

# The Art of Back-of-the-Envelope Paraxial Raytracing

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**Abstract**—Paraxial raytracing is a valuable tool for making quick “back-of-the-envelope” calculations in optical system design. Its popularity has been highly diminished as a consequence of the growing sophistication of computer-aided raytracing and the availability of powerful computers. The availability of raytracing computer programs does not harm in itself the usefulness of graphical raytracing, but it makes people forget, or never learn the basics of graphical raytracing. In this paper, some of the basic concepts of graphical raytracing are presented, along with some examples that illustrate how it can be applied to multielement systems. The mastering of the methods presented in this paper constitute some of the fundamentals required in the use of any optical software. Such knowledge is absolutely necessary for the conception of novel optical systems.

**Index Terms**—Hand raytracing, paraxial raytracing, ray sketching.

## I. INTRODUCTION

**G**EOMETRICAL optics is a part of optics involving phenomena that can be discussed within the framework of Fermat’s principle, from which laws of reflection, refraction, and propagation can be derived [1]. Starting with the definition of the optical path length as given by

$$OPL = \int_S n ds \tag{1}$$

where  $ds$  is an element of arc length along the propagation path and  $n$  is the index of refraction along that path and assuming two points in an isotropic medium, Fermat’s principle states that the optical path that the light actually takes is an extremum of all possible optical paths between these two points. In this paper, the reader should assume solely propagation in a homogeneous by parts and isotropic medium where the light can be shown to propagate along paths of infinite radii of curvature. Such paths provide a model for light propagation referred to as light rays [2]. The extremum reduces to a minimum for points in the regular neighborhood, that is, the region in space where one and only one ray passes through each point [3]. This region is limited by the envelope of the rays, known as the caustic, and illustrated in Fig. 1. For rays beyond the caustic, the optical path length is actually stationary [3].

The laws of reflection and refraction are illustrated in Fig. 2. The reader should assume that light is incident on the surface between two media. The point where the light intersects the surface is called the point of incidence. At any point of incidence, the normal  $\vec{N}$  to the surface can be established. If a ray propagates along the direction of the normal, a fraction of the

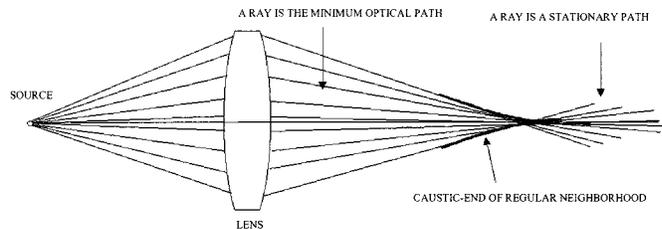


Fig. 1. Illustration of a longitudinal section of a caustic and regions of minimum versus stationary paths.

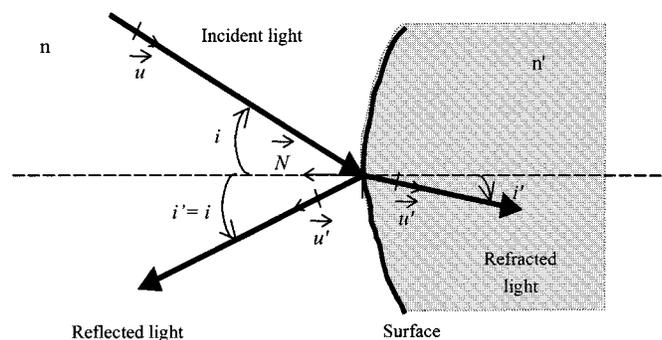


Fig. 2. Reflection and refraction of light at the surface between two media.

light is reflected on itself, while the other is transmitted undeviated. Otherwise, the angle subtended by the surface normal and the incident ray, known as the angle of incidence  $i$ , is nonzero and yields deviated reflected and refracted rays. The angle subtended by the surface normal and the reflected or refracted ray shall be denoted as  $i'$ .

Starting with Fermat’s principle, a vectorial law applicable to either reflection or refraction can be derived that is given by

$$n' \vec{u}' - n \vec{u} = (n' \cos i' - n \cos i) \vec{N} \tag{2}$$

where  $n$  and  $n'$  are the indexes of refraction of the incident medium and refracting medium, respectively, and  $\vec{u}$ ,  $\vec{u}'$  are unit vectors along the incident, and reflected or refracted ray, respectively [1]. The law given by (2) states that the reflected and transmitted rays are coplanar and lie in the plane being defined by  $\vec{u}$  and  $\vec{N}$ . This law finds applications not only in optical image formation but also in the field of computer graphics where raytracing is extensively used in simulation [4]. From the vectorial law, scalar laws of reflection and refraction may be derived. If a clockwise or a counter clockwise convention is selected to treat angles as signed entities, the scalar law of refraction, also known as Snell’s law, is given by

$$n \cdot \sin i = n' \cdot \sin i'. \tag{3}$$

The scalar law of reflection, obtained by taking  $n' = -n$ , states that the angle of reflection is numerically equal to the angle of

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incidence (i.e.,  $i' = -i$  upon reflection). Such a convention for changing the sign of the index of refraction upon reflection allows unification of the laws. In this respect such convention is advantageous over other possible conventions.

A large class of optical systems consists of a series of refracting or reflecting surfaces to provide the required deviation of the light rays together with appropriate stops and apertures to limit the angular and spatial extent of the rays. The optical designer must trace the path of selected rays through the system, which is done by repeated application of the vectorial reflection and refraction laws through the system. Such process is known as real raytracing.

