Multirate Simulation for High Fidelity Haptic Interaction with Deformable Objects in Virtual Environments

Murat Cenk Çavuşoğlu

Department of Electrical Engineering
and Computer Sciences
University of California, Berkeley
Berkeley, CA 94720
mcenk@robotics.eecs.berkeley.edu

Frank Tendick

Department of Surgery
University of California, San Francisco
San Francisco, CA 94143
frank@robotics.eecs.berkeley.edu

Abstract

Haptic interaction is an increasingly common form of interaction in virtual environment (VE) simulations. This medium introduces some new challenges. In this paper we study the problem arising from the difference between the sampling rate requirements of haptic interfaces and the significantly lower update rates of the physical models being manipulated. We propose a multirate simulation approach which uses a local linear approximation. The treatment includes a detailed analysis and experimental verification of the approach. The proposed method is also shown to improve the stability of the haptic interaction.

Keywords — Balanced model reduction, deformable models, haptic interfacing, virtual reality, virtual environments.

1 Introduction

Haptic interaction is an increasingly common form of interaction in virtual environment (VE) simulations, especially since the commercial availability of high fidelity haptic devices such as the Phantom (Sensable Technologies Inc., Cambridge, MA) and Impulse Engine (Immersion Corp., San Jose,CA). This relatively new medium introduces some new challenges, which are being studied in the literature. Ensuring stability of haptic interaction with the virtual environment is an important problem. Several groups have considered the effects of model sampling time on stability [8, 4]. Colgate *et.al.* point out the non-passive nature of the discrete implementations of virtual environments as a major source of instability [4], and propose a virtual coupling network to improve stability

[5]. Adams and Hannaford give a design algorithm to ensure stability of the haptic interface coupled to arbitrary passive virtual environments, therefore separating the design of the virtual environment and the haptic interface [1]. Simulation of stiff walls and hard contact is another interesting research topic. The penalty based approach is the most common way to simulate stiff walls. Zilles and Salisbury propose using "god-objects" to eliminate problems with penetration into the virtual objects in a penalty based approach [14], and Salcudean and Vlaar report that using a braking pulse greatly improves the perception of a stiff wall [9].

The authors' interest on this topic stems from the ongoing project of development of a VE based surgical training simulator [11, 7]. This application involves construction of realistic three dimensional geometric models of the anatomy; deformable object models simulated in real-time capable of manipulation, cutting, and suturing; as well as development of methods for teaching procedures, tasks, and skills transferable to actual surgery.

In a VE simulation of interaction with deformable bodies, for example in a surgical simulator, typically the physical model is updated at the visual update rates of 10 Hz order of magnitude. But haptic interfaces require much higher update rates, typically in the order of 1 kHz. It is not possible to increase the update rate of the physical model to the haptic rate with its full complexity due to computational limitations. The current practice is to apply the same force between the model updates, or to low-pass filter this generated force to the bandwidth of the model update rate. These effectively reduce the haptic update rate to the visual update rate, and therefore impair the fidelity of the haptic interaction. This is especially

significant when the high frequency interaction forces are significant, for example in nonlinear phenomena like contact.

Astley and Hayward propose to use a multiscale multirate finite element model to address this problem. In their method, a coarse linear finite element mesh models the behavior of the overall object and a finer finite element mesh running at a higher update rate is used locally where there is an interaction [2]. Their work is based on decoupling the coarse mesh and the fine mesh by using the Norton equivalents as interfaces. This is only applicable to the linear finite element case, and the update rates reported were still significantly below 1 kHz required by the haptics.

In this paper we propose a multirate simulation approach to handle the difference between the update rate requirements for the haptics and the physical model during haptic interaction in VE simulations, complete with theoretical and experimental verifications of the approach. The proposed method is justified by model reduction techniques from system theory, and the approach is applied to nonlinear physical models.

We will start our treatment in this paper with a demonstration of the problem. This will be followed by the description of the proposed method, analysis of the critical parts, implementation, a short discussion of stability implications, and concluding remarks.

The discussion here is limited to lumped element models (also referred to as mass-spring-damper models in the literature), but the arguments can easily be extended to deformable models based on finite element analysis.

2 Demonstration of the problem

We first consider a haptic interface interacting with a simulated nonlinear spring in one dimension, and evaluate the fidelity of the force output of different simulation schemes for a given stimulus. This simple analysis demonstrates the problems that arise from the low model update rate and illustrates the basic motivation of the method proposed in this paper.

