
$U[f] \oplus U[k]$ $\qquad$ $T[U[f] \oplus U[k]] = f \oplus k$

Figure 5.30 illustrates how gray scale dilation can be conceived by dilating the umbras and then taking the gray scale dilation to be the resulting top surface.

$$A \oplus B = \bigcup_{a \in A} B_a$$ ← for example, can use this binary definition



$U[f] \ominus U[k]$



$T[U[f] \ominus U[k]] = f \ominus k$

Figure 5.32 illustrates how gray scale erosion can be conceived by eroding the umbra of f by the umbra of k and then taking the gray scale erosion to be the resulting top.

$$A \ominus B = \{ x \in Z^N \mid B_x \subseteq A \}$$

Proof of gray-scale properties from binary morph properties.

- def'n $\qquad f \oplus k = T[U(f) \oplus U(k)]$ $\qquad$ (1)

- umbra homomorphism theorem $\qquad$ (2)

$$U[f \oplus k] = U[f] \oplus U[k]$$

$$\rightarrow f \oplus k = T[U(f) \oplus U(k)] \rightarrow$$
$$\rightarrow U(f \oplus k) = U[T[U(f) \oplus U(k)]]$$
$$\rightarrow U(f \oplus k) = U(f) \oplus U(k)$$

Proove that $\quad k_1 \oplus (k_2 \oplus k_3) = (k_1 \oplus k_2) \oplus k_3 \quad \leftarrow$ gray-scale

LHS:

$$k_1 \oplus (k_2 \oplus k_3) = T[U(k_1) \oplus U[k_2 \oplus k_3]] \quad \text{from 1}$$

$$= T[U(k_1) \oplus (U(k_2) \oplus U(k_3))] \quad \text{from 2}$$

$$= T[[U(k_1) \oplus U(k_2)] \oplus U(k_3)] \quad \text{from binary associativity}$$

$$= T[U(k_1 \oplus k_2) \oplus U(k_3)] \quad \text{from 2}$$

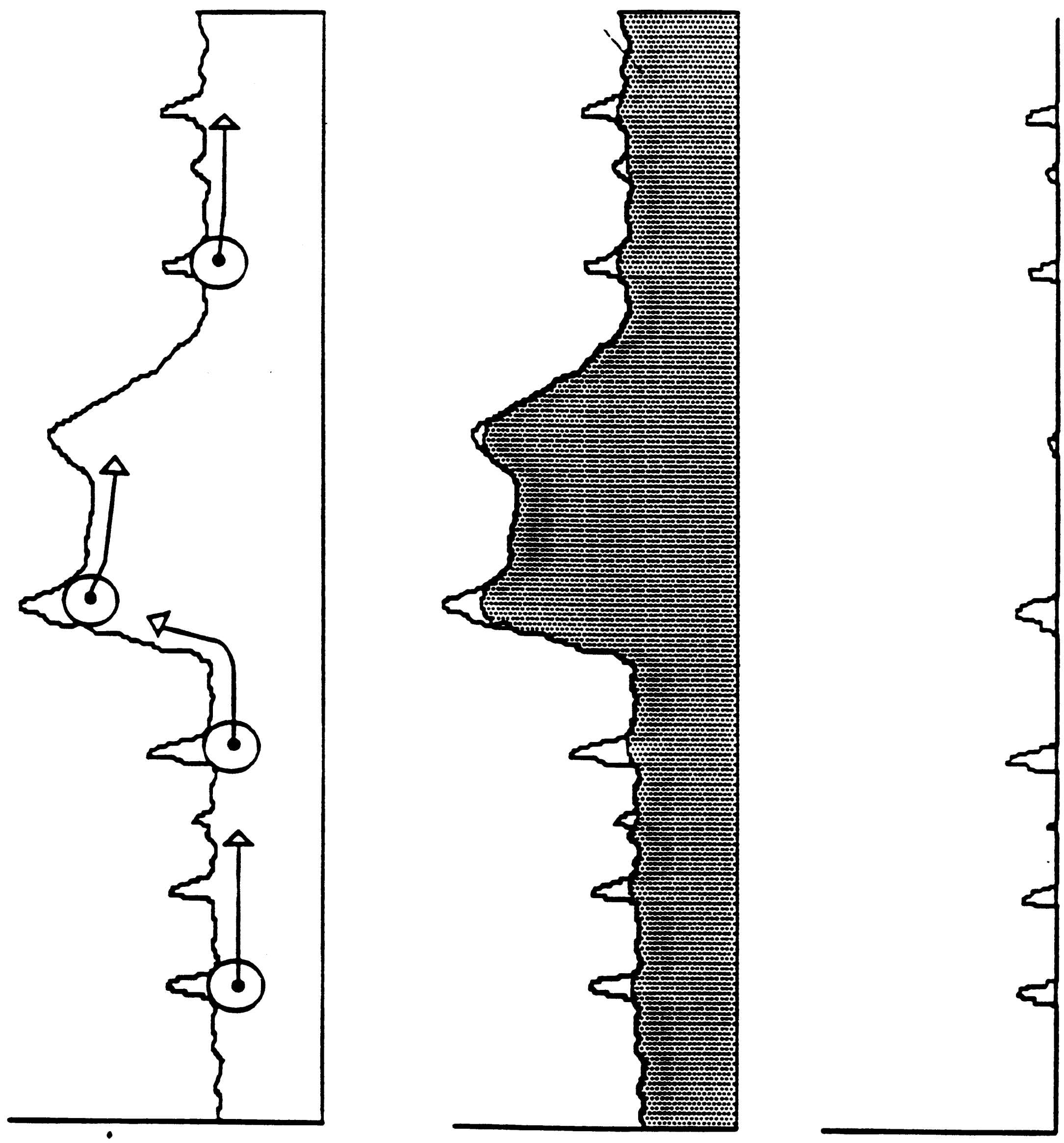
$$= (k_1 \oplus k_2) \oplus k_3$$

- Gray-scale calculation
- Slides
- rolling ball
- x-ray images
- etc.

Morphological Enhancement

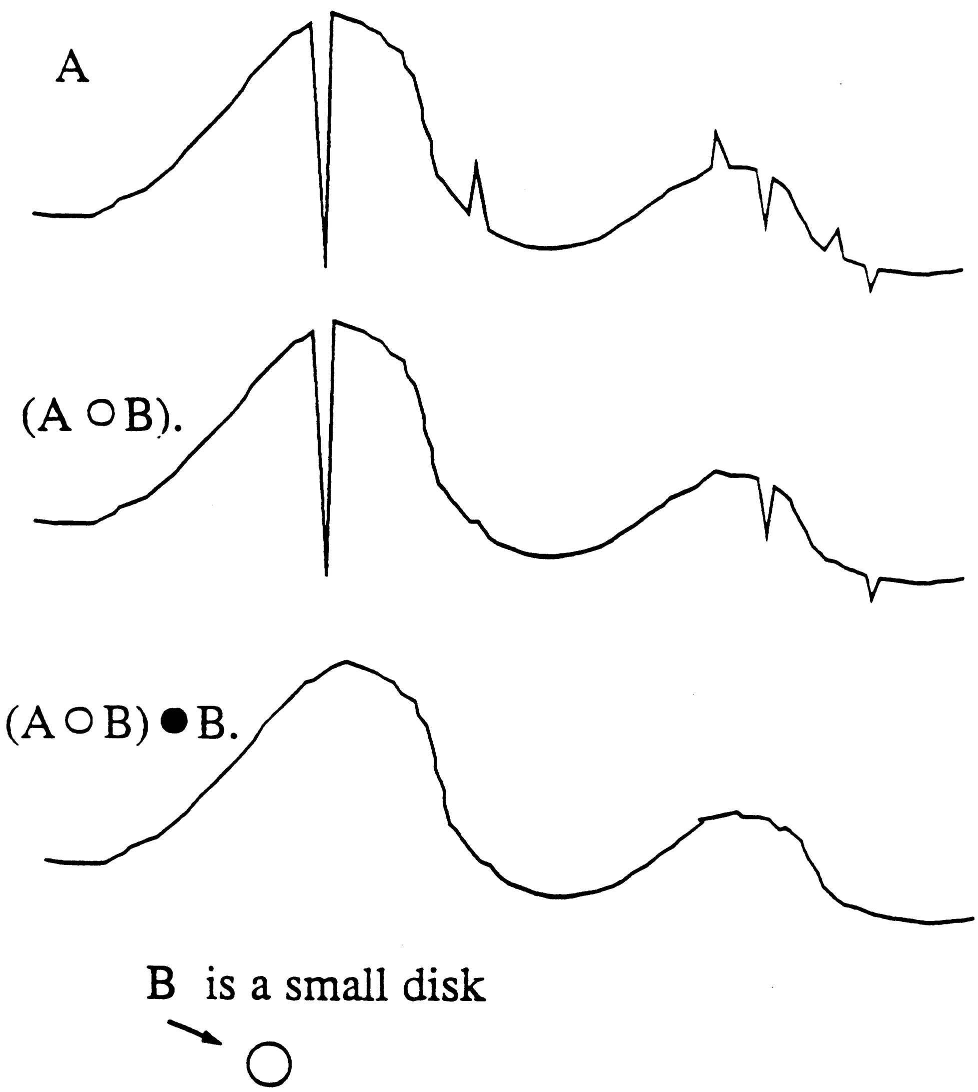
- **Create OPENED_MASK, $p(x, y)$, with an SE**
  - **Erode:** $e(x, y) = \text{min}_{i,j}[i(x - i, y - j) - s(-i, -j)]$
  - **Dilate:** $p(x, y) = \text{max}_{i,j}[e(x - i, y - j) + s(i, j)]$
- **ENHANCED_IMAGE = INPUT_IMAGE - OPENED_MASK**
- **Output image equation:**

  $o(x, y) = k_1[i(x, y) - p(x, y)] + (1 - k_2)p(x, y) + k_2 m$

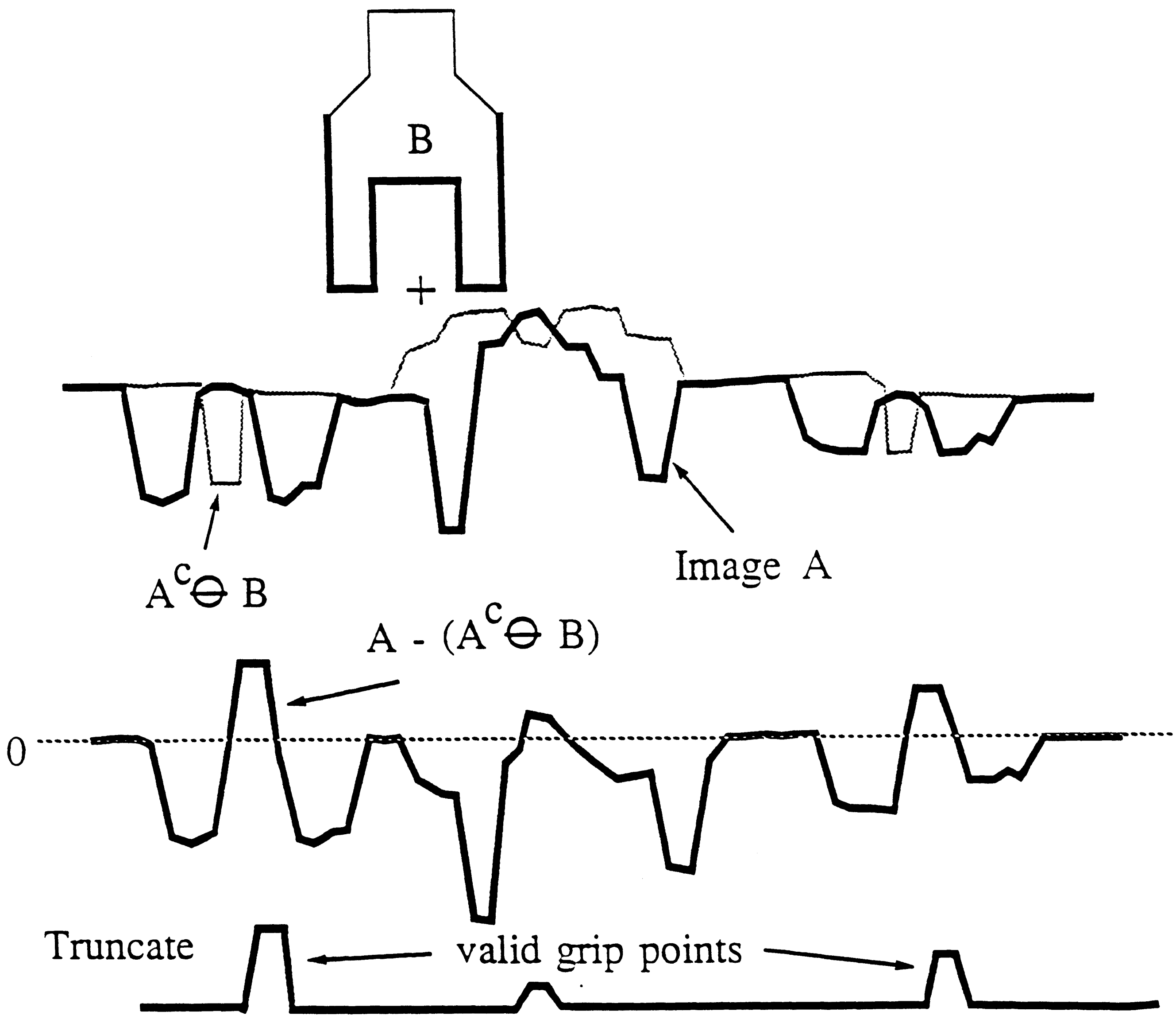


A grey scale salt and pepper filter.

open - closing: (A○B)●B.

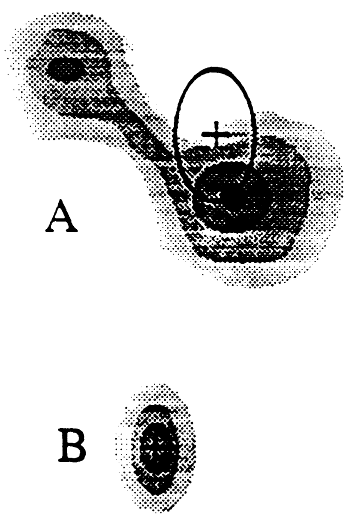


MODELING A ROBOT GRIPPER TO FIND GRIP POINTS IN A RANGE IMAGE



The weighted rank order filter.

There is one more obvious generalization:



	2	-	6
	0	+	1
	1	+	3
R'th →	32	-	12
	25	-	3
	21	+	1
	25	+	2
	18	+	10
	49	-	5
	45	+	0
	51	+	4
	57	+	5
	A		B

Informal definition:

1) Add the weights in the structuring element B to the corresponding image pixels A that are contacted.
2) Order the resulting sums.
3) Choose the Rth element in the ordered list.