In the current treatment, the authors only consider optical systems with a common axis of revolution, referred to as the optical axis. In the approximation of small angles of incidence on surfaces and with respect to the optical axis, the optical system is said to work in the Gauss or paraxial approximation, and raytracing under these assumptions is referred to as paraxial raytracing.

The art of exerting paraxial raytracing through lenses with no help of computers or calculators shall be referred to as graphical raytracing [5], [6]. In recent years the art of graphical raytracing has lost some of its early prominence as the consequence of the growing sophistication of computer-aided raytracing. This paper specifically focuses on redefining basic rules of paraxial raytracing without the use of computers or calculators in order to get quickly, back-of-the-envelope imaging solutions. Moreover, the ability to perform graphical raytracing may prove helpful in the conception of optical systems.

This paper will first establish a sign convention and briefly review some fundamentals such as real and virtual objects and images, and the cardinal points of optical systems. It will then lay out fundamental principles of graphical raytracing, and illustrate them with simple examples.

## II. SIGN CONVENTION

Assuming that light propagates from left to right, the authors define the sequence of left to right as the positive direction. All distances measured along this direction will be positive, and all distances measured in the opposite direction will be negative. All angles measured counter-clockwise from the optical axis will be positive, and all angles measured clockwise from the optical axis will be negative. The sign convention is represented as



## III. OBJECTS AND IMAGES

Beams of light rays impinging on an optical system from apparent unique locations define point objects that may lie either on axis (A) or off-axis (B), as shown in Fig. 3. Similarly, beams of light rays emerging from the optical system and intersecting at unique locations define corresponding images  $A'$  and  $B'$ . To define real and virtual objects and images, the first and last optical surfaces of the optical system are denoted as  $S_f$  and  $S_l$ , respectively, as shown in Fig. 3.

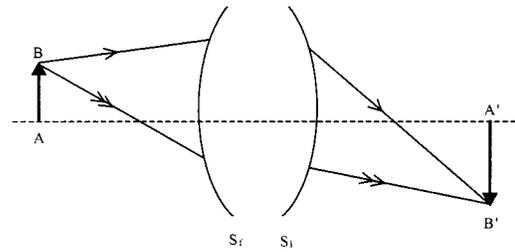


Fig. 3. On-axis, A, and off-axis, B, object points and respective images  $A'$  and  $B'$ .

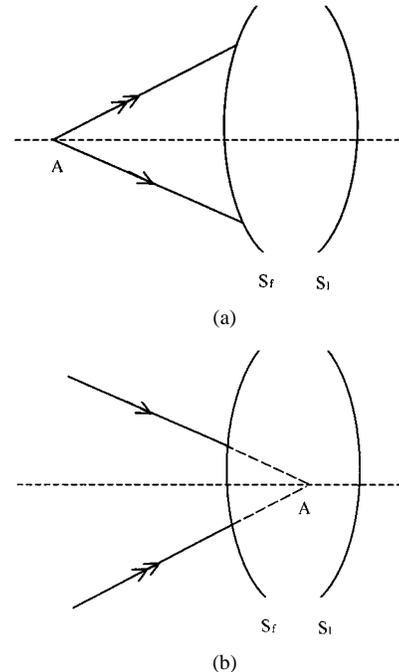


Fig. 4. (a) Illustration of real and (b) virtual objects.

An object is real if it is located before  $S_f$ , and it is virtual if located after  $S_f$ . Similarly, an image is real if it is located after  $S_l$ , and virtual if located before  $S_l$ . Real and virtual objects and images are illustrated in Figs. 4 and 5, respectively, providing that light propagates from left to right.

Finally, an aerial image must be distinguished from a recorded image. Light from an object produces a three-dimensional (3-D) distribution of light in image space. The aerial image is the distribution of light on a mathematical surface often about best focus, which is the locus of the image points resulting from object points. The aerial image is never the final goal. Ultimately, the light is captured by a detector device, and a recorded image is obtained. The recorded image varies with the position of the receiving surface due to possible defocus. The receiving surface is most often positioned to coincide with the best focus aerial image.

## IV. CARDINAL POINTS

Six axial points, called the cardinal points, can be used to represent a complete optical system. With these six points, one can determine the size and location of an image. This is true for both simple lenses and complex multielement lenses. These

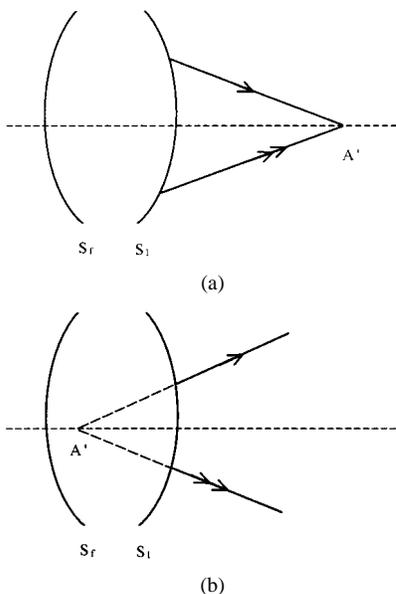


Fig. 5. (a) Illustration of real and (b) virtual images.