Four different simulation models, with 1kHz haptic update rates, are considered. In the first model, force feedback is generated by the nonlinear spring model updated at 1 kHz, which corresponds to the case where the model update rate is the same as haptic update rate. It is the baseline for the analysis as it is the ideal case. In the second model, the force is generated from the nonlinear spring model at an update rate of
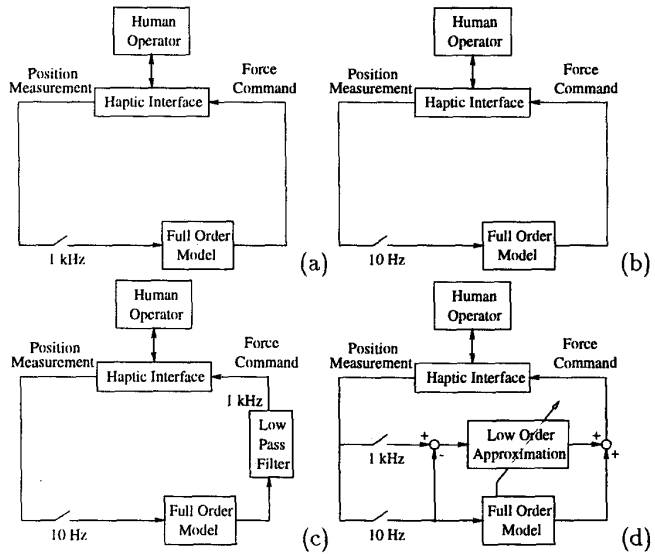

Figure 1: Simulation paradigms.

10 Hz, and maintained constant in between the model updates. This is the counterpart of the case where there is interaction with a deformable model simulated at a larger step size than the haptic sampling time. The third simulation model is an improved version of the second one. In this case, the nonlinear spring model is updated at 10 Hz, but the applied force is a low pass filtered version of the piecewise constant force generated from the nonlinear model. The bandwidth of the low pass filter is 10 Hz, and it is running at the haptic update rate of 1 kHz. In the last model, the nonlinear model is again updated at 10 Hz, as in the second and third models. However, the force output in between the nonlinear model updates is calculated from a linear spring model based on the tangent of the nonlinear spring at the last model update. To summarize

$$force_1[n] = f(x[n]) \tag{1}$$
$$force_2[n] = f(x[N]) \tag{2}$$
$$force_3[n] = f(x[N]) * lpf[n] \tag{3}$$
$$force_4[n] = f(x[N]) + f'(x[N])(x[n] - x[N]) \tag{4}$$

where n and N are respectively the haptic and model samples and $lpf[n]$ is the impulse response of the 10 Hz low-pass filter. Note that n runs at 1 kHz, and N runs at 10 Hz.

The nonlinear force-position characteristic of the spring used is based on the experimentally determined force deformation characteristics of the skin of the
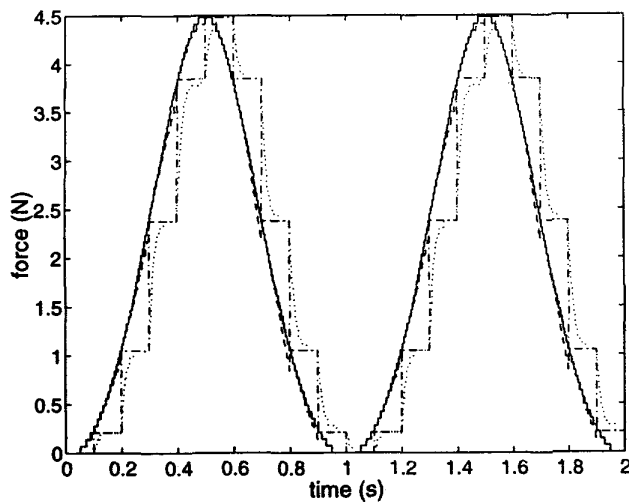
Figure 2: Interaction with a nonlinear spring in one dimension. Solid line is the 1kHz model, dash-dot line is the 10 Hz model, dotted line is the filtered 10 Hz model, and dashed line is the local tangent model.

thigh given in [6]:

$$f(x) = \frac{x}{5.4398 - 0.1418x} \qquad (5)$$

In the simulations, quantization noise is added to the position measurements and the force output. The quantization step size used is 0.3 mm for position measurements and 0.07 Newtons for force output. These values are the typical quantization values for the Phantom(TM) version 1.5 haptic interface [10].