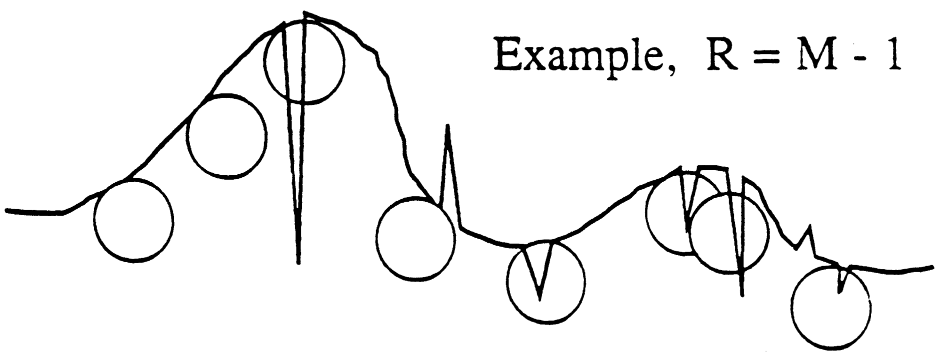
Note: the order will now depend on the weights.

Popular ranks:
R=1: dilation with a grey level structuring element.
R=M: erosion with a grey level structuring element.

GEOMETRICAL PICTURE OF THE WEIGHTED RANK ORDER FILTER

Opening and closing can not be defined (not idempotent, not antiextensive nor extensive.)

However, if the structuring element is a hemisphere (rolling ball), then an "erosion" by a weighted rank M-P filter followed by a dilation with a rank 1 filter with the same weights can be pictured as a rolling sponge, where P of the largest image noise spikes are allowed to pierce the surface of the sponge.



The rolling sponge

48

Filtering with multiple SE's — Binary Image

$K_i \rightarrow$

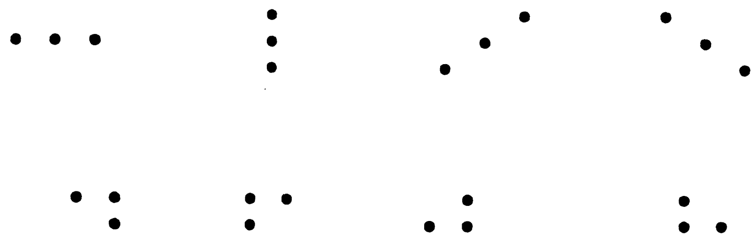


FIG. 8. The eight 3 × 3 structuring elements used in the filter of Fig. 9.

"The Analysis of Morphological Filter with Multiple Structuring Elements" Song + Delp; CVGIP [illegible]: 302-328.

$$B = \bigcup_{i=1}^{8} (A \circ K_i)$$

$$C = \bigcap_{i=1}^{8} (B \bullet K_i)$$

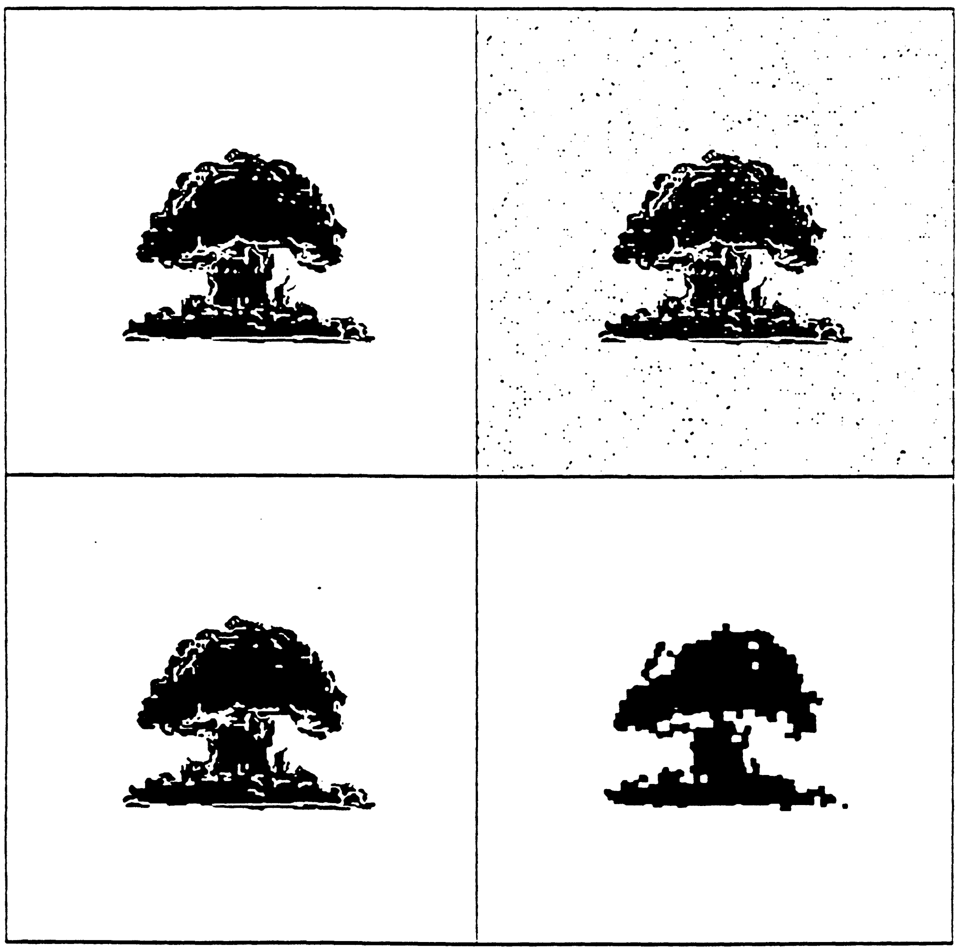


FIG. 9. Upper left: original image; upper right: noisy version of the original image; lower left: resultant image from a complex binary morphological filter using the eight structuring elements

Multiple SE : - Gray-scale case

$$g = \max_{i \in \{1, \dots, 8\}} (f \circ K_i),$$

$$h = \min_{i \in \{1, \dots, 8\}} (g \cdot K_i),$$

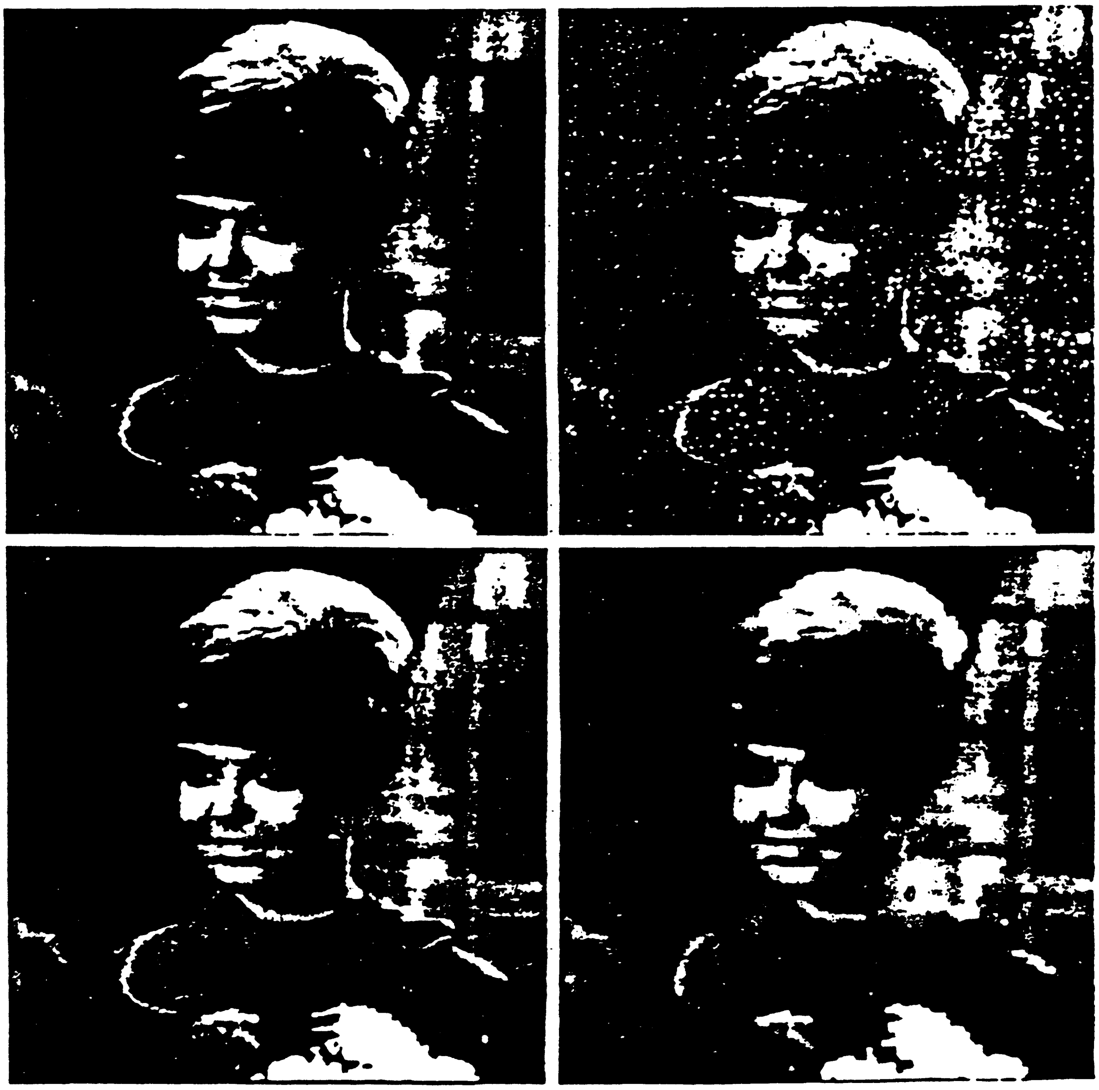
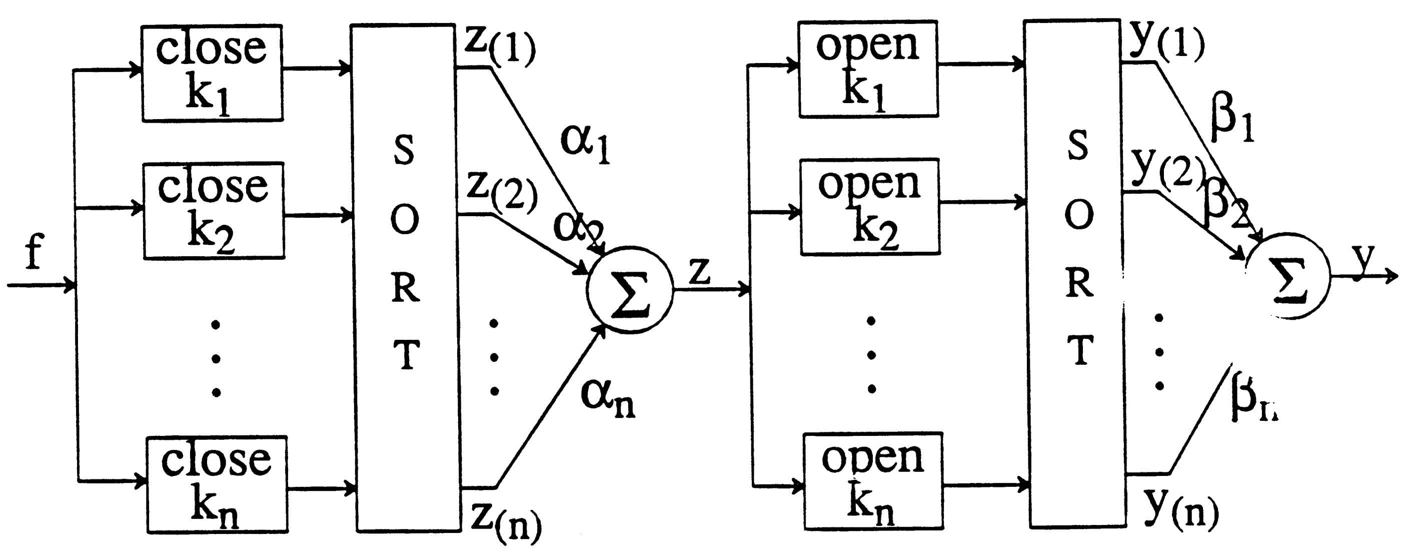


FIG. 10. Upper left: original image; upper right: noisy version of the original image; lower left: resultant image from a complex gray scale morphological filter using the eight structuring elements shown in Fig. 8; lower right: resultant image from a simple morphological filter using a single 3×3 square structuring element.

An Example of the GMF:



Conditional Dilation (segmentation method)

1. high gray scale threshold $\longrightarrow$ binary image J
2. low " " " $\longrightarrow$ " " I

(Image "I" will include extra noise pixels which are not connected to the real objects of interest.)

3. let $J_0 = J$
4. $J_1 = (J_0 \oplus D) \cap I$ where D is a small disk.

(This will grow J along I)

5. $J_n = (J_{n-1} \oplus D) \cap I$ repeat this recursive relation until $J_n = J_{n-1}$
6. END

The advantage of this algorithm is that isolated islands in I will not end up in J_n.

Edge Detection

Binary images

a) $X \setminus (X \ominus W)$
b) $(X \oplus W) \setminus X$

W is a small 2D, symmetric SE.

Gray-scale images

a) $f - (f \ominus W)$
b) $(f \oplus W) - f$

W is a flat, symmetric SE.