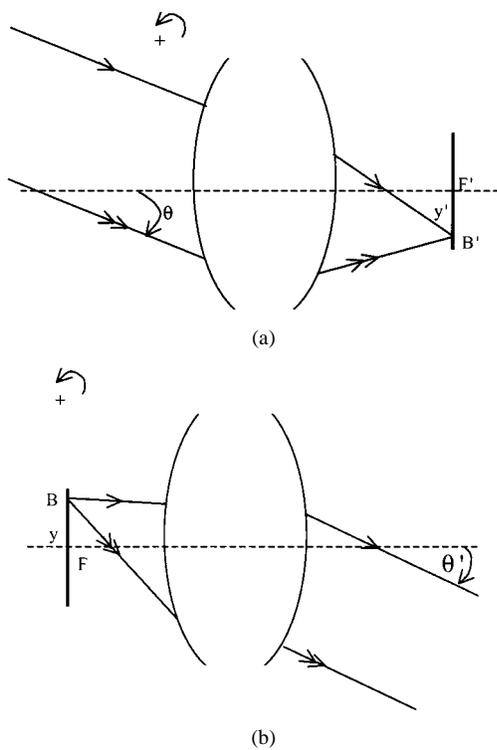


Fig. 6. (a) Focal plane image, (b) focal plane object.

six points are: the front and back focal points, the principal and antiprincipal points, and the nodal and antinodal points.

**A. Focal Points**

The focal points are the intersection of the focal planes with the optical axis. The focal plane image is defined as the plane where all parallel input rays, whether parallel to the optical axis to simulate a point on-axis at infinity, or at some angle with respect to the optical axis to simulate a point off-axis at infinity, are focused by the optical system. An off-axis beam is shown in Fig. 6(a). The intersection of the focal plane image with the

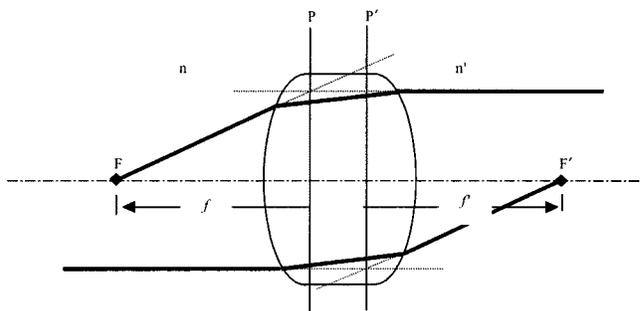


Fig. 7. Graphical construction of the principal planes P and P'.

optical axis defines the back focal point  $F'$  or image focal point. Similarly, a front focal point  $F$  can be defined (or focal point object) as the intersection of the focal plane object with the optical axis. The focal plane object is defined as the plane where object points yield parallel output rays as illustrated in Fig. 6(b).

**B. Principal and Antiprincipal Points**

The reader should define as conjugate planes two planes that are optical images of each other with respect to the optical system. The principal points are defined as the intersection of the principal planes with the optical axis, where the principal planes are conjugate planes corresponding to a transverse magnification of 1. They can be located within or outside the lens, and in any relative order. The principal planes can be thought of as mathematical constructs.

In Fig. 7 the authors show how to find the location of the principal planes by raytracing. First a paraxial ray is traced from object space parallel to the optical axis. This ray will exit the lens passing through the focal point image. If one extends the ray that enters the lens and the ray that exits the lens, the intersection will give the location of the principal plane image  $P'$ . Similarly, the principal plane object can be found by tracing backward a ray parallel to the optical axis and entering the lens from the image space instead. It is important to note that the rays intersecting through this technique may describe a surface instead of a plane as the impact height of the ray varies on the optics. The principal planes are indeed the tangents to these surfaces. Once found, the principal planes may be used to represent paraxial refraction or reflection through an optical system.

The focal length of a lens is defined in relation to the principal points. The front focal length  $f$  is defined as the distance from the principal point object to the front focal point and is denoted  $\overline{PF}$ . Similarly, the back focal length  $f'$  is defined as the distance from the principal point image to the back focal point and is denoted  $\overline{P'F'}$ . The front focal length and the back focal length are numerically equal only if the media surrounding the lens is the same on both sides ( $n = n'$ ). Otherwise the focal lengths are calculated as

$$C = n'/f' = -n/f. \tag{4}$$

The antiprincipal points are defined as the set of conjugate points with a transverse magnification of  $-1$ . It can be shown from collineation theory that the antiprincipal points are symmetrical to the principal points with respect to the corresponding focal points [7].

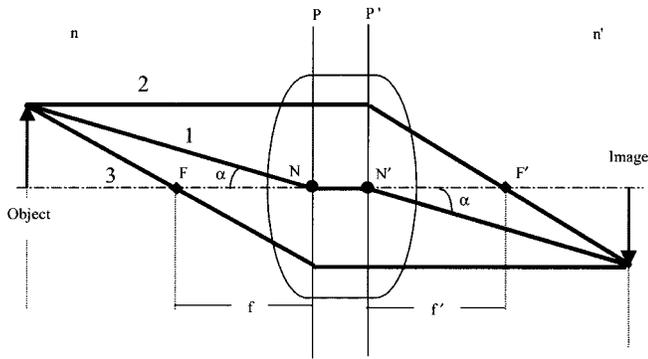


Fig. 8. The most commonly used rays in graphical raytracing are shown 1) a nodal ray; 2) a ray parallel to the optical axis; and 3) a ray passing through  $F$  and exiting parallel to the optical axis.