In the test simulations of Fig. 2, the haptic interface is following a sinusoidal path maintaining contact with the spring, and the interaction force generated by the simulation model versus time is shown in the plot.

The use of a low-pass filter to improve the performance of the constant output method seems to help by reducing contaminating noise at the harmonics of the model update rate. However, this approach has two main limitations. First, low pass filtering may eliminate useful high frequency force information. Second, the lag introduced by the low pass filter tends to destabilize the haptic interaction, or introduce oscillation.

The performance of the local tangent model gives the motivation of the method proposed in this paper for coping with the problems with the difference between the deformable model and haptic update rates.
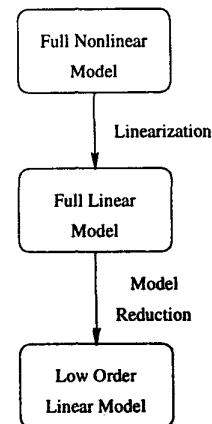

Figure 3: Construction of the low order model.

3 Using a Low Order Linear Approximation to Model Intersample Behavior

When the instrument interacts with the deformable model in a VE simulator, the haptic interface will displace the node(s) it is touching and display the reaction force. Therefore, from the haptic interfaces point of view, it will be interacting with a three-input three-output nonlinear dynamical system, considering the three components of translation and force respectively. However, the underlying dynamical system has a very high order as it includes the deformation of the whole body. For example, when interacting with a $10 \times 10 \times 10$ deformable cube, a mid-sized deformable model, the deformation will have $1000 \times 3 \times 2 = 6000$th order dynamics. This very high order dynamical system, which cannot be simulated in real time, needs to be replaced with a low order approximation for real time haptic performance.

The method we are proposing follows the local tangent approach described in section 2, shown in Fig. 1(d). In this approach, a low order approximation, running at the haptic update rate, is used on top of the full order model to estimate the intersample behavior. The low order approximation is updated by the full model after each step.

To analyze the construction of the low order approximation, we start with the paradigm given in Fig. 3. Linearization is a basic step. The linearized model gives the tangential behavior of the full model. As we want to capture the behavior in between the model updates, the deformation will be small.

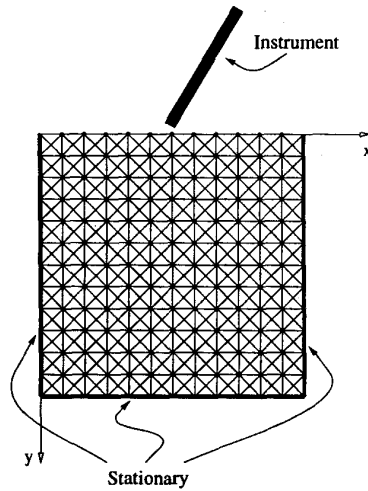The linearized system will have the same order as

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Figure 4: Two dimensional lumped element mesh.

Figure 5: Spatial dependence of the states of the reduced order model.

the full model, therefore the improvement is limited by just using a linear model, i.e. it will still be difficult if not impossible to simulate in real time. Therefore, model reduction is the critical step of the approach, as it is the means of getting a temporally local haptic model which can be simulated in real time.

4 Order Reduction

To evaluate the effectiveness of model reduction, consider a two dimensional 12 × 12 lumped element mesh being indented by an instrument (Fig. 4). Each node of the mesh has a lumped mass, which is connected to the neighboring nodes (diagonal as well as lateral neighbors) with spring and dampers. Three edges of the mesh are constrained to be stationary. Linearization of this system gives a two-input two-output 524th order linear dynamical system.