Morphological Correlation

linear: $\underbrace{\gamma = \sum_{n \in W} [f(n+k) - g(n)]^2}_{\text{minimize}} \rightarrow \underbrace{y = \sum_{n \in W} f(n+k)\, g(n)}_{\text{maximize}}$

morph: $\underbrace{\gamma = \sum_{n \in W} |f(n+k) - g(n)|}_{\text{minimize}} \longrightarrow \underbrace{y = \sum_{n \in W} \min[f(n+k), g(n)]}_{\text{maximize}}$

- minimizing $\sum |\cdot|$ is more robust than minimizing $\sum [\cdot]^2$. $\Rightarrow$ MORPH. may be better

Binary 3D Morphology

- voxels are either $\emptyset$ or 1. Gray-scale voxels are segmented.
- all definitions for $\ominus$ and $\oplus$ still apply
- $\circ$ and $\bullet$ can be done in 3D

Skeleton

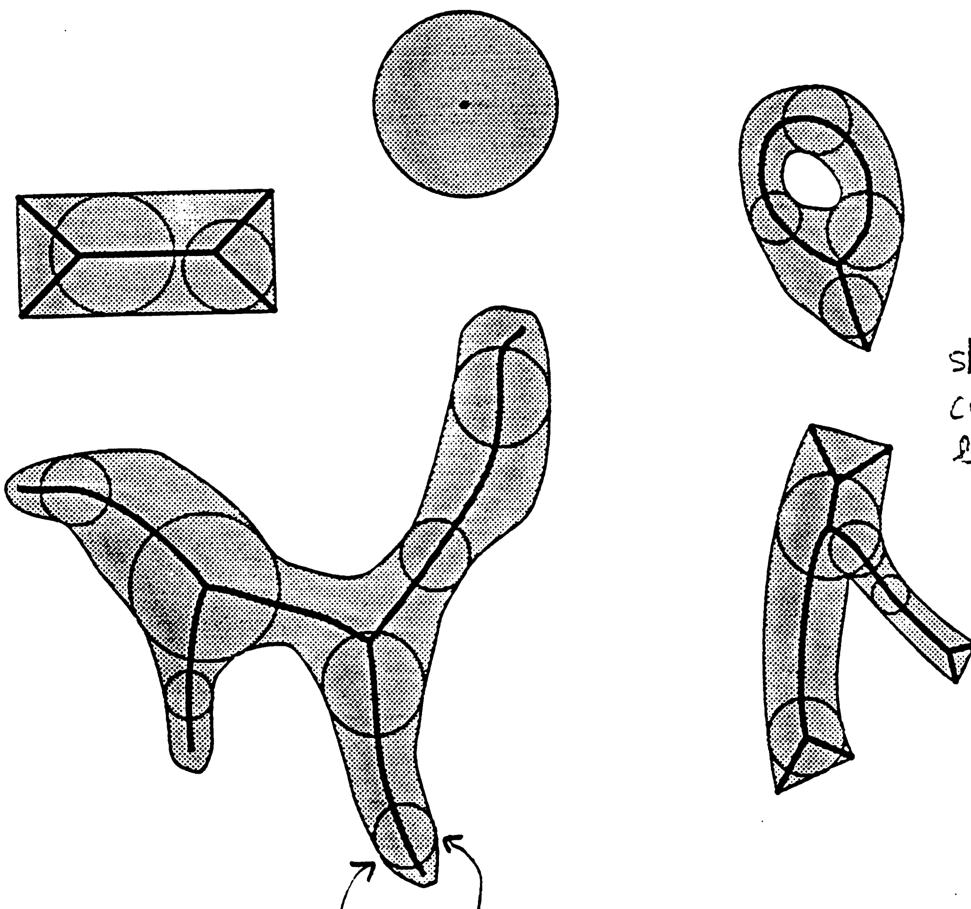
or

Medial Axis

The *skeleton* of a set X is the locus of the centers of its maximal balls:

maximal disk ⇒ no other disk contained within the object will enclose a maximal disk.

Examples:



skeleton will consist of straight lines and arcs

maximal disk always has at least 2 touch points

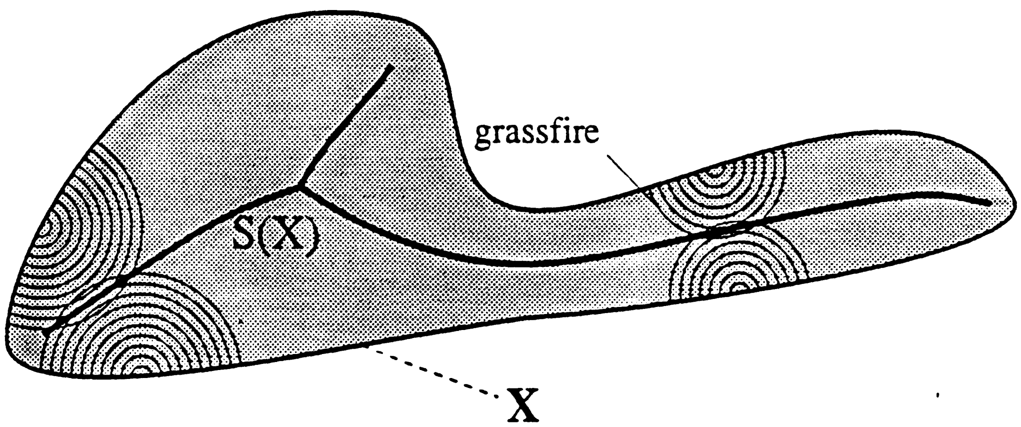
Skeletons using grassfires

Let X be a set such that each of its border points is the origin of a grassfire.

Skeleton of X =

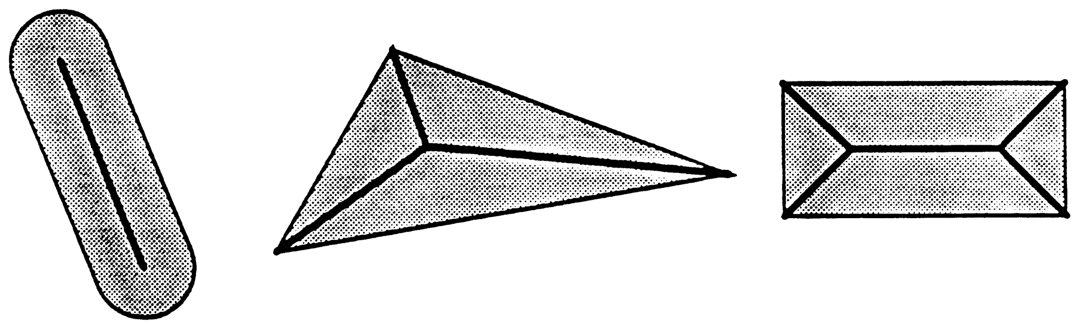
Locus of the points of X where two different *firefronts* meet.

Example:

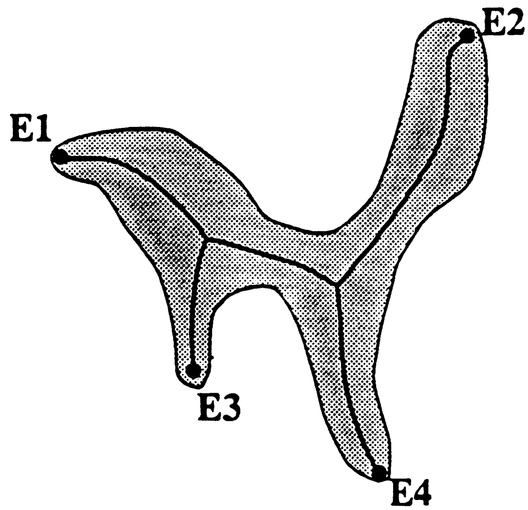


Use of the skeleton

- Shape description.
  $\longrightarrow$ Number of triple points and of extremities of the skeleton...



- Measurement of the *length* of particles.
- Detection of extremities:

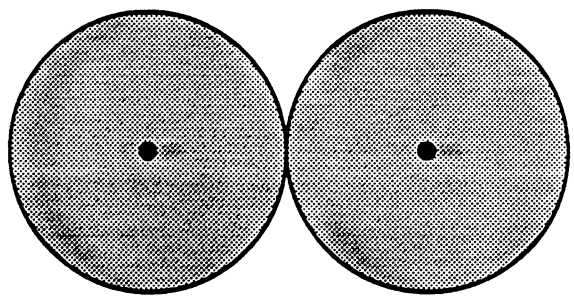


- Image compression

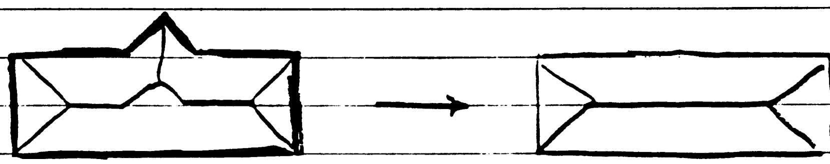
Some problems with the skeleton

Let X be a set in the plane $\mathbb{Z}^2$.

- The skeleton $S(X)$ is not necessarily connected, even though X is connected:



- Noise can affect the skeleton drastically



- The skeleton transformation is not guaranteed to be rotationally invariant.
- There are many skeletonization algorithms. They may not all give the same skeletons.

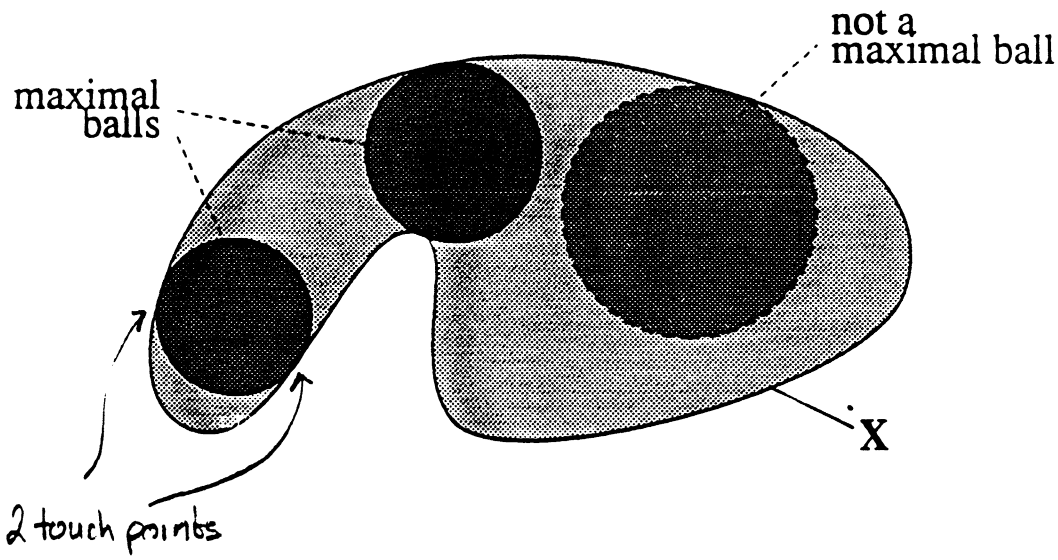
Maximal balls

Let $X \in \mathbb{R}^2$ be an arbitrary set.

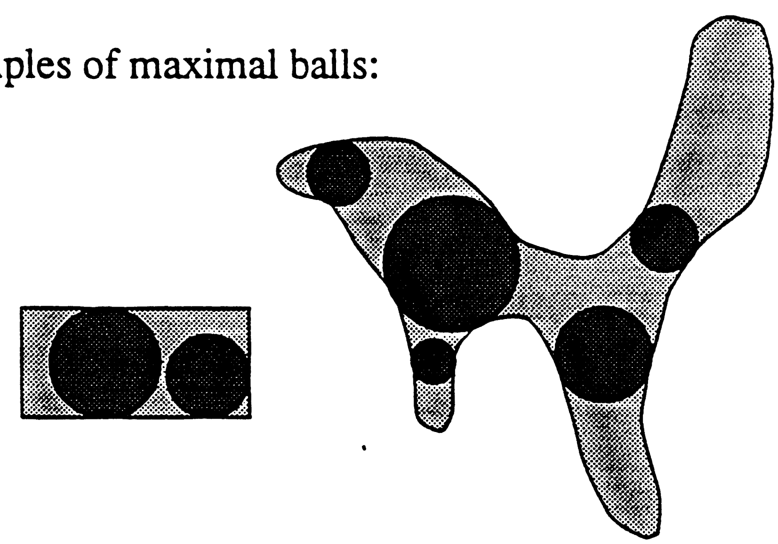
A ball $B \in X$ is said to be *maximal* in X iff:

$$\forall \text{ ball } B' \subseteq X, B \not\subseteq B'$$

<u>Example:</u>



Examples of maximal balls:



Some Algorithms for Computing Skeletons

1. Grass fires
2. Distance transforms
3. Morphological method as described by Maragos

definition

$$S_n = (X \ominus nB) \setminus [(X \ominus nB) \circ B] \quad n = 0, 1, \dots N$$

where

$$N = \max\{n \mid X \ominus nB \neq \emptyset\}$$

$$\text{Skeleton}\{X\} = \bigcup_{n=0,1,2\dots N} S_n$$

Implementation can make use of a decomposition -

$$nB = \underbrace{B \oplus B \oplus B \dots \oplus B}_{n \text{ times}}$$

$$\Rightarrow X \ominus nB = X \ominus [B \oplus B \dots \oplus B]$$

$$= X \ominus B \ominus B \ominus B \dots \ominus B$$

$$S_0 = X - X \circ B$$

$$S_1 = (X \ominus B) - [(X \ominus B) \circ B]$$

$$S_2 = (X \ominus B \ominus B) - [(X \ominus B \ominus B) \circ B]$$

$$S_3 = (\underbrace{X \ominus B \ominus B}_{} \ominus B) - [(\underbrace{X \ominus B \ominus B}_{} \ominus B) \circ B]$$

already computed!

$$S_N = (X \ominus \underbrace{B \ominus B \dots \ominus B}_{n \text{ times}}) - [(X \ominus \underbrace{B \ominus B \dots \ominus B}_{n \text{ times}}) \circ B]$$

Reconstruction from a Skeleton or Quench Function

1) partial reconstruction

$$X \circ kB = \bigcup_{k \le n \le N} S_n \oplus nB \qquad 0 \le k \le N$$

2) full reconstruction, $k = 0$

$$X = \bigcup_{0 \le n \le N} S_n \oplus nB$$

- Skeletonization can be used to compress an image and reconstruction can be used to decompress it.

- Skeletonization works better on hexagonal arrays because better small disks can be made. (Disks will tend to give rotational invariance)

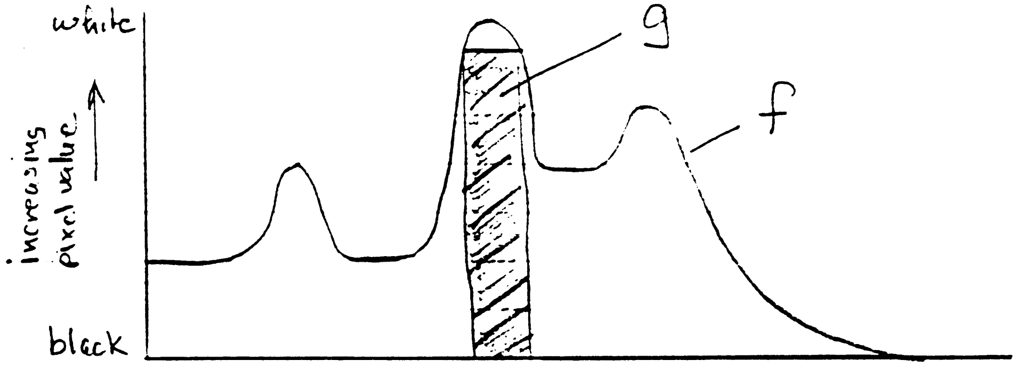
- On a rectilinear grid, small squares are normally used.

Gray-scale Geodesic Dilation

Gray-scale geodesic dilation is an extremely useful filtering technique for selectively removing or selectively including gray-scale features present in an input image into an output image.