### C. Nodal and Antinodal Points

The nodal points  $N$  and  $N'$  are defined as a pair of conjugate points with unit angular magnification. For each point in the field of view, the ray that has the same direction before reaching the lens and after leaving it, is named the nodal ray, and it intersects the optical axis at the two nodal points  $N$  and  $N'$ .

As a consequence, a lens, tilted around an axis that passes through the nodal point object and perpendicular to the optical axis, will yield a spatially invariant image focus image point as the tilt is varied. This concept is often used in the laboratory when testing optical systems using the nodal bench approach [8].

Whenever the refractive index in front of the lens is the same as the index behind it, the nodal points coincide with the principal points. If the refractive indexes are different, the nodal points move away from the principal planes, toward the side of the higher index [9].

The antinodal points are defined as the set of points with angular magnification of  $-1$ . Based on collineation theory, it can be shown that only in the case when  $n = n'$  are these points the symmetrical of the nodal points with respect to the corresponding focal points [7].

## V. GRAPHICAL RAYTRACING

The objective of raytracing is to determine the location, size, and orientation of an image formed by an optical system. This objective can be achieved by using a subset of the cardinal points defined in the previous section.

### A. Fundamentals

In order to locate an image by graphical raytracing, one needs to trace at least two rays and find where they intersect [10]. The most commonly traced rays are presented in Fig. 8.

The rays most commonly used for raytracing optical systems are:

- 1) rays going through the nodal points of the lens, that refracts without deviation (Note, however, the displacement of the ray from  $N$  to  $N'$ .);
- 2) parallel rays, which are parallel to the axis and pass through  $F'$  after refraction;

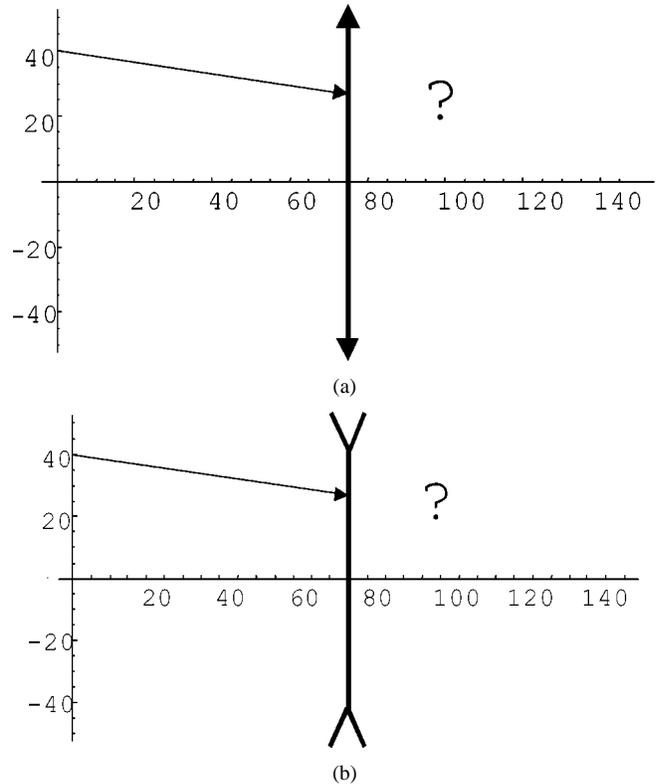


Fig. 9. A ray incident on (a) a thin positive  $f' = 50$  units or (b) a thin negative  $f' = -50$  units lens is shown. The aim is to trace the refracted ray.

- 3) focal rays, which pass through  $F$  and are parallel to the axis after refraction.

### B. Raytracing Examples

This section will first demonstrate how to raytrace an arbitrary ray through a single optical element. The methods will then be applied to the raytracing of a multielement optical system.

1) *Simple Positive and Negative Lenses:* The case depicted in Fig. 9 has either a positive, or a negative thin lens with a focal length  $f'$  (i.e.,  $f' = \pm 50$  arbitrary units in the example shown). First, the definitions of the cardinal points are applied to thin lenses;  $P$ ,  $P'$ ,  $N$  and  $N'$  are found to be located at the center of the lens. Given any ray incident on the lens, the question is what direction will that ray take after refraction?

Presented are two graphical methods shown in Fig. 10(a) and (b), respectively, that can be used to obtain the ray after refraction. Both approaches depend on one or more cardinal points. In both approaches, the focal length serves to define the exact position of the focal points. In the first approach, the ray to be traced is intersected by the focal plane object. A point source at that location would result in parallel rays after refraction. Therefore, the ray after refraction will also be parallel to that direction, where the direction can be found by tracing a nodal ray issued at the point of intersection.