When we perform a balanced model reduction [13] on this model, we can approximate the system's input-output response with a 10th order system, with the infinity norm of the error resulting from the approximation being less than 1.6×10^{-3}, less than 1% of the full order model. This is a significant reduction in computational complexity while virtually maintaining the accuracy of the model. The frequency responses of the original and reduced order systems are shown in Fig. 6. The responses of the two systems are essentially indistinguishable except in normal-tangential interactions, where the response magnitudes in both conditions are very small (less than -200 dB).
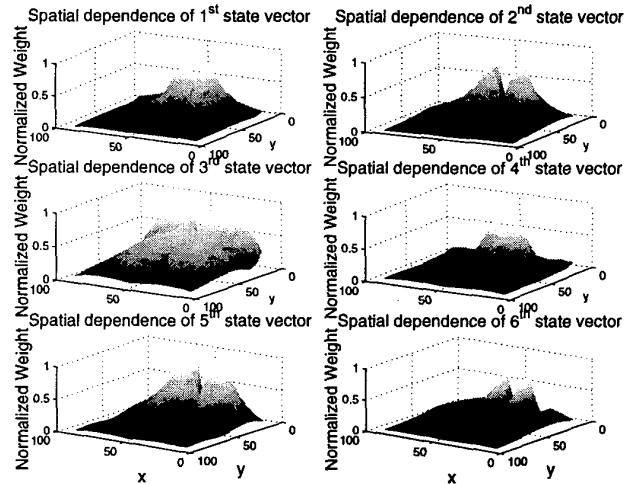
The original states of the system before order reduction are the positions and velocities of the lumped masses at the vertices of the mesh. To visualize the spatial properties of the reduced model, the states of the new model are shown in Fig. 5. The figure shows the magnitude of the components of the new states with respect to the location on the mesh. The input node is at coordinate $(50, 0)$, which is the top middle node.

The states of the new low order model show that it is a local approximation. This result is actually expected, because stress decays inversely proportional to the square of the distance from the load in a semi-infinite linear elastic body under a point load [12].

5 Towards a Real-Time Algorithm

It is important to note that balanced model reduction requires costly calculations as well, which prevents the use of this algorithm as a part of the on-line computation. However, the analysis in the previous section reveals that the approximation given by the balanced model reduction algorithm in a homogeneous medium is a local model, i.e. the force response depends mostly on the states spatially close to the interaction location. So, a natural way to construct a low order approximation with significantly less computation is to construct a local linear model directly from the full order model (Fig. 7). The local linear approximation we will demonstrate in this paper is shown in
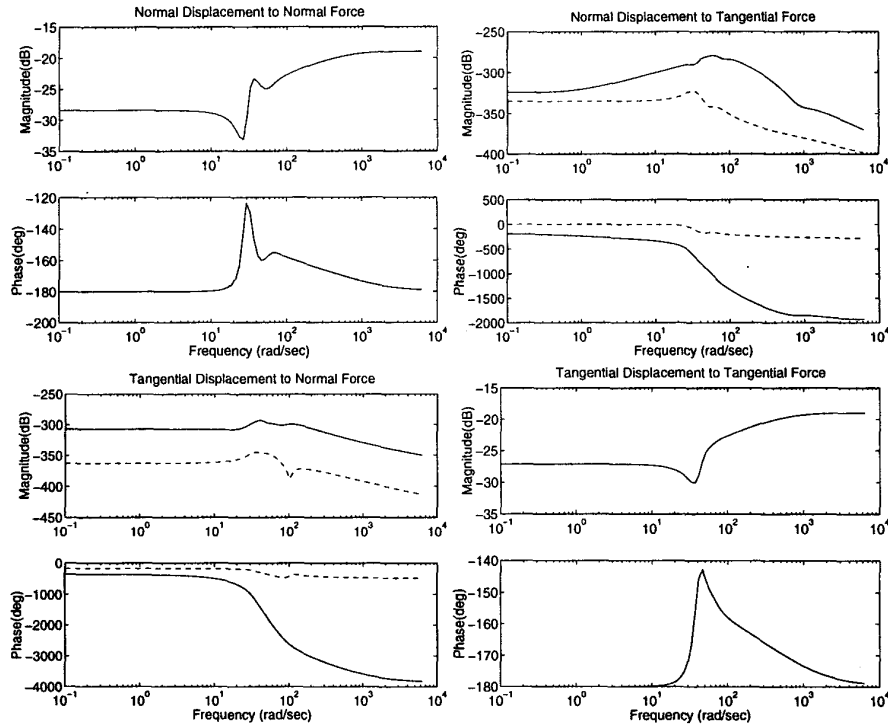
Figure 6: Frequency responses of the original and reduced order systems. Solid line is the reduced order model, dashed line is the full order model.
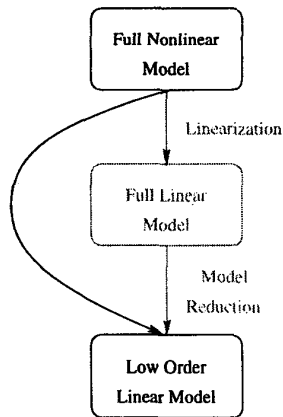

Figure 7: Construction of the low order model.