Two gray-scale input images are required:

f: input image = constraint image = feature image
g: marker image



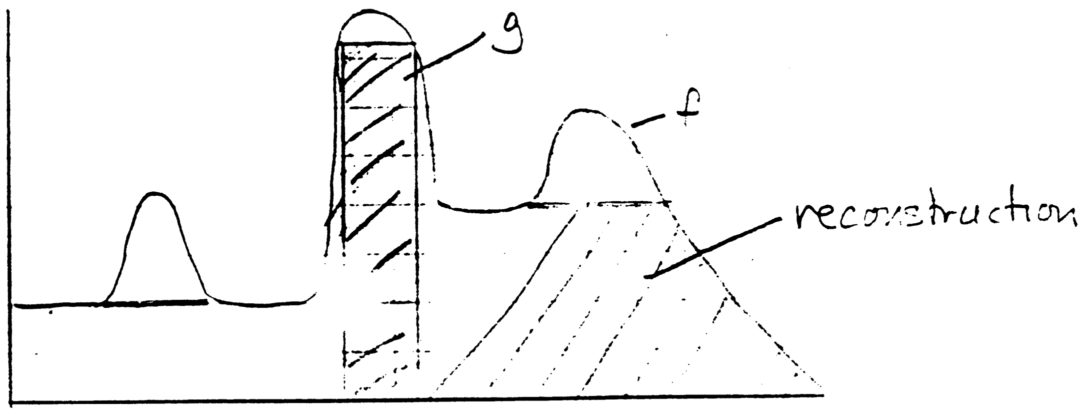
A single gray-scale dilation is obtained by dilating g with a flat SE, s, and taking the min with respect to g. That is,

$$gd\text{-}1 \longleftarrow \min[(g \oplus s), f]$$

The nth geodesic dilation is given by

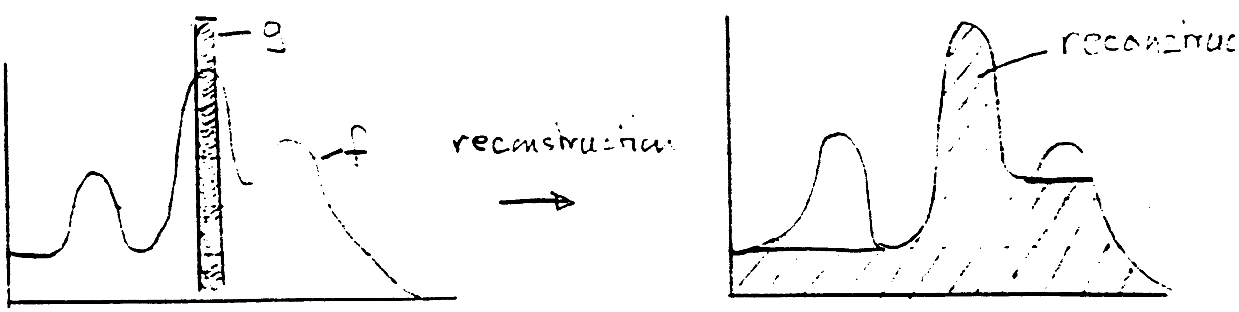
$$gd\text{-}n \longleftarrow \min[(gd\text{-}n\text{-}1 \oplus s), f].$$

The reconstruction of f from g is obtained by successively applying gray-scale geodesic dilations until there is no further change in the image. This occurs when the geodesic dilation reaches the borders.



Note that the gray-scale reconstruction is a monotonically decreasing function. This property is particularly useful when we want to threshold the image.

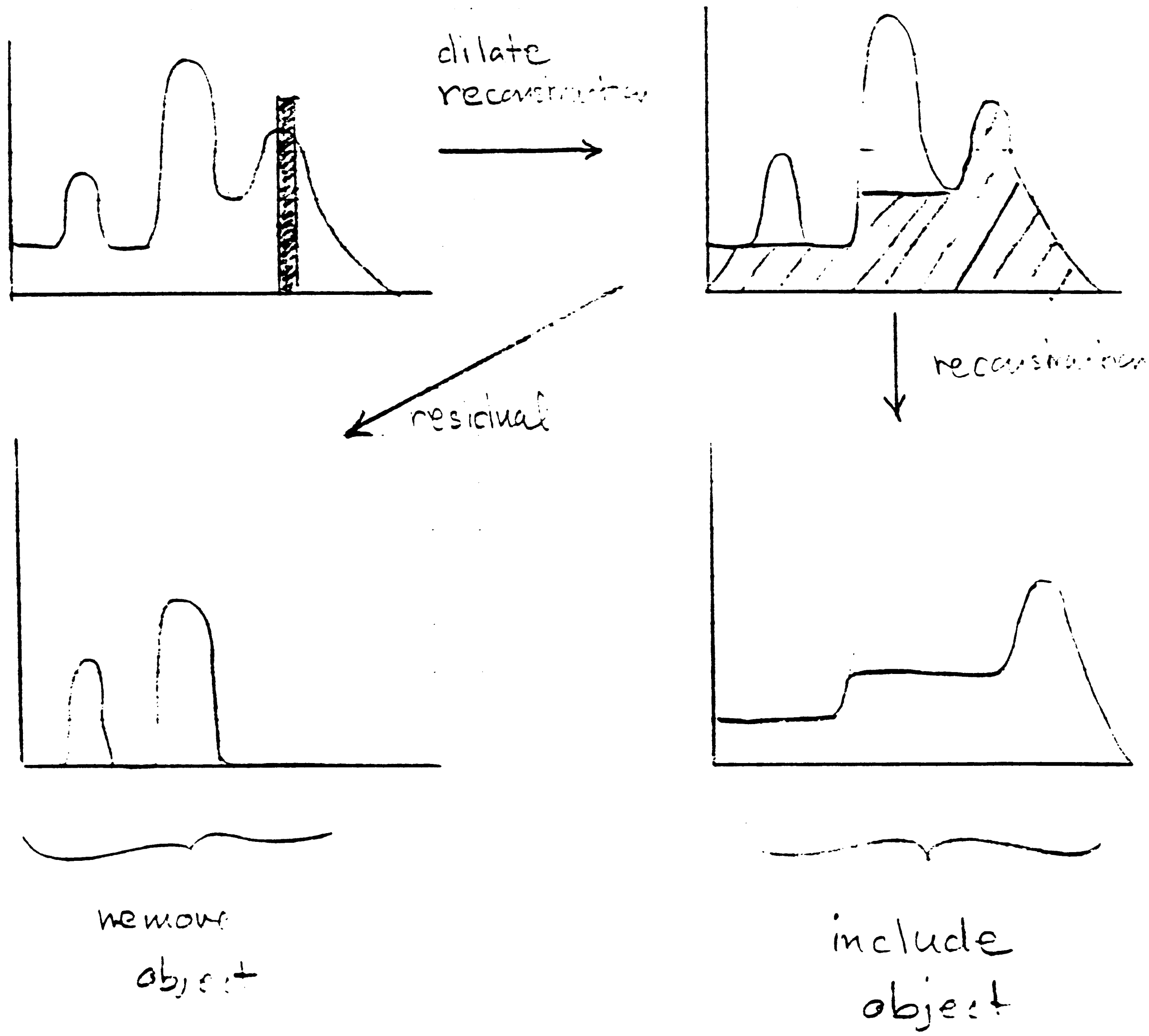
Often we will make the marker very large so as to reconstruct fully the marked object.



Gray-scale geodesic erosion is the dual operation, and it operates on the top of the gray-scale surface. It is a monotonically increasing function.

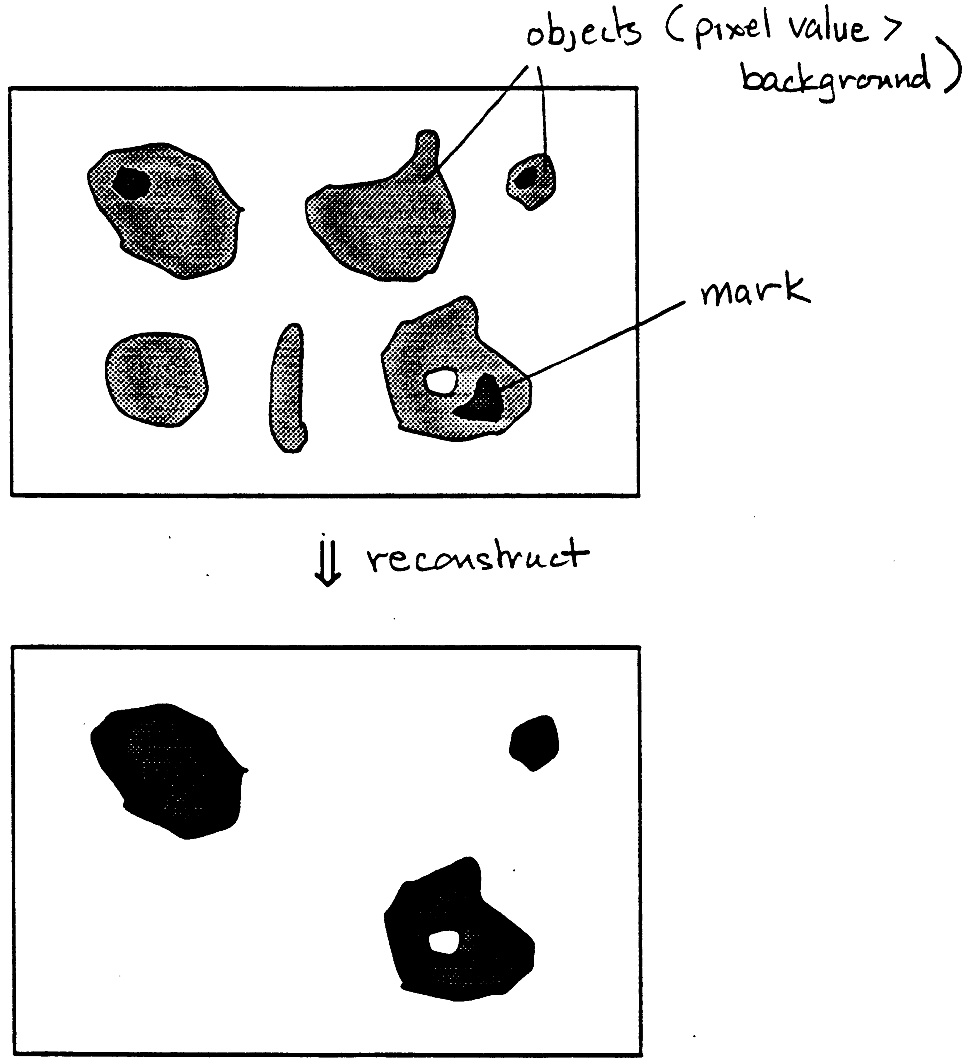
Applications of gray-scale geodesic dilation:

1. Selective removal or inclusion of a gray-scale object.



The trick is how to mark the object!

1. Selectively include objects (cont.)

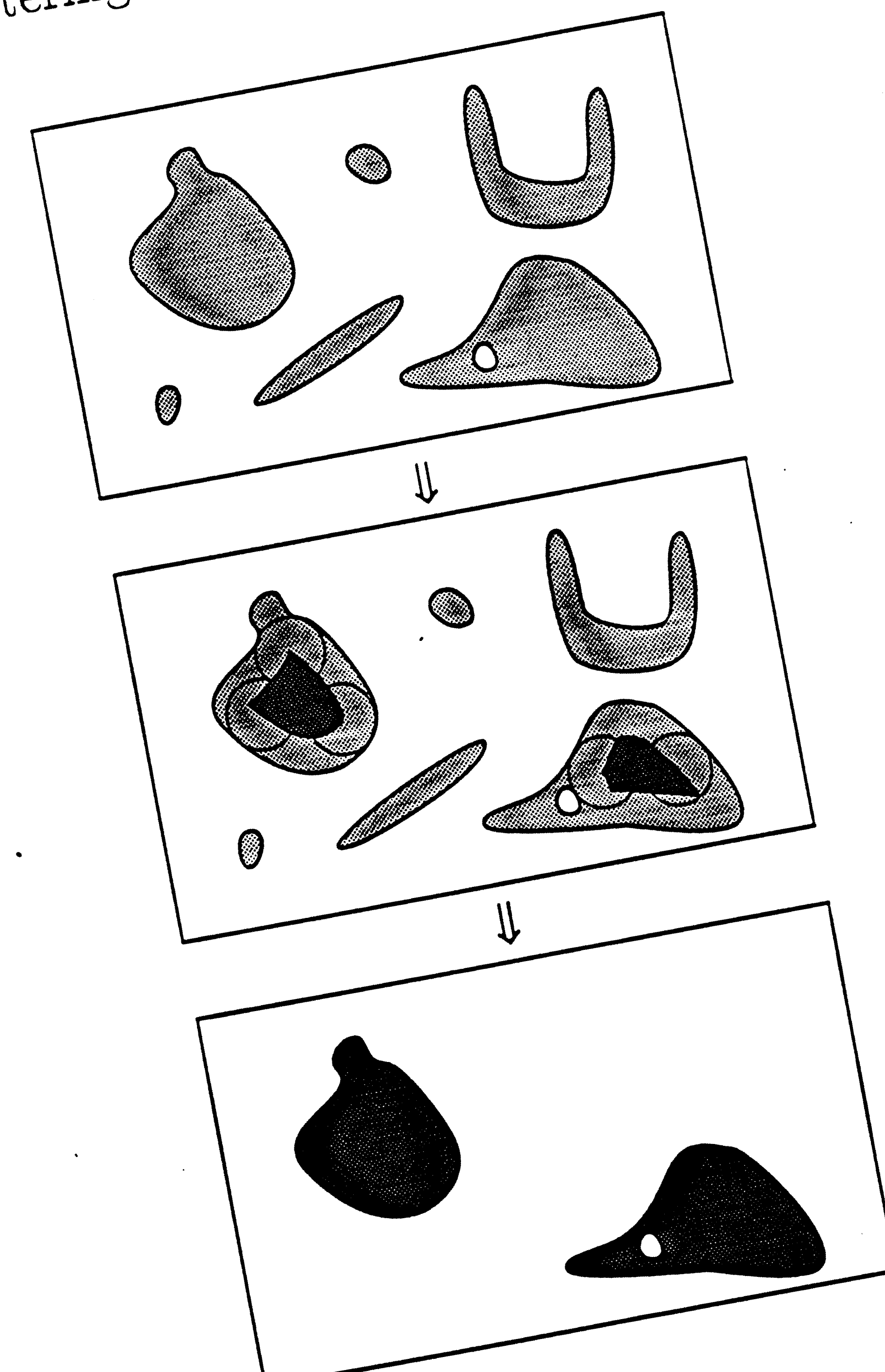


Reconstruction of "objet" X from "marker" Y:

$$R_X(Y) = D_X^{(+\infty)}(Y) = \lim_{r \to +\infty} D_X^{(r)}(Y)$$

⑤

2. Filtering by erosion-reconstruction

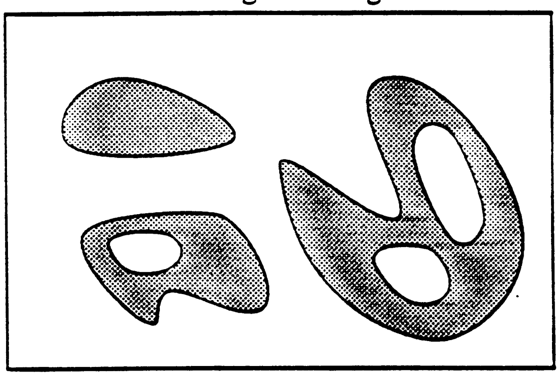


$\Longrightarrow$ Elimination of the connected components which cannot contain a disc of radius λ.