In Fig. 10(b) the focal plane image is considered. The focal plane image has the characteristic that parallel rays will focus at the focal plane image. Taking this into account, the authors trace an auxiliary ray parallel to the original ray that does not deviate after refraction; that is a nodal ray. The refracted ray will then

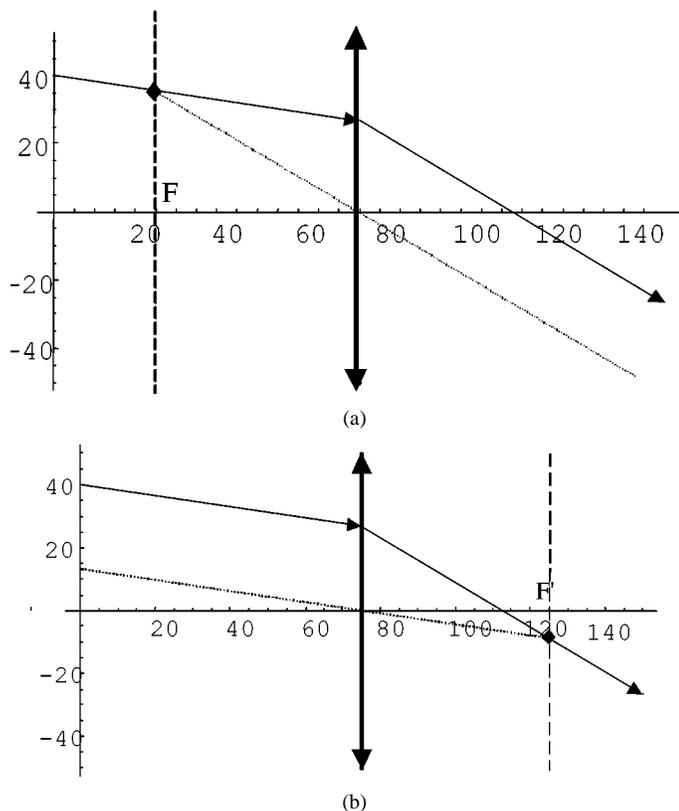


Fig. 10. How to find the direction of the refracted ray for a positive lens. (a) Method #1, (b) Method #2.

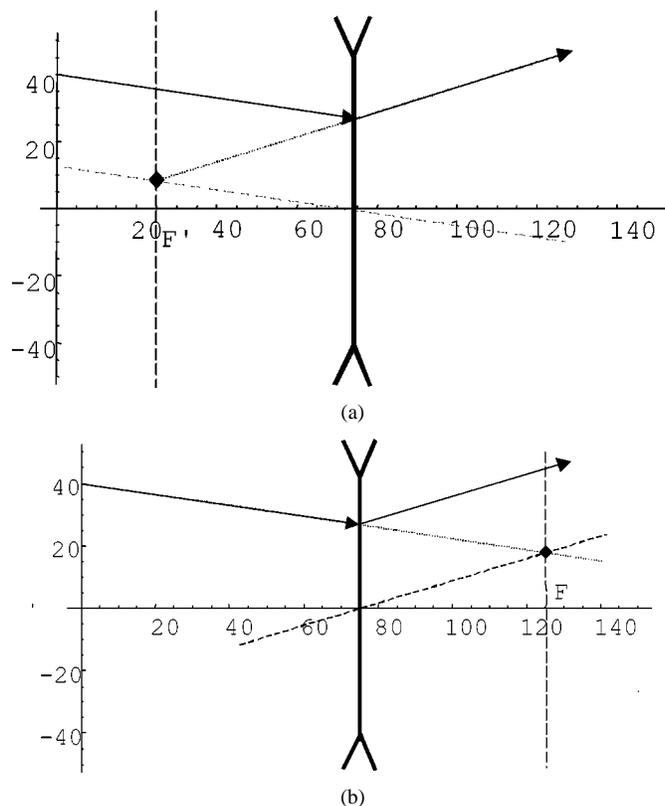


Fig. 11. Finding the direction of the refracted ray for a negative lens. (a) Method #1. (b) Method #2.

pass through the point formed by the intersection of the nodal ray with the focal plane image.

Using the same procedure as the one used for the positive lens, the refracted ray for a negative lens can be obtained as shown in Fig. 11(a) and (b).

By using these two simple methods of obtaining the ray direction after refraction one can trace graphically more complex systems.

2) *Simple Concave and Convex Mirrors:* Fig. 12 shows the four most commonly used rays for raytracing mirrors.

- 1) A ray that passes through the center of curvature of the mirror is reflected in its own initial path.
- 2) A ray parallel to the optical axis passes through the focal point of the mirror after reflection.
- 3) A ray that passes through the focal point object (or equivalently for a mirror the focal point image) is reflected parallel to the optical axis.
- 4) A ray that is reflected at the apex of the mirror is reflected in an angle equal (in magnitude) to the incident angle.

Similarly, as was done with the positive lens, one may find the direction of any ray reflecting off a mirror. One should consider the case of a ray hitting a concave mirror with a focal length of 50 arbitrary units.

Fig. 12 shows two graphical ways of finding the direction of the reflected ray by using auxiliary rays. Note that it is always convenient to choose a ray that does not change after reflection. In Fig. 12(a), a ray parallel to the original ray that goes through the center of curvature of the mirror is chosen as an auxiliary

ray. Such a ray is reflected along its path. Since parallel rays intersect at the same point in the focal plane image, the ray of interest will go through the intersection of the auxiliary ray and the focal plane image. The direction of the ray after reflection is thus obtained.

In Fig. 12(b), an auxiliary ray that reflects from the apex of the mirror is employed. The auxiliary ray that hits the apex of the mirror is reflected at an angle equal in magnitude to the incident angle. This ray intersects the focal plane image at a point where the ray to be traced will go through as well after reflection.

In Fig. 13, a similar approach is used to find reflected rays off convex mirrors.

3) *A System of Two Lenses:* The principles reviewed can be applied to a lens assembly. First, an optical system composed of two positive lenses shown in Fig. 14 is considered. The raytracing employed shows how to find the size and location of the image of a given object through this optical system.