Fig. 8. It models the local behavior of the mesh with the nodes, springs and dampers near the instrument.

The frequency response of the local linear approximation, along with the frequency responses of the full linear model and a reduced order system with the same

number of states as the local linear approximation calculated by balanced order reduction, are shown in Fig. 9. The local model approximates the behavior in the high frequency range, whereas its DC gain is significantly off. However, it is important to note that the local model is used only to estimate the intersample behavior of the full model, and therefore only needs to be close to the full model in the frequency range of around 10–1000 Hz, which is the case here. If necessary, it is possible to improve the low frequency accuracy by increasing the number of layers of nodes included around the instrument.

These results show that the local linear approximation is a suboptimal approximation, as expected. But it can be constructed on the fly with minimal computation and give sufficiently accurate behavior in the frequency range of interest.

6 Implementation

We have implemented the paradigm explained above in a real time VE simulation of manipulation
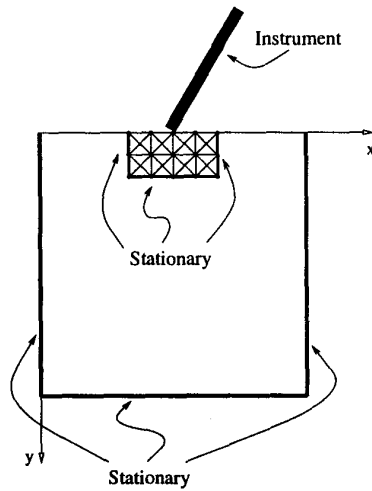
Figure 8: Local low order approximation.

of a deformable object. The object used is a $6 \times 6 \times 6$ lumped element model. The local low order model used is the three dimensional extension of the model shown in Fig. 8. In the simulation, the full nonlinear model is updated at 20 Hz, whereas the haptics and the local linear approximation is run at 1 kHz. It is important to note that the size of the full order nonlinear object model can be scaled without affecting the performance of the haptic interaction. The computational requirements of the construction of the local model and the haptic loop is fixed and independent of the size of the full order model.

The simulation is implemented in C++, using OpenGL as the graphic library. It is run on a dual processor SGI Octane computer. A Phantom(TM) version 1.5 manipulator is used as the haptic interface.

The force during interaction is shown in Fig. 10. The dashed line shows the force calculated by the low update rate model, and the solid line shows the force displayed by the local linear model at haptic update rate.

Stability Implications

Stability of haptic interaction with VEs is an important consideration for design of haptic interfaces and virtual objects. The update rate of the simulation is one of the critical determinants of the stability of interaction, where increasing the update rate of the model improves stability [8, 4]. In the method we are proposing, having the low order linear model running
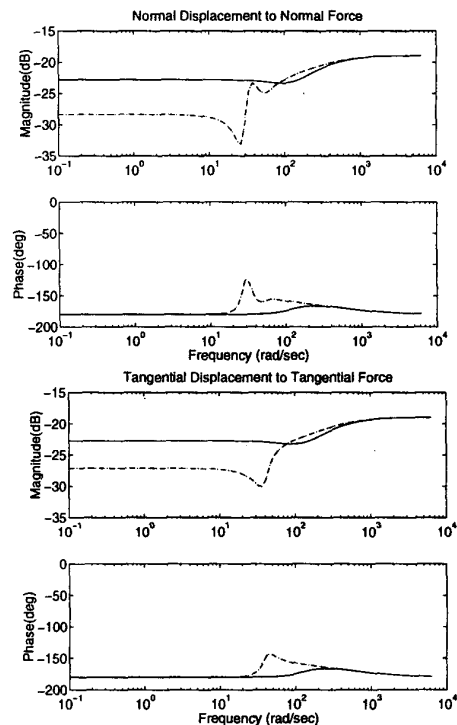

Figure 9: Frequency responses of the local linear approximation (solid line), full linear model (dashed line) and reduced order model (dotted line). Interactions between the normal and tangential directions are not plotted because the associated magnitudes are very small (less than -200 dB).

at a faster update improves the stability of the haptic interaction as the VE model runs at 1 kHz instead of 10 Hz.

