Analytic Derivation of the PI-AQM Stability Region

[Short Abstract]

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This talk reports on an analytic derivation of the *complete* stability region of the Proportional Integral (PI) Active Queue Management (AQM). The stability region represents the entire set of the feasible design parameters, i.e., the *proportional* and the *integral* gains, that stabilize the closed-loop TCP-AQM system.

AQM controls congestion by proactively marking or dropping packets before the inception of congestion, thus leading to low packet losses, short queueing delays, and high bandwidth utilization. The PI controller was proposed as an alternative AQM to the original RED AQM. The PI controller is a natural choice due to its robustness and its ability to eliminate the steady-state error in the queue length. Both theory and simulations showed that PI outperforms RED. The advent of PI as an AQM has spurred the proposition of several related controllers—such as PIP, PD, PID, and P^2I—all of which were proposed for a single goal: to speed up the responsiveness of the PI controller. However, a complete stability region for any of these controllers with the TCP-AQM model remained unexplored.

A closed-form derivation of the complete stability region for PI AQM is complicated by the combination of various factors. Some factors are intrinsic to the PI and TCP-AQM loop: the linearized TCP-AQM model is a second-order plant, includes time delays in the control loop, and is cascaded with PI, a relatively high-order controller. The resultant characteristic equation is a high-order quasi-polynomial whose stability analysis poses great theoretical challenges. Another source of complication pertains to the objective of attaining stability that is robust to approximate values of system parameters. In particular, the TCP-AQM closed-loop system should be stabilized not only for a single value of the delay but for a delay interval. All these factors render the stability analysis techniques of elementary Control Theory unusable. Previous work sidestepped the problem through assumptions and simplifications, ending up with only a subset of the whole space of stabilizing controller parameters. The original paper on PI AQM gave guidelines to choose only a single pair of the proportional gain, k_p, and the integral gain, k_i, that guarantees the stability of the closed-loop system. A follow-up paper expanded the stability set to a line segment in the k_i–k_p space. However, experiments in all of previous work used the guidelines prescribed by the original paper.

In our work, we exploit recent results on robust PI control theory for time-delay systems to obtain a complete stability region for the PI controller with the TCP-AQM model. We use the PI implementation in `ns-2` to validate the theoretical analysis via packet-level simulations. We show that the pair of k_p and k_i given by the original paper on PI can be excessively conservative when compared with the complete stability region. This, in turn, explains the sluggish responsiveness of PI observed in the literature. Further, we show that some provably stable controller parameters enjoy better performance than ones suggested by the previous literature.

The same stability analysis of PI can be utilized to obtain the complete stability region for other AQM algorithms, such as REM and PIP, with straightforward change of variables. One can then use the stability region to prove that PIP becomes unstable according to the original design.

Finally, the talk will present both theoretical and simulation results.