To find the image of an object, the intersection of two rays coming from the object must be formed. For simplicity a ray parallel to the optical axis (ray 1—single arrow) and a ray that passed through the center of the lens (ray 2—double arrow) are chosen. Ray 1 exits the first lens through the focal point image  $F_1'$ , and ray 2 passes through the first lens without being deviated. To find the direction of ray 1 after  $L_2$ , the method of Fig. 10(a) is employed. First, the intersection of ray 1 with the focal plane object of  $L_2$  ( $F_2$ ) is found. As shown in Fig. 10(a) and applied in Fig. 14, ray 1 (after refraction through  $L_2$ ) will be parallel to a ray initiated from that intersection and passing through the center of  $L_2$ . For ray 2, the method of Fig. 10(b) is

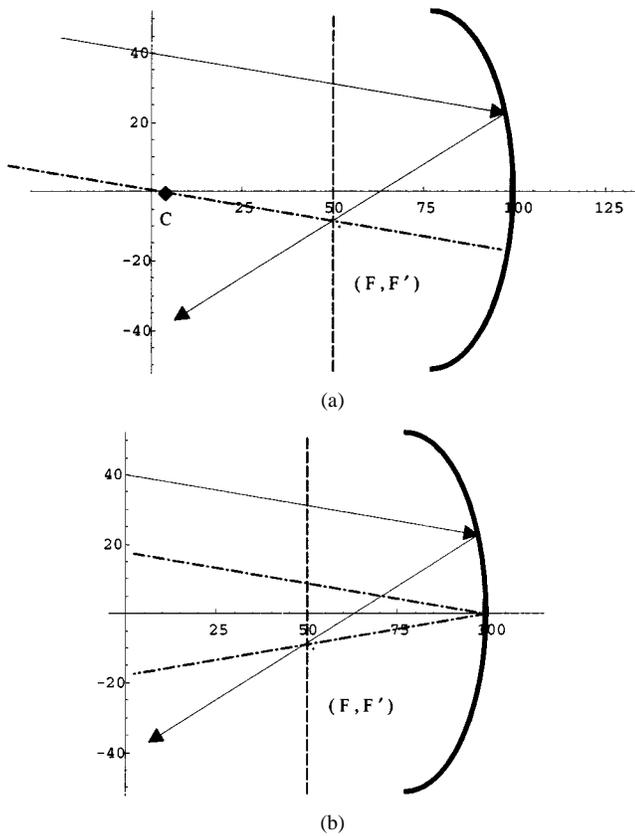


Fig. 12. How to find the direction of the reflected ray for a concave mirror. (a) By using an auxiliary ray that goes through the center of curvature. (b) By using the ray that intercepts the apex of the mirror.

demonstrated. First, the ray parallel to ray 2 that passes through the center of  $L_2$  is traced. Second, the intersection of this ray with the focal plane image of  $L_2$  ( $F_2'$ ) is found. Ray 2 will cross this intersection after refraction. The final image through the system of two lenses is found by establishing the intersection of ray 1 and ray 2 after refraction through both  $L_1$  and  $L_2$ .

4) *A Telescope Assembly:* As a second example of the raytracing techniques discussed above, the authors demonstrate how a six lenses astronomical telescope shown in Fig. 15 can be quickly raytraced without any software.

The telescope is composed of the following elements:

- A lens  $L_1$  of focal length 100 mm and diameter 15 mm;
- A lens  $L_2$  of focal length 50 mm, located 100 mm behind  $L_1$ ;
- A lens  $L_3$  of focal length 100 mm, located 100 mm behind  $L_2$ ;
- A lens  $L_4$  of focal length 150 mm;
- A reticule  $R$  located 150 mm behind the objective  $L_4$ , which is also the focal point image of  $L_4$  in this case. The reticule is also at the focal point object of the eyepiece assembly.

The reticule is observed through an eyepiece composed of two lenses  $L_5$  and  $L_6$ , such that the imaging conditions of a telescope are satisfied; a virtual image is thus formed so it can be perceived by the eye. Further, the reader should assume that the observer observes a virtual image at infinity. All lenses should be assumed to be thin. The approximate location of the front

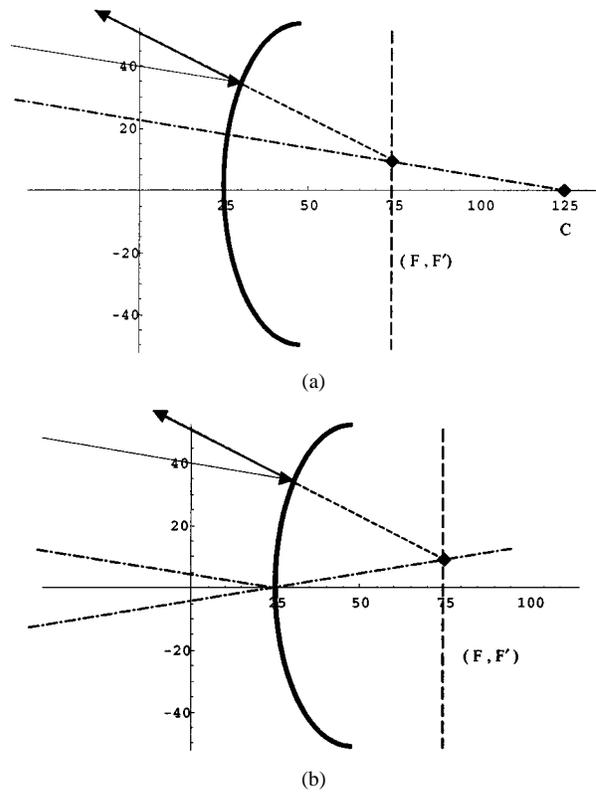


Fig. 13. How to find the direction of the reflected ray for a convex mirror. (a) By using an auxiliary ray that goes through the center of curvature. (b) By using the ray that intercepts the apex of the mirror.