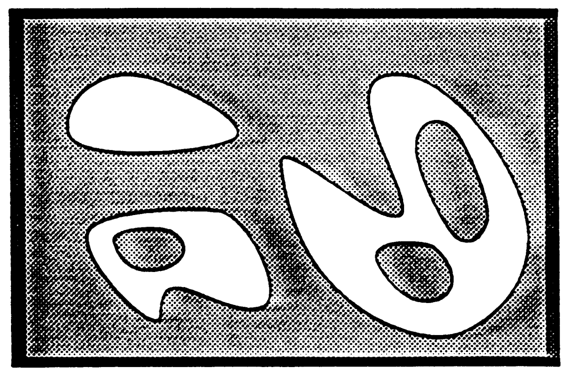
10c

3. Hole filling

1. Original image

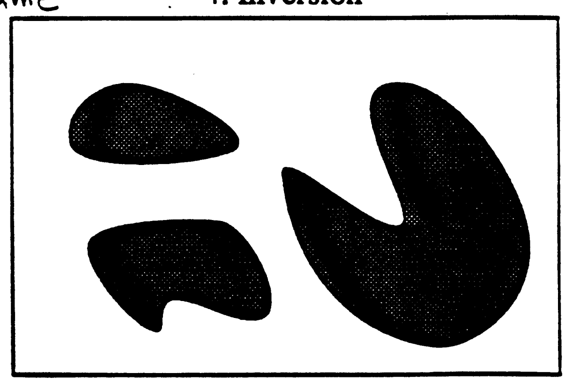


2. Inversion



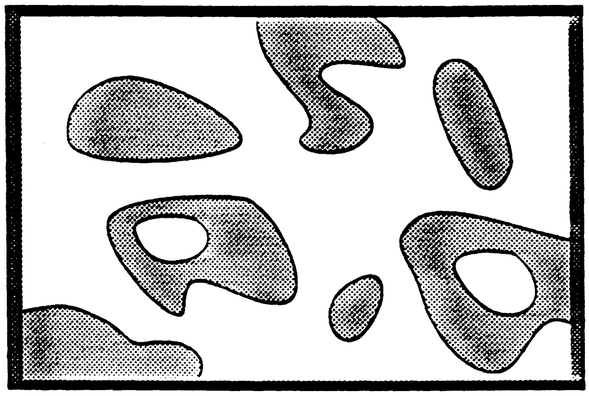
3. Reconstr. of the connected components which touch the edges of the ~~scope~~ image frame

4. Inversion

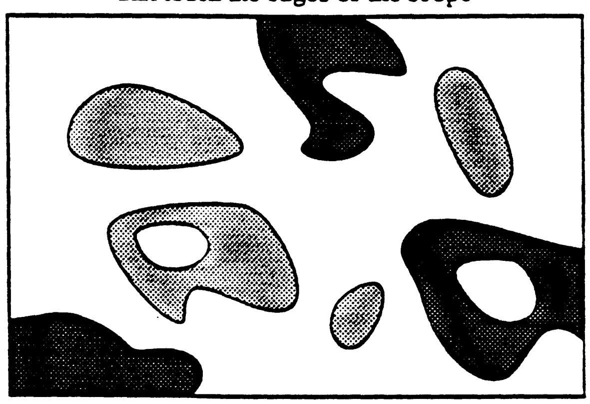


4. Elimination of the connected components that touch the edges of the ~~scope~~ image frame

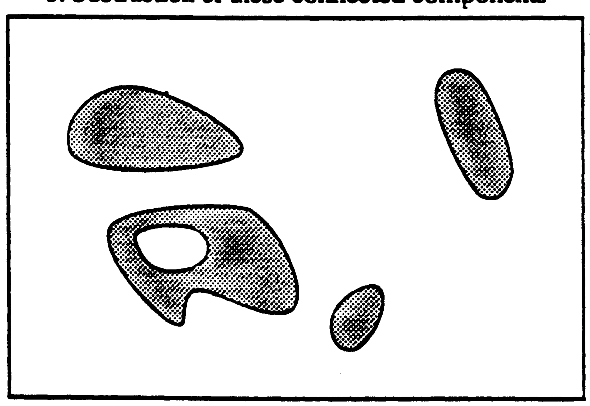
1. Original image



2. Reconstruction of the connected components that touch the edges of the scope



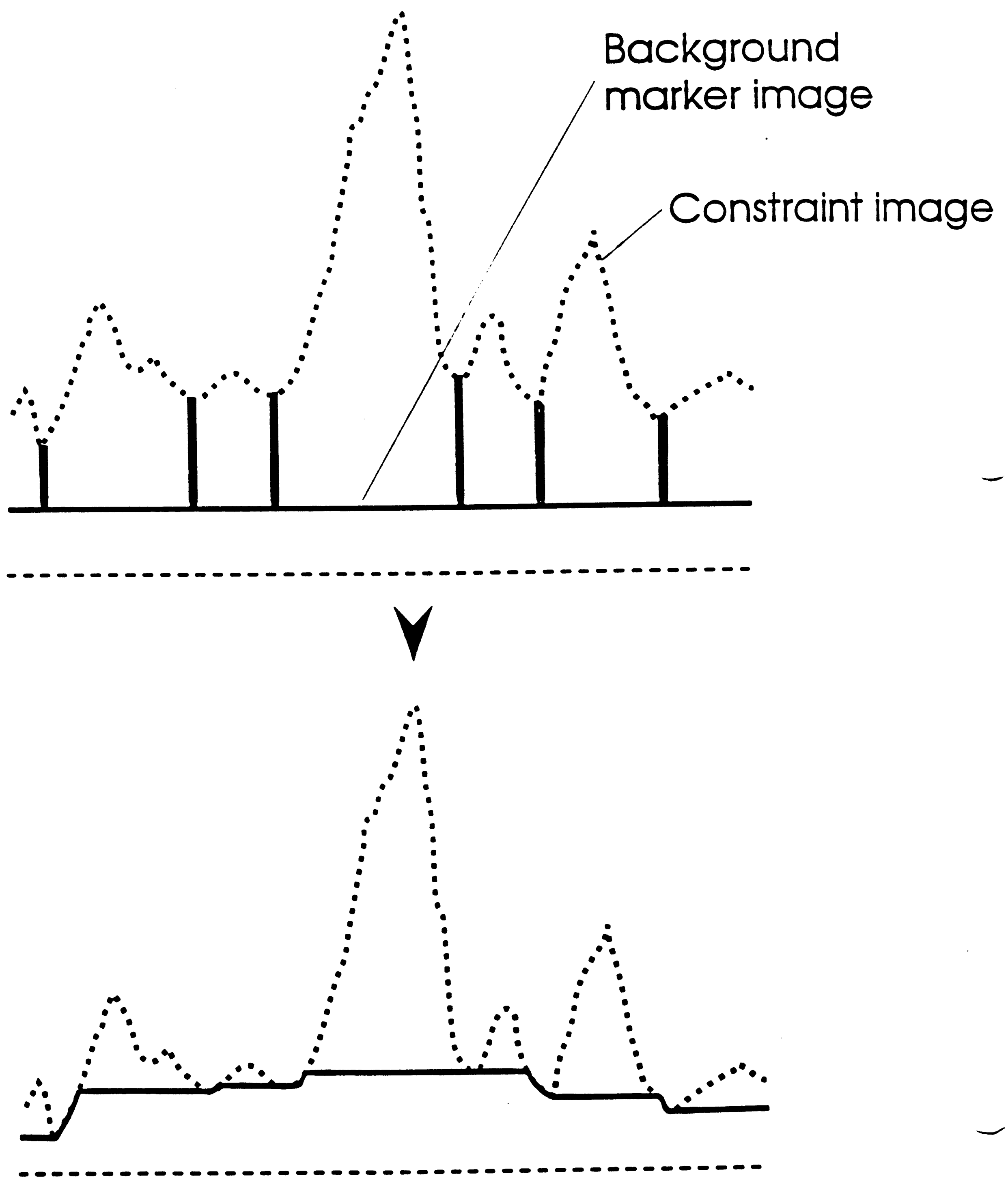
3. Subtraction of these connected components



5. Background Removal

Reconstruct background from minima

Subtract background from original



Useful tricks for setting image markers

1) Obtain regional maxima. This creates a binary image on the DIP station. Multiply this binary image by max-value.

2) Obtain regional maxima having values between two thresholds. This works for application 1 above.

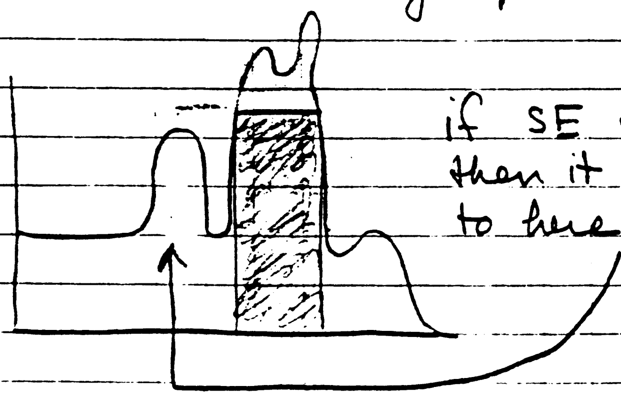
3) Threshold a gray-scale image making a binary image. Take the distance function and threshold it to get large objects.

4) Use the propagation function to mark elongated objects.

5)

Fast implementation on the DIP station —

- Use 4-connected dilation. Note that with larger SE's one can jump across boundaries.



if SE is this large "—", then it can jump across to here

- Pseudo-code

```
boolean image    RGM        ← regional maxima
image            MARKER
image            INIMAGE

find regional maxima of MARKER → RGM
put edges of RGM → FIFO

while (FIFO ! empty)
    for each pixel p ∈ FIFO
        for each neighbor n of p
            if MARKER[p] ≥ MARKER[n]
dilation →          temp = MAX( MARKER[p], MARKER[n])
min →               temp = MIN( temp, INIMAGE[n])
                    MARKER[N] = temp
use a binary } →    put n in the FIFO if not already there
image to track }
            endif
        endfor
    endfor
```

Issues :

1. FIFO can get large. 3/4 number of pixels.

2. Keep a boolean image that keeps track of the pixels in the FIFO, FIFO-PIXELS.

   When push a pixel, r, FIFO_PIXELS(r) = 1
   when pop a pixel, r, FIFO_PIXELS(r) = 0

3. The FIFO implementation is fast because only the pixels at the "edge" are operated on!

4. To find edges of RGM :

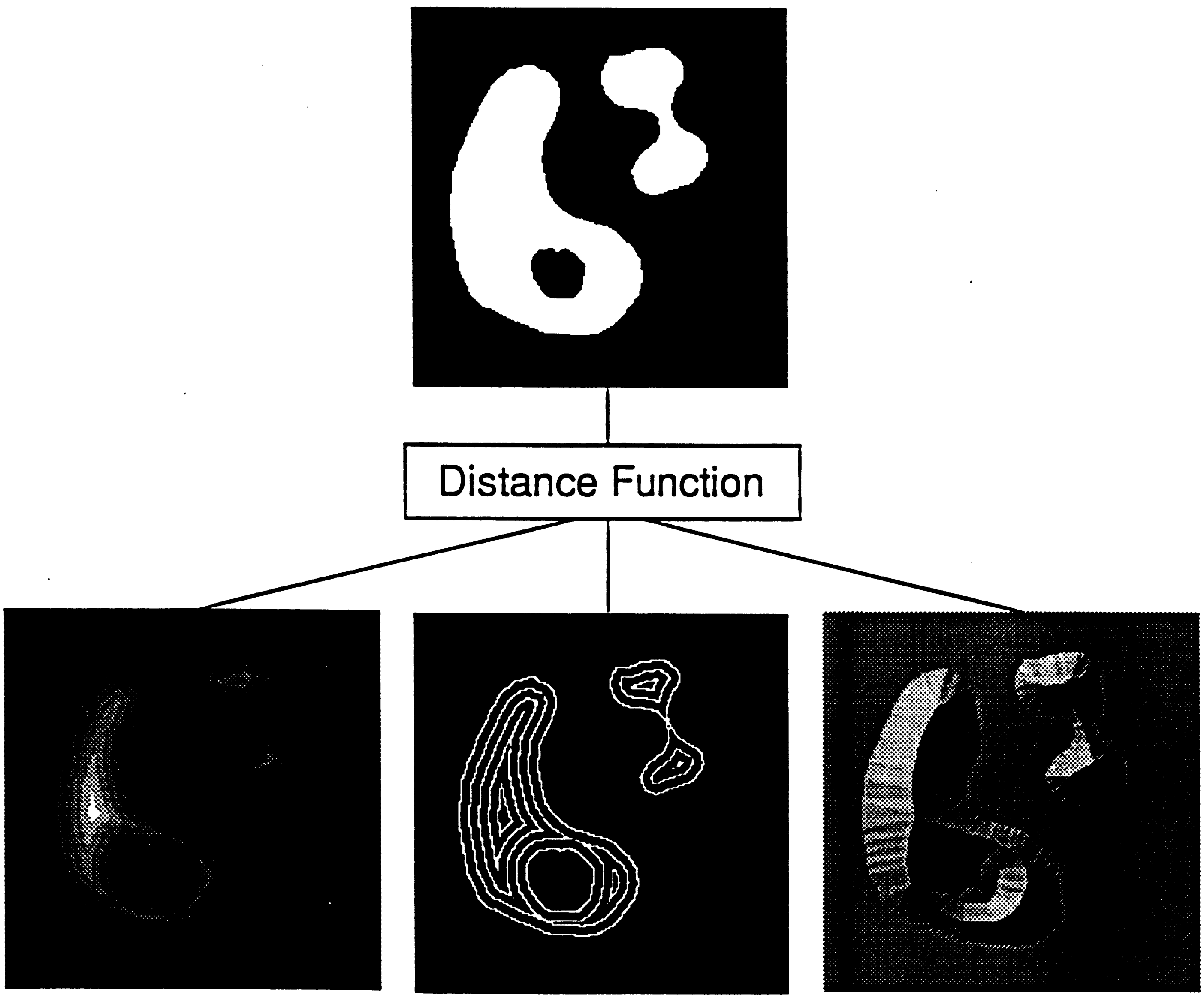
   operate with mask

	N_1	
N_4	P	N_2
	N_3	

   if p is 1 and N_1, N_2, N_3, or N_4 is 0, then p is an edge.

Distance Function

- Associate each point in X to the distance to the nearest edge.
- A definition that is easier to implement: associate each point in X to the sum of successive erosions of X at that point.



Distance Function

The distance function operates on a binary image and it gives the distance from all pixels within objects (value = 1) to the nearest object boundary.