This effect can also be observed in the implementation of our method described above. However, stability of our method is difficult to determine because the resulting system is a multirate nonlinear sampled-data system. In the simulation, if the local linear approximation is not used, the haptic interface tends to have oscillatory behavior when the operator loosens his/her grip (Fig. 11). This oscillatory behavior is not present with the local linear approximation even when the operator completely releases the instrument.

7 Discussion

In this paper, a multirate simulation approach to handle the difference between the sampling rate re-
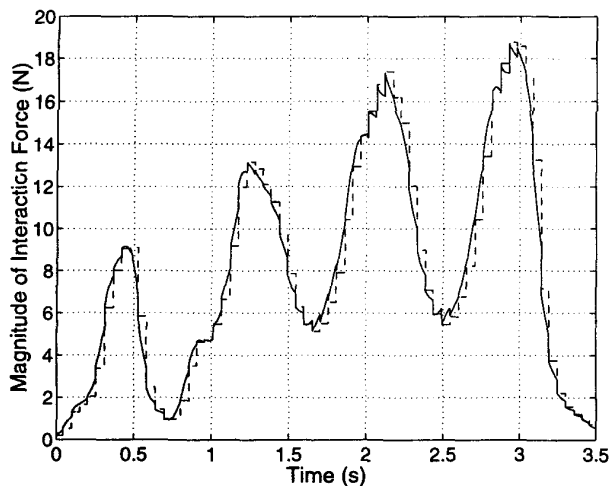
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Figure 10: Interaction force during manipulation of a deformable virtual object.

Figure 11: Oscillations observed when the local linear approximation is not used.

quirements of the haptics and the possible update rates of the physical models during haptic interaction with deformable objects in VE simulations is presented. The proposed method uses a linear approximation to model the intersample behavior of the nonlinear full order model. The natural choice of the linear approximation is to use the linearization of the nonlinear dynamics, which gives the tangent behavior of the dynamical system. However, this linearization does not completely solve the computational complexity problem since the order of the linearized model is still very high. We performed balanced model reduction on the linearized model and showed that it is possible to use a low order local approximation and still get an accurate input output response. Based on this analysis, we proposed a simple local linear approximation which can be computed in real-time and implemented it in a simulation to verify the method.

It is important to note that the local linear approximation used here is not necessarily the best choice. There is room for improvement. For example, it is possible to construct a local model which better approximates the low frequency behavior by extending the outer layer of spring and dampers of Fig. 8 to the outer edges of the body and suitably scaling their parameters to reflect the change in equivalent stiffness/damping as the mesh depth changes. Different local approximations are explored in detail in [3].

The major limit of this method is that the local states must be the dominant ones. This could be violated if the material was inhomogeneous, for example
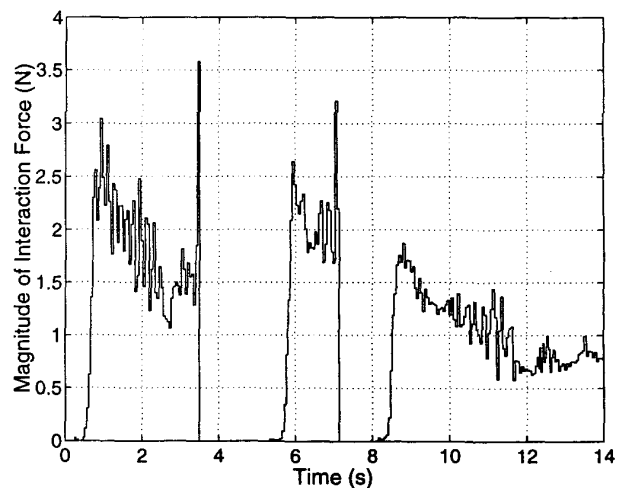
if the deep tissue were significantly more compliant than at the surface so that most of the deformation occurred in states far from the interaction. In this case, Astley and Hayward's method [2] would be useful, if the model was linear. Other effects that could violate the dominance of local modes include significant geometric nonlinearities or discontinuities in the tissue that produced large local stresses away from the instrument contact. However, the locality of the dominant modes can always be checked by performing off-line model reduction, as it is done here in section 4.

Acknowledgments

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