Optimization Congestion Control for Networked Control Systems

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I. INTRODUCTION

Technological advances in embedded systems and communication networks have given birth to devices with sensing, processing, actuating, and communication capabilities. When these sensing, processing, and actuating units are networked together, they can be used to monitor and control a remote physical environment. Sensors gather data and forward it to processing units, called controllers. Controllers process data and send decisions to actuators. Networked control finds applications in industrial automation, distributed instrumentations, unmanned vehicles, and home robotics.

In accord to [1] and all the resources contained therein, when these sensors, controllers, and actuators exist in a distributed fashion and communicate over IP networks, they are referred to by the term Networked Control Systems (or *NCSs*), see Fig. 1.

NCSs have several challenges, which are due to the nondeterministic nature of communication networks. For example, communication delays, delay jitter, and packet losses adversely affect the performance of the system, and may even cause instability.

Related research in this field has emerged into two distinct directions. The first direction aims at a control theoretical analysis while considering the network as a fixed parameter. The second direction targets the design of new communication network infrastructures, algorithms or protocols. Examples of the first approach include designing special controllers, and varying the sampling rate; and examples of the second include designing static and dynamic message scheduling algorithms.

In this document, we present our approach that combines both directions and relies on the well-established results in both communication networks and control theory.

We informally state our objective as simple as:

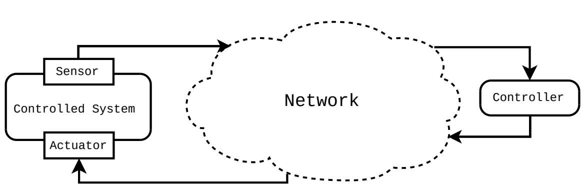

Fig. 1. An example of a NCS with one system and one controller

Allocate the network bandwidth among controlled systems to attain the maximum aggregate performance subject to stability and network constraints.

In contrast to the centralized approaches, such as FTT-CAN [2], we seek to realize our objective in a distributed manner in a much similar way TCP congestion control works. Therefore, both the end-systems, i.e., the controlled systems, and the network must coordinate, interact, and exchange information necessary to achieve this goal.

II. MATHEMATICAL FORMULATIONS

Due to their great generality and relative simplicity, we confine our attention to the case of scalar LTI systems with proportional controllers. Plant dynamics can be modeled by the following differential equation (in control theory , the term *plant* is commonly used to refer to the controlled system):

$$\dot{x}(t) = ax(t) + u(t) \tag{1}$$

where $x(t)$ is the system state; a is the system constant; $u(t) = -kx(t_j)$ is the control input supplied by the controller; k is the proportional gain; and t_j is the time at which the controller received sampled data number j.

We use a utility function to measure the system performance. This is a function of the system dynamics and the sampling period (the time difference between two consecutive samples of the sensed plant output. The author in [3] proposed a performance measure function that directly relates to the absolute error when a step input is applied to the system. For plant i, the function is defined as:

$$J_i(h_i) = \frac{-\bar{a}_i}{a_i} e^{a_i h_i} \tag{2}$$

where $\bar{a}_i = a_i - k_i$, and h_i is the sampling period.

Let $r_i = \frac{1}{h_i}$. Since h_i can be interpreted as the time in seconds to generate one sample and send it inside a packet, then r_i is the plant's transmission rate measured in packets per second. Based on $J_i(h_i)$ in (2), we then define a utility function, $U_i(r_i)$, as follows:

$$U_i(r_i) = \frac{\bar{a}_i}{a_i} e^{\frac{a_i}{r_i}} \tag{3}$$

Our objective stated verbally in section I can then be expressed mathematically as :

$$\begin{aligned} \text{maximize} \quad & \sum_i U_i(r_i), \\ \text{subject to} \quad & \sum_{i \in S(l)} r_i \leq C_l, l = 1, \ldots, L \end{aligned} \tag{4}$$

where $S(l)$ is set of plants whose end-to-end flow paths use link l, C_l is the capacity of link l, and L is the total number of links in the network.

The authors in [4] solved the same nonlinear optimization problem expressed in (4) in a distributed manner. The solution converges to the equilibrium state if each utility function satisfies certain conditions. $U_i(r_i)$ defined in (3) satisfies all of such conditions. Specifically, $U_i(r_i)$ is concave, is monotonically increasing, and is twice continuously differentiable; and $-U_i''(r_i) > 0$.

III. Basic Results

Due to the limited space, we present here a simple yet a demonstrative experiment. We used ns-2 to simulate three plants with three controllers sharing a single bottleneck link, see Fig. 2. Table I gives dynamics parameters, and sampling start and end times for the plants.

Figs. 3 shows how each plant adapts its sampling period, and consequently its transmission rate as other plants enter and leave the network. The three plants share the bottleneck link based on each's utility function, which depends on the dynamics for each plant, i.e., the values of a_i and k_i. Fig. 4 shows the plants' outputs in response to step inputs. The three plants all remain stable and converge to the steady value.

IV. Conclusion

This paper presented the following main contribution in the field of Networked Control Systems: we devised an integrated

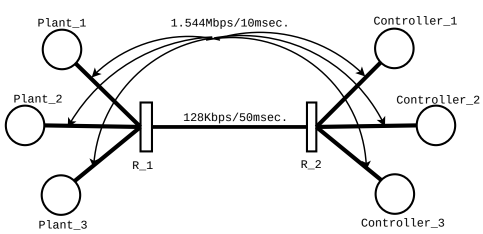

Fig. 2. Simulation Topology

TABLE I

PLANTS' SPECIFICATIONS

	Plant_1	Plant_2	Plant_3
a_i	0.01	0.005	0.0003
k_i	1.5	0.8	0.4
Sampling start time(sec)	0	80	40
Sampling end time(sec)	120	200	160

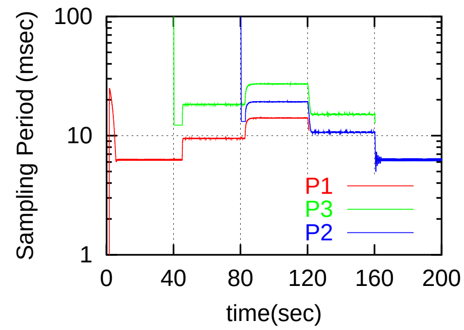

Fig. 3. Plants' Sampling Periods

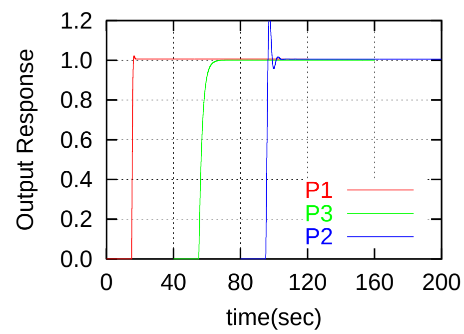

Fig. 4. Output Response

approach that combines well-established results in the two fields of control theory and of networking. Specifically, we use a REM-like AQM strategy (see [5]) to measure the congestion in the network and feed it back to plants. Plants use this feedback to correctly adjust their sampling rate. The primary objective of this integrated approach is to ensure the stability of control systems, to achieve maximum *aggregate* system performance, to use network resources in very efficient way, and to accommodate to dynamic changes in the network.

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