The distance for every pixel x in X is the distance to the nearest boundary of X. Since the boundary of X is where X^c "begins", we can also say it is the distance to the nearest pixel in X^c. Thus,

$$dist_x = d(x, X^c).$$

Computation:

The distance function can be evaluated by calculating its contours. The contours of the distance function are the successive erosions of X.

$$X/(X\ominus B) \longrightarrow \text{distance} = 1$$
$$(X\ominus B)/(X\ominus B\ominus B) \longrightarrow \text{distance} = 2$$
$$\vdots$$

Pseudocode for a distance function calculation follows.

```
read X
i=0
repeat
    i=i+1
    for all pixels,  X → TEMP
    for all pixels,  X ⊖ B → X
    for all pixels
        if (TEMP[n] - X[n]  = 1) then  DIST(n) = i
    end for
until all pixels in X = 0
```

Applications:

1) Identify big, round objects by thresholding the distance image.
2) Use to divide overlapping objects with watershed.

3.) Find a skeleton by following the crest-line.

The propogation function is a dual operation. It gives the distance from each pixel in X to the furthest boundary.

The propogation function is useful for identifying long, thin objects.

Thinning

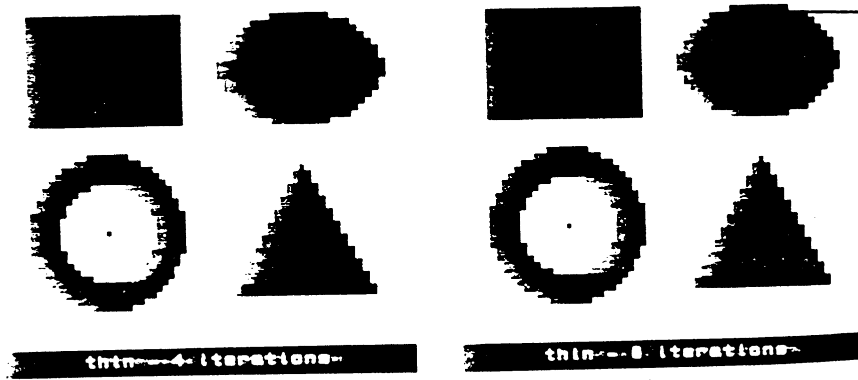
Thinning is an operation applied to binary images. One reason to thin an object is to get a skeleton. Another application is edge thinning when detected edges are too thick. Thick edges are often obtained from the threshold of a gradient-filtered image.

As with the skeleton algorithm, there are multiple thinning algorithms. Some references are:

1) Chapter 15, Digital Image Processing, Pratt
2) Image Analysis and Mathematical Morphology, Part II, Serra
3) Fundamentals of Digital Image Processing, Jain

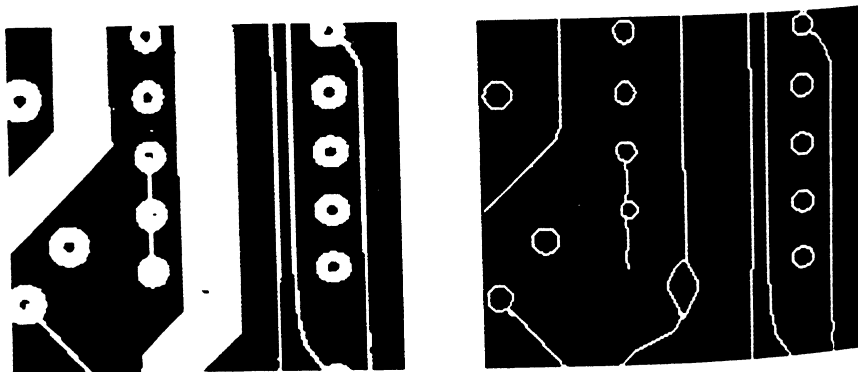
Desirable features of thinning are:

1) An object without holes will become thinner until there is a minimally connected stroke located equidistant from its boundaries.
2) An object with holes will give minimally connected rings lying midway between holes or between a hole and the boundary.



(a) 4 iterations (b) 8 iterations

FIGURE 15.3-3. Example of thinning of a binary image.

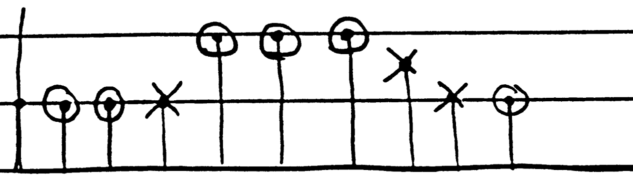


(a) Original PCB (b) Thinned PCB

FIGURE 15.3-4. Example of thinning of a printed circuit board.

Regional Maxima

Local maximum: a pixel is a local maximum if it is $\geq$ all of its neighbors. Neighbors can be 4-connected or 8-connected.

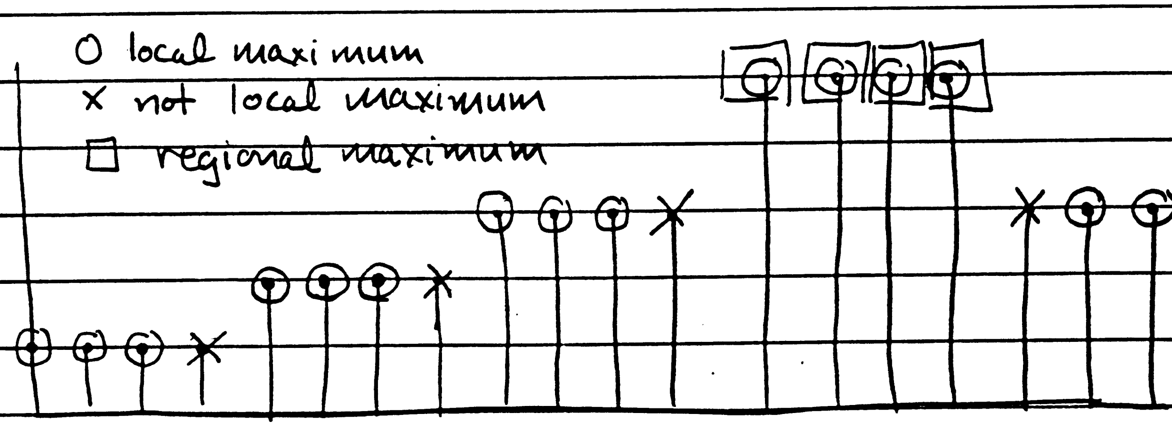


O = local maxima

X ≠ " "

Regional maximum:

- A regional maximum is a local maximum.
- If any neighbor of a local maximum is not a local maximum and is equal to the current pixel, then all adjoining equal pixels are not regional maxima.



Pseudocode

```
binary_image   RMax
image          InImage

read InImage
Local maximum of In Image → RMax
! RMax → FIFO
while ( FIFO ! empty)
      for p ∈ FIFO
          for neighbor n of p
              if (n ∈ RMax and InImage[n] = InImage[p]
                  n → FIFO
                  RMax[n] = ∅
              endif
          endfor
      endfor
```

candidate regional maxima

…t a regional maximum

Morphological Restoration of AFM Images

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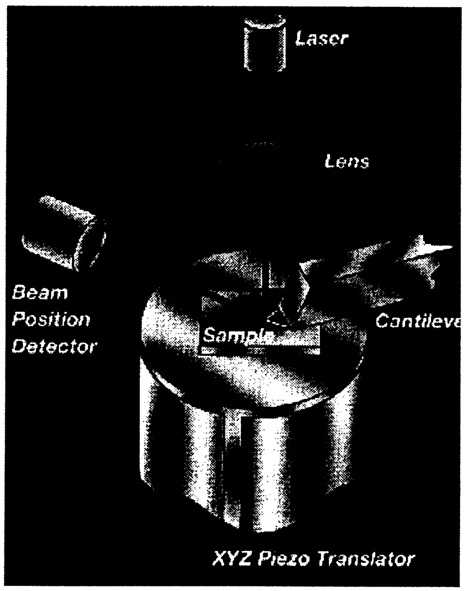
Cleveland, OH 44106

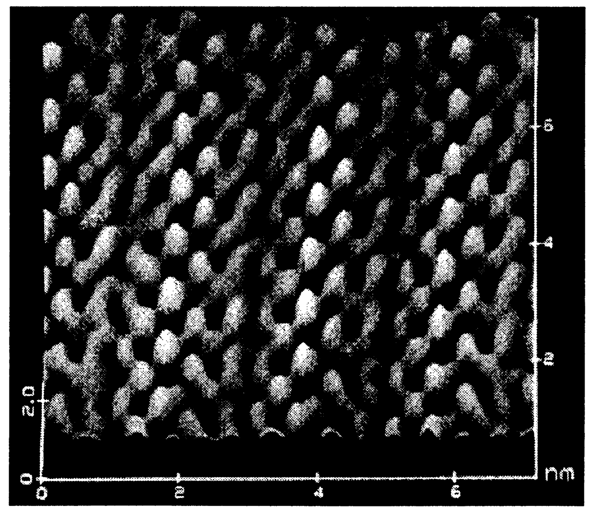
Outline

- AFM Imaging
- Morphological Model
- Morphological Restoration
- Modeling of Spheres
- Modeling of Biomolecules

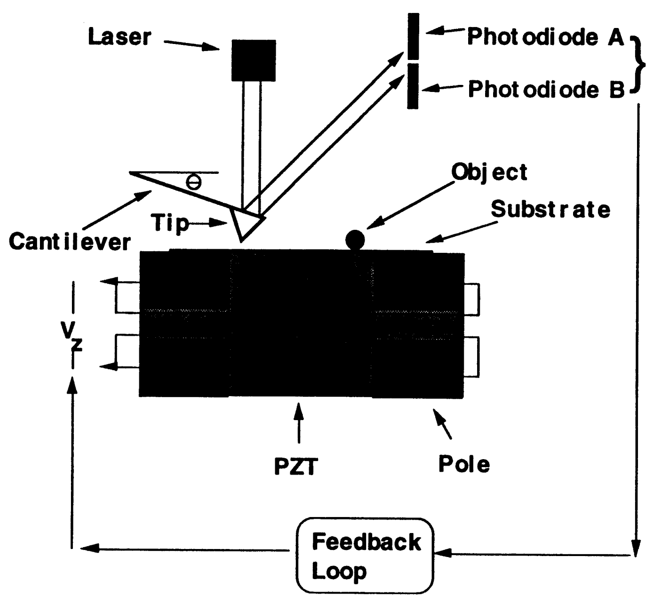
Atomic Force Microscope



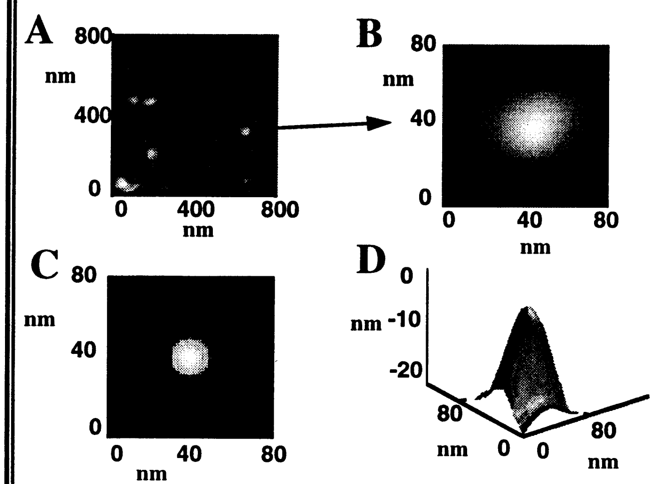
Atomic Structure of PbTeO



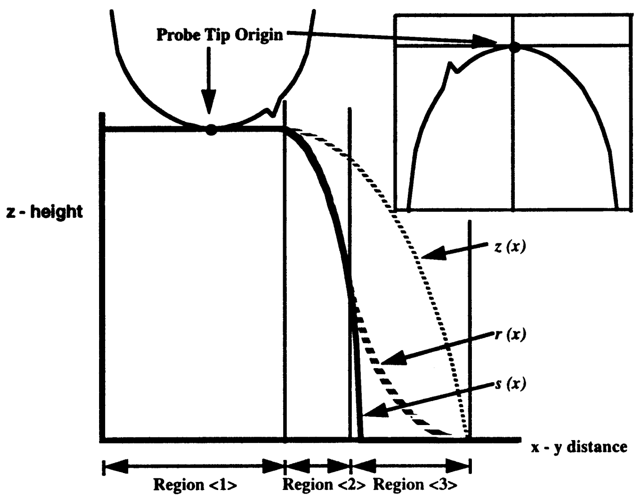
AFM Device



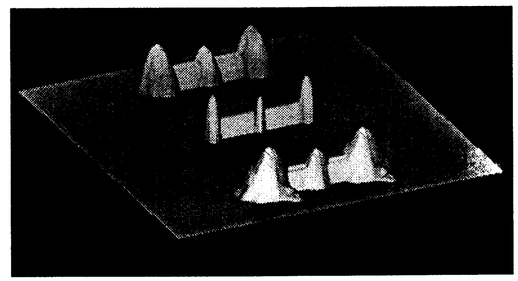
Measuring a Restored Tip

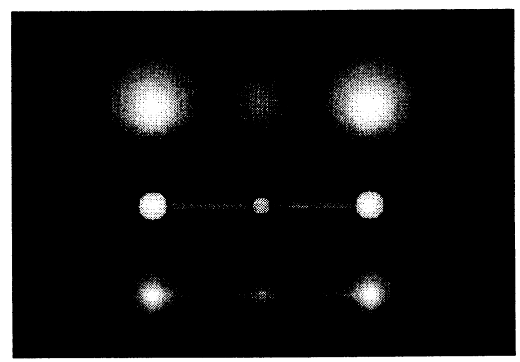


Tip Artifact in AFM Imaging



Morphological Modeling of AFM





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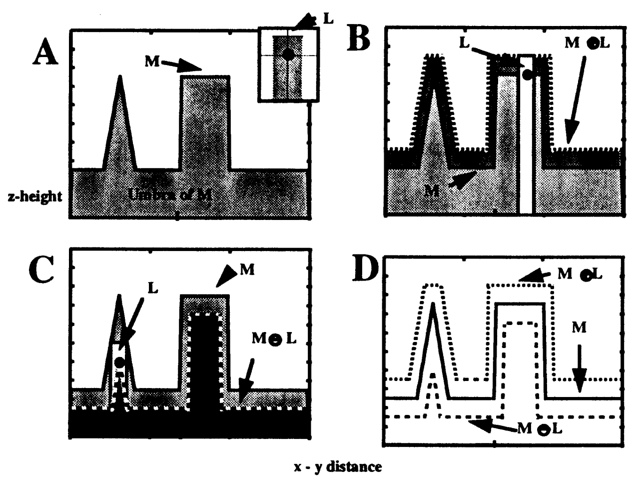
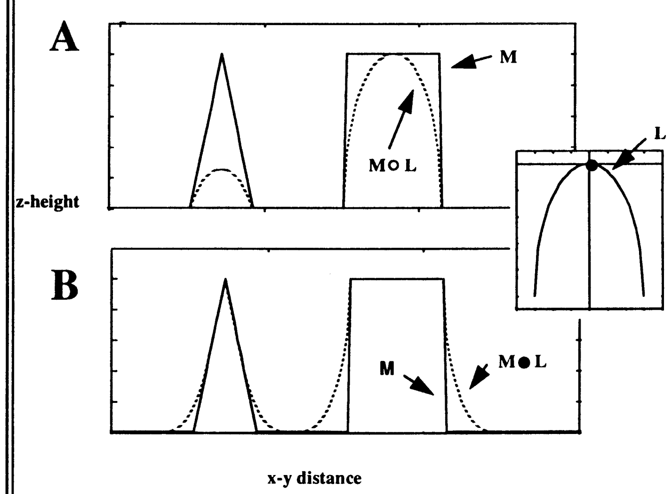
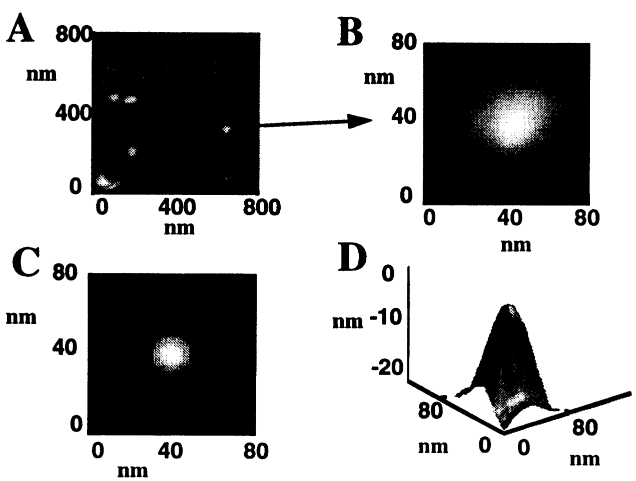
Gray-scale Erosion and Dilation