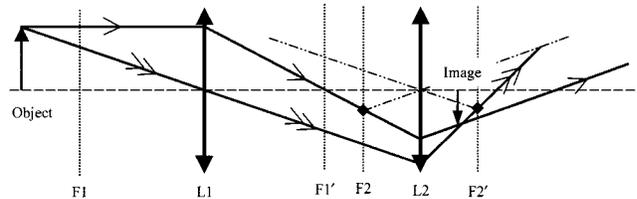


Fig. 14. Imaging system consisting of two lenses analyzed using graphical raytracing.

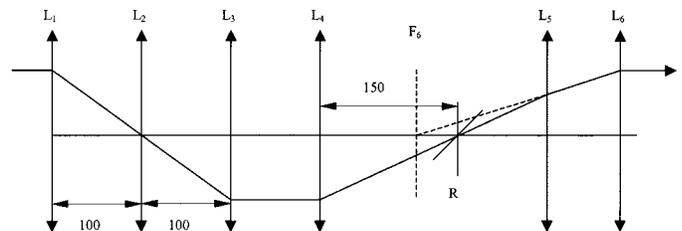


Fig. 15. Graphical raytracing of a telescope for a point source on the optical axis, located at infinity.

focal point of  $L_6$  is shown in Fig. 15 to satisfy the condition of a virtual final image.

First, the reader should examine this system carefully and make some observations about its properties. Notice that the lens  $L_2$  is located in the back focal plane of lens  $L_1$  and in the front focal point of lens  $L_3$ ; therefore,  $L_1$  and  $L_3$  are symmetrical with respect to  $L_2$ . Moreover, the focal length of  $L_2$  equals one-half that of lenses  $L_1$  and  $L_3$ . Thus,  $L_1$  and  $L_3$  are located

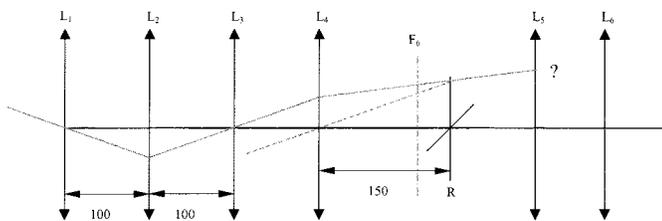


Fig. 16. Graphical raytracing of a telescope for a point source at an angle with respect to the optical axis.

at a distance  $d = |2f'_2|$  from  $L_2$ , which is also the location of the antinodal points of the  $L_2$ .

Given these observations the reader can now trace a ray parallel to the optical axis coming from a point source at infinity on axis as well as a ray coming from a point source off axis. The reader should begin with the relatively easier ray parallel to the optical axis, shown in Fig. 15. After encountering the first lens the ray is refracted such that it passes through its focal point image, which coincide with the vertex of the second lens. Therefore, the ray passes undeviated through the second lens. Upon encountering the third lens, it is refracted parallel to the optical axis. Then the ray encounters the fourth lens as a parallel ray to the optical axis and thus is refracted toward the focal point  $F'_4$ .

Because the reticule is at the focal point object of the assembly  $L_5$ – $L_6$  forming the eyepiece, the beam exiting the assembly will be parallel to the optical axis. Moreover, from the definition of the focal point object of a lens, the exiting rays will appear to emerge from point  $F_6$ . Therefore  $R$  and  $F_6$  are conjugate points through the lens  $L_5$ . This realization helps completing the raytracing on-axis as shown in Fig. 15, and will serve in performing off-axis raytracing as well.

For off-axis raytracing, the reader should trace a ray from a parallel beam coming from a point at infinity at some angle with respect to the optical axis, as shown in Fig. 16. The ray chosen here is the one passing through the center of  $L_1$ ; however, other rays parallel to this ray and intercepting  $L_1$  at some nonzero height could be traced as well, based on the raytracing techniques demonstrated in this paper. The ray first passes through the center of  $L_1$  undeviated. Since it is known that  $L_1$  and  $L_3$  are conjugate planes with respect to  $L_2$ , one can immediately deduce that the ray passes through the center of  $L_3$  as well. Another point, which can be seen immediately, is that the angle between the ray and the optical axis will be of the same magnitude but of the opposite sign since the locations of  $L_1$  and  $L_3$  are at the antinodal points of  $L_2$ . The ray then encounters  $L_4$ . In order to raytrace the ray through this lens, the reader should employ the concept discussed in Fig. 10(b), and should construct an auxiliary ray parallel to the ray of interest which passes through the center of  $L_4$ , and thus, is undeviated. Since the considered and the auxiliary rays are parallel, they focus at a point in the back focal plane of  $L_4$ . Therefore, the direction of the ray is readily determined.