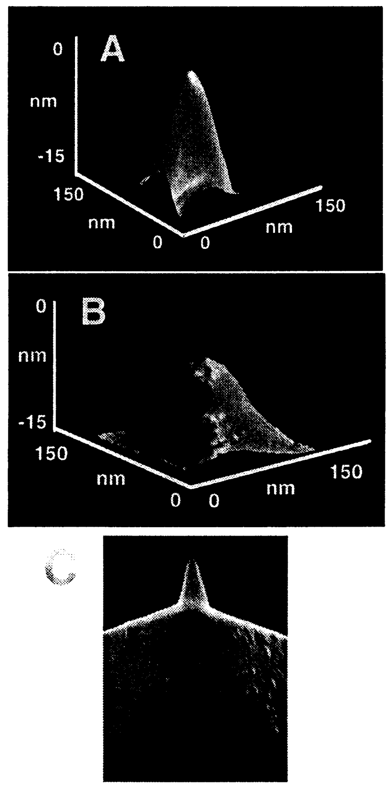
Gray-scale Opening and Closing



Measuring a Restored Tip



Tip from AFM and SEM



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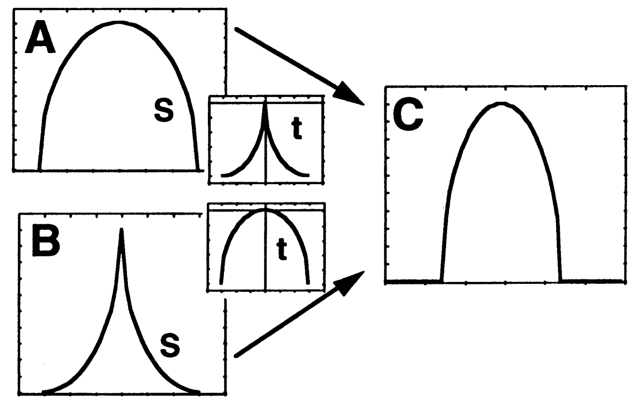
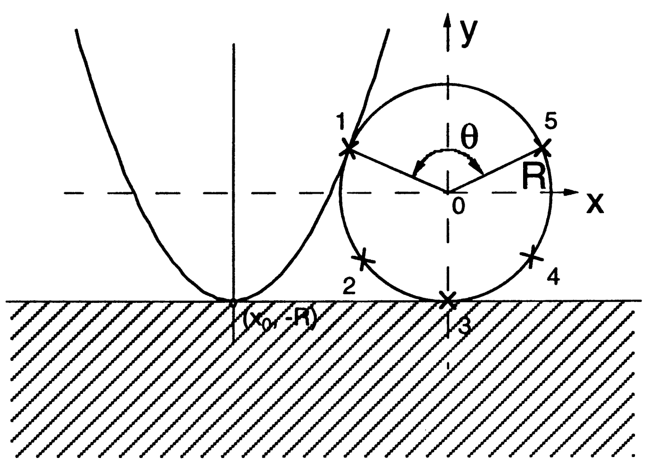
AFM Equations

- **All equations**
  $z = s \oplus t$
  $r = z \ominus t = s \oplus t \ominus t = s \bullet t$
  $z = s \oplus t = t \oplus s$
- **Generate AFM surface (z) from an object model surface (s) and tip (t)**
  $z = s \oplus t$
- **Generate restored surface from a model surface**
  $r = s \bullet t$
- **Generate restored surface from a measured surface (z)**
  $r = z \ominus t$
- **Generate a restored tip image**
  $z = t \oplus s$
  $r_{tip} = z \ominus s$

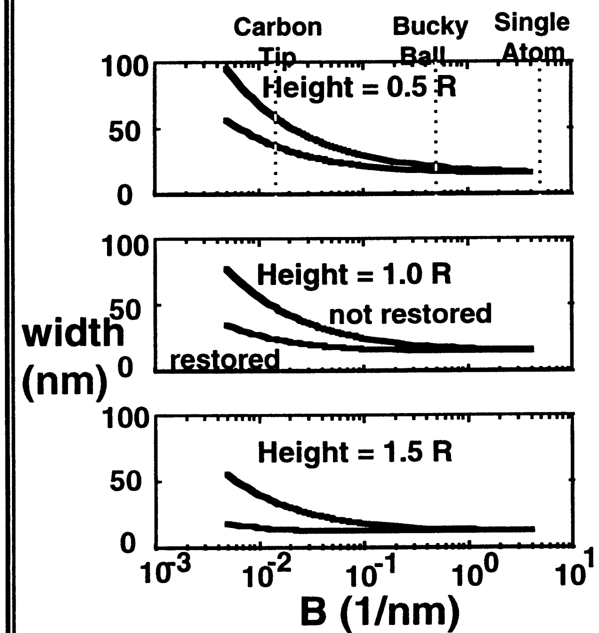
Commutative Property of Morphology



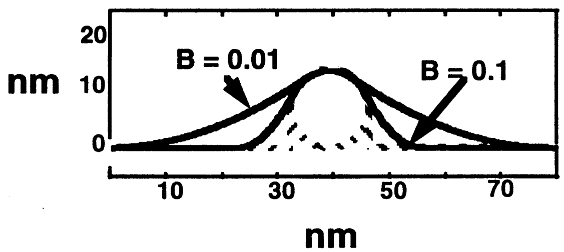
Initial Contact with Sphere



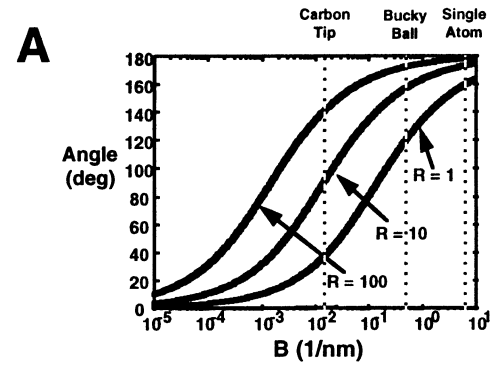
Measured Widths

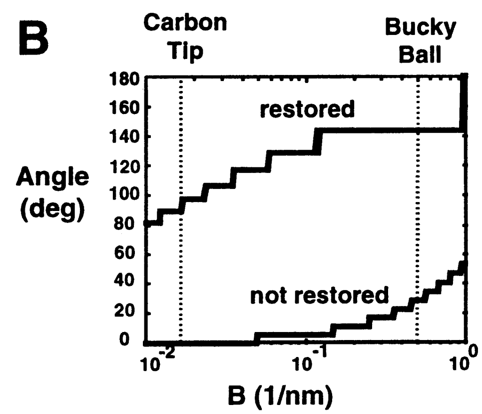


Broadening Due to Tip

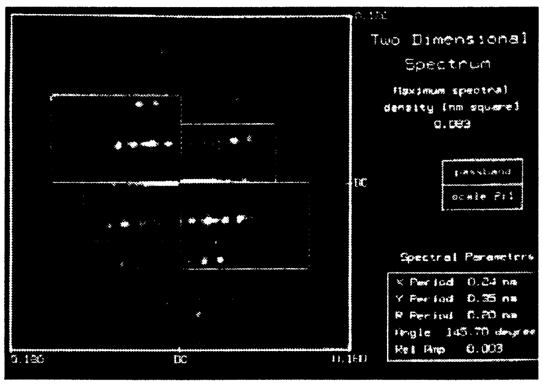


Perfectly Obtained Region





Spectrum Analysis and Filtering



The 2D DFT is displayed. Stopband and pass-band can be constructed interactively with a mouse. If one marks the bright dots in the 2D DFT, then periodic atomic images can be obtained from the inverse DFT.

Summary

- **Tips create a non-linear "blurring" of imaged objects. The height can be exactly obtained, but the width can be greatly increased.**
- **Morphology models the AFM imaging process.**
- **Morphological restoration can greatly improve measurements.**
- **Morphological modeling is being used to understand images of large biomolecules such as fibrinogen.**

Image Registration

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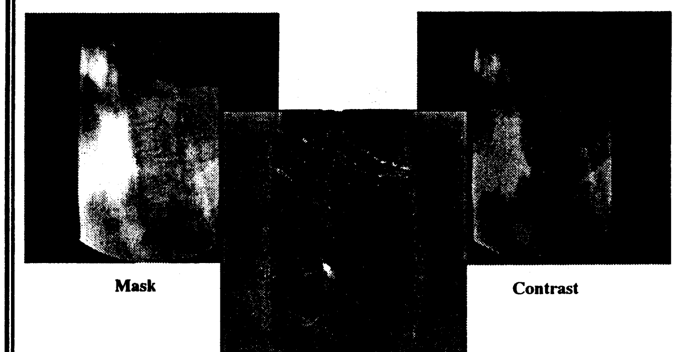
References

- Digital Image Processing. Pratt.
- Flexible mask subtraction for digital angiography. Luong Van Tran and Jack Sklansky. IEEE TMI 11:407-415.
- Automated registration of temporal image pairs for digital subtraction angiography. G. S. Cox and G. de Jager. SPIE Medical Imaging. 1994.
- Accurate three-dimensional registration of CT, PET, and/or MR images of the brain. Charles A. Pelizzari, George T. Y. Chen, Danny R. Spelbring, Ralph R. Weichselbaum, and Chin-Tu Chen. J. Comp. Assisted Tomog. 13:20-26. 1989.
- Rapid automated algorithm for aligning and reslicing PET images. Roger P. Woods, Simon R. Cherry, and John C. Mazziotta. J. Comp. Assisted Tomog. 16:620-633. 1992.
- MRI-PET registration with automated algorithm. Roger P. Woods, John C. Mazziotta, and Simon R. Cherry. J. Comp. Assisted Tomog. 17:536-546. 1993.

Outline

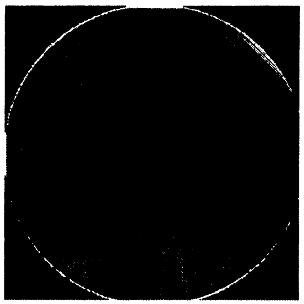
- 2D applications
- Basic approaches
- 2D similarity algorithm
- 3D applications
- 3D approaches
- 3D Woods algorithm

DSA Processing



DSA Registration

Un-registered DSA



Registered DSA

DSA tremendously improves artery conspicuity when there is no movement between the contrast and mask acquisitions. The injection of contrast causes discomfort. Often the patient moves, and subtraction artifacts result.

Registration consists of moving the mask frame with respect to the contrast frame in order to correct the motion.

2D Registration Applications

- **Digital subtraction angiography (DSA)**
- **Cellular florescence imaging**
- **Retinal imaging (images of blood vessels on surface and florescence dye images of vascular failures under the surface)**
- **Dental images (examine small changes in x-ray density over time)**
- **fMRI; images with and without a stimulus**
- **Register portal images in radiation therapy with predicted images from treatment plan**

Motion of 2D Images

- x and y translation
- rotation
- warping

In DSA, we normally have relatively small motions. For this reason, we will consider similarity measure methods that are particularly well-suited for this problem

Registration Methods

- Minimizing the squared error is the same as maximizing the cross-correlation

$$\varepsilon = \sum \sum_{x,y \in A} [I_1(x,y) - I_2(x+m,y+n)]^2$$

$$= \sum \sum_{x,y \in A} [I_1(x,y)]^2 + [I_2(x+m,y+n)]^2 - I_1(x,y) I_2(x+m,y+n)$$

$$c(m,n) = \sum \sum_{x,y \in A} I_1(x,y) I_2(x+m,y+n)$$

- Minimizing the squared error is the same as maximizing the cross-correlation
- Pratt suggests several modifications of the cross-correlation technique. In particular, he suggests to normalize the cross-correlation to remove some of the dependence upon mean value.
- Cross-correlation is time consuming to compute. One way to speed the calculation is to use a search technique and skip some evaluations.

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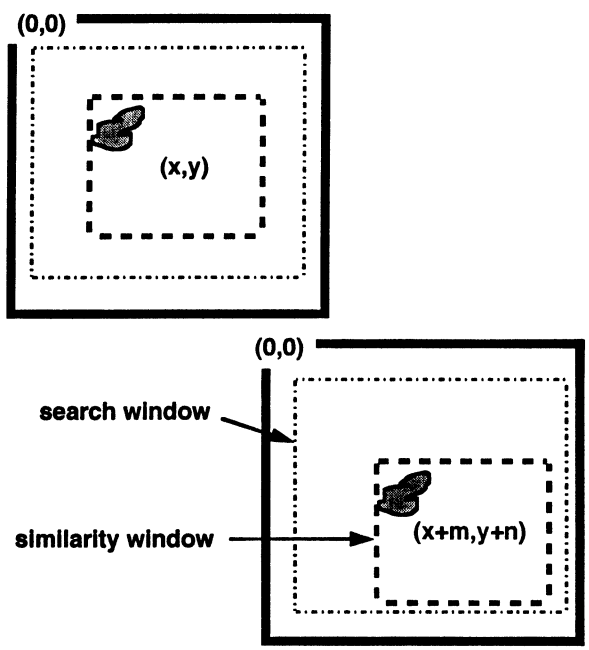
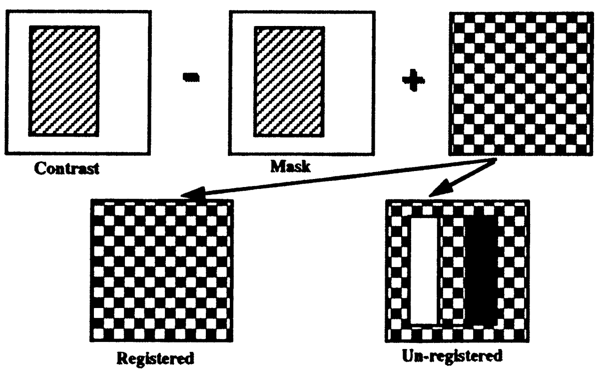
Registration: X and Y Translation



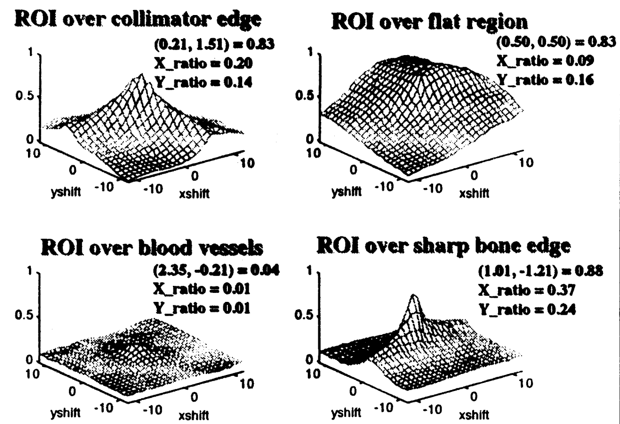
Some Similarity Measures in 2D Registration

- Minimize the sum of the absolute values
- Minimize the sum of the squared difference
- Maximize the sum of the pixel-by-pixel image product (correlation)
- Maximize the sum of the normalized pixel-by-pixel image product
- Minimize the max difference
- Maximize the number of sign changes

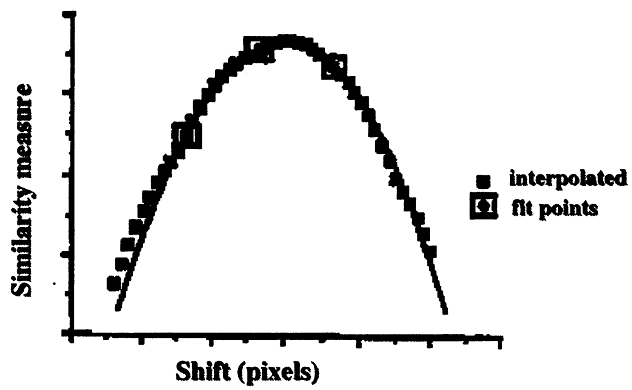
Sign Change Similarity

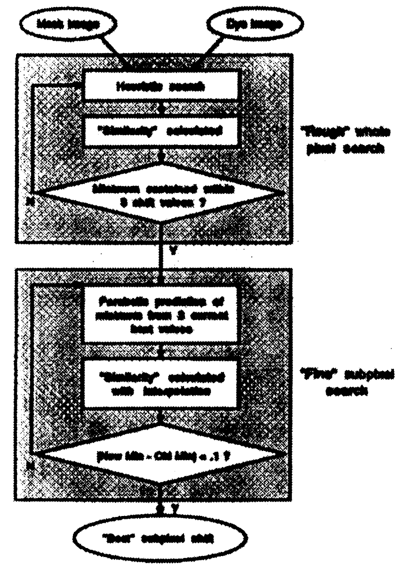


Similarity Surfaces



Parabolic Model Optimization

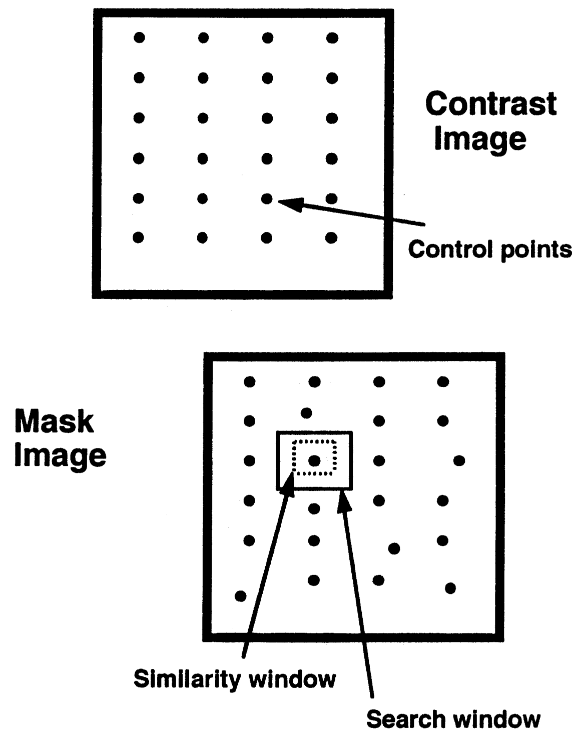




Similarity Registration

- **Pre-processing**
  - Sobel edge image eliminates effects of changes in mean gray-scale
  - morphological pre-processing can remove collimator edges
- **Similarity measure**
  - must correspond to visual subtraction artifacts
  - a smooth, highly peaked parameter surface with no local minima is desirable
  - insensitive to structures such as collimators, etc.
  - in DSA, contrast and mask images are not the same; desire a similarity measure which is robust to this.
  - a "goodness of registration" measure is highly desirable
- **Optimization routine**
  - fast
  - robust; no false minima

Warping Registration



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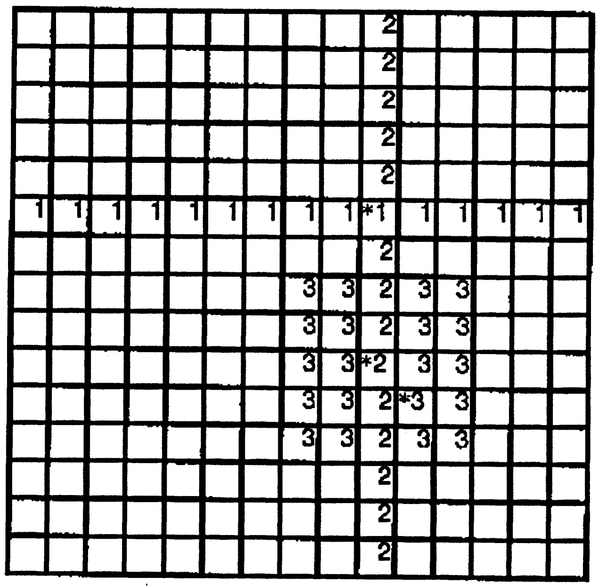
Warping Algorithm (Tran & Sklansky)

- Use control points such that similarity windows overlap slightly (1 pixel overlap)
- Similarity measure is the sum of absolute difference
- For each control point, translate in x and y.
- A rough, whole pixel search proceeds in 4 directions. The subpixel search starts at the best value and proceeds at 0.1 pixel increments.
- For each control point, an optimal x and y shift is obtained.
- Between control points, compute an interpolated shift vector.
- Compute a warped mask image and subtract it from the contrast image.
- Sometimes use an exclusion template to exclude dark arteries from the similarity. This is done by thresholding after an initial subtraction.
- Images are scaled to correct for gray-scale variation.

Warping Registration (CWRU-SCR)

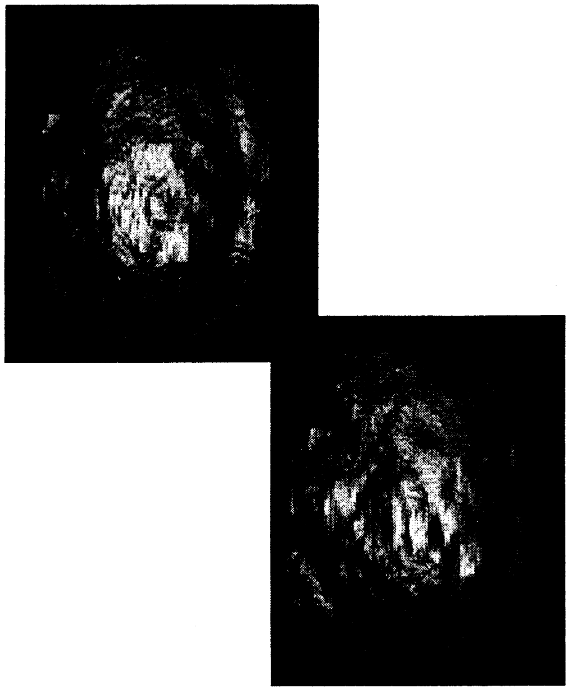
- Pre-processing
  - Sobel edge image
  - morphological filtering to remove collimator edges
- Non-overlapping similarity regions
- 8x8 regions on a 1024x1024 pixel image
- Search +- 15 pixels
- At each control point, x and y translation and rotation.
- One search strategy is Powell's method as described in Numerical Recipes.
- A much faster, more robust, approach is a heuristic whole pixel search followed by sub-pixel parabolic search.

Heuristic Whole Pixel Search



Results of Warping

3D Registration



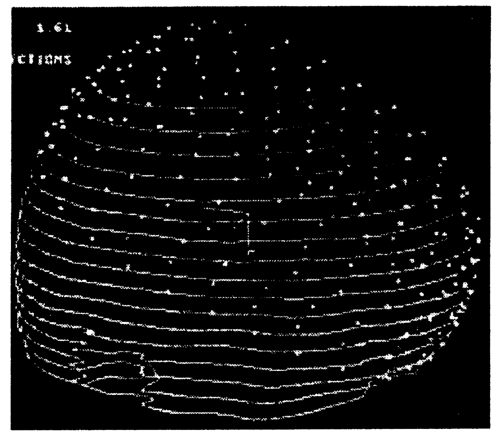
Applications of 3D Registration

- iMRI; compare images before, during, and after iMRI treatment
- fMRI; compare images with and without stimulus
- PET -PET; compare tumors following treatment
- PET-MRI; correlate tumor hot spots with anatomy
- PET-MRI; correlate brain function measurements with anatomical ROI's
- In PET-MRI, often create a color-coded visualization of PET count value and MRI anatomy
- MRI-MRI; changes with Alzheimer's disease progression

Methods for 3D Registration

- Externally attached fiducials
- Anatomical fiducials (bone structure, teeth, etc.)
- Hat-to-head, a surface fitting method (Pellizzari et al.)
- 3D correlation
- Similarity measure (σ_r / r_{mean}) for same-modality registration (Woods et al.)
- Similarity measure for crosss-modality registration (Woods et al.)

Hat to Head Surface Matching



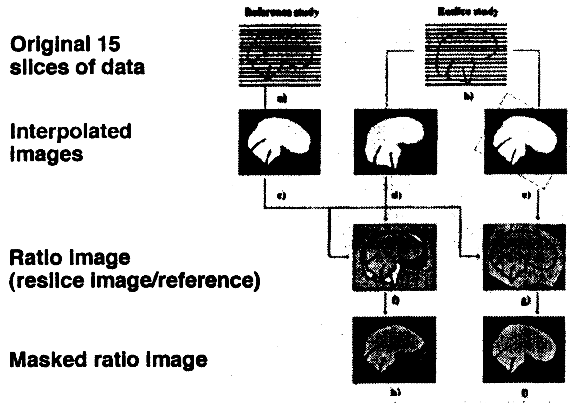
The "hat" are points from MRI slices while contours are the "head" from CT slices. An iterative scheme is used to minimize the distance between the head and hat.

PET-PET Registration Algorithm

General

The algorithm optimizes a similarity measure calculated over the data volume. There is a reference study and a reslice study. Data will be extracted from the reslice study assuming 6 degrees of freedom: x, y, and z translation and rotation about the x, y, and z axes. Given a new orientation parameter set, trilinear interpolation is used to extract new voxel data from the reslice data.

PET-PET Registration



PET-PET Registration

- One reference and one reslice volume
- Assume same voxel sizes in each volume. Since PET has thick slices, linearly interpolate slices along z to create isotropic voxels.
- Initial reslice parameters (x, y, and z rotations and translations) estimated by user
- A new reslice volume is obtained from a the set of reslice parameters using trilinear interpolation
- Each voxel in the resliced volume is divided by the corresponding voxel in the reference to give a ratio, r.
- The resliced and reference volumes may not totally overlap, and these regions must be excluded. The ratio volume is masked by assigning a value of zero to r values less than 21%. This threshold masks the dark regions in the figure.
- Over the remaining voxels, r_{mean} and σ_r are computed. The ratio, σ_r / r_{mean}, is computed.
- An iterative, Newton Raphson search algorithm is used to minimize this ratio.

Cross-Modality Registration

Cross-modality registration (e.g. MRI-PET) presents the problem that images may be very different. For example, brain white and gray matter will appear light and dark, respectively, in a T1-weighted MRI scan, but they will appear dark and light, respectively, in a PET study using FDG. Most similarity measures (sum of the absolute difference, sum of the square difference, etc.) will be penalized by this difference in light-dark structures.

If one could segment all structures of interest, then one could create a scaling factor for each tissue type that would correct this difference between structures. The algorithm created by Woods et al. for PET-MRI data does this in a brute force way using every MRI pixel value as a new tissue type. (The implicit assumption is that each MRI pixel value will be a unique tissue type with the same PET response.).

PET-MRI Registration (Woods et al.)

- For each voxel value j in the MRI volume, calculate the mean a_j and standard deviation σ_j of the corresponding PET voxels. Calculate the normalized standard deviation $\sigma_j' = \sigma_j / a_j$
- This ratio should be minimized when similar PET voxel values match a given MRI tissue type (a given MRI voxel value).
- To make this comparison over the entire volume, a weighted mean of the normalized standard deviations (σ'') is calculated. The weighting is based upon the proportion of the total number of voxels that have a particular MRI value.
- The Newton Raphson algorithm is used to minimize σ'' .

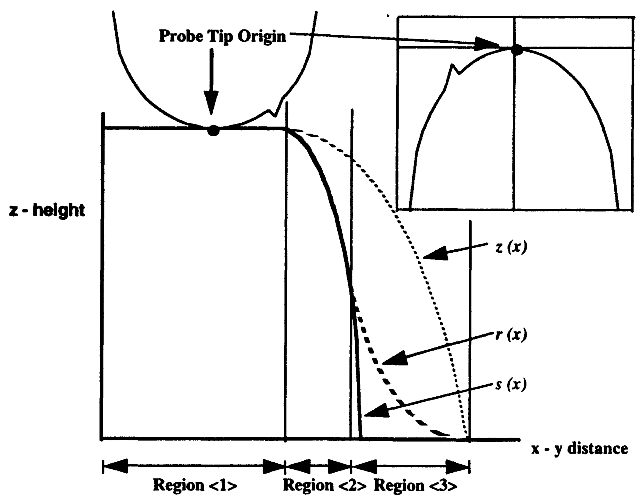
Note:

(1). e-mail Leda ≡ textbooks

(2). e-mail Cammy operator write up. due date, etc.

Jim Pohl ≡ images

Tip Artifact in AFM Imaging



Derivation of AFM Equations