Finally, in order to complete the raytracing it is necessary to determine how the ray will propagate through the eyepiece. The eyepiece and a ray incident on the eyepiece are shown in Fig. 17. Using the concept discussed in Fig. 10(a), the reader should consider the point  $A$  at which the ray to be traced intersects the plane

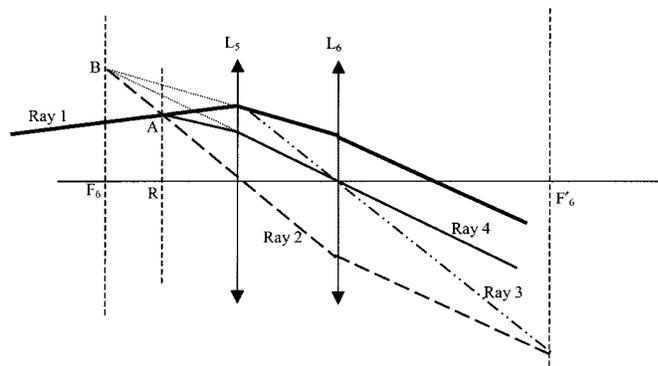


Fig. 17. Raytracing of the eyepiece for a point source off axis.

of the reticule. The point  $A$  can be thought of as a fictive point source, whose image after  $L_6$  is a parallel beam. In order to determine the direction of the ray of interest after  $L_5$ , the reader can utilize the fact that the focal plane object of the eyepiece assembly is conjugate to the focal plane object of the lens  $L_6$  through the lens  $L_5$ . Therefore, the reader can construct ray 2 emerging from point  $A$  and passing through the center of  $L_5$ . This ray is undeviated through  $L_5$  and incident upon  $L_6$  as a general ray off axis. To get its refraction through  $L_6$ , the reader should again employ the concept discussed in Fig. 10(b), and then construct ray 3 parallel to ray 2 and passing through the center of  $L_6$ . Ray 2 and ray 3 will intercept in the focal point image of  $L_6$ , as shown in Fig. 17. The final direction of ray 2 provides the direction of the parallel beam, off axis, that would be the image of the point  $A$  through the eyepiece.

Knowing that ray 1 intersects point  $A$  before  $L_5$ , and knowing its final direction after the eyepiece, its refraction through the  $L_5$ – $L_6$  assembly can now be established. To obtain the image of point  $A$  through  $L_5$ , the reader should trace ray 4 backward from the image space parallel to the output ray 2, and through the center of  $L_6$ . By definition this ray passes through point  $A$ . The intersection of the extension of ray 4 with the focal plane object of  $L_6$  gives the location of the image of point  $A$  through  $L_5$ . Thus, point  $B$  is determined as the image of point  $A$  through  $L_5$ . Now, the reader should consider only the lens  $L_6$ . Therefore, ray 1, which is the considered ray, is refracted through  $L_5$ , as if it comes from point  $B$ . The ray of interest (ray 1) will propagate after  $L_5$  along the line connecting point  $B$  with the point at which the ray encounters  $L_5$ . After encountering  $L_6$ , it will propagate parallel to the already constructed beam. Its refraction through  $L_6$  is such that the final direction of ray 1 is parallel to the previously established final direction of the beam.

## VI. CONCLUSION

In this paper, we have reviewed fundamentals of geometrical optics and specifically techniques for back-of-the-envelope paraxial raytracing. Importantly, the techniques presented are the same whether one traces rays through lenses or mirrors. Furthermore, the methods can be applied to systems of any complexity by sequential application of the methods. It is important to note that graphical raytracing provides a valuable tool not only to make quick “back-of-the-envelope” first-order solutions to optical design problems, but that the methods apply

equally well to computer graphics where optical raytracing is extensively employed.

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#### REFERENCES

- [1] A. Marechal, *Imagerie Geometrique: Aberrations*: Editions de la revue d'Optique Theorique et Instrumentale, 1952.
- [2] G. W. Forbes and M. A. Alonso, "What on Earth is a ray and how can we use them best?," in *Proc. Int. Lens Design Conf.*, 1998, pp. 22-31.
- [3] M. Born and E. Wolf, *Principles of Optics*, 7th ed. Cambridge, U.K.: Cambridge Univ. Press, 1999.
- [4] J. Foley, A. Van Dam, S. Feiner, and J. Hughes, *Computer Graphics Principles and Practice*, 2nd ed. Reading, MA: Addison-Wesley, 1996.
- [5] L. Silberstein, *Simplified Method of Tracing Rays Through Any Optical System of Lenses, Prisms, and Mirrors*. —AUTHOR, PLEASE LIST CITY OF PUBLISHER—: Longmans and Green, 1918.
- [6] L. E. W. Van Albada, *Graphical Design of Optical Systems*. New York: Pitman, 1955.
- [7] D. Goodman, "General principles of geometric optics," in *Handbook of Optics*, 2nd ed. New York: McGraw-Hill, 1995, vol. 1, ch. 1.
- [8] W. J. Smith, *Modern Optical Engineering: The Design of Optical Systems*, 2nd ed. New York: McGraw-Hill, 1990.
- [9] J. R. Meyer-Arendt, *Introduction to Classical and Modern Optics*, 4th ed. Englewood Cliffs, NJ: Prentice-Hall, 1995.
- [10] P. Mouroulis and J. Macdonald, *Geometrical Optics and Optical Design*. Oxford, U.K.: Oxford Univ. Press, 1997.

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